homework_2

August 24, 2022

0.1 Homework 2 and Financial Math Practice

[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
import sklearn
import datetime
import yfinance as yf
import scipy as scs
import cufflinks as cf
import plotly.offline as plyo
import seaborn as sns
from sklearn.linear_model import LinearRegression
import matplotlib.dates as mdates
from matplotlib.dates import DateFormatter
```

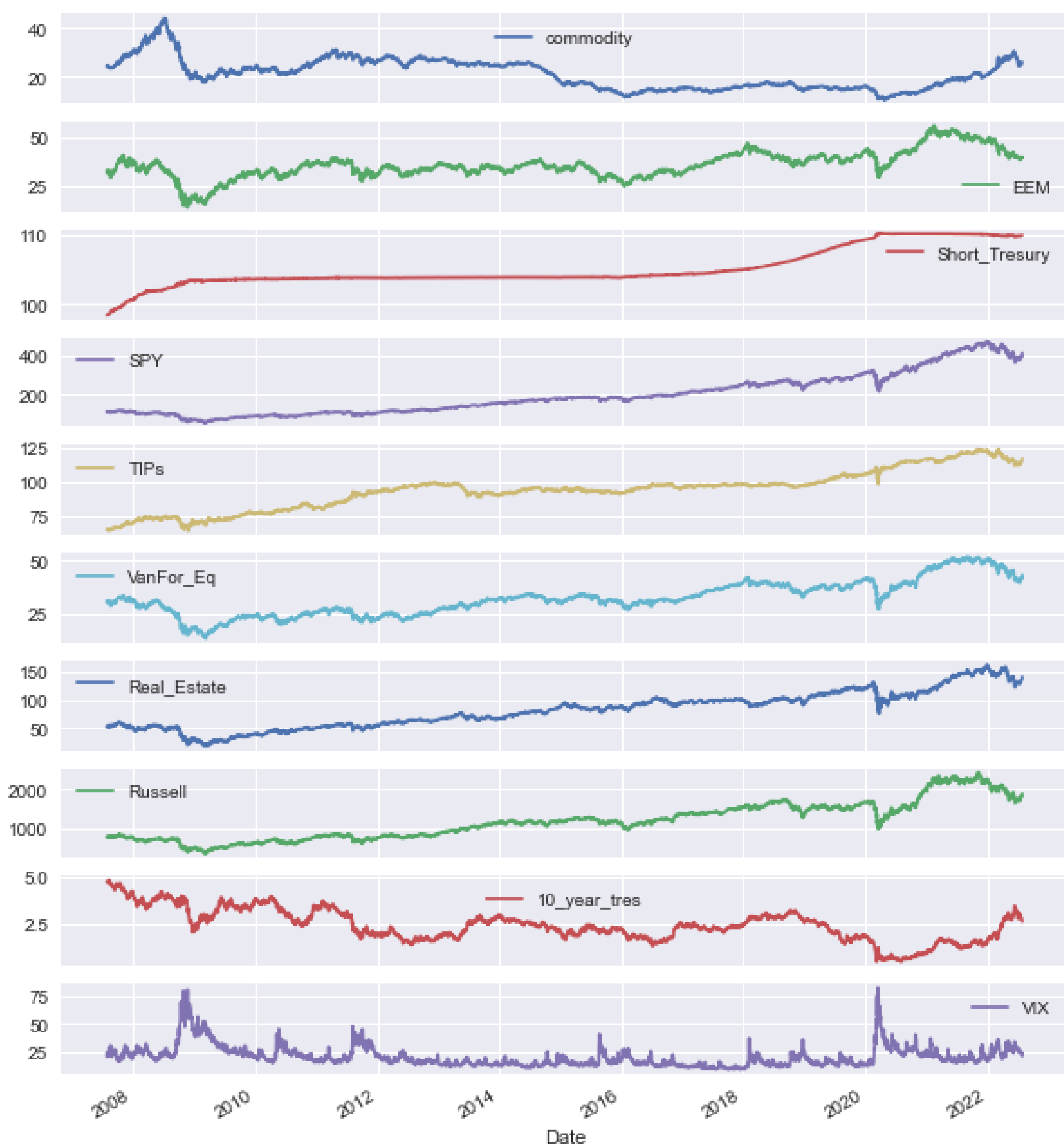
```
c:\Users\dcste\Anaconda\lib\site-packages\scipy\__init__.py:146: UserWarning: A
NumPy version >=1.16.5 and <1.23.0 is required for this version of SciPy
(detected version 1.23.1
  warnings.warn(f"A NumPy version >={np_minversion} and <{np_maxversion}"
```

[ ]:
```
tickers = "SPY EEM TIP ^RUT ^TNX DBC VGSLX VEA SHV ^VIX"
#Download adjusted close numbers
Start = "2005-12-31"
End = "2022-07-31"
adj_close = yf.download(tickers, start = "2005-12-31", end = "2022-7-31")["Adj␣
↪Close"]
adj_close.dropna(inplace = True)
adj_close.columns =␣
↪["commodity","EEM","Short_Tresury","SPY","TIPs","VanFor_Eq","Real_Estate","Russell","10_yea
↪"VIX"]
```

```
[*********************100%***********************]  10 of 10 completed
```

[ ]:
```
# Plot Prices
plt.style.use("seaborn")
adj_close.plot(figsize = (10,12), subplots = True)
```

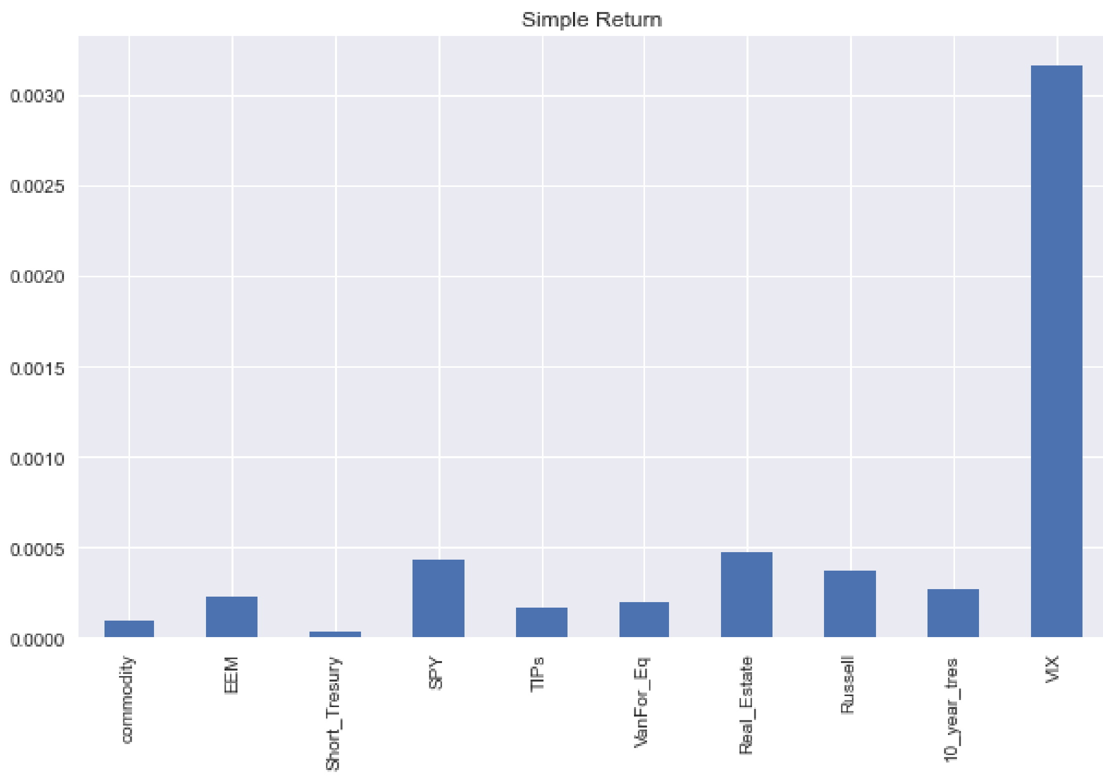
[ ]: array([<AxesSubplot:xlabel='Date'>, <AxesSubplot:xlabel='Date'>,
       <AxesSubplot:xlabel='Date'>, <AxesSubplot:xlabel='Date'>,
       <AxesSubplot:xlabel='Date'>, <AxesSubplot:xlabel='Date'>,
       <AxesSubplot:xlabel='Date'>, <AxesSubplot:xlabel='Date'>,
       <AxesSubplot:xlabel='Date'>, <AxesSubplot:xlabel='Date'>],
      dtype=object)



[ ]:
```
#Create a Data Frame that calculates the daily return

returns = adj_close.pct_change().round(4).dropna()
returns.mean().plot(kind = 'bar', figsize = (10,6))
```

