



One-Stop Analytics: Central Limit Theorem (CLT) & Confidence Interval (CI)

Case Study of Autism Spectrum Disorder (ASD) with R



ABOUT 1 IN 59 CHILDREN

WERE IDENTIFIED WITH AUTISM SPECTRUM DISORDER
AMONG A 2014 SAMPLE OF 8 YEAR OLDS FROM 11 US COMMUNITIES
IN CDC'S ADDM NETWORK

[United States]

Centers for Disease Control and Prevention (CDC) - Autism Spectrum Disorder (ASD)

Autism spectrum disorder (ASD) is a developmental disability that can cause significant social, communication and behavioral challenges. CDC is committed to continuing to provide essential data on ASD, search for factors that put children at risk for ASD and possible causes, and develop resources that help identify children with ASD as early as possible.

<https://www.cdc.gov/ncbdd/autism/data/index.html> (<https://www.cdc.gov/ncbdd/autism/data/index.html>)

[Singapore]

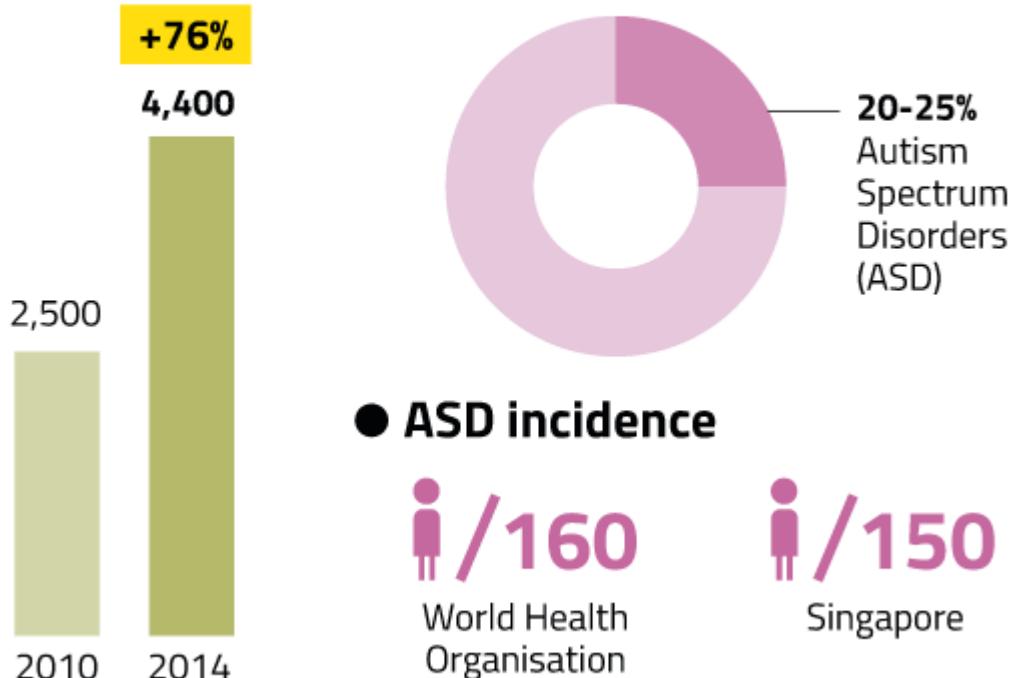
TODAY Online - More preschoolers diagnosed with developmental issues

Doctors cited better awareness among parents and preschool teachers, leading to early referrals for diagnosis.

<https://www.gov.sg/news/content/today-online-more-preschoolers-diagnosed-with-developmental-issues>
<https://www.gov.sg/news/content/today-online-more-preschoolers-diagnosed-with-developmental-issues>

Jump in preschoolers diagnosed with developmental issues

● New cases ● Types of diagnosed cases



Source: KK Women's and Children's Hospital, National University Hospital **TODAY**

The website for Pathlight School features a large banner at the top advertising it as the "1ST AUTISM-FOCUSED SCHOOL". The banner includes a photo of children playing outside. Below the banner, there are several navigation links and icons for different sections like Highlights, The Art Faculty, e-Learning Portals, and Parents' Corner.

1ST AUTISM-FOCUSED SCHOOL that offers a unique blend of mainstream academics & life readiness skills

Highlights
Latest events and happenings at Pathlight School.

The Art Faculty
Support the products by individuals with autism.

e-Learning Portals
» Learn for Life eCampus
» MC Online
» Student Learning Space

Parents' Corner
Useful resources and information for our parents

<https://www.pathlight.org.sg/> (<https://www.pathlight.org.sg/>)

Workshop Objective:

Use R to analyze Autism Spectrum Disorder (ASD) data from CDC USA.

<https://www.cdc.gov/ncbddd/autism/data/index.html> (<https://www.cdc.gov/ncbddd/autism/data/index.html>)

- **Data**
- **Sampling & Normality (CLT)**
- **Confidence Interval (CI)**
- **Appendices**

Data

Data - Get & Set working directory

Obtain current R **working directory**

```
In [1]: getwd()  
'/media/sf_vm_shared_folder/git/DDC/DDC-ASD/model_R'
```

Set new R working directory

```
In [2]: # setwd("/media/sf_vm_shared_folder/git/DDC/DDC-ASD/model_R")  
# setwd('~/Desktop/admin-desktop/vm_shared_folder/git/DDC-ASD/model_R')  
getwd()  
'/media/sf_vm_shared_folder/git/DDC/DDC-ASD/model_R'
```

Read in CSV data, storing as R **dataframe**

```
In [3]: # Read back in above saved file:
ASD_State <- read.csv("../dataset/ADV_ASD_State_R.csv")
# Convert Year_Factor to ordered.factor
ASD_State$Year_Factor <- factor(ASD_State$Year_Factor, ordered = TRUE)
ASD_State$Prevalence_Risk2 = factor(ASD_State$Prevalence_Risk2, ordered=TRUE,
                                    levels=c("Low", "High"))
ASD_State$Prevalence_Risk4 = factor(ASD_State$Prevalence_Risk4, ordered=TRUE,
                                    levels=c("Low", "Medium", "High", "Very Hi"))
```

```
In [4]: # Filter and create dataframe of different data sources, for easy data access
ASD_State_ADDM <- subset(ASD_State, Source == 'addm')
ASD_State_MEDI <- subset(ASD_State, Source == 'medi')
ASD_State_NSCH <- subset(ASD_State, Source == 'nsch')
ASD_State_SPED <- subset(ASD_State, Source == 'sped')
```

```
In [5]: # Adjust in-line plot size to M x N
options(repr.plot.width=8, repr.plot.height=4)
```

.0

Sampling & Normality

Sampling & Normality - Population & Sample

Create a **Population** of US. State level ASD Prevalence from Source SPED in Year 2016

```
In [6]: # -----
# Create a *Population* of US. State level ASD Prevalence from Source SPED in
# -----
ASD_State_SPED_2016 <- subset(ASD_State,
                               Source == 'sped' & Year == 2016, # conditions
                               select=c('State', 'Prevalence')) # selection
#
head(ASD_State_SPED_2016)
```

	State	Prevalence
1643	AL	9.1
1644	AK	10.1
1645	AZ	10.4
1646	AR	9.5
1647	CA	13.9
1648	CO	7.3

```
In [7]: dim(ASD_State_SPED_2016)
# *Population* mean Prevalence
mean(ASD_State_SPED_2016$Prevalence)
```

50 2

11.182

Define a function to calculate population std-dev (Omega):

```
In [8]: # Use sd() to calculate *sample* std-dev (S)
# Use sd.p() to calculate *population* std-dev (Omega)

# Define a function sd.p() to calculate *population* std-dev (Omega)
# https://www.dummies.com/education/math/statistics/standard-deviation-r/

sd.p = function(x) {sd(x) * sqrt((length(x)-1)/length(x))}

# Treat as sample:
cat('sd() of ASD_State_SPED_2016$Prevalence : ', sd(ASD_State_SPED_2016$Prevalence))

# Treat as population:
cat('\nsd.p() of ASD_State_SPED_2016$Prevalence : ', sd.p(ASD_State_SPED_2016$Prevalence))

sd() of ASD_State_SPED_2016$Prevalence : 3.233226
sd.p() of ASD_State_SPED_2016$Prevalence : 3.200731
```

Sampling & Normality - Central Limit Theorem (CLT)

Create a **Sample** of US. State level ASD Prevalence from Source SPED in Year 2016

