Homework 1

October 3, 2022

1 Mean Variance Optimization

• Import data and annualize the mea of monthly returns as well as the volatility of monthly returns with a scaling of $\sqrt(12)$

```
[]: import pandas as pd
     import numpy as np
     import matplotlib.pyplot as plt
     import statsmodels.api as sm
     import matplotlib as mpl
     import seaborn as sns
     import scipy as scs
     import math
[]: plt.style.use("seaborn")
     mpl.rcParams['font.family'] = 'serif'
     %matplotlib inline
[]: file path = "C:/Users/dcste/OneDrive/Portfolio_Theory/multi_asset_etf_data.xlsx"
     description = pd.read_excel(file_path,sheet_name = "descriptions")
     description
[]:
        ticker
                                       shortName quoteType currency
                                                                         volume
           SPY
                                    SPDR S&P 500
     0
                                                        ETF
                                                                 USD
                                                                      101107853
     1
           EFA
                           iShares MSCI EAFE ETF
                                                       ETF
                                                                 USD
                                                                       33352872
           EEM iShares MSCI Emerging Index Fun
     2
                                                       ETF
                                                                 USD
                                                                       47539498
     3
           PSP
                Invesco Global Listed Private E
                                                       ETF
                                                                 USD
                                                                         120371
     4
                IQ Hedge MultiIQ Hedge Multi-St
                                                       ETF
                                                                 USD
                                                                         138713
           QAI
     5
           HYG
                iShares iBoxx $ High Yield Corp
                                                       ETF
                                                                 USD
                                                                       48935762
     6
                Invesco DB Commodity Index Trac
           DBC
                                                       ETF
                                                                 USD
                                                                        2314311
     7
           IYR
                   iShares U.S. Real Estate ETF
                                                       ETF
                                                                 USD
                                                                       12097258
     8
           IEF
                iShares 7-10 Year Treasury Bond
                                                       ETF
                                                                 USD
                                                                        7992450
     9
           BWX
                SPDR Bloomberg International Tr
                                                        ETF
                                                                 USD
                                                                         369873
           TIP
     10
                           iShares TIPS Bond ETF
                                                       ETF
                                                                 USD
                                                                        2875478
```

totalAssets longBusinessSummary 358229114880 The Trust seeks to achieve its investment obje...

iShares Short Treasury Bond ETF

11

0

SHV

ETF

USD

3140935

```
1
          43798241280
                       The fund generally will invest at least 80% of...
     2
                       The fund generally will invest at least 80% of...
          25870192640
     3
            171932880
                       The fund generally will invest at least 90% of...
            707315584
                       The fund is a "fund of funds" which means it i...
     4
     5
          12276870144
                       The underlying index is a rules-based index co...
                       The fund pursues its investment objective by i...
     6
           3708376064
     7
           4077254400
                       The fund seeks to track the investment results...
                       The underlying index measures the performance ...
     8
          23017226240
                       The fund generally invests substantially all, ...
     9
            809217792
                       The fund will invest at least 80% of its asset...
     10
          29620422656
          19234586624
     11
[]: total_return = pd.read_excel(file_path, sheet_name = "total_returns")
     total_return = total_return.set_index("Date")
     total_return = total_return.drop("SHV", axis = 1)
     excess_return = pd.read_excel(file_path, sheet_name = "excess returns")
     excess_return = excess_return.set_index("Date")
```

1.1 Question 1

1. Calculate and display the summary statistics of each excess asset's return.

[]: total_return.columns = ["International Treasury ETF", "Commodity Index", |

→Real Estate", "Private Equity", "MultiStrat HedgeFund", "SPY", "TIPS"]

