hw_1

January 9, 2023

1 Homework 1| Danny Stein

[ ]:
```
import pandas as pd
import numpy as np
import datetime
import warnings
import statsmodels.api as sm

from sklearn.linear_model import LinearRegression
import scipy.optimize as optimize

import matplotlib.pyplot as plt
%matplotlib inline
plt.rcParams['figure.figsize'] = (12,6)
plt.rcParams['font.size'] = 15
plt.rcParams['legend.fontsize'] = 13
import seaborn as sns
sns.set(rc={'figure.figsize':(15, 10)})
import sys
sys.path.insert(0,'C:/Users/dcste/OneDrive/fixed_income/Fixed_Income/
↪treasury_cmds.py')
from treasury_cmds import *
```

[ ]:
```
df = pd.read_excel('C:/Users/dcste/OneDrive/fixed_income/Fixed_Income/
↪treasury_quotes_2022-09-30.xlsx')
df.columns = df.columns.str.upper()
df.sort_values('TMATDT', inplace = True)
df.set_index('KYTREASNO', inplace = True)
df.head()
```

[ ]:
```
                KYCRSPID_X      CALDT      TDBID      TDASK   TDNOMPRC  \
KYTREASNO
207892     20221004.400000 2022-09-30  99.973333  99.973444  99.973389
207774     20221006.400000 2022-09-30  99.960917  99.961083  99.961000
207893     20221011.400000 2022-09-30  99.924986  99.925292  99.925139
207868     20221013.400000 2022-09-30  99.907556  99.907917  99.907736
207430     20221015.201370 2022-09-30  99.921875  99.953125  99.937500
```

```
           TDNOMPRC_FLG TDSOURCR  TDACCINT      TDRETNUA     TDYLD  …  \
KYTREASNO                                                           …
207892                M        I  0.000000 -1.389258e-07  0.000067  …
207774                M        I  0.000000 -2.542594e-05  0.000065  …
207893                M        I  0.000000  1.473347e-05  0.000068  …
207868                M        I  0.000000 -8.270836e-05  0.000071  …
207430                M        I  0.631148  3.735728e-05  0.000079  …

            TFCALDT  TNOTICE  IYMCN  ITYPE  IUNIQ ITAX IFLWR TBANKDT  \
KYTREASNO
207892          NaN        0    NaN      4      0    1     1     NaN
207774          NaN        0    NaN      4      0    1     1     NaN
207893          NaN        0    NaN      4      0    1     1     NaN
207868          NaN        0    NaN      4      0    1     1     NaN
207430          NaN        0    NaN      2      0    1     1     NaN

           TSTRIPELIG TFRGNTGT
KYTREASNO
207892            NaN      NaN
207774            NaN      NaN
207893            NaN      NaN
207868            NaN      NaN
207430            NaN      NaN

[5 rows x 37 columns]
```

[ ]:
```
t_check = df['CALDT'].values[0]
if df['CALDT'].eq(t_check).all():
    t_current = t_check
else:
    warnings.warn('Quotes are from multiple dates.')
    t_current = None
f'These quotes are based on {pd.to_datetime(t_current):%Y-%m-%d}'
```

[ ]:
```
'These quotes are based on 2022-09-30'
```

[ ]:
```
treasury_metrics = df.
 ↪copy()[['TDATDT','TMATDT','TCOUPRT','TDYLD','TDDURATN','TDPUBOUT']]
treasury_metrics.columns = ['issue date','maturity date','coupon␣
 ↪rate','ytm','duration','outstanding']
treasury_metrics['ytm'] *= 365
treasury_metrics['duration'] /= 365
treasury_metrics['outstanding'] *= 1e6
treasury_metrics['ask-bid'] = df['TDASK'] - df['TDBID']
treasury_metrics['next cashflow date'] = (calc_cashflows(df) != 0).idxmax(1)
```

2 Problem 1

2.1 Part 1

For each issue, calculate its time-to-maturity, quoted in number of years, based on the maturity dates. Assume a year has exactly 365.25 days.

```
[ ]: treasury_metrics['Maturity'] = get_maturity_delta(treasury_metrics['maturity␣
     ↪date'],t_current)
     treasury_metrics['Maturity'].tail().to_frame('Maturity Dates')
     treasury_metrics['Periods'] = np.round(treasury_metrics['Maturity'] *2)
```

2.2 Part 2

- Calculate the dirty price for each bond issue as:

  $$\textbf{price} = \frac{1}{2}\ (\textbf{TBID+TASK})\ +\ \textbf{TDACCINT}$$

```
[ ]: treasury_metrics['price'] = .5*(df['TDBID'] + df['TDASK']) + df['TDACCINT']
     treasury_metrics['price'].to_frame("Dirty Price")
```

```
[ ]:            Dirty Price
     KYTREASNO
     207892       99.973389
     207774       99.961000
     207893       99.925139
     207868       99.907736
     207430      100.568648
     …                  …
     207808       66.640625
     207849       72.718750
     207850       62.910156
     207891       84.726562
     207934       86.507812

     [427 rows x 1 columns]
```

