Option Pricing

The Black-Scholes-Mertion equation for a **call option** with the dynamics:
spot price = S_t
annualized volatility = σ_t
Time-to-maturity = $\tau = T - t$
Risk-free-rate = r
Strike = K

equals the partial differential equation

$$e^{-r(T-t)}\mathbb{E}[(S_T - K|S_t)]^+ = S_t\Phi\Big(\frac{log(\frac{S_t}{K}) + (T-t)(r + \frac{\sigma^2}{2})}{\sigma\sqrt{T-t}}\Big) - Ke^{-r(T-t)}\Phi\Big(\frac{log(\frac{S_t}{K}) + (T-t)(r - \frac{\sigma^2}{2})}{\sigma\sqrt{(T-t)}}\Big)$$

$$C(K) = e^{-r(T-t)}\mathbb{E}(S_T - K|S_t)^+ = S_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$$

Where:

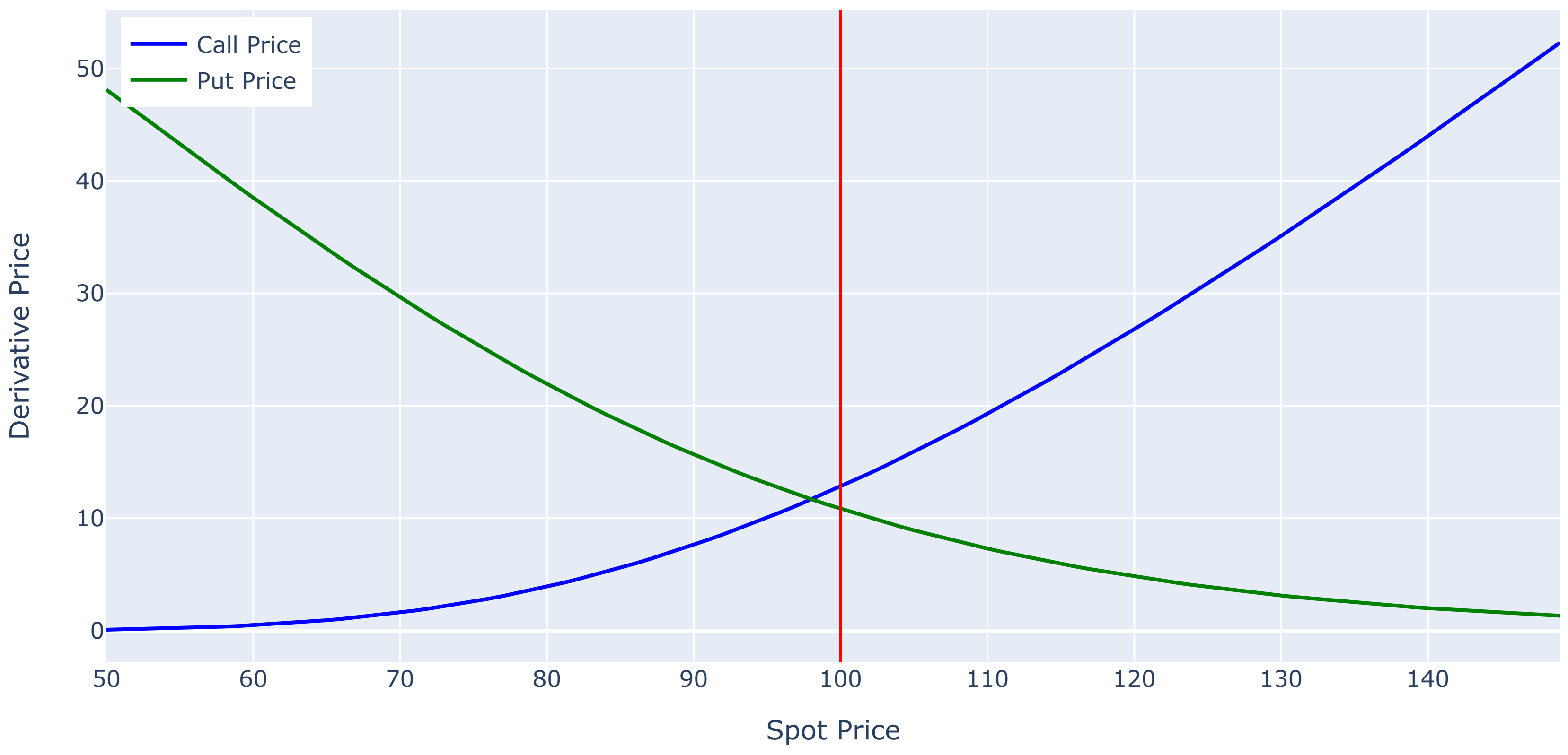
$d_1 = (\frac{log(\frac{S_t}{K})+(T-t)(r+\frac{\sigma^2}{2})}{\sigma\sqrt{T-t}})$ </br> and

$d_2 = (\frac{log(\frac{S_t}{K})+(T-t)(r-\frac{\sigma^2}{2})}{\sigma\sqrt{T-t}}) = d_1 - \sigma\sqrt{\tau}$

Similarly, the Black-Scholes-Merton equation for a **put option** is: </br>

$$P(K) = e^{-r(T-t)}\mathbb{E}(K - S_T|S_t)^+ = Ke^{-r(T-t)}\Phi(-d_2) - S_t\Phi(-d_1)$$

Option Price across Spot



Delta

$\Delta = \frac{\partial V}{\partial S_t}$

- **Delta** measures the change in the option prices per change in the underlying price.

$$\frac{\partial C}{\partial S_t} = \Phi(d1)$$

$$\frac{\partial P}{\partial S_t} = \Phi(-d1) = 1 - \Phi(d1)$$

Delta