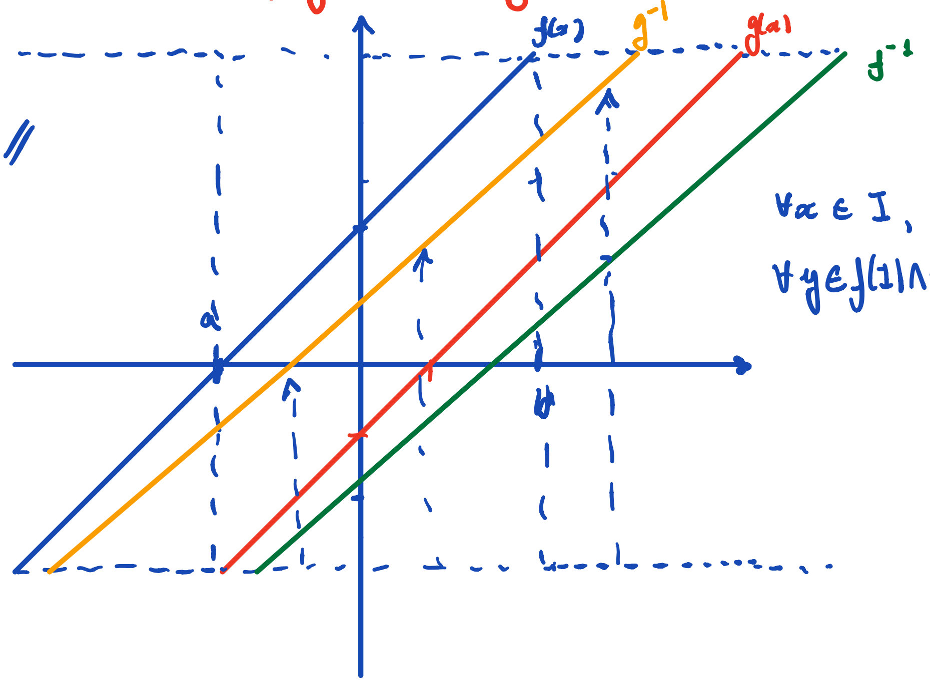
8. Let f, g be strictly increasing on $I \subseteq \mathbb{R}$ and $f(x) > g(x)$ $\forall x \in I$. If $y \in f(I) \cap g(I)$, show that $f^{-1}(y) < g^{-1}(y)$.



$\forall x \in I,\ f(x) > g(x)$

$\forall y \in f(I) \cap g(I),\ f^{-1}(y) < g^{-1}(y)$

$p: y \in f(I) \cap g(I)$ $\quad q: f^{-1}(y) < g^{-1}(y)$ $\quad \vdash p \to q$

continuous inv. theorem

$f: I \to \mathbb{R}$ $\quad g: I \to \mathbb{R}$, strictly monotone on I $\Rightarrow$

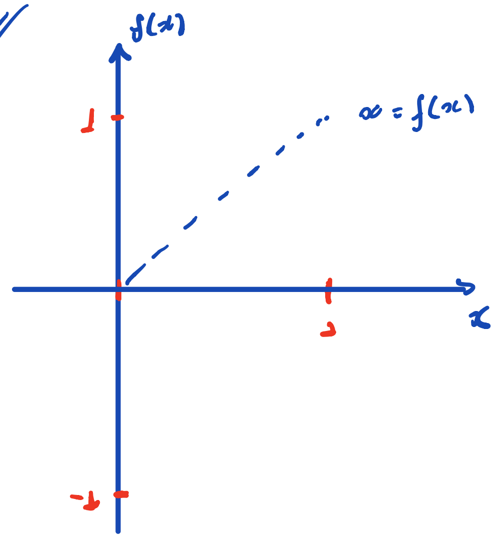
$\Rightarrow$ f^{-1} and g^{-1} $\begin{cases} \text{are strictly monotone} \\ \text{are continuous on } f(I) \text{ and } g(I) \text{, respectivly} \end{cases}$

$y \in f(I) \cap g(I) \Rightarrow f^{-1}$ and g^{-1} are continuous at y

$\lim_{x \to a^+} f > \lim_{x \to a^+} g > \lim_{x \to a^+} f^{-1}$, $\lim_{x \to a^-} f > \lim_{x \to a^-} g > \lim_{x \to a^-} f^{-1}$

12. Let $f: [0,1] \to \mathbb{R}$ be a continuous function that does not take on any of its values twice and with $f(0) < f(1)$. Show that f is strictly increasing on $[0,1]$.

//



$f:[0,1] \to \mathbb{R}$ continuous on $I := [0,1]$ and does not take the any $x_1, x_2 \in I \Rightarrow x_1 \neq x_2 \Rightarrow$ must be strictly increasing or decreasing. Since for $x_1 = 0$ and $x_2 = 1$ s.t. $x_1 < x_2$, we have that $f(x_1) < f(x_2) \Rightarrow f$ cannot be strictly decreasing $\therefore$ f is strictly increasing in $[0,1]$. //