2.3 Question 3 Yield-to-Maturity

- Calculate YTM for every bond issue. Sunday try to write a python function to calculate yield to maturity.

```
[ ]: def price(coupon_rate, periods, ytm, par = 100, freq = 2):
         if periods != 0:
             cfs = [(coupon_rate/freq)*(par/100)/(1+(ytm/freq)**(i+1)) for i in␣
     ↪range(periods)]
             discounted_face = par/(1+(ytm/freq))**periods
             return sum(cfs) + discounted_face
         else:
```

```
        return par
def ytm(coupon_rate, periods,_price, par = 100, freq = 2):
    if periods != 0:
        return optimize.newton(lambda y: price(coupon_rate, periods, y)-_price,.
↪1, maxiter= 100)
    else:
        return (par/_price)-1

ytm(1.5,9,89.7)
```

[ ]: 0.04193184471875915

```
[ ]: for i in treasury_metrics.index:
    coupon_r = treasury_metrics.loc[i,'coupon rate']
    p = treasury_metrics.loc[i,'Periods'].astype('int')
    PRICE = treasury_metrics.loc[i,'price']
    treasury_metrics.loc[i,'YLD']  = ytm(coupon_rate = coupon_r, periods␣
↪=p,_price = PRICE)
```

```
[ ]: treasury_metrics = treasury_metrics[(treasury_metrics['YLD'] >= 0) &␣
↪(treasury_metrics['YLD'] < 1) ]
```

2.4 Question 4| Summary Table

```
[ ]: treasury_metrics
```

[ ]:

```
           issue date maturity date  coupon rate       ytm   duration  \
KYTREASNO
207892     2022-06-07    2022-10-04        0.000  0.024286   0.010959
207774     2021-10-07    2022-10-06        0.000  0.023730   0.016438
207893     2022-06-14    2022-10-11        0.000  0.024850   0.030137
207868     2022-04-14    2022-10-13        0.000  0.025917   0.035616
207894     2022-06-21    2022-10-18        0.000  0.024805   0.049315
…                 …             …           …         …          …
207763     2021-08-15    2051-08-15        2.000  0.037968  20.262357
207808     2021-11-15    2051-11-15        1.875  0.037857  20.506232
207849     2022-02-15    2052-02-15        2.250  0.037800  19.982853
207850     2022-02-15    2052-02-15        0.125       NaN        NaN
207891     2022-05-15    2052-05-15        2.875  0.037612  18.901802

             outstanding   ask-bid next cashflow date   Maturity  Periods  \
KYTREASNO
207892               NaN  0.000111         2022-10-04   0.010951      0.0
207774               NaN  0.000167         2022-10-06   0.016427      0.0
207893               NaN  0.000306         2022-10-11   0.030116      0.0
207868               NaN  0.000361         2022-10-13   0.035592      0.0
207894               NaN  0.000500         2022-10-18   0.049281      0.0
```

```
…                   …         …                 …          …          …
207763      7.409800e+10  0.062500          2023-02-15  28.873374      58.0
207808      6.897200e+10  0.046875          2022-11-15  29.125257      58.0
207849      6.300400e+10  0.062500          2023-02-15  29.377139      59.0
207850      1.663300e+10  0.257812          2023-02-15  29.377139      59.0
207891      5.999400e+10  0.062500          2022-11-15  29.623546      59.0

               price       YLD
KYTREASNO
207892     99.973389  0.000266
207774     99.961000  0.000390
207893     99.925139  0.000749
207868     99.907736  0.000923
207894     99.877750  0.001224
…                …         …
207763     68.367188  0.079565
207808     66.640625  0.073582
207849     72.718750  0.095400
207850     62.910156  0.017837
207891     84.726562  0.249050

[419 rows x 12 columns]
```

[ ]:
```python
treasury_metrics = treasury_metrics[['Maturity','price','coupon rate','ytm','YLD','ask-bid', 'Periods']]
treasury_metrics
```

[ ]:
```
            Maturity      price  coupon rate       ytm       YLD   ask-bid  \
KYTREASNO
207892      0.010951  99.973389        0.000  0.024286  0.000266  0.000111
207774      0.016427  99.961000        0.000  0.023730  0.000390  0.000167
207893      0.030116  99.925139        0.000  0.024850  0.000749  0.000306
207868      0.035592  99.907736        0.000  0.025917  0.000923  0.000361
207894      0.049281  99.877750        0.000  0.024805  0.001224  0.000500
…                …          …          …          …        …         …
207763     28.873374  68.367188        2.000  0.037968  0.079565  0.062500
207808     29.125257  66.640625        1.875  0.037857  0.073582  0.046875
207849     29.377139  72.718750        2.250  0.037800  0.095400  0.062500
207850     29.377139  62.910156        0.125       NaN  0.017837  0.257812
207891     29.623546  84.726562        2.875  0.037612  0.249050  0.062500

           Periods
KYTREASNO
207892         0.0
207774         0.0
207893         0.0
207868         0.0
```

