Homework_1

October 3, 2022

1 Mean Variance Optimization

- Import data and annualize the mea of monthly returns as well as the volatility of monthly returns with a scaling of $\sqrt{(12)}$

[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
import matplotlib as mpl
import seaborn as sns
import scipy as scs
import math
```

[ ]:
```
plt.style.use("seaborn")
mpl.rcParams['font.family'] = 'serif'
%matplotlib inline
```

[ ]:
```
file_path = "C:/Users/dcste/OneDrive/Portfolio_Theory/multi_asset_etf_data.xlsx"
description = pd.read_excel(file_path,sheet_name = "descriptions")
description
```

[ ]:

	ticker	shortName	quoteType	currency	volume	\
0	SPY	SPDR S&P 500	ETF	USD	101107853	
1	EFA	iShares MSCI EAFE ETF	ETF	USD	33352872	
2	EEM	iShares MSCI Emerging Index Fun	ETF	USD	47539498	
3	PSP	Invesco Global Listed Private E	ETF	USD	120371	
4	QAI	IQ Hedge MultiIQ Hedge Multi-St	ETF	USD	138713	
5	HYG	iShares iBoxx $ High Yield Corp	ETF	USD	48935762	
6	DBC	Invesco DB Commodity Index Trac	ETF	USD	2314311	
7	IYR	iShares U.S. Real Estate ETF	ETF	USD	12097258	
8	IEF	iShares 7-10 Year Treasury Bond	ETF	USD	7992450	
9	BWX	SPDR Bloomberg International Tr	ETF	USD	369873	
10	TIP	iShares TIPS Bond ETF	ETF	USD	2875478	
11	SHV	iShares Short Treasury Bond ETF	ETF	USD	3140935	

	totalAssets	longBusinessSummary
0	358229114880	The Trust seeks to achieve its investment obje…

```
1    43798241280  The fund generally will invest at least 80% of…
2    25870192640  The fund generally will invest at least 80% of…
3      171932880  The fund generally will invest at least 90% of…
4      707315584  The fund is a "fund of funds" which means it i…
5    12276870144  The underlying index is a rules-based index co…
6     3708376064  The fund pursues its investment objective by i…
7     4077254400  The fund seeks to track the investment results…
8    23017226240  The underlying index measures the performance …
9      809217792  The fund generally invests substantially all, …
10   29620422656  The fund will invest at least 80% of its asset…
11   19234586624                                                r
```

[ ]:
```
total_return = pd.read_excel(file_path, sheet_name = "total returns")
total_return = total_return.set_index("Date")
total_return = total_return.drop("SHV", axis = 1)
```

[ ]:
```
total_return.columns = ["International Treasury ETF", "Commodity Index",␣
 ↪"Emerging", "MSCI EAFE","High Yield Index", "7-10 Year Treasury Bond","U.S.␣
 ↪Real Estate","Private Equity","MultiStrat HedgeFund","SPY","TIPS"]
total_return
```

[ ]:
```
            International Treasury ETF  Commodity Index  Emerging  MSCI EAFE  \
Date
2009-04-30                     0.008993        -0.001000  0.155582   0.115190
2009-05-31                     0.053672         0.162663  0.159400   0.131918
2009-06-30                     0.005149        -0.026259 -0.022495  -0.014050
2009-07-31                     0.031284         0.018568  0.110146   0.100415
2009-08-31                     0.007628        -0.040365 -0.013136   0.045031
…                                   …                …         …          …
2022-04-30                    -0.069696         0.056408 -0.061351  -0.067391
2022-05-31                     0.005460         0.046131  0.006135   0.019959
2022-06-30                    -0.046443        -0.075000 -0.051577  -0.087666
2022-07-31                     0.020443        -0.019895 -0.003491   0.051688
2022-08-31                    -0.051172         0.006128 -0.016767  -0.054778

