Tommy_Portfolio

September 14, 2022

0.1 Tommy's Optimal Portfolio

- Using just your assets I will find where the capital market line is tangent to the efficient frontier. The optimal combination of a set of portfolios that offer the lowest risk for a given return and the highest return for a given amount of risk is the **efficient frontier.** The *line* that is drawn tangently to the efficient frontier from the rate of return that is the risk-free is the capital makret line.

-This **Tangency Portfolio** I find maximizes the Sharpe ratio.

$\max_{x_1,x_2,\ldots,x_N} \mathrm{SR}_p = \frac{\mu_p - r_f}{\sigma_p}$

[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
from sklearn.linear_model import LinearRegression
import matplotlib as mpl
import seaborn as sns
import yfinance as yf
import scipy as scs
```

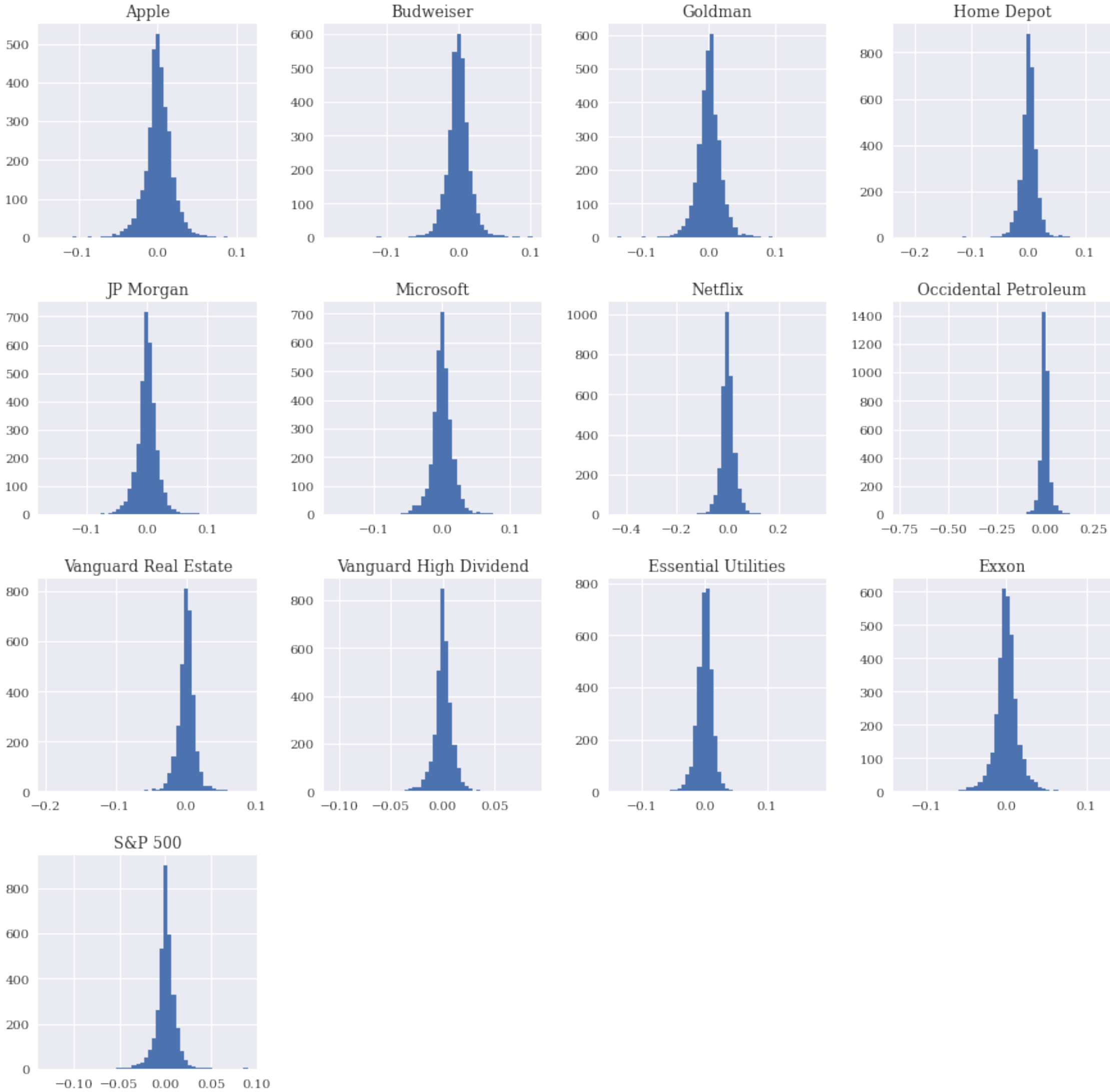
[ ]:
```
plt.style.use("seaborn")
mpl.rcParams['font.family'] = 'serif'
%matplotlib inline
```

[ ]:
```
# Getting Data
tickers = "AAPL MSFT GS VYM XOM JPM WTRG OXY ^GSPC BUD VNQ HD NFLX"
start = "2003-01-01"
end = "2022-09-01"
adj_close = pd.DataFrame(yf.download(tickers, start, end)["Adj Close"])
adj_close.dropna(inplace = True)
adj_close.columns = ["Apple", "Budweiser", "Goldman", "Home Depot", "JP␣
↪Morgan", "Microsoft", "Netflix", "Occidental Petroleum","Vanguard Real␣
↪Estate", "Vanguard High Dividend","Essential Utilities", "Exxon", "S&P 500"]
```

