Option Simplification

Put-Call Parity

Forward Contract: is a contract with a delivery price K that obligates its holder to buy one share of the stock at expiration time T in exchange for payment K. At expiration the value of the forward contract is $S(T) - K$. Let $f(t, x)$ denote the value of the forward contract at earlier times $t \in [0, T]$ if the stock price at time τ is $S(t) = x$, the value of the forward contract at time τ is :

$$f(\tau, x) = x - e^{-r(T-\tau)}K \tag{1}$$

- Using no arbitrage pricing the payoff of the forward contract agrees with the payoff of a portfolio that is *long one call* and *short one put*, which can be summarized using this mathematical notation:

$$f(t, x) = c(\tau, x) - p(\tau, x) \tag{2}$$
$$x - e^{-r(T-\tau)}K = c(\tau, x) - p(\tau, x) \tag{3}$$

Parity Relationships

- short put +short stock = short call

$$-P_0 - S_0 = -C_0 \tag{4}$$

- long put + long stock = long call

$$P_0 + S_0 = C_0 \tag{5}$$

- long call - short put = long stock

$$C_0 - P_0 = S_0 \tag{6}$$

- short call + long put = short stock

$$-C_0 + P_0 = -S_0 \tag{7}$$

- long call + short stock = long put

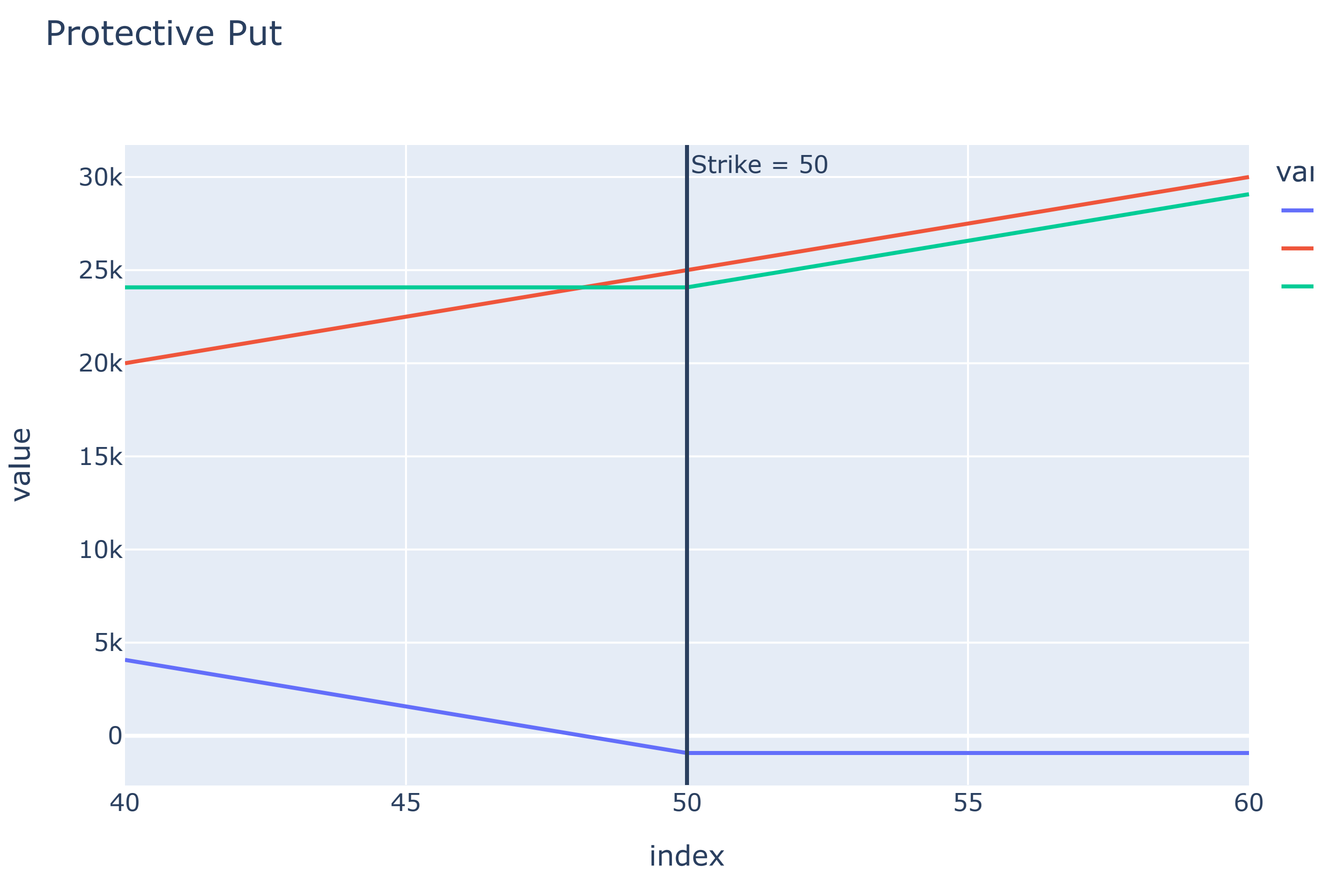
$$C_0 - S_0 = P_0 \tag{8}$$

- short call + long stock = short put

$$-C_0 + S_0 = -P_0 \tag{9}$$

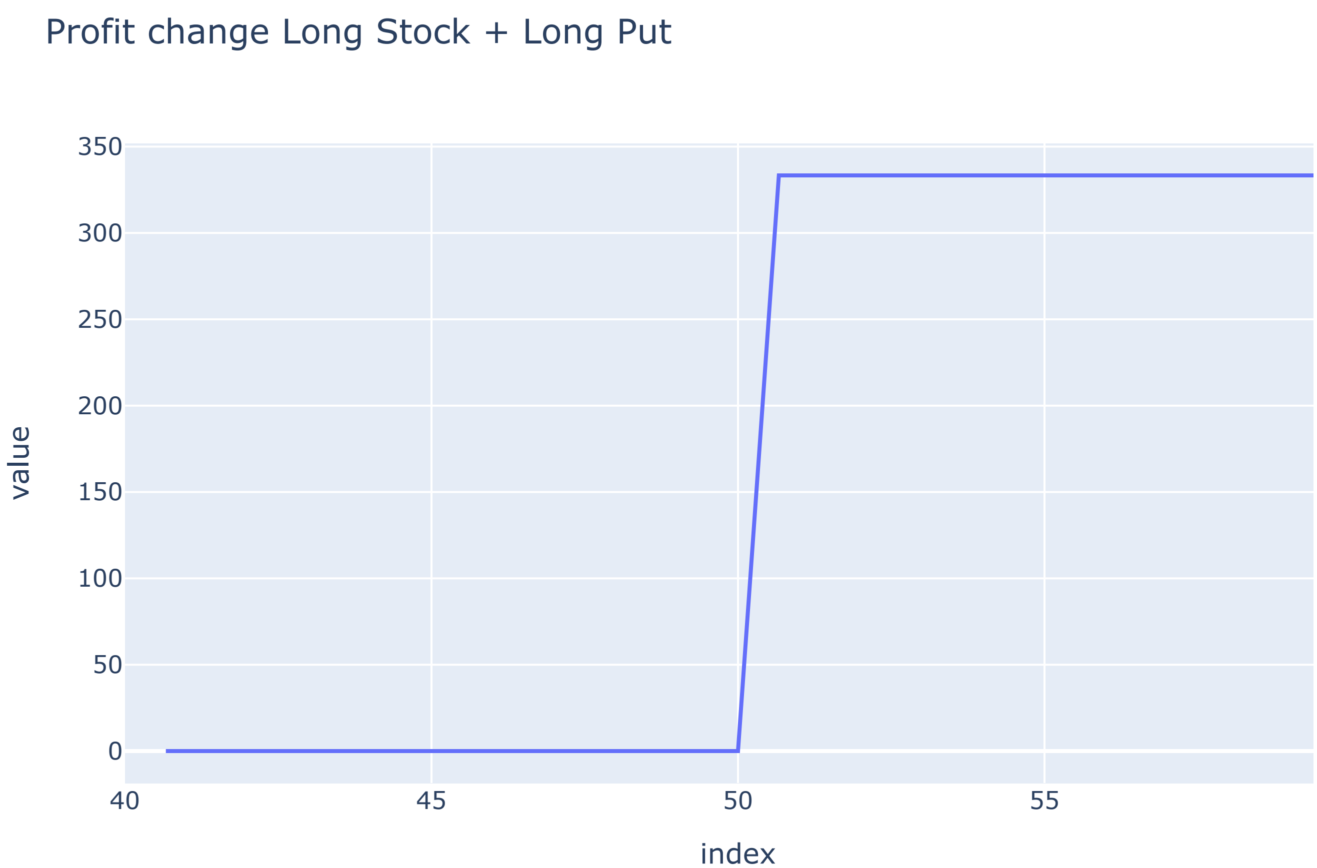
Protective Put

- Suppose you own 500 shares of Well Fargo, and buy 5 puts with a strike of 50 for a premium of $\$1.85$. The current stock price is $\$48.92$.

