Homework 2

February 6, 2023

1 Homework 2

1.1 FINM 37400 - 2023

1.1.1 UChicago Financial Mathematics

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2 1 HBS Case: Fixed-Income Arbitrage in a Financial Crisis (A): US Treasuries in November 2008

2.1 Data

- Use the data file `treasury_ts_2015-08-15.xlsx`.
- Examine the treasure issues with `kytreasno` of `204046` and `204047`. These are the bond and note (respectively) which mature on 2015-08-15.
- Look at the data on 2008-11-04.

2.2 1.1 The situation

Make a chart comparing the issues in the following features, (as of Nov 4, 2008.) * coupon rate * bid * ask * accrued interest * dirty price * duration (quoted in years, not days, assuming 365.25 days per year.) * modified duration * YTM

2.3 1.2 Hedge Ratio

Suppose you are building a trade to go long n_i bonds (`204046`) and short n_j notes (`204047`).

We can find an equation for n_j in terms of n_i such that the total holdings will have duration equal to zero. (Having zero duration also means have zero dollar duration, if helpful.)

Notation: * n_i: number of bonds purchased (or sold) * D_i: duration of bond i * $D_{\$,i}$: dollar duration of bond i, equal to $p_i D_i$

If we want the total duration of our holdings to be zero, then we need to size the trade such that n_i and n_j satisfy,

$$0 = n_i D_{\$,i} + n_j D_{\$,j}$$

$$n_j = -n_i \frac{D_{\$,i}}{D_{\$,j}}$$

Suppose you will use \$1mm of capital, leveraged 50x to buy \$50mm of the bonds (204046).

Use the ratio above to short a number of notes (204047) to keep zero duration.

Report the number of bonds and notes of your position, along with the total dollars in the short position.

```
[ ]: import pandas as pd
     import numpy as np
     import datetime
     import warnings
     import matplotlib as mpl
     import matplotlib.pyplot as plt
     from sklearn.linear_model import LinearRegression
     from scipy.optimize import minimize
     %matplotlib inline
     plt.style.use('seaborn')
     mpl.rcParams['font.family'] = 'serif'
     from treasury_cmds import *
     pd.options.display.float_format = '{:,.6f}'.format
```

```
[ ]: treasury_path = 'C:/Users/dcste/OneDrive/fixed_income/fixed_income_FORKED/
      ↪finm-fixedincome-2023/data/treasury_ts_2015-08-15.xlsx'
     t_path_2 = 'C:/Users/dcste/OneDrive/fixed_income/fixed_income_FORKED/
      ↪finm-fixedincome-2023/data/treasury_quotes_2022-09-30.xlsx'
     info = pd.read_excel(treasury_path, sheet_name='info', index_col = 0)
     ts_data = pd.read_excel(treasury_path, sheet_name= 'database')
     ts_data.columns = ts_data.columns.str.upper()
     treasury_df = pd.read_excel(t_path_2)
     treasury_df.columns = treasury_df.columns.str.upper()
     treasury_df.sort_values('TMATDT', inplace=True)
     treasury_df.set_index("KYTREASNO", inplace = True)
```

```
[ ]: mask = ts_data[ts_data.CALDT == '2008-11-04']
     metrics = mask.
      ↪copy()[['KYTREASNO','CALDT','TDBID','TDASK','TDNOMPRC','TDACCINT',␣
      ↪'TDYLD','TDDURATN','TDPUBOUT']].set_index('KYTREASNO')
     metrics.columns = ['CALDT','BID','ASK', 'Nominal_Price','Accrued␣
      ↪Interest','YTM','Duration','Outstanding']
```

```
[ ]: metrics['Dirty_Price'] = (metrics['BID']+metrics['ASK'])*.5 + metrics['Accrued␣
      ↪Interest']
```

```
[ ]: metrics['YTM'] *= 365.25
     metrics['Coupon_Rate'] = [4.25,10.625]
```

```
metrics['Duration'] /= 365.25
metrics['Modified Duration'] = metrics['Duration']/(1+(metrics['YTM']/2))
metrics['Dollar_Duration'] = metrics['Modified Duration']*metrics['Dirty_Price']
metrics = metrics.T
```

[ ]:
```
metrics
```

[ ]:
```
KYTREASNO                         204047               204046
CALDT                2008-11-04 00:00:00  2008-11-04 00:00:00
BID                           105.953125           141.859375
ASK                           105.984375           141.890625
Nominal_Price                 105.968750           141.875000
Accrued Interest                0.935462             2.338655
YTM                             0.032362             0.035753
Duration                        5.935706             5.230138
Outstanding                20,998.000000         2,852.000000
Dirty_Price                   106.904212           144.213655
Coupon_Rate                     4.250000            10.625000
Modified Duration               5.841189             5.138284
Dollar_Duration               624.447664           741.010749
```