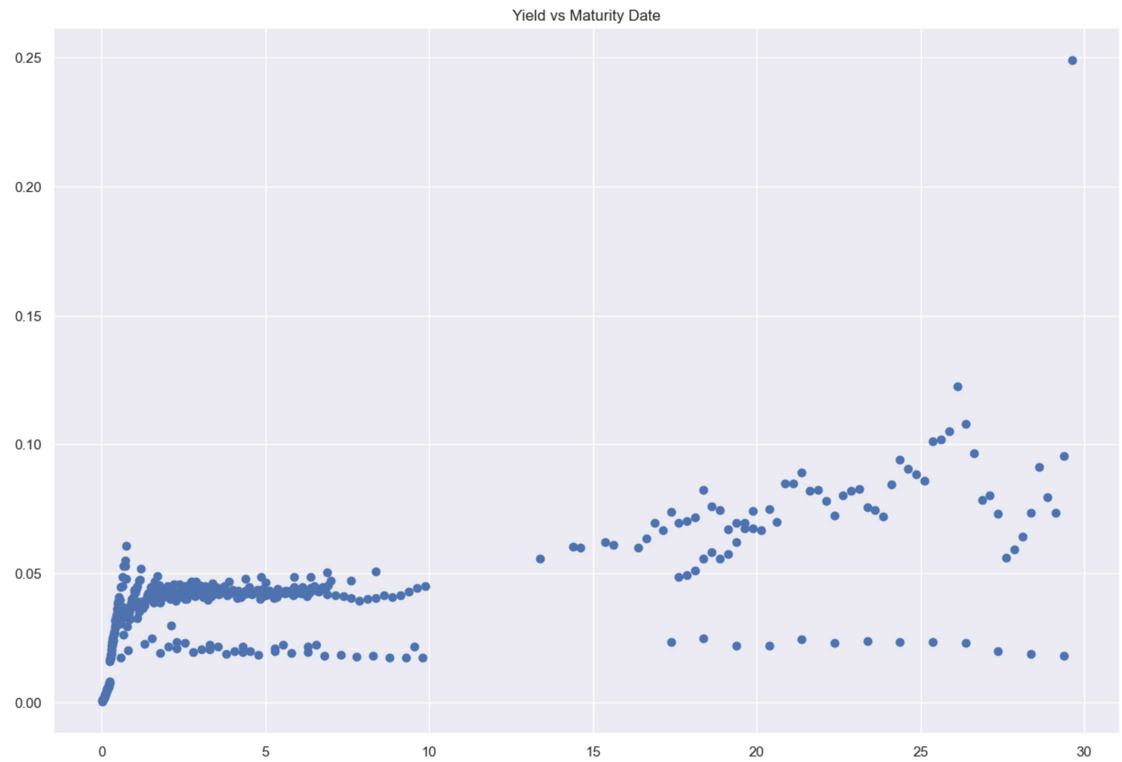
```
207894         0.0
…              …
207763        58.0
207808        58.0
207849        59.0
207850        59.0
207891        59.0

[419 rows x 7 columns]
```

[ ]:
```
fig = plt.figure()
ax = fig.add_subplot(1,1,1)
ax.set_title('Price vs Maturity (Measured in Years)')
ax.scatter(treasury_metrics['Maturity'], treasury_metrics['price'])
```

[ ]:
```
<matplotlib.collections.PathCollection at 0x13d93ade310>
```

[ ]:
```
fig = plt.figure()
ax = fig.add_subplot(1,1,1)
ax.set_title('Yield vs Maturity Date')
ax.scatter(treasury_metrics['Maturity'], treasury_metrics['YLD'])
```

[ ]:
```
<matplotlib.collections.PathCollection at 0x13d95cf7880>
```



2.5 Question 6| Bad Data

[ ]:
```
treasury_metrics[treasury_metrics['ytm'].isna()].describe()
```

[ ]:
```
        Maturity       price  coupon rate  ytm        YLD    ask-bid  \
count  49.000000   49.000000    49.000000  0.0  49.000000  49.000000
mean    9.584880   91.685945     0.875000  NaN   0.021024   0.107701
std     8.895883   10.981731     0.991829  NaN   0.002885   0.083852
min     0.292950   62.476562     0.125000  NaN   0.017118   0.007812
25%     3.293634   87.050781     0.125000  NaN   0.019117   0.042969
50%     5.790554   93.400391     0.500000  NaN   0.020879   0.082031
75%    17.377139   98.388672     1.000000  NaN   0.022600   0.207031
max    29.377139  113.630859     3.875000  NaN   0.034382   0.269531

         Periods
count  49.000000
mean   19.428571
std    17.792789
min     1.000000
25%     7.000000
50%    12.000000
75%    35.000000
```

```
max    59.000000
```

