Homework 3

May 7, 2023

1 Homework 3

1.1 FINM 37500 - 2023

1.1.1 UChicago Financial Mathematics

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2 1. Treasury Futures and Cheapest-to-Deliver

The file `data/fut_bond_data_FVU3_2023-04-21.xlsx` has market data on the following: * 5-year Treasury future, expiring September 2023 * The specifications of the deliverable treasury bonds

Market quotes are provided on the futures contract and the bond prices. These will be useful for some of the analysis questions, but you do not need them for your models as you are provided a BDT tree which is fit to swaps and caps. See below for more details on this BDT model.

Suppose the present date is `2023-04-21`.

[ ]:
```
import pandas as pd
import numpy as np
from Binomial_Fixed import binomial, ratecurves
from Binomial_Fixed import ficcvol
from treasury_cmds import *
import scipy
from scipy.optimize import fsolve
from scipy.stats import norm
from datetime import date
import datetime as dt
import matplotlib.pyplot as plt
import matplotlib as mpl
plt.style.use('seaborn')
mpl.rcParams['font.family'] = 'serif'
```

[ ]:
```
future_bonds = 'C:/Users/dcste/OneDrive/Fixed_Income_Derivatives/
↪finm-fiderivs-2023/data/fut_bond_data_FVU3_2023-04-21.xlsx'
bdt_path = 'C:/Users/dcste/OneDrive/Fixed_Income_Derivatives/finm-fiderivs-2023/
↪data/bdt_params_freq52_2023-04-21.xlsx'
```

```python
future_description = pd.read_excel(future_bonds, sheet_name='future')
bond_info = pd.read_excel(future_bonds,sheet_name='bonds')
bdt_params = pd.read_excel(bdt_path)
Future_Price = future_description.iloc[1,1]
```

[ ]:
```python
DATE = '2023-04-21'
FUT_DAYS_EXPIRE = 159
```

[ ]:
```python
future_description
```

[ ]:
```
                field           FVU3 Comdty
0      last_update_dt  2023-04-21 00:00:00
1             px_last           109.789062
2  last_tradeable_dt  2023-09-29 00:00:00
3     fut_dlv_dt_last  2023-10-04 00:00:00
4     fut_days_expire                  159
5             fut_ctd      T 3.875 11/30/27
6          fut_ctd_px           100.757812
7  fut_ctd_gross_basis          -17.074348
8    fut_ctd_net_basis             1.199828
```

[ ]:
```python
bond_info.head()
```

[ ]:
```
          ticker last_update_dt     px_last   maturity  days_to_mty    cpn  \
0  91282CFZ Govt     2023-04-21  100.757812 2027-11-30         1681  3.875
1  91282CGC Govt     2023-04-21  100.750000 2027-12-31         1712  3.875
2  91282CGH Govt     2023-04-21   99.195312 2028-01-31         1743  3.500
3  91282CGP Govt     2023-04-21  101.484375 2028-02-29         1772  4.000
4  91282CGT Govt     2023-04-21   99.828125 2028-03-31         1803  3.625

   nxt_cpn_dt  days_to_next_coupon   int_acc  accrued_days_between_cpn_dates  \
0 2023-05-31                   37  1.543613                             182
1 2023-06-30                   67  1.220304                             181
2 2023-07-31                   98  0.802486                             181
3 2023-08-31                  129  0.597826                             184
4 2023-09-30                  159  0.237705                             183

   days_acc  basis_mid  repo_implied_reporate  repo_reporate  conversion
0       145  13.138350               3.619994          4.815      0.9226
1       114  17.307175               3.644369          4.815      0.9212
2        83  24.180222               3.631045          4.815      0.9058
3        55  32.893100               3.550495          4.815      0.9234
4        24  38.039247               3.570506          4.815      0.9075
```

[ ]:
```python
bond_info['cash_price'] = bond_info['px_last'] + .
↪5*bond_info['cpn']*(bond_info['days_acc']/
↪(bond_info['accrued_days_between_cpn_dates']))
```

```
[ ]: def␣
     ↪forward_price(spot_price,cpn,repo,days_next_cpn,int_accrued,accrued_days_btwn_dates,DAY_COU
     ↪= 365.25):
        coupon = .5*(cpn)
        repo = repo*(1/100)
        I = int_accrued
        inner = spot_price-coupon*np.exp(-repo*(days_next_cpn/
     ↪accrued_days_btwn_dates))
        #inner = (spot_price-I)
        return inner*np.exp(repo*(159/365.25))

     def carry(cpn,repo, days_acc, days_fwd,accrued_days_between_cpns,DAY_COUNT =␣
     ↪360):
        cpn = .5*cpn
        return (cpn*(days_fwd/accrued_days_between_cpns) - cpn*(days_acc/
     ↪accrued_days_between_cpns)) - repo*(159/DAY_COUNT)
```

```
[ ]: bond_info['gross_basis'] =␣
     ↪bond_info['px_last']-bond_info['conversion']*Future_Price
     bond_info['carry'] =␣
     ↪(bond_info['cpn']-bond_info['repo_reporate'])*(FUT_DAYS_EXPIRE/365.
     ↪25)-bond_info['int_acc']
```

```
[ ]: bond_info['gross_basis'] = 32*(bond_info['px_last'] -␣
     ↪Future_Price*bond_info['conversion'])
     bond_info['net_basis'] = (bond_info['gross_basis'] - bond_info['carry'])
```

2.0.1 BDT Model

In this problem you will make use of a BDT modeled binomial tree.

