Homework_1

October 3, 2022

1 Mean Variance Optimization

- Import data and annualize the mea of monthly returns as well as the volatility of monthly returns with a scaling of $\sqrt{(12)}$

[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
import matplotlib as mpl
import seaborn as sns
import scipy as scs
import math
```

[ ]:
```
plt.style.use("seaborn")
mpl.rcParams['font.family'] = 'serif'
%matplotlib inline
```

[ ]:
```
file_path = "C:/Users/dcste/OneDrive/Portfolio_Theory/multi_asset_etf_data.xlsx"
description = pd.read_excel(file_path,sheet_name = "descriptions")
description
```

[ ]:
```
   ticker                        shortName quoteType currency     volume  \
0     SPY                   SPDR S&P 500       ETF      USD  101107853
1     EFA              iShares MSCI EAFE ETF       ETF      USD   33352872
2     EEM  iShares MSCI Emerging Index Fun       ETF      USD   47539498
3     PSP  Invesco Global Listed Private E       ETF      USD     120371
4     QAI  IQ Hedge MultiIQ Hedge Multi-St       ETF      USD     138713
5     HYG  iShares iBoxx $ High Yield Corp       ETF      USD   48935762
6     DBC  Invesco DB Commodity Index Trac       ETF      USD    2314311
7     IYR     iShares U.S. Real Estate ETF       ETF      USD   12097258
8     IEF  iShares 7-10 Year Treasury Bond       ETF      USD    7992450
9     BWX  SPDR Bloomberg International Tr       ETF      USD     369873
10    TIP              iShares TIPS Bond ETF       ETF      USD    2875478
11    SHV  iShares Short Treasury Bond ETF       ETF      USD    3140935

    totalAssets                              longBusinessSummary
0  358229114880  The Trust seeks to achieve its investment obje…
```

```
1    43798241280  The fund generally will invest at least 80% of…
2    25870192640  The fund generally will invest at least 80% of…
3      171932880  The fund generally will invest at least 90% of…
4      707315584  The fund is a "fund of funds" which means it i…
5    12276870144  The underlying index is a rules-based index co…
6     3708376064  The fund pursues its investment objective by i…
7     4077254400  The fund seeks to track the investment results…
8    23017226240  The underlying index measures the performance …
9      809217792  The fund generally invests substantially all, …
10   29620422656  The fund will invest at least 80% of its asset…
11   19234586624                                                r
```

[ ]:
```python
total_return = pd.read_excel(file_path, sheet_name = "total returns")
total_return = total_return.set_index("Date")
total_return = total_return.drop("SHV", axis = 1)
excess_return = pd.read_excel(file_path, sheet_name = "excess returns")
excess_return = excess_return.set_index("Date")
```

[ ]:
```python
total_return.columns = ["International Treasury ETF", "Commodity Index",␣
 ↪"Emerging", "MSCI EAFE","High Yield Index", "7-10 Year Treasury Bond","U.S.␣
 ↪Real Estate","Private Equity","MultiStrat HedgeFund","SPY","TIPS"]
total_return
```

[ ]:
```
            International Treasury ETF  Commodity Index  Emerging  MSCI EAFE  \
Date
2009-04-30                    0.008993        -0.001000  0.155582   0.115190
2009-05-31                    0.053672         0.162663  0.159400   0.131918
2009-06-30                    0.005149        -0.026259 -0.022495  -0.014050
2009-07-31                    0.031284         0.018568  0.110146   0.100415
2009-08-31                    0.007628        -0.040365 -0.013136   0.045031
…                                  …                …         …          …
2022-04-30                   -0.069696         0.056408 -0.061351  -0.067391
2022-05-31                    0.005460         0.046131  0.006135   0.019959
2022-06-30                   -0.046443        -0.075000 -0.051577  -0.087666
2022-07-31                    0.020443        -0.019895 -0.003491   0.051688
2022-08-31                   -0.051172         0.006128 -0.016767  -0.054778

            High Yield Index  7-10 Year Treasury Bond  U.S. Real Estate  \
Date
2009-04-30          0.138460                -0.027452          0.296151
2009-05-31          0.028555                -0.020773          0.022727
2009-06-30          0.033516                -0.005572         -0.024863
2009-07-31          0.069191                 0.008317          0.105799
2009-08-31         -0.016969                 0.007635          0.131939
…                        …                        …                 …
2022-04-30         -0.041803                -0.042283         -0.041305
2022-05-31          0.016299                 0.006184         -0.044434
```

