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A DUAL-MODE MODEL FOR THE NORMAL EYE TRACKING SYSTEM

AND THE SYSTEM WITH NYSTAGMUS

by

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The author wishes to dedicate this thesis to the memory of Leonidas Miltiadis Mantgiaris, whose formative years contributed greatly to those of the author, and also to the author's wife, Lynn. Her love and understanding together with the insights gleaned through close friendship with a person of Leonidas' sensitivity have been and continue to be, of immeasurable value in the search for "the good life".

Finally, the author wishes to express his deep appreciation to his parents for providing their love, understanding and encouragement throughout the long process of learning which, at this point, seems to be just beginning.

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CHAPTER I

INTRODUCTION

One method of studying a system involves the postulation and synthesis of a model to describe it. By studying such a model, one can often gain insight into the actual system of interest. This method of attack assumes greater importance when the system which is to be studied is one which is difficult to work with; due to the problems associated with presenting various inputs and measuring the responses under controlled conditions. The system which is the subject of this paper is the horizontal tracking mechanism of a normal subject and that of a subject with nystagmus. Physiological data will be taken from both subjects so that the responses may be compared. Based on this data and that of other investigators, a model will be postulated which will represent the normal tracking system. This model will then be modified to describe the system with nystagmus. Both models will be programmed on a computer and the transient responses will be compared to those of the biological system.

Throughout this paper previous works will be cited and discussed as each area of the total study is treated. In this way, a continuous transition from past investigations and conclusions to present thought on each particular subject will be presented.

The combination of control theory and extensive use of electronic computers has caused the emergence of the field of modeling physiological control systems so that the intricate balances present in the

living organism may be further studied. While one cannot expect to make an exact correlation between the electronics which comprise the functional black boxes of the model and the neural and muscular structures which make up the biological system, the existence of the functional correlations which gave rise to the model does enable the medical investigator to study a complex control system by considering its various functional sub-systems separately. Going beyond the normal system, the modeling of one which is abnormal extends this mode of investigation into the study of the pathological condition itself and how it affects the operation of the whole system. Whether or not the ultimate goal will be reached this way (that is, understanding the affliction to the point of discovering a cure) is something which remains to be seen. One other thing whose value should be pointed out is the quantitative data collected on both the normal subject and, for the first time (to the author's knowledge) the subject with nystagmus. These recorded responses and the methods employed to obtain them could become a valuable tool in treating this and other eye conditions.

1.1 ANATOMY

The eyeball is an almost spherical shell which lies in the orbital cavity in a surrounding bed of fibrous tissue and fat (1). It is filled with a viscous fluid called the vitreous humor. The hard shell is the sclera which is continuous except for the front where the cornea protrudes. The cornea is the front lens and has a power of 42 diopters and a radius of curvature of 8 mm. Behind the cornea is the iris which is a group of muscles whose contractions or relaxations vary the eye's

aperture or pupil. The lens lies just behind the iris and its variable power capability is used to focus an image on the retina which lies on the rear inner surface of the eye. In the retina are two types of photo-sensitive cells called the rods and the cones. The rods, which are predominantly distributed around the periphery of the retina, are used for night vision and the cones, more centrally located, are responsible for day vision as well as color vision. The highest acuity occurs at the fovea which is an area of about 60 microns in diameter (this corresponds to a visual angle of about 1°) and consists entirely of cones. The internal anatomy of the eye is shown in Fig. 1.1 along with notes on the physiology of its parts (2).

Connected to the eye are six muscles. These muscles are the extrinsic eye muscles or the extra-ocular muscles. They enable the eye to rotate and to properly position an image on the fovea of the retina. These muscles are arranged in three pairs. During any eye movement one of the muscles of a pair will contract (the agonist) and one will relax (the antagonist). Thus, the agonist pulls the eye while the antagonist maintains an opposing tension to the motion. The center of rotation of the eyeball is essentially at its geometric center (3). Normally these muscles give the eye a range of motion of about 45° left or right, 40° up and 60° down, and 30° in extorsion or intorsion. These muscles are attached at opposite ends of eyeball diameters which give them a large mechanical advantage. Also, they are 40 mm in length and 100 square millimeters in cross section which should give them the capability of exerting a force of nearly 1 kg weight. The actual force necessary for a moderate saccadic movement has been calculated

EYE

STRUCTURE   *FUNCTION of PARTS*

The Eyeball has three coats:-

1. OUTER or SCLEROTIC COAT — PROTECTIVE LAYER

Tough fibrous tissue of "White of Eye." --- *Preserves shape of eyeball and protects delicate inner layers.*

Transparent CORNEA in front. --- *Allows passage of LIGHT RAYS.*

[Extrinsic muscles are attached to sclera. --- *Permit and limit movements of eyeball within ORBIT.*]

2. MIDDLE or VASCULAR PIGMENTED COAT — LAYER of SUPPLY

Contains main arteries and veins of eyeball.

Circular opening at front – PUPIL.

Coloured muscular ring – IRIS – surrounds pupil. — *Controls size of pupil and amount of light entering eye.*

CILIARY BODY. — *Produces AQUEOUS HUMOUR.*

CILIARY MUSCLE. — *Contracts and moves forwards.*

SUSPENSORY LIGAMENT suspends CRYSTALLINE LENS. — *Relaxes to allow curvature of lens to alter for accommodation for NEAR VISION.*

CHOROID – Posterior ⅚ of Vascular Coat. — *Brings light rays to focus on light-sensitive RETINA.*

3. INNER or NERVOUS COAT – the RETINA — LIGHT-SENSITIVE LAYER

Lines back of eye. Contains RECEPTORS for VISION — *Highly specialized to respond to stimulation by light. Convert light energy into nerve impulses.* ⟶ OPTIC NERVE *Conveys these impulses to VISUAL CENTRES in OCCIPITAL (posterior) part of BRAIN.*

CORNEA

ANTERIOR CHAMBER (AQUEOUS HUMOUR)

IRIS

SUSPENSORY LIGAMENT

CILIARY BODY

LENS

(VITREOUS HUMOUR)

RETINA

CHOROID

SCLERA

Main blood vessels to retina enter and leave here.

CENTRAL ARTERY of the RETINA

Fig. 1.1 The eye (2)

at 100 g weight. This large reserve allows for extremely large velo-
cities (600°/sec) and also explains the fact that these muscles do not
normally fatigue (4). With the large number of innervating fibers pres-
ent in proportion to the muscle size, the brain can exercise very
fine control. The extra-ocular muscles have the shortest reaction time
(less than 10 msec) of any muscle in the body (5). Fig. 1.2 shows
these muscles, their points of attachment and the cranial nerves which
innervate them (2). Since this paper is concerned only with movements
in the horizontal plane, only the medial (internal) and lateral (exter-
nal) rectus muscles and cranial nerves III and VI will be considered.

1.2 TYPES OF EYE MOTION (6)
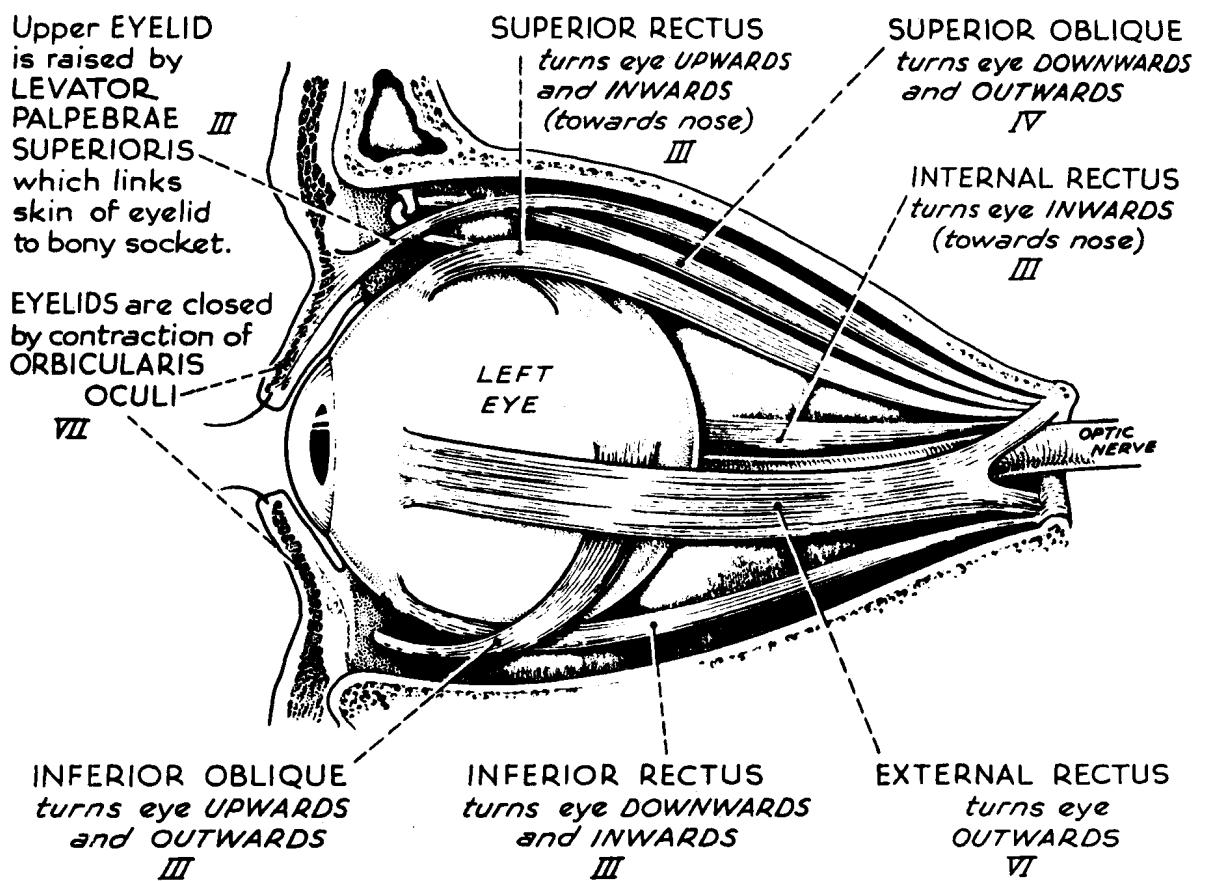
Before proceeding further into the anatomy and physiology involved
in the horizontal tracking mechanism of the eye, it would be advanta-
geous at this point to name and define the various types of motion the
eye is capable of and to point out those we will consider.

1.2.1 Saccadic Movements

A saccadic movement is one in which the eyes are moved conjugately
from one position to another in a step-like manner (also referred to
as a jerk or jump). Given an initial target displacement, the eye will
not respond for 120-250 msec (response time) (7,8). Then there will
be a period of acceleration, followed by a constant velocity segment
and then a period of deceleration. Many times there will be some over-
shoot before reaching the final value. At present there is evidence
that during a saccade vision is cut off, but this has not been proven.
There are two theories concerning the nature of this movement. One
school of thought pictures a saccade as a preprogrammed ballistic

MUSCLES of EYE

The EYEBALLS are moved by SMALL MUSCLES which link the SCLEROTIC COAT to the BONY SOCKET.

Upper EYELID is raised by LEVATOR PALPEBRAE III SUPERIORIS which links skin of eyelid to bony socket.

EYELIDS are closed by contraction of ORBICULARIS OCULI VII

SUPERIOR RECTUS
turns eye UPWARDS and INWARDS (towards nose)
III

SUPERIOR OBLIQUE
turns eye DOWNWARDS and OUTWARDS
IV

INTERNAL RECTUS
turns eye INWARDS (towards nose)
III

LEFT EYE

OPTIC NERVE

INFERIOR OBLIQUE
turns eye UPWARDS and OUTWARDS
III

INFERIOR RECTUS
turns eye DOWNWARDS and INWARDS
III

EXTERNAL RECTUS
turns eye OUTWARDS
VI

Acting together, the Extrinsic muscles of the Eyeballs can bring about ROTATORY movements of the Eyes.
The Extrinsic muscles are supplied by motor fibres from Cranial Nerves III, IV and VI.

Fig. 1.2 The eye muscles (2)

movement (9). This would be an essentially open loop system. The other
concept is that the saccade is controlled in a closed loop manner by
position feedback from the extra-ocular muscles. The question of the
existence of proprioceptive feedback is one which is widely disputed
and will be discussed in the section dealing with neuro-muscular
pathways.

1.2.2 Smooth Pursuit Movements

Smooth pursuit movements enable the eye to follow (or track) mov-
ing targets with angular velocities of from 1°/sec to $25\text{-}30^{\circ}$/sec (10,11).
As with saccades, there is a latent period associated with smooth track-
ing and it is about 120-250 msec. The object of smooth pursuit is to
keep the eye focused on a moving target. The angular velocity of the
eye is found to be linearly proportional to target velocity and the
initiation for this mode is a detection of retinal image motion, where-
as the saccadic mode is triggered by a position deviation on the retina.
It has been shown experimentally that pursuit is a semi-voluntary mode.
One can voluntarily follow a moving target but one cannot move his eyes
smoothly from one position to another without such a moving target to
follow (12).

1.2.3 Compensatory Movements

These movements help stabilize the retinal image in the presence
of head or body movements. They are smooth movements in a direction
opposite to the motion. These movements are limited to a velocity of
30°/sec and their latent period is about 100 msec (6,13).