[ ]:
```
treasury_metrics = treasury_metrics.dropna()
treasury_metrics.describe()
```

[ ]:

	Maturity	price	coupon rate	ytm	YLD
count	370.000000	370.000000	370.000000	370.000000	370.000000
mean	6.255693	93.485817	1.870608	0.039594	0.044359
std	7.945724	9.153625	1.581789	0.004024	0.022291
min	0.010951	56.304688	0.000000	0.023730	0.000266
25%	0.968515	88.663553	0.375000	0.038496	0.039213
50%	2.896646	95.895225	1.750000	0.040797	0.042494
75%	6.478439	98.831706	2.750000	0.042371	0.045987
max	29.623546	117.296875	7.625000	0.043562	0.249050

	ask-bid	Periods
count	370.000000	370.000000
mean	0.039229	12.516216
std	0.021525	15.892969
min	0.000111	0.000000
25%	0.031250	2.000000
50%	0.039062	6.000000
75%	0.046875	13.000000
max	0.164444	59.000000

- There are 49 rows where ytm is NaN. The average ask-bid spread is significantly higher for the dataset that includes all null values. This suggests these issues are more illiquid. Additionally, the average coupon rates are significantly lower for the NaN dataset.

3 Problem 2: Extract Discount Curve

3.1 Question 1|

- Create the cashflow matrix, **C**, where each row is a treasury issue, each column is a date, and each entry is the cash paid on a normalized face value of $100.

[ ]:
```
Cf = calc_cashflows(df)
display(Cf)
```

KYTREASNO	2022-10-04	2022-10-06	2022-10-11	2022-10-13	2022-10-15
207892	100	0	0	0	0.0000
207774	0	100	0	0	0.0000
207893	0	0	100	0	0.0000
207868	0	0	0	100	0.0000
207430	0	0	0	0	100.6875
…	…	…	…	…	…
207808	0	0	0	0	0.0000
207849	0	0	0	0	0.0000

```
207850              0           0           0           0      0.0000
207891              0           0           0           0      0.0000
207934              0           0           0           0      0.0000

           2022-10-18  2022-10-20  2022-10-25  2022-10-27  2022-10-30  …  \
KYTREASNO                                                              …
207892              0           0           0           0         0.0  …
207774              0           0           0           0         0.0  …
207893              0           0           0           0         0.0  …
207868              0           0           0           0         0.0  …
207430              0           0           0           0         0.0  …
…                …         …         …         …           …  …
207808              0           0           0           0         0.0  …
207849              0           0           0           0         0.0  …
207850              0           0           0           0         0.0  …
207891              0           0           0           0         0.0  …
207934              0           0           0           0         0.0  …

           2050-05-15  2050-08-15  2050-11-15  2051-02-15  2051-05-15  \
KYTREASNO
207892         0.0000      0.0000      0.0000      0.0000      0.0000
207774         0.0000      0.0000      0.0000      0.0000      0.0000
207893         0.0000      0.0000      0.0000      0.0000      0.0000
207868         0.0000      0.0000      0.0000      0.0000      0.0000
207430         0.0000      0.0000      0.0000      0.0000      0.0000
…                …         …         …         …           …
207808         0.9375      0.0000      0.9375      0.0000      0.9375
207849         0.0000      1.1250      0.0000      1.1250      0.0000
207850         0.0000      0.0625      0.0000      0.0625      0.0000
207891         1.4375      0.0000      1.4375      0.0000      1.4375
207934         0.0000      1.5000      0.0000      1.5000      0.0000

           2051-08-15  2051-11-15  2052-02-15  2052-05-15  2052-08-15
KYTREASNO
207892         0.0000      0.0000      0.0000      0.0000         0.0
207774         0.0000      0.0000      0.0000      0.0000         0.0
207893         0.0000      0.0000      0.0000      0.0000         0.0
207868         0.0000      0.0000      0.0000      0.0000         0.0
207430         0.0000      0.0000      0.0000      0.0000         0.0
…                …         …         …         …           …
207808         0.0000    100.9375      0.0000      0.0000         0.0
207849         1.1250      0.0000    101.1250      0.0000         0.0
207850         0.0625      0.0000    100.0625      0.0000         0.0
207891         0.0000      1.4375      0.0000    101.4375         0.0
207934         1.5000      0.0000      1.5000      0.0000       101.5

[427 rows x 340 columns]
```

```
[ ]: ((Cf.sum(axis = 0)).sort_values(ascending=False)).to_frame('Date with Max Sum␣
     ↪of Cashflow')
```

```
[ ]:             Date with Max Sum of Cashflow
     2023-01-31                        422.0625
     2023-02-15                        421.5000
     2023-08-15                        416.2500
     2025-02-15                        409.0000
     2025-08-15                        403.4375
     …                                      …
     2030-04-15                          1.6875
     2030-10-15                          1.6875
     2031-04-15                          1.6875
     2029-08-28                          1.5625
     2029-05-30                          1.3750

     [340 rows x 1 columns]
```

```
[ ]: ((Cf[Cf > 0].count()).sort_values(ascending= False)).to_frame('Date with most␣
     ↪cashflow')
```

```
[ ]:             Date with most cashflow
     2023-02-15                       87
     2023-08-15                       84
     2024-02-15                       81
     2024-08-15                       79
     2025-02-15                       77
     …                              …
     2022-10-06                        1
     2027-10-31                        1
     2027-12-31                        1
     2028-02-29                        1
     2052-08-15                        1

     [340 rows x 1 columns]
```