```
2022-06-30         -0.070499                   -0.008634         -0.068911
2022-07-31          0.066989                    0.029615          0.088606
2022-08-31         -0.037825                   -0.034538         -0.054829

            Private Equity  MultiStrat HedgeFund       SPY      TIPS
Date
2009-04-30        0.230202              0.022882  0.099346 -0.017952
2009-05-31        0.053892              0.027865  0.058454  0.019967
2009-06-30        0.045449             -0.003436 -0.000655  0.001982
2009-07-31        0.143247              0.015326  0.074606  0.000879
2009-08-31        0.033413             -0.004151  0.036939  0.008413
…                      …                   …         …        …
2022-04-30       -0.125679             -0.033398 -0.087769 -0.021831
2022-05-31        0.015084             -0.004025  0.002257 -0.009922
2022-06-30       -0.132477             -0.033681 -0.082460 -0.031155
2022-07-31        0.108961              0.018822  0.092087  0.043098
2022-08-31       -0.080808             -0.008553 -0.033447 -0.018330

[161 rows x 11 columns]
```

1.1 Question 1

1. Calculate and display the summary statistics of each excess asset's return.
2. Which assets have the best and worst sharpe ratios?

[ ]:
```
excess_return.columns = ["International Treasury ETF", "Commodity Index",
 ↪"Emerging", "MSCI EAFE","High Yield Index", "7-10 Year Treasury Bond","U.S.
 ↪Real Estate","Private Equity","MultiStrat HedgeFund","SPY","TIPS"]
summary_excess = pd.DataFrame((excess_return.mean()*12), columns = ["Excess
 ↪Annual Return"])
summary_excess["Volatility"] = excess_return.std()*np.sqrt(12)
summary_excess["Sharpe Ratio"] = summary_excess["Excess Annual Return"]/
 ↪summary_excess["Volatility"]
summary_excess.sort_values(by = "Sharpe Ratio", ascending = False)
```

[ ]:

	Excess Annual Return	Volatility	Sharpe Ratio
SPY	0.145643	0.145260	1.002640