1.2.4 Vergence Movements

These movements are an important part of binocular vision since
it is the amount of convergence plus the amount of lens accomodation

which accounts for the perception of distance (14). Vergence movements differ from saccadic, pursuit, and compensatory movements by causing the eyes to move in opposite directions instead of in the same direction as conjugate movements do. Thus, as a target moves toward or away from the subject, his eyes will converge or diverge respectively so that the image falls on corresponding parts of the retina. Experiments have shown that the vergence and tracking systems are independent and that vergence movements are smooth and quite a bit slower than conjugate movements (15,16). Also, a target displacement which would require a step change in convergence gives rise to velocities in the order of $10^{o}/\text{sec}$ and the time necessary for the change is about 1 sec.

1.2.5 Torsional Movements

These movements are rotations about an axis running horizontally from the center of the pupil back through the retina. They are compensatory in nature and they serve to maintain the image on the fovea as the subject rotates his head about the line of sight. The muscles which are mainly involved in this rotation are the inferior and superior oblique muscles (17). It has been observed that the eyes may lag behind the head by as much as 30°.

1.2.6 Miniature Movements

During fixation on a stationary target, in the absence of large movements caused by some pathological condition, three types of very small movements (less than 1°) can be observed (18,19). While they may also be present during large movements, both voluntary tracking and involuntary excursions of the eye, it is at present extremely difficult to verify this experimentally.

(1) Drift: This is a random movement of the eyes which occurs during "steady" fixation. The velocity of this random motion is usually less than several minutes of an arc per second (20).

(2) Saccades: Small saccades in a direction opposite to the drift have been observed to occur within 30 msec of each other (21). The purpose of these saccades is to keep the image focused within the foveal region of about 1° (22).

(3) Tremor: Tremor is a high frequency vibration of the eyes (23). The range of frequencies of this movement is from 30 to 150 Hz with a peak in energy between 70 and 90 Hz and an amplitude which varies from 5 to 15 sec/arc (24). There is evidence that tremor is independent for each eye. This would suggest that the mechanism involved is not the same one that is responsible for conjugate tracking movements. The purpose of tremor is perhaps brought out by experiments which have been performed using an optical method to negate the effects of tremor on the retina. When such tests were made, the subject reported that the image faded out after several seconds, and then continually appeared and disappeared (25). From this it has been postulated that the purpose of tremor is to prevent the saturation of any one cone by continually sweeping across several cones. Another, or perhaps simultaneous, purpose might be to help one to accurately determine boundaries that would be difficult to perceive because of the random way in which rods and cones are distributed on the retina.

1.2.7 Nystagmus

Movements which are oscillatory or unstable are referred to as nystagmus movements (26). There are several types of nystagmus which have been observed, some of which are described below (27,28,29,30,31).

(1) Vestibular Spontaneous Nystagmus: This type of nystagmus is always of the jerk type and is more prominent with lid closure and tends to diminish during fixation. It is more prominent in a fully aroused subject and diminishes as his fatigue increases (32). Respiration also affects this type of nystagmus. Vestibular nystagmus can arise from either central or peripheral locations. Related types of nystagmus are positional head-shaking, and toxic nystagmus. Vestibular nystagmus can be induced by rotation of the subject (33) or by irrigating the ear canals with warm or cool water (34).

(2) Gaze Nystagmus: There are two types of gaze nystagmus; gaze paretic and gaze direction nystagmus. The paretic type is accompanied by partial gaze paresis in the direction of the quick phase and is usually at frequencies of 1 to 2 Hz. The directional type is usually at frequencies of 3 to 8 Hz and its amplitude increases with increasing gaze deflection from the straight-ahead position.

(3) Fixation Nystagmus: Fixation nystagmus is almost always congenital and is the type of nystagmus that will be of interest in the present study. More specifically, we will be concerned with horizontal fixation nystagmus, which is oscillation in the horizontal plane only. While it is possible for a person with fixation nystagmus to have spontaneous nystagmus with closed eyes, the subject used in this study did not have this additional condition so it will not be investigated at this time. Typical fixation nystagmus does, however, show up in complete darkness when the gaze is directed in a particular direction by a stimulus other than an optical one (i.e. an acoustic stimulus).

Nystagmus can be induced in a subject by directing his gaze to a rotating drum which has alternating black and white vertical bars on

it and which is rotating about a vertical axis so that the bars pass
from one side of the subject's visual field to the other. In this way,
the movement of the visual field from left to right or right to left
will induce the eyes to conjugately track a part of the field as it
moves until the eyes reach an uncomfortable position at one end of the
range of eye excursion. At this point, the eyes will jump back to the
central position and the above sequence repeated. This is referred to
as optokinetic nystagmus and it is a valuable diagnostic tool.

1.3 OCULOMOTOR ORGANIZATION

This section is devoted to the cranial nerves, neural pathways,
and sites in the brain which make up the oculomotor system. An attempt
will be made to present the various dissenting views which are associ-
ated with this topic and, where possible, try to arrive at the conclu-
sions that are most likely to yield a true picture of this very complex
and controversial system.

1.3.1 Cranial Nerves

Five of the twelve cranial nerves are involved, directly or in-
directly, in controlling the movements of the eyes. They are the optic
(II), the oculomotor (III), the trochlear (IV), the trigeminal (V), and
the abducens (VI), nerves.

(1) Optic: This nerve is the nerve which is responsible for
vision. It arises from cells in the retina, exits the eye in the rear
at a point on the retina known as the "blind spot" and enters the skull
through the optic foramen (1,35). The two optic nerves join at the op-
tic chiasma where fibers from the nasal half of each retina cross over
to the other side and fibers from the temporal half continue in the

optic tract of the same side. The fibers of the optic tract then pro-
ceed to the brain stem where they terminate. From the brain stem other
fibers carry the visual impulses to the visual cortex at the back of
the occipital lobe.

(2) Oculomotor: The point of origin of the oculomotor nerve is
the under surface of the brain stem in front of the pons. From there
it goes forward through the superior orbital fissure into the orbit
where it supplies the superior rectus, inferior rectus, medial rectus,
inferior oblique, and levator palpebrae superioris muscles. The smooth
muscles of the ciliary body and iris are also served by this nerve.

(3) Trochlear: The dorsal surface of the midbrain is the point
of origin of the trochlear nerve. From there it passes around the side
of the brain stem and goes forward through the superior orbital fissure
into the orbit where it supplies the superior oblique muscle.

(4) Trigeminal: There are two roots of this nerve, a sensory and
a motor, which are attached close together on the side of the pons.
The semilunar (Gasserian) ganglion lies on the sensory root at the
point where it passes into the middle fossa of the skull. There are
three large sensory branches which bring fibers into this ganglion:
(a) the opthalmic, bringing fibers from the orbit, upper eyelid, bridge
of the nose, and scalp as far as the crown of the head; (b) the maxil-
lary, with fibers from the lower eyelid, the lower portions of the nose,
the cheek, the upper lip, jaw, and palate; and (c) the mandibular,
with sensations from the lower lip and jaw, lower part of the face,
and front of the ear. The opthalmic nerve passes through the superior
orbital fissure, the maxillary through the foramen rotundum, and

mandibular through the foramen ovale. The motor root lies below the semilunar ganglion and it is continued into the mandibular nerve at the foramen ovale. The fibers go to the muscles of mastication, the masseter muscle, and the temporal muscle. We will not be interested in the motor portion of the trigeminal nerve in this paper.

(5) Abducens: The abducens nerve originates from the under side of the brain stem behind the pons. It passes into the orbit through the superior orbital fissure and supplies the lateral rectus muscle.
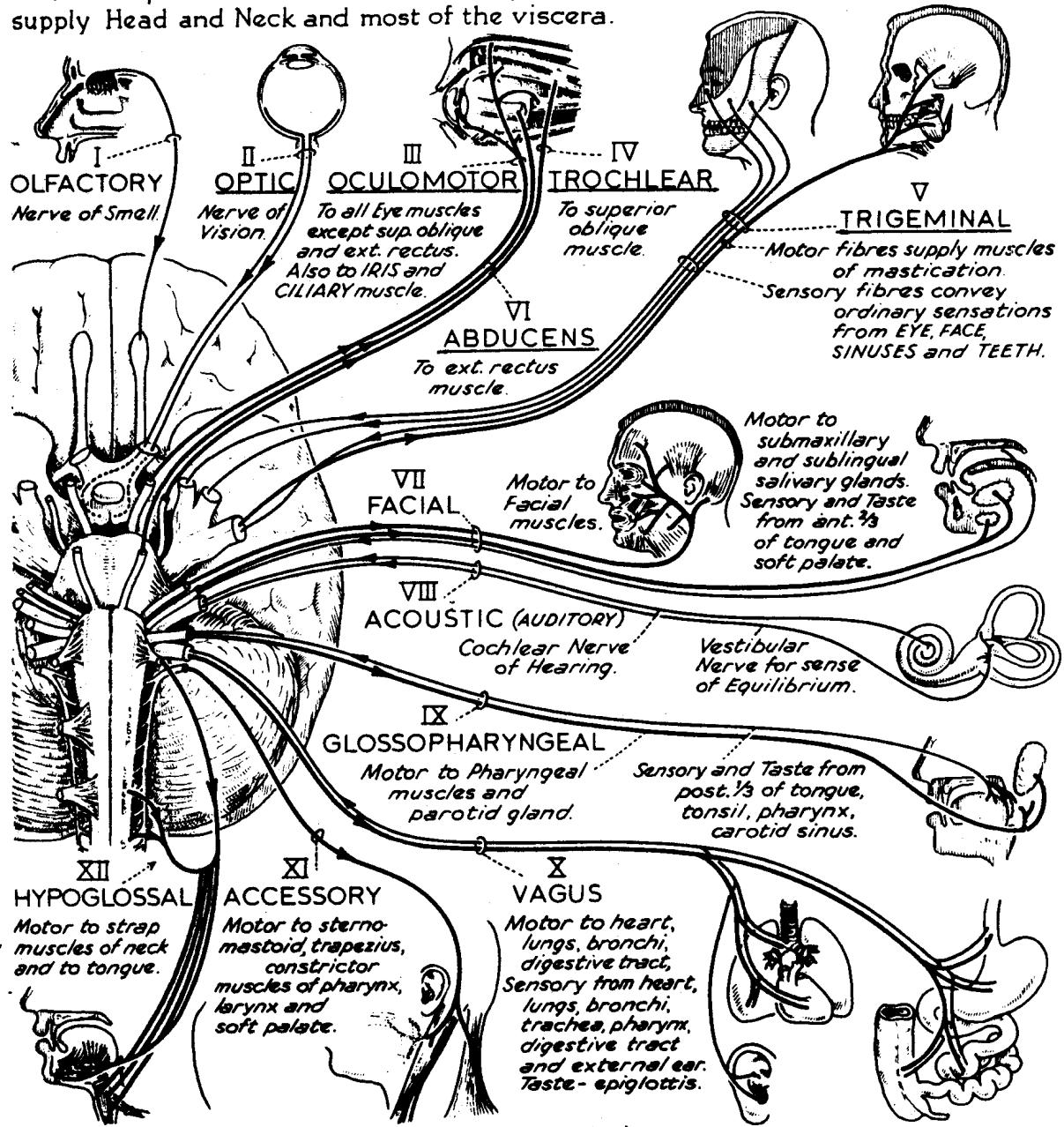
The above descriptions will be more meaningful to the reader if they are used in conjunction with Fig. 1.3 (2). Also shown in this figure are the points of insertion of the nerves into the brain.

1.3.2 Brain Sites (35,36,37)

Since the various areas of the brain will be referred to when discussing the neural pathways involved in the transmittance of visual and command information, it will help to become familiar with the relative positions of these areas. To do this most efficiently, Fig. 1.4 and Fig. 1.5 have been provided (2). The areas of most concern are: the frontal lobe, the parietal lobe, the occipital lobe, the pons, the midbrain, and the cerebellum. Also of interest are the thalamus and corpora quadrigemina, shown in Fig. 1.4, and the various areas of the cerebrum which are shown in Fig. 1.5. Some areas of special interest are: (1) the lateral geniculate body; (2) the superior colliculus; (3) the pretectal region; (4) the motor nucleus of the oculomotor nerve (MN-III); (5) the motor nucleus of the abducens nerve (MN-VI); and (6) the proprioceptive nucleus of the trigeminal nerve (MN-V) which is also referred to as mesencephalic nucleus V.

CRANIAL NERVES

Twelve pairs of nerves arise directly from the undersurface of the Brain to supply Head and Neck and most of the viscera.

I
OLFACTORY
Nerve of Smell.

II
OPTIC
Nerve of Vision.

III
OCULOMOTOR
To all Eye muscles except sup. oblique and ext. rectus. Also to IRIS and CILIARY muscle.

IV
TROCHLEAR
To superior oblique muscle.

V
TRIGEMINAL
Motor fibres supply muscles of mastication. Sensory fibres convey ordinary sensations from EYE, FACE, SINUSES and TEETH.

VI
ABDUCENS
To ext. rectus muscle.

VII
FACIAL
Motor to Facial muscles.
Motor to submaxillary and sublingual salivary glands. Sensory and Taste from ant. ⅔ of tongue and soft palate.

VIII
ACOUSTIC (AUDITORY)
Cochlear Nerve of Hearing.
Vestibular Nerve for sense of Equilibrium.

IX
GLOSSOPHARYNGEAL
Motor to Pharyngeal muscles and parotid gland.
Sensory and Taste from post. ⅓ of tongue, tonsil, pharynx, carotid sinus.

XII
HYPOGLOSSAL
Motor to strap muscles of neck and to tongue.

XI
ACCESSORY
Motor to sterno-mastoid, trapezius, constrictor muscles of pharynx, larynx and soft palate.

X
VAGUS
Motor to heart, lungs, bronchi, digestive tract, Sensory from heart, lungs, bronchi, trachea, pharynx, digestive tract and external ear. Taste - epiglottis.

