Homework 2

February 5, 2023

1 Homework 2

1.1 FINM 37400 - 2023

1.1.1 UChicago Financial Mathematics

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2 1 HBS Case: Fixed-Income Arbitrage in a Financial Crisis (A): US Treasuries in November 2008

2.1 Data

- Use the data file `treasury_ts_2015-08-15.xlsx`.
- Examine the treasure issues with `kytreasno` of `204046` and `204047`. These are the bond and note (respectively) which mature on 2015-08-15.
- Look at the data on 2008-11-04.

2.2 1.1 The situation

Make a chart comparing the issues in the following features, (as of Nov 4, 2008.) * coupon rate * bid * ask * accrued interest * dirty price * duration (quoted in years, not days, assuming 365.25 days per year.) * modified duration * YTM

2.3 1.2 Hedge Ratio

Suppose you are building a trade to go long n_i bonds (`204046`) and short n_j notes (`204047`).

We can find an equation for n_j in terms of n_i such that the total holdings will have duration equal to zero. (Having zero duration also means have zero dollar duration, if helpful.)

Notation: * n_i: number of bonds purchased (or sold) * D_i: duration of bond i * $D_{\$,i}$: dollar duration of bond i, equal to $p_i D_i$

If we want the total duration of our holdings to be zero, then we need to size the trade such that n_i and n_j satisfy,

$$0 = n_i D_{\$,i} + n_j D_{\$,j}$$

$$n_j = -n_i \frac{D_{\$,i}}{D_{\$,j}}$$

Suppose you will use \$1mm of capital, leveraged 50x to buy \$50mm of the bonds (204046).

Use the ratio above to short a number of notes (204047) to keep zero duration.

Report the number of bonds and notes of your position, along with the total dollars in the short position.

[ ]:
```
import pandas as pd
import numpy as np
import datetime
import warnings
import matplotlib as mpl
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression
from scipy.optimize import minimize
%matplotlib inline
plt.style.use('seaborn')
mpl.rcParams['font.family'] = 'serif'
from treasury_cmds import *
pd.options.display.float_format = '{:,.6f}'.format
```

[ ]:
```
treasury_path = 'C:/Users/dcste/OneDrive/fixed_income/fixed_income_FORKED/
↪finm-fixedincome-2023/data/treasury_ts_2015-08-15.xlsx'
t_path_2 = 'C:/Users/dcste/OneDrive/fixed_income/fixed_income_FORKED/
↪finm-fixedincome-2023/data/treasury_quotes_2022-09-30.xlsx'
info = pd.read_excel(treasury_path, sheet_name='info', index_col = 0)
ts_data = pd.read_excel(treasury_path, sheet_name= 'database')
ts_data.columns = ts_data.columns.str.upper()
treasury_df = pd.read_excel(t_path_2)
treasury_df.columns = treasury_df.columns.str.upper()
treasury_df.sort_values('TMATDT', inplace=True)
treasury_df.set_index("KYTREASNO", inplace = True)
```

[ ]:
```
mask = ts_data[ts_data.CALDT == '2008-11-04']
metrics = mask.
↪copy()[['KYTREASNO','CALDT','TDBID','TDASK','TDNOMPRC','TDACCINT',␣
↪'TDYLD','TDDURATN','TDPUBOUT']].set_index('KYTREASNO')
metrics.columns = ['CALDT','BID','ASK', 'Nominal_Price','Accrued␣
↪Interest','YTM','Duration','Outstanding']
```

[ ]:
```
metrics['Dirty_Price'] = (metrics['BID']+metrics['ASK'])*.5 + metrics['Accrued␣
↪Interest']
```

[ ]:
```
metrics['YTM'] *= 365.25
metrics['Coupon_Rate'] = [4.25,10.625]
```

```
metrics['Duration'] /= 365.25
metrics['Modified Duration'] = metrics['Duration']/(1+(metrics['YTM']/2))
metrics['Dollar_Duration'] = metrics['Modified Duration']*metrics['Dirty_Price']
metrics = metrics.T
```

[ ]:
```
metrics
```

[ ]:
```
KYTREASNO                        204047               204046
CALDT               2008-11-04 00:00:00  2008-11-04 00:00:00
BID                          105.953125           141.859375
ASK                          105.984375           141.890625
Nominal_Price                105.968750           141.875000
Accrued Interest               0.935462             2.338655
YTM                            0.032362             0.035753
Duration                       5.935706             5.230138
Outstanding               20,998.000000         2,852.000000
Dirty_Price                  106.904212           144.213655
Coupon_Rate                    4.250000            10.625000
Modified Duration              5.841189             5.138284
Dollar_Duration              624.447664           741.010749
```

