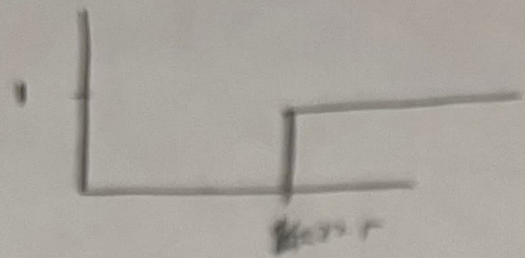
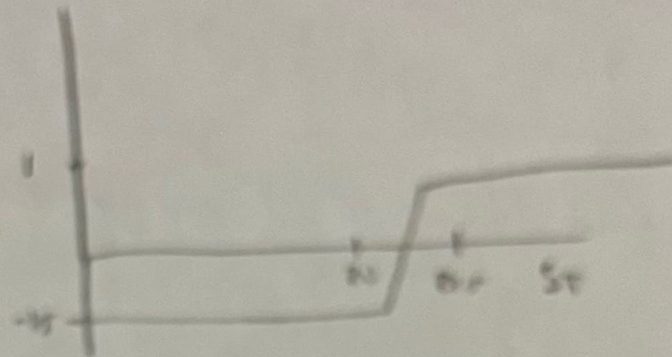
Part 1

a. $C(K=22.5) = \begin{cases} 1 & S_T \geq K \\ 0 & S_T < K \end{cases}$



$\theta = \begin{bmatrix} -1.5B \\ C_0(K=20) \\ -C_0(K=22.5) \end{bmatrix} =$



Bounds of Binary Call

$-1.5 \leq C(K=22.5) \leq 1$

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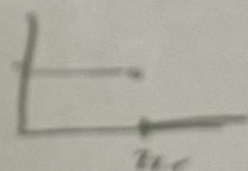
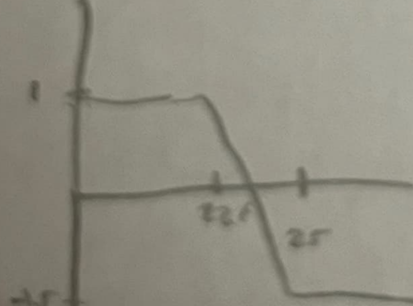
b. $P(K=22.5) = \begin{cases} 0 & S_T \geq K \\ 1 & S_T < K \end{cases} =$



$\theta = \begin{bmatrix} -1B \\ -C_0(22.5) \\ C_0(25) \end{bmatrix} =$



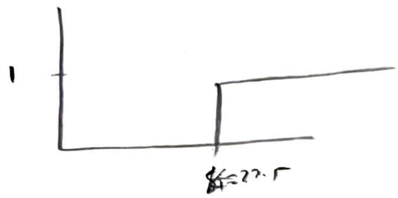
Bounds of Put option:

$1 \geq P(K=22.5) \geq$

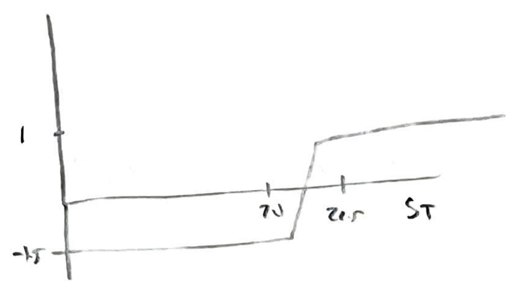
~~Que~~ Part 1

a.

$$C(K=22.5) = \begin{cases} 1 & S_T \geq K \\ 0 & S_T < K \end{cases}$$



$$\theta = \begin{bmatrix} -1.5B \\ Co(K=20 \\ -Co(K=22.5) \end{bmatrix} =$$

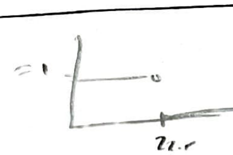


Bounds of Binary Call

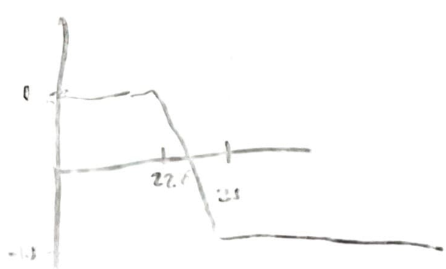
$-1.5 \leq C(K=22.5) \leq 1$

---

b. $P(K=22.5) = \begin{cases} 0 & S_T \geq K \\ 1 & S_T < K \end{cases}$



$$\theta = \begin{bmatrix} -1B \\ -Co([illegible]) \\ Co([illegible]) \end{bmatrix} =$$