To save you some time, you are provided the parameters of a BDT tree fit to both swaps and caps. * Use the file `bdt_params_freq52_2023-04-21.xlsx` * With these σ and θ parameters, you should be able to build a BDT tree with $T = 5$ and $dt = 1/52$.

Note If interested in how this was done, find the data and files used to get these parameters. In particular, * The market quotes interpolated to weekly frequency: `cap_curves_2023-04-21_freq_52.xlsx`. * The file to estimate the model is `Parameterize BDT.ipynb`.

2.1 1.1 Trading Bonds

Give brief answers to these based on the market quotes provided, ### 1.1.1 Calculate the * gross basis * carry * net basis for each bond

2.1.1 1.1.2

Which bond seems most likely to be CTD?

2.1.2 1.1.3

If you were required to put on a position today * long one of the bonds * short the future

which would you choose based on the data provided in the spreadsheet?

[ ]:
```
#1.1.1
bond_info.iloc[:,-3:]
```

[ ]:
```
   gross_basis     carry  net_basis
0   -17.074450 -1.952812 -15.121638
1   -12.405900 -1.629503 -10.776397
2    -8.051850 -1.374930  -6.676920
3     3.364950 -0.952610   4.317560
4     6.225625 -0.755734   6.981359
```

2.2 1.2 Conversion Factors

Calculate the conversion factor for each bond. Report it to 6 decimal places.

Do they match the conversion factor provided by Bloomberg?

[ ]:
```
tmat = (bond_info['days_to_mty'] - FUT_DAYS_EXPIRE)/365.25
conversion_factor = pd.DataFrame(ratecurves.price_bond(.
 ↪06,tmat,bond_info['cpn']/(100),cpnfreq = 2,face = 100, accr_frac=0)/100,␣
 ↪columns=['Conversion Factor'])
conversion_factor = conversion_factor.merge(bond_info['conversion'],␣
 ↪left_index=True, right_index=True)
conversion_factor.index = bond_info['ticker']
converts = ratecurves.price_bond(.06,tmat,bond_info['cpn']/(100),cpnfreq =␣
 ↪2,face = 100, accr_frac=0)/100
```

[ ]:
```
```

[ ]:
```
conversion_factor
```

[ ]:
```
               Conversion Factor  conversion
ticker
91282CFZ Govt           0.922669      0.9226
91282CGC Govt           0.921283      0.9212
91282CGH Govt           0.905770      0.9058
91282CGP Govt           0.923408      0.9234
91282CGT Govt           0.907522      0.9075
```

2.3 1.3 BDT Tree

Report the number of steps for * each bond's maturity * the futures contract expiration

Build the interest-rate tree and display it.

[ ]:
```
FREQUENCY = 52
def number_steps(num_days, frequency):
    years = num_days/365.25
    tree_steps = round(round(years*frequency)/frequency,6)
    return tree_steps
```

[ ]:
```
coupons = number_steps(bond_info['days_to_next_coupon'], frequency=52)
maturities = number_steps(bond_info['days_to_mty'], frequency=52)
Future_Expiry = number_steps(159,frequency=FREQUENCY)
maturities
```

[ ]:
```
0    4.596154
1    4.692308
2    4.769231
3    4.846154
4    4.942308
Name: days_to_mty, dtype: float64
```

[ ]:
```
for i in maturities:
    print(f'tsteps {int(52*i)}')
```

```
tsteps 239
tsteps 244
tsteps 248
tsteps 252
tsteps 257
```

- Each bond will have 5 time steps

2.4 1.4 Bond Pricing

Use the tree to price each bond. Report * time-0 dirty and clean price of each bond * terminal (clean) value of each bond at futures expiration, for each state of the tree.

Thus, to report the terminal values you will need to grab the expiration column of each bond's (clean) pricing tree and adjust (inflate) it for the conversion factor.

[ ]:
```
bdt_params = bdt_params.set_index('maturity')
```

[ ]:
```
Future_Expiry
```

[ ]:
```
0.442308
```

[ ]:
```
bdt_params.head()
```

[ ]:

maturity	discount	fwd vol	theta
0.019231	0.999060	0.234111	0.426576
0.038462	0.998113	0.234111	0.462743

```
0.057692  0.997159  0.211844  0.422369
0.076923  0.996200  0.203757  0.358536
0.096154  0.995236  0.201575  0.284977
```

[ ]:
```
ratetree=  binomial.BDTtree(bdt_params['theta'],sigmas=bdt_params['fwd␣
↪vol'],px_bond0=bdt_params['discount'].iloc[0], dt = 1/FREQUENCY)
ratetree.loc[:,:Future_Expiry].head()
```