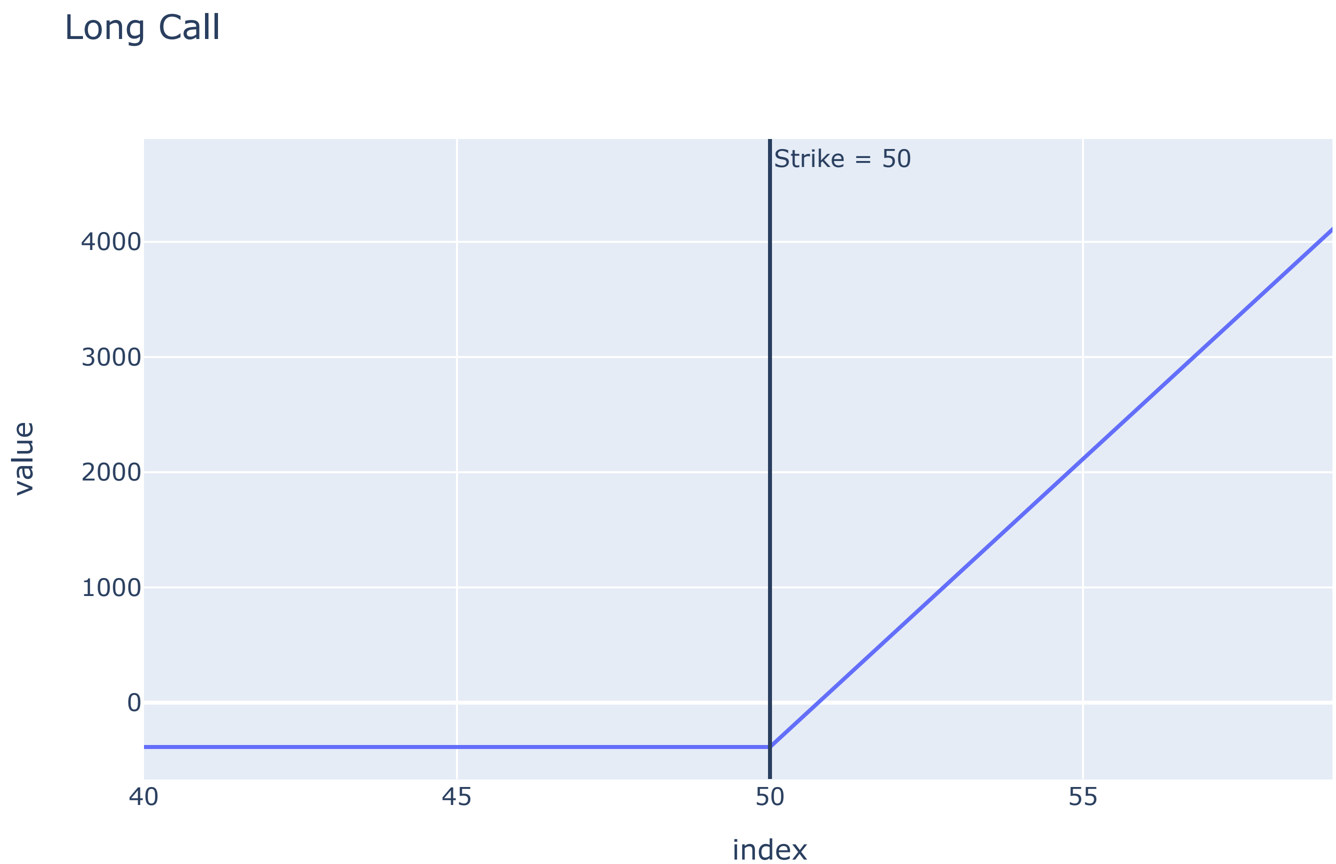


- Notice the *green* line and how your losses are protected if the stock price drops below $\$50.00$, however, your breakeven point is actually at $\$48.15$ because you paid $\$1.85$ for the put premium.
- The *green* line is also known as a synthetic **long call with strike equal to 50**. This syntetic long call is priced at $\$0.77$. Why?
  - Because:

$$C_o = P_o + S_o - K \tag{10}$$



- This is the payoff graph of the portfolio.