Bounds of Put option:

$1 \geq P(K=22.5) \geq -1.5$

c.

Find the time-0 price of the contract $\max(2.5, S_T - 22.5)$

Since we know $C_0(22.5) = \max(0, S_T - 22.5) = 4.15$

We need to price this call in relation to the market

so no arbitrage exists.

Thus, $C_0(2.5), S_t - 22.5) = 2.5 + 4.15 = \boxed{6.65}$

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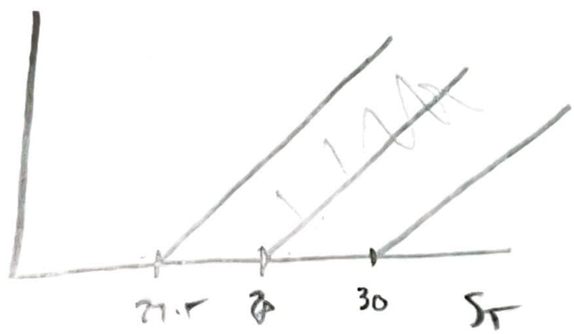
d. Find the upper and lower bounds of T-expiry 28 strike vanilla

call $C_0(K=30) \leq C_0(K=28) \leq C_0(K=22.5) \;=\; .8 \leq C_0(K=28) \leq 1.$



Homework 1 Question 2

For $S > K_*$

$$f(s) = f(K_*) + f'(K_*)(S-K) + \int_{K_*}^{\infty} f''(k)(s-k)^+ \, dk$$

$$= f(K_*) + f'(K_*)(S-K) + \int_{K^*}^{S} f''(k)(s-k)\,dk + \int_{S}^{\infty} f''(k)\overset{0}{(S-K)}\,dk$$

$$= f(K_*) + f'(K_*)(S-K) + \Big[f'(K)(S-K) + f(K)\Big]_{K^*}^{S}$$

$$= f(K_*) + f'(K_*)(S-K) + \Big[f'(s)\overset{0}{(s-s)} + f(s) - \big(f'(K_*)(S-K_*) + f(K_*)\big)\Big]$$

$$= \cancel{f(K_*)} + f'(K_*)(S-K) + f(s) - \cancel{f'(K_*)(S-K_*)} - \cancel{f(K_*)}$$

$$= f(s)$$

For $S < K_*$

$$f(s) = f(K_*) + f'(K_*)(S-K) + \int_{0}^{K_*} f''(k)(k-s)^+ \, dk$$

$$= f(s) - f(K_*) + f'(K_*)(S-K) + \int_{0}^{S} f''\overset{0}{(K-S)}\,dk + \int_{S}^{K} f''(K)(K-S)\,dk$$

$$= f(K_*) + f'(K_*)(S-K) + \Big[f'(K)(K-S) - f(K)\Big]_{S}^{K_*}$$

$$= f(K_*) + f'(K_*)(S-K) + \Big[f'(K_*)(K_*-S) - f(K_*) - \big(f(S)\underset{0}{(S-S)} - f(S)\big)\Big]$$

$$= f(K_*) + \cancel{f'(K_*)}(S-K) + \cancel{f'(K_*)}(K-S) - f(K_*) + f(S)$$

$$= f(S)$$

Part b

$S = -2\ln(S_T)$

$S' = -2 S_T^{-1}$

$S'' = 2 S_T^{-2}$

How many puts with strike 1950 should I hold with $k^* = 1960$

$$= \sum_{i=S_T}^{1960} (K - S_i) =$$