(After Frank H. NETTER, M.D., The Ciba Collection of Medical Illustrations)

Fig. 1.3 The cranial nerves (2)

VERTICAL SECTION through BRAIN

This is a Vertical Section through the LONGITUDINAL FISSURE— a deep cleft which separates the two Cerebral Hemispheres. At the bottom of this cleft are tracts of nerve fibres which link up the different LOBES of each hemisphere and also link the two hemispheres with each other — the CORPUS CALLOSUM.

FOREBRAIN

Cerebral Hemisphere
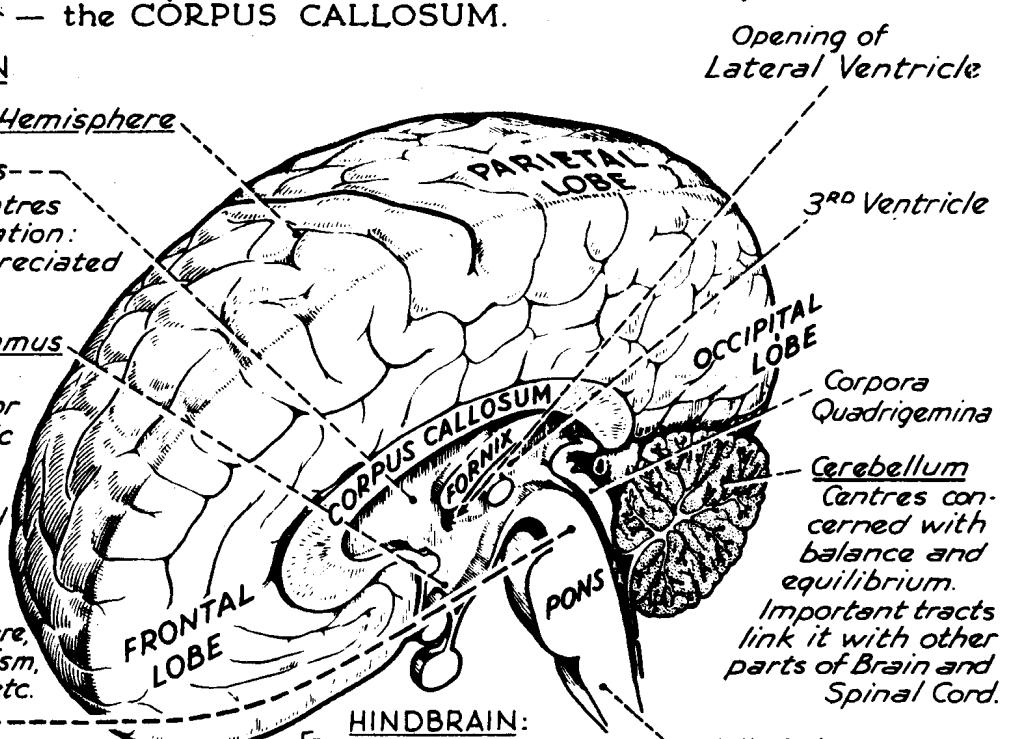
Thalamus-
-relay centres for sensation: pain appreciated here.

Hypothalamus
-contains centres for Autonomic Nervous System. e.g. Control of Heart, Blood pressure, Temperature, Metabolism, etc.

Opening of Lateral Ventricle

PARIETAL LOBE

CORPUS CALLOSUM

FORNIX

OCCIPITAL LOBE

FRONTAL LOBE

PONS

3RD Ventricle

Corpora Quadrigemina

Cerebellum
Centres concerned with balance and equilibrium. Important tracts link it with other parts of Brain and Spinal Cord.

MIDBRAIN
Receives impulses from Retina and Ear. Serves as a centre for Visual and Auditory Reflexes. In the Grey Matter are nerve cell bodies of III, IV Cranial nerves and the Red Nucleus which helps to control skilled muscular movements. The White Matter carries nerve fibres linking Red Nucleus with Cerebral Cortex, Thalamus, Cerebellum, Corpus Striatum and Spinal Cord. It also carries Ascending Sensory fibres in Lateral and Medial Lemnisci, and Descending Motor fibres on their way to Pons and Spinal Cord.

HINDBRAIN:
[PONS, CEREBELLUM, MEDULLA OBLONGATA]

Pons: Groups of Neurones form sensory nucleus of V and also nuclei of VI and VII Cranial nerves. Other nerve cells here relay impulses along their axons to Cerebellum and Cerebrum. Rubrospinal tract. Lateral and Medial Lemnisci pass through Pons and nerve fibres linking Cerebral Cortex with Medulla Oblongata and Spinal Cord.
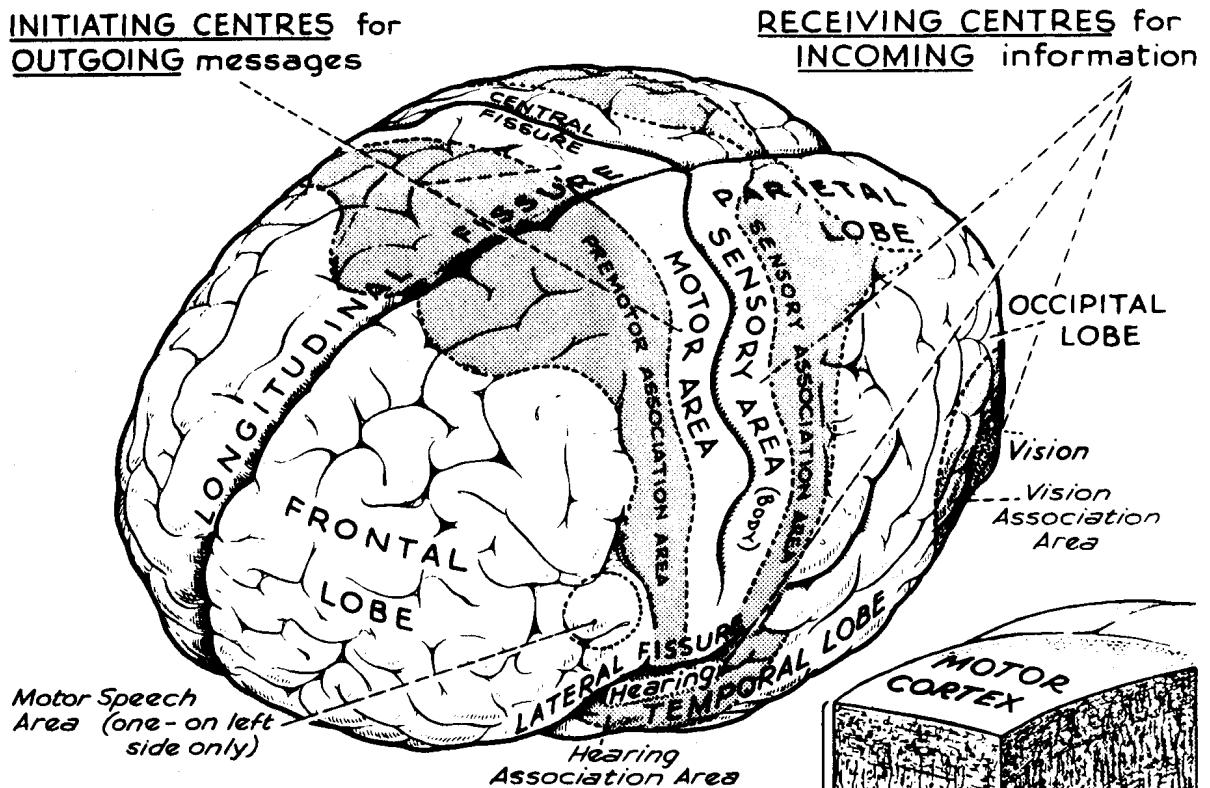
Medulla Oblongata:
Groups of Neurones form Nuclei of VIII, IX, X, XI, XII Cranial nerves. Gracile and Cuneate Nuclei -second sensory neurones in cutaneous pathways. Tracts of Sensory fibres decussate and ascend to other side of Cerebral Cortex. Some fibres remain uncrossed. The larger part of each Motor pyramidal tract crosses and descends in other side of Spinal Cord.

Fig. 1.4 Vertical section through the brain (2)

CEREBRUM

The largest part of the human brain is the CEREBRUM — made up of
2 CEREBRAL HEMISPHERES. Each of these is divided into LOBES.

INITIATING CENTRES for
OUTGOING messages

RECEIVING CENTRES for
INCOMING information

Motor Speech
Area (one - on left
side only)

Hearing
Association Area

Large uncharted areas of the Cerebral Hemispheres
are probably concerned with MENTAL PROCESSES
such as *Intelligence, Memory, Judgement,
Imagination, Creative* and *Conscious Thought.*

The surface of the brain shows many folds or
CONVOLUTIONS. This has the effect of
increasing the amount of GREY MATTER present.
The GREY MATTER forms the outer layer or
CORTEX. It contains the cell bodies of the
NEURONES arranged in many interconnecting
layers to form a 3-dimensional network.

About 90% of all Nerve Cells are
in the Cerebral Cortex.
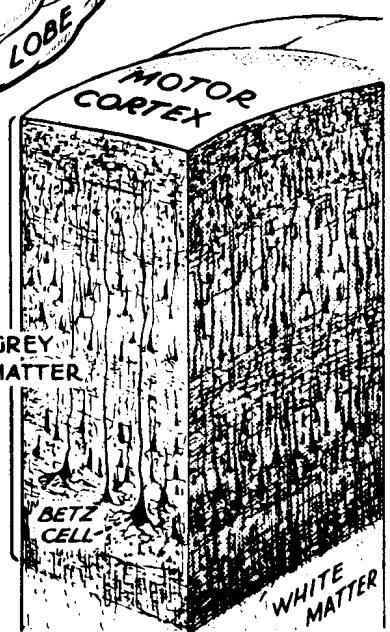
Fig. 1.5 The cerebrum (2)

(1) Lateral Geniculate Body: This is an oval-shaped body which is located lateral to the posterior end of the thalamus. It is at the level of the pineal body as can be seen in Fig. 1.6 (35).

(2) Superior Colliculus: This body is located on the upper part of the corpora quadrigemina just below the pineal body (see Fig. 1.6).
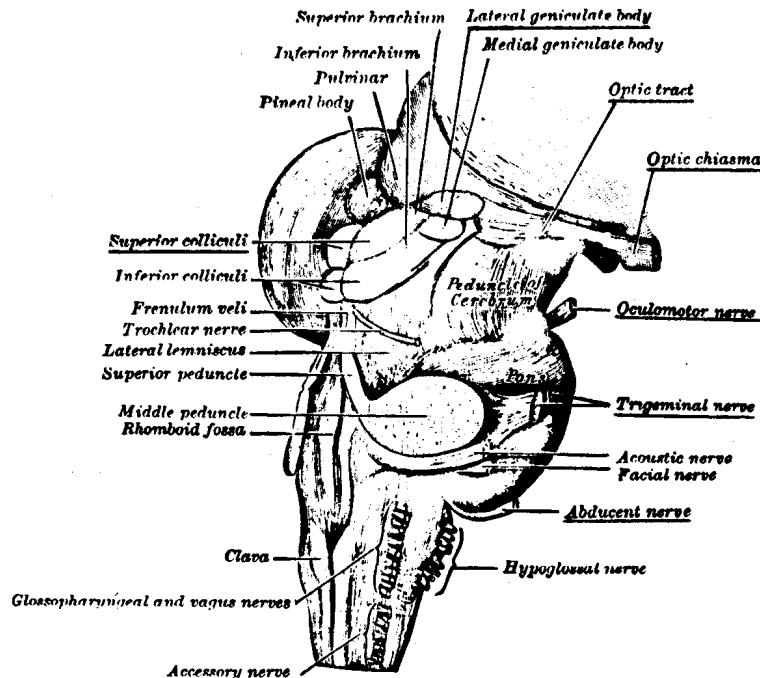
Fig. 1.6 Hind- and midbrains; postero-lateral view (35)

(3) Pretectal Region: The pretectal region is a transition zone which lies between the thalamus and the superior colliculus.

(4) MN-III: The oculomotor nucleus consists of two parts, the somatic and the autonomic nuclei. The somatic nucleus consists of definite groups of neurons for the individual muscles which can be mapped topographically. The autonomic nucleus lies dorsal to the somatic and is composed of a lateral portion called the Edinger-Westphal nucleus and a medial portion which contains simlar small, motor-type cells.

The oculomotor nucleus lies in the gray matter ventral to the aquaduct and is from 6 to 10 mm in length. Adjacent to this, as well as all the nuclei of the extra-ocular muscles, is the mesencephalic nucleus of the trigeminal nerve which supplies the proprioceptive fibers of cranial nerves III and VI.

(5) MN-VI: This nucleus is a small spherical mass in the lower portion of the pons and is the most caudal of the nuclei of the motor nerves to the eye.

(6) MN-V: The trigeminal nerve contains somatic sensory, special visceral efferent, and proprioceptive fibers. Since we are only interested in the proprioceptive fibers we will discuss the nuclei of the mesencephalic root. The mesencephalic root arises from unipolar cells which are located in groups at the lateral edge of the gray matter that surrounds the upper end of the fourth ventricle. It is from this nucleus that the proprioceptive fibers of the III and VI cranial nerves are supplied.

1.3.3 Neural Pathways

The tracing of the neural paths involved in the transmission of visual impulses from the eye to the brain, the internal brain-paths which interpret the visual signals and cause motor commands to be sent back to the eyes, as well as the paths for the command signals is a necessary part of this study even though some of the information available is contradictory. An effort will be made to trace only those paths related to the horizontal tracking system and to provide, in those areas where a controversy exists, references in support of and in opposition to the author's contentions.