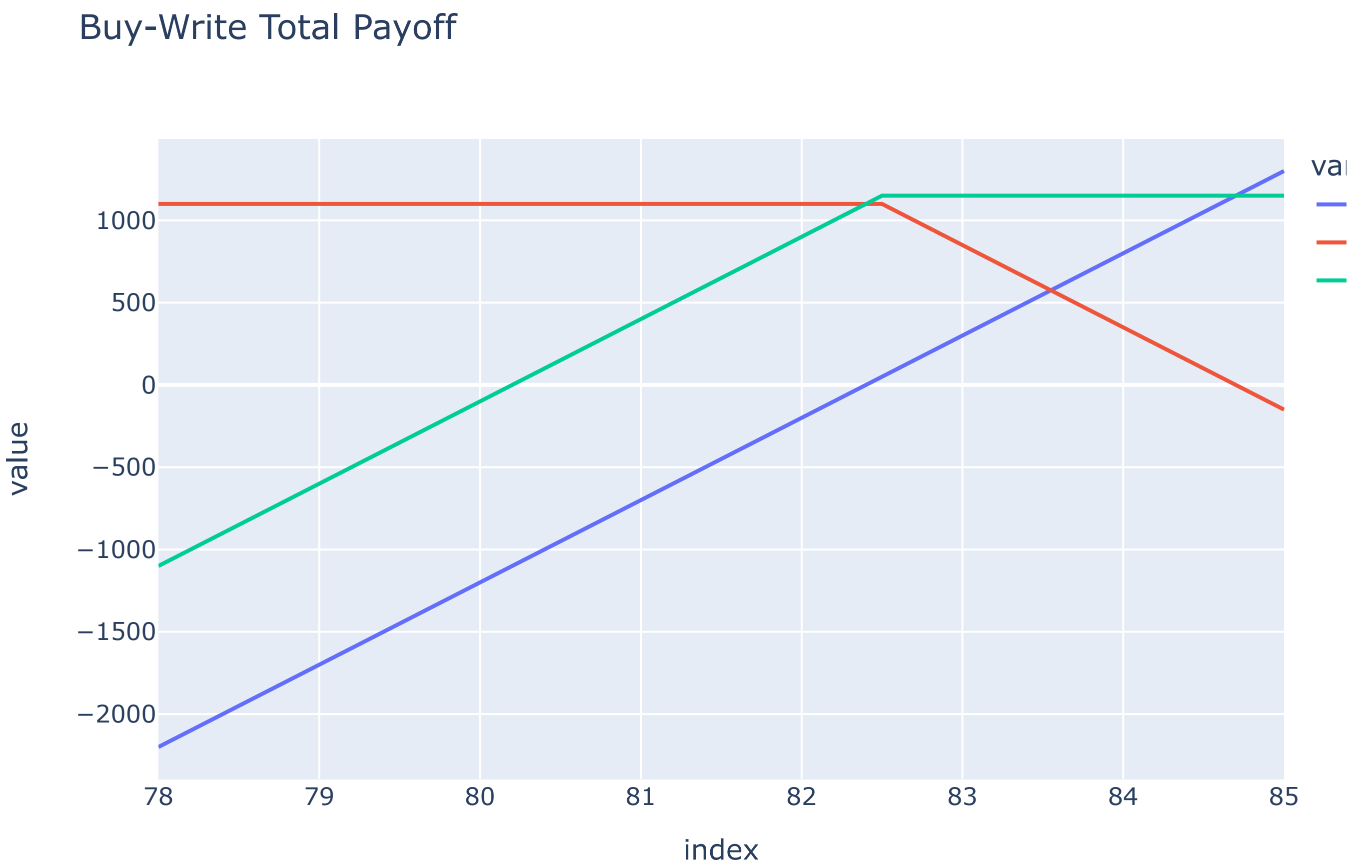


Buy-Write

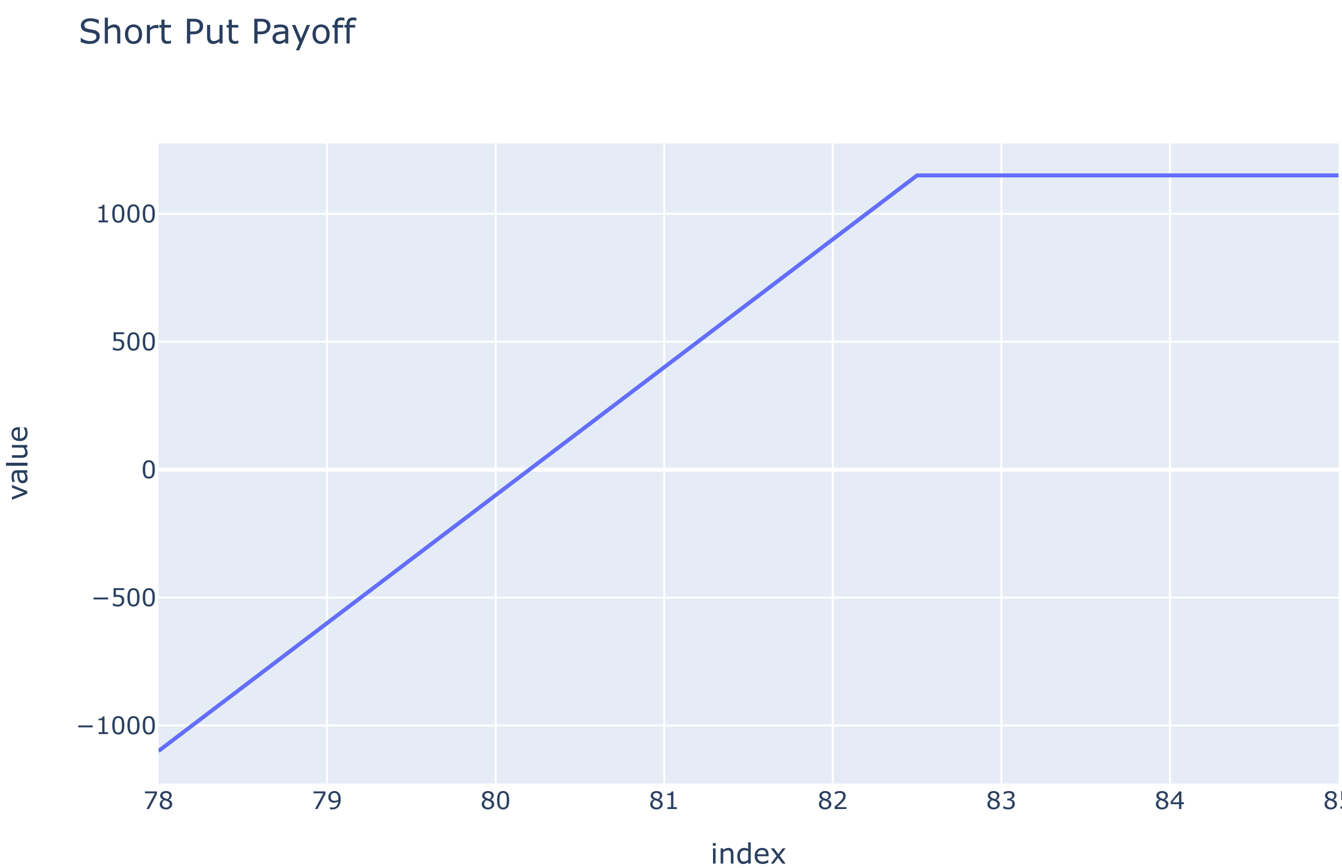
- A portfolio with a short call option and long stock position
- This is equivalent to a short put option

<u>Example</u>

$S_0 = \$82.40$ and the number of shares bought is $\Delta = 500$. Additionally, you sold $5C(K = 82.5) = \$2.20$



- In order to simulate this exact payoff we need to sell a put for a price of $\$2.30$
- $P_0 = C_0 - S_0 + K$



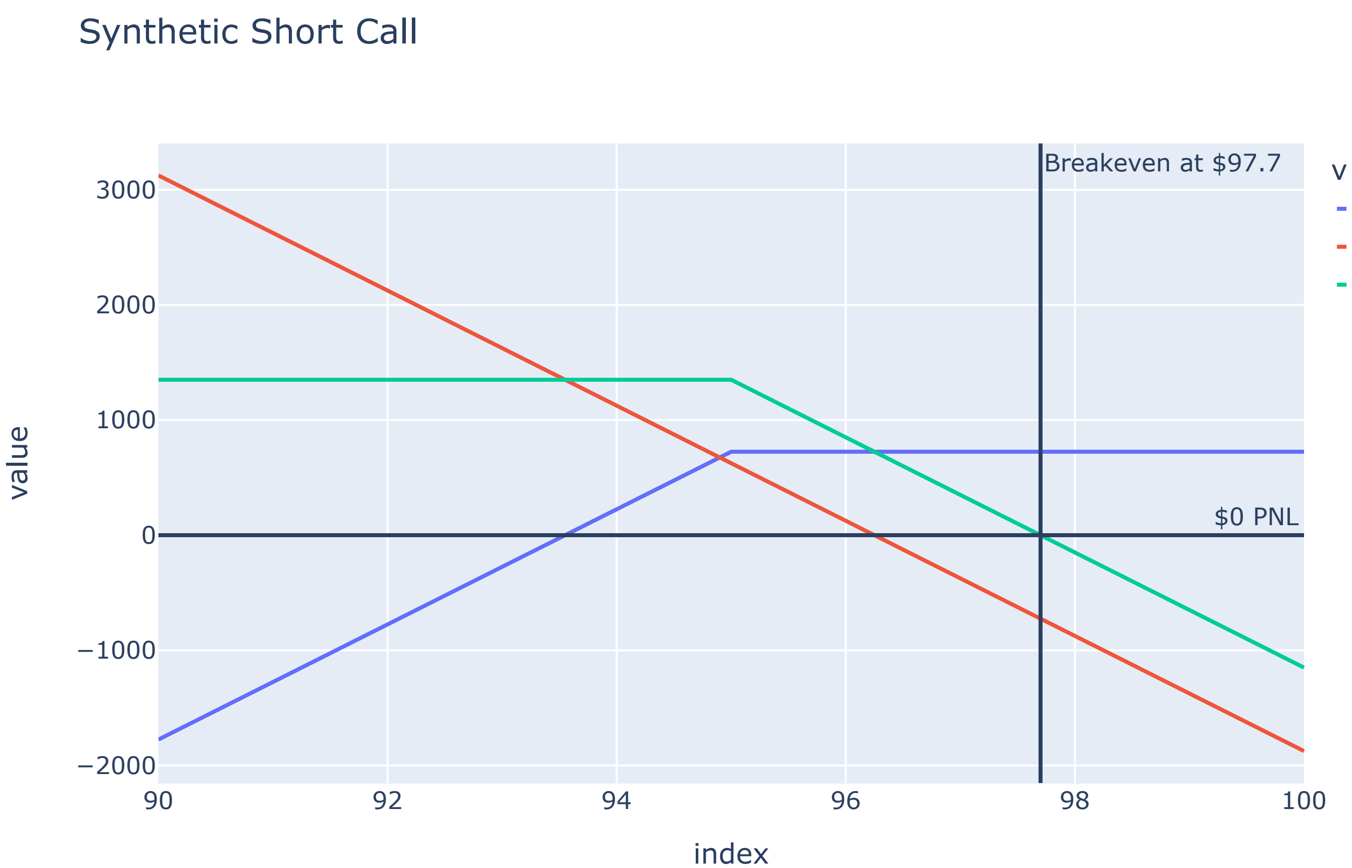
- This chart provides the same payoff of selling 5 puts.
- Thus buying 500 shares of stock and selling 500 calls is equivalent to selling 5 puts with the same strike price.

Short Puts Plus Short Stock

short put plus short stock equals short call

Suppose you sell 500 deltas of at $S_0 = \$96.25$ and simultaneoulsy short $5P(K = 95) = \$1.45$

- Since this put is all *time value* the price of the corresponding call is equal to the put's time value plus the differential between the strike price and spot price. This equals $(96.25 - 95) + 1.45 = \$2.70$.
- Similarly, if you know the price of the ITM call is priced at 2.7, the price of the put with the same strike is equal to the call's **time value** of $\$1.45$.
- This portfolio payoff is exactly equal to $-5C(K = 95) = -\$2.7$