U.S. Real Estate	0.145477	0.184744	0.787452
High Yield Index	0.066938	0.089701	0.746233
TIPS	0.030317	0.047681	0.635828
Private Equity	0.128622	0.221773	0.579971
MSCI EAFE	0.076474	0.162298	0.471197
MultiStrat HedgeFund	0.018212	0.049174	0.370346
7-10 Year Treasury Bond	0.021182	0.059387	0.356685
Emerging	0.067971	0.192071	0.353884
Commodity Index	0.034196	0.180663	0.189279
International Treasury ETF	0.000003	0.078307	0.000034

[ ]:
```
# Scaling excess monthly return to yearly return
# This is if the risk free rate is equal to 0
annualized_mean = pd.DataFrame((total_return.mean()*12), columns = ["Annual␣
↪Return"])
annualized_mean["Volatility"] = total_return.std()*np.sqrt(12)
annualized_mean["Sharpe Ratio"] = annualized_mean["Annual Return"]/
↪annualized_mean["Volatility"]
annualized_mean.sort_values(by = "Sharpe Ratio",ascending = False)
```

[ ]:

	Annual Return	Volatility	Sharpe Ratio
SPY	0.150293	0.144811	1.037857
IYR	0.150128	0.184407	0.814113
HYG	0.071588	0.089403	0.800730
TIP	0.034967	0.047833	0.731032
PSP	0.133272	0.221299	0.602227
EFA	0.081124	0.161885	0.501125
QAI	0.022862	0.048879	0.467723
IEF	0.025833	0.060077	0.429996
EEM	0.072621	0.191787	0.378655
DBC	0.038846	0.180186	0.215590
BWX	0.004653	0.078535	0.059248

The best Sharpe Ratio is SPY with a value of 1.0026. The worst sharpe ratio is the International Treasury ETF.

1.2 Question 2

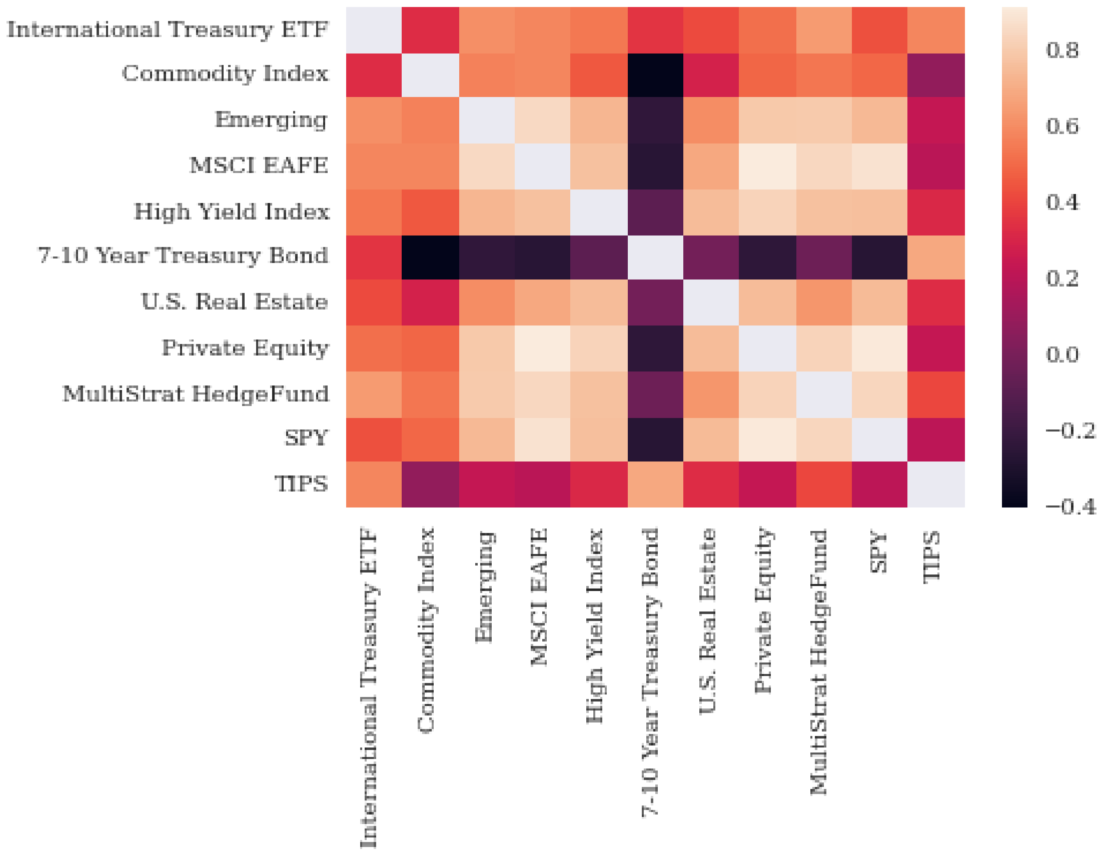
a. Calculate the correlation matrix of the returns. Which pair has the highest and lowest correlation?

b. How well have TIPs done in the sample? Hve they outperformed domestic or foreign bonds?

c. Based on the data, do TIPs seem to expand the investment opportunity set, implying that Harvard should consider them as a separate asset?

[ ]:
```
corr_mat = total_return.corr()
corr_mat[corr_mat == 1] = None
sns.heatmap(corr_mat)
```

[ ]: <AxesSubplot:>