            High Yield Index  7-10 Year Treasury Bond  U.S. Real Estate  \
Date
2009-04-30          0.138460                -0.027452          0.296151
2009-05-31          0.028555                -0.020773          0.022727
2009-06-30          0.033516                -0.005572         -0.024863
2009-07-31          0.069191                 0.008317          0.105799
2009-08-31         -0.016969                 0.007635          0.131939
…                        …                        …                 …
2022-04-30         -0.041803                -0.042283         -0.041305
2022-05-31          0.016299                 0.006184         -0.044434
2022-06-30         -0.070499                -0.008634         -0.068911
2022-07-31          0.066989                 0.029615          0.088606
```

```
2022-08-31         -0.037825                -0.034538         -0.054829

            Private Equity  MultiStrat HedgeFund       SPY      TIPS
Date
2009-04-30        0.230202              0.022882  0.099346 -0.017952
2009-05-31        0.053892              0.027865  0.058454  0.019967
2009-06-30        0.045449             -0.003436 -0.000655  0.001982
2009-07-31        0.143247              0.015326  0.074606  0.000879
2009-08-31        0.033413             -0.004151  0.036939  0.008413
…                      …                     …         …         …
2022-04-30       -0.125679             -0.033398 -0.087769 -0.021831
2022-05-31        0.015084             -0.004025  0.002257 -0.009922
2022-06-30       -0.132477             -0.033681 -0.082460 -0.031155
2022-07-31        0.108961              0.018822  0.092087  0.043098
2022-08-31       -0.080808             -0.008553 -0.033447 -0.018330