```
plt.title("Simple Return")
```

[ ]: Text(0.5, 1.0, 'Simple Return')



[ ]:
```
#Compute Log-Returns
rets_log = np.log(adj_close/adj_close.shift(1))
plyo.iplot(
    rets_log.iplot(kind = 'hist',
                   subplots = True,
                   bins = 225,
                   asFigure= True),
    image = 'png',
    filename = 'hist_log_rets'
)
```

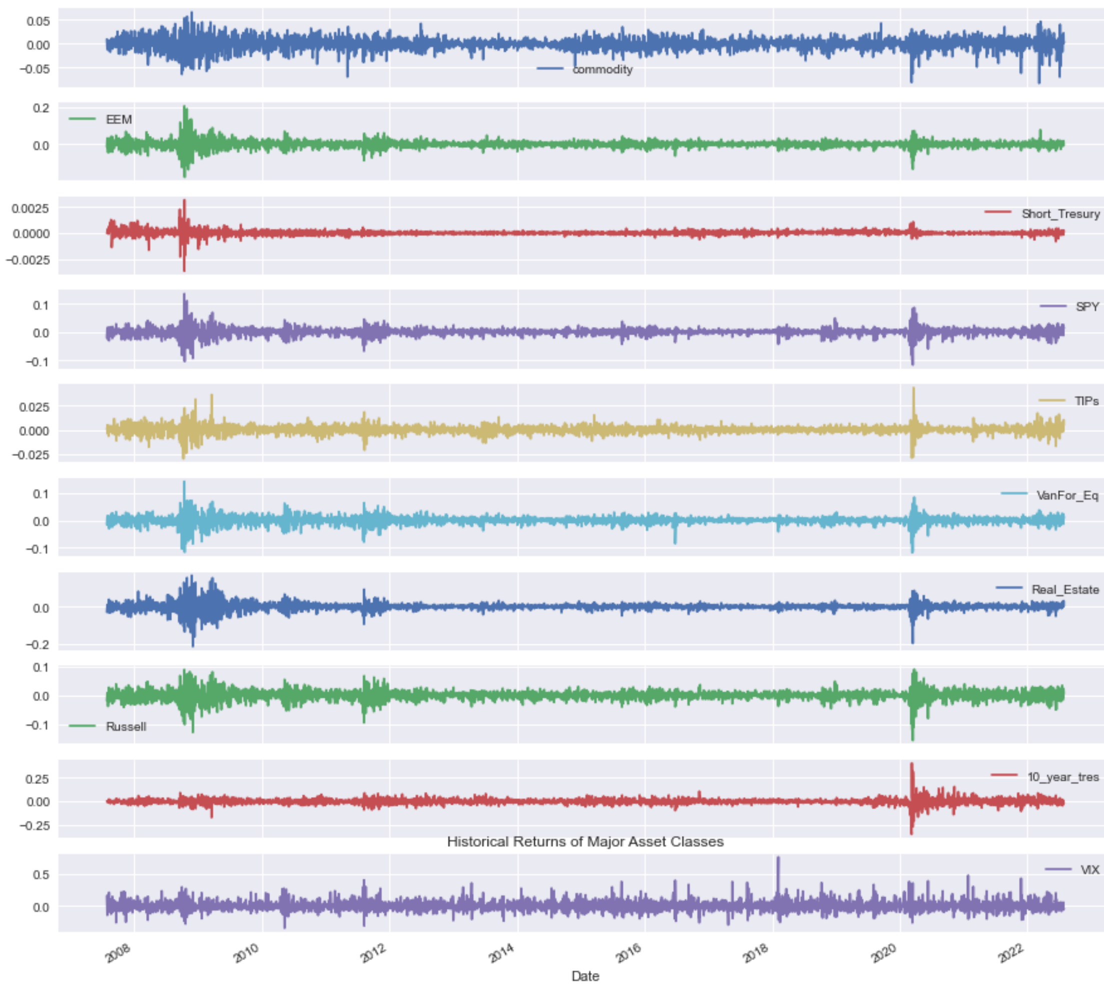
0.1.1 Computing the Summary Statistics of the log-returns

[ ]:
```
rets_log.describe().round(6)
plt.style.use('seaborn')
fig, ax = plt.subplots(figsize = (14,14))
plt.title("SPY Historical Log Returns")
ax.plot(rets_log["SPY"], c = "green", label = "SPY", linewidth = 3)
```

```python
ax.plot(rets_log["commodity"], c = "red", label = "Commodity Index")
plt.legend(shadow = True, loc = "upper right")
plt.xlabel("Date")
plt.ylabel("Returns")
plt.show()
```

[ ]:
```python
plt.style.use("seaborn")
rets_log.plot(subplots = True, figsize = (15,15))
plt.title("Historical Returns of Major Asset Classes")
```

[ ]: Text(0.5, 1.0, 'Historical Returns of Major Asset Classes')



- Now replicate SPY Returns as a linear combination of the other asset classes

0.2 UChicago Homework Practice

- Use the data provided which has return data from various asset classes.

0.2.1 Correlation

- First download the data from excel and display a matrix of the returns

Which pair has the highest correlation? Which pair has the smallest (most negative) correlation?

[ ]:
```
## Download the Data

file_path = "C:/Users/dcste/OneDrive/Uchicago_practice/multi_asset_etf_data.
↪xlsx"
info = pd.read_excel(file_path, sheet_name = "descriptions")
info.rename(columns = {"Unnamed: 0": "Symbol"}, inplace = True)
```

```python
info.set_index("Symbol", inplace = True)

rets = pd.read_excel(file_path, sheet_name = "total returns")
rets.set_index("Date", inplace =True)
rets.drop(columns = "SHV")

rets_excess = pd.read_excel(file_path, sheet_name = "excess returns")
rets_excess.set_index("Date", inplace = True)
# Sort columns by order of description

symbol_list = info.index.drop("SHV")
rets = rets[symbol_list]
rets_excess = rets_excess[symbol_list]

rets.head()
```

[ ]:

```
                 SPY       EFA       EEM       PSP       QAI       HYG  \
Date
2009-04-30  0.099346  0.115190  0.155583  0.230202  0.022882  0.138460
2009-05-31  0.058454  0.131918  0.159400  0.053893  0.027865  0.028555
2009-06-30 -0.000655 -0.014049 -0.022495  0.045449 -0.003437  0.033517
2009-07-31  0.074606  0.100415  0.110146  0.143247  0.015326  0.069190
2009-08-31  0.036940  0.045031 -0.013136  0.033413 -0.004151 -0.016969

                 DBC       IYR       IEF       BWX       TIP
Date
2009-04-30 -0.001000  0.296151 -0.027452  0.008993 -0.017951
2009-05-31  0.162663  0.022727 -0.020773  0.053672  0.019966
2009-06-30 -0.026259 -0.024863 -0.005571  0.005148  0.001982
2009-07-31  0.018568  0.105799  0.008316  0.031285  0.000879
2009-08-31 -0.040365  0.131939  0.007634  0.007628  0.008413
```

[ ]:

```python
info
```

[ ]:

```
                                         ETF Description
Symbol
SPY                          Domestic Equity SPDR S&P500
EFA                          Foreign Equity iShares EAFE
EEM                             iShares Emerging Markets
PSP          Private Equity Invesco Global Private Equity
QAI                          Absolute Return IQ Multi-Strat
HYG           High Yield iShares High Yield Corporate Bond
DBC                Invesco DB Commodity Index Tracking Fund
IYR                    Real Estate iShares US Real Estate
IEF               Domestic Bonds iShares 7-10 Year Treasury
BWX      Foreign Bonds SPDR Bloomberg Barclay Internati…
TIP                     Inflation-Indexed iShares TIPS Bond
```