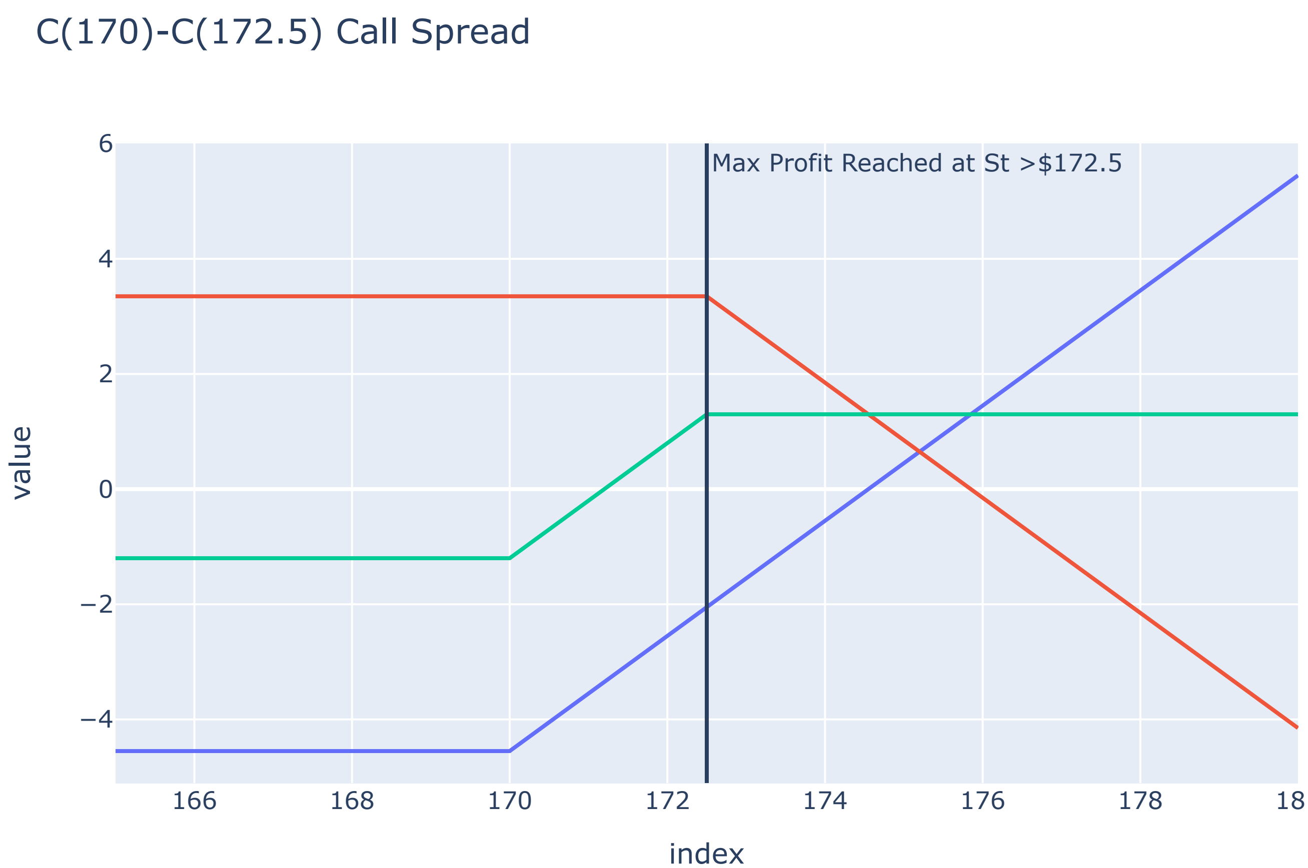
- For $S \in [0, 25]$: $Vp = 5(\Delta_s)$ where $\Delta_s = (S - \$13.012)$. I am long 500 shares for a price of 13.
- For $S \in [25, \text{inf}]$: $Vp = -12(\Delta_s)$ where $\Delta_s = (S - \$29.995)$. I am short 1200 shares for a selling price of nearly 30 dollars.

Bull Call Spread Example

- Remember short the lower strike call option and simultaneoulsy sell the lower call strike

- Consider:

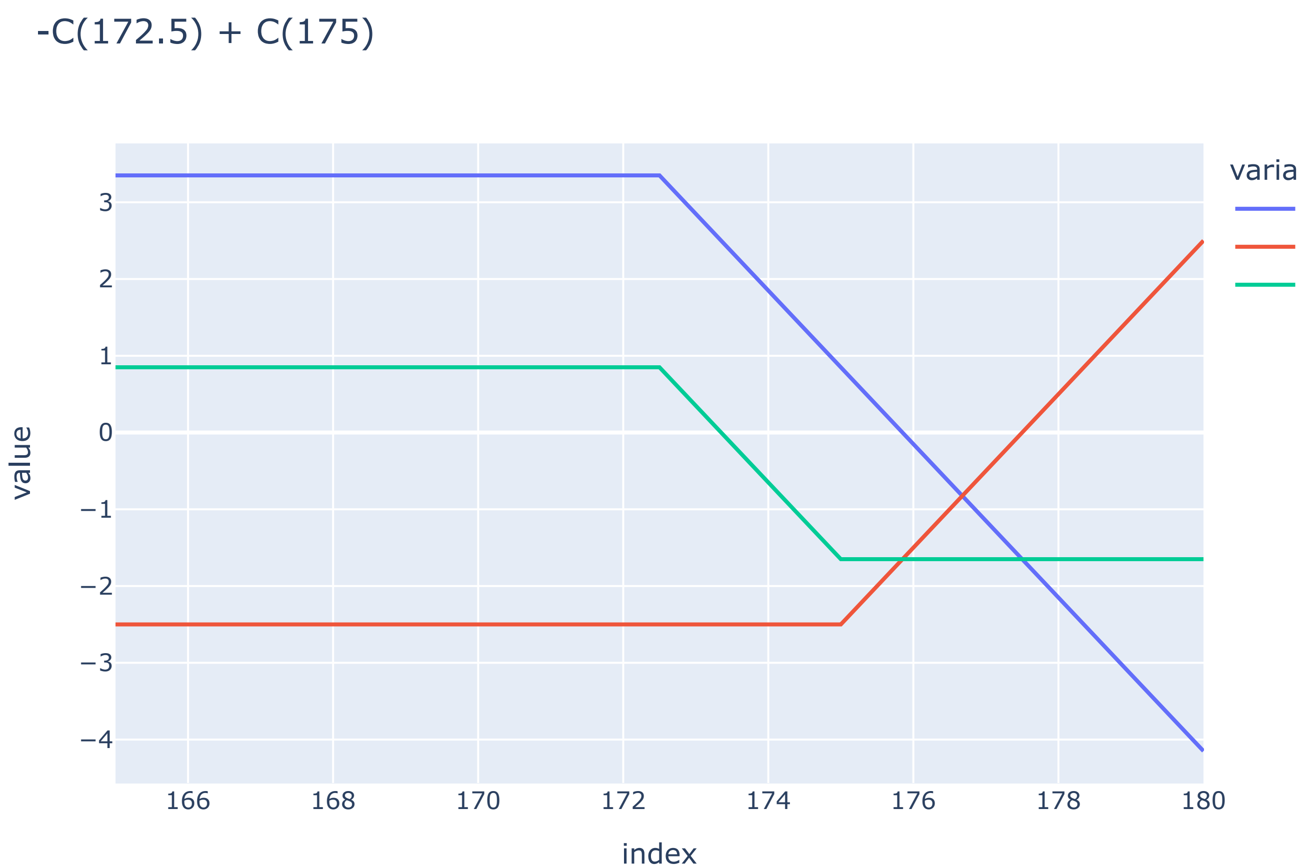
$$V_{p,\tau_0} = C(170) - C(172.5) - 1.20 \tag{11}$$
$$\text{Where } C(170) = \$4.55 \text{ and } C(172.5) = \$3.35 \tag{12}$$



Short Bull Call Spread

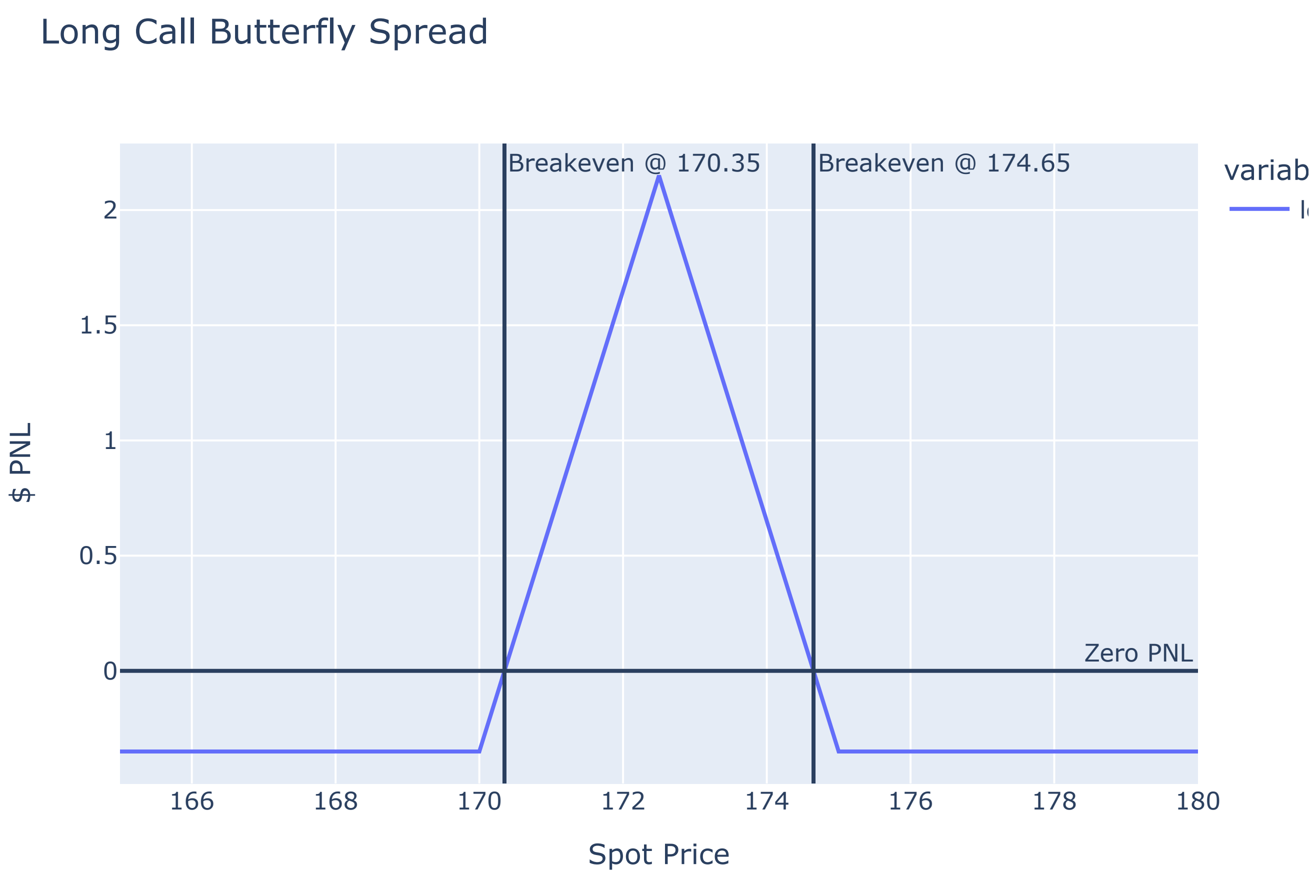
- $C(K = 172.5) = \$3.35$ and $C(K = 175) = \$2.50$.

$$V_{p,\tau_0} = -C(K = 172.5) + C(K = 175) + \$0.85 \tag{13}$$



When you *buy the Sep 170-172.5 call spreads* for $\$1.20$ and *sell an equal number of Sep 175-172.5 call spreads* for $\$0.85$ you have created a **long call butterfly spread.**

- Let's visualize our payoffs.