- On February 15, 2023, there are 87 issues paying a cashflow.
- On January 31, 2023 the most cashflow occurs on this date.

3.2 Part 2| Bootstrap

- Bootstrap the discount factors
- Report the first 5 head and tail rows of $\tilde{C}$
- Solve the system of equations $P = \tilde{C} * z + \epsilon$
- $\tilde{C}$ is a filtered cashflow matrix with the following properties:

1. For dates with multiple bonds maturing, keep only that with the smallest bid-ask. If there are still multiple bonds maturing on the date, keep only that with the smallest coupon.
2. Only keep the first **L** columns of the cashflow matrix, such that every column has a bond maturing.
3. Eliminate any bonds, (rows,) that had maturity beyond column **L**.

[ ]:
```
MODEL = bootstrap

RESTRICT_YLD = True
RESTRICT_TIPS = True

RESTRICT_DTS_MATURING = True
RESTRICT_REDUNDANT = True
data = filter_treasuries(df, t_date=t_current, filter_yld = RESTRICT_YLD,␣
 ↪filter_tips = RESTRICT_TIPS, drop_duplicate_maturities=RESTRICT_REDUNDANT)
Cf =␣
 ↪filter_treasury_cashflows(calc_cashflows(data),filter_maturity_dates=RESTRICT_DTS_MATURING)

price_mat = Cf.join(treasury_metrics['price'], how = 'inner' )
price_mat
```

[ ]:
```
           2022-10-04 00:00:00  2022-10-06 00:00:00  2022-10-11 00:00:00  \
KYTREASNO
207892                     100                    0                    0
207774                       0                  100                    0
207893                       0                    0                  100
207868                       0                    0                    0
207894                       0                    0                    0
…                          …                    …                    …
207761                       0                    0                    0
207806                       0                    0                    0
207847                       0                    0                    0
207889                       0                    0                    0
207932                       0                    0                    0

           2022-10-13 00:00:00  2022-10-15 00:00:00  2022-10-18 00:00:00  \
KYTREASNO
207892                       0                  0.0                    0
207774                       0                  0.0                    0
207893                       0                  0.0                    0
207868                     100                  0.0                    0
207894                       0                  0.0                  100
…                          …                    …                    …
207761                       0                  0.0                    0
207806                       0                  0.0                    0
207847                       0                  0.0                    0
```