```
SHV                           iShares Short Treasury Bond
```

[ ]:
```
rets
```

[ ]:
```
                 SPY       EFA       EEM       PSP       QAI       HYG  \
Date
2009-04-30  0.099346  0.115190  0.155583  0.230202  0.022882  0.138460
2009-05-31  0.058454  0.131918  0.159400  0.053893  0.027865  0.028555
2009-06-30 -0.000655 -0.014049 -0.022495  0.045449 -0.003437  0.033517
2009-07-31  0.074606  0.100415  0.110146  0.143247  0.015326  0.069190
2009-08-31  0.036940  0.045031 -0.013136  0.033413 -0.004151 -0.016969
…                …         …         …         …         …         …
2022-01-31 -0.052741 -0.036350 -0.000205 -0.086028 -0.020761 -0.026549
2022-02-28 -0.029517 -0.034292 -0.043202 -0.073602 -0.006746 -0.008590
2022-03-31  0.037590  0.005190 -0.033811 -0.007721 -0.002587 -0.012871
2022-04-30 -0.087769 -0.067391 -0.061351 -0.125679 -0.033398 -0.041803
2022-05-31 -0.054296 -0.012675 -0.029259 -0.043478 -0.016437 -0.022563

                 DBC       IYR       IEF       BWX       TIP
Date
2009-04-30 -0.001000  0.296151 -0.027452  0.008993 -0.017951
2009-05-31  0.162663  0.022727 -0.020773  0.053672  0.019966
2009-06-30 -0.026259 -0.024863 -0.005571  0.005148  0.001982
2009-07-31  0.018568  0.105799  0.008316  0.031285  0.000879
2009-08-31 -0.040365  0.131939  0.007634  0.007628  0.008413
…                …         …         …         …         …
2022-01-31  0.078922 -0.082314 -0.021130 -0.026176 -0.020588
2022-02-28  0.064674 -0.045881 -0.003041 -0.011197  0.008557
2022-03-31  0.091747  0.068646 -0.040609 -0.042240 -0.018755
2022-04-30  0.056408 -0.041305 -0.042283 -0.069696 -0.021830
2022-05-31  0.015619 -0.085494  0.011248  0.000496 -0.012753

[158 rows x 11 columns]
```

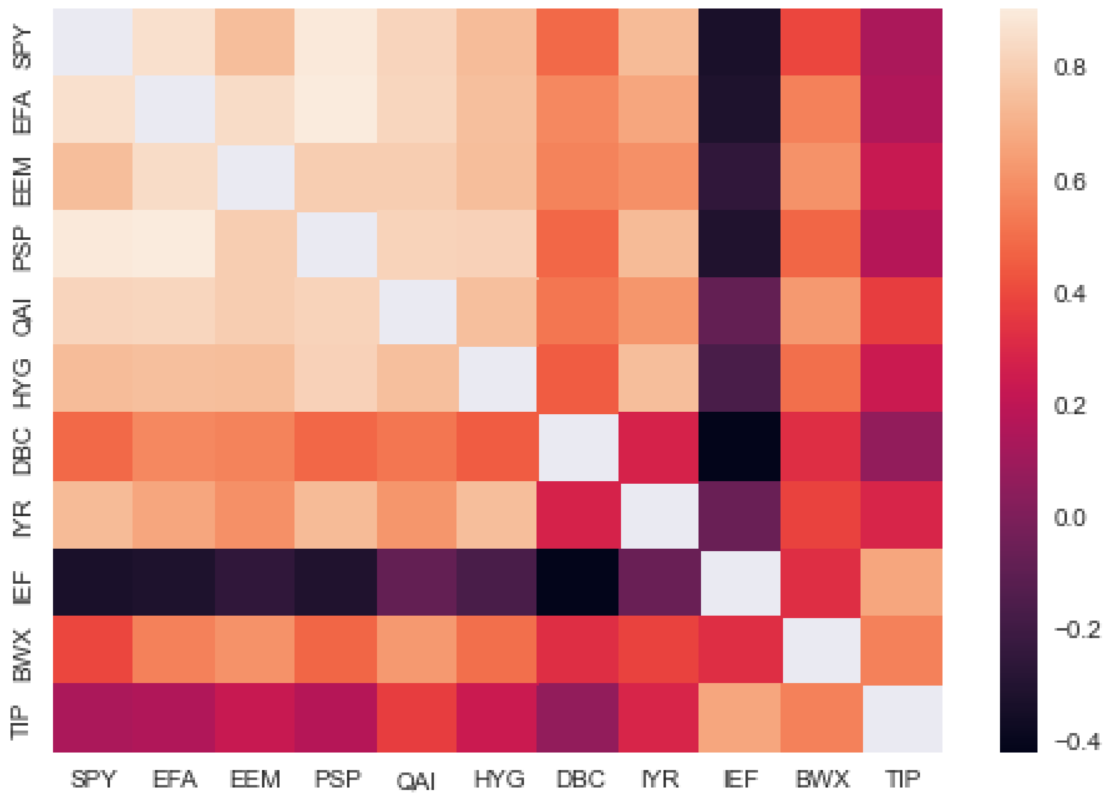
[ ]:
```
rets_excess
```

[ ]:
```
                 SPY       EFA       EEM       PSP       QAI       HYG  \
Date
2009-04-30  0.098793  0.114637  0.155029  0.229649  0.022329  0.137907
2009-05-31  0.058925  0.132390  0.159872  0.054364  0.028337  0.029027
2009-06-30 -0.001255 -0.014649 -0.023094  0.044849 -0.004036  0.032918
2009-07-31  0.074633  0.100443  0.110173  0.143274  0.015353  0.069218
2009-08-31  0.036504  0.044595 -0.013572  0.032977 -0.004587 -0.017405
…                …         …         …         …         …         …
2022-01-31 -0.051926 -0.035535  0.000610 -0.085213 -0.019946 -0.025734
2022-02-28 -0.029154 -0.033929 -0.042840 -0.073240 -0.006383 -0.008228
2022-03-31  0.037953  0.005552 -0.033449 -0.007359 -0.002225 -0.012509
```

```
2022-04-30 -0.087479 -0.067101 -0.061061 -0.125388 -0.033108 -0.041513
2022-05-31 -0.054868 -0.013247 -0.029831 -0.044050 -0.017009 -0.023135

                 DBC       IYR       IEF       BWX       TIP
Date
2009-04-30 -0.001553  0.295598 -0.028005  0.008440 -0.018504
2009-05-31  0.163134  0.023199 -0.020301  0.054144  0.020438
2009-06-30 -0.026858 -0.025462 -0.006171  0.004549  0.001383
2009-07-31  0.018595  0.105827  0.008344  0.031312  0.000907
2009-08-31 -0.040800  0.131503  0.007198  0.007192  0.007977
…                …         …         …         …         …
2022-01-31  0.079737 -0.081499 -0.020315 -0.025361 -0.019773
2022-02-28  0.065037 -0.045518 -0.002678 -0.010834  0.008919
2022-03-31  0.092110  0.069009 -0.040247 -0.041877 -0.018393
2022-04-30  0.056699 -0.041014 -0.041992 -0.069406 -0.021540
2022-05-31  0.015047 -0.086066  0.010676 -0.000076 -0.013325

[158 rows x 11 columns]
```

[ ]:
```python
# Calculating the correlation

corrmat = rets.corr()
corrmat[corrmat == 1] = None
sns.heatmap(corrmat)

corr_rank = corrmat.unstack().sort_values().dropna()
print(f"The smallest correlation is {corr_rank.index[-1]} and the highest␣
 ↪correlation is {corr_rank.index[0]}")
```

```
The smallest correlation is ('PSP', 'EFA') and the highest correlation is
('DBC', 'IEF')
```



0.2.2 Multivariate Regression

Suppose we want to decompose PSP into a linear combination of the other asset classes. * PSP is a benchmark of private equity returns. * Estimate the equation:

```
$r_{t}^{PSP} = \alpha + \beta r_{t} + e$

* Where **rt** denotes the vector of all other returns (exclusing PSP) at time **t.**
```

- Report the estimated alpha, betas, and the r-squared

[ ]:
```
y = rets["PSP"]
X_full = rets.drop(columns=["PSP"])


mod_1 = LinearRegression(fit_intercept = True).fit(X_full, y)
mod_1.coef_
estimation = sm.OLS(y, sm.add_constant(X_full)).fit()
estimation.summary()
```

[ ]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    PSP   R-squared:                       0.898
Model:                            OLS   Adj. R-squared:                  0.891
Method:                 Least Squares   F-statistic:                     129.4
Date:                Wed, 24 Aug 2022   Prob (F-statistic):           1.24e-67
Time:                        13:04:17   Log-Likelihood:                 394.20
No. Observations:                 158   AIC:                            -766.4
Df Residuals:                     147   BIC:                            -732.7
Df Model:                          10
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0020      0.002     -1.043      0.299      -0.006       0.002
SPY            0.4408      0.109      4.041      0.000       0.225       0.656
EFA            0.5755      0.100      5.741      0.000       0.377       0.774
EEM           -0.0213      0.066     -0.320      0.749      -0.153       0.110
QAI            0.2511      0.299      0.840      0.402      -0.340       0.842
HYG            0.5281      0.128      4.114      0.000       0.274       0.782
DBC           -0.1185      0.046     -2.584      0.011      -0.209      -0.028
IYR            0.0650      0.056      1.152      0.251      -0.047       0.177
IEF           -0.4933      0.208     -2.366      0.019      -0.905      -0.081
BWX            0.0469      0.132      0.355      0.723      -0.214       0.308
TIP            0.3356      0.222      1.509      0.133      -0.104       0.775
==============================================================================
Omnibus:                        5.018   Durbin-Watson:                   2.055
Prob(Omnibus):                  0.081   Jarque-Bera (JB):                4.829
Skew:                          -0.307   Prob(JB):                       0.0894
Kurtosis:                       3.598   Cond. No.                         190.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

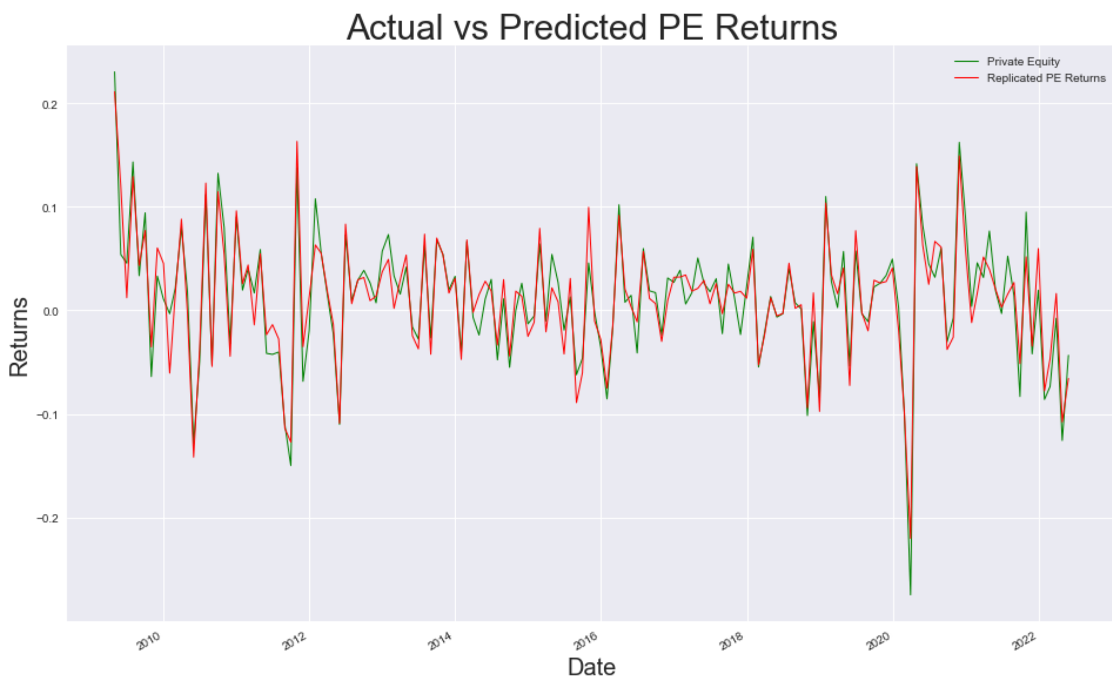
0.2.3 Replicating PE with a three asset classes

- Try replicationg private equity returns with a few assets. I would suggest that PE returns can be heavily explained by SPY, Absolute Return, and Real Estate.

[ ]:
```
X_small = sm.add_constant(rets[["SPY","QAI", "IYR"]])
estimation_small = sm.OLS(y, X_small).fit()
estimation_small.summary()
```

[ ]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                    PSP   R-squared:                       0.837
Model:                            OLS   Adj. R-squared:                  0.834
Method:                 Least Squares   F-statistic:                     263.0
Date:                Wed, 24 Aug 2022   Prob (F-statistic):           2.28e-60
Time:                        13:04:18   Log-Likelihood:                 357.03
No. Observations:                 158   AIC:                            -706.1
Df Residuals:                     154   BIC:                            -693.8
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0041      0.002     -1.898      0.060      -0.008       0.000
SPY            0.8824      0.102      8.613      0.000       0.680       1.085
QAI            1.0639      0.260      4.088      0.000       0.550       1.578
IYR            0.1964      0.057      3.460      0.001       0.084       0.309
==============================================================================
Omnibus:                        0.178   Durbin-Watson:                   1.775
Prob(Omnibus):                  0.915   Jarque-Bera (JB):                0.157
Skew:                          -0.074   Prob(JB):                        0.925
Kurtosis:                       2.957   Cond. No.                         133.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

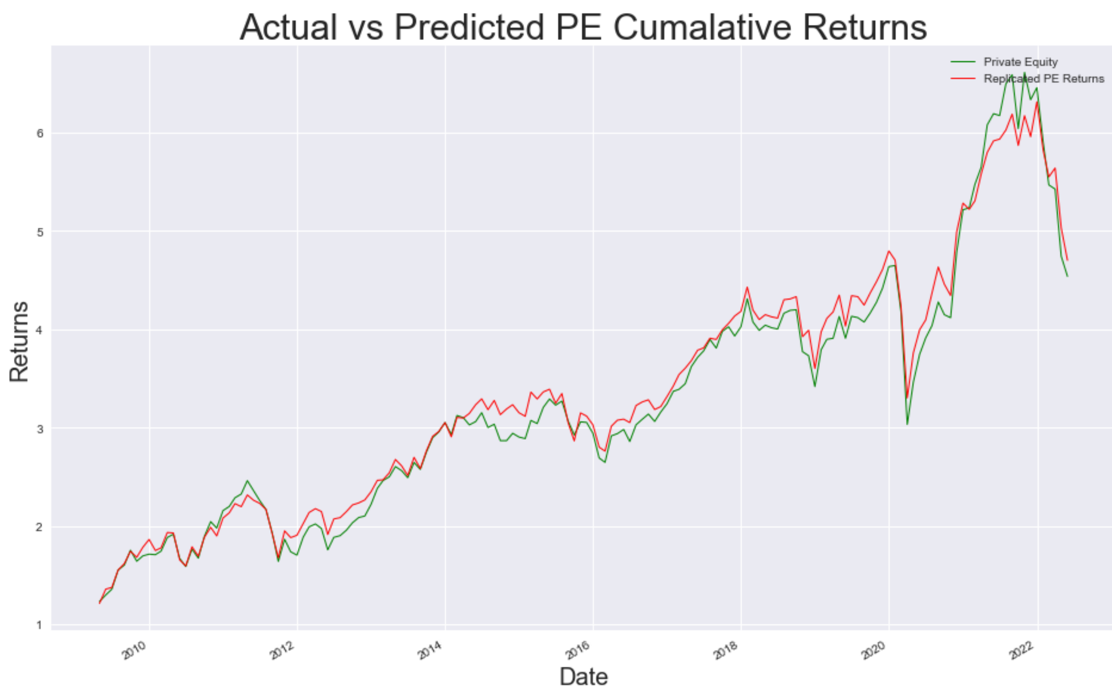
[ ]:
```
# Graph of full model
full_model = pd.concat([y, pd.DataFrame(data = estimation.predict(sm.
 ↪add_constant(X_full)), index = X_full.index, columns= ["Replicated"])], axis␣
 ↪= 1)
plt.style.use("seaborn")
fig , ax = plt.subplots(figsize = (16,10))
ax.plot(full_model["PSP"], c = "green", label = "Private Equity", linewidth = 1)
ax.plot(full_model["Replicated"], c = "red", label = "Replicated PE Returns",␣
 ↪linewidth = 1)
plt.legend(loc = "upper right", shadow = True)
ax.set_ylabel("Returns", fontsize = 20)
ax.set_xlabel("Date", fontsize = 20)
ax.set_title("Actual vs Predicted PE Returns", fontsize = 30)
fig.autofmt_xdate()
```



Replicating Cumlative Returns vs Actual Cumulative Returns We know cumulative returns is equal to this equation:

$$r_{it} = \prod_{i=1}^{T}(1 + r_{it}) - 1$$

[ ]:
```
cum_returns = (full_model +1).cumprod()
plt.style.use("seaborn")
fig , ax = plt.subplots(figsize = (16,10))
ax.plot(cum_returns["PSP"], c = "green", label = "Private Equity", linewidth =␣
 ↪1)
ax.plot(cum_returns["Replicated"], c = "red", label = "Replicated PE Returns",␣
 ↪linewidth = 1)
plt.legend(loc = "upper right", shadow = True)
ax.set_ylabel("Returns", fontsize = 20)
ax.set_xlabel("Date", fontsize = 20)
ax.set_title("Actual vs Predicted PE Cumalative Returns", fontsize = 30)
fig.autofmt_xdate()
```