```
[*********************100%***********************]  13 of 13 completed
```

[ ]:
```
# Computing Log Returns
noa = 13
log_returns = np.log(adj_close/adj_close.shift(1))
log_returns.dropna(inplace = True)
log_returns.hist(bins = 50, figsize = (15,15))
```

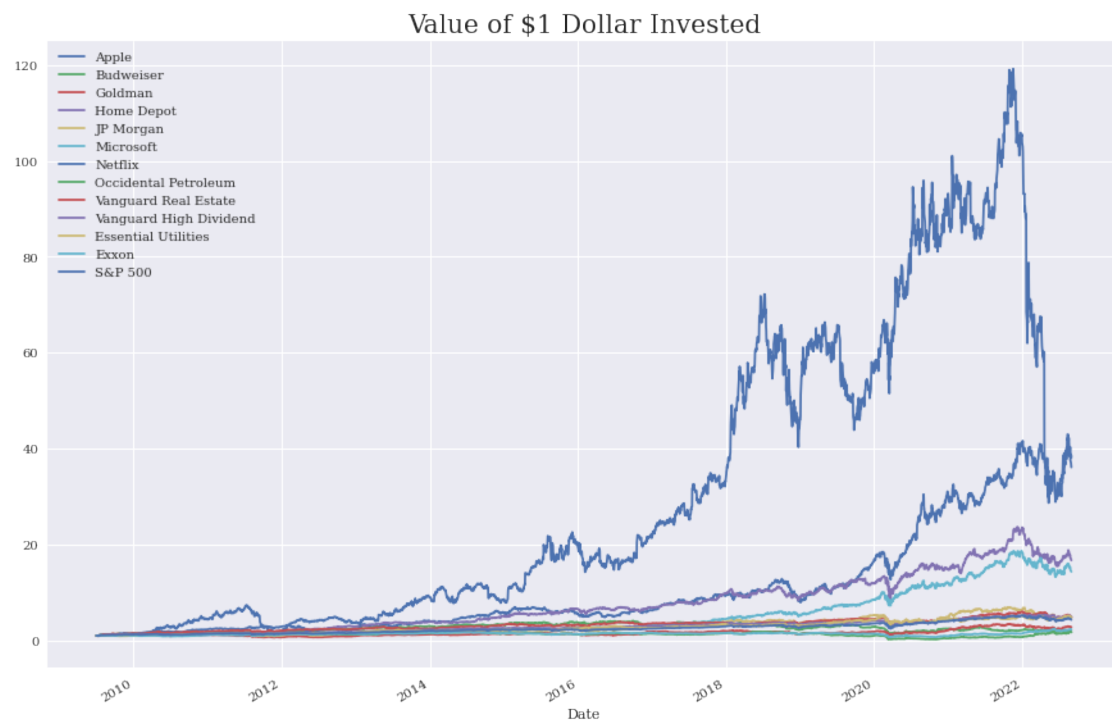
[ ]:
```
array([[<AxesSubplot:title={'center':'Apple'}>,
        <AxesSubplot:title={'center':'Budweiser'}>,
        <AxesSubplot:title={'center':'Goldman'}>,
        <AxesSubplot:title={'center':'Home Depot'}>],
       [<AxesSubplot:title={'center':'JP Morgan'}>,
        <AxesSubplot:title={'center':'Microsoft'}>,
        <AxesSubplot:title={'center':'Netflix'}>,
        <AxesSubplot:title={'center':'Occidental Petroleum'}>],
       [<AxesSubplot:title={'center':'Vanguard Real Estate'}>,
        <AxesSubplot:title={'center':'Vanguard High Dividend'}>,
        <AxesSubplot:title={'center':'Essential Utilities'}>,
        <AxesSubplot:title={'center':'Exxon'}>],
       [<AxesSubplot:title={'center':'S&P 500'}>, <AxesSubplot:>,
        <AxesSubplot:>, <AxesSubplot:>]], dtype=object)
```



[ ]:
```
plt.figure(figsize = (20,15))
log_returns.cumsum().apply(np.exp).plot(figsize = (15,10))
plt.title("Value of $1 Dollar Invested", size = 20)
```

[ ]:
```
Text(0.5, 1.0, 'Value of $1 Dollar Invested')
```