```
[ ]: corr_rank = pd.DataFrame(corr_mat.unstack().sort_values().dropna(), columns =
     ↪["Correlation"])
     corr_rank
```

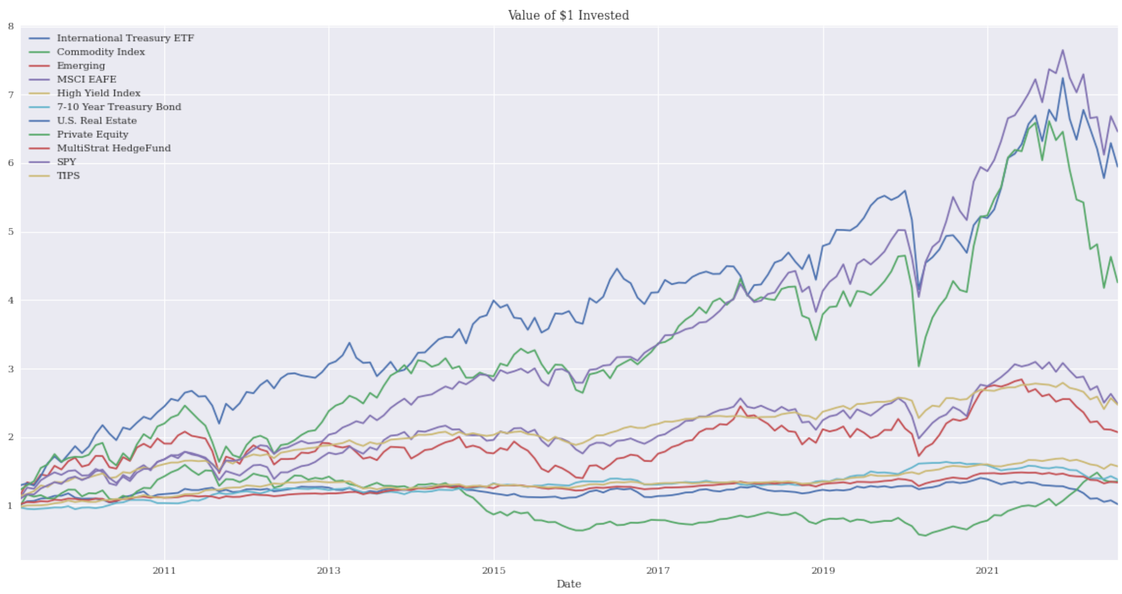
```
[ ]:                                                 Correlation
     7-10 Year Treasury Bond Commodity Index            -0.405431
     Commodity Index         7-10 Year Treasury Bond    -0.405431
     SPY                     7-10 Year Treasury Bond    -0.269163
     7-10 Year Treasury Bond SPY                        -0.269163
     MSCI EAFE               7-10 Year Treasury Bond    -0.264846
     …                                                      …
                             SPY                         0.874024
     Private Equity          SPY                         0.903421
     SPY                     Private Equity              0.903421
     Private Equity          MSCI EAFE                   0.908746
     MSCI EAFE               Private Equity              0.908746

     [110 rows x 1 columns]
```

1. As you can see the pair that has the higest correlation is MSCI EAFE and Private Equity

being 0.90987. The pair that has the smallest correlation is the 7-10 Year Treasury Bond and Commodity Index with a correlation of −.405431.

[ ]:
```
cum_returns = (total_return + 1).cumprod()
cum_returns.plot(figsize = (20,10), title = "Value of $1 Invested")
```

[ ]: <AxesSubplot:title={'center':'Value of $1 Invested'}, xlabel='Date'>



[ ]:
```
cum_returns[["International Treasury ETF","7-10 Year Treasury Bond","TIPS"]].
↪tail(3)
```

[ ]:

Date	International Treasury ETF	7-10 Year Treasury Bond	TIPS
2022-06-30	1.054727	1.388467	1.536603
2022-07-31	1.076289	1.429586	1.602828
2022-08-31	1.021213	1.380211	1.573448

[ ]:
```
annualized_mean.loc[["International Treasury ETF", "7-10 Year Treasury␣
↪Bond","TIPS"]].sort_values(by = "Sharpe Ratio")
```

[ ]:

	Annual Return	Volatility	Sharpe Ratio
International Treasury ETF	0.004653	0.078535	0.059248
7-10 Year Treasury Bond	0.025833	0.060077	0.429996
TIPS	0.034967	0.047833	0.731032

1. All in all *Treasury Inflation-Protected Securities* do not perform exceedingly well between 2010-2022, with an average annual return of around 3. Compared to the International Treasury ETF and the 7-10 Year Domestic Treasury performance, TIPS do outperform in all measures - cumulative returns, annualized returns, and have a higher sharpe ratio.

2. Based on the data, **TIPS** definitely expand the investment opportunity offering any portfolio a better risk-return profile. The reason why TIPS expand the investment opportunity set because traditional fixed income assets respond to unanticipated inflation with a declining price (because the **ytm increases**). In contrast, inflation indexed bonds respond to unancipated inflation with an increasing price since the principal is increases in proportion to inflation. When two assets respond in an opposite fashion to an important variable , it is important to categorize them in separate asset classes.

- Yes, *Harvard* should consider **Treasury Inflation-Protected Securities as a separate asset class.**

1.3 Question 3

1. Compute and display the weights of the tangency portfolios: w^t
2. Compute the mean, volatility, and sharpe ratio for the tangency portfolio corresponding to w^t.

[ ]:
```
def compute_tangency(df_tilde, diagonalize_Sigma=False):
    Sigma = df_tilde.cov()