[161 rows x 11 columns]
```

1.1 Question 1

1. Calculate and display the summary statistics of each asset's return.
2. Which assets have the best and worst sharpe ratios?

[ ]:
```
# Scaling excess monthly return to yearly return
annualized_mean = pd.DataFrame((total_return.mean()*12), columns = ["Annual␣
↪Return"])
annualized_mean["Volatility"] = total_return.std()*np.sqrt(12)
annualized_mean["Sharpe Ratio"] = annualized_mean["Annual Return"]/
↪annualized_mean["Volatility"]
annualized_mean.sort_values(by = "Sharpe Ratio",ascending = False)
```

[ ]:
```
                           Annual Return  Volatility  Sharpe Ratio
SPY                             0.150293    0.144811      1.037857
U.S. Real Estate                0.150128    0.184407      0.814113
High Yield Index                0.071588    0.089403      0.800730
TIPS                            0.034967    0.047833      0.731032
Private Equity                  0.133272    0.221299      0.602227
MSCI EAFE                       0.081124    0.161885      0.501125
MultiStrat HedgeFund            0.022862    0.048879      0.467723
7-10 Year Treasury Bond         0.025833    0.060077      0.429996
Emerging                        0.072621    0.191787      0.378655
Commodity Index                 0.038846    0.180186      0.215590
International Treasury ETF      0.004653    0.078535      0.059248
```

The best Sharpe Ratio is SPY with a value of 1.0026. The worst sharpe ratio is the International Treasury ETF.

1.2 Question 2

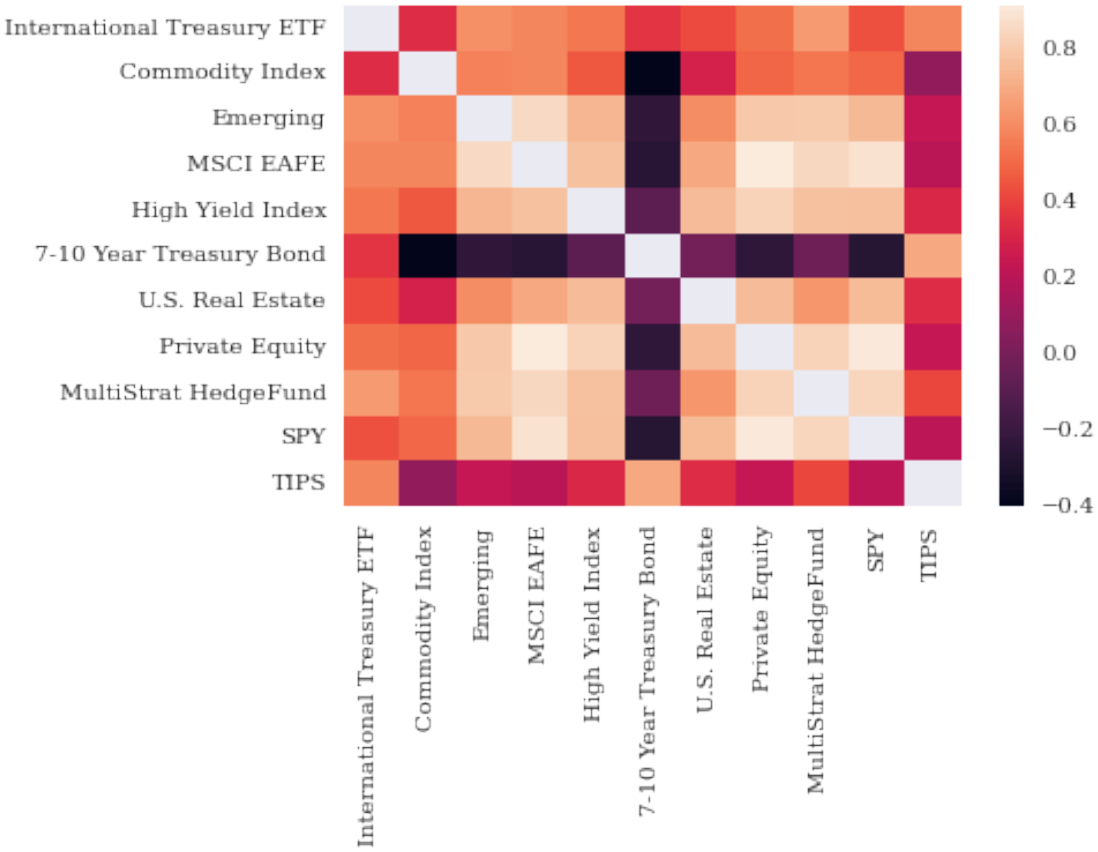
a. Calculate the correlation matrix of the returns. Which pair has the highest and lowest correlation?

b. How well have TIPs done in the sample? Hve they outperformed domestic or foreign bonds?

c. Based on the data, do TIPs seem to expand the investment opportunity set, implying that Harvard should consider them as a separate asset?

[ ]:
```
corr_mat = total_return.corr()
corr_mat[corr_mat == 1] = None
sns.heatmap(corr_mat)
```

[ ]:
```
<AxesSubplot:>
```



[ ]:
```
corr_rank = pd.DataFrame(corr_mat.unstack().sort_values().dropna(), columns =␣
↪["Correlation"])
corr_rank
```

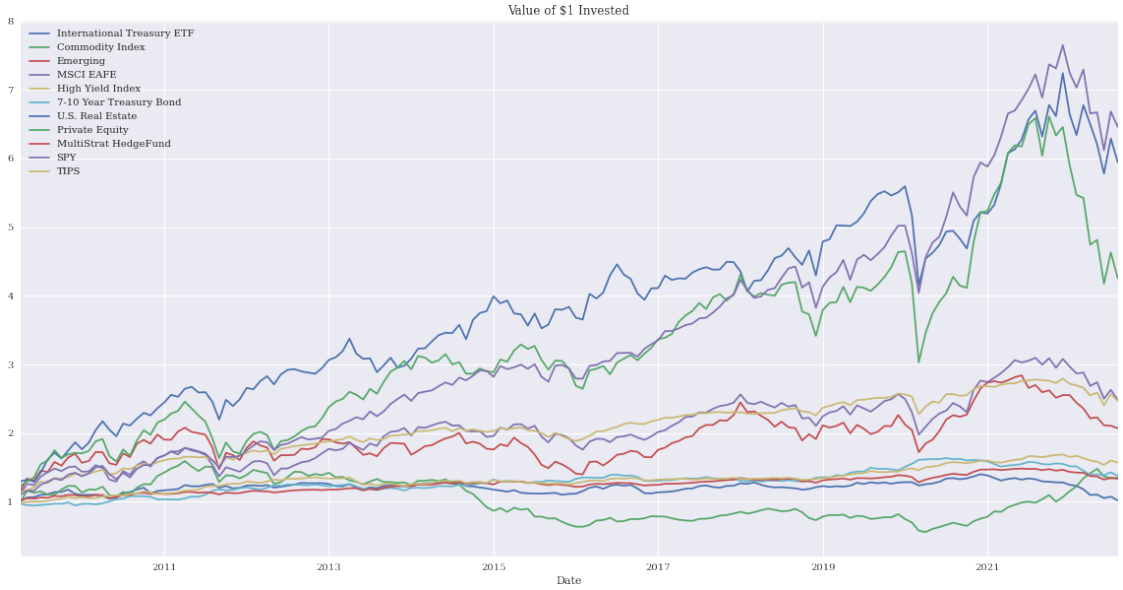
[ ]:
```
                                                  Correlation
7-10 Year Treasury Bond Commodity Index             -0.405431
Commodity Index         7-10 Year Treasury Bond     -0.405431
SPY                     7-10 Year Treasury Bond     -0.269163
7-10 Year Treasury Bond SPY                         -0.269163
MSCI EAFE               7-10 Year Treasury Bond     -0.264846
…                                                        …
                        SPY                          0.874024
Private Equity          SPY                          0.903421
SPY                     Private Equity               0.903421
Private Equity          MSCI EAFE                    0.908746
MSCI EAFE               Private Equity               0.908746