```
<Figure size 1440x1080 with 0 Axes>
```



[ ]:
```
#Annualized Return
log_returns.mean()*252
```

[ ]:
```
Apple                     0.272617
Budweiser                 0.040266
Goldman                   0.077406
Home Depot                0.213954
JP Morgan                 0.117054
Microsoft                 0.202316
Netflix                   0.277567
Occidental Petroleum      0.040698
Vanguard Real Estate      0.120498
Vanguard High Dividend    0.120616
Essential Utilities       0.119643
Exxon                     0.061280
S&P 500                   0.110587
dtype: float64
```

[ ]:
```
#Covariance Matrix
log_returns.cov()*252
```

[ ]:
```
                             Apple  Budweiser   Goldman  Home Depot  JP Morgan  \
Apple                     0.080014   0.024835  0.036270    0.030896   0.033779
Budweiser                 0.024835   0.072059  0.035179    0.026542   0.037337
```

```
Goldman                 0.036270   0.035179  0.085295    0.034759   0.069094
Home Depot              0.030896   0.026542  0.034759    0.054803   0.035099
JP Morgan               0.033779   0.037337  0.069094    0.035099   0.082813
Microsoft               0.042017   0.025659  0.036575    0.031713   0.035314
Netflix                 0.041700   0.022850  0.035134    0.030335   0.029560
Occidental Petroleum    0.036545   0.047041  0.061355    0.033710   0.066297
Vanguard Real Estate    0.027527   0.029163  0.035056    0.030411   0.038014
Vanguard High Dividend  0.025925   0.025857  0.034722    0.025890   0.036712
Essential Utilities     0.020292   0.020486  0.022158    0.023208   0.025193
Exxon                   0.023503   0.029955  0.039500    0.023345   0.041505
S&P 500                 0.033900   0.027458  0.038098    0.029720   0.038764

                        Microsoft   Netflix  Occidental Petroleum  \
Apple                    0.042017  0.041700              0.036545
Budweiser                0.025659  0.022850              0.047041
Goldman                  0.036575  0.035134              0.061355
Home Depot               0.031713  0.030335              0.033710
JP Morgan                0.035314  0.029560              0.066297
Microsoft                0.065452  0.044486              0.035119
Netflix                  0.044486  0.267063              0.027919
Occidental Petroleum     0.035119  0.027919              0.207250
Vanguard Real Estate     0.028115  0.024550              0.042608
Vanguard High Dividend   0.027132  0.022874              0.042682
Essential Utilities      0.022478  0.016470              0.028723
Exxon                    0.024060  0.017386              0.080929
S&P 500                  0.033900  0.033316              0.042557