    # N is the number of assets

    N = Sigma.shape[0]

    Sigma_adj = Sigma.copy()

    if diagonalize_Sigma:

        Sigma_adj.loc[:,:] = np.diag(np.diag(Sigma_adj))



    mu_tilde = df_tilde.mean()

    Sigma_inv = np.linalg.inv(Sigma_adj)

    weights = Sigma_inv @ mu_tilde / (np.ones(N) @ Sigma_inv @ mu_tilde)

    # For convenience, I'll wrap the solution back into a pandas.Series object.

    omega_tangency = pd.Series(weights, index=mu_tilde.index)

    return omega_tangency, mu_tilde, Sigma_adj


omega_tangency, mu_tilde, Sigma = compute_tangency(total_return)

omega_tangency.to_frame('Tangency Weights')
```

[ ]:

	Tangency Weights
International Treasury ETF	-1.335168
Commodity Index	0.239151
Emerging	0.339786
MSCI EAFE	-0.117068
High Yield Index	1.070489
7-10 Year Treasury Bond	2.457952
U.S. Real Estate	-0.307783
Private Equity	-0.513078
MultiStrat HedgeFund	-3.955222
SPY	2.430623
TIPS	0.690317

[ ]:
```
omega_tangency
```

[ ]:
```
International Treasury ETF   -1.335168
Commodity Index               0.239151
Emerging                      0.339786
MSCI EAFE                    -0.117068
High Yield Index              1.070489
7-10 Year Treasury Bond       2.457952
U.S. Real Estate             -0.307783
Private Equity               -0.513078
MultiStrat HedgeFund         -3.955222
SPY                           2.430623
TIPS                          0.690317
dtype: float64
```

The weights above reflect the weights of the portfolio tangent to mean-volatitility frontier.

[ ]:
```
def portfolio_stats(df_tilde, omega, annualize_fac):
    # Mean
    mean = df_tilde.mean() @ omega * annualize_fac

    # Volatility
    vol = (df_tilde @ omega).std() * np.sqrt(12)


    # Sharpe ratio
    sharpe_ratio = mean / vol

    return round(pd.DataFrame(data = [mean, vol, sharpe_ratio],
                              index = ['Mean', 'Volatility', 'Sharpe'],
                              columns = ['Portfolio Stats']), 4)

portfolio_stats(total_return,omega_tangency, 12)
```

[ ]:

	Portfolio Stats
Mean	0.3428
Volatility	0.1759
Sharpe	1.9493

The stats above reflect the mean, variance, and sharpe ratio of portfolio w^t that is tangent to the mean-volatiltiy frontier.

1.4 Question 4 - The Allocation

Here I will calculate the optimized portfolio allocation with a target return of 1.5%.

[ ]:
```
def target_mv_portfolio(df_tilde,tangency_weights, target_return = 0.01):
    mu = df_tilde.mean()
    sigma_ = df_tilde.cov()
    sigma_inv = np.linalg.inv(sigma_)
    n = sigma_.shape[0]
    weight_v = (sigma_inv @ np.ones(n))/(np.ones(n) @ sigma_inv @ np.ones(n))
    weight_t =  tangency_weights
    omega = (target_return - mu.T@weight_v)/(mu.T@weight_t - mu.T@weight_v)
    omega_star = omega*weight_t + (1-omega)*weight_v
    return pd.Series(omega_star, index = tangency_weights.index )