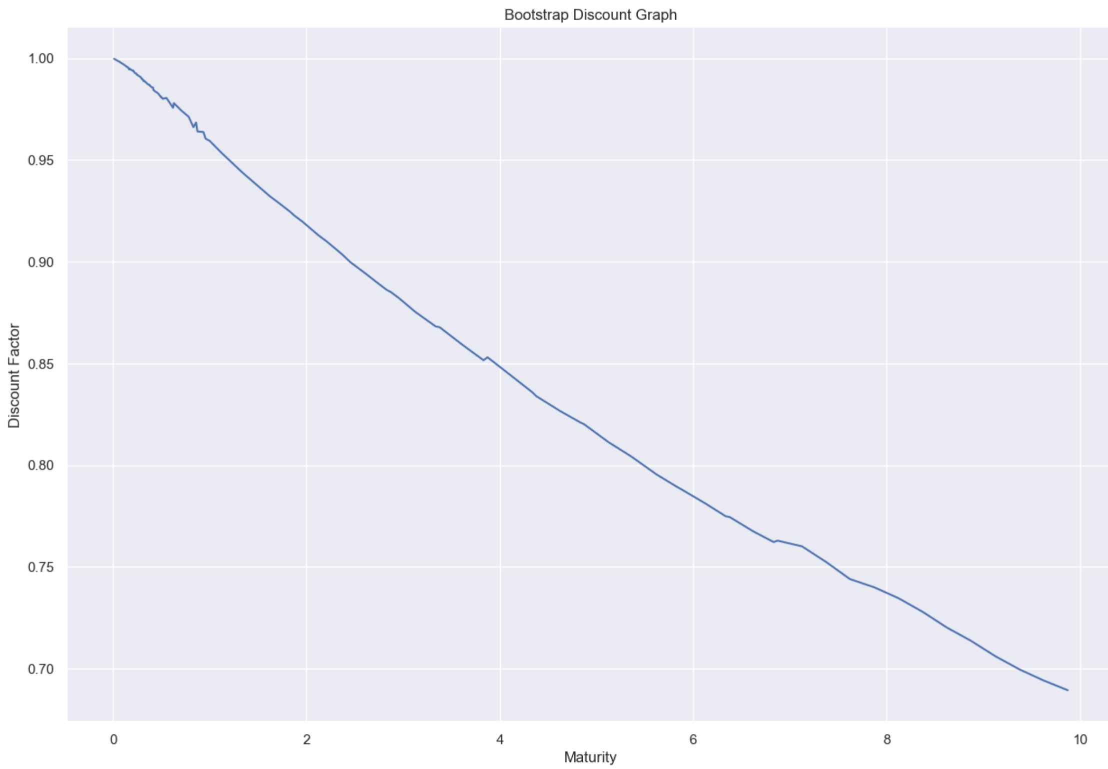
```
207889                 0                   0.0                     0
207932                 0                   0.0                     0

           2022-10-20 00:00:00  2022-10-25 00:00:00  2022-10-27 00:00:00  \
KYTREASNO
207892                 0                    0                    0
207774                 0                    0                    0
207893                 0                    0                    0
207868                 0                    0                    0
207894                 0                    0                    0
…                    …                  …                  …
207761                 0                    0                    0
207806                 0                    0                    0
207847                 0                    0                    0
207889                 0                    0                    0
207932                 0                    0                    0

           2022-10-31 00:00:00  …  2030-08-15 00:00:00  2030-11-15 00:00:00  \
KYTREASNO                       …
207892                 0  …               0.0000               0.0000
207774                 0  …               0.0000               0.0000
207893                 0  …               0.0000               0.0000
207868                 0  …               0.0000               0.0000
207894                 0  …               0.0000               0.0000
…                    …  …                …                  …
207761                 0  …               0.6250               0.0000
207806                 0  …               0.0000               0.6875
207847                 0  …               0.9375               0.0000
207889                 0  …               0.0000               1.4375
207932                 0  …               1.3750               0.0000

           2031-02-15 00:00:00  2031-05-15 00:00:00  2031-08-15 00:00:00  \
KYTREASNO
207892            0.0000               0.0000               0.0000
207774            0.0000               0.0000               0.0000
207893            0.0000               0.0000               0.0000
207868            0.0000               0.0000               0.0000
207894            0.0000               0.0000               0.0000
…                    …                  …                  …
207761            0.6250               0.0000             100.6250
207806            0.0000               0.6875               0.0000
207847            0.9375               0.0000               0.9375
207889            0.0000               1.4375               0.0000
207932            1.3750               0.0000               1.3750

           2031-11-15 00:00:00  2032-02-15 00:00:00  2032-05-15 00:00:00  \
KYTREASNO
```

```
207892              0.0000         0.0000         0.0000
207774              0.0000         0.0000         0.0000
207893              0.0000         0.0000         0.0000
207868              0.0000         0.0000         0.0000
207894              0.0000         0.0000         0.0000
…                     …            …             …
207761              0.0000         0.0000         0.0000
207806            100.6875         0.0000         0.0000
207847              0.0000       100.9375         0.0000
207889              1.4375         0.0000       101.4375
207932              0.0000         1.3750         0.0000

           2032-08-15 00:00:00      price
KYTREASNO
207892                   0.000  99.973389
207774                   0.000  99.961000
207893                   0.000  99.925139
207868                   0.000  99.907736
207894                   0.000  99.877750
…                          …        …
207761                   0.000  80.765625
207806                   0.000  81.531250
207847                   0.000  84.710938
207889                   0.000  93.253906
207932                 101.375  91.523438

[119 rows x 122 columns]
```

[ ]:
```python
price = price_mat['price'][price_mat.index]
price_mat = price_mat.drop(columns='price')
params_boot = estimate_rate_curve(MODEL, price_mat, t_current = t_current, prices = price)
boot_df = pd.DataFrame({'maturity':params_boot[0],'Boot Disc': params_boot[1]}).set_index('maturity')
boot_df = boot_df[boot_df['Boot Disc']>.5]
```

[ ]:
```python
plt.plot(boot_df)
plt.ylabel('Discount Factor')
plt.xlabel('Maturity')
plt.title('Bootstrap Discount Graph')
```