[ ]:
```
trade_pair = pd.DataFrame(data = None, columns=['204047','204046'])
trade_pair.loc['YTM'] = [0.032362,0.035753]
trade_pair.loc['Dirty_Price']  = [106.904212,144.213655]

trade_pair.loc['Modified_Duration'] = [5.841189,5.13824]
trade_pair.loc['dollar_duration'] = [624.447664,741.010749]
```

[ ]:
```
trade_pair
```

[ ]:
```
                      204047      204046
YTM                 0.032362    0.035753
Dirty_Price       106.904212  144.213655
Modified_Duration   5.841189    5.138240
dollar_duration   624.447664  741.010749
```

[ ]:
```
long_security = trade_pair.loc['YTM'].idxmax()
short_security = trade_pair.loc['YTM'].idxmin()

short_security
```

[ ]:
```
'204047'
```

[ ]:
```
p = trade_pair.loc['Dirty_Price']
p.loc['204046']
```

[ ]:
```
144.213655
```

3 Calculating Hedge Ratio

3.1 Financing Assumptions

- In this trade we take a long position in the bond which has the higher yield and short the note that has a lower yield.

```
[ ]: def hedge_ratio(dollar_duration_long, dollar_duration_short, size_long = None):
         if size_long == None:
             return -dollar_duration_long/dollar_duration_short
         else:
             return (-size_long*(dollar_duration_long))/dollar_duration_short
```

```
[ ]: LONG_EQUITY = 1e6
     def trade_balance_sheet(prices, durations, haircuts, long_equity, short_equity):
         balance_sheet = pd.DataFrame(data =␣
      ↪None,columns=['Equity','Asset_Value','Contracts','Dollar_Duration','Dirty_Price'],␣
      ↪index = [long_equity,short_equity])
         long_price = prices.loc[long_equity]
         short_price = prices.loc[short_equity]
         duration_long = durations.loc[long_equity]
         duration_short = durations.loc[short_equity]
         haircut_short = haircuts.loc['short']
         haircut_long = haircuts.loc['long']
         balance_sheet.loc[long_equity,'Dollar_Duration'] = duration_long
         balance_sheet.loc[short_equity,'Dollar_Duration'] = duration_short
         balance_sheet.loc[long_equity,'Equity'] = LONG_EQUITY
         balance_sheet.loc[long_equity,'Asset_Value'] = 5e7
         balance_sheet.loc[long_equity,'Contracts'] = balance_sheet.
      ↪loc[long_equity,'Asset_Value']/long_price
         balance_sheet.loc[short_equity,'Contracts'] =␣
      ↪hedge_ratio(dollar_duration_long=duration_long,dollar_duration_short=duration_short,\
             size_long=balance_sheet.loc[long_equity,'Contracts'])
         balance_sheet.loc[short_equity,'Asset_Value'] = balance_sheet.
      ↪loc[short_equity,'Contracts']*short_price
         balance_sheet.loc[short_equity,'Equity'] = balance_sheet.
      ↪loc[short_equity,'Asset_Value']*haircut_short
         balance_sheet.loc[short_equity,'Dirty_Price'] = short_price
         balance_sheet.loc[long_equity,'Dirty_Price'] = long_price

         return balance_sheet
```

```
[ ]: financing = pd.DataFrame(dtype='float64',index=['long','short'])
     financing['haircut'] = [.02,.02]
     financing['repo'] = [.0015,.0010]
     financing
```

