Cubic Spline on 2 data pts w/ dv/dt

y_1' and y_2' are given

So we have 4 data, so we can solve for 4 coefficients of taylor $\Rightarrow$ our friend the cubic spline.

$$f(x_1+\Delta x) = f(x_1) + f'(x_1)\Delta x + f''(x_1)\Delta x^2/2 + f'''(x_1)\Delta x^3/3!$$

$y_1 = f(x_1)$ $\quad y_1' = f'(x_1)$ So we only have to find $f''(x_1)$ and $f'''(x_1)$ by solving linear system of 2 eqs. ☺

$$y_2 = f(x_1) + f'(x_1)(x_2-x_1) + f''(x_1)(x_2-x_1)^2/2 + f'''(x_2-x_1)^3/3!$$

$$y_2' = f'(x_1) + f''(x_1)(x_2-x_1) + f'''(x_2)(x_2-x_1)^2/2$$

$$\Rightarrow \begin{pmatrix} (x_2-x_1)^2/2 & (x_2-x_1)^3/6 \\ x_2-x_1 & (x_2-x_1)^2/2 \end{pmatrix} \begin{pmatrix} f''(x_1) \\ f'''(x_1) \end{pmatrix} = \begin{pmatrix} y_2 - y_1 - y_1'(x_2-x_1) \\ y_2' - y_1' \end{pmatrix}$$

$$\vdots$$

$$M$$

$$\Rightarrow \begin{pmatrix} f''(x_1) \\ f'''(x_1) \end{pmatrix} = M^{-1} \begin{pmatrix} y_2 - y_1 - y_1'(x_2-x_1) \\ y_2' - y_1' \end{pmatrix}$$