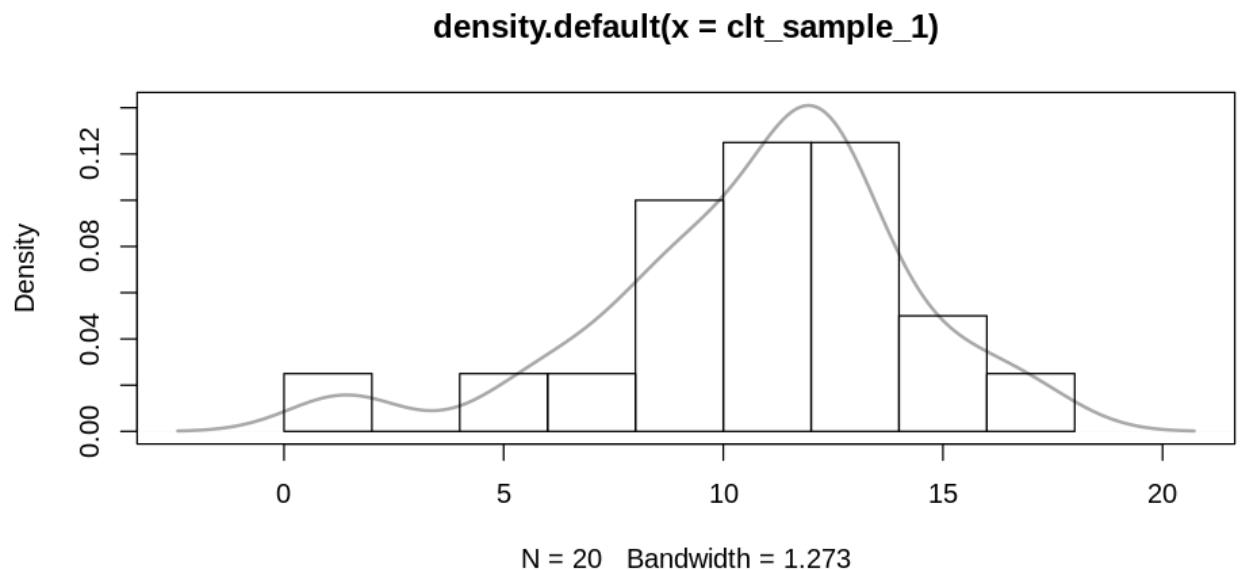
```
In [9]: # Create a *Sample* from ASD_State_SPED_2016$Prevalence,
# with sample size n =
clt_n = 20
# clt_n = 40

set.seed(88)
clt_sample_1 = sample(x = ASD_State_SPED_2016$Prevalence, size = clt_n, replace = TRUE)

11.2 9.5 16.9 6.9 11.2 8.2 12.7 9.5 1.4 12.7 10.1 14.2 11.9 11.9 8.5 12.1 12.1
15.4 13 5.5
```

```
In [10]: # Adjust in-line plot size to M x N
options(repr.plot.width=8, repr.plot.height=4)
```

```
In [11]: plot(density(clt_sample_1), col="darkgrey", lwd=2)
hist(clt_sample_1, probability = T, add = T)
```



Draw a large k number of samples, with sample size = n:

```
In [12]: # Repeatedly sample for k times, create a matrix/array to store these samples  
clt_k = 10000 # or called 'N', but this can be confusing due to N can also be  
  
set.seed(88) # Repeatable sampling using pseudo random method  
clt_sample_k <- (replicate(clt_k, sample(x = ASD_State_SPED_2016$Prevalence, s  
  
# first few samples  
clt_sample_k[, 1:6]
```

```
11.2   9.5   11.2   8.3   8.3   6.9  
 9.5  11.9  10.4  11.9  11.9  15.2  
16.9   6.9   9.3   1.4  15.4  19.4  
 6.9  13.0  19.4  12.7  8.5  15.1  
11.2   8.3  12.1   9.1  11.2  12.1  
 8.2  12.1  10.1  14.2  10.2  1.4  
12.7  10.8  12.1   8.6   9.5  15.1  
 9.5  15.2  14.1  14.1  10.8   6.9  
 1.4   8.7  12.5  16.7   9.8   9.5  
12.7   9.0   5.5  13.9  19.4  14.2  
10.1   9.3   9.5   9.5  14.2  19.4  
14.2  12.1  16.7  12.5  14.1  16.9  
11.9   1.4   9.6  10.8  11.9   9.6  
11.9  11.9  10.2  10.3   5.5  14.1  
 8.5  10.8  15.2   6.9  10.3  14.2  
12.1  14.1  15.1   1.4  15.4  19.4  
12.1  15.4  16.9  14.1  14.2   8.6  
15.4   8.5  12.1  14.1  13.0   9.5  
13.0   1.4  19.4  12.1  15.4   9.0  
 5.5   8.5  12.7  10.8  11.9  19.4
```

```
In [13]: # last sample  
clt_sample_k[, clt_k]
```

```
8.6  14.1  12.1  19.4  12.7  10.3  10.3  11.2  10.4  9  10.4  13.9  9.3  14.2  11  12.1  9.5  
10.1  13  15.2
```

```
In [14]: # mean values of first few samples  
mean(clt_sample_k[, 1])  
mean(clt_sample_k[, 2])  
mean(clt_sample_k[, 3])  
mean(clt_sample_k[, 4])  
mean(clt_sample_k[, 5])  
mean(clt_sample_k[, 6])  
  
# or use apply() function to loop  
apply(clt_sample_k[, 1:6], 2, mean)
```

10.745
9.94
12.705
10.67
12.045
12.795
10.745 9.94 12.705 10.67 12.045 12.795

```
In [15]: # std-dev values of first few samples  
sd(clt_sample_k[, 1])  
sd(clt_sample_k[, 2])  
sd(clt_sample_k[, 3])  
sd(clt_sample_k[, 4])  
sd(clt_sample_k[, 5])  
sd(clt_sample_k[, 6])  
  
# or use apply() function to loop  
apply(clt_sample_k[, 1:6], 2, sd)
```

3.52158382430107
3.7529497170822
3.57115269611191
4.01026314926038
3.17895630009202
5.01140541270873
3.52158382430107 3.7529497170822 3.57115269611191 4.01026314926038
3.17895630009202 5.01140541270873

k sample's distributions (k many)

In [16]:

```
# -----  
# k sample's distributions (k many)  
# -----  
# Show the first few sample's histogram  
par(mfrow=c(2, 3))  
apply(clt_sample_k[, 1:6], 2, FUN=hist)  
# Reset  
par(mfrow=c(1, 1))  
  
[[1]]  
$breaks  
[1] 0 2 4 6 8 10 12 14 16 18  
  
$counts  
[1] 1 0 1 1 4 5 5 2 1  
  
$density  
[1] 0.025 0.000 0.025 0.025 0.100 0.125 0.125 0.050 0.025  
  
$mids  
[1] 1 3 5 7 9 11 13 15 17  
  
$xname  
[1] "newX[, i]"  
  
$equidist  
[1] TRUE
```

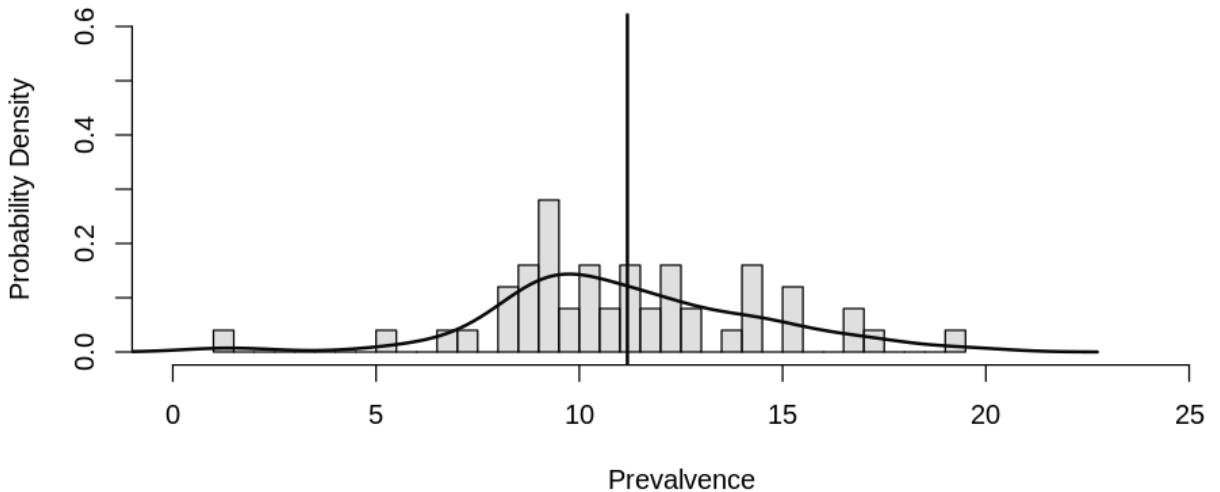
Show the first few sample's density, together with Population

```
In [17]: # Show the first few sample's density, together with Population
# Population (Prevalence) histogram in probability
hist(ASD_State_SPED_2016$Prevalence, probability = T,
      col=rgb(0.75,0.75,0.75,0.5), breaks = 50,
      xlab = 'Prevalence', xlim = (c(0, 25)),
      ylab = 'Probability Density',
      ylim = (c(0, 0.6)),
      main = 'Visualize Population & Samples')

# Overlay curve:
# Population (Prevalence) density
lines(density(ASD_State_SPED_2016$Prevalence), col="grey4", lwd=2)

# Overlay line:
# mean = mean of Population (Prevalence)
abline(v=mean(ASD_State_SPED_2016$Prevalence), col="grey4", lwd=2)
```