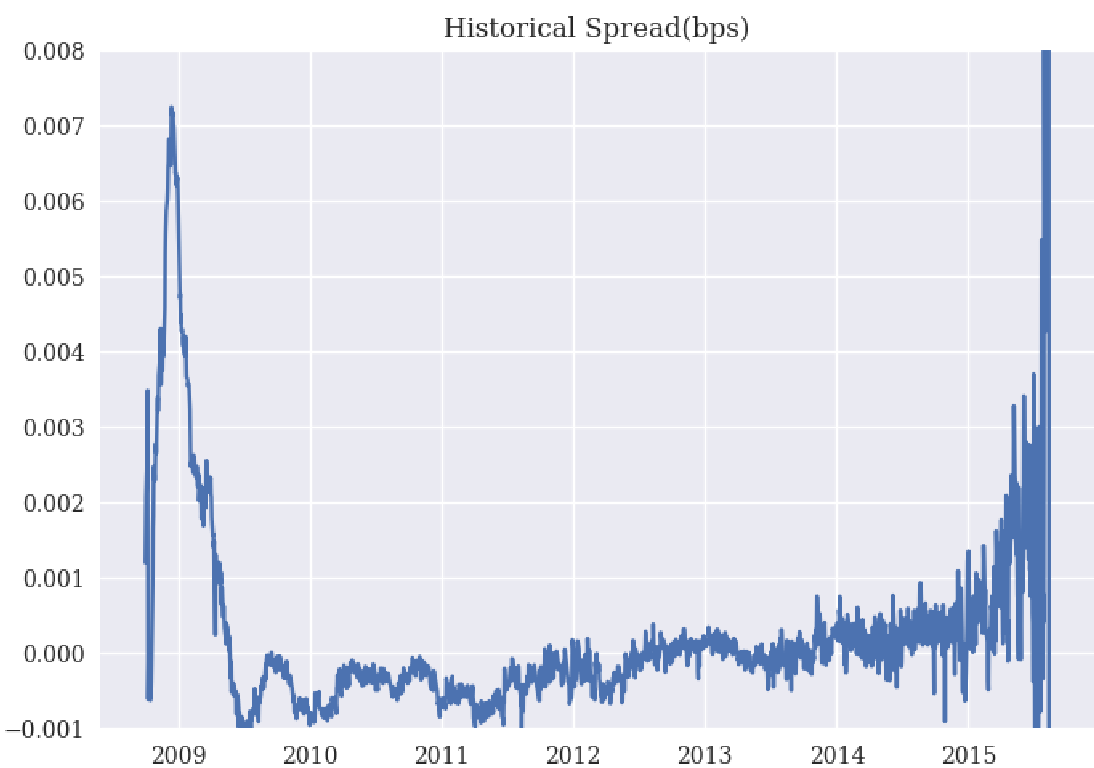
```
[ ]:         haircut      repo
     long   0.020000 0.001500
     short  0.020000 0.001000
```

```
[ ]: balance_sheet = trade_balance_sheet(prices=trade_pair.loc['Dirty_Price'],␣
     ↪durations=trade_pair.loc['dollar_duration'],haircuts = financing['haircut'],\
         long_equity = long_security, short_equity = short_security )
```

```
[ ]: spread_df = ts_data.pivot_table(values='TDYLD', index =␣
     ↪'CALDT',columns='KYTREASNO')
     spread_df.columns = ['204046','204047','206524']
     spread_df['206524'] = spread_df['204046']- spread_df['204047']
     spread_df = spread_df.rename(columns={'206524':'Spread'})
     # I need to format the yields
     spread_df *= 365.25
     spread_df = spread_df.loc['2006':,:]
```

```
[ ]: plt.plot(spread_df.loc['2008-10':,'Spread'])
     plt.title('Historical Spread(bps)')
     plt.ylim(-.001,.008)
```

```
[ ]: (-0.001, 0.008)
```



[ ]:
```
(spread_df*100).describe()
```

[ ]:

	204046	204047	Spread
count	2,410.000000	2,410.000000	2,410.000000
mean	2.009363	1.990174	0.019189
std	1.719906	1.709289	0.182395
min	-6.077314	0.007719	-6.085032
25%	0.353566	0.345058	-0.026247
50%	1.727539	1.718937	0.001298
75%	3.465700	3.398835	0.020286
max	5.221068	5.229953	2.657635

3.2 1.3 Profit Opportunity

Using the concept of **modified duration**, how much profit or loss (PnL) would you expect to make for every basis point of convergence in the spread? Specifically, assume the convergence is symmetric: the bond's (`204046`) ytm goes down 0.5bp and the note (`204047`) ytm goes up 0.5bp.

Describe the PnL you would expect to achieve on your position should this happen. Specify the PnL of the long position, the short position, and the net total.

Suppose the spread in YTM between the two securities disappears, due to a symmetric move of roughly ~17bps in each security's YTM. What is the PnL? (This is just a linearly scaling of your prior answer for a 1bp convergence.)

3.3 1.4 Result in 2008

Calculate the profit (or loss) on the position on the following two dates: * 2008-11-25 * 2008-12-16

To calculate the pnl on each date, simply use the prices of the securities on those dates along with your position sizes, (n_i, n_j). No coupon is being paid in November or December, so all you need is the "dirty" price on these two dates.

Does the pnl make sense (approximately) given your results in 1.3 with regard to the sensitivity of pnl to moves in the YTM spread?

3.4 1.5 *Optional*: Examining the Trade through June 2009

Calculate the pnl of the trade for the following dates: * 2009-01-27 * 2009-03-24 * 2009-06-16

Did the trade do well or poorly in the first six months of 2009?

Calculate the YTM spreads on these dates. Does the YTM spread correspond to pnl roughly as we would expect based on the calculation in 1.3?