(1) Visual Information: From the retina both position and rate of change (velocity) information are sent via the optic nerve to the optic chiasma (9,38,39). At the chiasma the fibers of the nerves divide as described in section 1.3.1. The information is now carried in the optic tracts to the lateral geniculate bodies. Most of the fibers terminate in the lateral geniculate body but some go on to the superior colliculus and the pretectal region (35). Wiesel has shown the existence of a mapping between the retina and the lateral geniculate body (40). Fibers go to the visual cortex in the occipital lobe from the lateral geniculate body. The fibers from the pretectal region go to the Edinger-Westphal nucleus where they form part of the pupillary constriction reflex arc which will not be considered in this paper. The superior colliculus is the center for the coordination of eye movements.

(2) Interpretation: Area 17 of the visual cortex is the area in which the visual signals are interpreted for color, form, size, motion, illumination, and transparency. Areas 18 and 19, which surround the visual sensory area, are the visual psychic areas (see Fig. 1.7) (35). It is here that visual impressions are elaborated and associated with past experience for recognition and identification. Eye movements are also related to visual impressions in these areas (35,41).

(3) Motor Commands: At this time there is no definite evidence or agreement as to the layer of the visual cortex in which the motor fibers begin (39,41). Some authors consider the beginnings of the motor path to lie in area 17, but Hines has presented evidence that the efferent path begins in area 18 and area 19 (42). From the area, or areas, of origin the signals pass through the superior colliculus to
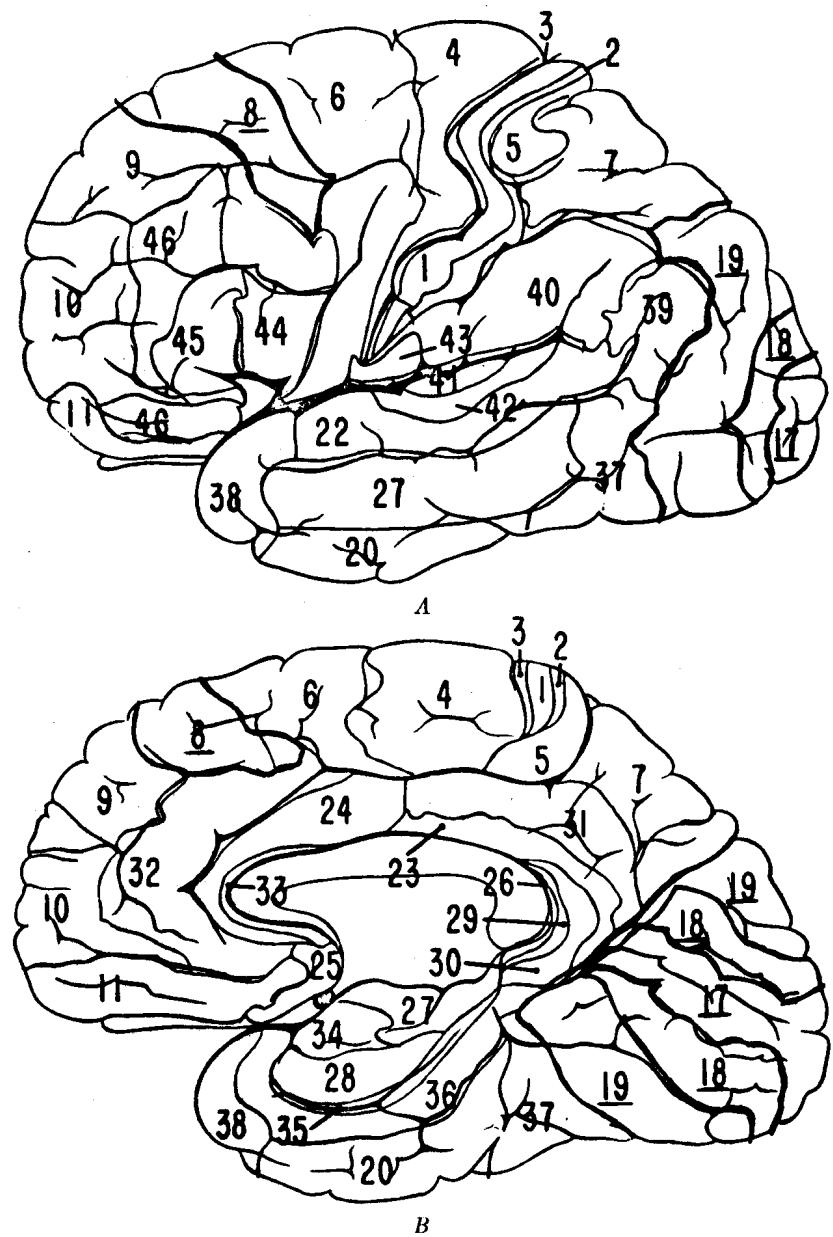
Fig. 1.7 Areas of the cerebrum (35) - A) lateral, B) medial

the vestibular nucleus where the commands for conjugate movement are
sent to the motor nuclei of cranial nerves III and VI (43,44). Volun-
tary movements of the eye are a result of signals from the second and
third frontal convolutions of each hemisphere (area 8). The signals
then pass through the oculogyric centers in the frontal lobe to the
motor nuclei of the oculomotor nerves. Some clinicians feel that there
is a pontine center for lateral gaze but there is no histologic evidence
for such a center (39). The signals arriving at the motor nuclei are
then sent to the oculomotor muscles via cranial nerves III and VI where
they cause the eye to move in the desired direction. This completes
the forward path in the system along with the main feedback path which
is the displacement signal that results from the retinal motion.

(4) Proprioception: The question of whether or not proprioception
exists in the extra-ocular muscles and, if it does, what its role might
be is one which has been the subject of controversy and discussion
since 1867 when H. von Helmholtz published his now famous experiments
on the subject (45,46). His observations are subjective and the experi-
ments are simple enough so that anyone may repeat them. He pointed out
that if one is looking at stationary objects in the surroundings and
moves his point of fixation from one object to another these objects
do not appear to have moved even though their retinal images have un-
dergone a displacement. Apparently there is some mechanism which
allows for eye and head movements when interpreting retinal images be-
cause, if there weren't, the retinal image motion would give rise to the
sensation of object movement in the real world. Alternately, if one
fixes his gaze on a stationary object and then physically moves his eye

by means of his finger the sensation is one of motion of the surround-
ings. Thus, retinal image motion gives rise to apparent object move-
ment in the latter case and to non-movement in the former case. This
experiment is cited by Helmholtz as proof that the mechanism involved
is not one of proprioception from the extra-ocular muscles since in
both cases such proprioceptive signals would have been sent to the brain
as a result of the muscle movement. The explanation put forth by Helm-
holtz was that one was aware of eye movements by a knowledge of the
innervation sent to the extra-ocular muscles to effect the movement.
Experiments with after-images recently conducted by Brindley support
this contention (47). Brindley also stated that the movement of one
eye does not cause a reflex movement in the other. These results were
also found by Merton who used after-images to show the absence of cons-
cious position sense in the dark (48,49). In 1936, Irvine made a
histological study of the extra-ocular muscles in which he found little
anatomic evidence for a proprioceptive sense (50). The same conclu-
sions were reached by McCouch in 1932 and by Hyde in 1960 although Hyde
did concede there was some evidence for proprioception (51,52). In the
1930's, extensive work was done by Irvine and Ludvigh in which they
claimed that there was neither a conscious nor an unconscious pro-
prioceptive sense (53). They also stated that proprioception played
no part in projection or motion interpretation. Later work by Ludvigh
in 1952 substantiated his conclusions concerning the absence of a cons-
cious position sense but he postulated a system with parametric feed-
back operating at an unconscious level (54,55).

At the same time the above-mentioned investigators were at work trying to support Helmholtz's "outflow" theory of position sense, others were conducting experiments whose results tended to dispute Helmholtz's contentions. One of the first persons to do so was William James (56). He hypothesized that signals from an eye which has moved contain position information and he used this to explain why a person with one paralyzed eye perceives target motion when he tries to move his eye toward an object located in a part of the field to which he has difficulty moving his eye. Since his good eye is being covered, signals from it, according to James, account for the apparent motion and inability to correctly point to the object (subject always points past the object). The reason is that the good eye has moved a greater distance from the center line than would have been necessary in an effort to move the paralyzed eye. Thus, the object seems further to the side than it actually is. Later investigations proved that positions taken up by the sound eye were not related to the false projection. The most powerful arguments to the "outflow" theory, if judged by the impact they had on future study, were those of Sherrington and later, Tozer (57,58,59,60, 61,62). These experiments, published from 1893 to 1918, presented evidence which supposedly verified the existence of a position sense which enabled very accurate direction of gaze in total darkness. Another experiment which Sherrington designed consisted of two sets of three dots on a wall in front of a subject. Each set consisted of the three dots arranged in a vertical line. One set was directly in front of the observer and the other was up and to the right of the straight-ahead position. When the subject directed his gaze straight ahead to the

first set of vertical dots he perceived them just as they were; a ver-
tical array of three dots. He knew this by the fact that their images
fell on parts of each retina which, experience had taught him, repre-
sented verticality. If the subject then directed his eyes up and to
the right to the other set of dots he received the correct impression
that they were a set of three dots arranged in a vertical line and situ-
ated up and to the right of the first set. It was shown that, due to
torsional movements, the images no longer fell on the same receptors
in the retina as they did in the previous case. The images actually
fell on areas which lie obliquely to the original areas. This should
have created the impression that the dots lie on an oblique line from
the vertical. Sherrington postulated that proprioceptive position in-
formation was responsible for the correct impression of the dot place-
ment. This is an over-simplification. As Irvine and Ludvigh pointed
out, just because retinal receptors, which should produce a sense of
the oblique, are stimulated there is no reason to suppose that they in
fact do so or, that there exists another mechanism which alters their
message (53). The factor of experience can modify perception greatly
under varying conditions. As is pointed out in Adler's text, a dinner
plate viewed from an angle is still perceived as being round even though
the retinal image is that of an ellipse (39).

More recently Cooper, Daniel, and Whitteridge have, by a thorough
histologic examination, proven the existence of proprioceptive fibers
in the extra-ocular muscles (63,64,65,66,67). They found many spiral
nerve endings in these muscles. The signals from these endings were
traced to MN-V, the superior cerebellar peduncle, the ventral tegumental

region, the central tegumental tract, the superior colliculus, and the medial longitudinal fasticulus. They showed that proprioceptive sense was involved with all eye movements and that the brain stem records all eye motion. It is postulated that this proprioceptive feedback plays an important part in control of efferent nerve fibers. They also traced the impulses back to the brain stem and found that the high-threshold receptors sent back signals via cranial nerves III, IV, and VI, while the low-threshold receptors used cranial nerve V. They showed conclusively the existence of an unconscious proprioceptive feedback loop which plays a part in eye movement control. Graphs, which plotted proprioceptive impulse frequency as a function of time, showed that the signal frequency was proportional to the first derivative of the input signals to the muscles. Also found was a continuous signal which was related to stretch. In 1955, Wolter found six different types of sensory endings in the extra-ocular muscles (68). Thus, while the "inflow" theory of Sherrington seems to be disputed by more recent work and the current feelings favor Helmholtz's "outflow" theory, the presence of the anatomical apparatus and the physiological signals for proprioception increase speculation on the purpose of proprioception in the extra-ocular muscles. In 1952, Ludvigh presented a possible system which would account for present observations (55). As can be seen in Fig. 1.8, the proprioceptive feedback takes place at a level which is unperceived in agreement with the accepted "outflow" theory's assertion that there is no conscious position sense in the extra-ocular system. This feedback does exist however, and does exert some effect on the innervating signals to the extra-ocular muscles.
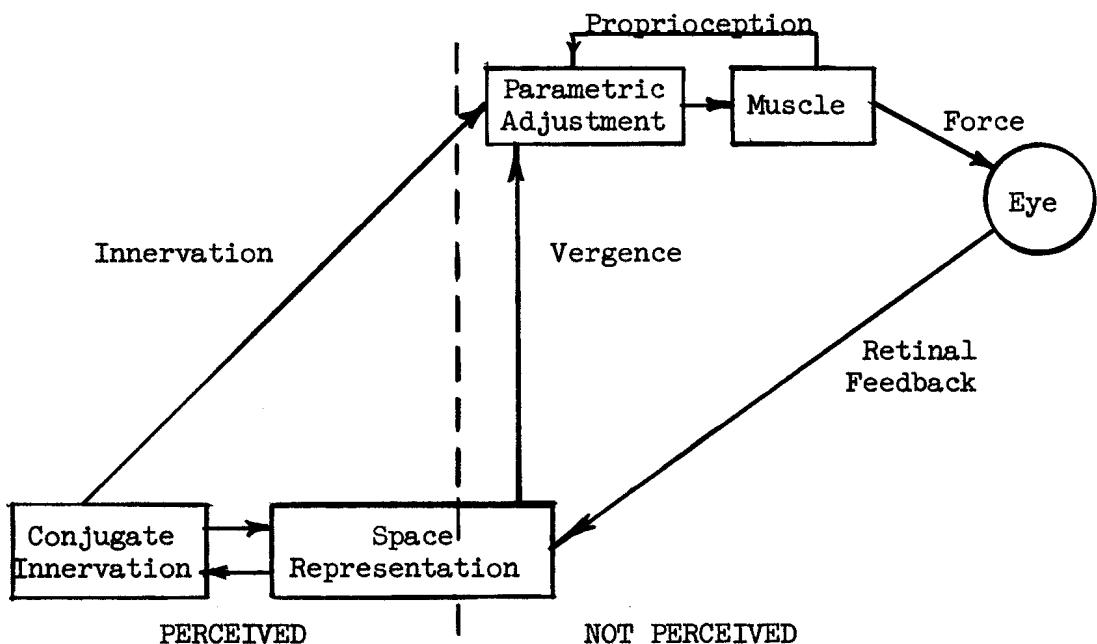
Fig. 1.8 Ludvigh's system (55)

At present, it is generally agreed that the extra-ocular muscles
have a well-developed proprioceptive stretch reflex system with muscle
spindles and afferent nerves running in the III, IV, and V cranial
nerves. The fibers go to the MN-V and from there signals have been
traced to the cerebellum and thalamus (69). The arguments concerning
conscious proprioception are, in a sense, irrelevant to this type of
proprioceptive feedback where the highest level reached is the thalamus.
Further discussion of this topic can be found in Christman's paper and
in Whitteridge's chapter in the Handbook of Physiology (70,44).
Fig. 1.9 shows, in a schematic way, the main neural pathways involved
in horizontal tracking.