                        Vanguard Real Estate  Vanguard High Dividend  \
Apple                               0.027527                0.025925
Budweiser                           0.029163                0.025857
Goldman                             0.035056                0.034722
Home Depot                          0.030411                0.025890
JP Morgan                           0.038014                0.036712
Microsoft                           0.028115                0.027132
Netflix                             0.024550                0.022874
Occidental Petroleum                0.042608                0.042682
Vanguard Real Estate                0.047560                0.027125
Vanguard High Dividend              0.027125                0.025886
Essential Utilities                 0.030900                0.022253
Exxon                               0.026820                0.029237
S&P 500                             0.029562                0.026978

                        Essential Utilities     Exxon   S&P 500
Apple                              0.020292  0.023503  0.033900
Budweiser                          0.020486  0.029955  0.027458
Goldman                            0.022158  0.039500  0.038098
Home Depot                         0.023208  0.023345  0.029720
```

JP Morgan	0.025193	0.041505	0.038764
Microsoft	0.022478	0.024060	0.033900
Netflix	0.016470	0.017386	0.033316
Occidental Petroleum	0.028723	0.080929	0.042557
Vanguard Real Estate	0.030900	0.026820	0.029562
Vanguard High Dividend	0.022253	0.029237	0.026978
Essential Utilities	0.051080	0.021258	0.022936
Exxon	0.021258	0.062415	0.028377
S&P 500	0.022936	0.028377	0.031070

0.2 Generating Risk-Return Profiles for a given allocation of Assets

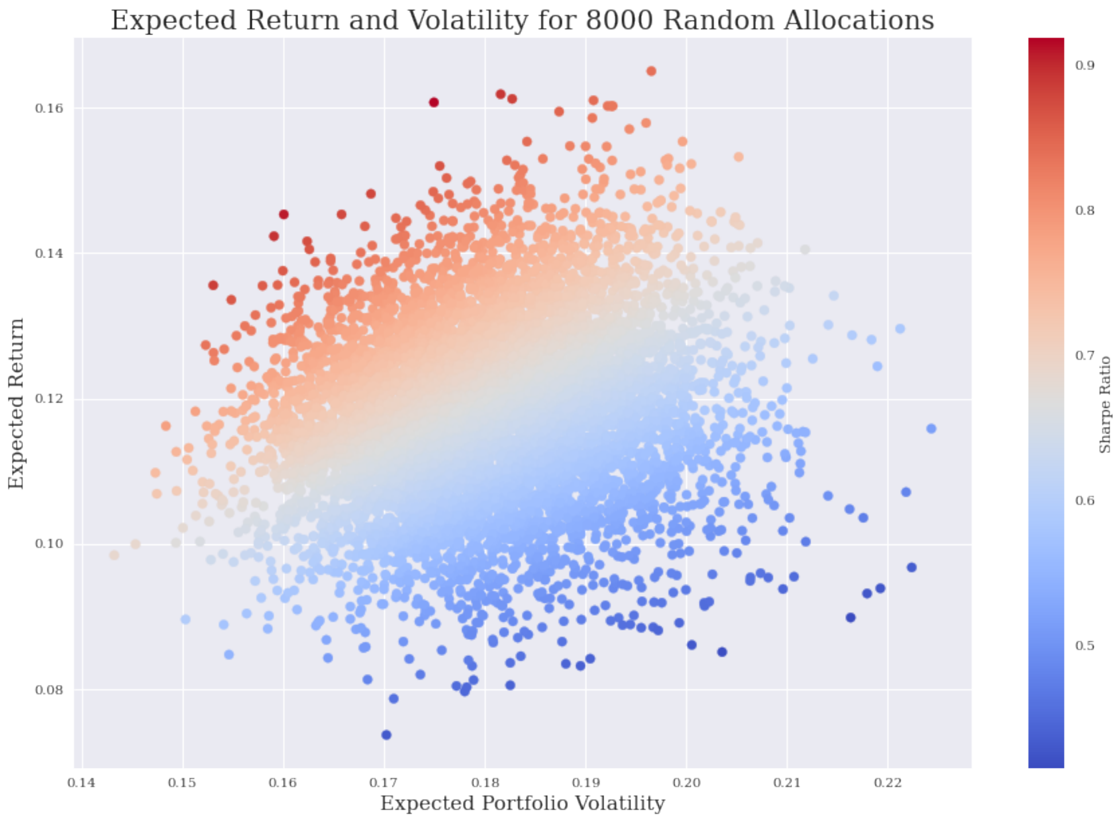
Here I simulate allocations and the code records the resulting expected portfolio return and variance.

[ ]:
```
weights = np.random.random(noa)
weights /= np.sum(weights)
```

[ ]:
```
def port_ret(weights):
    return np.sum(log_returns.mean() *weights)*252
def port_vol(weights):
    return np.sqrt(np.dot(weights.T,np.dot(log_returns.cov()*252, weights)))
prets = []
pvols = []
for p in range(8000):
    weights = np.random.random(noa)
    weights /= np.sum(weights)
    prets.append(port_ret(weights))
    pvols.append(port_vol(weights))
prets = np.array(prets)
pvols = np.array(pvols)
```

[ ]:
```
from scipy.ndimage import label