[110 rows x 1 columns]
```

1. As you can see the pair that has the higest correlation is MSCI EAFE and Private Equity being 0.90987. The pair that has the smallest correlation is the 7-10 Year Treasury Bond and Commodity Index with a correlation of $-.405431$.

[ ]:
```
cum_returns = (total_return + 1).cumprod()
cum_returns.plot(figsize = (20,10), title = "Value of $1 Invested")
```

[ ]:
```
<AxesSubplot:title={'center':'Value of $1 Invested'}, xlabel='Date'>
```



[ ]:
```
cum_returns.tail()
```

[ ]:
```
            International Treasury ETF  Commodity Index  Emerging  MSCI EAFE  \
Date
```

```
2022-04-30                    1.100091         1.416716  2.216338   2.688641
2022-05-31                    1.106097         1.482071  2.229935   2.742304
2022-06-30                    1.054727         1.370916  2.114923   2.501896
2022-07-31                    1.076289         1.343642  2.107539   2.631215
2022-08-31                    1.021213         1.351875  2.072202   2.487082

            High Yield Index  7-10 Year Treasury Bond  U.S. Real Estate  \
Date
2022-04-30          2.549639                 1.391952          6.496916
2022-05-31          2.591196                 1.400560          6.208233
2022-06-30          2.408520                 1.388467          5.780419
2022-07-31          2.569864                 1.429586          6.292600
2022-08-31          2.472660                 1.380211          5.947585

            Private Equity  MultiStrat HedgeFund       SPY      TIPS
Date
2022-04-30        4.743946              1.375366  6.656517  1.601910
2022-05-31        4.815505              1.369830  6.671543  1.586015
2022-06-30        4.177563              1.323692  6.121404  1.536603
2022-07-31        4.632756              1.348606  6.685109  1.602828
2022-08-31        4.258392              1.337072  6.461509  1.573448
```

2. All in all *Treasury Inflation-Protected Securities* do not perform exceedingly well between 2010-2022, with an average excess return of around 3. However, this does not mean it shouldn't be included in a portfolio. TIPs do outperform domestic bonds over this investment horizon and they also outperform International Treasury Bonds.

3. Based on the data, **TIPS** definitely expand the investment opportunity offering any portfolio a better risk-return profile. The reason why TIPS expand the investment opportunity set because traditional fixed income assets respond to unanticipated inflation with a declining price (because the **ytm increases**). In contrast, inflation indexed bonds respond to unancipated inflation with an increasing price since the principal is increases in proportion to inflation. When two assets respond in an opposite fashion to an important variable , it is important to categorize them in separate asset classes.

- Yes, *Harvard* should consider **Treasury Inflation-Protected Securities as a separate asset class.**

1.3 Question 3

1. Compute and display the weights of the tangency portfolios: w^t
2. Compute the mean, volatility, and sharpe ratio for the tangency portfolio corresponding to w^t.

[ ]:
```python
def compute_tangency(df_tilde, diagonalize_Sigma=False):
    Sigma = df_tilde.cov()