[ ]:
```
Text(0.5, 1.0, 'Bootstrap Discount Graph')
```



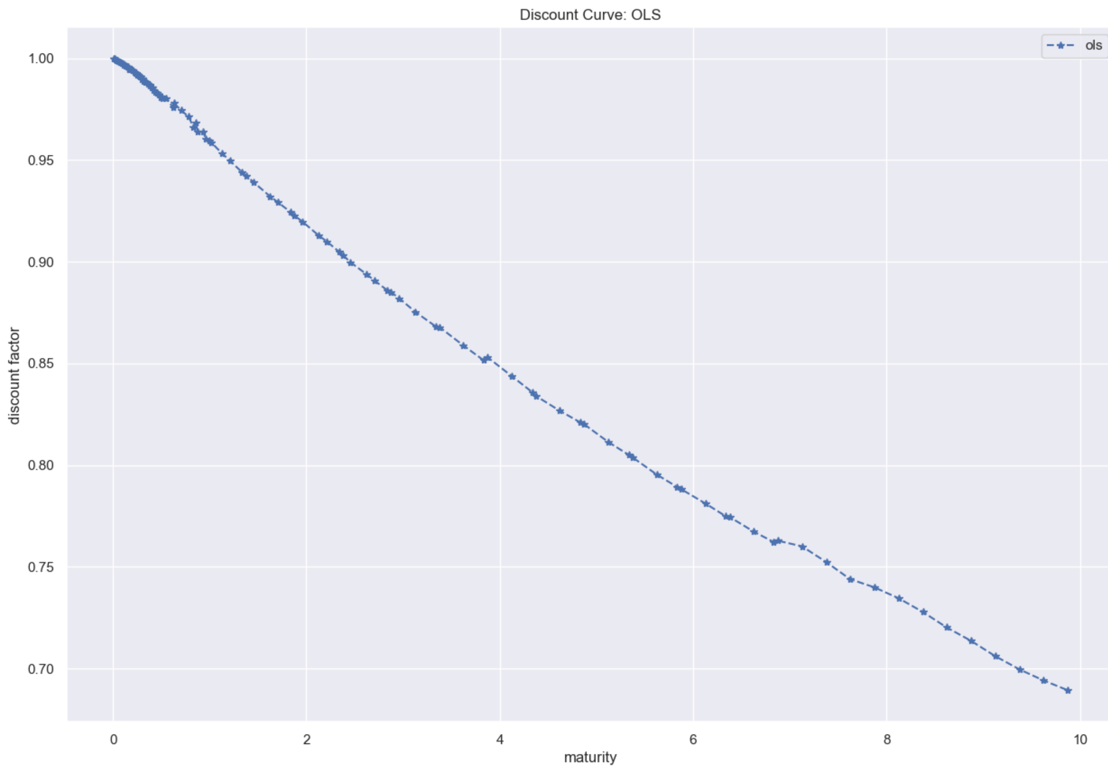
3.3 OLS

- Estimate the discount factors using OLS.

$$\mathbf{p} = \hat{\mathbf{C}}\,\mathbf{z}_{\text{ols}} + \epsilon$$

[ ]:
```
params_OLS = estimate_rate_curve(MODEL, price_mat, t_current = t_current,␣
↪prices = price)
```

[ ]:
```
disc_ols = pd.DataFrame({'maturity':params_OLS[0], 'ols': params_OLS[1]}).
↪set_index('maturity')
disc_ols = disc_ols[disc_ols['ols']>.5]
disc_ols.plot(marker='*',linestyle='--',ylabel='discount␣
↪factor',title='Discount Curve: OLS')
plt.show()
```



3.4 Part 4| Factors and Rates

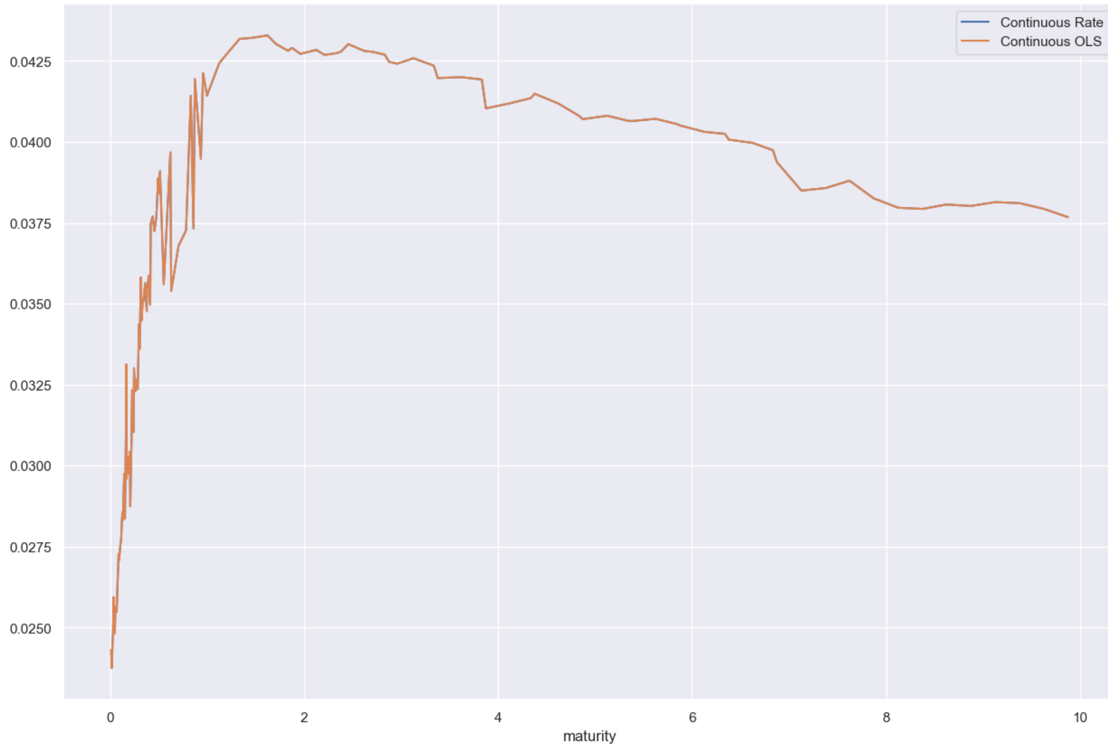
Convert the bootstrap and ols discount factors to continous discount rates and semiannually discount rates.