[ ]:
```
trade_pair = pd.DataFrame(data = None, columns=['204047','204046'])
trade_pair.loc['YTM'] = [0.032362,0.035753]
trade_pair.loc['Dirty_Price']  = [106.904212,144.213655]

trade_pair.loc['Modified_Duration'] = [5.841189,5.13824]
trade_pair.loc['dollar_duration'] = [624.447664,741.010749]
```

[ ]:
```
trade_pair
```

[ ]:
```
                       204047      204046
YTM                  0.032362    0.035753
Dirty_Price        106.904212  144.213655
Modified_Duration    5.841189    5.138240
dollar_duration    624.447664  741.010749
```

[ ]:
```
long_security = trade_pair.loc['YTM'].idxmax()
short_security = trade_pair.loc['YTM'].idxmin()

short_security
```

[ ]:
```
'204047'
```

[ ]:
```
p = trade_pair.loc['Dirty_Price']
p.loc['204046']
```

[ ]:
```
144.213655
```

3 Calculating Hedge Ratio

3.1 Financing Assumptions

- In this trade we take a long position in the bond which has the higher yield and short the note that has a lower yield.

[ ]:
```
def hedge_ratio(dollar_duration_long, dollar_duration_short, size_long = None):
    if size_long == None:
        return -dollar_duration_long/dollar_duration_short
    else:
        return (-size_long*(dollar_duration_long))/dollar_duration_short
```

[ ]:
```
LONG_EQUITY = 1e6
def trade_balance_sheet(prices, durations, haircuts, long_equity, short_equity):
    balance_sheet = pd.DataFrame(data =␣
  ↪None,columns=['Equity','Asset_Value','Contracts','Dollar_Duration','Dirty_Price'],␣
  ↪index = [long_equity,short_equity])
    long_price = prices.loc[long_equity]
    short_price = prices.loc[short_equity]
    duration_long = durations.loc[long_equity]
    duration_short = durations.loc[short_equity]
    haircut_short = haircuts.loc['short']
    haircut_long = haircuts.loc['long']
    balance_sheet.loc[long_equity,'Dollar_Duration'] = duration_long
    balance_sheet.loc[short_equity,'Dollar_Duration'] = duration_short
    balance_sheet.loc[long_equity,'Equity'] = LONG_EQUITY
    balance_sheet.loc[long_equity,'Asset_Value'] = 5e7
    balance_sheet.loc[long_equity,'Contracts'] = balance_sheet.
  ↪loc[long_equity,'Asset_Value']/long_price
    balance_sheet.loc[short_equity,'Contracts'] =␣
  ↪hedge_ratio(dollar_duration_long=duration_long,dollar_duration_short=duration_short,\
        size_long=balance_sheet.loc[long_equity,'Contracts'])
    balance_sheet.loc[short_equity,'Asset_Value'] = balance_sheet.
  ↪loc[short_equity,'Contracts']*short_price
    balance_sheet.loc[short_equity,'Equity'] = balance_sheet.
  ↪loc[short_equity,'Asset_Value']*haircut_short
    balance_sheet.loc[short_equity,'Dirty_Price'] = short_price
    balance_sheet.loc[long_equity,'Dirty_Price'] = long_price

    return balance_sheet
```

[ ]:
```
financing = pd.DataFrame(dtype='float64',index=['long','short'])
financing['haircut'] = [.02,.02]
financing['repo'] = [.0015,.0010]
financing
```