    # N is the number of assets
```

```
    N = Sigma.shape[0]

    Sigma_adj = Sigma.copy()

    if diagonalize_Sigma:

        Sigma_adj.loc[:,:] = np.diag(np.diag(Sigma_adj))



    mu_tilde = df_tilde.mean()

    Sigma_inv = np.linalg.inv(Sigma_adj)

    weights = Sigma_inv @ mu_tilde / (np.ones(N) @ Sigma_inv @ mu_tilde)

    # For convenience, I'll wrap the solution back into a pandas.Series object.

    omega_tangency = pd.Series(weights, index=mu_tilde.index)

    return omega_tangency, mu_tilde, Sigma_adj


omega_tangency, mu_tilde, Sigma = compute_tangency(total_return)

omega_tangency.to_frame('Tangency Weights')
```

[ ]:

	Tangency Weights
International Treasury ETF	-1.335168
Commodity Index	0.239151
Emerging	0.339786
MSCI EAFE	-0.117068
High Yield Index	1.070489
7-10 Year Treasury Bond	2.457952
U.S. Real Estate	-0.307783
Private Equity	-0.513078
MultiStrat HedgeFund	-3.955222
SPY	2.430623
TIPS	0.690317

The weights above reflect the weights of the portfolio tangent to mean-volatitility frontier.

[ ]:
```
def portfolio_stats(omega, mu_tilde, Sigma, annualize_fac):
    # Mean
    mean = (mu_tilde @ omega) * annualize_fac

    # Volatility
    vol = np.sqrt(omega @ Sigma @ omega) * np.sqrt(annualize_fac)
```

```
    # Sharpe ratio
    sharpe_ratio = mean / vol

    return round(pd.DataFrame(data = [mean, vol, sharpe_ratio],
                              index = ['Mean', 'Volatility', 'Sharpe'],
                              columns = ['Portfolio Stats']), 4)

portfolio_stats(omega_tangency, mu_tilde, Sigma, 12)
```

[ ]:

	Portfolio Stats
Mean	0.3428
Volatility	0.1759
Sharpe	1.9493

The stats above reflect the mean, variance, and sharpe ratio of portfolio w^t that is tangent to the mean-volatiltiy frontier.

1.4 Question 4 - The Allocation

Here I will calculate the optimized portfolio allocation with a target return of 1.5%.

[ ]:
```
def target_mv_portfolio(df_tilde,target_return = 0.01):
    mu = df_tilde.mean()
    sigma = df_tilde.cov()
    sigma_inv = np.linalg.inv(sigma)
    n = sigma.shape[0]
    weight_v = (sigma_inv @ np.ones(n))/(np.ones(n) @ sigma_inv @ np.ones(n))
    weight_t =  compute_tangency(df_tilde)[0]
    omega = (target_return - mu.T@weight_v)/(mu.T@weight_t - mu.T@weight_v)
    omega_star = omega*weight_t + (1-omega)*weight_v
    return pd.Series(omega_star, index = mu_tilde.index)

optimized_portfolio = target_mv_portfolio(total_return, target_return = 0.015)
```

[ ]:
```
optimized_portfolio
```

[ ]:
```
International Treasury ETF   -0.743896
Commodity Index               0.125074
Emerging                      0.140120
MSCI EAFE                    -0.043110
High Yield Index              0.606047
7-10 Year Treasury Bond       1.303991
U.S. Real Estate             -0.163098
Private Equity               -0.316038
MultiStrat HedgeFund         -1.545644
SPY                           1.210202
```