[ ]:
```
time   0.000000  0.019231  0.038462  0.057692  0.076923  0.096154  0.115385  \
state
0      0.048894  0.050923  0.053074  0.055102  0.057073  0.059013  0.060938
1           NaN  0.047722  0.049737  0.051638  0.053485  0.055303  0.057107
2           NaN       NaN  0.046610  0.048391  0.050122  0.051827  0.053517
3           NaN       NaN       NaN  0.045630  0.047262  0.048869  0.050463
4           NaN       NaN       NaN       NaN  0.044666  0.046184  0.047691

time   0.134615  0.153846  0.173077  …  0.269231  0.288462  0.307692  \
state                                …
0      0.062859  0.064785  0.066724  …  0.076833  0.078980  0.081182
1      0.058908  0.060712  0.062529  …  0.072003  0.074015  0.076079
2      0.055204  0.056896  0.058598  …  0.067476  0.069362  0.071296
3      0.052054  0.053649  0.055254  …  0.063626  0.065404  0.067227
4      0.049194  0.050701  0.052218  …  0.060130  0.061810  0.063534

time   0.326923  0.346154  0.365385  0.384615  0.403846  0.423077  0.442308
state
0      0.083446  0.085778  0.088183  0.090670  0.093245  0.095916  0.098691
1      0.078200  0.080385  0.082639  0.084970  0.087383  0.089886  0.092487
2      0.073284  0.075331  0.077444  0.079628  0.081889  0.084235  0.086673
3      0.069102  0.071033  0.073025  0.075084  0.077217  0.079429  0.081727
4      0.065305  0.067130  0.069013  0.070959  0.072974  0.075064  0.077236

[5 rows x 24 columns]
```

[ ]:
```
FV = 100
compound = FREQUENCY
dt = 1/compound
cpn_freq = 2
```

[ ]:
```
terminal_values = pd.DataFrame(dtype=float, index = ratetree.index, columns =␣
↪bond_info.index)
px_bonds = pd.DataFrame(dtype=float, index =bond_info.index ,␣
↪columns=['clean_price'])
px_dirty = pd.DataFrame(dtype = float, index = bond_info.index,␣
↪columns=['Dirty_Price'])
```

```
[ ]: for idx, bond in enumerate(bond_info.index):
         print(bond_info.loc[bond,'cpn'])
```

```
3.875
3.875
3.5
4.0
3.625
```

```
[ ]: for idx, bond in enumerate(bond_info.index):
         time_steps = round(maturities[idx]/dt)
         cpn = bond_info.loc[bond,'cpn']/100

         wrapper_function = lambda rate : binomial.payoff_bond(rate, dt,␣
      ↪facevalue=FV*(1+cpn/cpn_freq))

         cftree = binomial.construct_bond_cftree(maturities[idx], compound =␣
      ↪compound, cpn = cpn)
         if coupons[idx] == 0:
             cftree.loc[0,0] += cpn/2

         bondtree = binomial.bintree_pricing(payoff=wrapper_function,␣
      ↪ratetree=ratetree.iloc[:time_steps,:time_steps],cftree=cftree)
         accrued_int_tree = binomial.construct_accint(timenodes=bondtree.columns.
      ↪values,freq = compound, cpn = cpn)
         #dirty_tree = bondtree+accrued_int_tree
         px_dirty.loc[bond] = bondtree.iloc[0,0]
         clean_tree = np.maximum(bondtree - accrued_int_tree,0)
         px_bonds.loc[bond] = clean_tree.iloc[0,0]
         terminal_values[bond] = clean_tree[Future_Expiry]
```

```
[ ]: bond_info
```

```
[ ]:           ticker last_update_dt     px_last   maturity  days_to_mty    cpn  \
     0  91282CFZ Govt     2023-04-21  100.757812 2027-11-30         1681  3.875
     1  91282CGC Govt     2023-04-21  100.750000 2027-12-31         1712  3.875
     2  91282CGH Govt     2023-04-21   99.195312 2028-01-31         1743  3.500
     3  91282CGP Govt     2023-04-21  101.484375 2028-02-29         1772  4.000
     4  91282CGT Govt     2023-04-21   99.828125 2028-03-31         1803  3.625

         nxt_cpn_dt  days_to_next_coupon   int_acc  accrued_days_between_cpn_dates  \
     0 2023-05-31                   37  1.543613                             182
     1 2023-06-30                   67  1.220304                             181
     2 2023-07-31                   98  0.802486                             181
     3 2023-08-31                  129  0.597826                             184
     4 2023-09-30                  159  0.237705                             183
```

```
   days_acc  basis_mid  repo_implied_reporate  repo_reporate  conversion  \
0       145  13.138350               3.619994          4.815      0.9226
1       114  17.307175               3.644369          4.815      0.9212
2        83  24.180222               3.631045          4.815      0.9058
3        55  32.893100               3.550495          4.815      0.9234
4        24  38.039247               3.570506          4.815      0.9075

   cash_price  gross_basis     carry  net_basis
0  102.301425   -17.074450 -1.952812 -15.121638
1  101.970304   -12.405900 -1.629503 -10.776397
2   99.997799    -8.051850 -1.374930  -6.676920
3  102.082201     3.364950 -0.952610   4.317560
4  100.065830     6.225625 -0.755734   6.981359
```

[ ]:
```
px_bonds
```

[ ]:
```
   clean_price
0   100.715335
1   100.739787
2    99.142046
3   101.320874
4    99.686176
```

[ ]:
```
px_dirty
```

[ ]:
```
   Dirty_Price
0   102.280239
1   101.932095
2    99.949739
3   101.936259
4    99.895311
```