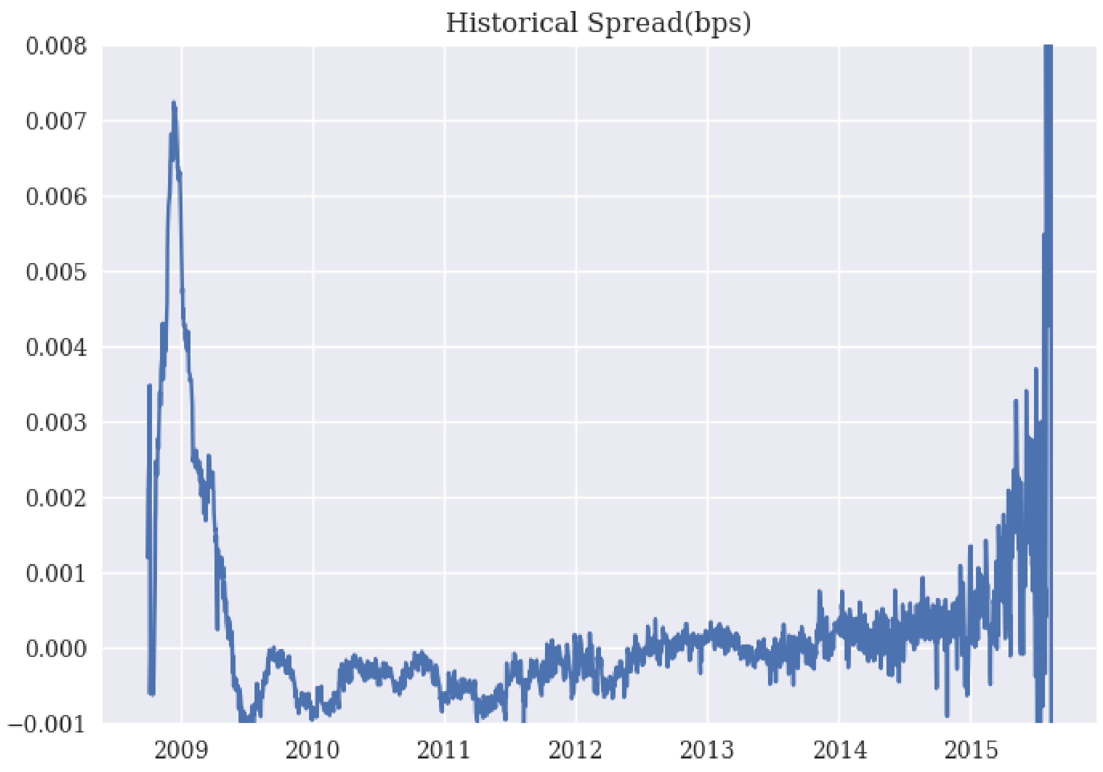
```
[ ]:        haircut     repo
     long   0.020000 0.001500
     short  0.020000 0.001000
```

```
[ ]: balance_sheet = trade_balance_sheet(prices=trade_pair.loc['Dirty_Price'],␣
     ↪durations=trade_pair.loc['dollar_duration'],haircuts = financing['haircut'],\
         long_equity = long_security, short_equity = short_security )
```

```
[ ]: spread_df = ts_data.pivot_table(values='TDYLD', index =␣
     ↪'CALDT',columns='KYTREASNO')
     spread_df.columns = ['204046','204047','206524']
     spread_df['206524'] = spread_df['204046']- spread_df['204047']
     spread_df = spread_df.rename(columns={'206524':'Spread'})
     # I need to format the yields
     spread_df *= 365.25
     spread_df = spread_df.loc['2006':,:]
```

```
[ ]: plt.plot(spread_df.loc['2008-10':,'Spread'])
     plt.title('Historical Spread(bps)')
     plt.ylim(-.001,.008)
```

```
[ ]: (-0.001, 0.008)
```



```
[ ]: (spread_df*100).describe()
```

```
[ ]:            204046      204047      Spread
     count 2,410.000000 2,410.000000 2,410.000000
     mean      2.009363     1.990174     0.019189
     std       1.719906     1.709289     0.182395
     min      -6.077314     0.007719    -6.085032
     25%       0.353566     0.345058    -0.026247
     50%       1.727539     1.718937     0.001298
     75%       3.465700     3.398835     0.020286
     max       5.221068     5.229953     2.657635
```

3.2 1.3 Profit Opportunity

Using the concept of **modified duration**, how much profit or loss (PnL) would you expect to make for every basis point of convergence in the spread? Specifically, assume the convergence is symmetric: the bond's (204046) ytm goes down 0.5bp and the note (204047) ytm goes up 0.5bp.

Describe the PnL you would expect to achieve on your position should this happen. Specify the PnL of the long position, the short position, and the net total.

Suppose the spread in YTM between the two securities disappears, due to a symmetric move of roughly ~17bps in each security's YTM. What is the PnL? (This is just a linearly scaling of your prior answer for a 1bp convergence.)

3.3 1.4 Result in 2008

Calculate the profit (or loss) on the position on the following two dates: * 2008-11-25 * 2008-12-16

To calculate the pnl on each date, simply use the prices of the securities on those dates along with your position sizes, (n_i, n_j). No coupon is being paid in November or December, so all you need is the "dirty" price on these two dates.

Does the pnl make sense (approximately) given your results in 1.3 with regard to the sensitivity of pnl to moves in the YTM spread?

3.4 1.5 *Optional*: Examining the Trade through June 2009

Calculate the pnl of the trade for the following dates: * 2009-01-27 * 2009-03-24 * 2009-06-16

Did the trade do well or poorly in the first six months of 2009?