Visualize Population & Samples



```
In [18]: # Show the first few sample's density, together with Population
# Population (Prevalence) histogram in probability
hist(ASD_State_SPED_2016$Prevalence, probability = T,
      col=rgb(0.75,0.75,0.75,0.5), breaks = 50,
      xlab = 'Prevalence', xlim = (c(0, 25)),
      ylab = 'Probability Density',
      ylim = (c(0, 0.6)),
      main = 'Visualize Population & Samples')

# Overlay curve:
# Population (Prevalence) density
lines(density(ASD_State_SPED_2016$Prevalence), col="grey4", lwd=2)

# Overlay line:
# mean = mean of Population (Prevalence)
abline(v=mean(ASD_State_SPED_2016$Prevalence), col="grey4", lwd=2)

# Overlay:
# First few sample's density & mean
lines(density(clt_sample_k[, 1]), col="blue", lwd=1)
abline(v=mean(clt_sample_k[, 1]), col="blue", lwd=1)

lines(density(clt_sample_k[, 2]), col="blue", lwd=1)
abline(v=mean(clt_sample_k[, 2]), col="blue", lwd=1)

lines(density(clt_sample_k[, 3]), col="blue", lwd=1)
abline(v=mean(clt_sample_k[, 3]), col="blue", lwd=1)

lines(density(clt_sample_k[, 4]), col="blue", lwd=1)
abline(v=mean(clt_sample_k[, 4]), col="blue", lwd=1)

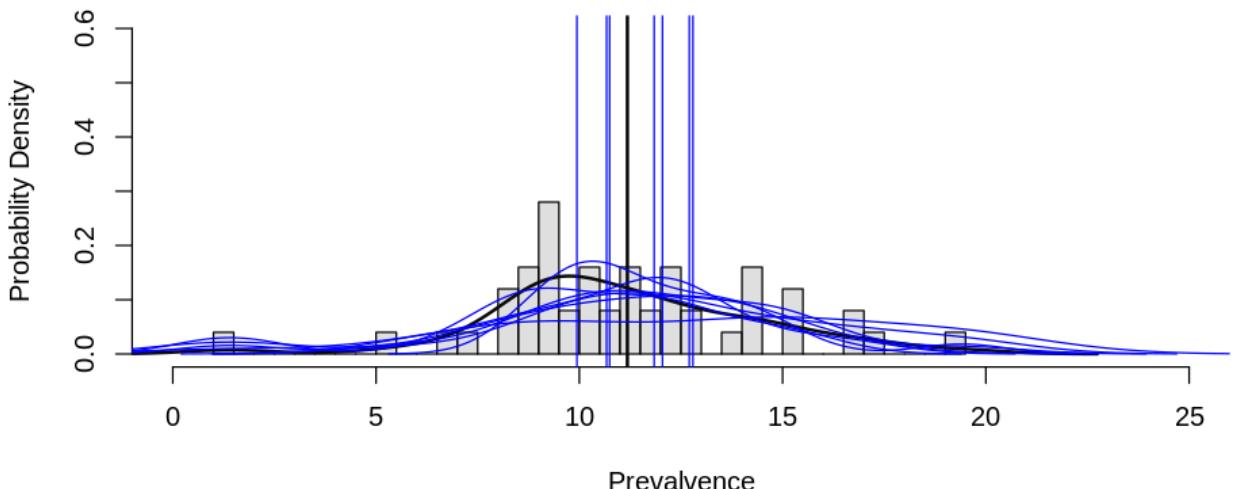
lines(density(clt_sample_k[, 5]), col="blue", lwd=1)
abline(v=mean(clt_sample_k[, 5]), col="blue", lwd=1)

lines(density(clt_sample_k[, 6]), col="blue", lwd=1)
abline(v=mean(clt_sample_k[, 6]), col="blue", lwd=1)

lines(density(clt_sample_k[, clt_k]), col="blue", lwd=1)
abline(v=mean(clt_sample_k[, clt_k]), col="blue", lwd=1)

# We can see that sample's distributions are all different.
```

Visualize Population & Samples



[Tips] We notice that sample's distributions are all different.

Create Sampling Distribution (only one):

```
In [19]: # -----  
# Sampling distribution (only one)  
# -----  
# Calculate sample mean value for k samples  
clt_sample_k_mean <- apply(clt_sample_k, 2, mean)  
# Show first few sample means  
clt_sample_k_mean[1:6]
```

10.745 9.94 12.705 10.67 12.045 12.795

```
In [20]: # Calculate sample std-dev value for each individual sample (totally k std-dev  
clt_sample_k_sd <- apply(clt_sample_k, 2, sd)  
# Show first few samples' std-dev  
clt_sample_k_sd[1:6]
```

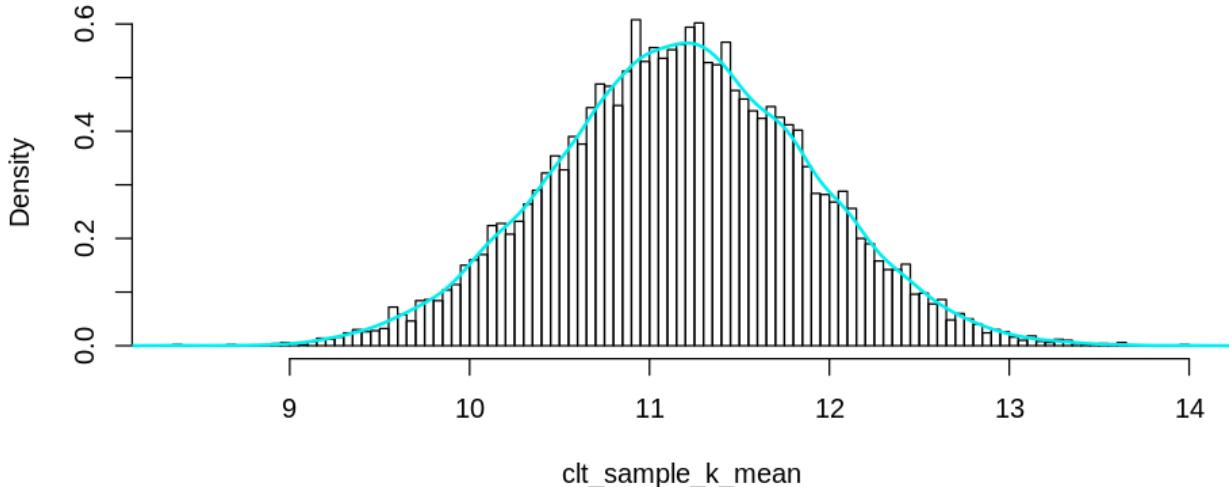
3.52158382430107 3.7529497170822 3.57115269611191 4.01026314926038
3.17895630009202 5.01140541270873

```
In [21]: # Calculate std-dev value for Sampling DIstdtribution (only one std-dev)  
sd(clt_sample_k_mean)
```

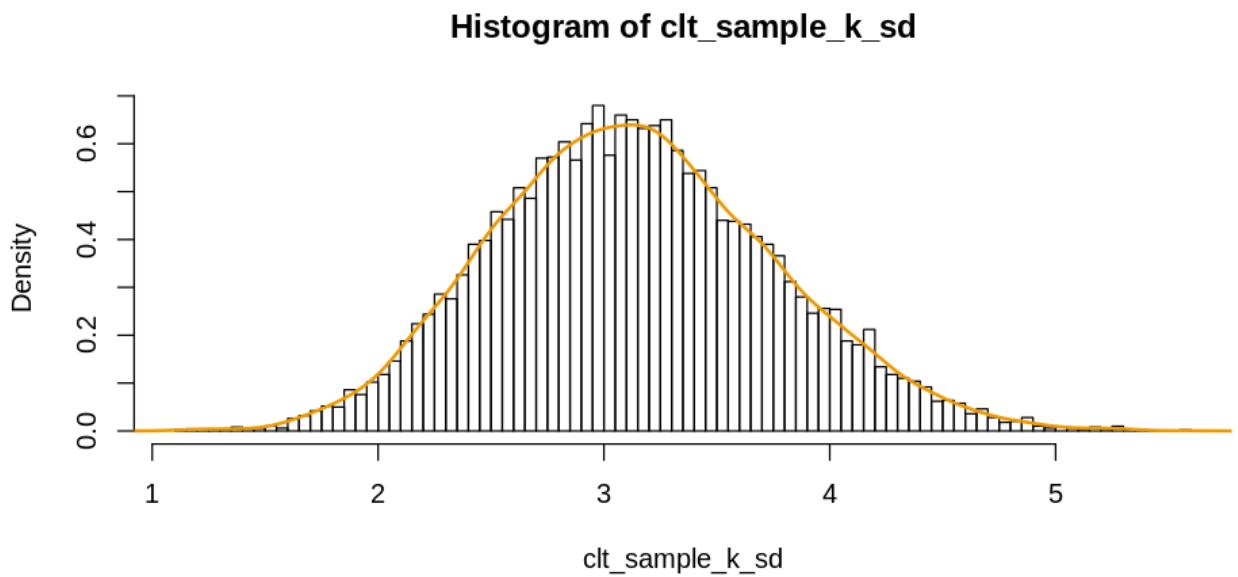
0.713336703117785

```
In [22]: # histogram of sample means (Sampling distribution of the mean)  
hist(clt_sample_k_mean, probability = T, breaks = 100)  
lines(density(clt_sample_k_mean), col="cyan2", lwd=2)
```