optimized_portfolio = target_mv_portfolio(total_return,omega_tangency,␣
 ↪target_return = 0.015)
```

The weights above relfect the optimized asset allocation of the mean-variance portfolu with a targeted expected return of 1.5%

[ ]:
```
portfolio_stats(total_return, optimized_portfolio, 12)
```

[ ]:

	Portfolio Stats
Mean	0.1800
Volatility	0.0934
Sharpe	1.9271

The stats above refelect the mean, standard deviation, and sharpe ratio of the allocation weights w^p.

Discuss the allocation:

- The assets in which you are most long in are SPY and High Yield Bond Index. Some of the allocations are greater $|1|$, which is unrealistic since this would involve investing with a high degree of levarage.
- The positions in which you are most short are BWX (SPDR Bloomberg International Treasury), Private Equity, and MultiQ Hedge Multi-Strategy Index. This involves taking **negative positions** by borrowing shares from prime broker an then immediately *selling* the asses with the intention of buying them back at a later date to profit from price declines.

Does the w^p allocation align with the assets that have the strongest Sharpe Ratios?

Answer: Yes, I the w^p portfolio allocations do align with the sharpe ratios. As you can see below, we there is a positive correlation between sharpe ratio and asset allocation. Specifically the correlation is 0.42. All this means is that, generally, higher sharpe values tend to have *positive* allocations. SPY has a sharpe ratio of 1.03-so we can expect to see a postive allocation-with our data we have an allocation 1.21. For Multi-Strat Hedge, we have the largest negative allocation of -1.54. Althougth Multi-Strat Hedge doe not have the lowest sharpe, its asset class does not offer great risk-adjusted returns

[ ]:
```
print(annualized_mean)
print()
print(optimized_portfolio)
```

```
                             Annual Return  Volatility  Sharpe Ratio
International Treasury ETF        0.004653    0.078535      0.059248
Commodity Index                   0.038846    0.180186      0.215590
Emerging                          0.072621    0.191787      0.378655
MSCI EAFE                         0.081124    0.161885      0.501125
High Yield Index                  0.071588    0.089403      0.800730
7-10 Year Treasury Bond           0.025833    0.060077      0.429996
U.S. Real Estate                  0.150128    0.184407      0.814113
Private Equity                    0.133272    0.221299      0.602227
MultiStrat HedgeFund              0.022862    0.048879      0.467723
SPY                               0.150293    0.144811      1.037857
TIPS                              0.034967    0.047833      0.731032

International Treasury ETF   -0.743896
Commodity Index               0.125074
Emerging                      0.140120
MSCI EAFE                    -0.043110
High Yield Index              0.606047
7-10 Year Treasury Bond       1.303991
U.S. Real Estate             -0.163098
Private Equity               -0.316038
MultiStrat HedgeFund         -1.545644
SPY                           1.210202
TIPS                          0.426352
dtype: float64
```

[ ]:
```
np.corrcoef(annualized_mean["Sharpe Ratio"], optimized_portfolio)
```

[ ]:
```
array([[1.        , 0.44202232],
       [0.44202232, 1.        ]])
```

1.5 Simple Portfolios

- A) Calculate the performance of an equally-weighted portfolio over the sample. Rescale the entire weighting vector to have a $u^p = 0.015$. Report its mean, volatility, and Sharpe ratio.

[ ]:
```
equal_weights = np.repeat(1/11,11)
portfolio_stats(total_return,equal_weights,12)
```

[ ]:

	Portfolio Stats
Mean	0.0715
Volatility	0.0999
Sharpe	0.7152

[ ]:
```
target_return = 0.015
equal_weight_scaled = equal_weights *(target_return/(total_return.mean() @
    ↪equal_weights))
portfolio_stats(total_return,equal_weight_scaled,12)
```

[ ]:

	Portfolio Stats
Mean	0.1800
Volatility	0.2517
Sharpe	0.7152

Calculating the performance of the risk parity portfolio with weights

- $w^i = \frac{1}{\sigma_i}$

[ ]:
```
w = 1/(total_return.std())
target_mean = 0.015
factor = 1/((w @ total_return.mean())/target_mean)
w = factor *w
pd.DataFrame(w, columns = ["Weights"])
```

[ ]:

	Weights
International Treasury ETF	0.379571
Commodity Index	0.165439
Emerging	0.155431
MSCI EAFE	0.184142
High Yield Index	0.333430
7-10 Year Treasury Bond	0.496193
U.S. Real Estate	0.161652
Private Equity	0.134704
MultiStrat HedgeFund	0.609866
SPY	0.205852
TIPS	0.623205

[ ]:
```
portfolio_stats(total_return,w,12)
```

[ ]:

	Portfolio Stats
Mean	0.1800
Volatility	0.2356
Sharpe	0.7640

- How does this compare tothe MV portfolio from problem 2.4?

Answer:

- The *risk parity portfolio* is a lot more **inefficient** than the mean-variance portfolio calculated in 2.4. We see that both porfolios have the same expected return of 18%, however, the Mean-Variance porfolio lies on the efficient frontier, whereas the risk parity porfolio lies inside the efficient frontier.
- In fact the sharpe ratio for the MV portfolio is 2.522 times greater than the risk parity porfolio.
- It would be unwise to allocate according to the risk parity porfolio weights.

1.6 Question 6 (TIPS Affect on Sharpe Ratio)

- Assess the performance of the MV Portfolio if we drop TIPS from the investment set relative to the original MV porfolio statistics.

[ ]:
```
total_return_2 = total_return.drop(columns=["TIPS"])
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
c:
↪\Users\dcste\OneDrive\Portfolio_Theory\Homework_Jupyter\portfolio_theory\Homework_1.
↪ipynb Cell 43 in <cell line: 3>()
      <a href='vscode-notebook-cell:/c%3A/Users/dcste/OneDrive/Portfolio_Theory/
↪Homework_Jupyter/portfolio_theory/Homework_1.ipynb#X60sZmlsZQ%3D%3D?line=0'>1</
↪a> total_return_2 = total_return.drop(columns=["TIPS"])
----> <a href='vscode-notebook-cell:/c%3A/Users/dcste/OneDrive/Portfolio_Theory/
↪Homework_Jupyter/portfolio_theory/Homework_1.ipynb#X60sZmlsZQ%3D%3D?line=2'>3</
↪a> del sigma_2, mu_tilde_2, tangency_weights_no_TIP