```
[ ]: X_SMALL = X_small.drop(columns = "const")
```

0.3 1.3 Multicollinearity

Should we be worred about multicollinearity in the full model? Calculate some metrics about $X^{`}X$. This has to do with rank of the matrix.

```
[ ]: print(f"The conditioning number for the full model is: {np.linalg.cond(X_full.T␣
      ↪@ X_full):.5f}.\n The corresponding determinant is: {np.linalg.det(X_full.T␣
      ↪@ X_full):.9f}.")
     print(f"The conditioning number for the the smaller model is: {np.linalg.
      ↪cond(X_SMALL.T @ X_SMALL):.5f} and the corresponding determinant is:\n {np.
      ↪linalg.det(X_SMALL.T @ X_SMALL):.8f}.")
```

```
The conditioning number for the full model is: 328.87178.
 The corresponding determinant is: 0.000000000.
The conditioning number for the the smaller model is: 74.25950 and the
corresponding determinant is:
 0.00059184.
```

1 Quantifying Relationships for OIL, VIX, SPY, and Russell 200 Index

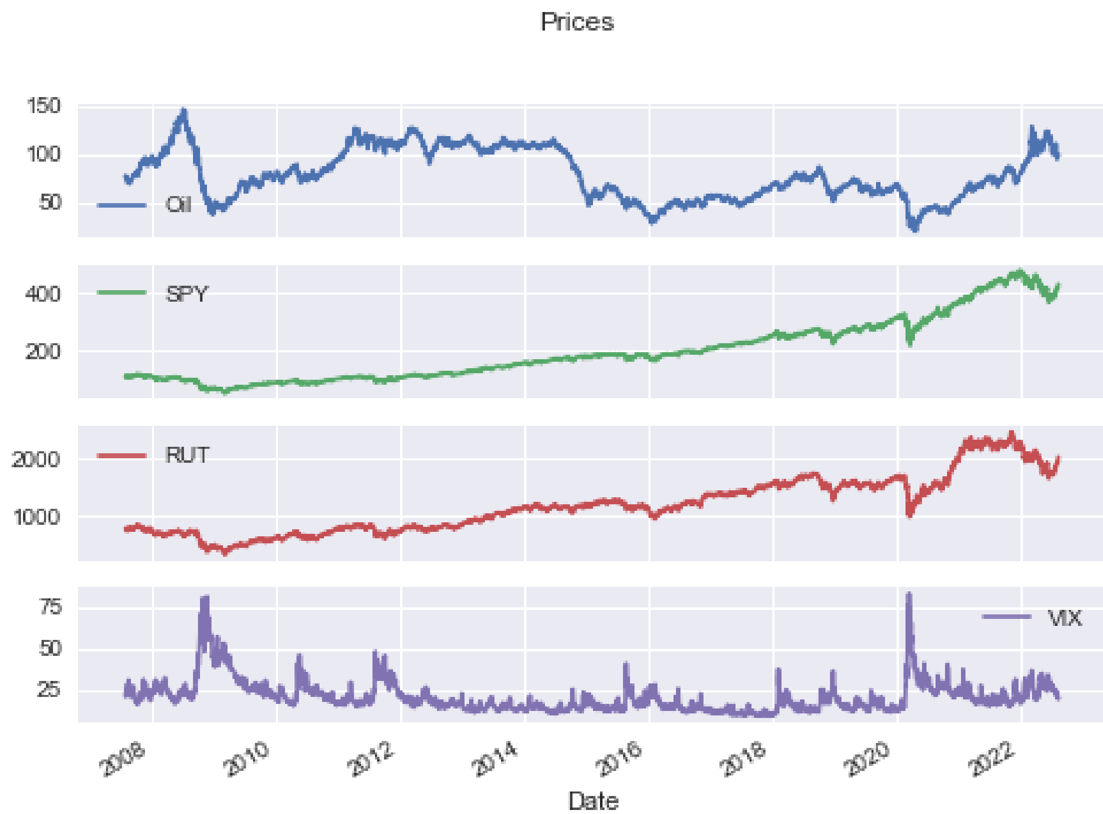
- First Graph the Price of oil starting before the pandemic along with its corresponding volatility. Then compute the multiple linear regression equation:

$$r_{it}^{spy} = \alpha + \beta_1 r_{it}^{oil} + \beta_2 r_{it}^{vix} + \beta_3 r_{it}^{rut} + \epsilon$$

[ ]:
```
tixs = "SPY BZ=F ^VIX ^RUT"
prices = pd.DataFrame(yf.download(tixs, start = "2005-01-01", end =␣
 ↪"2022-08-13")["Adj Close"])
prices.dropna(inplace = True)
prices.columns = ["Oil", "SPY", "RUT", "VIX"]
prices.plot(subplots = True, title = "Prices")
```

```
[*********************100%***********************]  4 of 4 completed
```

[ ]:
```
array([<AxesSubplot:xlabel='Date'>, <AxesSubplot:xlabel='Date'>,
       <AxesSubplot:xlabel='Date'>, <AxesSubplot:xlabel='Date'>],
      dtype=object)
```



[ ]:
```
prices.info()
```

```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 3728 entries, 2007-07-30 to 2022-08-12
Data columns (total 4 columns):
 #   Column  Non-Null Count  Dtype
---  ------  --------------  -----
```

```
 0   Oil      3728 non-null   float64
 1   SPY      3728 non-null   float64
 2   RUT      3728 non-null   float64
 3   VIX      3728 non-null   float64
dtypes: float64(4)
memory usage: 145.6 KB
```

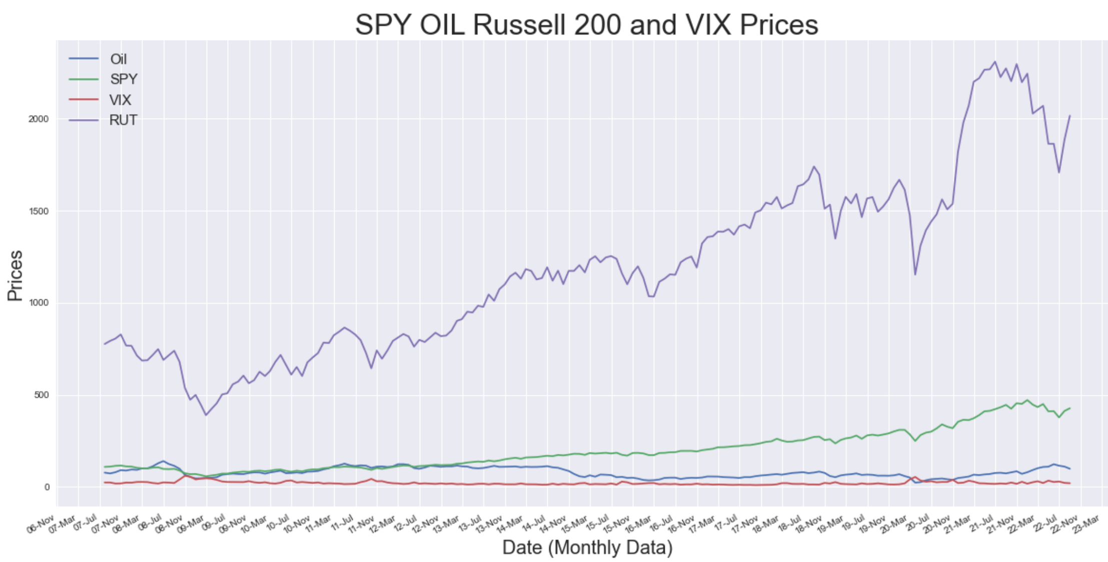
[ ]:
```
print(prices["2022-05-01":].mean())
print(prices["2022-05-01":].std())
```