[ ]:
```
trade_pair
```

[ ]:

	204047	204046
YTM	0.032362	0.035753
Dirty_Price	106.904212	144.213655
Modified_Duration	5.841189	5.138240

```
dollar_duration   624.447664 741.010749
```

[ ]:
```python
spread_convergence = .0001

trade_pair.loc['YTM'].idxmax()
```

[ ]:
```
'204046'
```

[ ]:
```python
def duration_pnl(dollar_duration, delta_rate, no_contracts):
    delta_bond_price = -dollar_duration*delta_rate
    PNL = no_contracts*delta_bond_price
    return PNL
```

[ ]:
```python
balance_sheet
```

[ ]:
```
                 Equity         Asset_Value        Contracts Dollar_Duration  \
204046 1,000,000.000000  50,000,000.000000  346,707.806553      741.010749
204047   -879,664.154632 -43,983,207.731591 -411,426.331187      624.447664

       Dirty_Price
204046  144.213655
204047  106.904212
```

[ ]:
```python
def pnl_spread_trade(spread_conv, dollar_duration,prices,size_pos, long_equity,␣
 ↪short_equity):
    spread_conv /= 2
    pnl_df = pd.DataFrame(data = None, index =␣
 ↪[long_equity,short_equity,'total'])
    long_price = prices.loc[long_equity]

    short_price = prices.loc[short_equity]

    long_d_duration = dollar_duration.loc[long_equity]
    short_d_duration = dollar_duration.loc[short_equity]
    no_contracts_long = size_pos.loc[long_equity]
    no_contracts_short = size_pos.loc[short_equity]
    pnl_df.loc[long_equity,'YTM_Change'] = -spread_conv
    pnl_df.loc[short_equity,'YTM_Change'] = spread_conv
    pnl_df.loc[long_equity,'Dollar_Duration'] = long_d_duration
    pnl_df.loc[short_equity,'Dollar_Duration'] = short_d_duration
    pnl_df.loc[long_equity,'Contracts'] = no_contracts_long
    pnl_df.loc[short_equity,'Contracts'] = no_contracts_short
    pnl_df.loc[long_equity,'PNL'] =␣
 ↪duration_pnl(long_d_duration,-spread_conv,no_contracts_long)
    pnl_df.loc[short_equity,'PNL'] =␣
 ↪duration_pnl(short_d_duration,spread_conv,no_contracts_short)
    pnl_df.loc['total','PNL'] = pnl_df['PNL'].sum()
```

```python
    return pnl_df.fillna(0)
```

[ ]:
```python
balance_sheet.Dirty_Price
```

[ ]:
```
204046    144.213655
204047    106.904212
Name: Dirty_Price, dtype: object
```

[ ]:
```python
# if the spread converges by 1 basis point you are projected to make $25,000
pnl_1bs = pnl_spread_trade(spread_convergence,balance_sheet.
 ↪Dollar_Duration,balance_sheet.Dirty_Price,balance_sheet.
 ↪Contracts,long_security,short_security)
```

[ ]:
```python
pnl_1bs
```

[ ]:

	YTM_Change	Dollar_Duration	Contracts	PNL
204046	-0.000050	741.010749	346,707.806553	12,845.710571
204047	0.000050	624.447664	-411,426.331187	12,845.710571
total	0.000000	0.000000	0.000000	25,691.421142

[ ]:
```python
pnl_1bs.loc['total','PNL']
```

[ ]:
```
25691.421141777464
```

4 Calculating the PNL on 2008-11-25 and 2008-12-16

[ ]:
```python
pnl_spread_trade(st1,balance_sheet.Dollar_Duration,balance_sheet.
 ↪Dirty_Price,balance_sheet.Contracts,long_security,short_security)
```

[ ]:

	YTM_Change	Dollar_Duration	Contracts	PNL
204046	0.000955	741.010749	346,707.806553	-245,234.069145
204047	-0.000955	624.447664	-411,426.331187	-245,234.069145
total	0.000000	0.000000	0.000000	-490,468.138290

[ ]:
```python

```

[ ]:
```python
st1 = -(spread_df.loc['2008-11-25']['Spread']-spread_df.
 ↪loc['2008-11-04']['Spread'])
st2 = -(spread_df.loc['2008-12-16']['Spread'] - spread_df.
 ↪loc['2008-11-04']['Spread'])
```

[ ]:
```python
pnl_spread_trade(st2,balance_sheet.Dollar_Duration,balance_sheet.
 ↪Dirty_Price,balance_sheet.Contracts,long_security,short_security)
```

[ ]:

	YTM_Change	Dollar_Duration	Contracts	PNL
204046	0.001888	741.010749	346,707.806553	-485,037.913422
204047	-0.001888	624.447664	-411,426.331187	-485,037.913422
total	0.000000	0.000000	0.000000	-970,075.826844

---

5 2 Calculating Duration

Use the data file `../data/treasury_quotes_2022-09-30.xlsx`.