NameError: name 'sigma_2' is not defined
```

[ ]:
```
tangency_weight_no_tip = compute_tangency(total_return_2)
tangency_weight_no_tip
```

[ ]:
```
(International Treasury ETF   -1.425461
 Commodity Index               0.310110
 Emerging                      0.392642
 MSCI EAFE                    -0.188777
 High Yield Index              1.141159
 7-10 Year Treasury Bond       3.154791
 U.S. Real Estate             -0.315906
 Private Equity               -0.524867
 MultiStrat HedgeFund         -4.217960
 SPY                           2.674270
 dtype: float64,
 International Treasury ETF    0.000388
 Commodity Index               0.003237
```

```
Emerging                         0.006052
MSCI EAFE                        0.006760
High Yield Index                 0.005966
7-10 Year Treasury Bond          0.002153
U.S. Real Estate                 0.012511
Private Equity                   0.011106
MultiStrat HedgeFund             0.001905
SPY                              0.012524
dtype: float64,
                            International Treasury ETF  Commodity Index  \
International Treasury ETF                    0.000514         0.000386
Commodity Index                               0.000386         0.002706
Emerging                                      0.000757         0.001626
MSCI EAFE                                     0.000610         0.001403
High Yield Index                              0.000314         0.000605
7-10 Year Treasury Bond                       0.000138        -0.000366
U.S. Real Estate                              0.000497         0.000787
Private Equity                                0.000737         0.001608
MultiStrat HedgeFund                          0.000205         0.000389
SPY                                           0.000405         0.001067

                            Emerging  MSCI EAFE  High Yield Index  \
International Treasury ETF  0.000757   0.000610          0.000314
Commodity Index             0.001626   0.001403          0.000605
Emerging                    0.003065   0.002190          0.001036
MSCI EAFE                   0.002190   0.002184          0.000920
High Yield Index            0.001036   0.000920          0.000666
7-10 Year Treasury Bond    -0.000228  -0.000215         -0.000040
U.S. Real Estate            0.001765   0.001694          0.001026
Private Equity              0.002782   0.002713          0.001359
MultiStrat HedgeFund        0.000619   0.000552          0.000277
SPY                         0.001700   0.001707          0.000817