1.4 GENETICS (71,72)

Congenital nystagmus is a condition with well-established heredi-
tary characteristics but unknown cause. It probably arises from cen-
tral disorders in the vestibulo-oculomotor apparatus (71). The actual
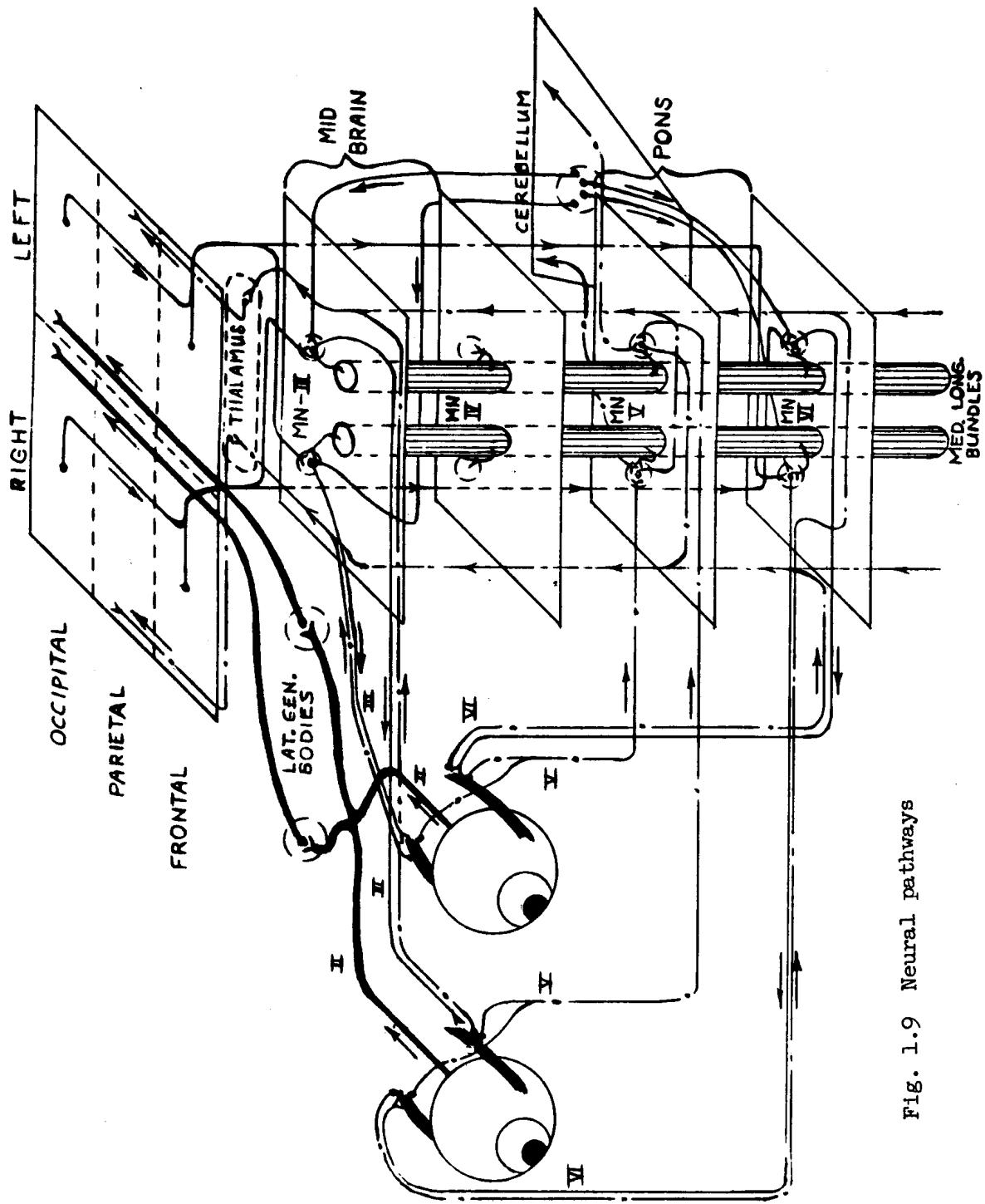
Fig. 1.9 Neural pathways
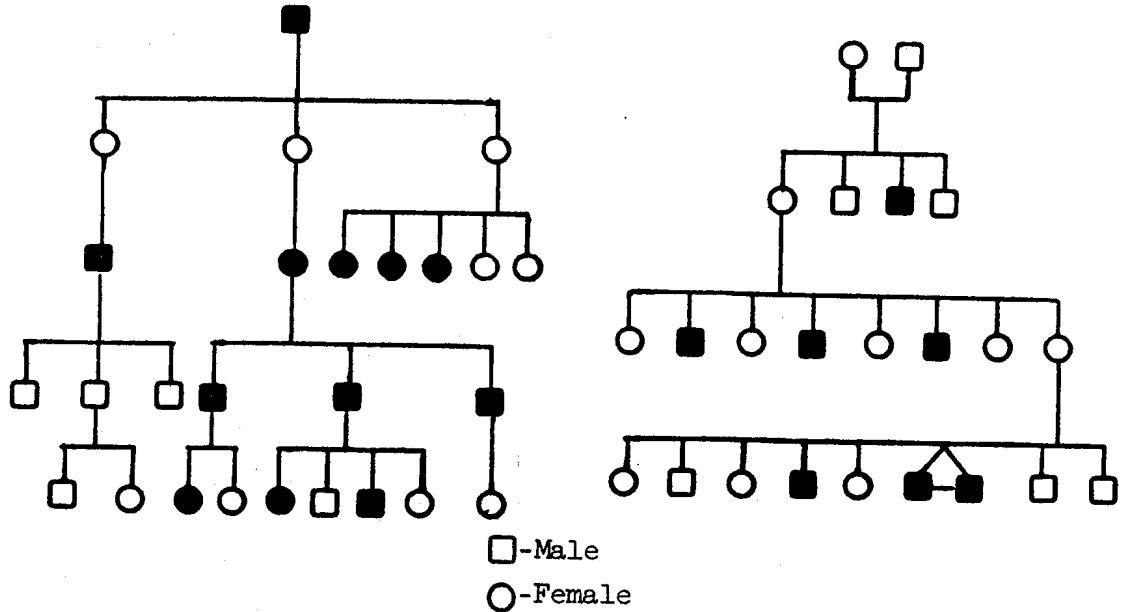
manifestation of the disease varies considerably from one case to another. It could be an imperceptible eye tremor or a large, rapid oscillation which makes vision extremely difficult. It may be unilateral, although in most cases it's bilateral. Amblyopia may or may not occur simultaneously and nystagmus may be the result of other diseases or the subject may be completely normal except for the nystagmus. However, the hereditary element is strong and there are several types of transmission.

(1) Ambi-sexual, irregularly dominant: This affects both sexes with a predominance in the male. In about one half of the cases, head movements occur while pigmentary defects occur only in rare cases.

(2) Recessive, sex-linked, male-limited and gynephoric: In this, a smaller group, head movements are less common but other defects are more common. These include color-blindness, pigmentary failure or optic atrophy.

(3) Simple recessive: This very rare form occurs in one generation, frequently where there is consanguinity in the parentage. It may be unilateral, and the subject may have normal color vision, ocular pigmentation and a normal fovea. This suggests a non-ocular, central origin. Head oscillations may also be present. Fig. 1.10 shows some histories of types (1) and (2).
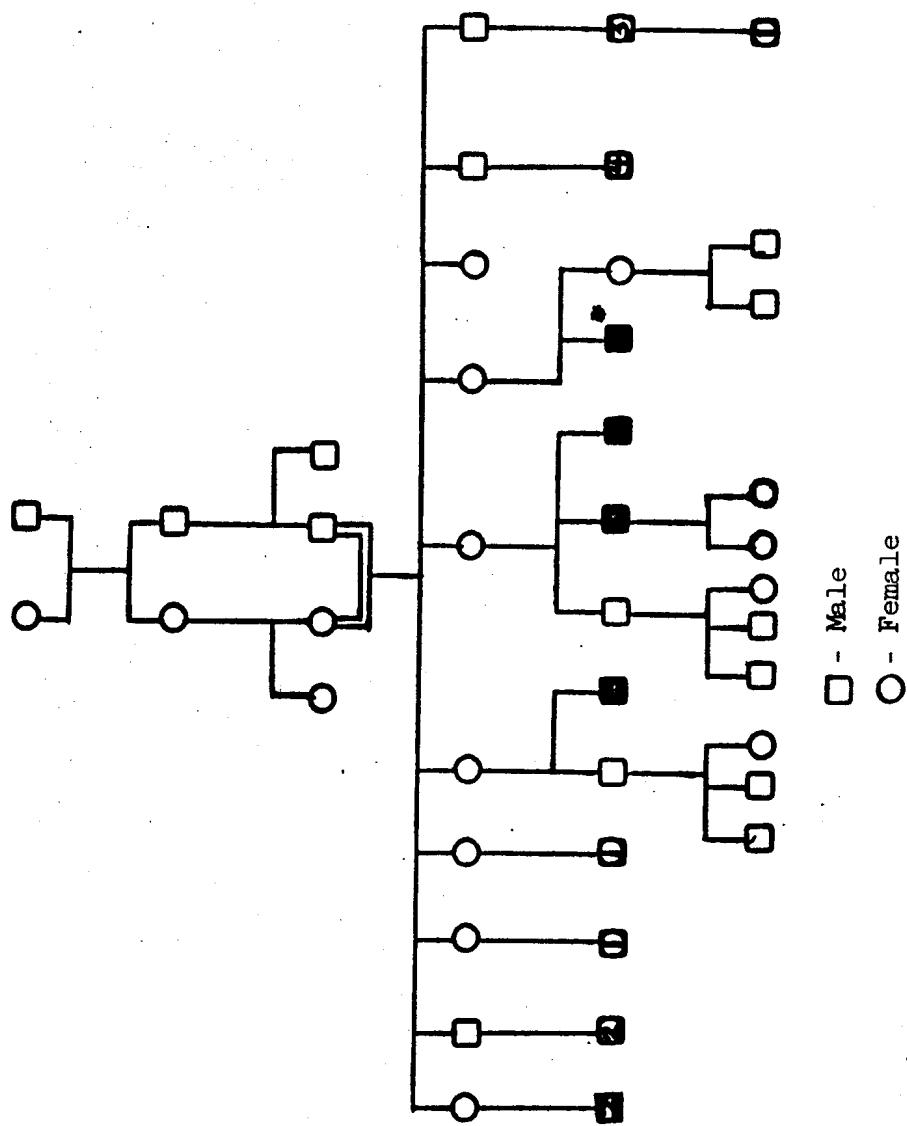
The genetic history of the subject tested is shown in Fig. 1.11. Since only one generation has been afflicted with nystagmus as of the date of this paper, it is difficult to properly identify the type of genetic transmission involved. It has only appeared in male offspring of three sisters and therefore, it may be the recessive, sex-linked,

(1) Ambi-sexual dominant   (2) Sex-limited
Fig. 1.10 Genetic histories of nystagmus

male-limited and gynephoric type or, it may even be the simple reces-

sive type (note the presence of a consanguineous marriage in the

parentage).

This is a sex-linked transmission. The most probable offspring

to be affected would be the male offspring of the subject's sister.

To date, she has had two male offspring and both are unaffected. It

will probably take one more generation to accurately classify the

type of transmission involved.

* - Subject

Fig. 1.11 Subject's genetic history

☐ - Male

○ - Female

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CHAPTER II

OCULOGRAPHY AND PHYSIOLOGICAL DATA

2.1 METHODS OF OCULOGRAPHY

Many methods have been developed and used to measure eye position

and velocity. The work done in this area will be briefly discussed

and those interested in a more detailed history of this area are re-

ferred to the works of Young, Carmichael and Dearborn, and Shackel

(12,74,75,76). Historically, eye movement monitoring has developed as

follows:

(a) The first method, which is still in use clinically, is that

of direct observation of the subject's eyes by the investigator. This

is usually sufficient to detect any gross abnormalities of the eye

tracking system. This method is a subjective one and one which yields

no permanent quantitative record of eye movement.

(b) The early work in this field was done using after-images pro-

duced by bright light (77,78). A regularly flashing light leaves a

series of after-images on the retina and the subject reports their

number and spacing. This requires an experienced subject and was a

very subjective method. Again, no permanent record resulted.

(c) To satisfy the need for permanent records, several mechanical

methods were devised. One typical mechanism consisted of a rod attached

to the eye using a plaster of Paris ring. This rod moved levers which

recorded motion on the smoked paper of a rotating kymograph (79).