[ ]:
```
bondtree.head()
```

[ ]:
```
time    0.000000    0.019231    0.038462    0.057692    0.076923    0.096154  \
state
0      99.895311   98.727797   97.487076   96.170555   94.775408   93.299222
1            NaN  101.250771  100.161981   99.002701   97.769627   96.459753
2            NaN         NaN  102.525489  101.512961  100.432499   99.280730
3            NaN         NaN         NaN  103.721899  102.782449  101.777977
4            NaN         NaN         NaN         NaN  104.843464  103.973843

time     0.115385    0.134615    0.153846    0.173077  …  4.750000  4.769231  \
state                                                  …
0       91.739985   90.096121   88.366552   86.550751  …       0.0       0.0
1       95.070348   93.598996   92.043648   90.402680  …       0.0       0.0
2       98.054445   96.750634   95.366532   93.899674  …       0.0       0.0
```

```
3      100.705016   99.560193   98.340271  97.042197  …       0.0       0.0
4      103.042331  102.045394  100.979544  99.841370  …       0.0       0.0

time   4.788462  4.807692  4.826923  4.846154  4.865385  4.884615  4.903846  \
state
0           0.0       0.0       0.0       0.0       0.0       0.0       0.0
1           0.0       0.0       0.0       0.0       0.0       0.0       0.0
2           0.0       0.0       0.0       0.0       0.0       0.0       0.0
3           0.0       0.0       0.0       0.0       0.0       0.0       0.0
4           0.0       0.0       0.0       0.0       0.0       0.0       0.0

time   4.923077
state
0           0.0
1           0.0
2           0.0
3           0.0
4           0.0

[5 rows x 257 columns]
```

[ ]:
```
bond_info['px_last']
```

[ ]:
```
0    100.757812
1    100.750000
2     99.195312
3    101.484375
4     99.828125
Name: px_last, dtype: float64
```

[ ]:
```
terminal_values = terminal_values.dropna()
```

2.5 1.5 CTD

Use your terminal values calculated above to state which bond is CTD in each interest-rate state (at this expiration node.)

Report the duration of each bond (as of today's price, not recomputed for the interest-rate nodes.) Do you see a relationship between the time-0 duration and the at-expiration CTD?

[ ]:
```
def highlight_min(s):
    is_min = s == s.min()
    return ['background-color: green' if v else '' for v in is_min]
```

[ ]:
```
bond_info
```

[ ]:
```
          ticker last_update_dt     px_last   maturity  days_to_mty    cpn  \
0  91282CFZ Govt     2023-04-21  100.757812 2027-11-30         1681  3.875
```

```
1  91282CGC Govt     2023-04-21  100.750000 2027-12-31          1712  3.875
2  91282CGH Govt     2023-04-21   99.195312 2028-01-31          1743  3.500
3  91282CGP Govt     2023-04-21  101.484375 2028-02-29          1772  4.000
4  91282CGT Govt     2023-04-21   99.828125 2028-03-31          1803  3.625

   nxt_cpn_dt  days_to_next_coupon   int_acc  accrued_days_between_cpn_dates  \
0 2023-05-31                   37  1.543613                             182
1 2023-06-30                   67  1.220304                             181
2 2023-07-31                   98  0.802486                             181
3 2023-08-31                  129  0.597826                             184
4 2023-09-30                  159  0.237705                             183

   days_acc  basis_mid  repo_implied_reporate  repo_reporate  conversion  \
0       145  13.138350               3.619994          4.815      0.9226
1       114  17.307175               3.644369          4.815      0.9212
2        83  24.180222               3.631045          4.815      0.9058
3        55  32.893100               3.550495          4.815      0.9234
4        24  38.039247               3.570506          4.815      0.9075

   cash_price  gross_basis     carry  net_basis
0  102.301425   -17.074450 -1.952812 -15.121638
1  101.970304   -12.405900 -1.629503 -10.776397
2   99.997799    -8.051850 -1.374930  -6.676920
3  102.082201     3.364950 -0.952610   4.317560
4  100.065830     6.225625 -0.755734   6.981359
```

[ ]:
```
Future_Price*conversion_factor['Conversion Factor'].values
```

[ ]:
```
array([101.29896327, 101.14684537,  99.44368169, 101.38014068,
        99.63593948])
```

[ ]:
```
t_values = terminal_values - Future_Price*conversion_factor['Conversion␣
↪Factor'].values
```

[ ]:
```
cdts = terminal_values.style.apply(highlight_min,axis = 1)
cdts
```

[ ]:
```
<pandas.io.formats.style.Styler at 0x23ba929f1c0>
```

[ ]:
```
bondstats = pd.DataFrame(dtype=float, index = bond_info.index,␣
↪columns=['ticker','px_last','cpn','ytm','duration','conversion'])
for idx, bond in enumerate(bond_info.index):
    T = bond_info.loc[idx,'days_to_mty']/365.25
    bondstats.loc[bond,'ticker'] = bond_info.loc[bond,'ticker']
    cpn =bond_info.loc[bond,'cpn']/100
    bondstats.loc[idx,'cpn'] = cpn
```

```
    accfrac = bond_info.loc[bond,'days_acc']/bond_info.
↪loc[bond,'accrued_days_between_cpn_dates']
    bondstats.loc[bond,'px_last'] = bond_info.loc[bond,'px_last']
    p = bond_info.loc[bond,'px_last']
    bondstats.loc[bond,'conversion'] = bond_info.loc[bond,'conversion']
    bondstats.loc[bond,'ytm'] = ratecurves.ytm(p,T = T, cpn =␣
↪cpn,cpnfreq=2,face = 100,accr_frac=accfrac)
    bondstats.loc[bond,'duration'] = ratecurves.duration_closed_formula(tau =␣
↪T, ytm = bondstats.loc[bond,'ytm'],cpnrate=cpn,freq = 2)
```