Histogram of clt_sample_k_mean



```
In [23]: # histogram of sample std-dev  
hist(clt_sample_k_sd, probability = T, breaks = 100)  
lines(density(clt_sample_k_sd), col="orange2", lwd=2)
```



```
In [24]: # k *Sample* (sample size = n) mean Prevalence  
mean(clt_sample_k_mean)
```

11.1788145

```
In [25]: # *Population* mean Prevalence  
mean(ASD_State_SPED_2016$Prevalence)
```

11.182

[Tips] We notice that the above two means are close.

Visualise: Central Limit Theorem (CLT)

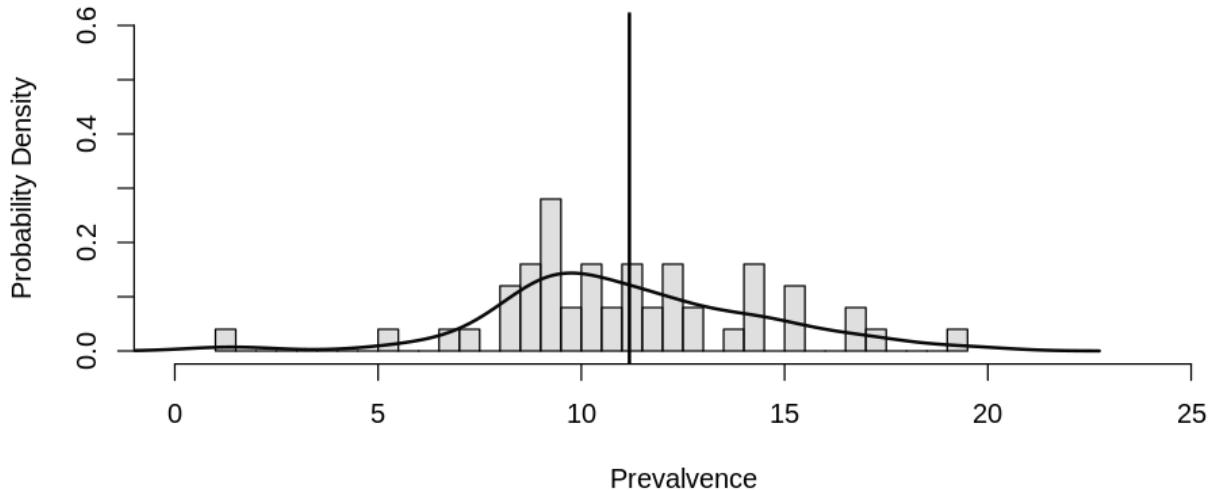
In [26]:

```
# -----
# Sampling distribution vs. Population distribution vs. Z-Norm
# -----
# Create:
# Population (Prevalence) histogram in probability
hist(ASD_State_SPED_2016$Prevalence, probability = T,
      col=rgb(0.75,0.75,0.75,0.5), breaks = 50,
      xlab = 'Prevalvence', xlim = (c(0, 25)),
      ylab = 'Probability Density', ylim = (c(0, 0.6)),
      main = 'Visualize Central Limit Theorem (CLT)')

# Overlay curve:
# Population (Prevalence) density
lines(density(ASD_State_SPED_2016$Prevalence), col="grey4", lwd=2)

# Overlay line:
# mean = mean of Population (Prevalence)
abline(v=mean(ASD_State_SPED_2016$Prevalence), col="black", lwd=2)
```

Visualize Central Limit Theorem (CLT)



Overlay: Sample means histogram in probability (Sampling disribution)

In [27]:

```
# -----
# Sampling distribution vs. Population distribution vs. Z-Norm
# -----
# Create:
# Population (Prevalence) histogram in probability
hist(ASD_State_SPED_2016$Prevalence, probability = T,
      col=rgb(0.75,0.75,0.75,0.5), breaks = 50,
      xlab = 'Prevalvence', xlim = (c(0, 25)),
      ylab = 'Probability Density', ylim = (c(0, 0.6)),
      main = 'Visualize Central Limit Theorem (CLT)')

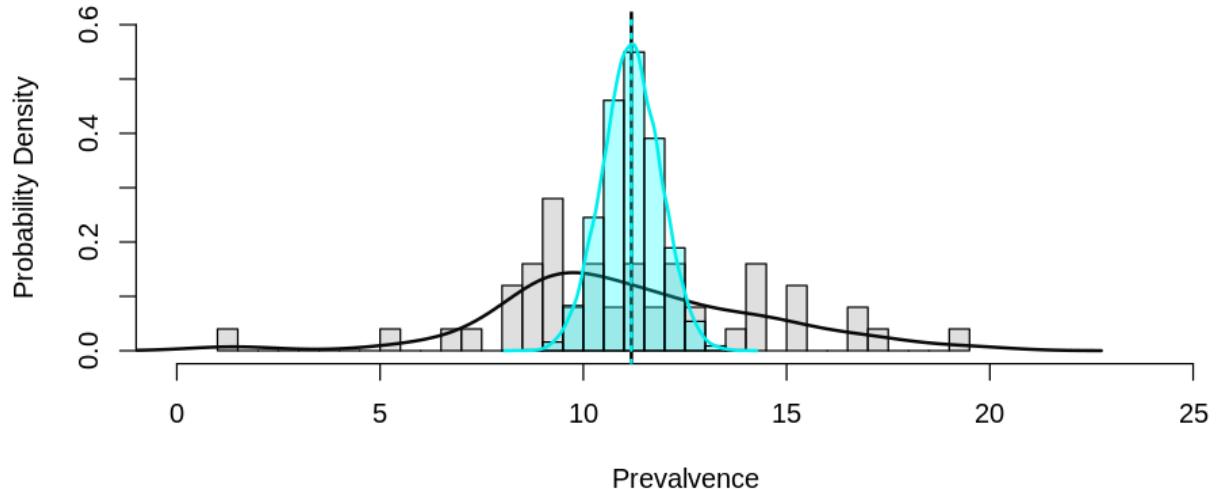
# Overlay curve:
# Population (Prevalence) density
lines(density(ASD_State_SPED_2016$Prevalence), col="grey4", lwd=2)

# Overlay line:
# mean = mean of Population (Prevalence)
abline(v=mean(ASD_State_SPED_2016$Prevalence), col="black", lwd=2)

# Overlay line:
# Sample means histogram in probability (Sampling disribution)
hist(clt_sample_k_mean, probability = T,
      col=rgb(0,1,1,0.3), # https://www.dataanalytics.org.uk/make-transparent-c
      add=T)

# Overlay curve:
# Sample (Prevalence) density (Sampling disribution)
lines(density(clt_sample_k_mean), col="cyan2", lwd=2)
# Overlay line:
# mean of Sampling distribution (of Prevelance, sample size n)
abline(v=mean(clt_sample_k_mean), col="cyan2", lwd=2, lty=3)
```

Visualize Central Limit Theorem (CLT)



< How to make transparent colors in R > <https://www.dataanalytics.org.uk/make-transparent-colors-in-r/>
[\(https://www.dataanalytics.org.uk/make-transparent-colors-in-r/\)](https://www.dataanalytics.org.uk/make-transparent-colors-in-r/)

In [28]: `col2rgb(c("cyan", "grey", "purple", "orange")) / 255`

```
red  0  0.745098  0.6274510  1.0000000
green 1  0.745098  0.1254902  0.6470588
blue 1  0.745098  0.9411765  0.0000000
```

```
In [29]: # Recall:  
# k *Sample* (sample size = n) mean Prevalence  
mean(clt_sample_k_mean)  
# *Population* mean Prevalence  
mean(ASD_State_SPED_2016$Prevalence)  
# We see that the above two means are close. Good estimation!
```

11.1788145

11.182

[Tips] We notice that the above two means are close. Good estimation!