Later work used pressure transducers and membranes placed against the

closed eyelid of one eye as the other eye viewed the target. The cor-
neal bulge activated these transducers as the eyes moved (80). Mechan-
ical methods succeeded in providing permanent records. However, they
did interfere with normal eye motion and thus caused changes in eye
response.

(d) Photography eliminated the mechanical interference associated
with mechanical transducers and provided permanent records of eye
motion (81). From total eye photography emerged methods employing
the photographing of tiny bright objects attached to the cornea.
Those used were a small flake of chinese white and small, bright silver
beads (15,82,6). These methods consumed large quantities of film and
required very rigid head and camera mounting.

(e) An offshoot of direct photography is the corneal reflection
method. This technique is based upon the fact that the cornea is
actually a bulge on an otherwise spherical shape. Thus it will reflect
light, focused on a spot on the cornea, at an angle from its normal at
that spot equal to the angle from the normal that the incident light
falls. As the eye moves, the normal to the cornea changes position
and so the reflected light beam moves also. This light beam can be
focused on photographic film for a record. The basic techniques were
constantly refined as the need for greater accuracy arose and to suit
special conditions (83,84,85,86). Unfortunately, the use of film for
a record prevented real-time monitoring of data. This was changed by
a method proposed by Young and which was used in this study (12).
Corneal reflection also requires rigid head clamping to prevent errors
due to lateral motion.

(f) To increase sensitivity of the corneal reflection method a contact lens with a tiny mirror mounted on it was used (87). This method can record eye movements of less than 10$''$ arc. Lamps have also been used in place of mirrors (88). Here, the light is passed through two knife edges and falls on the surface of a photomultiplier whose output voltage is an indication of eye position. These methods involve changing the dynamics of the eye and the results are subject to errors due to slippage of the contact lens.

(g) Work in 1922 showed that eye movement had associated electrical changes (89). It was recently however, 1936, that the electronics were developed to measure and record these changes (90). Electrodes were placed on the skin around the eyes and a potential measured between them. This potential is the result of the potential difference between the retina and cornea which sets up an electrostatic field which rotates with the eye. Due to the higher metabolic rate of the retina it is about 1 mv negative with respect to the cornea. EEG electrodes have been used to record these potentials and since they are fixed to the head, no artifacts due to head motion are present (91,92, 13). The disadvantages associated with this method have been non-linearity of voltage in relation to eye movement and a low signal to noise ratio due to muscle action potentials. Shielding problems have been overcome by balanced input and AC amplifiers (93,94).

(h) Another technique which is in use requires electronic photo-sensitive devices which detect the position of the limbus (the boundary between the white sclera and dark iris). This information is then sent, in the form of a voltage signal, to a recorder where a record of eye

position results. Several different techniques using this basic idea have been developed and are discussed by Young (95,9,96,97).

2.2 METHOD USED

The method used is a variation of the corneal reflection technique which was developed by Richter and used by Young in his work (98). It consists of placing two small lamps in a position which enables them to illuminate circular patches on either side of the pupil of the eye (see Fig. 2.1). At the side of each lamp is a photosensitive device which responds to the reflected light from the eye. The voltage outputs from these photosensors are a function of the angle of gaze and by taking the difference between them a voltage results which is related to the angle of gaze and which is minimally affected by non-linearities or ambient changes. The lamps used were Number 224 which were adjusted to illuminate a spot approximately 1.5 cm in diameter. This spot was mainly on the sclera with a little overlap onto the cornea when the eyes were in the straight ahead position. The photo-sensitive devices were RCA type 1P42 phototubes. Their small size and head-on cathode made them ideal for this application. One other advantage was their fast dynamic response. Young used photo resistors which had a much slower response and he recommended the use of phototubes for studies of saccadic jumps and nystagmus. The bandwidth of the 1P42 is 1 kHz compared to 10 Hz for the photoresistors. The system output is linear to $\pm 15^{\circ}$ of horizontal eye movement and the resolution is approximately $1/4^{\circ}$. The lamps were powered by a constant current power supply adjusted to supply 210 ma (Hewlett-Packard model 721A). The phototubes formed two arms of a bridge circuit whose other arms were

5 megohm resistors. The bridge output was fed to a differential pre-amplifier (Argonaut model LRA 042) whose output was recorded on one channel of a Sanborn dual channel, hot-stylus recorder. The target motion was recorded on the other channel to provide real-time corres-pondence. The lamps, phototubes and bridge resistors were all mounted on a clear piece of plexiglass through which the subject viewed the target. The subject's head was stabilized by means of a bite-bar and a forehead rest which were rigidly clamped down (see Fig. 2.2). A more detailed description of the construction of the apparatus can be found in a paper by Doescher (99).

2.3 TARGET DISPLAY

The target consisted of the horizontally moving spot of an oscil-loscope. The specific types of motion necessary were generated by a Hewlett-Packard model 202A Low Frequency Function Generator and asso-ciated circuitry where needed. The 10 cm screen was positioned 27.1 cm from the eye so that 5 cm on the screen was equivalent to a 10° deflec-tion. Thus, since the radius of the eye is approximately 1.3 cm,

$$\theta = \tan^{-1} \frac{5}{28.4} = 10^{\circ} \tag{2-1}$$

In both cases, the subjects were instructed to look at the target at all times. No instructions were given regarding anticipation of target motion.

2.4 RESPONSE TO PREDICTABLE SINUSOIDS

In an effort to compare the measuring apparatus against other methods, the responses to sinusoidal target motion were recorded for frequencies from 0.1 to 2.5 Hz. The amplitude of the motion was 20°
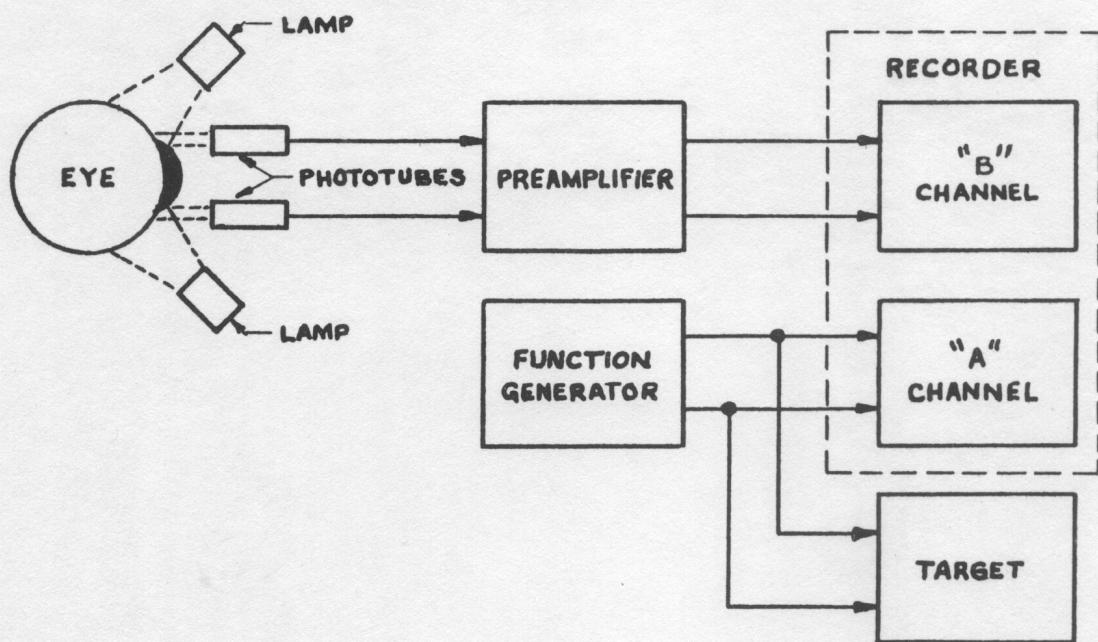
Fig. 2.1 Block diagram of tracking device
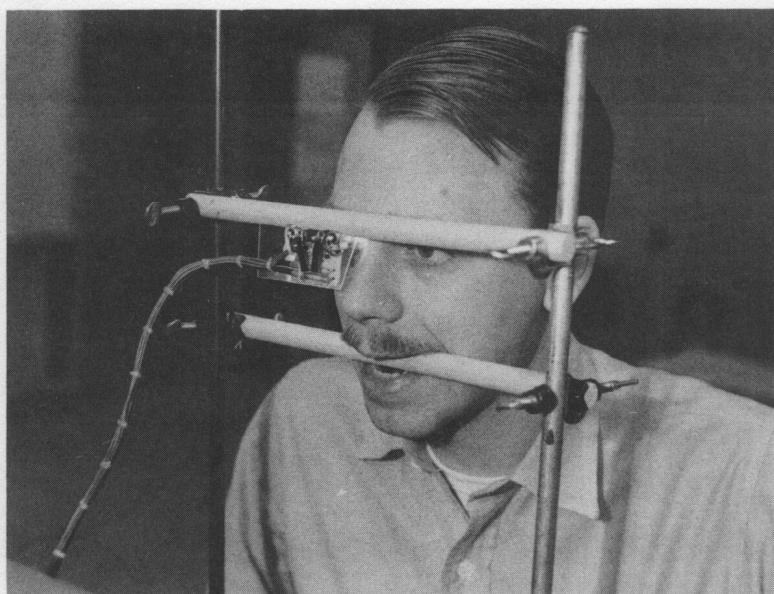
Fig. 2.2 Subject using the eye-tracking device

peak to peak ($\pm 10^{\circ}$ from straight ahead). This was done for both subjects to further provide a basis of comparison between the ability of the normal system to follow these predictable target motions and that of the subject with nystagmus. It must be pointed out that the gain and phase plots resulting do not represent the responses to unpredictable target motion which is the topic of this paper and for which a model will be postulated. Fig. 2.3 shows the gain and phase plots for the normal subject tested and the nystagmus inflicted subject. For comparison, the results obtained by Young are included.

2.5 TRANSIENT RESPONSES

The following data will illustrate the responses of both subjects to unpredictable target motion.

2.5.1 Pulse Response

The pulse in target position was generated using a step change and an R-C differentiating circuit. As can be seen in Fig. 2.4, the response to a short pulse in target position consists of an initial delay followed by a pulse whose width is greater than or equal to the delay. In the case of the subject with nystagmus, we see the 3 Hz sawtooth oscillation present, before, during and after eye movement.

2.5.2 Step Response

The step response of the human eye tracking system is shown in Fig. 2.5. Basically it is a delayed step change in eye position. At times some overshoot is noted. Again, a 3 Hz oscillation appears in the response of the subject with nystagmus.