This data reports duration as `TDDURATN`. It quotes the duration in days, so I recommend dividing by 365 to get the duration in its usual format.

5.1 2.1

Set up the cashflow matrix.

5.2 2.2

Build a discount curve assuming that the spot rate is 2% per year, continuously compounded.

Plot the discount curve and the associated spot curve out to 30 years maturity.

Note, you do not need to properly extract a spot curve and associated discount factors; rather, you are simply assuming a flat term structure of spot rates at 2% and using that to figure out discount rates.

5.3 2.3

For each treasury issue, calculate the duration as the weighted average of the (discounted!) cashflow maturity.

Report the summary statistics of the durations. (Use `.describe()` from pandas.)

5.4 2.4

How close are your duration estimates to the imputed durations given in the data source, (column `TDDURATN`)?

Report the summary statistics of the imputed durations minus your calculated durations from above.

Why might they be different?

5.5 2.5

Continue using your assumed discount rates of 2% to calculate the convexity of each issue.

Report the summary statistics of these convexity calculations.

5.6 2.6 (Optional)

Re-do the duration and convexity calculations using an extracted discount curve instead of a discount curve based on a constant (arbitrary) spot rate.

[ ]:
```
t_current = treasury_df.CALDT.values[0]
rawprice = (treasury_df['TDASK']+ treasury_df['TDBID'])*.5 +␣
↪treasury_df['TDACCINT']
rawprice.name = 'Prices'
```

[ ]:
```
maturity_delta = get_maturity_delta(treasury_df.TMATDT,t_current=t_current)
```

[ ]:
```
RESTRICT_YLD = True
RESTRICT_TIPS = True

RESTRICT_DTS_MATURING = False
RESTRICT_REDUNDANT = False
treasury_filtered = filter_treasuries(treasury_df, t_date=t_current,␣
↪filter_yld=RESTRICT_YLD,\
    filter_tips=RESTRICT_TIPS,drop_duplicate_maturities=RESTRICT_REDUNDANT)
CF = filter_treasury_cashflows(calc_cashflows(treasury_filtered),␣
↪filter_maturity_dates=RESTRICT_DTS_MATURING)
```