Calculate the YTM spreads on these dates. Does the YTM spread correspond to pnl roughly as we would expect based on the calculation in 1.3?

```
[ ]: trade_pair
```

```
[ ]:                      204047     204046
     YTM                0.032362   0.035753
     Dirty_Price      106.904212 144.213655
     Modified_Duration  5.841189   5.138240
```

```
dollar_duration   624.447664 741.010749
```

[ ]:
```
spread_convergence = .0001

trade_pair.loc['YTM'].idxmax()
```

[ ]:
```
'204046'
```

[ ]:
```
def duration_pnl(dollar_duration, delta_rate, no_contracts):
    delta_bond_price = -dollar_duration*delta_rate
    PNL = no_contracts*delta_bond_price
    return PNL
```

[ ]:
```
balance_sheet
```

[ ]:
```
                 Equity       Asset_Value       Contracts  Dollar_Duration  \
204046 1,000,000.000000  50,000,000.000000  346,707.806553       741.010749
204047  -879,664.154632 -43,983,207.731591 -411,426.331187       624.447664

       Dirty_Price
204046  144.213655
204047  106.904212
```

[ ]:
```
def pnl_spread_trade(spread_conv, dollar_duration,prices,size_pos, long_equity,␣
↪short_equity):
    spread_conv /= 2
    pnl_df = pd.DataFrame(data = None, index =␣
↪[long_equity,short_equity,'total'])
    long_price = prices.loc[long_equity]

    short_price = prices.loc[short_equity]

    long_d_duration = dollar_duration.loc[long_equity]
    short_d_duration = dollar_duration.loc[short_equity]
    no_contracts_long = size_pos.loc[long_equity]
    no_contracts_short = size_pos.loc[short_equity]
    pnl_df.loc[long_equity,'YTM_Change'] = -spread_conv
    pnl_df.loc[short_equity,'YTM_Change'] = spread_conv
    pnl_df.loc[long_equity,'Dollar_Duration'] = long_d_duration
    pnl_df.loc[short_equity,'Dollar_Duration'] = short_d_duration
    pnl_df.loc[long_equity,'Contracts'] = no_contracts_long
    pnl_df.loc[short_equity,'Contracts'] = no_contracts_short
    pnl_df.loc[long_equity,'PNL'] =␣
↪duration_pnl(long_d_duration,-spread_conv,no_contracts_long)
    pnl_df.loc[short_equity,'PNL'] =␣
↪duration_pnl(short_d_duration,spread_conv,no_contracts_short)
    pnl_df.loc['total','PNL'] = pnl_df['PNL'].sum()
    return pnl_df.fillna(0)
```

```
[ ]: balance_sheet.Dirty_Price
```

```
[ ]: 204046    144.213655
     204047    106.904212
     Name: Dirty_Price, dtype: object
```

```
[ ]: # if the spread converges by 1 basis point you are projected to make $25,000
     pnl_1bs = pnl_spread_trade(spread_convergence,balance_sheet.
      ↪Dollar_Duration,balance_sheet.Dirty_Price,balance_sheet.
      ↪Contracts,long_security,short_security)
```

```
[ ]: pnl_1bs
```

```
[ ]:         YTM_Change  Dollar_Duration       Contracts           PNL
     204046   -0.000050       741.010749  346,707.806553 12,845.710571
     204047    0.000050       624.447664 -411,426.331187 12,845.710571
     total     0.000000         0.000000        0.000000 25,691.421142
```

```
[ ]: pnl_1bs.loc['total','PNL']
```

```
[ ]: 25691.421141777464
```

4 Calculating the PNL on 2008-11-25 and 2008-12-16

```
[ ]: pnl_spread_trade(st1,balance_sheet.Dollar_Duration,balance_sheet.
      ↪Dirty_Price,balance_sheet.Contracts,long_security,short_security)
```

```
[ ]:         YTM_Change  Dollar_Duration       Contracts            PNL
     204046    0.000955       741.010749  346,707.806553 -245,234.069145
     204047   -0.000955       624.447664 -411,426.331187 -245,234.069145
     total     0.000000         0.000000        0.000000 -490,468.138290
```

```
[ ]:
```

```
[ ]: st1 = -(spread_df.loc['2008-11-25']['Spread']-spread_df.
      ↪loc['2008-11-04']['Spread'])
     st2 = -(spread_df.loc['2008-12-16']['Spread'] - spread_df.
      ↪loc['2008-11-04']['Spread'])
```

```
[ ]: pnl_spread_trade(st2,balance_sheet.Dollar_Duration,balance_sheet.
      ↪Dirty_Price,balance_sheet.Contracts,long_security,short_security)
```

```
[ ]:          YTM_Change  Dollar_Duration       Contracts            PNL
     204046     0.001888       741.010749  346,707.806553 -485,037.913422
     204047    -0.001888       624.447664 -411,426.331187 -485,037.913422
     total      0.000000         0.000000        0.000000 -970,075.826844
```

---

5 2 Calculating Duration

Use the data file `../data/treasury_quotes_2022-09-30.xlsx`.