plt.figure(figsize = (15,10))
plt.scatter(pvols, prets, c = prets/pvols,marker = 'o', cmap = "coolwarm")
plt.xlabel("Expected Portfolio Volatility", fontsize = 15)
plt.ylabel("Expected Return", fontsize = 15)
plt.title("Expected Return and Volatility for 8000 Random Allocations", size =␣
 ↪20)
plt.colorbar(label = "Sharpe Ratio")
plt.show()
```



0.3 Finding the Efficient Frontier

[ ]:
```
import scipy.optimize as sco

def min_func_sharpe(weights):
    return -port_ret(weights)/port_vol(weights)
cons = ({'type': 'eq', 'fun': lambda x: np.sum(x)-1})
bnds = tuple((0,1) for x in range(noa))
eweights = np.array(noa*[1./noa,])
eweights
min_func_sharpe(eweights)
```

[ ]: -0.7111579641783428

[ ]:
```
opts = sco.minimize(min_func_sharpe, eweights, method = "SLSQP", constraints =␣
 ↪cons)
pd.DataFrame(opts['x'].round(3), index = ["Apple", "Budweiser", "Goldman",␣
 ↪"Home Depot", "JP Morgan", "Microsoft", "Netflix", "Occidental Petroleum",␣
 ↪"Vanguard Real Estate", "Vanguard High Dividend","Essential Utilities",␣
 ↪"Exxon", "S&P 500"], columns = ["Weights"])
```

```
[ ]:                          Weights
     Apple                      0.770
     Budweiser                 -0.189
     Goldman                   -0.210
     Home Depot                 0.639
     JP Morgan                  0.136
     Microsoft                  0.523
     Netflix                    0.167
     Occidental Petroleum      -0.066
     Vanguard Real Estate       0.119
     Vanguard High Dividend     4.123
     Essential Utilities       -0.004
     Exxon                     -0.200
     S&P 500                   -4.807
```

```
[ ]: print("The resulting portfolio return and portfolio volatility from the optimal␣
     ↪weights are", np.round(port_ret(opts['x']),4), "and",np.
     ↪round(port_vol(opts["x"]),4), "respectively.")
```

```
The resulting portfolio return and portfolio volatility from the optimal weights
are 0.4553 and 0.2666 respectively.
```

You see that some of these weights are **negative** this is because if you were *short* meaning a negative allocation and long on the other's you **maximize** the sharpe ratio for that particular given **allocation**. You'll see below that we will eliminate this assumption so that you are only in **long positions.**

- Here I will compute the **Minimization of the Variance** *for given allocation.*

```
[ ]: optv = sco.minimize(port_vol, eweights, method = "SLSQP", bounds = bnds,␣
     ↪constraints = cons)
     pd.DataFrame(optv['x'].round(3), index = ["Apple", "Budweiser", "Goldman",␣
     ↪"Home Depot", "JP Morgan", "Microsoft", "Netflix", "Occidental Petroleum",␣
     ↪"Vanguard Real Estate", "Vanguard High Dividend","Essential Utilities",␣
     ↪"Exxon", "S&P 500"], columns = ["Weights"])
```