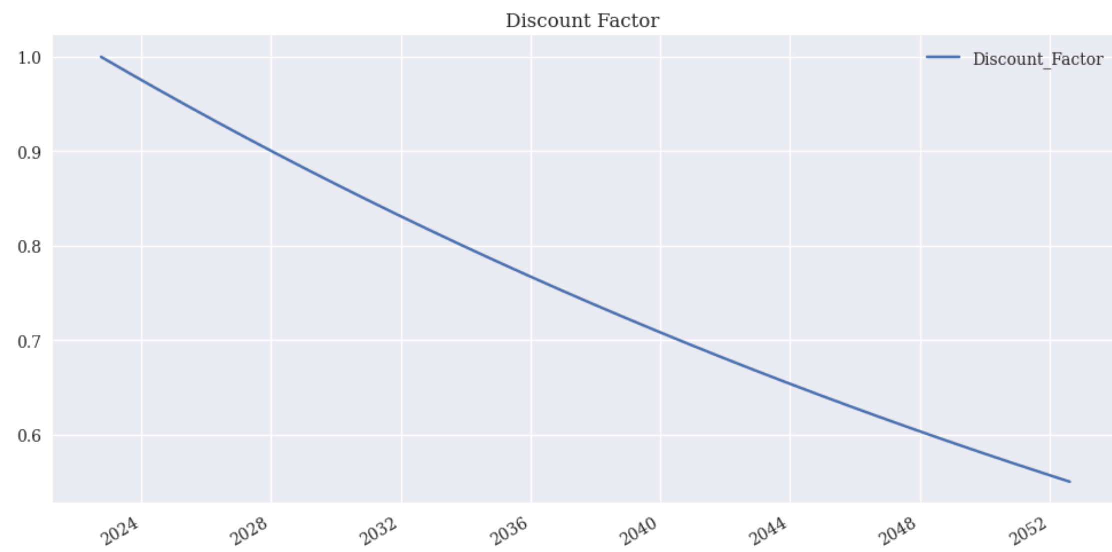
[ ]:
```
CF
```

[ ]:
```
            2022-10-04  2022-10-06  2022-10-11  2022-10-13  2022-10-15  \
KYTREASNO
207892             100           0           0           0    0.000000
207774               0         100           0           0    0.000000
207893               0           0         100           0    0.000000
207868               0           0           0         100    0.000000
207430               0           0           0           0  100.687500
…                  …          …          …          …         …
207763               0           0           0           0    0.000000
207808               0           0           0           0    0.000000
207849               0           0           0           0    0.000000
207891               0           0           0           0    0.000000
207934               0           0           0           0    0.000000

            2022-10-18  2022-10-20  2022-10-25  2022-10-27  2022-10-30  …  \
KYTREASNO                                                               …
207892               0           0           0           0    0.000000  …
207774               0           0           0           0    0.000000  …
207893               0           0           0           0    0.000000  …
207868               0           0           0           0    0.000000  …
207430               0           0           0           0    0.000000  …
…                  …          …          …          …         …  …
207763               0           0           0           0    0.000000  …
```

```
207808             0          0          0          0   0.000000  …
207849             0          0          0          0   0.000000  …
207891             0          0          0          0   0.000000  …
207934             0          0          0          0   0.000000  …

           2050-05-15  2050-08-15  2050-11-15  2051-02-15  2051-05-15  \
KYTREASNO
207892       0.000000    0.000000    0.000000    0.000000    0.000000
207774       0.000000    0.000000    0.000000    0.000000    0.000000
207893       0.000000    0.000000    0.000000    0.000000    0.000000
207868       0.000000    0.000000    0.000000    0.000000    0.000000
207430       0.000000    0.000000    0.000000    0.000000    0.000000
…                 …          …          …          …          …
207763       0.000000    1.000000    0.000000    1.000000    0.000000
207808       0.937500    0.000000    0.937500    0.000000    0.937500
207849       0.000000    1.125000    0.000000    1.125000    0.000000
207891       1.437500    0.000000    1.437500    0.000000    1.437500
207934       0.000000    1.500000    0.000000    1.500000    0.000000

           2051-08-15  2051-11-15  2052-02-15  2052-05-15  2052-08-15
KYTREASNO
207892       0.000000    0.000000    0.000000    0.000000    0.000000
207774       0.000000    0.000000    0.000000    0.000000    0.000000
207893       0.000000    0.000000    0.000000    0.000000    0.000000
207868       0.000000    0.000000    0.000000    0.000000    0.000000
207430       0.000000    0.000000    0.000000    0.000000    0.000000
…                 …          …          …          …          …
207763     101.000000    0.000000    0.000000    0.000000    0.000000
207808       0.000000  100.937500    0.000000    0.000000    0.000000
207849       1.125000    0.000000  101.125000    0.000000    0.000000
207891       0.000000    1.437500    0.000000  101.437500    0.000000
207934       1.500000    0.000000    1.500000    0.000000  101.500000

[378 rows x 312 columns]
```

[ ]:
```
RATE = 0.02
maturity_grid = pd.Series(get_maturity_delta(CF.columns.
 ↪values,t_current=t_current), index = CF.columns)
discount_factors = np.exp(-RATE*maturity_grid)
curve = pd.DataFrame(discount_factors,index = CF.columns,␣
 ↪columns=['Discount_Factor'])
curve.plot(figsize=(12,6))
plt.title('Discount Factor ')
```

[ ]:
```
Text(0.5, 1.0, 'Discount Factor ')
```




```
[ ]: weight = CF
```

```
[ ]: 2022-10-04   0.999781
     2022-10-06   0.999672
     2022-10-11   0.999398
     2022-10-13   0.999288
     2022-10-15   0.999179
                    …
     2051-08-15   0.561318
     2051-11-15   0.558498
     2052-02-15   0.555691
     2052-05-15   0.552959
     2052-08-15   0.550181
     Name: Discount_Factor, Length: 312, dtype: float64
```

```
[ ]: print(CF.shape,curve.shape)
```

```
(378, 312) (312, 1)
```

```
[ ]: weight = CF.mul(curve['Discount_Factor'], axis  = 1)
     #divide each present value of CF by the present value of the bond
     weight = weight.div(weight.sum(axis =1),axis = 0)
```

```
[ ]: print(weight.shape, maturity_grid.shape)
```

```
(378, 312) (312,)
```

```
[ ]: duration = weight@maturity_grid.to_frame().rename(columns={0:'Duration'})
```

```
[ ]: duration
```

```
[ ]:            Duration
KYTREASNO
207892      0.010951
207774      0.016427
207893      0.030116
207868      0.035592
207430      0.041068
…                …
207763     22.009445
207808     22.292807
207849     21.825479
207891     20.847305
207934     20.927847

[378 rows x 1 columns]
```

```
[ ]: freq = 365.25
     dif = pd.DataFrame(treasury_df['TDDURATN']/365.25 - duration['Duration'],␣
      ↪columns=['Difference'])
```

```
[ ]: dif.describe()
```

```
[ ]:        Difference
count   378.000000
mean     -0.212790
std       0.469856
min      -1.958441
25%      -0.034362
50%      -0.002948
75%      -0.000018
max       0.000037
```

6 Calculating Convexity

```
[ ]: duration['Convexity'] = weight@(maturity_grid**2)
```

```
[ ]: duration
```

```
[ ]:            Duration   Convexity
KYTREASNO
207892      0.010951    0.000120
207774      0.016427    0.000270
207893      0.030116    0.000907
207868      0.035592    0.001267
207430      0.041068    0.001687
…                …           …
207763     22.009445  574.894240
```

```
207808    22.292807 589.750695
207849    21.825479 573.468303
207891    20.847305 539.912461
207934    20.927847 543.799863

[378 rows x 2 columns]
```

7 3 Hedging Duration

Import `treasury_ts_issue_duration_(207392, 207391, 207457).xlsx`.