```
TIPS                          0.426352
dtype: float64
```

The weights above relfect the optimized asset allocation of the mean-variance portfolu with a targeted expected return of 1.5%

[ ]:
```
portfolio_stats(optimized_portfolio, mu_tilde,Sigma, 12)
```

[ ]:
```
            Portfolio Stats
Mean                 0.1800
Volatility           0.0934
Sharpe               1.9271
```

The stats above refelect the mean, standard deviation, and sharpe ratio of the allocation weights w^p.

Discuss the allocation:

- The assets in which you are most long in are SPY and High Yield Bond Index. Some of the allocations are greater $|1|$, which is unrealistic since this would involve investing with a high degree of levarage.
- The positions in which you are most short are BWX (SPDR Bloomberg International Treasury), Private Equity, and MultiQ Hedge Multi-Strategy Index. This involves taking **negative positions** by borrowing shares from prime broker an then immediately *selling* the asses with the intention of buying them back at a later date to profit from price declines.

Does the w^p allocation align with the assets that have the strongest Sharpe Ratios?

Answer: Yes, I the w^p portfolio allocations do align with the sharpe ratios. As you can see below, we there is a positive correlation between sharpe ratio and asset allocation. Specifically the correlation is 0.42. All this means is that, generally, higher sharpe values tend to have *positive* allocations. SPY has a sharpe ratio of 1.03-so we can expect to see a postive allocation-with our data we have an allocation 1.21. For Multi-Strat Hedge, we have the largest negative allocation of -1.54. Althougth Multi-Strat Hedge doe not have the lowest sharpe, its asset class does not offer great risk-adjusted returns

[ ]:
```
print(annualized_mean)
print()
print(optimized_portfolio)
```

```
                          Annual Return  Volatility  Sharpe Ratio
International Treasury ETF      0.004653    0.078535      0.059248
Commodity Index                 0.038846    0.180186      0.215590
Emerging                        0.072621    0.191787      0.378655
MSCI EAFE                       0.081124    0.161885      0.501125
High Yield Index                0.071588    0.089403      0.800730
7-10 Year Treasury Bond         0.025833    0.060077      0.429996
U.S. Real Estate                0.150128    0.184407      0.814113
Private Equity                  0.133272    0.221299      0.602227
MultiStrat HedgeFund            0.022862    0.048879      0.467723
SPY                             0.150293    0.144811      1.037857
```

```
TIPS                              0.034967     0.047833       0.731032
```

```
International Treasury ETF   -0.743896
Commodity Index               0.125074
Emerging                      0.140120
MSCI EAFE                    -0.043110
High Yield Index              0.606047
7-10 Year Treasury Bond       1.303991
U.S. Real Estate             -0.163098
Private Equity               -0.316038
MultiStrat HedgeFund         -1.545644
SPY                           1.210202
TIPS                          0.426352
dtype: float64
```

[ ]:
```
np.corrcoef(annualized_mean["Sharpe Ratio"], optimized_portfolio)
```

[ ]:
```
array([[1.        , 0.44202232],
       [0.44202232, 1.        ]])
```

1.5 Simple Portfolios

- A) Calculate the performance of an equally-weighted portfolio over the sample. Rescale the entire weighting vector to have a $u^p = 0.015$. Report its mean, volatility, and Sharpe ratio.

[ ]:
```
equal_weights = np.repeat(1/11,11)
portfolio_stats(equal_weights,mu_tilde, Sigma,12)
```

[ ]:
```
            Portfolio Stats
Mean                 0.0715
Volatility           0.0999
Sharpe               0.7152
```

[ ]:
```
target_return = 0.015
equal_weight_scaled = equal_weights *(target_return/(total_return.mean() @␣
 ↪equal_weights))
portfolio_stats(equal_weight_scaled, mu_tilde, Sigma, 12)
```

[ ]:
```
            Portfolio Stats
Mean                 0.1800
Volatility           0.2517
Sharpe               0.7152
```