```
[ ]:                          Weights
     Apple                      0.000
     Budweiser                  0.004
     Goldman                    0.000
     Home Depot                 0.000
     JP Morgan                  0.000
     Microsoft                  0.000
     Netflix                    0.013
     Occidental Petroleum       0.000
     Vanguard Real Estate       0.000
     Vanguard High Dividend     0.870
     Essential Utilities        0.113
     Exxon                      0.000
```

```
S&P 500                    0.000
```

This makes sense because obviously a TIP fund is way less volatile, hence you will minimize your portfolio variance a.k.a **risk.**

```
[ ]: print("Expected return", port_ret(optv['x']),"and minimizing portfolio␣
      ↪volatility is", port_vol(optv['x']))
```

```
Expected return 0.12226489416136486 and minimizing portfolio volatility is
0.15947944969195965
```

- The expected return for maximized return for the best *sharpe ratio* is about $\mu_p = 0.452$ with a portfolio variance of $\sigma_p = .1602$
- The expected return for the minimum variance portfolio about $\mu_p = 0.1205$ with a minimum variance of $\sigma_p = 0.15958$.

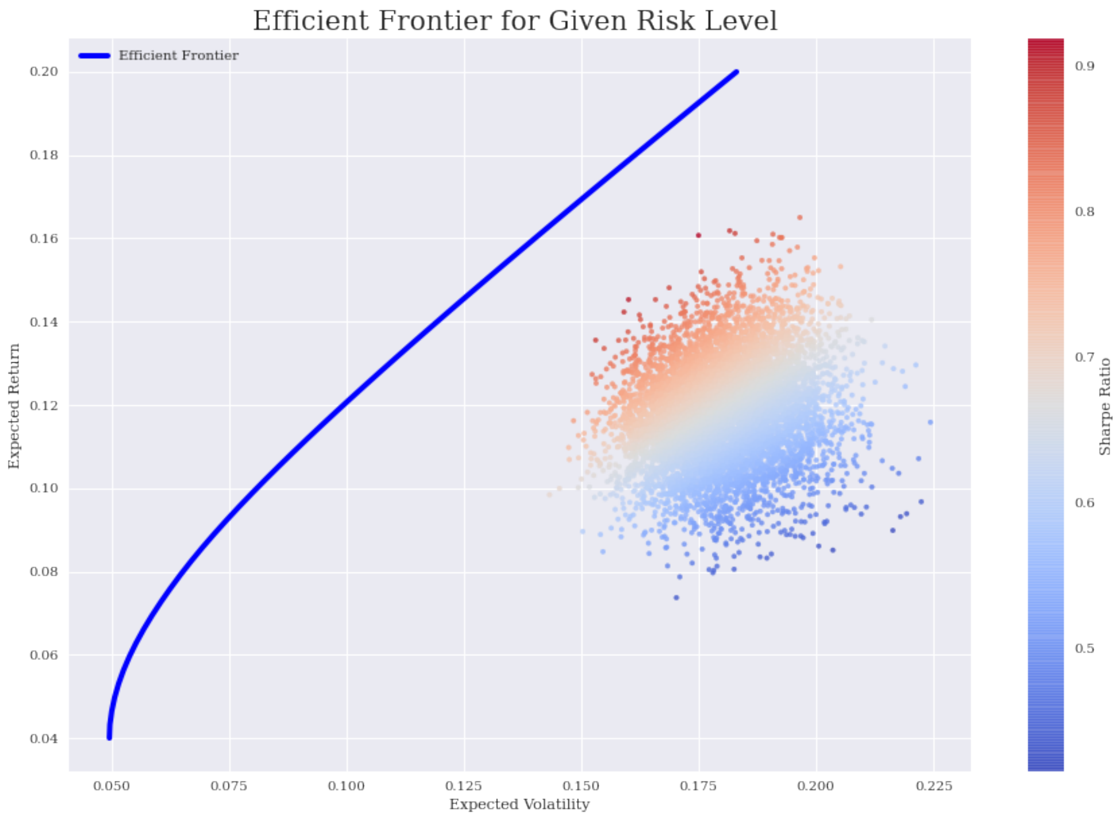
```
Your portfolio has high volatility, which is not necessarily a bad thing.
```

0.4 Graph of Efficient Frontier