I suggest using code such as

- `tsdata = pd.read_excel(filepath_tsdata,sheet_name='ts')`
- `tsdata.columns = tsdata.columns.str.upper()`
- `px = tsdata.pivot_table(index='CALDT',columns='KYTREASNO',values='TDASK').dropna()`
- `duration = tsdata.pivot_table(index='CALDT',columns='KYTREASNO',values='TDDURATN').dropna(`

7.1 3.1

Suppose you have a portfolio of 10,000 USD long in security `207391` on the last day of the sample.

If you want to manage interest rate exposure using duration, how large of a short position should you hold in `207392`?

(Duration is the column `TDDURATN` in the raw data.)

7.2 3.2

Step through the time-series, doing the following:

- Starting at the end of the first day, set the hedged position according to the relative given durations.
- Use the second day's price data to evaluate the net profit or loss of the hedged position.
- Reset the the hedged position using the end-of-second-day durations. Again fix the long position of security `207391` to be 10,000.
- Repeat throughout the timeseries.

Report * the total profit (or loss.) * the mean, standard deviation, min, and max of the daily profit or loss.

```
[ ]: ts_q3_path = 'C:/Users/dcste/OneDrive/fixed_income/fixed_income_FORKED/
     ↪finm-fixedincome-2023/data/treasury_ts_issue_duration_(207392, 207391,␣
     ↪207457).xlsx'
     treasury_ts_info = pd.read_excel(ts_q3_path, sheet_name='info').
     ↪set_index('kytreasno')
     treasury_ts_data = pd.read_excel(ts_q3_path,sheet_name='ts')
```

[ ]:
```
size = 1e4
IDLONG = 207391
IDSHORT = 207392
FREQ = 365.25
```

[ ]:
```
treasury_ts_info
```

[ ]:
```
           issue date maturity date  coupon rate  security type
kytreasno
207392     2019-08-15    2049-08-15     2.250000              1
207391     2019-08-15    2029-08-15     1.625000              2
207457     2019-12-15    2022-12-15     1.625000              2
```

[ ]:
```
treasury_ts_data.columns = treasury_ts_data.columns.str.upper()
px = treasury_ts_data.pivot_table(index = 'CALDT',columns = 'KYTREASNO',values␣
↪= 'TDASK').dropna().drop(columns=207457)
duration = (treasury_ts_data.pivot_table(index =␣
↪'CALDT',columns='KYTREASNO',values = 'TDDURATN').dropna().
↪drop(columns=207457))/FREQ
```

[ ]:
```
position = pd.DataFrame(index = duration.index, dtype='float')
position['long'] = size/px[IDLONG]
position['Hedge_Ratio'] = (duration[IDLONG]/duration[IDSHORT])*(px[IDLONG]/
↪px[IDSHORT])
position['short'] = -position['Hedge_Ratio']*position['long']
position[['long ($)','short ($)']] =␣
↪position[['long','short']]*px[[IDLONG,IDSHORT]].values
```

[ ]:
```
position['net ($)'] = position[['long ($)','short ($)']].sum(axis = 1)
wts = position[['long ($)', 'short ($)']].div(position[['long ($)', 'short␣
↪($)']].sum(axis = 1), axis = 0)
position['duration'] = (wts*duration[[IDLONG,IDSHORT]].values).sum(axis = 1)
position[['duration']].describe()
```