[ ]:
```
boot_df['Continuous Rate'] = -np.log(boot_df['Boot Disc'])/boot_df.index
boot_df['Semiannual Rate'] = 2 * (1/(boot_df['Boot Disc']**(1/(2*boot_df.
    ↪index)))-1)
```

[ ]:
```
disc_ols['Continuous OLS'] =  -np.log(disc_ols['ols'])/disc_ols.index
disc_ols['Semiannual OLS Rate'] = 2 * (1/(disc_ols['ols']**(1/(2*disc_ols.
    ↪index)))-1)
```

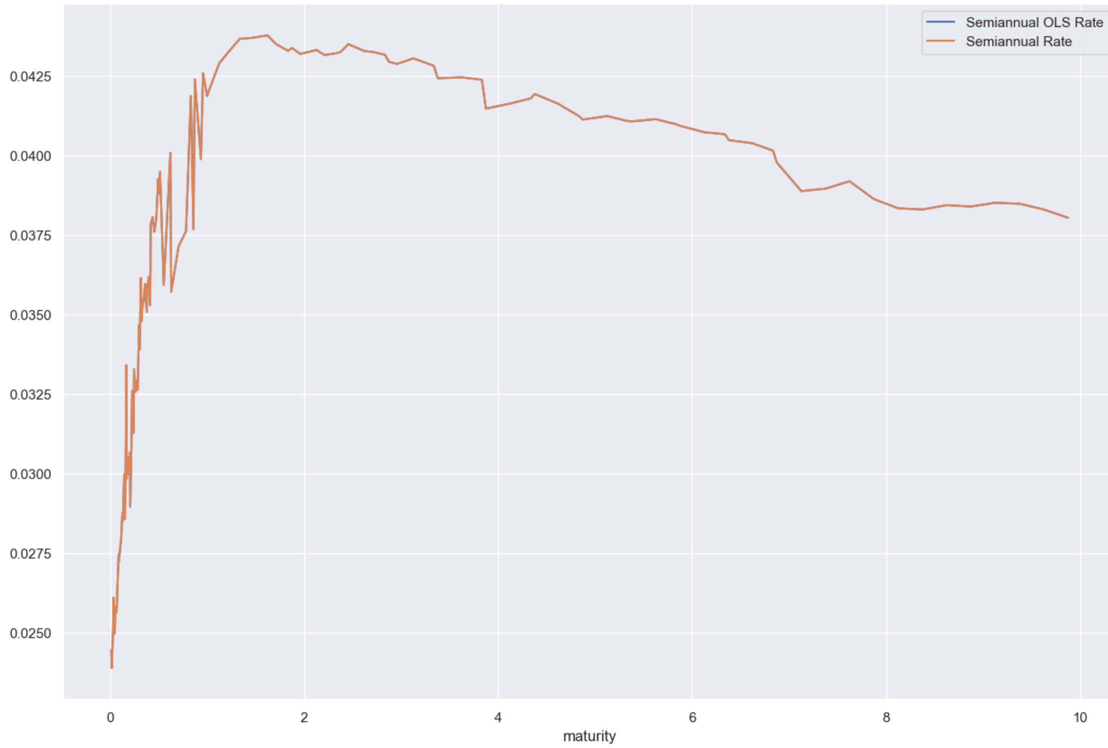
[ ]:
```
continous_rates = pd.merge(boot_df['Continuous Rate'], disc_ols['Continuous␣
    ↪OLS'], right_index= True, left_index = True)
continous_rates.plot()
```

[ ]: <AxesSubplot:xlabel='maturity'>



[ ]:
```python
semiannual_rates = pd.merge(disc_ols['Semiannual OLS Rate'],␣
  ↪boot_df['Semiannual Rate'], left_index = True, right_index = True)
semiannual_rates.plot()
```

[ ]: <AxesSubplot:xlabel='maturity'>


Semiannual OLS Rate
Semiannual Rate
0.0425
0.0400
0.0375
0.0350
0.0325
0.0300
0.0275
0.0250
0
2
4
6
8
10
maturity