⇔"Emerging", "MSCI EAFE", "High Yield Index", "7-10 Year Treasury Bond", "U.S.⊔

2. Which assets have the best and worst sharpe ratios?

[]:	Excess	Annual Return	Volatility	Sharpe Ratio
SPY		0.145643	0.145260	1.002640
U.S. Real Estate		0.145477	0.184744	0.787452
High Yield Index		0.066938	0.089701	0.746233
TIPS		0.030317	0.047681	0.635828
Private Equity		0.128622	0.221773	0.579971
MSCI EAFE		0.076474	0.162298	0.471197
MultiStrat HedgeFund		0.018212	0.049174	0.370346
7-10 Year Treasury Bond		0.021182	0.059387	0.356685
Emerging		0.067971	0.192071	0.353884

```
        Commodity Index
        0.034196
        0.180663
        0.189279

        International Treasury ETF
        0.000003
        0.078307
        0.000034
```

```
[]:
          Annual Return Volatility
                                      Sharpe Ratio
     SPY
               0.150293
                            0.144811
                                           1.037857
     IYR
               0.150128
                            0.184407
                                           0.814113
    HYG
               0.071588
                            0.089403
                                           0.800730
     TIP
               0.034967
                            0.047833
                                           0.731032
    PSP
               0.133272
                            0.221299
                                           0.602227
     EFA
               0.081124
                            0.161885
                                           0.501125
     OAI
               0.022862
                            0.048879
                                           0.467723
     IEF
               0.025833
                            0.060077
                                           0.429996
               0.072621
     EEM
                            0.191787
                                           0.378655
    DBC
               0.038846
                            0.180186
                                           0.215590
     BWX
               0.004653
                            0.078535
                                           0.059248
```

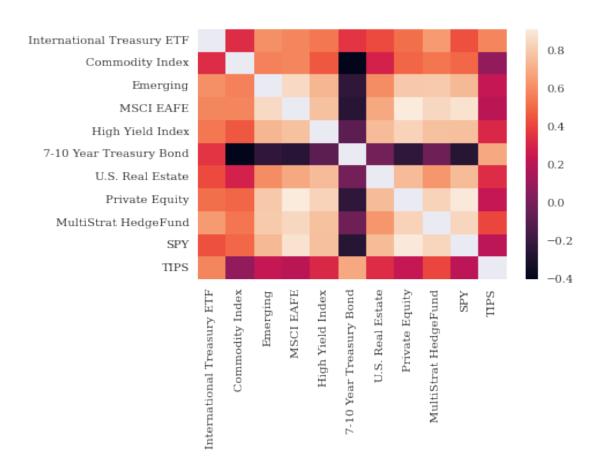
The best Sharpe Ratio is SPY with a value of 1.0026. The worst sharpe ratio is the International Treasury ETF.

1.2 Question 2

- a. Calculate the correlation matrix of the returns. Which pair has the highest and lowest correlation?
- b. How well have TIPs done in the sample? Hve they outperformed domestic or foreign bonds?
- c. Based on the data, do TIPs seem to expand the investment opportunity set, implying that Harvard should consider them as a separate asset?

```
[]: corr_mat = total_return.corr()
corr_mat[corr_mat == 1] = None
sns.heatmap(corr_mat)
```

[]: <AxesSubplot:>



[]:			Correlation
	7-10 Year Treasury Bond	l Commodity Index	-0.405431
	Commodity Index	7-10 Year Treasury Bond	-0.405431
	SPY	7-10 Year Treasury Bond	-0.269163
	7-10 Year Treasury Bond	l SPY	-0.269163
	MSCI EAFE	7-10 Year Treasury Bond	-0.264846
			•••
		SPY	0.874024
	Private Equity	SPY	0.903421
	SPY	Private Equity	0.903421
	Private Equity	MSCI EAFE	0.908746
	MSCI EAFE	Private Equity	0.908746

[110 rows x 1 columns]

1. As you can see the pair that has the higest correlation is MSCI EAFE and Private Equity

being 0.90987. The pair that has the smallest correlation is the 7-10 Year Treasury Bond and Commodity Index with a correlation of -.405431.

```
[]: cum_returns = (total_return + 1).cumprod()
cum_returns.plot(figsize = (20,10), title = "Value of $1 Invested")
```

[]: <AxesSubplot:title={'center':'Value of \$1 Invested'}, xlabel='Date'>



```
[]: cum_returns[["International Treasury ETF","7-10 Year Treasury Bond","TIPS"]].
```

```
[]: International Treasury ETF 7-10 Year Treasury Bond TIPS
Date
2022-06-30 1.054727 1.388467 1.536603
2022-07-31 1.076289 1.429586 1.602828
2022-08-31 1.021213 1.380211 1.573448
```

```
[]: annualized_mean.loc[["International Treasury ETF", "7-10 Year Treasury

⇔Bond", "TIPS"]].sort_values(by = "Sharpe Ratio")
```