```
Oil     109.482778
SPY     396.355497
RUT    1809.696392
VIX      26.662083
dtype: float64
Oil      7.895581
SPY     15.144973
RUT     83.152801
VIX      3.627232
dtype: float64
```

[ ]:
```
#Convert the Prices to monthly periods
prices_month = pd.DataFrame(prices.resample("1m",label = "right").last())
prices_month.plot()
```

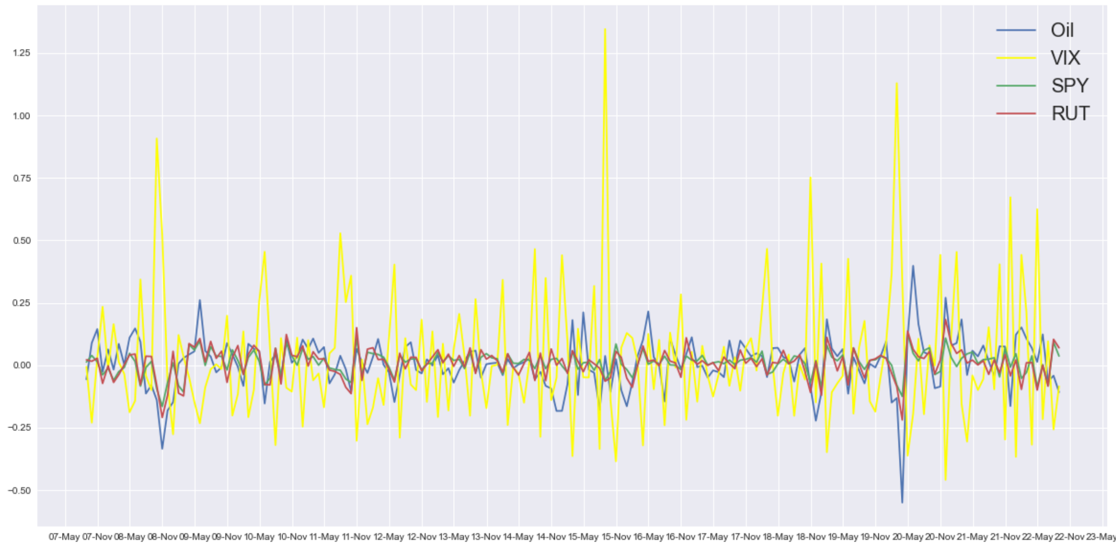
[ ]: <AxesSubplot:xlabel='Date'>

[ ]:
```
plt.figure(figsize = (20,10))
plt.plot(prices_month["Oil"], label = "Oil")
plt.plot(prices_month["SPY"], label = "SPY")
plt.plot(prices_month["VIX"], label = "VIX")
plt.plot(prices_month["RUT"], label = "RUT")
plt.legend(loc = 2, fontsize = 15)
plt.ylabel("Prices", fontsize = 20)
plt.xlabel("Date (Monthly Data)", fontsize = 20)
plt.title("SPY OIL Russell 200 and VIX Prices", fontsize = 30)
ax = plt.gca()

ax.xaxis.set_major_locator(mdates.MonthLocator(interval=4))
ax.xaxis.set_major_formatter(mdates.DateFormatter('%y-%b'))

plt.gcf().autofmt_xdate()
```



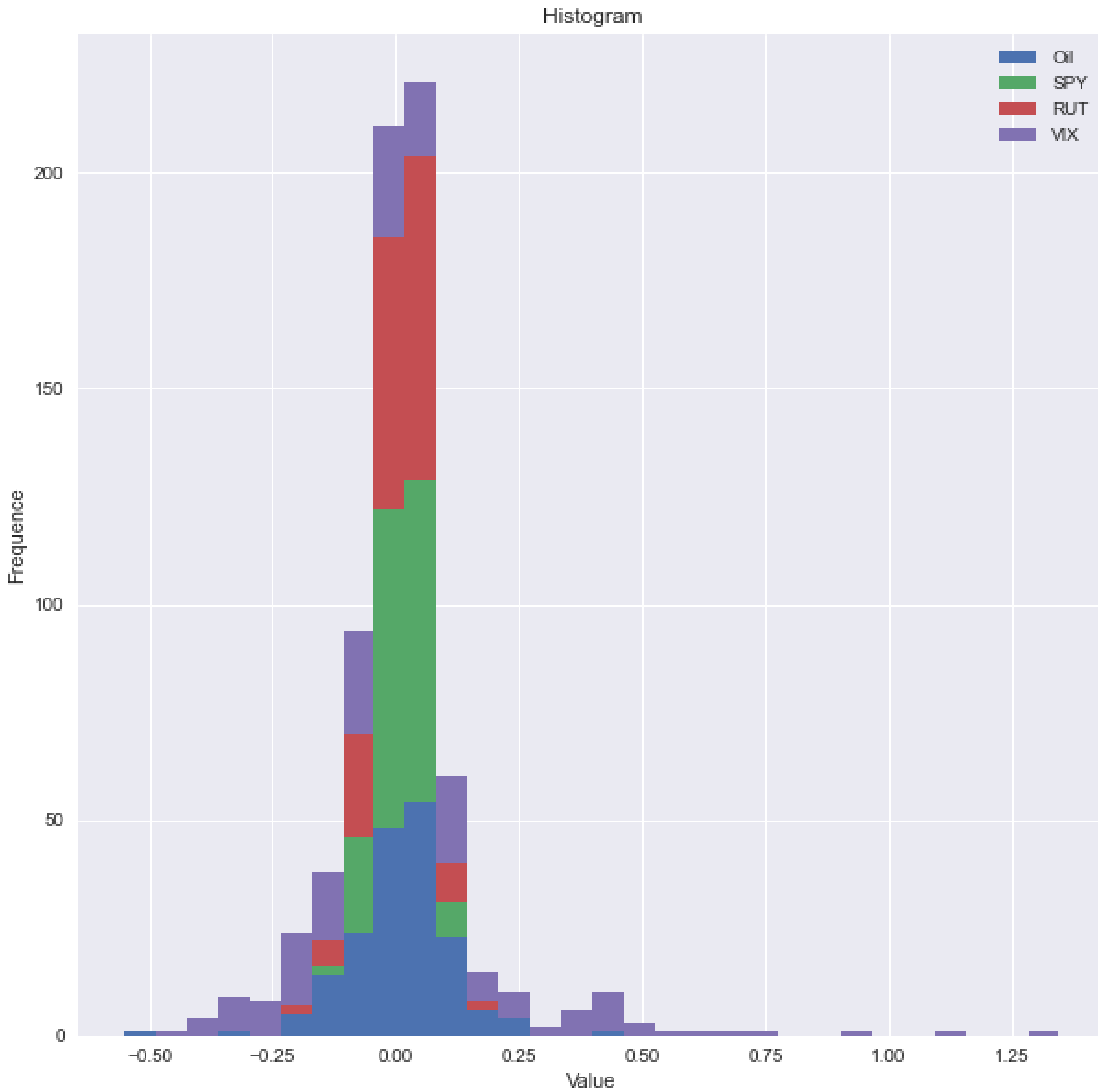
[ ]:
```
returns_monthly = prices_month.pct_change().round(4).dropna()
fig, ax = plt.subplots(figsize= (20,10))
ax.plot(returns_monthly["Oil"], label = "Oil")
ax.plot(returns_monthly["VIX"], label = "VIX", c = "yellow")
ax.plot(returns_monthly["SPY"], label = "SPY")
ax.plot(returns_monthly["RUT"], label = "RUT")
ax.xaxis.set_major_locator(mdates.MonthLocator(interval=6))
ax.xaxis.set_major_formatter(mdates.DateFormatter('%y-%b'))
plt.legend(loc = 0, shadow = True, fontsize = 20)
```

[ ]:
```
<matplotlib.legend.Legend at 0x1bc049e4550>
```



[ ]:
```python
#Computing Monthly Returns From Prices Month Object

