Black-Scholes

- There are 5 parameters in Black Scholes
  - S
  - K
  - r
  - σ
  - T
- **Call Option** equals:

$$C(S,t) = e^{-r\tau}\left[S_0 \cdot N(d_1) - K \cdot N(d_2)\right] \quad \text{(1)}$$

- **Put Option** equals:

$$P(S,t) = e^{-r\tau}\left[K \cdot N(-d_2) - S_0 \cdot N(-d_1)\right] \quad \text{(2)}$$

Where

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} \quad \text{(3)}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + (r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} \quad \text{(4)}$$

$$\text{Put Option} = K \cdot e^{-r\tau} \cdot N(-d_2) - S_0 \cdot N(-d_1) \quad \text{(5)}$$

$$\text{Call Option} = S_0 \cdot N(d_1) - K \cdot e^{-r\tau} \cdot N(d_2) \quad \text{(6)}$$

$N(x)$ is the **standard normal cumulative density** function. The probability that $Z \leq x, P(Z \leq x) = \Phi(x) = 0.92$

- In put-call parity for $S_0 = K$, S this means the value of the option is the value of the underlying less the option value **decreases** by $1 - \Delta$, Δ increases by Φ, then the price of the put option theoretically declines by Φ (for a NEW contract)

Decreases as the option does more like the delta of a call and puts prices change with respect to their put counts.

- As an approximation need to buy Δ shares of stock while simultaneously short Δ of those calls.

```
STOCK: $100, STRIKE: $100
STOCK:$100, EXTERNAL: $-0.0000001
```

- The model PRICES, $S_0 = 100$, that I want to buy a lower contribution of calls and the underlying such that my portfolio's **delta neutral**. This is known generally as the **Greeks** parameters.
- The $C(S)$ has the following partial:

Price	Delta	Gamma	Vega	Theta
$C(S) = 100$	0.5000	0.0199	0.3989	-0.0499

$$\frac{\partial C(S)}{\partial S} = N(d_1) \quad \text{(7)}$$

$$\frac{\partial^2 C(S)}{\partial S^2} = \frac{N'(d_1)}{S_0 \cdot \sigma\sqrt{\tau}} \quad \text{(8)}$$

$$\frac{\partial C(S)}{\partial \sigma} = S_0 \cdot N'(d_1) \cdot \sqrt{\tau} \quad \text{(9)}$$

$$\frac{\partial C(S)}{\partial \tau} = -\frac{S_0 \cdot N'(d_1) \cdot \sigma}{2\sqrt{\tau}} \quad \text{(10)}$$

- Solving the system we find the unknown

$$\Delta_{call} = 100 \cdot 0.5000$$
$$\Delta_{put} = 100 \cdot (0.5000 - 1)$$

- So we approximately need to buy 50 shares of stock while simultaneously short 0 of those calls.

```
STOCK:$ 0.00000000, $-0.0000001
STOCK:$100, EXTERNAL: $-0.0000001
```

- The model PRICES, $S_0 = 100$, that I want to buy a lower contribution of calls and the underlying such that my portfolio's **delta neutral**. This is known generally as the **Greeks** parameters.
- Put **Delta** has the following partial:
- Put **Gamma** has the following partial:

Price	Delta	Gamma	Vega	Theta
$C(S) = 100$	0.0000	0.0199	0.3989	-0.0499

$$\frac{\partial C(S)}{\partial S} = N(d_1) - 1 \quad \text{(11)}$$

$$\frac{\partial^2 C(S)}{\partial S^2} = \frac{N'(d_1)}{S_0 \cdot \sigma\sqrt{\tau}} \quad \text{(12)}$$

- Solving the system we find the unknown

$$\Delta_{call} = 100 \cdot 0.0000$$
$$\Delta_{put} = 100 \cdot (0.0000 - 1)$$

- So we approximately need to buy 100 shares of stock while simultaneously short 0 of those calls.

The Greeks

Delta Δ

- $\Delta = \frac{\partial V}{\partial S}$
  - **Delta** measures the change in the option price per change in the underlying price. It is the slope of the curve option curve with respect to the price.
  - The delta of a call option always positive while the delta of a put option is always negative.

$$\text{Call Option} = N(d_1) \quad \text{(13)}$$

$$\text{Put Option} = -N(-d_1) = N(d_1) - 1 \quad \text{(14)}$$

Gamma Γ

- If buy $N(d_1)$ DELTA, $N(d_2)$ DELTA and want to buy a lower contribution of calls and the underlying such that my portfolio's **delta neutral**. This is known generally as the **Gamma** parameter.
- Put **Gamma** has the following partial:

Price	Delta	Gamma	Vega	Theta
$C(S) = 100$	0.5000	0.0199	0.3989	-0.0499

$$\frac{\partial^2 C(S)}{\partial S^2} = \frac{N'(d_1)}{S_0 \cdot \sigma\sqrt{\tau}} \quad \text{(15)}$$

- Solving the system we find the unknown

$$\Delta_{call} = 100 \cdot 0.5000$$
$$\Delta_{put} = 100 \cdot (0.5000 - 1)$$

- So we approximately need to buy 50 shares of stock while simultaneously short 0 of those calls.

Δ wrt. σ_{τ}

$$\frac{\partial C(S)}{\partial \sigma} = S_0 \cdot N'(d_1) \cdot \sqrt{\tau} \quad \text{(16)}$$

$$\frac{\partial C(S)}{\partial \tau} = -\frac{S_0 \cdot N'(d_1) \cdot \sigma}{2\sqrt{\tau}} \quad \text{(17)}$$

- **STRIKE DETAILS Derivatives**
  - $K = S_0$

$$d_1 = \frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} = \frac{(r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} \quad \text{(18)}$$

$$d_2 = \frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} = \frac{(r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} \quad \text{(19)}$$

- For **Put** on τ

$$\frac{\partial P}{\partial \tau} = -\frac{S_0 \cdot N'(d_1) \cdot \sigma}{2\sqrt{\tau}} \quad \text{(20)}$$

- For **Call** on τ

$$C_t = S_0 N(d_1) - Ke^{-r\tau} N(d_2) \quad \text{(21)}$$

$$\frac{\partial C_t}{\partial \tau} = -\frac{S_0 \cdot N'(d_1) \cdot \sigma}{2\sqrt{\tau}} \quad \text{(22)}$$

- very Symmetric intervals remains 0, 2000
- Non-convex-concave-convex-concave-convex-convex-convex

What if we want to hedge Γ too?

- Then we need to add in the market equation because the **fundamental theorem of asset pricing** states that in order to dynamically hedge three parameters we need three assets to make the market complete and arb-free.

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CALCULATE RISKS
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- We need to short 250,000 Γ shares of stock, buy 2142.116 call options, and sell 2384.807 put options.