```
[ ]: cons = ({'type':'eq', 'fun': lambda x: port_ret(x)-tret}, {'type':'eq', 'fun':␣
      ↪lambda x: np.sum(x)-1})
     bnds = tuple((0,1)for x in weights)
     trets = np.linspace(.04,.2,50)
     tvols = []
     for tret in trets:
         res = sco.minimize(port_vol,eweights, method = "SLSQP", bounds = bnds,␣
      ↪constraints = cons)
         tvols.append(res['fun'])
     tvols = np.array(tvols)
```

```
[ ]: plt.figure(figsize = (15,10))
     plt.scatter(pvols, prets, c = prets/pvols, marker = '.', alpha = 0.8, cmap =␣
      ↪'coolwarm')
     plt.plot(tvols, trets, 'b',label = "Efficient Frontier",lw =4.0)
     plt.xlabel("Expected Volatility")
     plt.ylabel("Expected Return")
     plt.legend(loc = "upper left")
     plt.colorbar(label = "Sharpe Ratio")
     plt.title("Efficient Frontier for Given Risk Level ", fontsize = 20)
```

```
[ ]: Text(0.5, 1.0, 'Efficient Frontier for Given Risk Level ')
```



0.5 Capital Market Line and Efficient Frontier for Optimal Portfolio

[ ]:
```
import scipy.interpolate as sci
ind = np.argmin(tvols)
evols = tvols[ind:]
erets = trets[ind:]
tck = sci.splrep(evols, erets)

def f(x):
    return sci.splev(x, tck, der = 0)
def df(x):
    #First derivative of efficient frontier
    return sci.splev(x,tck, der = 1)
```

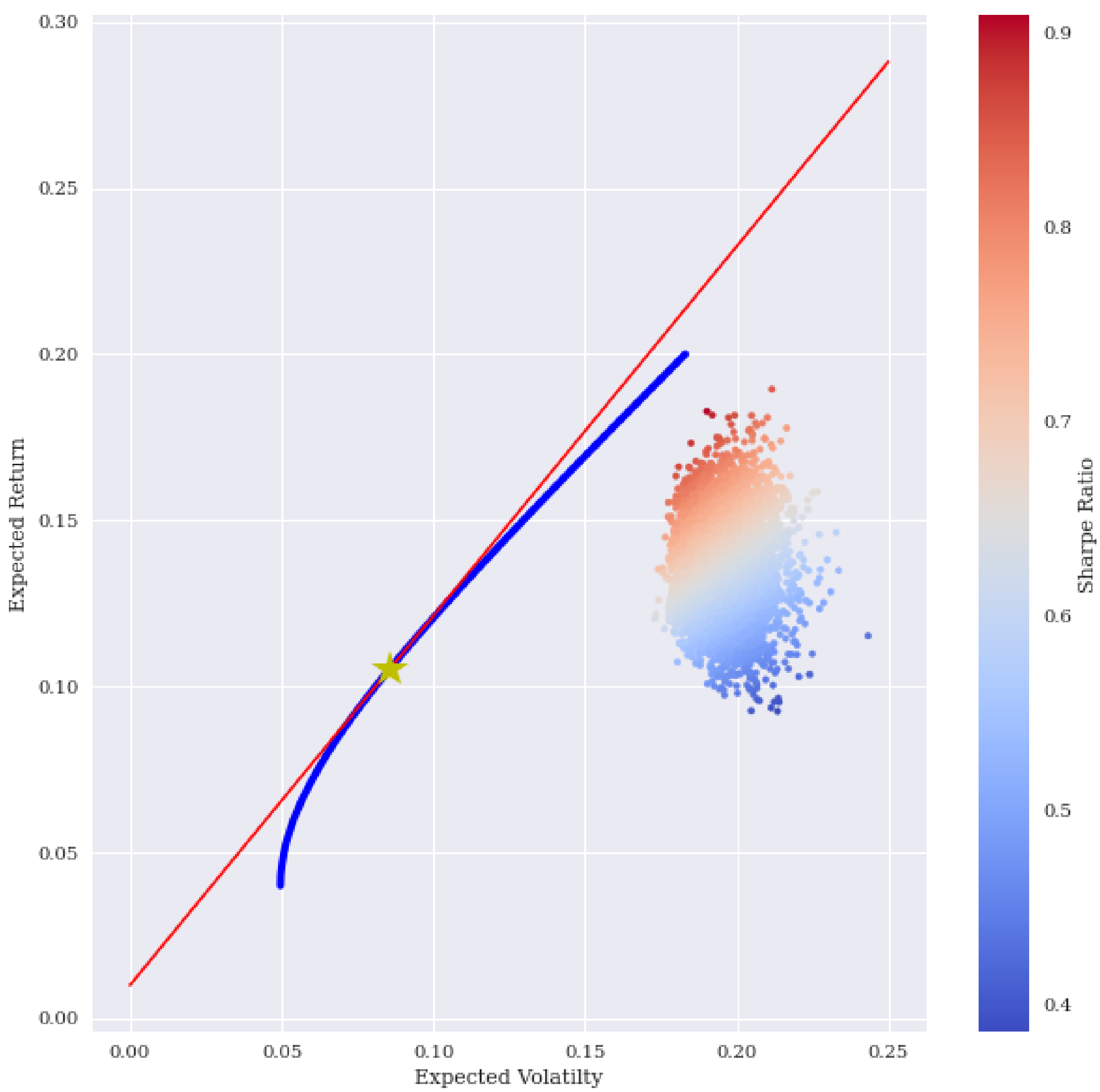
[ ]:
```
def equations(p, rf = 0.01):
    eq1 = rf - p[0]
    eq2 = rf +p[1]*p[2] - f(p[2])
    eq3 = p[1] - df(p[2])
    return eq1, eq2, eq3
opt = sco.fsolve(equations, [.01,.5,.15])
```