2.5.3 Ramp Response

The response to a target that moves with constant velocity at some time t = 0 has been shown by Young to be a delayed ramp of velocity
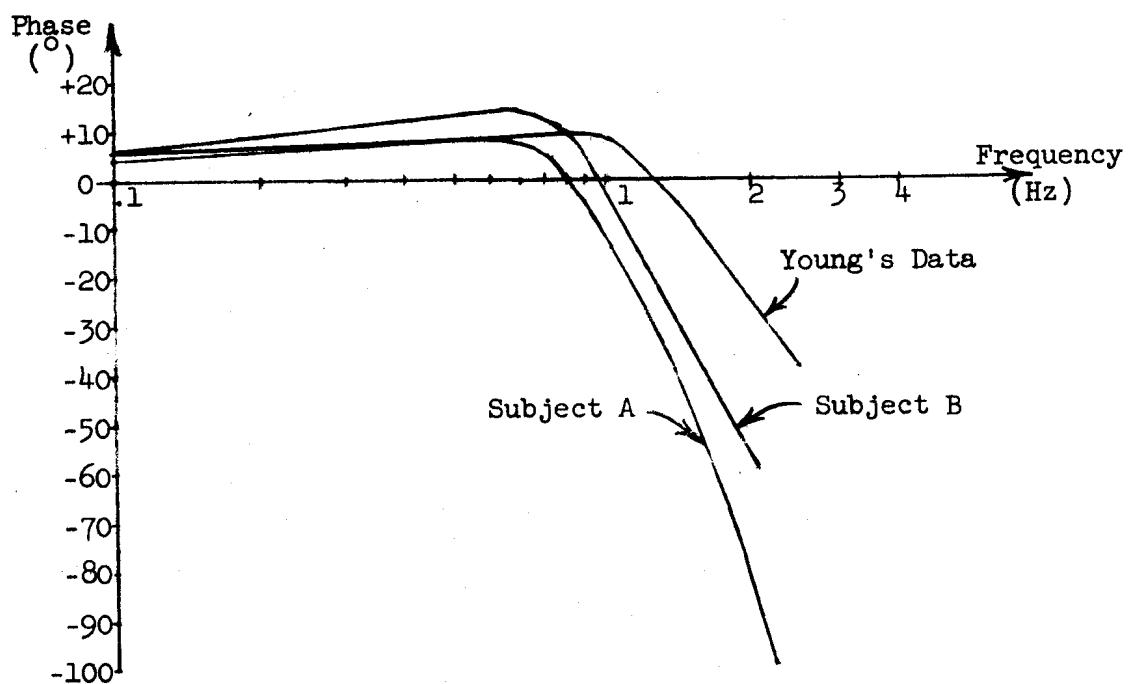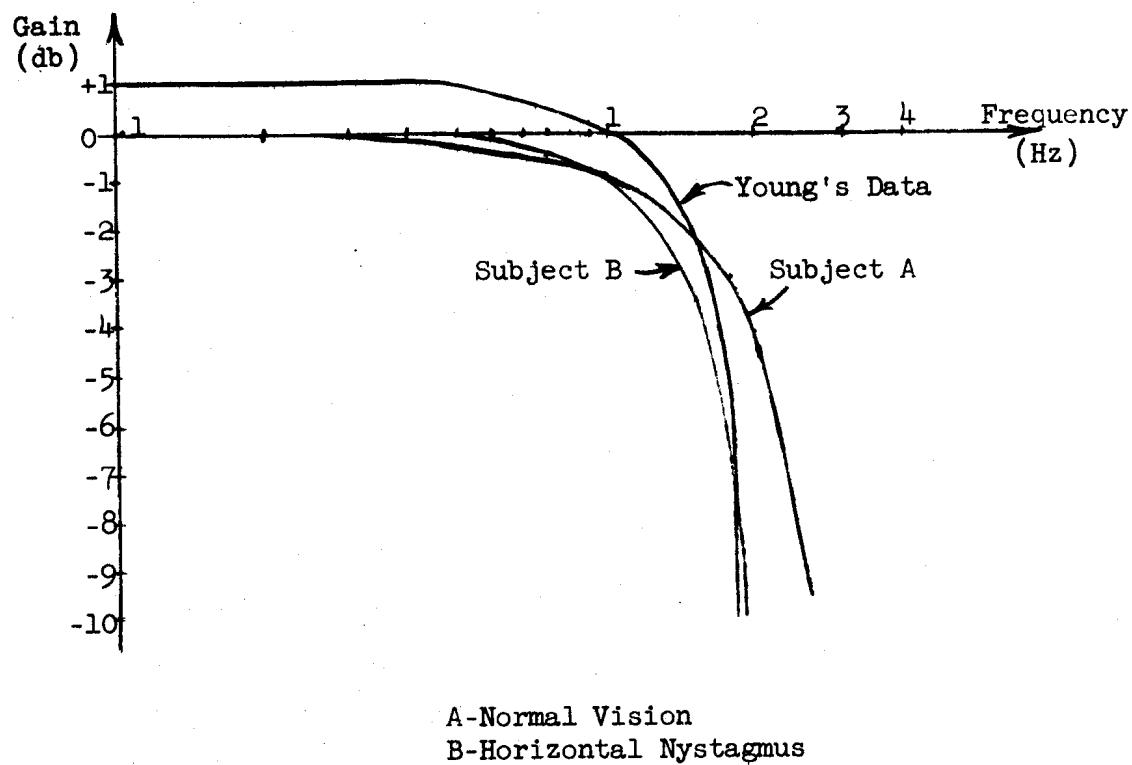
Fig. 2.3 Gain and phase plots for predictable target motion
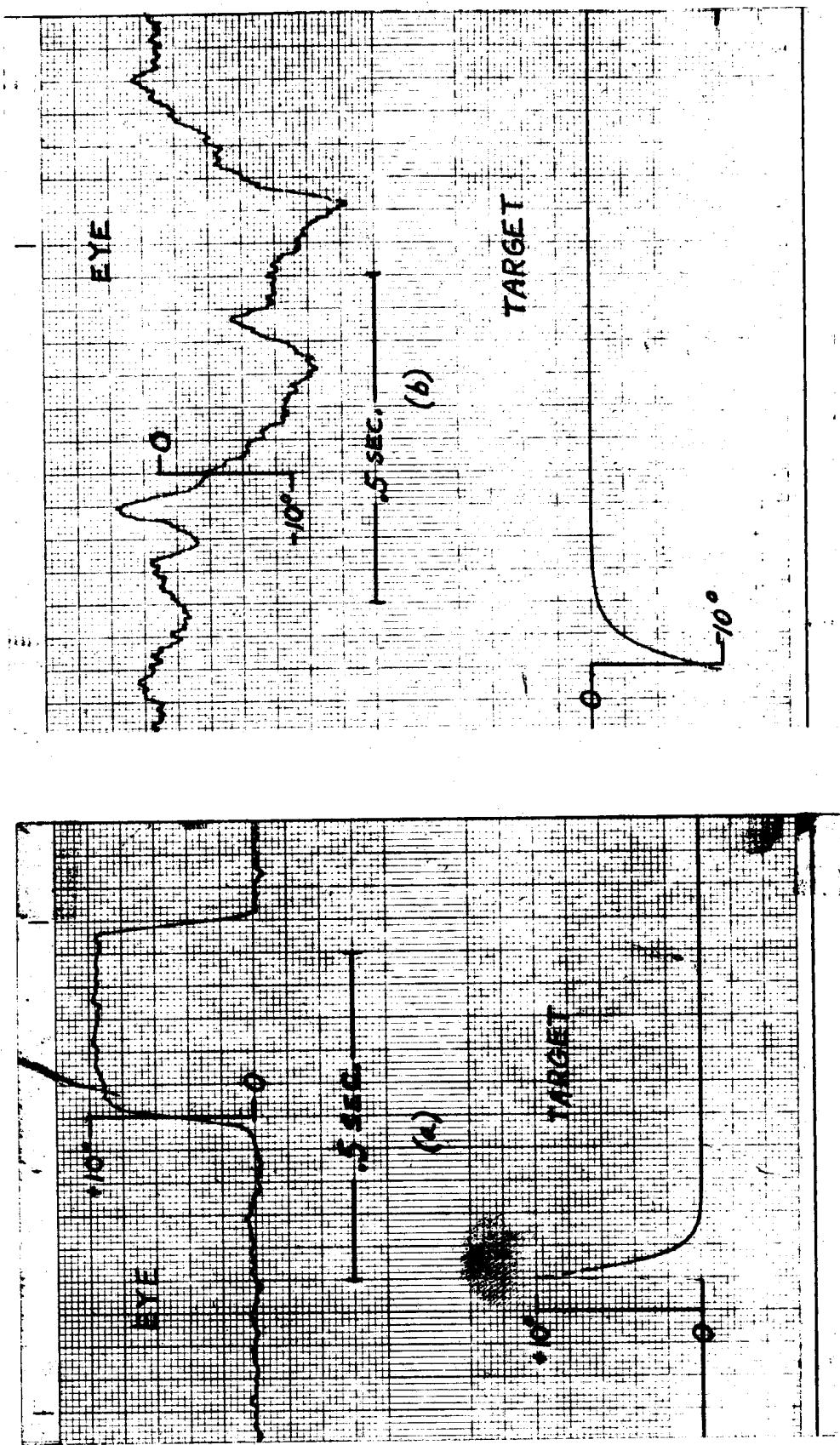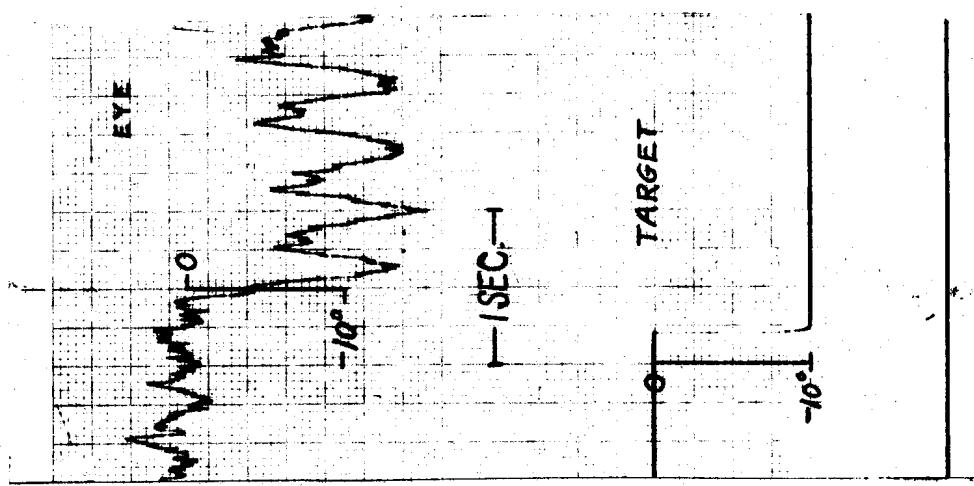
Fig. 2.4 Pulse response (a) normal (b) with nystagmus

(b)

(a)
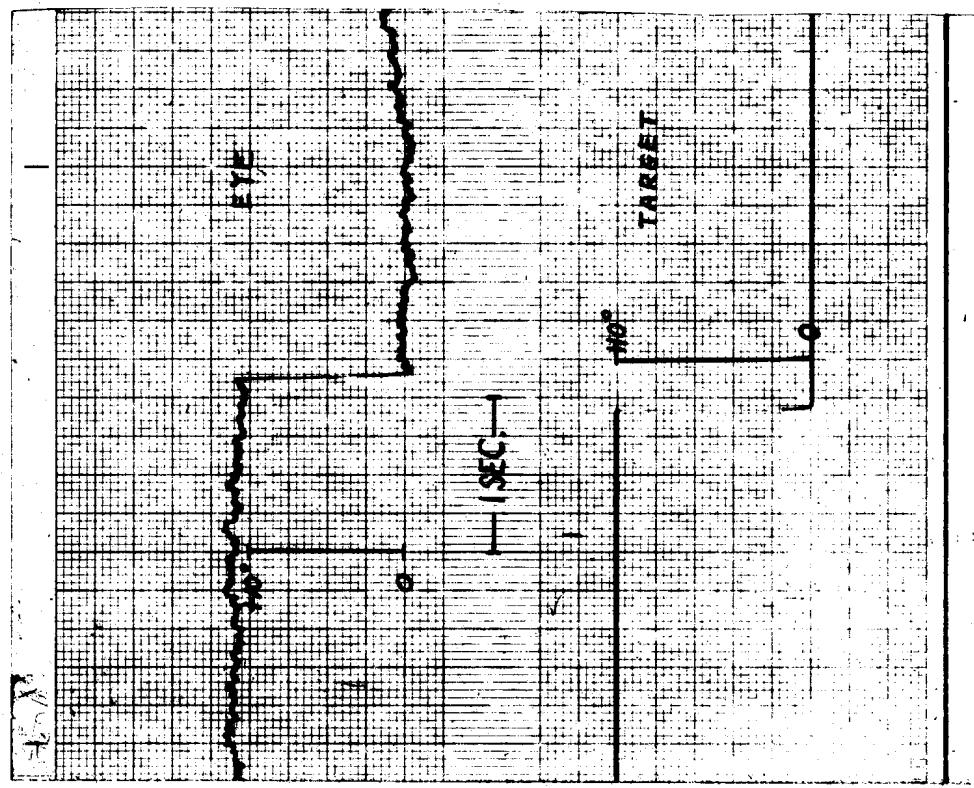
Fig. 2.5 Step response (a) normal (b) with nystagmus

equal to the target velocity with a step correction occuring approxi-

mately one refractory period after eye motion begins (see Fig. 2.6).

2.5.4 Parabolic Response

The response to a constantly accelerating target has been shown

by Young to be a series of ramps of increasing magnitude with correc-

tive step changes in position (see Fig. 2.7).

2.5.5 Responses to Various Velocities

To illustrate the difference in responses to slowly moving targets

and to those that move more rapidly, both subjects were presented with

triangular waveshapes of various frequencies from 0.1 to 3 Hz which

corresponded to velocities of from $2^{o}/sec$ to $60^{o}/sec$ respectively.

Fig. 2.8 shows the normal response to $4^{o}/sec$ and the response with nys-

tagmus to $2^{o}/sec$. Fig. 2.9 shows both responses to $10^{o}/sec$.

2.5.6 Step-Ramp Response

This target motion was reported on by Young to demonstrate the

independence of the saccadic and smooth tracking mechanisms of the eye.

It is essentially the superposition of the step response and the ramp

response. Fig. 2.10 is a drawing made from Young's data.

2.6 DISCUSSION OF DATA

It should be noted at this point that the directions of gaze to

the left and right of straight-ahead were given the arbitrary signs of

+ and - respectively with straight-ahead denoted as an angle of 0°.

This applies to the model as well as to the physiological responses.