Standard Error (SE) (of mean prevalence), can be estimated as: std-dev of the Sampling distribution (of mean prevalence):

```
In [30]: # -----  
# Standard Error (SE) (of mean prevalence), can be estimated as:  
# std-dev of the Sampling distribution (of mean prevalence)  
# -----  
# https://en.wikipedia.org/wiki/Sampling_distribution  
  
# [1] Actual SE: When Population std-dev is known, SE using Population standard deviation  
sd.p(ASD_State_SPED_2016$Prevalence) / sqrt(clt_n)  
  
# [2] Estimated SE with k samples: When Population std-dev is NOT known, but we have k samples  
sd(clt_sample_k_mean)  
  
# [3] Estimated SE with only one sample: When Population std-dev is NOT known,  
clt_sample_k_sd[1] / sqrt(clt_n)
```

0.715705106870141

0.713336703117785

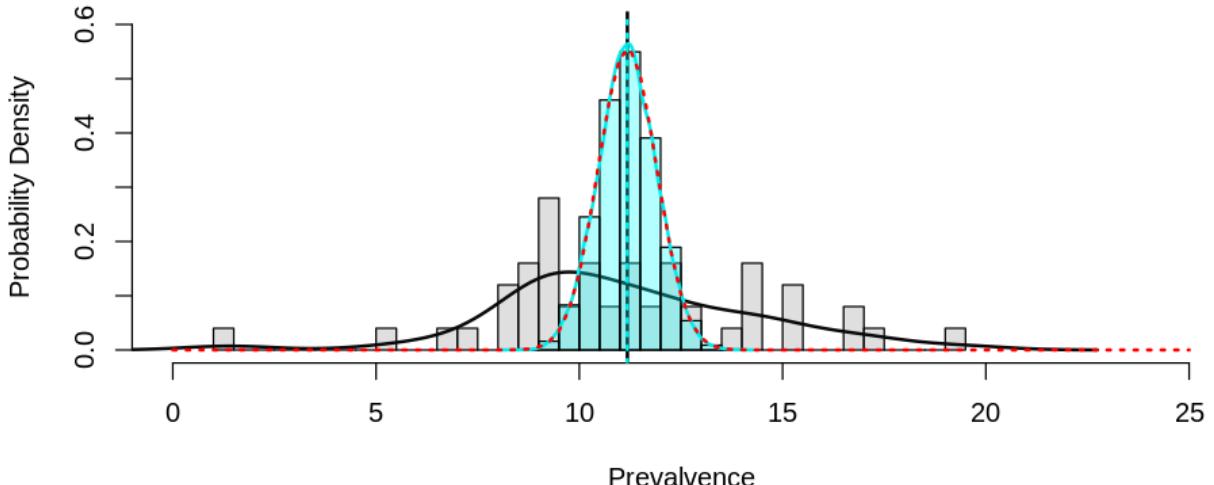
0.787450081960087

Overlay: Theoretic Sampling Distribution with population mean & std-dev = Actual SE:

In [31]:

```
# -----  
# Sampling distribution vs. Population distribution vs. Z-Norm  
# -----  
# Create:  
# Population (Prevalence) histogram in probability  
hist(ASD_State_SPED_2016$Prevalence, probability = T,  
    col=rgb(0.75,0.75,0.75,0.5), breaks = 50,  
    xlab = 'Prevalvence', xlim = (c(0, 25)),  
    ylab = 'Probability Density', ylim = (c(0, 0.6)),  
    main = 'Visualize Central Limit Theorem (CLT)')  
  
# Overlay curve:  
# Population (Prevalence) density  
lines(density(ASD_State_SPED_2016$Prevalence), col="grey4", lwd=2)  
  
# Overlay line:  
# mean = mean of Population (Prevalence)  
abline(v=mean(ASD_State_SPED_2016$Prevalence), col="black", lwd=2)  
  
# Overlay line:  
# Sample means histogram in probability (Sampling disribution)  
hist(clt_sample_k_mean, probability = T,  
    col=rgb(0,1,1,0.3), # https://www.dataanalytics.org.uk/make-transparent-c  
    add=T)  
  
# Overlay curve:  
# Sample (Prevalence) density (Sampling disribution)  
lines(density(clt_sample_k_mean), col="cyan2", lwd=2)  
# Overlay line:  
# mean of Sampling distribution (of Prevelance, sample size n)  
abline(v=mean(clt_sample_k_mean), col="cyan2", lwd=2, lty=3)  
  
# Overlay curve:  
# *Theoretic Sampling Distribution* with population mean & std-dev = Actual SE  
# mean = mean of Population (Prevalence) & std-dev = std-dev of Population (Pr  
curve(dnorm(x,  
    mean(ASD_State_SPED_2016$Prevalence), # Actual Population mean  
    sd.p(ASD_State_SPED_2016$Prevalence) / sqrt(clt_n)), # Actual SE (Pr  
    add=TRUE, col="red", lwd=2, lty=3)
```

Visualize Central Limit Theorem (CLT)

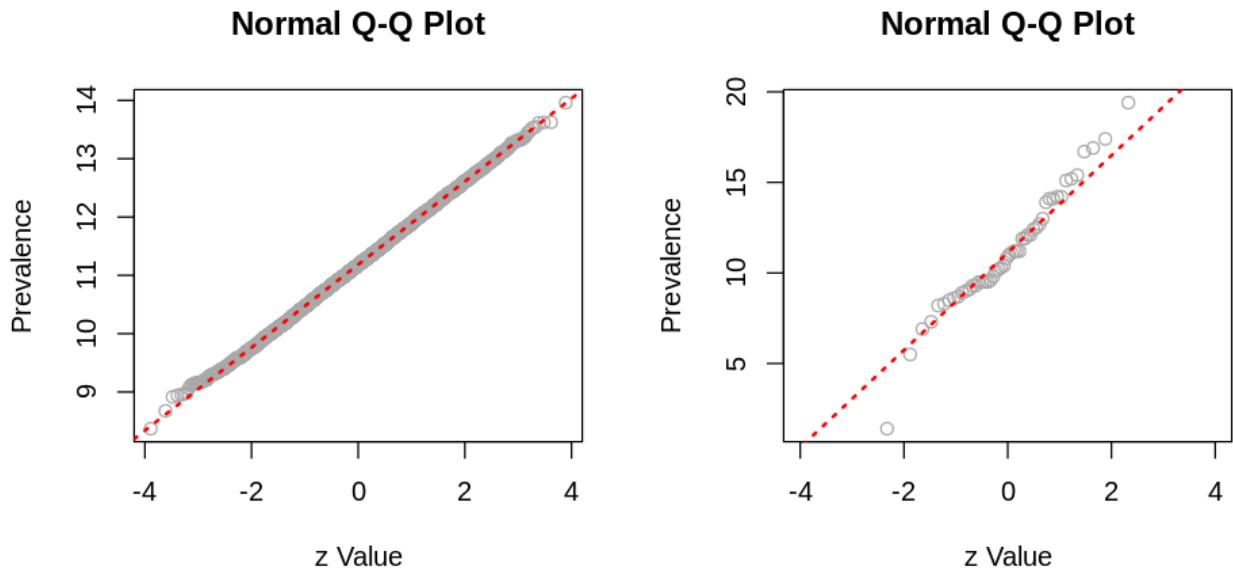


Use QQ Plot

< Construct a Quantile-Quantile Plot (QQ plot) > <https://youtu.be/okjYjCISjQg>
[\(https://youtu.be/okjYjCISjQg\)](https://youtu.be/okjYjCISjQg)

In [32]:

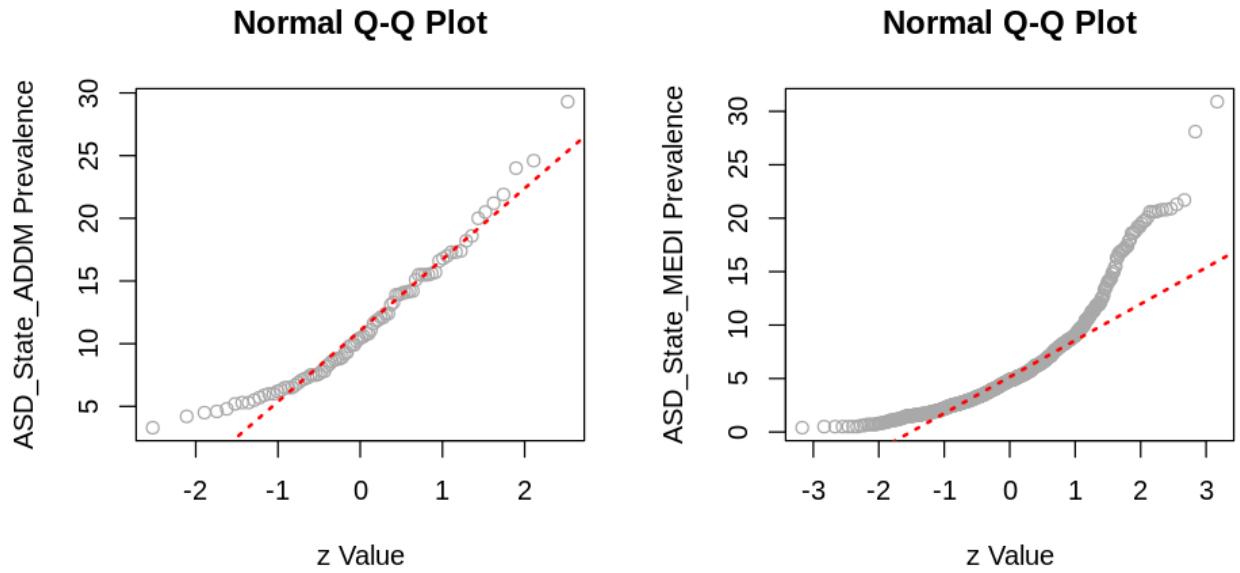
```
# -----  
# Evaluate normality  
# -----  
# Construct a Quantile-Quantile Plot (QQ plot)  
# https://youtu.be/okjYjCISjQg  
  
par(mfrow=c(1, 2))  
# Sample means  
qqnorm(clt_sample_k_mean, col="darkgrey",  
#       xlim=(c(-4, 4)), ylim=(c(0, 20)),  
#       xlab="z Value", ylab="Prevalence")  
qqline(clt_sample_k_mean, col="red", lwd=2, lty=3)  
# Population  
qqnorm(ASD_State_SPED_2016$Prevalence, col="darkgrey",  
#       xlim=(c(-4, 4)),  
#       xlab="z Value", ylab="Prevalence")  
qqline(ASD_State_SPED_2016$Prevalence, col="red", lwd=2, lty=3)  
# Reset  
par(mfrow=c(1, 1))
```



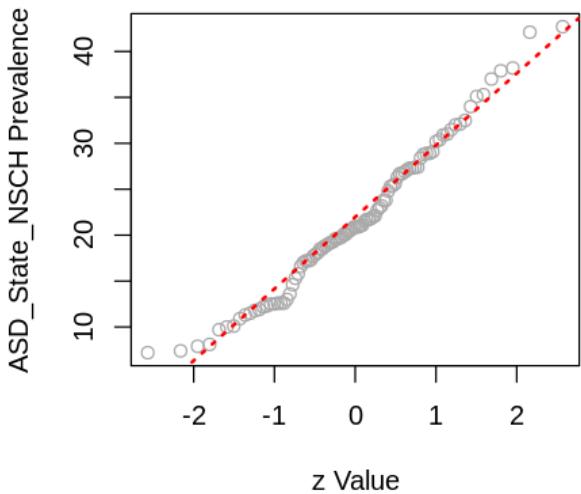
[Tips] If most/all data points are aligned with the red straight line, then the underlying data points are normally distributed.

In [33]:

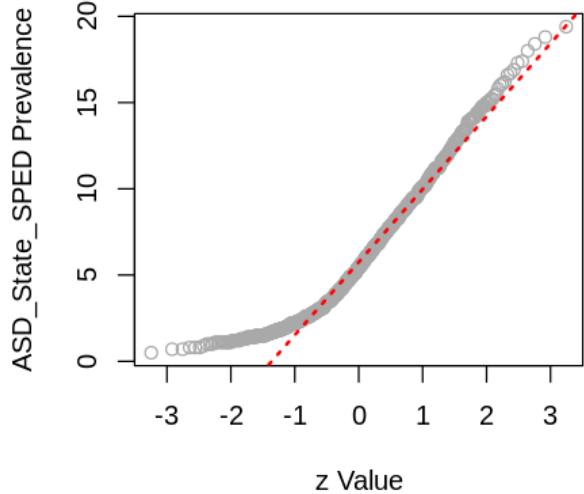
```
# -----  
# Evaluate normality  
# -----  
par(mfrow=c(1, 2))  
  
# ASD_State_ADDM$Prevalence  
qqnorm(ASD_State_ADDM$Prevalence, col="darkgrey",  
#       xlim=(c(-4, 4)), ylim=(c(0, 20)),  
#       xlab="z Value", ylab="ASD_State_ADDM Prevalence")  
qqline(ASD_State_ADDM$Prevalence, col="red", lwd=2, lty=3)  
# plot(density(ASD_State_ADDM$Prevalence))  
  
# ASD_State_MEDI$Prevalence  
qqnorm(ASD_State_MEDI$Prevalence, col="darkgrey",  
#       xlim=(c(-4, 4)), ylim=(c(0, 20)),  
#       xlab="z Value", ylab="ASD_State_MEDI Prevalence")  
qqline(ASD_State_MEDI$Prevalence, col="red", lwd=2, lty=3)  
  
# ASD_State_NSCH$Prevalence  
qqnorm(ASD_State_NSCH$Prevalence, col="darkgrey",  
#       xlim=(c(-4, 4)), ylim=(c(0, 20)),  
#       xlab="z Value", ylab="ASD_State_NSCH Prevalence")  
qqline(ASD_State_NSCH$Prevalence, col="red", lwd=2, lty=3)  
  
# ASD_State_SPED$Prevalence  
qqnorm(ASD_State_SPED$Prevalence, col="darkgrey",  
#       xlim=(c(-4, 4)), ylim=(c(0, 20)),  
#       xlab="z Value", ylab="ASD_State_SPED Prevalence")  
qqline(ASD_State_SPED$Prevalence, col="red", lwd=2, lty=3)  
  
# Reset  
par(mfrow=c(1, 1))
```



Normal Q-Q Plot



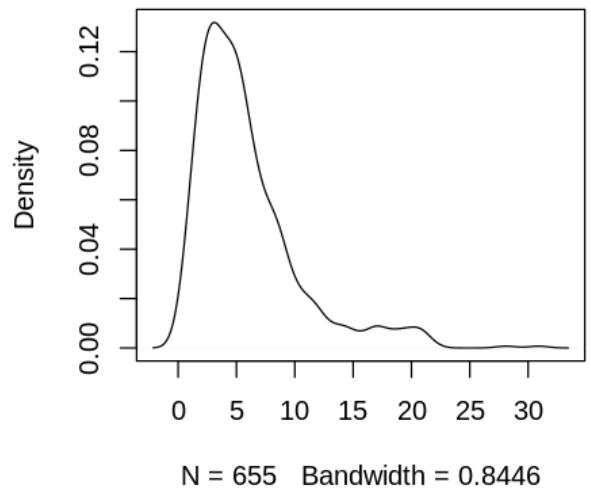
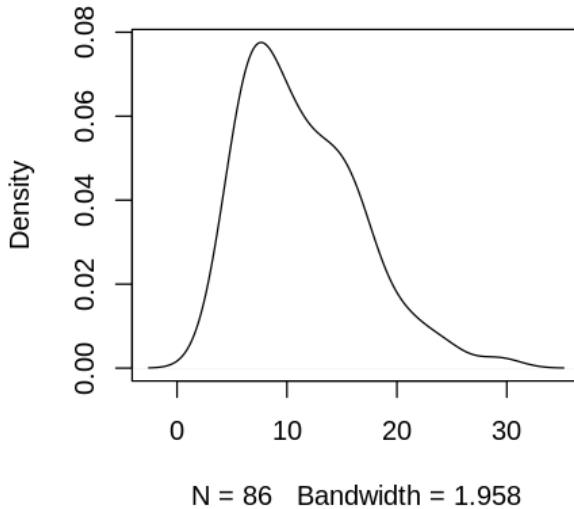
Normal Q-Q Plot