plt.figure(figsize = (10,10))
plt.hist(returns_monthly, stacked = True, bins = 30, label = ["Oil", "SPY",␣
↪"RUT","VIX"] )
plt.legend(loc = 0)
plt.title("Histogram")
plt.xlabel("Value")
plt.ylabel("Frequence")
```

[ ]:
```
Text(0, 0.5, 'Frequence')
```



As you can see the *VIX* index is very volatile with higher frequencies of positive and negative returns in the tails of the histogram. The other assets appear to be normal, however, we know financial data is not normally distributed.

- Let's measure this equation: $r_{it}^{spy} = \alpha + \beta_1 r_{it}^{oil} + \beta_2 r_{it}^{vix} + \beta_3 r_{it}^{rut} + \epsilon$

$ starting with the full data set and then slicing the data to measure the relationship just before the pandemic started.

[ ]:
```
from textwrap import indent


r_spy = returns_monthly["SPY"]
independ_var = sm.add_constant(returns_monthly.drop(columns = "SPY"))
```

```python
full_equation = sm.OLS(r_spy, independ_var).fit()
full_equation.summary()
```

[ ]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                    SPY   R-squared:                       0.854
Model:                            OLS   Adj. R-squared:                  0.852
Method:                 Least Squares   F-statistic:                     345.8
Date:                Wed, 24 Aug 2022   Prob (F-statistic):           9.41e-74
Time:                        13:04:24   Log-Likelihood:                 475.64
No. Observations:                 181   AIC:                            -943.3
Df Residuals:                     177   BIC:                            -930.5
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0056      0.001      4.127      0.000       0.003       0.008
Oil            0.0144      0.015      0.983      0.327      -0.015       0.043
RUT            0.5728      0.031     18.472      0.000       0.512       0.634
VIX           -0.0386      0.006     -6.081      0.000      -0.051      -0.026
==============================================================================
Omnibus:                        0.179   Durbin-Watson:                   1.813
Prob(Omnibus):                  0.915   Jarque-Bera (JB):                0.322
Skew:                           0.046   Prob(JB):                        0.851
Kurtosis:                       2.815   Cond. No.                         24.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

- Here I am going to fit the model on training data until the pandemic starts and then test its performance on data after that time period.

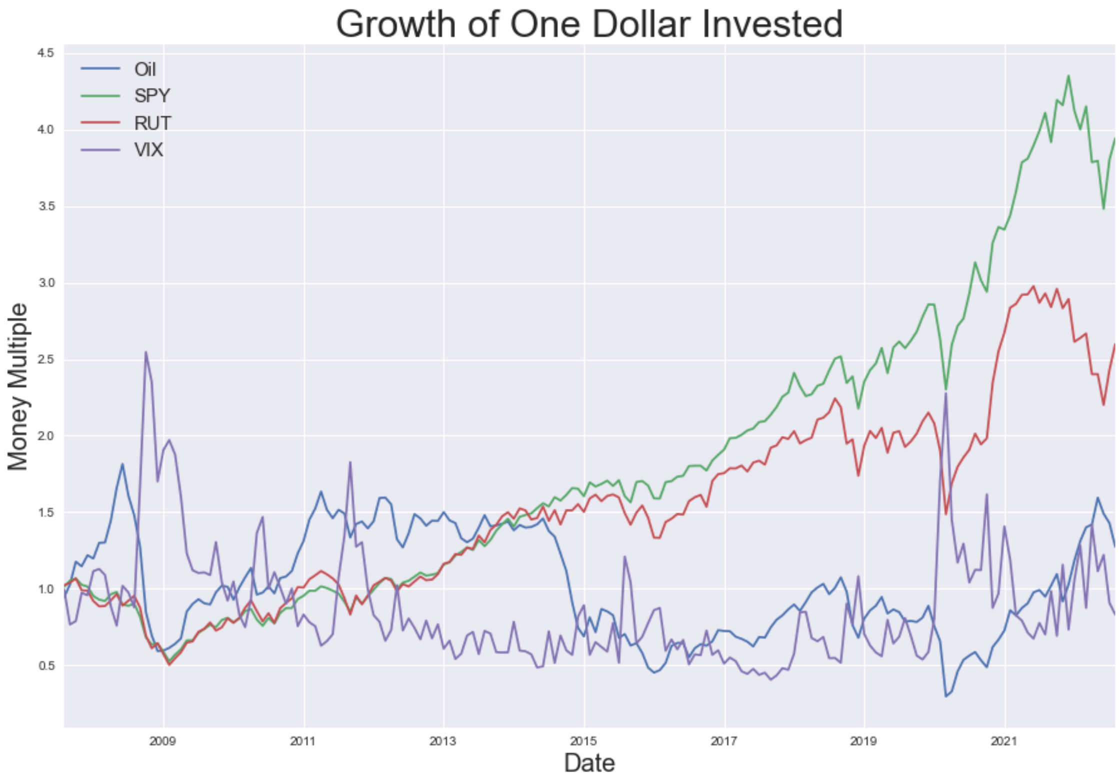
[ ]:
```python
train_data = independ_var[:"2019-10-01"]
train_dependent = r_spy[:"2019-10-01"]
train_model = sm.OLS(train_dependent,train_data).fit()
train_model.summary()
```

[ ]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                    SPY   R-squared:                       0.863
```

```
Model:                            OLS   Adj. R-squared:                  0.860
Method:                 Least Squares   F-statistic:                     299.0
Date:                Wed, 24 Aug 2022   Prob (F-statistic):           3.80e-61
Time:                        13:04:24   Log-Likelihood:                 397.02
No. Observations:                 146   AIC:                            -786.0
Df Residuals:                     142   BIC:                            -774.1
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0046      0.001      3.365      0.001       0.002       0.007
Oil            0.0379      0.017      2.236      0.027       0.004       0.071
RUT            0.5773      0.033     17.544      0.000       0.512       0.642
VIX           -0.0341      0.007     -4.777      0.000      -0.048      -0.020
==============================================================================
Omnibus:                        0.578   Durbin-Watson:                   1.926
Prob(Omnibus):                  0.749   Jarque-Bera (JB):                0.329
Skew:                          -0.103   Prob(JB):                        0.848
Kurtosis:                       3.108   Cond. No.                         25.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

[ ]:
```
## Plotting Cumalative Returns on graph
(returns_monthly + 1).cumprod().plot(figsize = (15,10))
plt.xlabel("Date", fontsize = 20)
plt.ylabel("Money Multiple", fontsize = 20)
plt.title("Growth of One Dollar Invested", fontsize = 30)
plt.legend(fontsize = 15)
```

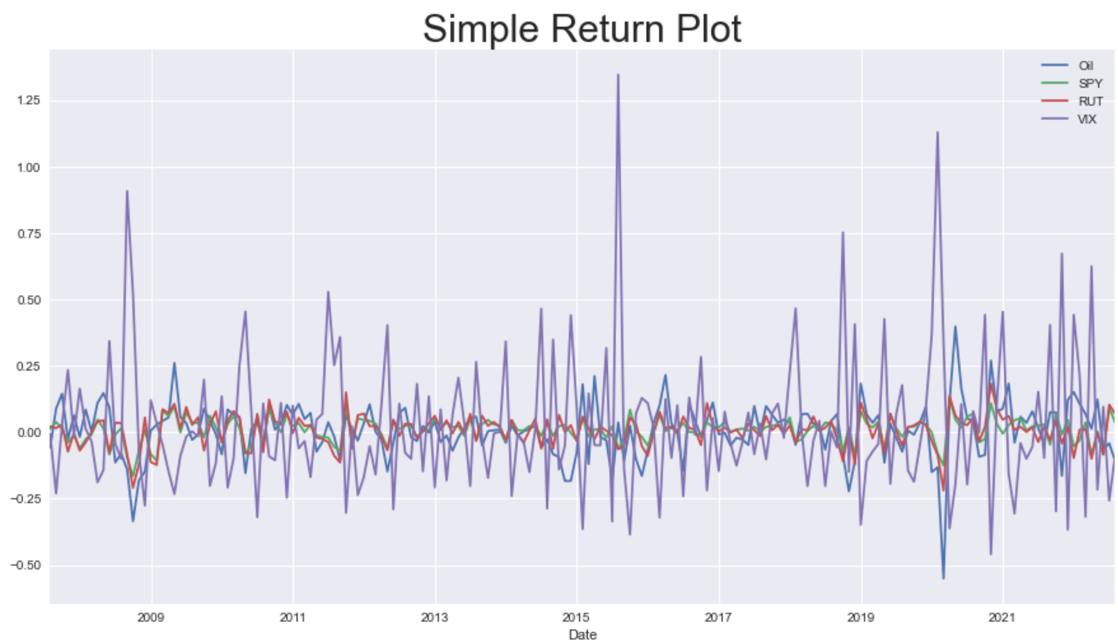
[ ]: <matplotlib.legend.Legend at 0x1bc044de4f0>



[ ]:
```
returns_monthly.plot(figsize = (15,8))