```
np.round(equations(opt), 6)
```

[ ]: array([0., 0., 0.])

[ ]:
```
plt.figure(figsize = (10,10))
plt.plot(evols, erets, 'b', lw=4, label = "Efficient Frontier")
plt.scatter(pvols,prets, c = (prets -.01)/pvols,marker = '.', cmap = 'coolwarm')
cx = np.linspace(0,.250)
plt.plot(cx, opt[0]+opt[1]*cx, 'r', lw =1.5)
plt.plot(opt[2], f(opt[2]), 'y*', markersize = 20.0)
plt.grid(True)
plt.xlabel("Expected Volatilty")
plt.ylabel("Expected Return")
plt.colorbar(label = "Sharpe Ratio")
```

[ ]: <matplotlib.colorbar.Colorbar at 0x26e2664f040>



[ ]:
```
cons = ({'type':'eq', 'fun': lambda x: port_ret(x)-f(opt[2])}, {'type':'eq',␣
 ↪'fun': lambda x: np.sum(x)-1})
res = sco.minimize(port_vol, eweights, method = "SLSQP", bounds = bnds,␣
 ↪constraints = cons)

pd.DataFrame(res['x'].round(3), index =["Apple", "Budweiser", "Goldman", "Home␣
 ↪Depot", "JP Morgan", "Microsoft", "Netflix", "Occidental Petroleum",␣
 ↪"Vanguard Real Estate", "Vanguard High Dividend","Essential Utilities",␣
 ↪"Exxon", "S&P 500"] , columns = ["Weights"])
```

[ ]:

	Weights
Apple	0.000
Budweiser	0.145
Goldman	0.000
Home Depot	0.000
JP Morgan	0.000
Microsoft	0.000
Netflix	0.000
Occidental Petroleum	0.000
Vanguard Real Estate	0.000
Vanguard High Dividend	0.660
Essential Utilities	0.131
Exxon	0.064
S&P 500	0.000

[ ]:
```
pd.DataFrame([port_ret(res['x']).round(4), port_vol(res['x']).round(4)], index␣
 ↪= ["Portfolio Return", "Portfolio Volatility"], columns = ["Portfolio␣
 ↪Characteristics"])
```

[ ]:

	Portfolio Characteristics
Portfolio Return	0.105
Portfolio Volatility	0.164

The reason I took out *TIPs* is because you want to be aggressive. The optimal portfolio allocation with TIPs would definitely limit risk but you would not have sustained returns. Does the allocation of Vanguard High Dividend make sense? It does because it's sharpe ratio is the highest, so you are better off always allocating more in there than holding considerable apple shares.