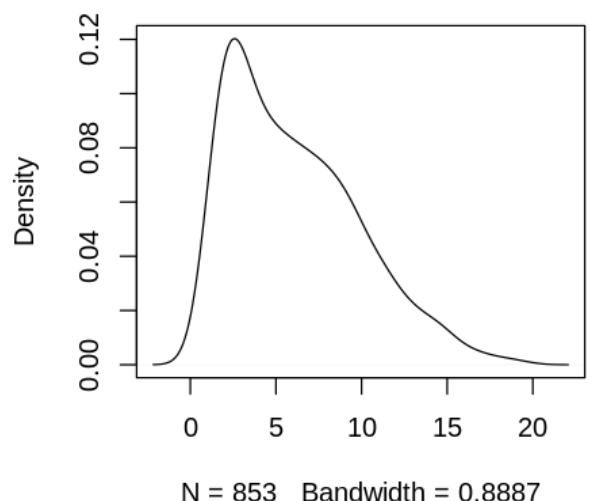
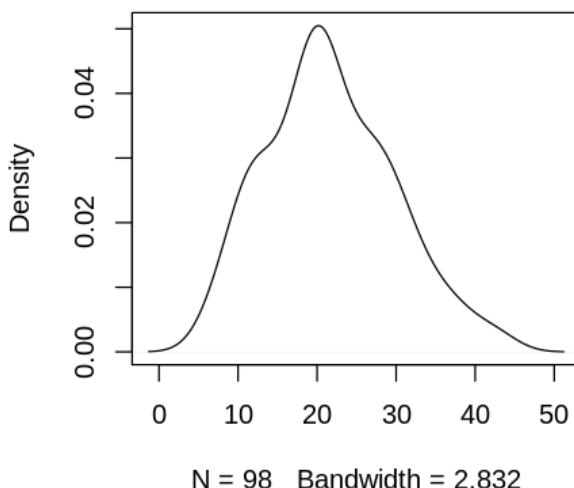
In [34]:

```
# -----  
# Evaluate normality  
# -----  
par(mfrow=c(1, 2))  
plot(density(ASD_State_ADDM$Prevalence))  
plot(density(ASD_State_MEDI$Prevalence))  
plot(density(ASD_State_NSCH$Prevalence))  
plot(density(ASD_State_SPED$Prevalence))  
# Reset  
par(mfrow=c(1, 1))
```

density.default(x = ASD_State_ADDM\$Prevalence) density.default(x = ASD_State_MEDI\$Prevalence)



density.default(x = ASD_State_NSCH\$Prevalence) density.default(x = ASD_State_SPED\$Prevalence)



```
In [35]: # Alternatively, use shapiro.test() to test Normality
set.seed(88)

# Test data of k sample's means (Sampling Distribution data):
shapiro.test(sample(x = clt_sample_k_mean, size = 1000))

# Test data of population's Prevalence values (Population Distribution data):
shapiro.test(ASD_State_SPED_2016$Prevalence)
```

```
Shapiro-Wilk normality test

data: sample(x = clt_sample_k_mean, size = 1000)
W = 0.99904, p-value = 0.8907
```

```
Shapiro-Wilk normality test

data: ASD_State_SPED_2016$Prevalence
W = 0.96985, p-value = 0.2282
```

[Tips] General speaking, if **p-value** is greater than **0.05** (meaning more than 5% chance of being normally distributed), then the underlying data points are normally distributed.

Confidence Interval (CI)

Confidence Interval (CI) - Mean Estimation & Its CI

Use a **Sample statistic (e.g. mean)** to estimate a **population statistic (e.g. mean)**. And quantitatively calculate the confidence of the estimation.

In [36]:

```
# -----  
# Use a sample of a few US. State's ASD prevalence (mean) to estimate:  
# Average prevalence of ALL US. States (the *Population*) [Source SPED, Year 2  
# -----  
dim(ASD_State_SPED_2016)  
#  
ASD_State_SPED_2016 # This is considered as a population now.
```

50 2

	State	Prevalence
1643	AL	9.1
1644	AK	10.1
1645	AZ	10.4
1646	AR	9.5
1647	CA	13.9
1648	CO	7.3
1649	CT	15.4
1650	DE	11.1
1651	DC	11.9
1652	FL	12.1
1653	GA	10.3
1654	HI	8.3
1655	ID	9.5
1656	IL	11.0
1657	IN	14.2
1658	IA	1.4
1659	KS	8.2
1660	KY	9.3
1661	LA	6.9
1662	ME	16.7
1663	MD	11.9
1664	MA	17.4
1665	MI	11.2
1666	MN	19.4
1667	MS	9.5
1668	MO	12.4
1669	MT	5.5
1670	NE	10.8
1671	NV	12.7
1672	NH	14.1
1673	NJ	14.1
1674	NM	8.5
1675	NY	13.0
1676	NC	11.2
1677	ND	9.8

	State	Prevalence
1678	OH	12.5
1679	OK	8.9
1680	OR	15.1
1681	PA	16.9
1682	RI	15.2
1683	SC	9.6
1684	SD	9.0
1685	TN	9.5
1686	TX	10.2
1687	UT	8.7
1688	VT	12.1
1689	VA	14.2
1690	WA	11.2
1691	WV	8.6
1692	WY	9.3

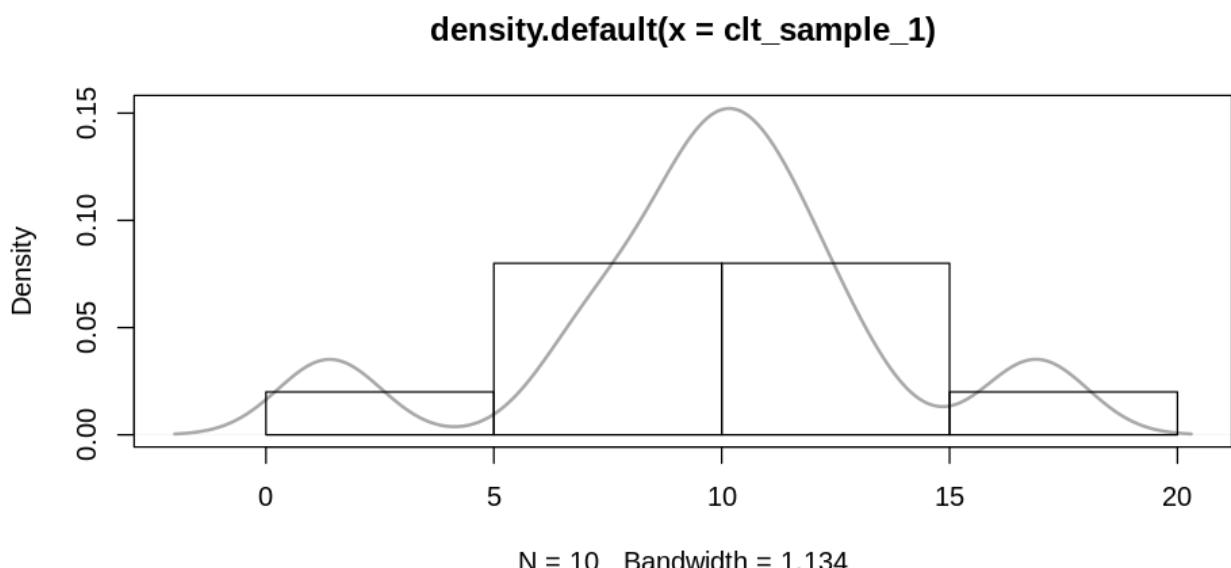
Draw a **Sample** from ASD_State_SPED_2016\$Prevalence

```
In [37]: # Create a *Sample* from ASD_State_SPED_2016$Prevalence,
# with sample size n =
clt_n = 10
# Try 20 or 40, larger sample size, narrower the CI (more confident at xx% lev
# clt_n = 20

set.seed(88)
clt_sample_1 = sample(x = ASD_State_SPED_2016$Prevalence, size = clt_n, replace = T)
clt_sample_1
```

11.2 9.5 16.9 6.9 11.2 8.2 12.7 9.5 1.4 10.2

```
In [38]: plot(density(clt_sample_1), col="darkgrey", lwd=2)
hist(clt_sample_1, probability = T, add = T)
```



```
In [39]: # Sample mean Prevalence  
mean(clt_sample_1)
```

```
# *Population* mean Prevalence  
mean(ASD_State_SPED_2016$Prevalence)
```

9.77

11.182

1. Calculate Confidence Interval of mean estimation: CI using Z (Standard Normal) distribution

```
In [40]: # -----  
# CI using Z (Standard Normal) distribution  
# -----  
# sample mean  
sample_mean = mean(clt_sample_1)  
sample_mean
```

9.77

```
In [41]: # sample size n  
sample_size_n = length(clt_sample_1)  
sample_size_n
```

10

```
In [42]: # sample standard deviation  
sample_sd = sd(clt_sample_1)  
sample_sd
```

4.00833853083516

```
In [43]: # sample standard error  
sample_se = sample_sd / sqrt(sample_size_n)  
sample_se
```

1.26754793904522

```
In [44]: # 95% quantile (z score)  
z_score = qnorm(p = 0.975)  
z_score
```

1.95996398454005

```
In [45]: # ?qnorm
```

```
In [46]: # CI using Z distribution  
sample_ci = z_score * sample_se  
sample_ci
```

2.4843483092066

```
In [47]: # Lower CI: mean + CI  
sample_mean - sample_ci  
  
# Upper CI: mean + CI  
sample_mean + sample_ci  
  
# Display  
cat('\t< Confidence Interval (Prevalence) >\n', '\tLower CI : ',  
    sample_mean - sample_ci, '\tMean : ',  
    sample_mean, '\tUpper CI : ',  
    sample_mean + sample_ci)
```

7.2856516907934

12.2543483092066

```
< Confidence Interval (Prevalence) >  
Lower CI : 7.285652      Mean : 9.77      Upper CI : 12.25435
```

[?] Is the population mean in this CI range?