[ ]:
```
bondstats
```

[ ]:

	ticker	px_last	cpn	ytm	duration	conversion
0	91282CFZ Govt	100.757812	0.03875	0.040795	4.257591	0.9226
1	91282CGC Govt	100.750000	0.03875	0.039922	4.329998	0.9212
2	91282CGH Govt	99.195312	0.03500	0.038884	4.432692	0.9058
3	91282CGP Govt	101.484375	0.04000	0.037915	4.459884	0.9234
4	91282CGT Govt	99.828125	0.03625	0.037173	4.563210	0.9075

2.6 1.6 Futures Price

Model the futures price with the tree approach. * Use the CTD terminal value for each rate. * Step backward through the tree.

As you step backward remember that for a futures contract * no discounting by the riskfree rate * the futures contract has no capital requirement and thus an expected P&L of zero under this measure.

Thus, each node is the simple average of the two nodes at the following step.

Report the futures price.

2.6.1 Compare

How does it compare to * the quoted futures price * the modeled bond prices

[ ]:
```
converts
```

[ ]:
```
0    0.922669
1    0.921283
2    0.905770
3    0.923408
4    0.907522
dtype: float64
```

[ ]:
```
payoff_func = lambda rate: (terminal_values/converts).min(axis = 1).values
ratetree_fwd_measure = ratetree.copy().loc[:,:Future_Expiry].dropna(how = 'all')
ratetree_fwd_measure *= 0
```

```
future_tree = binomial.
  ↪bintree_pricing(payoff=payoff_func,ratetree=ratetree_fwd_measure)
future_tree.head()
```

[ ]:

```
time      0.000000    0.019231    0.038462    0.057692    0.076923    0.096154  \
state
0       109.733681  108.483577  107.146463  105.717779  104.192974  102.567635
1              NaN  110.983785  109.820690  108.575147  107.242583  105.818314
2              NaN         NaN  112.146879  111.066233  109.907712  108.666853
3              NaN         NaN         NaN  113.227525  112.224754  111.148570
4              NaN         NaN         NaN         NaN  114.230297  113.300938

time      0.115385    0.134615    0.153846    0.173077  …    0.269231  \
state                                                   …
0       100.837666   98.999555   97.050700   94.989779  …   83.079495
1       104.297603  102.675777  100.948410   99.111620  …   88.240055
2       107.339025  105.919430  104.403143  102.785200  …   93.029135
3       109.994681  108.758619  107.435717  106.021086  …   97.391946
4       112.302460  111.230743  110.081521  108.850349  …  101.291408

time     0.288462   0.307692   0.326923   0.346154   0.365385   0.384615  \
state
0       80.413661  77.669611  74.854741  71.976973  69.045180  66.069144
1       85.745330  83.157711  80.484480  77.732509  74.908765  72.021216
2       90.734780  88.332949  85.830941  83.236451  80.556253  77.796314
3       95.323490  93.136611  90.834958  88.425430  85.916649  83.316192
4       99.460402  97.510370  95.438264  93.244485  90.934212  88.517106

time     0.403846   0.423077   0.442308
state
0       63.059492  60.027637  56.985739
1       69.078796  66.091347  63.069535
2       74.963636  72.066245  69.113160
3       80.628992  77.861027  75.019331
4       86.003392  83.396958  80.702724

[5 rows x 24 columns]
```

[ ]:

```
future_comps = pd.DataFrame({'Quote':[Future_Price],'Model':[future_tree.
  ↪iloc[0,0]]},index = ['Price'])
future_comps
```

[ ]:

```
             Quote       Model
Price  109.789062  109.733681
```

[ ]:

```
quality = terminal_values.copy().divide(converts,axis = 1)
quality.index = ratetree.loc[:,Future_Expiry].dropna()
```

```
quality.columns = bond_info.ticker
quality
```