plt.title("Simple Return Plot", fontsize  =30)
returns_monthly.aggregate([min, np.mean, np.std,np.median, max])
```

[ ]:

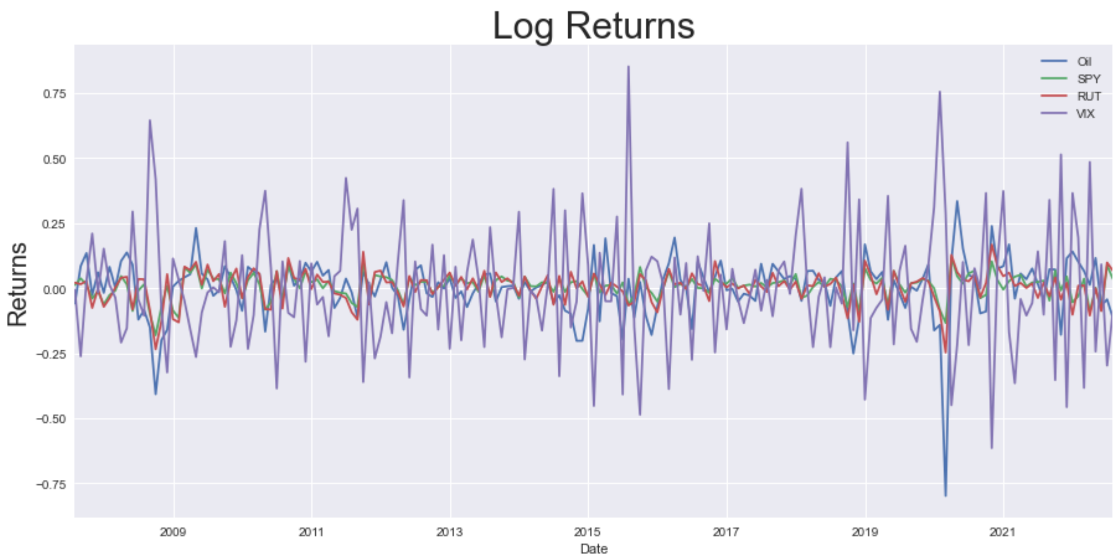
	Oil	SPY	RUT	VIX
min	-0.549900	-0.165200	-0.219000	-0.459000
mean	0.007319	0.008671	0.007120	0.029141
std	0.104568	0.045910	0.059967	0.267850
median	0.013400	0.015100	0.016400	-0.016200
max	0.398100	0.127000	0.182900	1.345700



[ ]:
```python
log_returns = np.log(prices_month/prices_month.shift(1)).dropna()
log_returns.plot(figsize = (15,7))
plt.title("Log Returns", fontsize = 30)
plt.ylabel("Returns", fontsize = 20)
log_returns.aggregate([min, np.mean, np.std,np.median, max])
```

[ ]:

	Oil	SPY	RUT	VIX
min	-0.798244	-0.180547	-0.247173	-0.614279
mean	0.001337	0.007576	0.005276	-0.001027
std	0.113386	0.046308	0.061027	0.239624
median	0.013286	0.015006	0.016237	-0.016311
max	0.335115	0.119545	0.167943	0.852588



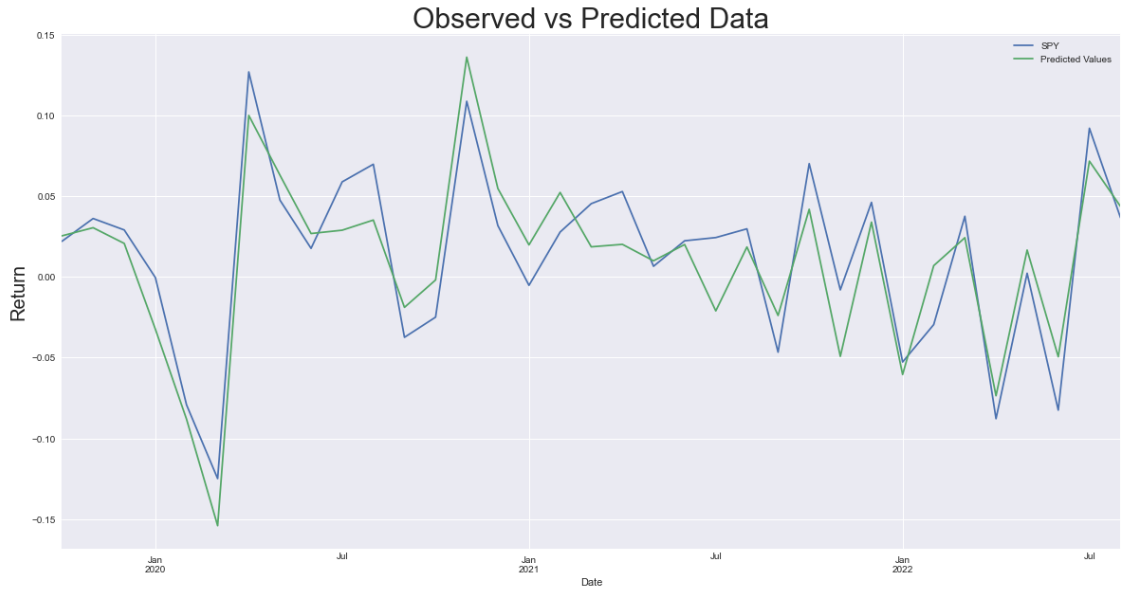
Since 2008, we can clearly see that the most volatile security is the **VIX** index with a monthly standard deviation of approximately *26%.*

SPY averages a monthly return of *0.75%* with a monthly standard deviation of *4.5% since 2008.

- Let's see how accurate our model is when we train our full model on the test data. The test data begins **2019-10-01** and goes until the last date in **2022.**

[ ]:
```
test_data = independ_var["2019-10-01":]
predicted = pd.DataFrame(train_model.predict(test_data), columns = ["Predicted␣
↪Values"])
observed = pd.DataFrame(r_spy["2019-10-01":])
df1 = pd.concat((observed, predicted), axis = 1)
df1.plot(figsize = (20,10))
plt.ylabel("Return", fontsize = 20)
plt.title("Observed vs Predicted Data", fontsize = 30)
```

[ ]:
```
Text(0.5, 1.0, 'Observed vs Predicted Data')
```



To test the performance of this model we need to calculate the R^2. Using algebra the R^2 is equal to the squared correlation coefficient between the **actual** y_i and the *fitted values* y_i.

[ ]:
```
# R-squared from training model was 0.864
df1.corr()**2
```

[ ]:

	SPY	Predicted Values
SPY	1.000000	0.826623
Predicted Values	0.826623	1.000000

This values above indicate performance of the training model on test data. We see that $R^2 = Corr(y_i, \hat{y})^2 = 0.826623$ This means our training model is accurate for predicting monthly returns. In fact, we can say *82%* of the sample variation in SPY returns can be explained by returns of **OIL, RUSSELL 2000, and VIX.**