This data reports duration as `TDDURATN`. It quotes the duration in days, so I recommend dividing by 365 to get the duration in its usual format.

5.1 2.1

Set up the cashflow matrix.

5.2 2.2

Build a discount curve assuming that the spot rate is 2% per year, continuously compounded.

Plot the discount curve and the associated spot curve out to 30 years maturity.

Note, you do not need to properly extract a spot curve and associated discount factors; rather, you are simply assuming a flat term structure of spot rates at 2% and using that to figure out discount rates.

5.3 2.3

For each treasury issue, calculate the duration as the weighted average of the (discounted!) cashflow maturity.

Report the summary statistics of the durations. (Use `.describe()` from pandas.)

5.4 2.4

How close are your duration estimates to the imputed durations given in the data source, (column `TDDURATN`)?

Report the summary statistics of the imputed durations minus your calculated durations from above.

Why might they be different?

5.5 2.5

Continue using your assumed discount rates of 2% to calculate the convexity of each issue.

Report the summary statistics of these convexity calculations.

5.6 2.6 (Optional)

Re-do the duration and convexity calculations using an extracted discount curve instead of a discount curve based on a constant (arbitrary) spot rate.

```
[ ]: t_current = treasury_df.CALDT.values[0]
     rawprice = (treasury_df['TDASK']+ treasury_df['TDBID'])*.5 +␣
      ↪treasury_df['TDACCINT']
     rawprice.name = 'Prices'
```

```
[ ]: maturity_delta = get_maturity_delta(treasury_df.TMATDT,t_current=t_current)
```

```
[ ]: RESTRICT_YLD = True
     RESTRICT_TIPS = True

     RESTRICT_DTS_MATURING = False
     RESTRICT_REDUNDANT = False
     treasury_filtered = filter_treasuries(treasury_df, t_date=t_current,␣
      ↪filter_yld=RESTRICT_YLD,\
         filter_tips=RESTRICT_TIPS,drop_duplicate_maturities=RESTRICT_REDUNDANT)
     CF = filter_treasury_cashflows(calc_cashflows(treasury_filtered),␣
      ↪filter_maturity_dates=RESTRICT_DTS_MATURING)
```

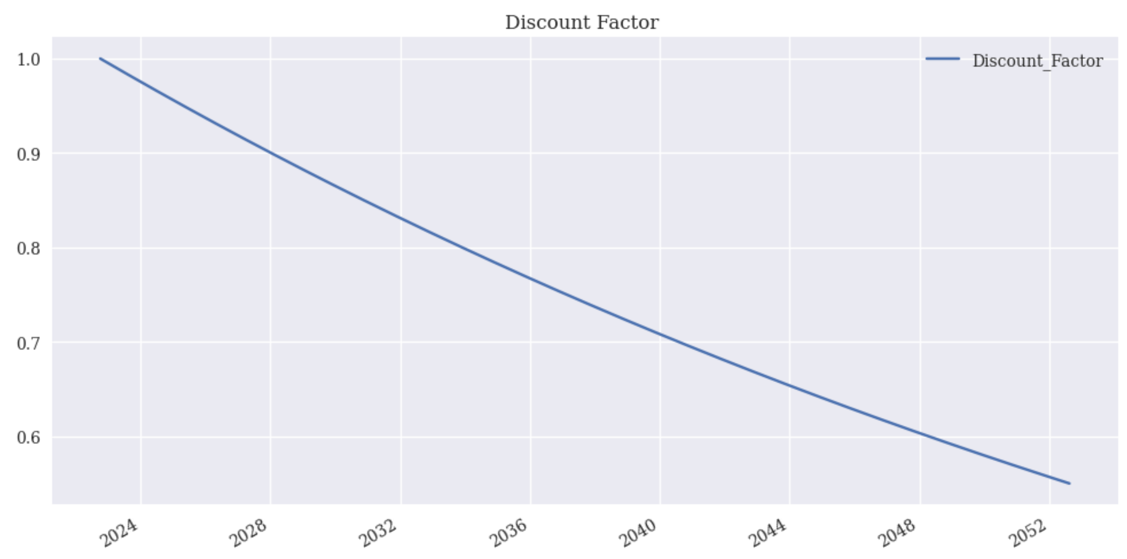
```
[ ]: CF
```

```
[ ]:            2022-10-04  2022-10-06  2022-10-11  2022-10-13  2022-10-15  \
     KYTREASNO
     207892            100           0           0           0    0.000000
     207774              0         100           0           0    0.000000
     207893              0           0         100           0    0.000000
     207868              0           0           0         100    0.000000
     207430              0           0           0           0  100.687500
     …                 …           …           …           …         …
     207763              0           0           0           0    0.000000
     207808              0           0           0           0    0.000000
     207849              0           0           0           0    0.000000
     207891              0           0           0           0    0.000000
     207934              0           0           0           0    0.000000

                2022-10-18  2022-10-20  2022-10-25  2022-10-27  2022-10-30  …  \
     KYTREASNO                                                              …
     207892              0           0           0           0    0.000000  …
     207774              0           0           0           0    0.000000  …
     207893              0           0           0           0    0.000000  …
     207868              0           0           0           0    0.000000  …
     207430              0           0           0           0    0.000000  …
     …                 …           …           …           …         …  …
     207763              0           0           0           0    0.000000  …
```

```
207808               0           0           0           0    0.000000  …
207849               0           0           0           0    0.000000  …
207891               0           0           0           0    0.000000  …
207934               0           0           0           0    0.000000  …

           2050-05-15  2050-08-15  2050-11-15  2051-02-15  2051-05-15  \
KYTREASNO
207892       0.000000    0.000000    0.000000    0.000000    0.000000
207774       0.000000    0.000000    0.000000    0.000000    0.000000
207893       0.000000    0.000000    0.000000    0.000000    0.000000
207868       0.000000    0.000000    0.000000    0.000000    0.000000
207430       0.000000    0.000000    0.000000    0.000000    0.000000
…                 …         …         …         …         …
207763       0.000000    1.000000    0.000000    1.000000    0.000000
207808       0.937500    0.000000    0.937500    0.000000    0.937500
207849       0.000000    1.125000    0.000000    1.125000    0.000000
207891       1.437500    0.000000    1.437500    0.000000    1.437500
207934       0.000000    1.500000    0.000000    1.500000    0.000000

           2051-08-15  2051-11-15  2052-02-15  2052-05-15  2052-08-15
KYTREASNO
207892       0.000000    0.000000    0.000000    0.000000    0.000000
207774       0.000000    0.000000    0.000000    0.000000    0.000000
207893       0.000000    0.000000    0.000000    0.000000    0.000000
207868       0.000000    0.000000    0.000000    0.000000    0.000000
207430       0.000000    0.000000    0.000000    0.000000    0.000000
…                 …         …         …         …         …
207763     101.000000    0.000000    0.000000    0.000000    0.000000
207808       0.000000  100.937500    0.000000    0.000000    0.000000
207849       1.125000    0.000000  101.125000    0.000000    0.000000
207891       0.000000    1.437500    0.000000  101.437500    0.000000
207934       1.500000    0.000000    1.500000    0.000000  101.500000

[378 rows x 312 columns]
```