[ ]:

```
ticker    91282CFZ Govt  91282CGC Govt  91282CGH Govt  91282CGP Govt  \
0.442308
0.098691      58.732674      58.266055      57.631254      57.685177
0.092487      64.717878      64.273279      63.694682      63.713960
0.086673      70.617355      70.207746      69.696654      69.689486
0.081727      76.338263      75.975385      75.541271      75.516265
0.077236      81.801717      81.495497      81.145590      81.111245
0.073037      86.943373      86.701534      86.440612      86.404857
0.069043      91.715501      91.543352      91.373701      91.343536
0.065205      96.087365      95.987780      95.909305      95.890552
0.061496     100.044353     100.017907     100.028384     100.025555
0.057899     103.586174     103.631411     103.726826     103.743113
0.054406     106.724446     106.838237     107.013188     107.050531
0.051015     109.480002     109.657940     109.906049     109.965283
0.047726     111.880184     112.116987     112.431301     112.512341
0.044541     113.956338     114.246222     114.619566     114.721610
0.041463     115.741654     116.078660     116.503932     116.625664
0.038499     117.269425     117.647683     118.118085     118.257853
0.035650     118.571741     118.985665     119.494866     119.650844
0.032922     119.678590     120.123014     120.665254     120.835562
0.030318     120.617314     121.087573     121.657710     121.840511
0.027841     121.412349     121.904314     122.497836     122.691399
0.025493     122.085187     122.595259     123.208265     123.411007
0.023275     122.654482     123.179573     123.808728     124.019232
0.021188     123.136258     123.673750     124.316234     124.533255
0.019232     123.544169     124.091861     124.745311     124.967772

ticker    91282CGT Govt
0.442308
0.098691      56.985739
0.092487      63.069535
0.086673      69.113160
0.081727      75.019331
0.077236      80.702724
0.073037      86.091191
0.069043      91.128463
0.065205      95.775158
0.061496     100.008493
0.057899     103.820928
0.054406     107.218070
0.051015     110.216142
0.047726     112.839295
0.044541     115.117019
0.041463     117.081816
```

```
0.038499     118.767244
0.035650     120.206381
0.032922     121.430705
0.030318     122.469353
0.027841     123.348697
0.025493     124.092177
0.023275     124.720318
0.021188     125.250879
0.019232     125.699077
```

[ ]:
```
quality.style.highlight_min(color = 'green', axis = 1)
```

[ ]:
```
<pandas.io.formats.style.Styler at 0x23ba92bd610>
```

[ ]:
```
bondstats
```

[ ]:

	ticker	px_last	cpn	ytm	duration	conversion
0	91282CFZ Govt	100.757812	0.03875	0.040795	4.257591	0.9226
1	91282CGC Govt	100.750000	0.03875	0.039922	4.329998	0.9212
2	91282CGH Govt	99.195312	0.03500	0.038884	4.432692	0.9058
3	91282CGP Govt	101.484375	0.04000	0.037915	4.459884	0.9234
4	91282CGT Govt	99.828125	0.03625	0.037173	4.563210	0.9075

[ ]:
```
bond_comps = pd.concat([bond_info['px_last'], px_bonds], axis=1).
  ↪rename(columns={'px_last':'Quote','clean_price':'Model'})
bond_comps_converted = bond_comps.copy().divide(converts,axis = 0)
fut_vs_bonds = pd.concat([future_comps,bond_comps_converted],axis = 0)
fut_vs_bonds
```

[ ]:

	Quote	Model
Price	109.789062	109.733681
0	109.202557	109.156519
1	109.358310	109.347224
2	109.514855	109.456047
3	109.901942	109.724880
4	110.000832	109.844419

[ ]:
```
bond_comps_converted
```

[ ]:

	Quote	Model
0	109.202557	109.156519
1	109.358310	109.347224
2	109.514855	109.456047
3	109.901942	109.724880
4	110.000832	109.844419

2.7 1.7 Early Delivery

Optional Above we modeled the terminal value at the futures expiration. Now consider if early delivery would be better.

Which periods in the tree are eligible to deliver based on the parameters of the 5-year futures contract?

Based on your model, does it make sense to deliver early in any of the nodes of the tree?

2.8 1.8 Option-Adjusted Spread

Optional

Calculate and report the option-adjusted spread (OAS) for the future.

Note that you * do NOT need to recalculate the bond prices * will simply add a constant rate (at every node) for discounting the futures price in the previous problem.

What does the OAS indicate?

---

3 2. Fed Funds Futures

The file `data/fedfutures_2023-04-21.xlsxjj` has market data on the following: * Fed Fund Futures Chain out 18 months. * Dates of upcoming Fed meetings (approximated in 2024.) * Spot Fed Funds data * Prices of the futures chain on a historic date.

Suppose the present date is `2023-04-21`.

[ ]:
```
fed_file = 'C:/Users/dcste/OneDrive/Fixed_Income_Derivatives/finm-fiderivs-2023/
↪data/fedfutures_2023-04-21.xlsx'
fed_futures = pd.read_excel(fed_file, sheet_name = 'fed futures')
fed_funds = pd.read_excel(fed_file,sheet_name='fed funds')
fed_futures_historic = pd.read_excel(fed_file,sheet_name='fed futures historic')
fed_meetings = pd.read_excel(fed_file,sheet_name='fed meetings')
fed_futures_historic
```

[ ]:

	ticker	2022-10-25 00:00:00
0	FFJ3 Comdty	95.105
1	FFK3 Comdty	95.085
2	FFM3 Comdty	95.100
3	FFN3 Comdty	95.115
4	FFQ3 Comdty	95.155
5	FFU3 Comdty	95.175
6	FFV3 Comdty	95.220
7	FFX3 Comdty	95.330
8	FFZ3 Comdty	95.395
9	FFF4 Comdty	95.435
10	FFG4 Comdty	95.510

```
11   FFH4 Comdty                95.545
12   FFJ4 Comdty                95.620
13   FFK4 Comdty                95.695
14   FFM4 Comdty                95.765
15   FFN4 Comdty                95.810
16   FFQ4 Comdty                95.855
17   FFU4 Comdty                95.890
```

[ ]:

3.1 2.1 Chart the Fed Futures Rates

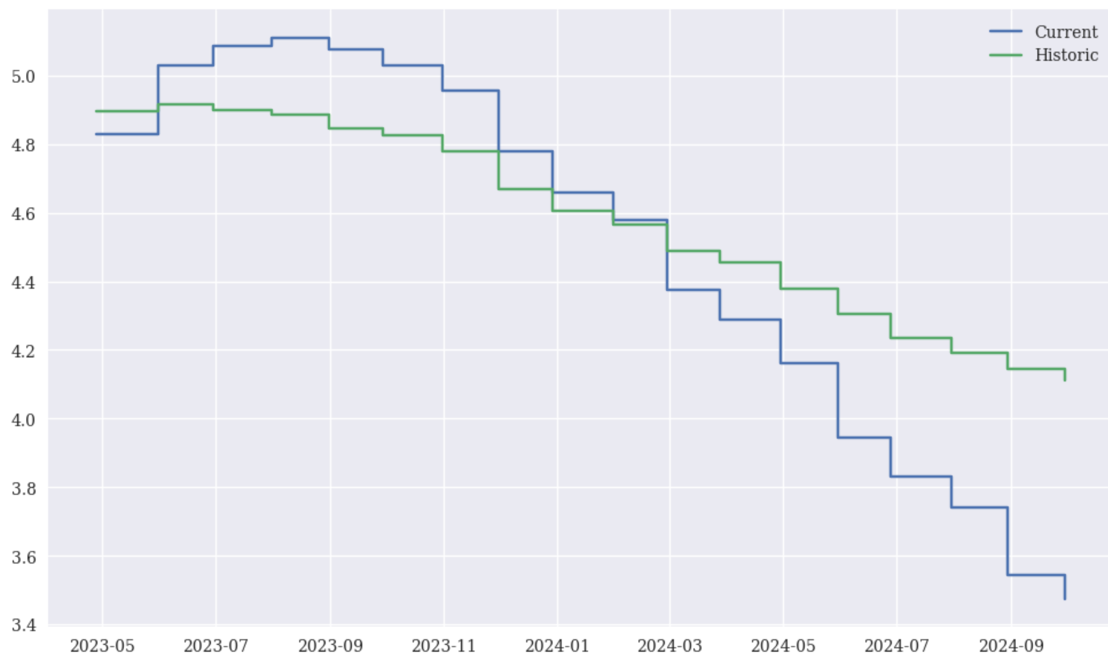
Chart the Fed Funds curve at * the present date * the historic date

Note that you are charting the implied Fed Funds Futures *rate*, not price.

Comment on how today's **open interest** varies across the chain.

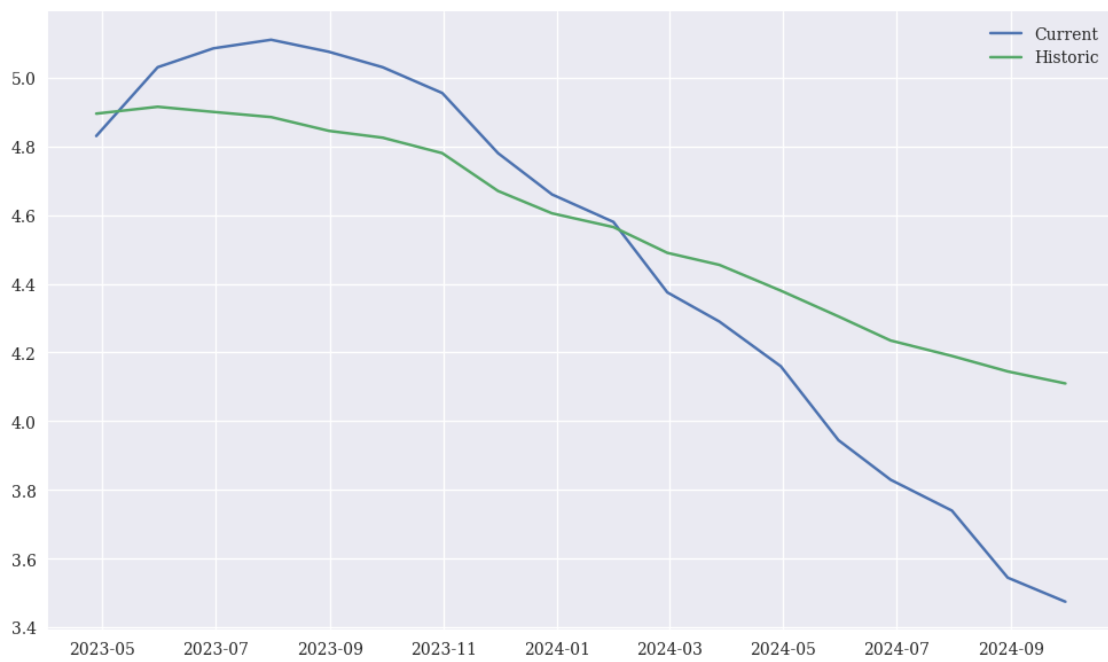
[ ]:
```
plt.figure(figsize = (12,7))
plt.step(fed_futures['last_tradeable_dt'],100-fed_futures['px_last'],where =␣
↪'post', label = 'Current')
plt.step(fed_futures['last_tradeable_dt'] , 100 - fed_futures_historic.iloc[:
↪,1], where = 'post',label = "Historic")

plt.legend(loc = 0)
```