[]:	Annual Return	Volatility	Sharpe Ratio
International Treasury ETF	0.004653	0.078535	0.059248
7-10 Year Treasury Bond	0.025833	0.060077	0.429996
TIPS	0.034967	0.047833	0.731032

1. All in all *Treasury Inflation-Protected Securities* do not perform exceedingly well between 2010-2022, with an average annual return of around 3. Compared to the International Treasury ETF and the 7-10 Year Domestic Treasury performance, TIPS do outperform in all measures - cumulative returns, annualized returns, and have a higher sharpe ratio.

- 2. Based on the data, **TIPS** definitely expand the investment opportunity offering any portfolio a better risk-return profile. The reason why TIPS expand the investment opportunity set because traditional fixed income assets respond to unanticipated inflation with a declining price (because the **ytm increases**). In contrast, inflation indexed bonds respond to unancipated inflation with an increasing price since the principal is increases in proportion to inflation. When two assets respond in an opposite fashion to an important variable, it is important to categorize them in separate asset classes.
- Yes, *Harvard* should consider **Treasury Inflation-Protected Securities as a separate** asset class.

1.3 Question 3

- 1. Compute and display the weights of the tangency portfolios: w^t
- 2. Compute the mean, volatility, and sharpe ratio for the tangency portfolio corresponding to w^t .

```
[]: def compute_tangency(df_tilde, diagonalize_Sigma=False):
         Sigma = df_tilde.cov()
         # N is the number of assets
         N = Sigma.shape[0]
         Sigma_adj = Sigma.copy()
         if diagonalize_Sigma:
             Sigma_adj.loc[:,:] = np.diag(np.diag(Sigma_adj))
         mu_tilde = df_tilde.mean()
         Sigma_inv = np.linalg.inv(Sigma_adj)
         weights = Sigma inv @ mu tilde / (np.ones(N) @ Sigma inv @ mu tilde)
         # For convenience, I'll wrap the solution back into a pandas. Series object.
         omega_tangency = pd.Series(weights, index=mu_tilde.index)
         return omega_tangency, mu_tilde, Sigma_adj
     omega_tangency, mu_tilde, Sigma = compute_tangency(total_return)
     omega_tangency.to_frame('Tangency Weights')
```

```
[]:
                                 Tangency Weights
     International Treasury ETF
                                         -1.335168
     Commodity Index
                                          0.239151
    Emerging
                                          0.339786
    MSCI EAFE
                                         -0.117068
    High Yield Index
                                          1.070489
     7-10 Year Treasury Bond
                                          2.457952
    U.S. Real Estate
                                         -0.307783
    Private Equity
                                         -0.513078
    MultiStrat HedgeFund
                                         -3.955222
     SPY
                                          2.430623
     TIPS
                                          0.690317
```

[]: omega_tangency

```
[]: International Treasury ETF
                                  -1.335168
     Commodity Index
                                   0.239151
     Emerging
                                   0.339786
    MSCI EAFE
                                  -0.117068
    High Yield Index
                                   1.070489
    7-10 Year Treasury Bond
                                   2.457952
    U.S. Real Estate
                                  -0.307783
    Private Equity
                                  -0.513078
    MultiStrat HedgeFund
                                  -3.955222
    SPY
                                   2.430623
    TIPS
                                   0.690317
    dtype: float64
```

The weights above reflect the weights of the portfolio tangent to mean-volatitility frontier.

```
[]: Portfolio Stats

Mean 0.3428

Volatility 0.1759

Sharpe 1.9493
```

The stats above reflect the mean, variance, and sharpe ratio of portfolio w^t that is tangent to the mean-volatility frontier.