[ ]:
```
RATE = 0.02
maturity_grid = pd.Series(get_maturity_delta(CF.columns.
 ↪values,t_current=t_current), index = CF.columns)
discount_factors = np.exp(-RATE*maturity_grid)
curve = pd.DataFrame(discount_factors,index = CF.columns,␣
 ↪columns=['Discount_Factor'])
curve.plot(figsize=(12,6))
plt.title('Discount Factor ')
```

[ ]:
```
Text(0.5, 1.0, 'Discount Factor ')
```



```
[ ]: weight = CF
```

```
[ ]: 2022-10-04   0.999781
     2022-10-06   0.999672
     2022-10-11   0.999398
     2022-10-13   0.999288
     2022-10-15   0.999179
                    …
     2051-08-15   0.561318
     2051-11-15   0.558498
     2052-02-15   0.555691
     2052-05-15   0.552959
     2052-08-15   0.550181
     Name: Discount_Factor, Length: 312, dtype: float64
```

```
[ ]: print(CF.shape,curve.shape)
```

```
(378, 312) (312, 1)
```

```
[ ]: weight = CF.mul(curve['Discount_Factor'], axis  = 1)
     #divide each present value of CF by the present value of the bond
     weight = weight.div(weight.sum(axis =1),axis = 0)
```

```
[ ]: print(weight.shape, maturity_grid.shape)
```

```
(378, 312) (312,)
```

```
[ ]: duration = weight@maturity_grid.to_frame().rename(columns={0:'Duration'})
```

```
[ ]: duration
[ ]:             Duration
     KYTREASNO
     207892      0.010951
     207774      0.016427
     207893      0.030116
     207868      0.035592
     207430      0.041068
     …                …
     207763     22.009445
     207808     22.292807
     207849     21.825479
     207891     20.847305
     207934     20.927847

     [378 rows x 1 columns]
```

```
[ ]: freq = 365.25
     dif = pd.DataFrame(treasury_df['TDDURATN']/365.25 - duration['Duration'],␣
      ↪columns=['Difference'])
```

```
[ ]: dif.describe()
```

```
[ ]:        Difference
     count  378.000000
     mean    -0.212790
     std      0.469856
     min     -1.958441
     25%     -0.034362
     50%     -0.002948
     75%     -0.000018
     max      0.000037
```