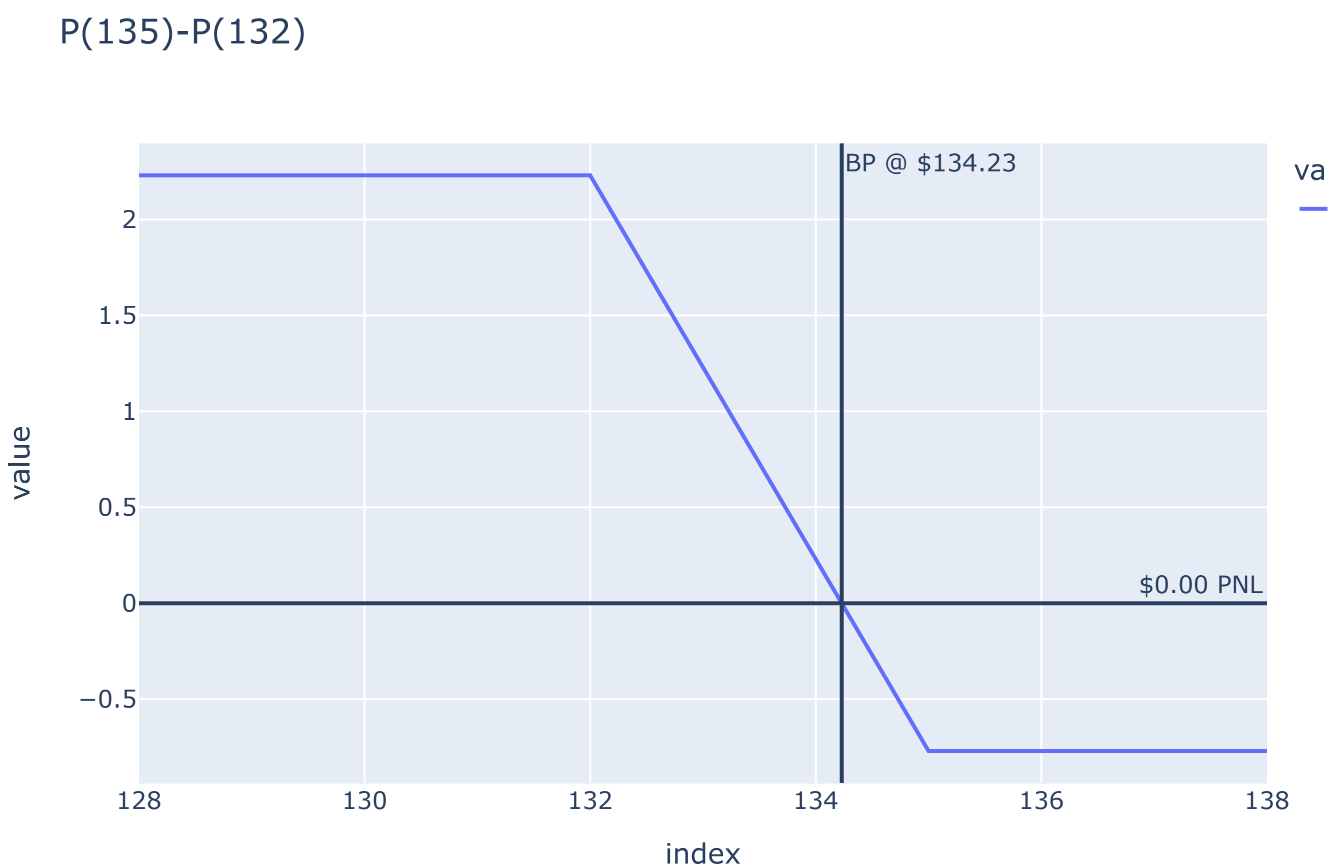


Put-Spread (Bearish Put Vertical Spread)

- The overall delta is negative for Bearish Put Vertical Spreads
- Consider $P(135) = \$1.83$ and $P(132) = \$1.06$

Consider the portfolio:

$$V_{p,\tau_0} = P(135) - P(132) - \$0.77 \tag{14}$$

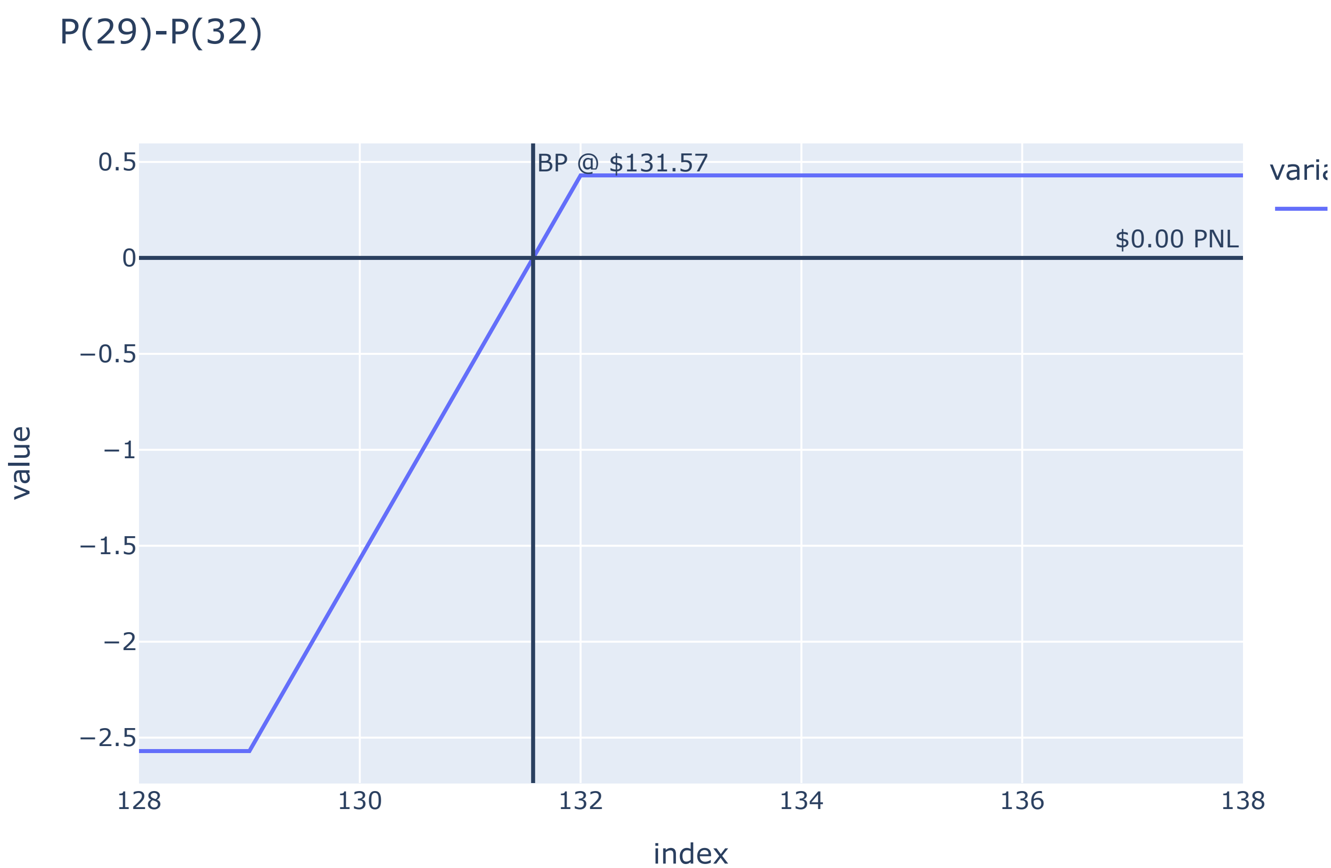


Bull Put-Spread

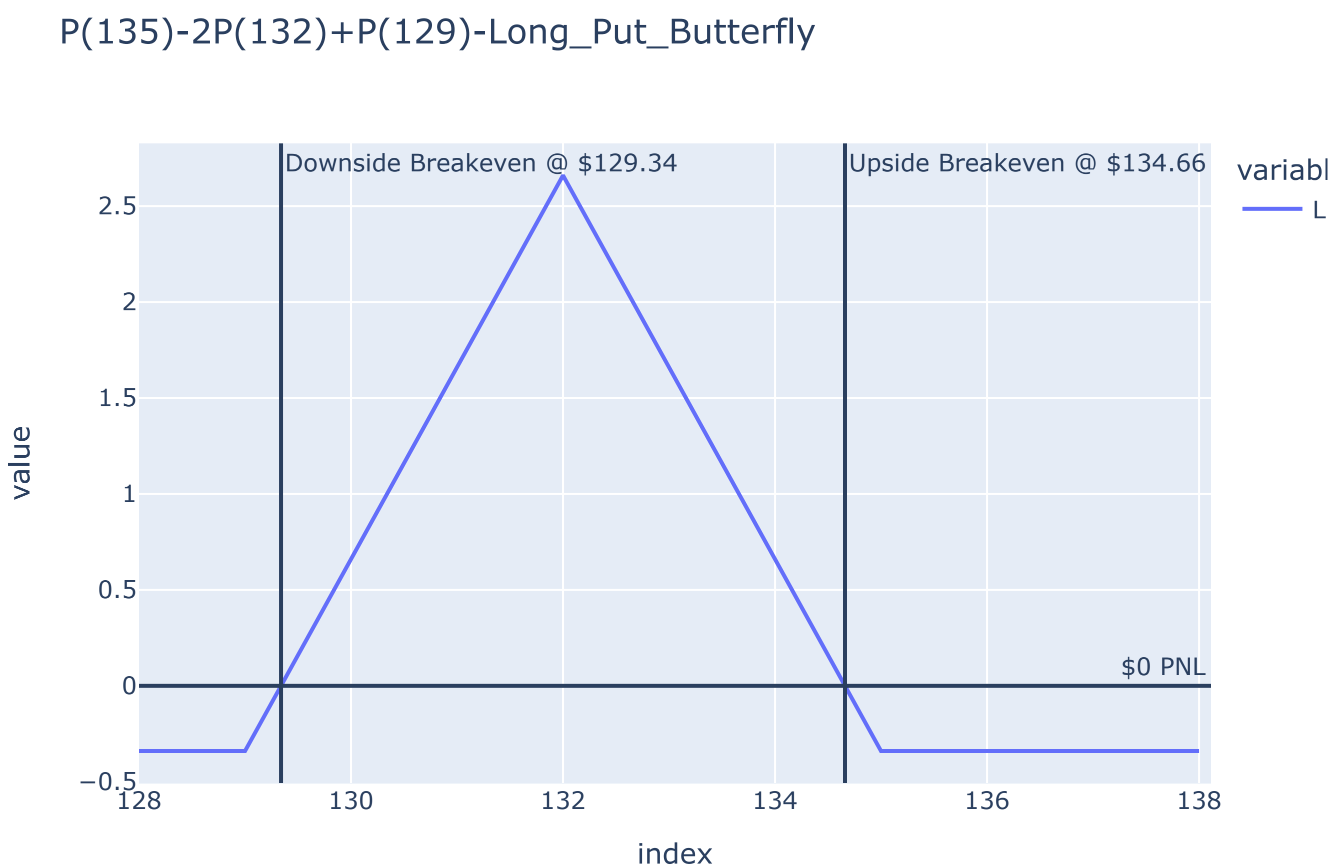
- Essentially, think of creating a portfolio that is $+\Delta$ with two puts.
- Consider $P(129) = \$0.63$ and $P(132) = \$1.06$

Consider the portfolio:

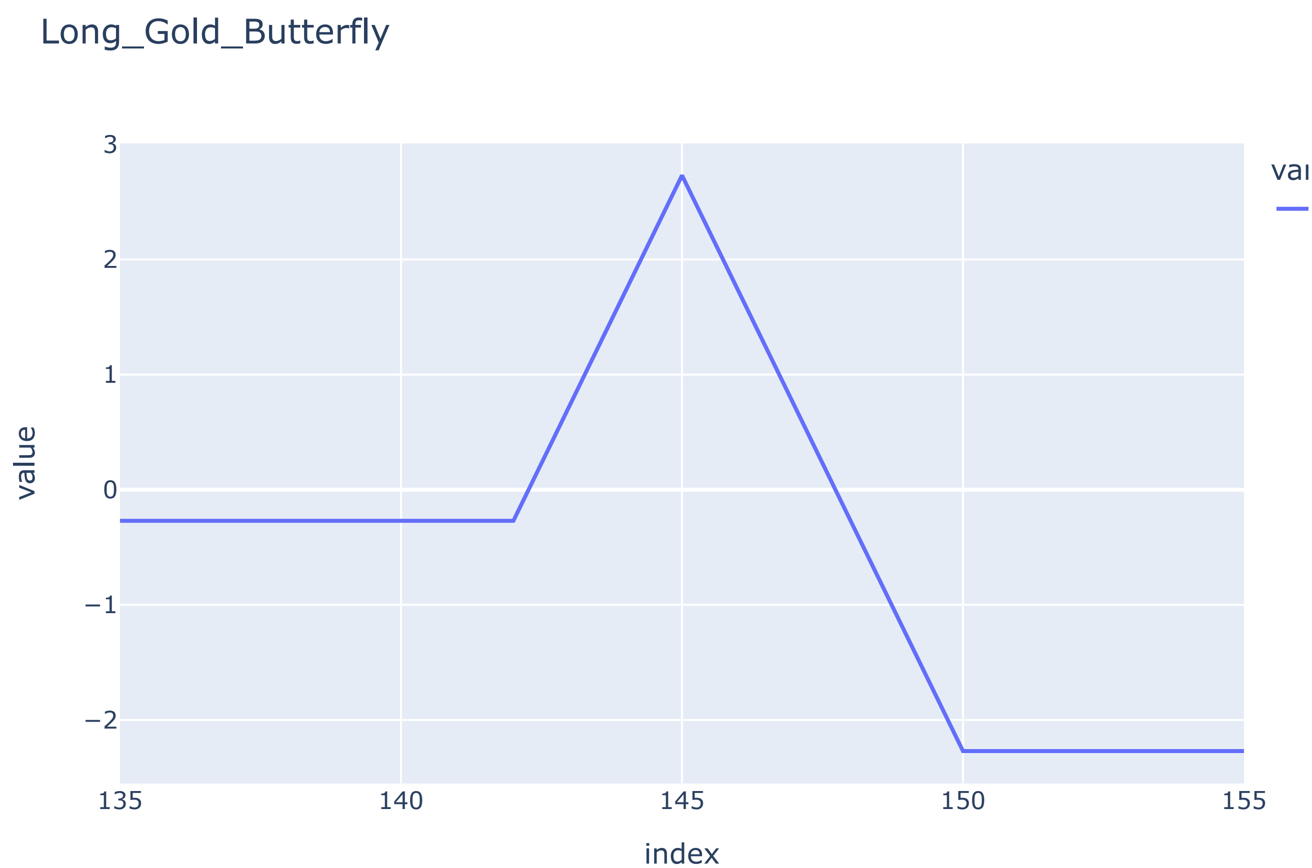
$$V_{p,\tau_0} = P(129) - P(132) + \$0.43 \tag{15}$$



- When you buy the September 135-132 put spread for $\$0.77$ and sell the September 129 -132 put spread for $\$0.43$ you have created a **long put butterfly spread.**
- The net premium is a $\$0.34$ debit.



When you buy the gold 142-145 call spread for $\$1.15$ and sell an equal amount of the the 150-145 call spread for $\$0.88$ you have created a **long split strike call butterfly.**