1.4 Question 4 - The Allocation

Here I will calculate the optimized portfolio allocation with a target return of 1.5%.

```
def target_mv_portfolio(df_tilde,tangency_weights, target_return = 0.01):
    mu = df_tilde.mean()
    sigma_ = df_tilde.cov()
    sigma_inv = np.linalg.inv(sigma_)
    n = sigma_.shape[0]
    weight_v = (sigma_inv @ np.ones(n))/(np.ones(n) @ sigma_inv @ np.ones(n))
    weight_t = tangency_weights
    omega = (target_return - mu.T@weight_v)/(mu.T@weight_t - mu.T@weight_v)
    omega_star = omega*weight_t + (1-omega)*weight_v
    return pd.Series(omega_star, index = tangency_weights.index )

optimized_portfolio = target_mv_portfolio(total_return,omega_tangency,u)
    otarget_return = 0.015)
```

The weights above reflect the optimized asset allocation of the mean-variance portfolu with a targeted expected return of 1.5%

```
[]: portfolio_stats(total_return, optimized_portfolio, 12)
```

```
[]: Portfolio Stats

Mean 0.1800

Volatility 0.0934

Sharpe 1.9271
```

The stats above refelect the mean, standard deviation, and sharpe ratio of the allocation weights w^p .

Discuss the allocation:

- The assets in which you are most long in are SPY and High Yield Bond Index. Some of the allocations are greater |1|, which is unrealistic since this would involve investing with a high degree of levarage.
- The positions in which you are most short are BWX (SPDR Bloomberg International Treasury), Private Equity, and MultiQ Hedge Multi-Strategy Index. This involves taking **negative positions** by borrowing shares from prime broker an then immediately *selling* the asses with the intention of buying them back at a later date to profit from price declines.

Does the w^p allocation align with the assets that have the strongest Sharpe Ratios?

Answer: Yes, I the w^p portfolio allocations do align with the sharpe ratios. As you can see below, we there is a positive correlation between sharpe ratio and asset allocation. Specifically the correlation is 0.42. All this means is that, generally, higher sharpe values tend to have positive allocations. SPY has a sharpe ratio of 1.03-so we can expect to see a postive allocation-with our data we have an allocation 1.21. For Multi-Strat Hedge, we have the largest negative allocation of -1.54. Although Multi-Strat Hedge doe not have the lowest sharpe, its asset class does not offer great risk-adjusted returns

```
[]: print(annualized_mean)
print()
print(optimized_portfolio)
```

	Annual Return	Volatility	Sharpe Ratio
International Treasury ETF	0.004653	0.078535	0.059248
Commodity Index	0.038846	0.180186	0.215590
Emerging	0.072621	0.191787	0.378655
MSCI EAFE	0.081124	0.161885	0.501125
High Yield Index	0.071588	0.089403	0.800730
7-10 Year Treasury Bond	0.025833	0.060077	0.429996
U.S. Real Estate	0.150128	0.184407	0.814113
Private Equity	0.133272	0.221299	0.602227
MultiStrat HedgeFund	0.022862	0.048879	0.467723
SPY	0.150293	0.144811	1.037857
TIPS	0.034967	0.047833	0.731032

```
International Treasury ETF
                              -0.743896
Commodity Index
                               0.125074
Emerging
                               0.140120
MSCI EAFE
                              -0.043110
High Yield Index
                               0.606047
7-10 Year Treasury Bond
                               1.303991
U.S. Real Estate
                              -0.163098
Private Equity
                              -0.316038
MultiStrat HedgeFund
                              -1.545644
SPY
                               1.210202
TIPS
                               0.426352
```

dtype: float64

```
[]: np.corrcoef(annualized_mean["Sharpe Ratio"], optimized_portfolio)
```