6 Calculating Convexity

```
[ ]: duration['Convexity'] = weight@(maturity_grid**2)
```

```
[ ]: duration
```

```
[ ]:             Duration   Convexity
     KYTREASNO
     207892      0.010951    0.000120
     207774      0.016427    0.000270
     207893      0.030116    0.000907
     207868      0.035592    0.001267
     207430      0.041068    0.001687
     …                …          …
     207763     22.009445 574.894240
```

```
207808     22.292807 589.750695
207849     21.825479 573.468303
207891     20.847305 539.912461
207934     20.927847 543.799863

[378 rows x 2 columns]
```

7 3 Hedging Duration

Import `treasury_ts_issue_duration_(207392, 207391, 207457).xlsx`.

I suggest using code such as

- `tsdata = pd.read_excel(filepath_tsdata,sheet_name='ts')`
- `tsdata.columns = tsdata.columns.str.upper()`
- `px = tsdata.pivot_table(index='CALDT',columns='KYTREASNO',values='TDASK').dropna()`
- `duration = tsdata.pivot_table(index='CALDT',columns='KYTREASNO',values='TDDURATN').dropna(`

7.1 3.1

Suppose you have a portfolio of 10,000 USD long in security `207391` on the last day of the sample.

If you want to manage interest rate exposure using duration, how large of a short position should you hold in `207392`?

(Duration is the column `TDDURATN` in the raw data.)

7.2 3.2

Step through the time-series, doing the following:

- Starting at the end of the first day, set the hedged position according to the relative given durations.
- Use the second day's price data to evaluate the net profit or loss of the hedged position.
- Reset the the hedged position using the end-of-second-day durations. Again fix the long position of security `207391` to be 10,000.
- Repeat throughout the timeseries.

Report * the total profit (or loss.) * the mean, standard deviation, min, and max of the daily profit or loss.

```
[ ]: ts_q3_path = 'C:/Users/dcste/OneDrive/fixed_income/fixed_income_FORKED/
       ↪finm-fixedincome-2023/data/treasury_ts_issue_duration_(207392, 207391,␣
       ↪207457).xlsx'
     treasury_ts_info = pd.read_excel(ts_q3_path, sheet_name='info').
       ↪set_index('kytreasno')
     treasury_ts_data = pd.read_excel(ts_q3_path,sheet_name='ts')
```

```
[ ]: size = 1e4
     IDLONG = 207391
     IDSHORT = 207392
     FREQ = 365.25
```

```
[ ]: treasury_ts_info
```

```
[ ]:            issue date maturity date  coupon rate  security type
     kytreasno
     207392     2019-08-15    2049-08-15     2.250000              1
     207391     2019-08-15    2029-08-15     1.625000              2
     207457     2019-12-15    2022-12-15     1.625000              2
```

```
[ ]: treasury_ts_data.columns = treasury_ts_data.columns.str.upper()
     px = treasury_ts_data.pivot_table(index = 'CALDT',columns = 'KYTREASNO',values␣
      ↪= 'TDASK').dropna().drop(columns=207457)
     duration = (treasury_ts_data.pivot_table(index =␣
      ↪'CALDT',columns='KYTREASNO',values = 'TDDURATN').dropna().
      ↪drop(columns=207457))/FREQ
```

```
[ ]: position = pd.DataFrame(index = duration.index, dtype='float')
     position['long'] = size/px[IDLONG]
     position['Hedge_Ratio'] = (duration[IDLONG]/duration[IDSHORT])*(px[IDLONG]/
      ↪px[IDSHORT])
     position['short'] = -position['Hedge_Ratio']*position['long']
     position[['long ($)','short ($)']] =␣
      ↪position[['long','short']]*px[[IDLONG,IDSHORT]].values
```

```
[ ]: position['net ($)'] = position[['long ($)','short ($)']].sum(axis = 1)
     wts = position[['long ($)', 'short ($)']].div(position[['long ($)', 'short␣
      ↪($)']].sum(axis = 1), axis = 0)
     position['duration'] = (wts*duration[[IDLONG,IDSHORT]].values).sum(axis = 1)
     position[['duration']].describe()
```