Strangles

- The simultaneous purchase of *OTM Call* and *OTM Put* is a **strangle.**

Short Strangle

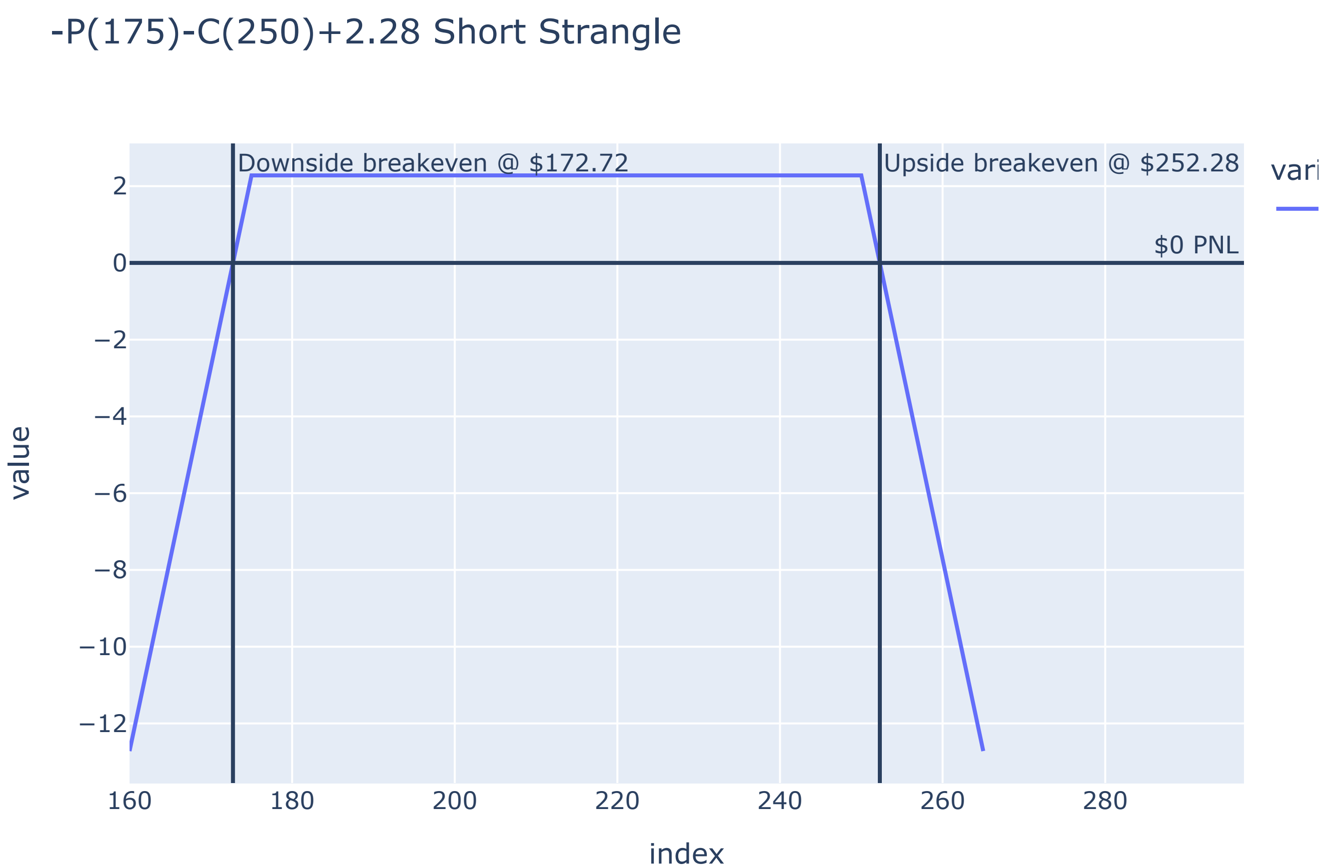
- Consider $P(175) = \$1.60$ and $C(250) = \$0.68$
- $S_0 = \$212.5$

Consider the portfolio:

$$V_{p,\tau_0} = -P(175) - C(250) + \$2.28 \tag{16}$$

Breakeven is at $S_T = \$252.28$ and $S_T = \$172.72$

- Your position benefits with decreasing volatility and decreasing demand for options.



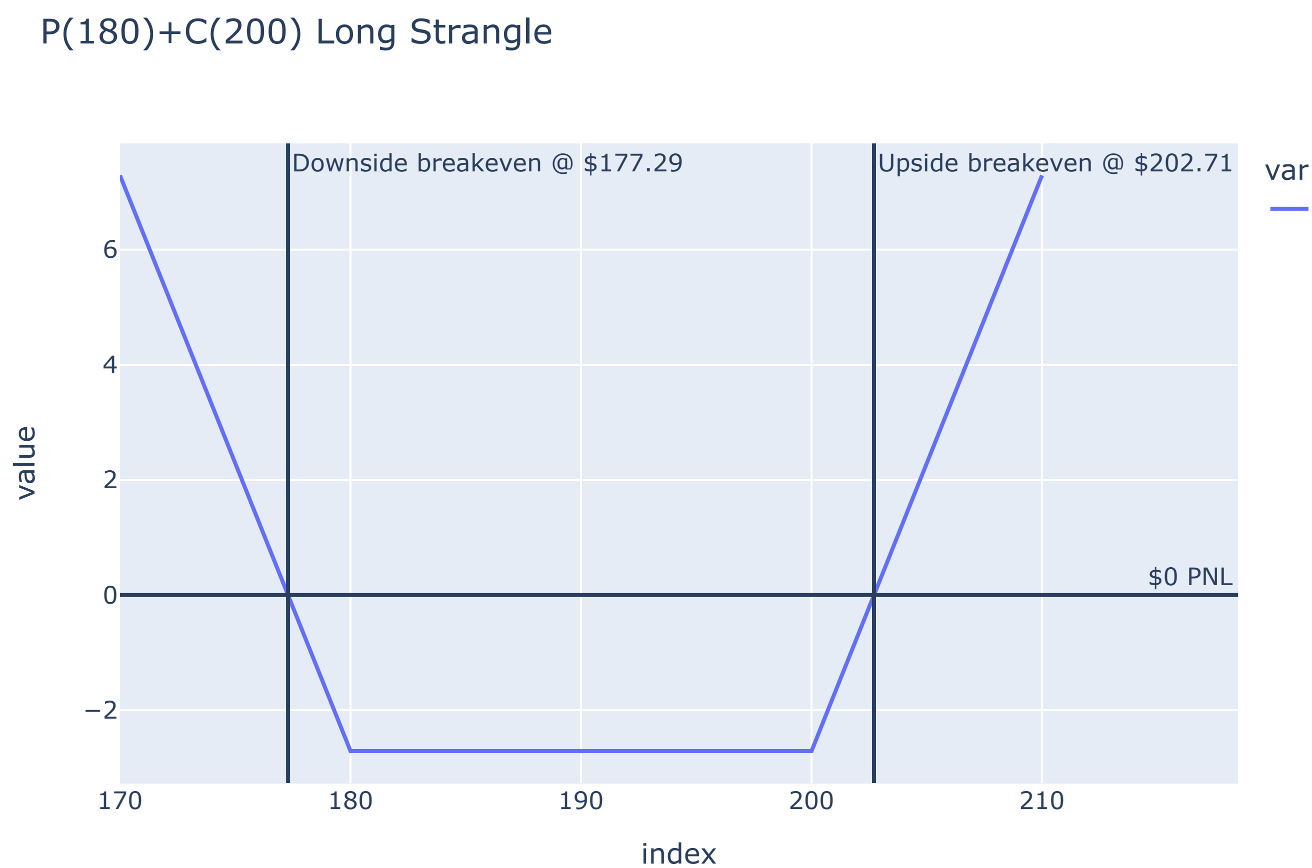
Long Strangle

- Consider $P(180) = \$1.55$ and $C(200) = \$1.16$
- $S_0 = \$212.5$

Consider the portfolio:

$$V_{p,\tau_0} = P(180) + C(200) - \$2.71 \tag{17}$$

Breakeven is at $S_T = \$202.71$ and $S_T = \$177.29$



Long Iron Butterfly

- A *long iron butterfly* = **short straddle + long strangle**

A short straddle as unlimited potential upside losses and massive potential downside losses. When you buy a strangle, you are defining your maximum potential upside and downside losses.

Consider the portfolio:

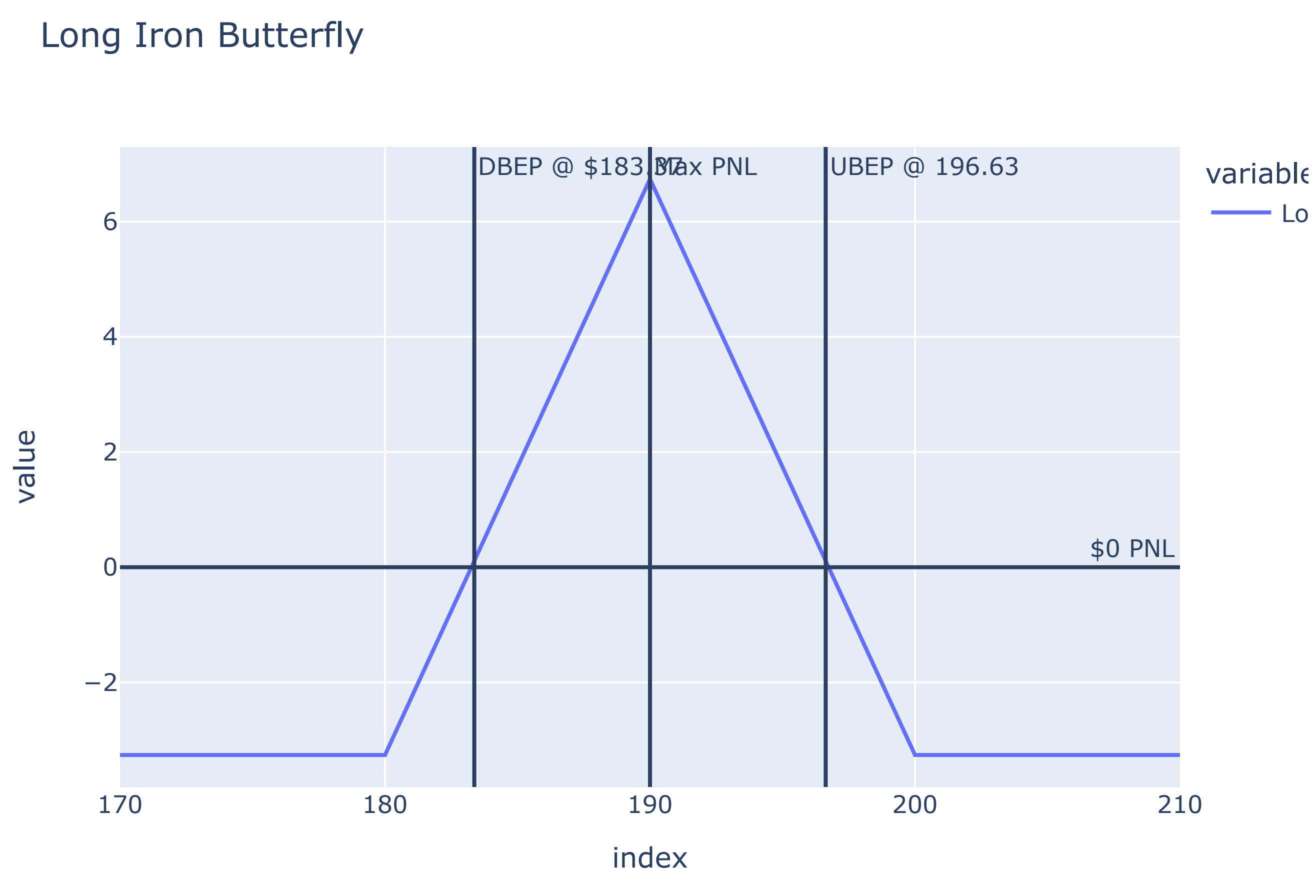
$$V_{p,\tau_0} = P(180) + C(200) - C(190) - P(190) + \$6.74 \tag{18}$$