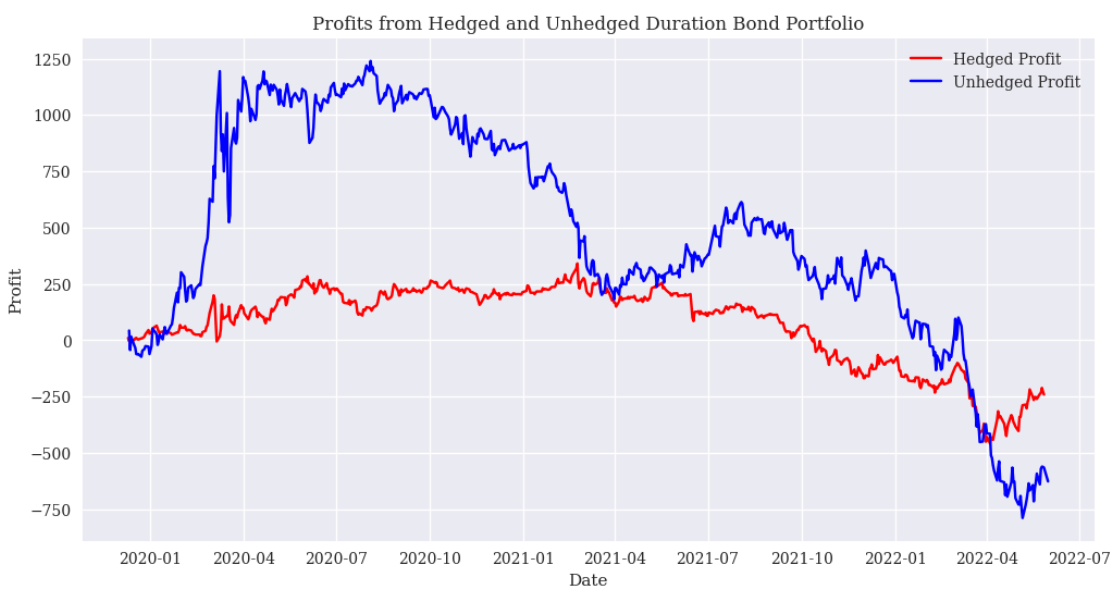
[ ]:
```
         duration
count  621.000000
mean    -0.000000
std      0.000000
min     -0.000000
25%     -0.000000
50%      0.000000
75%      0.000000
max      0.000000
```

[ ]:
```
position[['long ($) realized', 'short ($) realized']] = \
    position[['long','short']]*px[[IDLONG,IDSHORT]].shift(-1).values
```

```
position[['long pnl', 'short pnl']] = position[['long ($) realized', 'short ($)␣
↪realized']] - position[['long ($)', 'short ($)']].values
position['profit ($) hedge'] = position['long pnl'] + position['short pnl']
position['profit'] = px[IDLONG].diff() *px[IDLONG].shift()
```

[ ]:
```
fig, ax = plt.subplots(figsize=(12,6))
ax.plot(position['profit ($) hedge'].cumsum(), color = 'red', label ='Hedged␣
↪Profit')
ax.plot(position['profit'].cumsum(), color = 'blue',label = 'Unhedged Profit')
ax.set_ylabel('Profit')
ax.set_xlabel('Date')
ax.set_title('Profits from Hedged and Unhedged Duration Bond Portfolio')
ax.legend()
```

[ ]: <matplotlib.legend.Legend at 0x1b440a11e20>



[ ]:
```
position[['profit ($) hedge','profit']].describe()
```

[ ]:

	profit ($) hedge	profit
count	620.000000	620.000000
mean	-0.388564	-1.010402
std	18.797647	47.651475
min	-98.317955	-374.706299
25%	-9.776653	-26.452835
50%	-0.077860	-1.345947
75%	9.250157	24.240128
max	107.636148	297.649902

---

8 4 Other Interest-Rate Risks

8.1 *Optional*

No need to submit this problem, but if we discuss it, then you are expected to know it.

8.2 4.1 Other Yield Curve Movements

Use the yield curve time-series data in `'../data/yields_2022-11-30.xlsx` to calculate the time-series of the **level**, **slope**, and curvature** factors.

Calculate the yield-curve factors. For each point in time, calculate the following three factors:

$$x_t^{\text{level}} = \frac{1}{N_{\text{yields}}} \sum_{i=1}^{N_{\text{yields}}} y_t^{(i)} \tag{1}$$

$$x_t^{\text{slope}} = y_t^{(30)} - y_t^{(1)} \tag{2}$$

$$x_t^{\text{curvature}} = -y_t^{(1)} + 2y_t^{(10)} - y_t^{(30)} \tag{3}$$

Report the mean and volatility of each factor.

Report the correlation matrix of the factors.

8.3 4.2 Factor Duration

Calculate the factor duration of the treasuries from `treasury_ts_issue_duration_(207392, 207391, 207457).xlsx`.

Run a multivariate regression of the bond prices on all three factors constructed above from the yield factors: level, slope, and curvature.

Estimate the regression in the form of day-over-day differences for both bond prices and factors. That is, we are using regression to approximate the factor duration equation,

$$\frac{dP}{P} = \beta_1 dz_1 + \beta_2 dz_2 + \beta_3 dz_3 + \epsilon \tag{4}$$

Report the betas for each of these factors, for each of the bond prices.