[ ]: <matplotlib.legend.Legend at 0x23ba8b2d9a0>



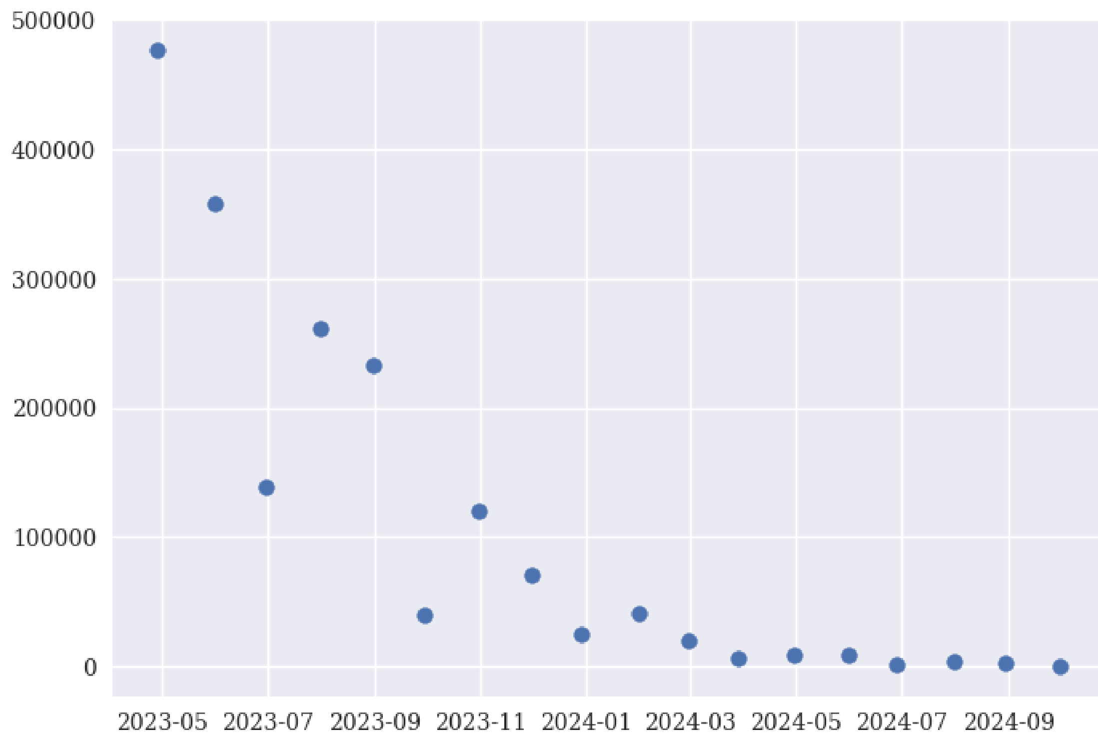
[ ]:
```
plt.figure(figsize = (12,7))
plt.plot(fed_futures['last_tradeable_dt'],100-fed_futures['px_last'], label =␣
↪'Current')
plt.plot(fed_futures['last_tradeable_dt'] , 100 - fed_futures_historic.iloc[:
↪,1], label = "Historic")
plt.legend(loc = 0)
```

[ ]: <matplotlib.legend.Legend at 0x23ba97886d0>



[ ]:
```
plt.scatter(fed_futures['last_tradeable_dt'],fed_futures['open_int'])
```

[ ]: <matplotlib.collections.PathCollection at 0x23ba9b15be0>



- The STIR futures with the most open interest are ones that mature in the front end because they are the most liquid and traded.

3.2 2.2 Extracting the Expected Path of Fed Funds Rates

The Fed has a great deal of control over the Fed Funds Rate. We simplify by assuming the Fed * sets the rate exactly at its list of meeting dates. * does not change the rate betweeen meeting dates.

Use the present data to calculate–and plot–the implied set of expected Fed Funds rates as of each meeting date.

Note One (minor) assumption: * Consider months, t, where there is a meeting, but such that in month $t+1$ there is no meeting. * There will be two reasonable ways to extract the expected fed funds rate: 1. Use the futures rate from the $t+1$-contract 2. Calculate the implied rate for the remainder of month t, knowing the expected rate at the end of month $t+1$. * These are both reasonable and will likely not differ much. * Here, use the simpler method #1–that is, for months with no meeting in the following month, the calculation is very simple.

3.3 2.3 Compare to the Historic Curve

Use the price data in the historic tab to extract the expectations at the previous date. * Note that you do not need to "bootstrap" up from the historic date to the current date. * There was no meeting in the current month, so its futures price is enough to get started.

Compare this to the answer in the previous problem, for the current data.

3.4 2.4 Analyzing the Expected Path

These questions are both conceptual–no calculation required.

3.4.1 2.4.1

Conceptually, is the path extracted above the **expected path**? In what sense is it or is it not?

3.4.2 2.4.2

Probability Distributions

The implied path above is not representative of any single actual path of Fed rates, which are typically changed by 25bps at a time.

Conceptually, what would you need to make probability statements about the Fed moving rates up/down by 25bps on any given meeting date? For instance, as seen in the `probabilities` tab of the CME FedWatch Tool?

Do not quantitatively solve this–just a conceptual answer is fine.
