

Homework 6

1 The Shared Birthday Problem

1.1 Introduction

The purpose of this script is to determine the average minimum number of people in a group where 2 of their birthdays are within a week of each other. Assuming that there is an equal chance of being born on any given day, a random number generator can accomplish this task. In order to determine the average, I ran 10,000 iterations of the random number generator to get a more definitive answer. I also printed a histogram of the 10,000 data points to model the distribution.

1.2 Models and Methods

I first created a vector to store all the minimum numbers for all the 10,000 iterations and a vector to store the random numbers with the `zeros` function. Only one is shown below because they both use the same format.

```
group = zeros(1,52);
```

Next I created a `for` loop to iterate through all 10,000 trials and defined a boolean variable to check that the while loop conditions are still true. I also defined the variable that the loop will check for birthdays near. Next I created a `while` loop to loop until a match is found. I then increased the *index* value and created a random number for every index value. Next I created a `for` loop to loop thru all the people in the group less than the index value to check for matches. In the `for` loop I used an `if` statement to check if any birthdays are within a week of each other. If so, the match variable was changed, and the while loop was broken. This is all shown below.

```

for m = 1:top
    index = 1;
    match = 0;
    group(index) = randi([1,365]);
    while match == 0
        index = index + 1;
        group(index) = randi([1,365]);
        for j = 1:index-1
            if abs(group(index) - group(j)) < 7 || abs(group(index) -
                group(j)) > 358
                N_vec(m) = index;
                match = 1;
            end
        end
    end
end

```

Finally, I calculated the median and printed it out using the `median` and `fprintf` functions. I also created a histogram to model the distribution with the `histogram` function. I added labels to the histogram with the functions `xlabel` and `ylabel`. This is shown below.

```

x = median(N_vec);
fprintf('Median Number of People = %1.0f',x);
histogram(N_vec);
xlabel('Amount of People');
ylabel('Number of times occurred');

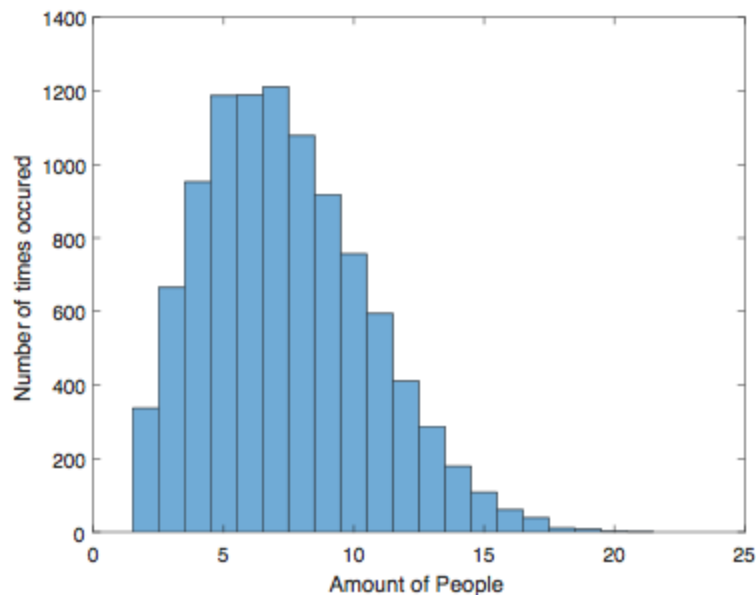
```

1.3 Calculations and Results

After running the script the following is printed.

```
Median Number of People = 7
```

A histogram of the data is also created, shown below.



1.4 Discussion

As shown above, the median minimum number of people in a group to have 2 people with birthdays within a week of each other is 7 people. This means that there is greater than a 50% chance of having a birthday within a week of someone in a group of larger than 7 people. This value makes sense because in my experience it is very likely to know several people in a group of friends whose birthdays lie within a week of each other. I expect this value to go down in real life because it is a lot more common for some days of the year than others. For example, a good percentage of people have birthdays approximately 9 months after Valentine's Day for obvious reasons. It is the same for 9 months after New Years Eve.

2 Simulated Annealing

2.1 Introduction

The purpose of this script is to find the shortest, fully closed path between a set of N random points on a 2-D grid. I can do this by using a simulated annealing algorithm. This algorithm uses a function to find the path distance and iterates until it finds the smallest distance. After finding this distance, I plotted a graph of the random points with the final path and a plot of the distance vs the iteration number.

2.2 Models and methods

I first created a function to calculate the path distance for each specific iteration. This was done in a separate script. In this script I first used a function call to create the function `newpathdistance`. Next I defined the initial conditions and then created a `for` loop to iterate through the entire length of the input vectors. In the loop, I updated the distance calculation based on the given formulas. Most of this code is shown below. There is an `else` statement for the case when $k = N$ but this is almost exactly the same code as the `if` statement so I left it out.

```
function dist = getpathdistance(x, y, path)
N = length(path);
dist = 0;
for k = 1:N
    if k ~= N
        distance = sqrt((x(path(k))-x(path(k+1)))^2 +
(y(path(k))-y(path(k+1)))^2);
        dist = dist + distance;
```

This finished the function, so I created a new script for the problem. In this script I set the random number generator to default before starting the code. I then required the user to input the

number of points with the `input` function. Next I defined the `x`, `y`, and `path` vectors with the `zeros` function, `rand` function, `randperm` function, and a `for` loop. This is shown below.

```
rng('default');
N = input('Please input value for N: ');
x = zeros(1,N);
y = zeros(1,N);
path = randperm(N);
for k = 1:N
    x(k) = rand;
    y(k) = rand;
end
```