What would the greeks be of this position?

- You establish a **credit.**

Well since they are all at the same expiration cycle, and if $S_0 = \$190$. You would be **short** *delta*, **short** *gamma*, **short** *vega*, and **long** *theta*.

	Price	Delta	Gamma	Vega	Theta
CALL(K = 190)	-5.644750	-54.097849	-3.044645	-25.861577	10.265074
PUT(K = 190)	-4.752733	45.902151	-3.044645	-25.861577	7.263360
CALL(K = 200)	2.405142	28.107388	2.352242	21.978241	-8.964131
PUT(K = 180)	2.118135	-22.508713	1.917938	19.549426	-7.200408
Net_Position	-5.874207	-2.597024	-1.819111	-10.195487	1.363895



Iron Condor

<u>Example</u>

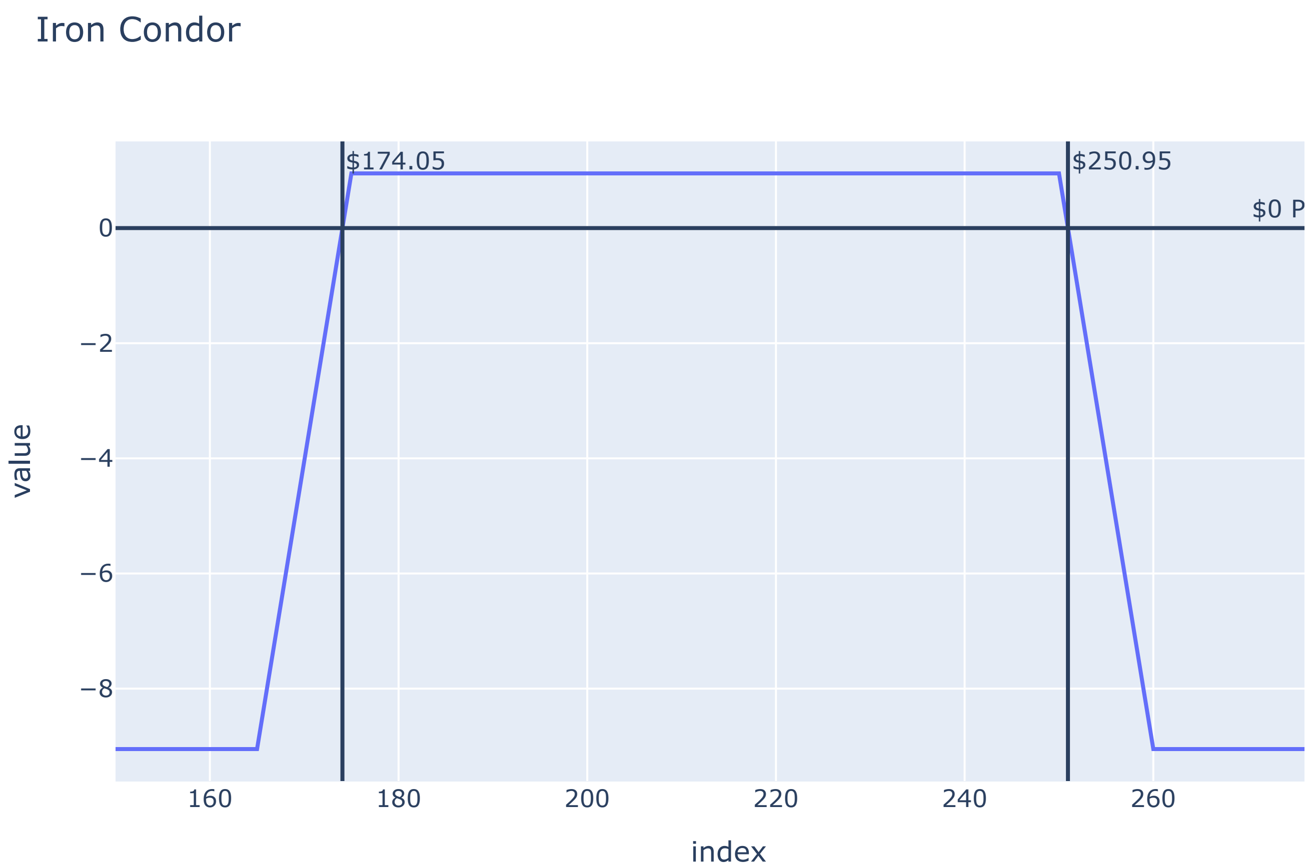
$$V_{p_1,\tau_0} = P(165) + C(260) - \$1.33 \tag{19}$$
$$V_{p_2,\tau_0} = -P(175) - C(250) + \$2.28 \tag{20}$$
$$V_{p,\tau_0} = V_{p_1,\tau_0} + V_{p_2,\tau_0} + \$0.95 \tag{21}$$

- *Iron Condor* = **Long 165-260 strangle** + **short 175-250 call strangle**

- The iron condor is equivalent to a short *OTM vertical put spread* and and short *OTM vertical call spread*.

These spreads have a very high probability of success, but all it takes is one big move in the market and several months of profits can be wiped away.

Overall, this initial position is **short** *delta*, **short** *gamma*, **short** *vega*, and **long** *theta*.



Long Butterfly Spreads

- They can be created in two ways with $K_1 < K_2 < K_3$

$$V_{p,\tau_0} = C(K_1) - 2C(K_2) + C(K_3) \tag{22}$$
$$V_{p,\tau_0} = P(K_1) - 2P(K_2) + P(K_3) \tag{23}$$

1. Long call butterfly layer a higher strike bear spread on top of a lower strike bull spread. The strike price for the short call of the bear spread is the same as the strike price as the short call in the bull spread.

2. Long put butterfly layer a lwoer strike bull spread on top of a higher strike bear spread. The strike price for the short put of the bull spread is the same as the short put in the bear spread.
3. The differential in the strike prices is the same for both vertical spreads.
4. Established for a debit.
5. Debit is maximum possible loss.
6. Maximum value for long call butterfly and long put butterfly is at the **middle strike price**
7. Maximum profit is the maximum value less the debit accrued.
8. ATM long butterfly **wants minimual volatility.**
9. OTM call butterfly is bullish.
10. OTM put butterfly is bearish.

Calendar Spreads

Long Time Value Spreads:

1. Sell option closer to expiration cycle.
2. Buy option in more nearby expiration cycle.
3. Benefit most when stock is near the strike price when nearby option expires.
4. Hurt the most when the stock is far above or below the strike price when nearby option expires.
5. Position morphs into *long call* or *synthetic long call (put spread)* when stock is below strike at the expiration of nearby option.
6. Postion morphs into a *long put* or *synthetic long put (call spread)* when stock is above strike at the expiration of nearby option.
7. Buying OTM call spread is *bullish*.
8. Buying OTM put spread is *bearish*.

	Price	Delta	Gamma	Vega	Theta
CALL(K = 315)	-8.638790	-46.243023	-1.547196	-42.481574	19.130956
CALL(K = 315)	20.791532	52.992962	0.753456	73.700131	-13.946753
Position	12.152741	6.749939	-0.793739	31.218557	5.184203

- Initially, we establish a debit for $\$1,215.27$. We expect the spot price to increase but not by much in 3 months.

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1215.2741289706541
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500.3481610664976
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- If at the end of the first expiration cycle, the spot price is close to $\$315.00$, then our theoretical profits would be about: $\$500.34$ per calendar spread bought.