2. Calculate Confidence Interval of mean estimation: CI using T distribution

```
In [48]: # -----  
# CI using T distribution  
# -----  
# sample mean  
sample_mean = mean(clt_sample_1)  
sample_mean  
# sample size n  
sample_size_n = length(clt_sample_1)  
sample_size_n  
# sample standard deviation  
sample_sd = sd(clt_sample_1)  
sample_sd  
# sample standard error  
sample_se = sample_sd / sqrt(sample_size_n)  
sample_se
```

9.77

10

4.00833853083516

1.26754793904522

```
In [49]: # 95% quantile (t score)  
t_score = qt(p = 0.975, df = sample_size_n - 1)  
t_score
```

2.2621571627982

```
In [50]: # ?qt
```

```
In [51]: # CI using T distribution
sample_ci = t_score * sample_se
sample_ci

# Lower CI: mean + CI
sample_mean - sample_ci

# Upper CI: mean + CI
sample_mean + sample_ci

# Display
cat('\t< Confidence Interval (Prevalence) >\n', '\tLower CI : ',
    sample_mean - sample_ci, '\tMean : ',
    sample_mean, '\tUpper CI : ',
    sample_mean + sample_ci)
```

2.86739264950124

6.90260735049876

12.6373926495012

< Confidence Interval (Prevalence) >
 Lower CI : 6.902607 Mean : 9.77 Upper CI : 12.63739

[?] Is the population mean in this CI range?

[?] Compare CIs of Z and T distribution, which CI has wider range? Is it reasonable?

```
In [52]: # Alternatively, calculate CI using t.test() function
t.test(clt_sample_1, conf.level = 0.95)
```

One Sample t-test

data: clt_sample_1
 $t = 7.7078$, df = 9, p-value = 2.976e-05
 alternative hypothesis: true mean is not equal to 0
 95 percent confidence interval:
 6.902607 12.637393
 sample estimates:
 mean of x
 9.77

```
In [53]: # Two group hypothesis test : sample mean vs. population mean
t.test(clt_sample_1, conf.level = 0.95, mu = mean(ASD_State_SPED_2016$Prevalen
```

One Sample t-test

data: clt_sample_1
 $t = -1.114$, df = 9, p-value = 0.2942
 alternative hypothesis: true mean is not equal to 11.182
 95 percent confidence interval:
 6.902607 12.637393
 sample estimates:
 mean of x
 9.77

Quiz:

Obtain CI using smaller/larger sample size (clt_n) at 99% confidence. Compare CI width.

Observe: larger sample size, narrower the CI (more confident at xx% level)

In [54]: # Write your code below and press Shift+Enter to execute

Double-click **here** for the solution.

Confidence Interval (CI) - Proportion Estimation & Its CI

Use a **Sample statistic (e.g. proportion)** to estimate a **population statistic (e.g. proportion)**. And quantitatively calculate the confidence of the estimation.

In [55]: # -----
Use a sample of one US. State's ASD prevalence (proportion) to estimate:
Prevalence of THAT US. State's ALL Children (the *Population*) [Source SPED,

No. Children with ASD
ASD <- ASD_State_SPED\$Numerator_ASD[ASD_State_SPED\$Year == 2016]

str(ASD)

```
int [1:50] 6140 1204 10746 4181 79041 5902 7391 1383 782 30920 ...
```

In [56]: # No. Children with ASD of first US. State (AL-Alabama)
ASD[1]

```
6140
```

In [57]: # No. Children surveyed
Children <- ASD_State_SPED\$Denominator[ASD_State_SPED\$Year == 2016]

str(Children)

```
int [1:50] 674701 119217 1033241 440130 5686400 808556 479961 124609 65732 2  
555399 ...
```

In [58]: # No. Children surveyed of first US. State (AL-Alabama)
Children[1]

```
674701
```

1. Calculate Confidence Interval of proportion estimation: **CI using Z score interval (standard normal distribution)**

```
In [59]: # -----
# CI using Z score interval (standard normal distribution)
# https://en.wikipedia.org/wiki/Binomial_proportion_confidence_interval
# -----

# sample proportion of first US. State (AL-Alabama) in year 2016 of source SPED
sample_proportion = ASD[1] / Children[1]
sample_proportion # p
1 - sample_proportion # q = 1 - p
```

0.00910032740428723
0.990899672595713

```
In [60]: # sample size n
sample_size_n = Children[1]
sample_size_n
```

674701

```
In [61]: # 95% quantile (z score)
z_score = qnorm(p = 0.975)
z_score
```

1.95996398454005

```
In [62]: sample_ci = z_score * sqrt(sample_proportion * (1 - sample_proportion)) / sample_size_n
sample_ci
```

0.000226587404757579

```
In [63]: # Lower CI: mean + CI
sample_proportion - sample_ci

# Upper CI: mean + CI
sample_proportion + sample_ci

# Display
cat('< Confidence Interval >\n', '\tLower CI : ', sample_proportion - sample_ci, '\tMean : ', sample_proportion, '\tUpper CI : ', sample_proportion + sample_ci)
```

0.00887373999952965
0.00932691480904481
< Confidence Interval >
Lower CI : 0.00887374 Mean : 0.009100327 Upper CI : 0.009326915

```
In [64]: # Display * 1000 -> Prevalence
cat('< Confidence Interval (Prevalence) >\n', '\tLower CI : ', 1000*(sample_proportion - sample_ci), '\tMean : ', 1000*sample_proportion, '\tUpper CI : ', 1000*(sample_proportion + sample_ci))
```

< Confidence Interval (Prevalence) >
Lower CI : 8.87374 Mean : 9.100327 Upper CI : 9.326915

[Tips] Based above calculation upon [Source: SPED] [Year: 2016] data, we have 95%

confidence that: The actual AL-Alabama state level ASD prevalence (if ALL childrens in Alabama state were surveyed) would be in the above calculated CI range 95% times.

Or, assuming there are 100 different Alabama states exist in 100 parallel universes, we obtained 100 actual prevalence proportions. 95 of them will likely fall into the CI range.

2. Calculate Confidence Interval of proportion estimation: [CI using Wilson score interval](#)

In [65]:

```
# -----  
# CI using Wilson score interval  
# https://en.wikipedia.org/wiki/Binomial_proportion_confidence_interval  
# -----  
  
sample_proportion # = ASD[1] / Children[1]  
  
# Yates' chi-squared test = Wilson score interval with continuity correction -  
prop.test(ASD[1], Children[1], conf.level = 0.95)  
  
# Pearson's chi-squared test = Wilson score interval - wilson  
prop.test(ASD[1], Children[1], conf.level = 0.95, correct = FALSE)
```

0.00910032740428723

1-sample proportions test with continuity correction

```
data: ASD[1] out of Children[1], null probability 0.5  
X-squared = 650363, df = 1, p-value < 2.2e-16  
alternative hypothesis: true p is not equal to 0.5  
95 percent confidence interval:  
 0.008875786 0.009330477  
sample estimates:  
      p  
0.009100327
```

1-sample proportions test without continuity correction

```
data: ASD[1] out of Children[1], null probability 0.5  
X-squared = 650365, df = 1, p-value < 2.2e-16  
alternative hypothesis: true p is not equal to 0.5  
95 percent confidence interval:  
 0.008876518 0.009329726  
sample estimates:  
      p  
0.009100327
```

Quiz:

Obtain CI of Male.Prevalence proportion [Source: ADDM] [Year: 2014] at 99% confidence.

In [66]: [# Write your code below and press Shift+Enter to execute](#)

Double-click **here** for the solution.

Quiz:

Obtain CI of Female.Prevalence propotion [Source: ADDM] [Year: 2014] at 99% confidence.

Then Compare CI range with Male children's CI range. Which gender has statistically higher ASD prevalence/proportion?

In [67]: # Write your code below and press Shift+Enter to execute

Double-click **here** for the solution.

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Excellent! You have completed the workshop notebook!

Connect with the author:

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Sam (https://www.iss.nus.edu.sg/about-us/staff/detail/201/GU_Zhan) is currently a lecturer in [Institute of Systems Science](https://www.iss.nus.edu.sg/) (<https://www.iss.nus.edu.sg/>) in [National University of Singapore](https://www.nus.edu.sg/) ([http://www.nus.edu.sg/](https://www.nus.edu.sg/)). He devotes himself into pedagogy & andragogy, and is very passionate in inspiring next generation of artificial intelligence lovers and leaders.

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Appendices

Interactive workshops: < Learning R inside R > using swirl() (in R/RStudio)

<https://github.com/telescopeuser/S-SB-Workshop> (<https://github.com/telescopeuser/S-SB-Workshop>)

(<https://github.com/dd-consulting>)