I then called the function `getpathdistance` to find the initial path, and defined some constants and a vector to put all the distances in.

```
dist = getpathdistance(x, y, path);
T = 1000;
index = 0;
D_vec = zeros(1,1379);
```

Finally I created a `while` loop to iterate until the temperature dropped below 1. In the loop, I randomly swapped two values in the `path` and created a new vector `pathNew` for this swapped path. I then calculated the new distance from this path and the function `getpathdistance`. Then I calculated the difference between the old path and the new path and calculated the probability of the exploration metric p . I then used an `if elseif` statement to update the distance and path based on the exploration metric. Finally I updated the temp T , the index, and put the distance value in a vector. This is all shown below.

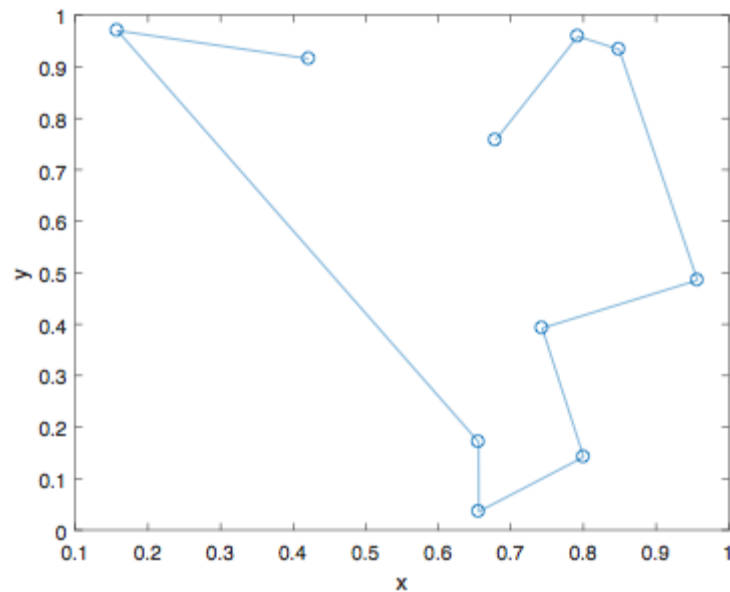
```
while T > 1
    swap = randperm(N,2);
    pathNew = path;
    pathNew(swap(1)) = path(swap(2));
    pathNew(swap(2)) = path(swap(1));
    distNew = getpathdistance(x,y,pathNew);
    diff = distNew - dist;
    p = exp(-1000*diff/T);
    z = rand;
    if diff < 0
        path = pathNew;
        dist = distNew;
    elseif p > z
        path = pathNew;
        dist = distNew;
    end
    T = T*(1 - 0.005);
    index = index + 1;
    D_vec(index) = dist;
end
```

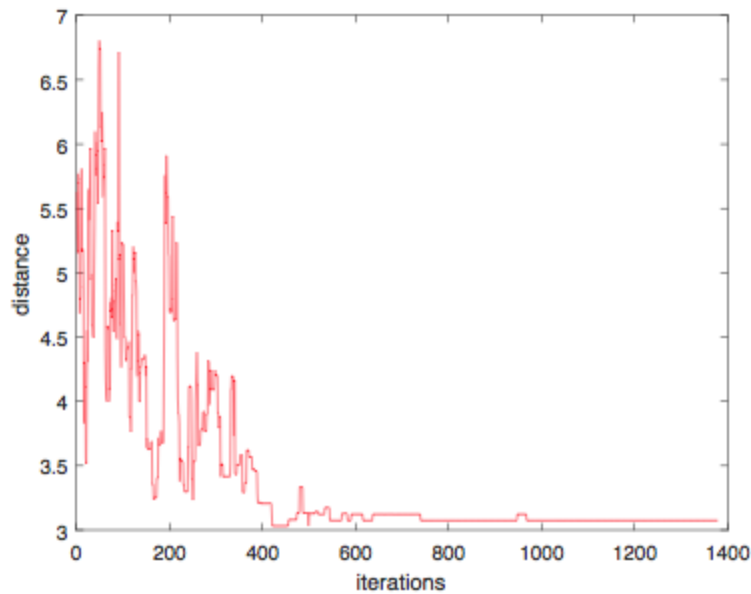
Finally I created vectors to graph this data with the `linspace` and `zeros` functions and a `for` loop. Then I graphed the two plots with the `plot`, `xlabel`, and `ylabel` functions. This is shown below.

```
iterations = linspace(1,index,index);
x2 = zeros(1,N);
y2 = zeros(1,N);
for k = 1:N
    x2(k) = x(path(k));
    y2(k) = y(path(k));
end
plot(x2,y2,'o-');
xlabel('x');
ylabel('y');
figure;
plot(iterations, D_vec,'r');
xlabel('iterations');
ylabel('distance');
```

2.3 Calculations and Results

After running the script and inputting 10 for the number of points, the following graphs are printed.





2.4 Discussion

As shown above, the code does accurately find the shortest path between all the points. Just looking by eye, I cannot think of another combination of path choices that would result in a shorter path. As N gets larger, the final path choices becomes more questionable whether it is actually the shortest path. If λ was smaller and c was larger, these results would be improved because it would take more iterations to get the answer.