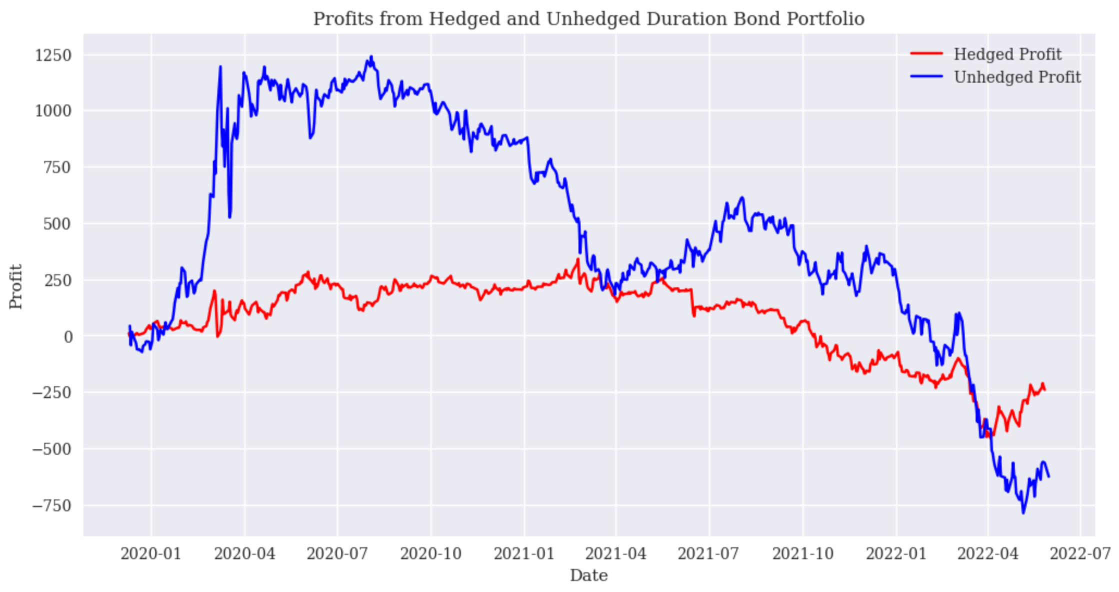
```
[ ]:         duration
     count 621.000000
     mean    -0.000000
     std      0.000000
     min     -0.000000
     25%     -0.000000
     50%      0.000000
     75%      0.000000
     max      0.000000
```

```
[ ]: position[['long ($) realized', 'short ($) realized']] = \
         position[['long','short']]*px[[IDLONG,IDSHORT]].shift(-1).values
```

```
position[['long pnl', 'short pnl']] = position[['long ($) realized', 'short ($)␣
↪realized']] - position[['long ($)', 'short ($)']].values
position['profit ($) hedge'] = position['long pnl'] + position['short pnl']
position['profit'] = px[IDLONG].diff() *px[IDLONG].shift()
```

[ ]:
```
fig, ax = plt.subplots(figsize=(12,6))
ax.plot(position['profit ($) hedge'].cumsum(), color = 'red', label ='Hedged␣
↪Profit')
ax.plot(position['profit'].cumsum(), color = 'blue',label = 'Unhedged Profit')
ax.set_ylabel('Profit')
ax.set_xlabel('Date')
ax.set_title('Profits from Hedged and Unhedged Duration Bond Portfolio')
ax.legend()
```

[ ]:
```
<matplotlib.legend.Legend at 0x1b440a11e20>
```



[ ]:
```
position[['profit ($) hedge','profit']].describe()
```

[ ]:
```
       profit ($) hedge      profit
count        620.000000  620.000000
mean          -0.388564   -1.010402
std           18.797647   47.651475
min          -98.317955 -374.706299
25%           -9.776653  -26.452835
50%           -0.077860   -1.345947
75%            9.250157   24.240128
max          107.636148  297.649902
```

---

8 4 Other Interest-Rate Risks

8.1 *Optional*

No need to submit this problem, but if we discuss it, then you are expected to know it.

8.2 4.1 Other Yield Curve Movements

Use the yield curve time-series data in `'../data/yields_2022-11-30.xlsx` to calculate the time-series of the **level**, **slope**, and curvature** factors.

Calculate the yield-curve factors. For each point in time, calculate the following three factors:

$$x_t^{\text{level}} = \frac{1}{N_{\text{yields}}} \sum_{i=1}^{N_{\text{yields}}} y_t^{(i)} \tag{1}$$

$$x_t^{\text{slope}} = y_t^{(30)} - y_t^{(1)} \tag{2}$$

$$x_t^{\text{curvature}} = -y_t^{(1)} + 2y_t^{(10)} - y_t^{(30)} \tag{3}$$

Report the mean and volatility of each factor.

Report the correlation matrix of the factors.

8.3 4.2 Factor Duration

Calculate the factor duration of the treasuries from `treasury_ts_issue_duration_(207392, 207391, 207457).xlsx`.

Run a multivariate regression of the bond prices on all three factors constructed above from the yield factors: level, slope, and curvature.

Estimate the regression in the form of day-over-day differences for both bond prices and factors. That is, we are using regression to approximate the factor duration equation,

$$\frac{dP}{P} = \beta_1 dz_1 + \beta_2 dz_2 + \beta_3 dz_3 + \epsilon \tag{4}$$

Report the betas for each of these factors, for each of the bond prices.