1.5 Simple Portfolios

• A) Calculate the performance of an equally-weighted portfolio over the sample. Rescale the entire weighting vector to have a $u^p = 0.015$. Report its mean, volatility, and Sharpe ratio.

```
[]: equal_weights = np.repeat(1/11,11)
     portfolio_stats(total_return,equal_weights,12)
[]:
                 Portfolio Stats
     Mean
                           0.0715
     Volatility
                           0.0999
     Sharpe
                           0.7152
[]: target_return = 0.015
     equal_weight_scaled = equal_weights *(target_return/(total_return.mean() @__
      ⇔equal_weights))
     portfolio_stats(total_return,equal_weight_scaled,12)
[]:
                 Portfolio Stats
    Mean
                           0.1800
     Volatility
                           0.2517
                           0.7152
     Sharpe
    Calculating the performance of the risk parity portfolio with weights
       • w^i = \frac{1}{\sigma_i}
[]: w = 1/(total_return.std())
     target_mean = 0.015
     factor = 1/((w @ total_return.mean())/target_mean)
     w = factor *w
     pd.DataFrame(w, columns = ["Weights"])
[]:
                                   Weights
     International Treasury ETF
                                  0.379571
     Commodity Index
                                  0.165439
    Emerging
                                  0.155431
    MSCI EAFE
                                  0.184142
    High Yield Index
                                  0.333430
     7-10 Year Treasury Bond
                                  0.496193
    U.S. Real Estate
                                  0.161652
    Private Equity
                                  0.134704
    MultiStrat HedgeFund
                                  0.609866
     SPY
                                  0.205852
     TIPS
                                  0.623205
[]: portfolio_stats(total_return,w,12)
[]:
                 Portfolio Stats
                           0.1800
     Mean
     Volatility
                           0.2356
     Sharpe
                           0.7640
```

• How does this compare to the MV portfolio from problem 2.4?

Answer:

- The risk parity portfolio is a lot more **inefficient** than the mean-variance portfolio calculated in 2.4. We see that both portfolios have the same expected return of 18%, however, the Mean-Variance portfolio lies on the efficient frontier, whereas the risk parity portfolio lies inside the efficient frontier.
- In fact the sharpe ratio for the MV portfolio is 2.522 times greater than the risk parity porfolio.
- It would be unwise to allocate according to the risk parity porfolio weights.

1.6 Question 6 (TIPS Affect on Sharpe Ratio)

• Assess the performance of the MV Portfolio if we drop TIPS from the investment set relative to the original MV porfolio statistics.

```
[]: total_return_2 = total_return.drop(columns=["TIPS"])
```

```
Traceback (most recent call last)
     NameError
     c:
       →\Users\dcste\OneDrive\Portfolio_Theory\Homework_Jupyter\portfolio_theory\Home ork_1.
       ⇒ipynb Cell 43 in <cell line: 3>()
            <a href='vscode-notebook-cell:/c%3A/Users/dcste/OneDrive/Portfolio Theory
       →Homework_Jupyter/portfolio_theory/Homework_1.ipynb#X60sZmlsZQ%3D%3D?line=0'>1:/

d> total_return_2 = total_return.drop(columns=["TIPS"])

      ----> <a href='vscode-notebook-cell:/c%3A/Users/dcste/OneDrive/Portfolio Theory
       -Homework_Jupyter/portfolio_theory/Homework_1.ipynb#X60sZmlsZQ%3D%3D?line=2'>3:/
       →a> del sigma_2, mu_tilde_2, tangency_weights_no_TIP
     NameError: name 'sigma_2' is not defined
[]: tangency_weight_no_tip = compute_tangency(total_return_2)
     tangency_weight_no_tip
[]: (International Treasury ETF
                                   -1.425461
     Commodity Index
                                    0.310110
     Emerging
                                    0.392642
```

MSCI EAFE -0.188777High Yield Index 1.141159 7-10 Year Treasury Bond 3.154791 U.S. Real Estate -0.315906 Private Equity -0.524867MultiStrat HedgeFund -4.217960 SPY 2.674270 dtype: float64, International Treasury ETF 0.000388 Commodity Index 0.003237

