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Business cycle and stock volatility

1. Introduction

- Papers that relate/analyze the asset volatility relationship:
  - Black (1976): Stock returns have a negative cross sectional price dependence
  - (1976): Fama, Schwert, Bollerslev (1989): The equilibrium of stock prices is risk dependent
- Black Hypothesis: Leverage effects contribute to this relationship:
  - (1976): Fama, Schwert: Investors are often risk averse (asymmetric risk)
  - (1989): Fama, Schwert: Bollerslev (1989): Non-parametric asymmetric models
- Prior Work: Evidence that the asset volatility relationship has been a subjecting discussion in the literature. Hence asset volatility exposure build from strong asset returns are specifications would indicate good or bad
- Contribution: Asset volatility relationship has been a subjecting discussion in the literature. Hence asset volatility exposure build from strong asset returns are specifications would indicate good or bad

2. Theoretical asset volatility

- Model assumptions:
  - Asset prices:
    - Fama et al. (1989)
    - Brennan and Xia (1998)
  - Overview of asset volatility relationship:
    - Merton (1980)
    - Campbell, Lo, and MacKinlay (1997)

Asset volatility in a real stock relationship

- Consider the following two equations:
  - Consider that asset is defined as the present/terminal value related the underlying factors will be measure.
  - Consider the factors that to be subject to the stock
  - Conditional power to measure the impact of the asset volatility relationship:

$$\sigma_{R_{t+1}} = \sigma_{R_t} + \sigma_{\Delta R_{t+1}}$$

$\sigma_{R_{t+1}}$: Asset volatility

σ_{R_t}: Conditional expected volatility which has some variations

- Calculate $E[\sigma_R]$ on the stock asset
- Realized $\sigma_{\Delta R_{t+1}}$: $E[\sigma_{\Delta R_{t+1}}] = \lambda(\sigma + \beta\sigma_{R_{t+1}})$
- Unexpected $\sigma_{\Delta R_{t+1}}$: $E[\sigma_{\Delta R_{t+1}}] = \lambda(\sigma + \beta\sigma_{R_{t+1}} - \beta\sigma_{R_t})$

Conditional change in an real stock relationship

- Consider the following two equations:
  - Consider that asset is defined as the present/terminal value related the underlying factors will be measure.
  - Consider the factors that to be subject to the stock
  - Conditional power to measure the impact of the asset volatility relationship:

$$\sigma_{R_{t+1}} = \sigma_{R_t} + \sigma_{\Delta R_{t+1}}$$

$\sigma_{R_{t+1}}$: Asset volatility

σ_{R_t}: Conditional expected volatility which has some variations

$\sigma_{\Delta R_{t+1}}$: Realized $\sigma_{\Delta R_{t+1}}$: $E[\sigma_{\Delta R_{t+1}}] = \lambda(\sigma + \beta\sigma_{R_{t+1}})$

$\sigma_{\Delta R_{t+1}}$: Unexpected $\sigma_{\Delta R_{t+1}}$: $E[\sigma_{\Delta R_{t+1}}] = \lambda(\sigma + \beta\sigma_{R_{t+1}} - \beta\sigma_{R_t})$

Statistical properties of measured returns

$\Sigma = \text{cov}(\cdot)$

$\sigma_t \in \mathbb{R}$: Measurement on one of the stock returns

$\sigma_t \in \mathbb{R}_+$: Measurement of the underlying factors

β_{t+1}: Size on exclusion factors

$\sigma_{R_t} \in \mathbb{R}_+$: Coverage of shares on the exclusion factors

$\sigma_{R_t} = \{\sigma_{R_t}\}_{t=1}^{T}$: Coverage of shares on the stock factors

$$\underbrace{\sigma_{R_t}}_{\text{exclusion factors}} \xrightarrow[\sigma_t]{\beta_{t+1}} \underbrace{\sigma_t}_{\text{stock returns}}$$

$\Sigma = \{\Sigma_{R_t}, \Sigma_{\Delta R_t}, \ldots, \Sigma_{R_{t+1}}\}_{t=1}^{T}$: Matrix of exclusion factors factors

$\Sigma_t^{\Delta R} = \{\Sigma_{R_t}, \Sigma_{\Delta R_t}, \ldots, \Sigma_{R_{t+1}}\}_{t=1}^{T}$: Matrix of stock factors factors

$X_t = \{\Sigma, \Sigma_t^{\Delta R}\}_{t=1}^{T}$: Data factors for the exclusion factors factors

- The GDP Hypothesis: $\sigma_{R_t} = \{\sigma_{R_t}\}_{t=1}^{T}$ $\sigma_t = \{\sigma_t\}_{t=1}^{T}$ $\Sigma_t = \{\Sigma_t\}_{t=1}^{T}$ $\Sigma_t^{\Delta R} = \{\Sigma_t^{\Delta R}\}_{t=1}^{T}$

$$\Sigma_t^{\Delta R} = \sum_{t=1}^{T} \frac{\sigma_{R_t}}{\sigma_t} \Sigma_t + \sum_{t=1}^{T} \frac{\sigma_t}{\sigma_t} \Sigma_t^{\Delta R}$$

$\Sigma_t^{\Delta R}$: Data factors for the stock factors

Σ_t: Data factors for the exclusion factors factors

$\Sigma_t^{\Delta R}$: Data factors for the stock factors factors

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$\Sigma_t^{\Delta R}$: Data factors for the stock factors factors

- The stock factors factors factors