                            7-10 Year Treasury Bond  U.S. Real Estate  \
International Treasury ETF                 0.000138          0.000497
Commodity Index                           -0.000366          0.000787
Emerging                                  -0.000228          0.001765
MSCI EAFE                                 -0.000215          0.001694
High Yield Index                          -0.000040          0.001026
7-10 Year Treasury Bond                    0.000301         -0.000017
U.S. Real Estate                          -0.000017          0.002834
Private Equity                            -0.000272          0.002536
MultiStrat HedgeFund                      -0.000010          0.000468
SPY                                       -0.000195          0.001648

                            Private Equity  MultiStrat HedgeFund       SPY
International Treasury ETF        0.000737              0.000205  0.000405
```

```
 Commodity Index                   0.001608              0.000389  0.001067
 Emerging                          0.002782              0.000619  0.001700
 MSCI EAFE                         0.002713              0.000552  0.001707
 High Yield Index                  0.001359              0.000277  0.000817
 7-10 Year Treasury Bond          -0.000272             -0.000010 -0.000195
 U.S. Real Estate                  0.002536              0.000468  0.001648
 Private Equity                    0.004081              0.000743  0.002413
 MultiStrat HedgeFund              0.000743              0.000199  0.000491
 SPY                               0.002413              0.000491  0.001748   )
```

[ ]:
```
optimized_no_tip = target_mv_portfolio(total_return_2,␣
↪tangency_weight_no_tip[0],0.015)
pd.DataFrame(optimized_no_tip, columns = ["Allocation"])
```

[ ]:

	Allocation
International Treasury ETF	-0.739181
Commodity Index	0.155252
Emerging	0.151514
MSCI EAFE	-0.077601
High Yield Index	0.602196
7-10 Year Treasury Bond	1.597092
U.S. Real Estate	-0.153831
Private Equity	-0.303840
MultiStrat HedgeFund	-1.465329
SPY	1.233728

[ ]:
```
print(portfolio_stats(total_return_2,optimized_no_tip,12))
print()
print(portfolio_stats(total_return,optimized_portfolio,12))
```

```
            Portfolio Stats
Mean                 0.1800
Volatility           0.0941
Sharpe               1.9131

            Portfolio Stats
Mean                 0.1800
Volatility           0.0934
Sharpe               1.9271
```

- As you can see there is a very slight increase in volatility and a slight decline in the sharpe ratio when you remove *TIPS* from the portfolio suggesting including *Treasury-Inflation Protected Securities* offers better risk-adjusted returns.

1.7 Out of Sample Performance

- Using only data through the end of 2021, compute w^p with a $\mu^p = 0.15$, allocating to all 11 assets.

- Using the weights w^p, calculate the portfolio's sharpe ratio within that sample through the end of 2021.
- Again using those weights, calculate the portfolio's Sharpe ratio based ont he performance in 2022

[ ]:
```
train_data = total_return[:'2021-12-31']
test_data = total_return['2022-01-31':]
train_tangency = compute_tangency(train_data)
train_tangency_p = train_tangency[0]
target_train_p = target_mv_portfolio(train_data, train_tangency_p,0.015)
pd.DataFrame(target_train_p, columns = ["Weights through end of 2021"])
```

[ ]:

	Weights through end of 2021
International Treasury ETF	-0.260550
Commodity Index	-0.013134
Emerging	0.003842
MSCI EAFE	-0.058326
High Yield Index	0.665042
7-10 Year Treasury Bond	1.186294
U.S. Real Estate	-0.257773
Private Equity	-0.087996
MultiStrat HedgeFund	-1.587112
SPY	1.109137
TIPS	0.300576

[ ]:
```
portfolio_stats(train_data,target_train_p,12)
```

[ ]:

	Portfolio Stats
Mean	0.180
Volatility	0.078
Sharpe	2.308

The weights above are the tangency weights for the portfolio w^p throught the end of 2021. Additionally, you can see the resulting portfolio statistics across these asset classes. The Sharpe ratio is significantly higher, but that would make sense given bullish market through the end of 2021.

[ ]:
```
portfolio_stats(test_data, target_train_p, 12)
```

[ ]:

	Portfolio Stats
Mean	-0.1707
Volatility	0.2299
Sharpe	-0.7423

This portfolio is significantly different, but we would these statitics given how the market has behaved with increased selling pressures across all financial markets and fear arising from impending recession.