Emerging MSCI EAFE High Yield Index 7-10 Year Treasury Bond U.S. Real Estate Private Equity MultiStrat HedgeFund SPY dtype: float64,	0.006052 0.006760 0.005966 0.002153 0.012511 0.011106 0.001905 0.012524
International Treasury ETF Commodity Index Emerging MSCI EAFE High Yield Index 7-10 Year Treasury Bond U.S. Real Estate Private Equity MultiStrat HedgeFund SPY	International Treasury ETF Commodity Index \
International Treasury ETF Commodity Index Emerging MSCI EAFE High Yield Index 7-10 Year Treasury Bond U.S. Real Estate Private Equity MultiStrat HedgeFund SPY	Emerging MSCI EAFE High Yield Index \ 0.000757 0.000610 0.000314 0.001626 0.001403 0.000605 0.003065 0.002190 0.001036 0.002190 0.002184 0.000920 0.001036 0.000920 0.000666 -0.000228 -0.000215 -0.000040 0.001765 0.001694 0.001026 0.002782 0.002713 0.001359 0.000619 0.000552 0.000277 0.001700 0.001707 0.000817
International Treasury ETF Commodity Index Emerging MSCI EAFE High Yield Index 7-10 Year Treasury Bond U.S. Real Estate Private Equity MultiStrat HedgeFund SPY	7-10 Year Treasury Bond U.S. Real Estate \ 0.000138
International Treasury ETF	Private Equity MultiStrat HedgeFund SPY 0.000737 0.000205 0.000405

```
Commodity Index
                                  0.001608
                                                        0.000389 0.001067
Emerging
                                  0.002782
                                                        0.000619
                                                                  0.001700
MSCI EAFE
                                  0.002713
                                                        0.000552
                                                                  0.001707
High Yield Index
                                  0.001359
                                                        0.000277
                                                                  0.000817
7-10 Year Treasury Bond
                                 -0.000272
                                                       -0.000010 -0.000195
U.S. Real Estate
                                  0.002536
                                                        0.000468 0.001648
Private Equity
                                  0.004081
                                                        0.000743 0.002413
MultiStrat HedgeFund
                                  0.000743
                                                        0.000199
                                                                  0.000491
SPY
                                                        0.000491 0.001748 )
                                  0.002413
```

```
[]:
                                 Allocation
     International Treasury ETF
                                   -0.739181
     Commodity Index
                                    0.155252
     Emerging
                                    0.151514
    MSCI EAFE
                                   -0.077601
    High Yield Index
                                    0.602196
     7-10 Year Treasury Bond
                                    1.597092
    U.S. Real Estate
                                   -0.153831
    Private Equity
                                   -0.303840
    MultiStrat HedgeFund
                                   -1.465329
     SPY
                                    1.233728
```

```
[]: print(portfolio_stats(total_return_2,optimized_no_tip,12))
    print()
    print(portfolio_stats(total_return,optimized_portfolio,12))
```

	Portfolio Stats
Mean	0.1800
Volatility	0.0941
Sharpe	1.9131

Portfolio Stats
Mean 0.1800
Volatility 0.0934
Sharpe 1.9271

• As you can see there is a very slight increase in volatility and a slight decline in the sharpe ratio when you remove *TIPS* from the portfolio suggesting including *Treasury-Inflation Protected Securities* offers better risk-adjusted returns.

1.7 Out of Sample Performance

• Using only data through the end of 2021, compute w^p with a $\mu^p = 0.15$, allocating to all 11 assets.

- Using the weights w^p , calculate the portfolio's sharpe ratio within that sample through the end of 2021.
- Again using those weights, calculate the portfolio's Sharpe ratio based ont he performance in 2022

```
[]: train_data = total_return[:'2021-12-31']
  test_data = total_return['2022-01-31':]
  train_tangency = compute_tangency(train_data)
  train_tangency_p = train_tangency[0]
  target_train_p = target_mv_portfolio(train_data, train_tangency_p,0.015)
  pd.DataFrame(target_train_p, columns = ["Weights through end of 2021"])
```

[]: Weights through end of 2021 International Treasury ETF -0.260550 Commodity Index -0.013134 Emerging 0.003842 MSCI EAFE -0.058326 High Yield Index 0.665042 7-10 Year Treasury Bond 1.186294 U.S. Real Estate -0.257773Private Equity -0.087996 MultiStrat HedgeFund -1.587112SPY 1.109137

[]: portfolio_stats(train_data,target_train_p,12)

0.300576

[]: Portfolio Stats

Mean 0.180

Volatility 0.078

Sharpe 2.308

TIPS

The weights above are the tangency weights for the portfolio w^p throught the end of 2021. Additionally, you can see the resulting portfolio statistics across these asset classes. The Sharpe ratio is significantly higher, but that would make sense given bullish market through the end of 2021.

```
[]: portfolio_stats(test_data, target_train_p, 12)
```

[]: Portfolio Stats

Mean -0.1707

Volatility 0.2299

Sharpe -0.7423

This portfolio is significantly different, but we would these statitics given how the market has behaved with increased selling pressures across all financial markets and fear arising from impending recession.