Comparison of the gain and phase plots of the normal subject with those

of Young provides the assurance that the measuring apparatus is func-

tioning well. The only difference in the gain curves is at the low
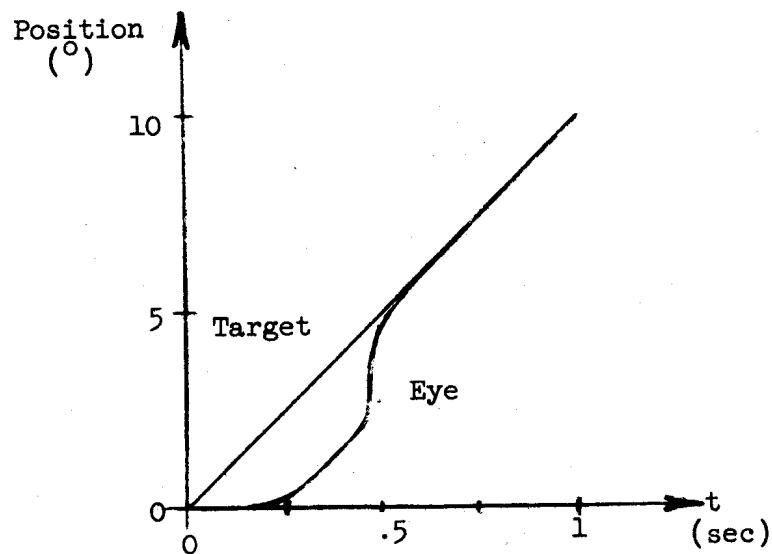
Fig. 2.6   Sketch of ramp response (from Young)
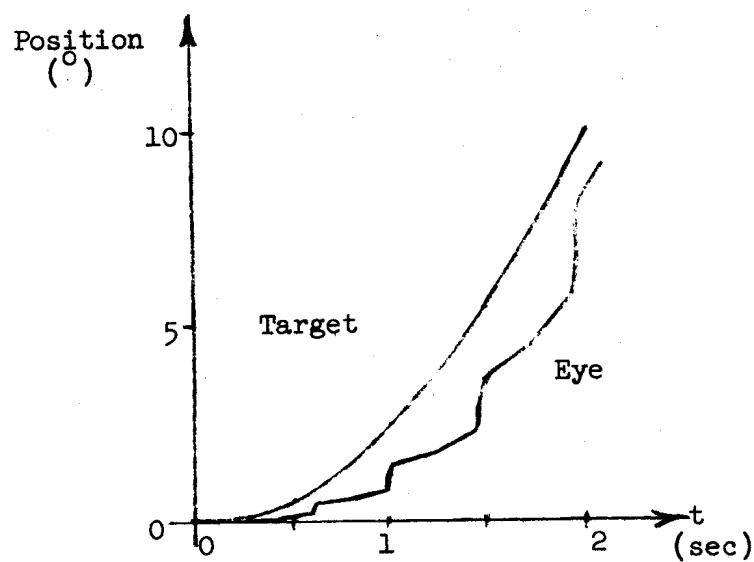
Fig. 2.7   Sketch of parabolic response (from Young),
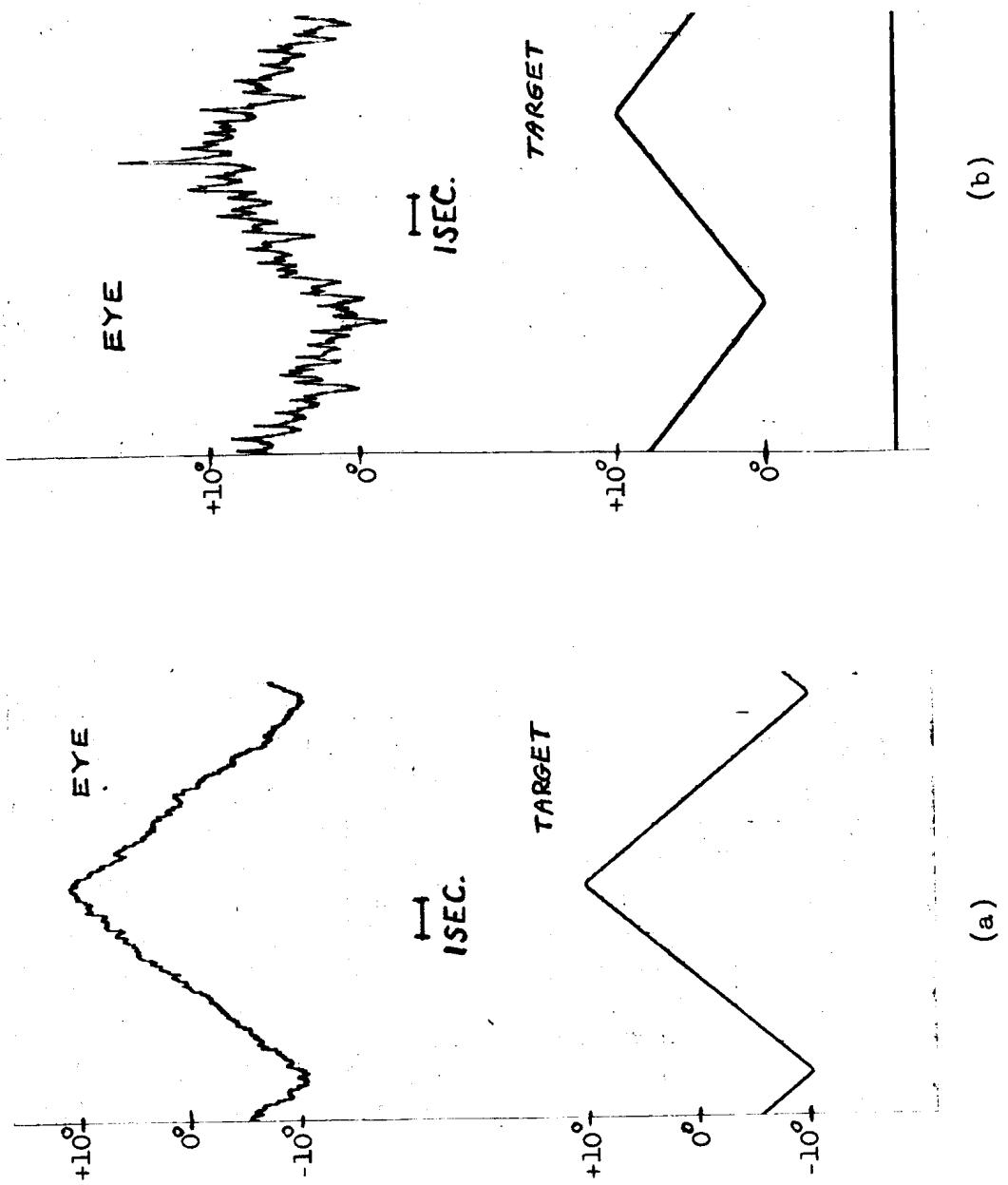
Fig. 2.8 Low velocity tracking (a) normal (4°/sec) (b) with nystagmus (2°/sec)
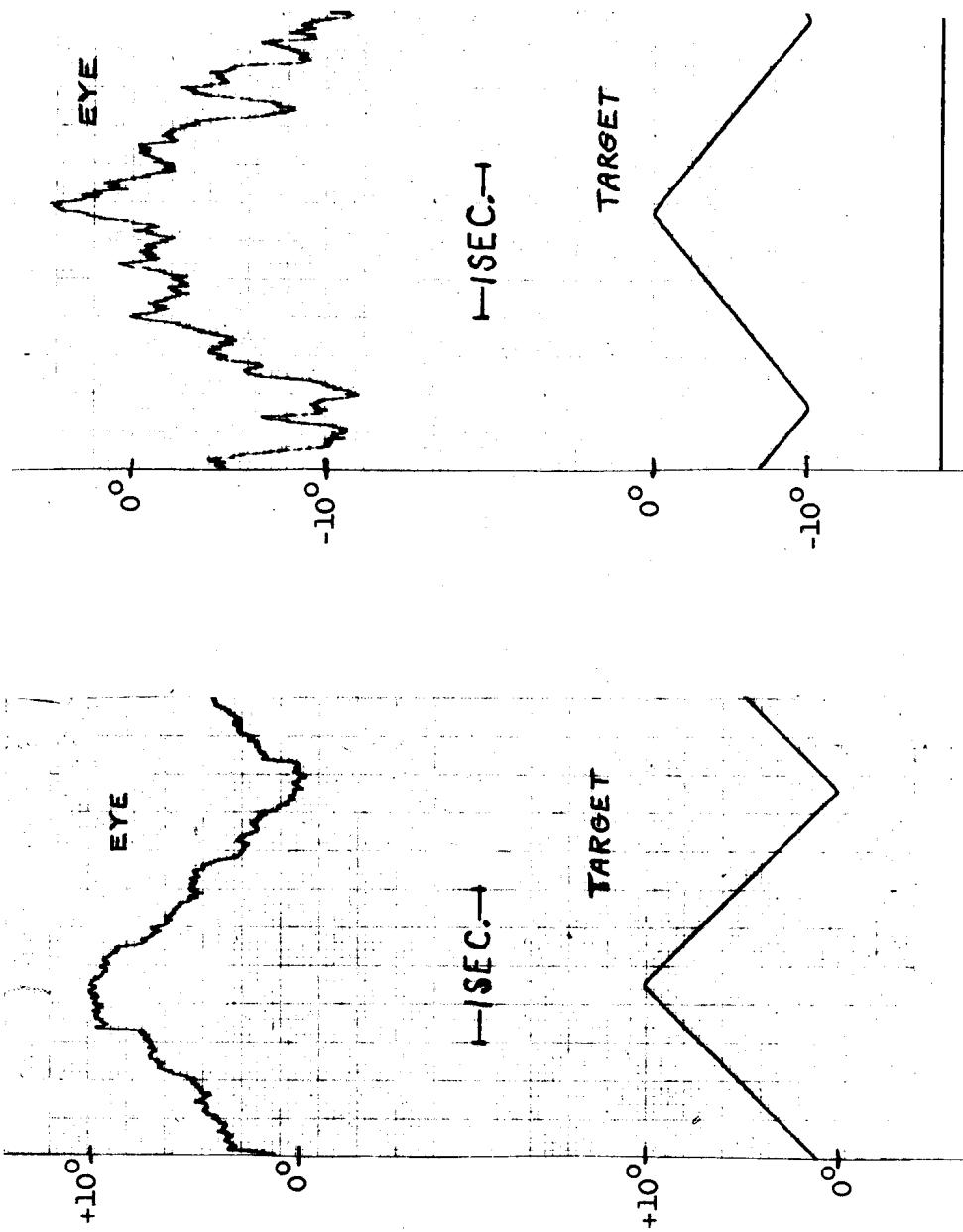
Fig. 2.9 Tracking a target moving at 10°/sec (a) normal (b) with nystagmus

frequencies where Young reported a gain greater than one and the results of this investigation show a gain of zero. This is midway between Young's data and the value of -3 presented by Fender and Nye (100). In any case, all three sets of data show the ease of tracking low-frequency predictable sinusoids. The phase plots for the normal subject and from Young's data both show a phase lead at low-frequencies which reaches a peak of approximately 10° at about 0.8 Hz and then drops sharply. This is the result of a predictive mechanism which is causing the eyes to actually lead the target. It works on past information and expected repetition. The response to unpredictable target motion obviously cannot make use of this mechanism and the gain and phase plots obtained by Young using such target motion reflect this. Fig. 2.3 also shows that the responses of the subject with nystagmus are quite indistinguishable from normal responses. This fact supports the author's hypothesis that both systems are equivalent in response with the only difference being the 3 Hz sawtooth oscillation on the nystagmus response.

The response to a pulse change in eye position illustrates the lack of control present in the system once a decision has been made. The eye sees the target returning to its initial position long before the eye has moved and yet the eye does move to where the target was but is no longer. It can return only when one refractory period has elapsed. The response of the subject with nystagmus shows how the amplitude of this 3 Hz sawtooth varies with position. At the 0° position it is smaller than at the -10° (10° to the right) position. For both subjects it was found that the delay varied from 150 to 250 msec

and the pulse width, which was slightly larger, varied from 200 to 300 msec. The term "delay" is taken to mean the time from the start of target motion to that point when the eye has reached 50% of its final value and the term "pulse width" is that period of time between the 50% point on the leading edge of the pulse and the 50% point on the trailing edge. The rise-time varied from 15 to 25 msec. Here, "rise-time" is that period of time in which the eye goes from 10% to 90% of its final value. These figures are for target pulses whose duration is substantially less than one refractory period (about 200 msec). The response to long pulses is merely the superposition of the responses to two step changes in target position.

The response to a step change in target position is a step change in eye position which is delayed approximately 150-250 msec. The movement of the eye appears to be similar to a critically damped second-order system. The rise-time exhibited by the eye was 15-25 msec as would be expected based on the data given for a pulse response. The nystagmus response shows clearly the variation of the amplitude of the oscillation with eye position. The general shape of the sawtooth is also apparent from this trace. The so called "quick phase" (the direction in which the eye moves fastest) is to the left in both gaze positions.

The eye tracks a target that moves with a constant velocity, which starts at some unpredictable point in time, by assuming that velocity after a delay of 150-250 msec. This is followed by a saccadic change in eye position, which occurs approximately one refractory period after the onset of eye motion, to correct for the position error caused by the delay in the onset of eye motion.

A parabolic input, which is equivalent to a constantly acceler-
ating target, produces a response which is a series of ramps (constant
velocity segments) with corrective saccades interspaced among them.
This type of response will always lag the target since it results
from velocity commands which were made one refractory period before.
These velocity segments change in magnitude approximately every 200
msec.

As the responses of the subject with nystagmus to constant velocity
targets show, the presence of the oscillation masks the corrective
saccades. They are clearly visible in the normal response to a target
moving at 10°/sec. However, a comparison of the shape of the nystag-
mus response to this input with the response to a 2°/sec velocity,
where no saccades are present, brings out the fact that they are pres-
ent in the former response. Again, the quick phase is to the left
and amplitude increases as the gaze deviates from the straight-ahead
position.

If a target has a step change in position in one direction and a
constant velocity in the other, at some point in time (a step-ramp)
the eye will, after a delay of 150-250 msec, exhibit a typical step
response superimposed on a typical ramp response. Thus, there is no
interaction of the two. This establishes the fact that the two systems,
saccadic and pursuit, are functionally independent. The saccadic
mechanism responds to target position and the pursuit mechanism re-
sponds to target velocity.

Since it was found that the amplitude of the nystagmus was a
function of eye position, data were taken with the subject focusing on

stationary targets placed at points from +12 to -12 degrees. Fig. 2.11 illustrates the peak to peak magnitude of the oscillation as a function of eye position. Measurement of the slopes involved in the nystagmus showed the motion to the left to be approximately twice as fast as the motion to the right. Thus, assuming that the nystagmus is an asymmetrical triangular wave, the slope of motion to the left would be twice that to the right. The period was found to vary between 250 and 400 msec depending upon factors such as fatigue, eye irritation, anxiety, etc. During the tests run, the frequency most prevalent was approximately 3 Hz. No correlation between frequency and eye position was apparent. The actual wave shape consisted of a sharp movement to the left followed (or preceded) by a slower movement to the right. This movement to the right usually had a notch somewhere in it. The size and/or position of this notch appeared to have some dependence on eye position but the experiments conducted were not designed to study such small variations in response but rather the gross movements associated with the tracking of well-defined target motions. Thus, in this paper the nystagmus will be approximated by an asymmetrical triangular wave with no notch.
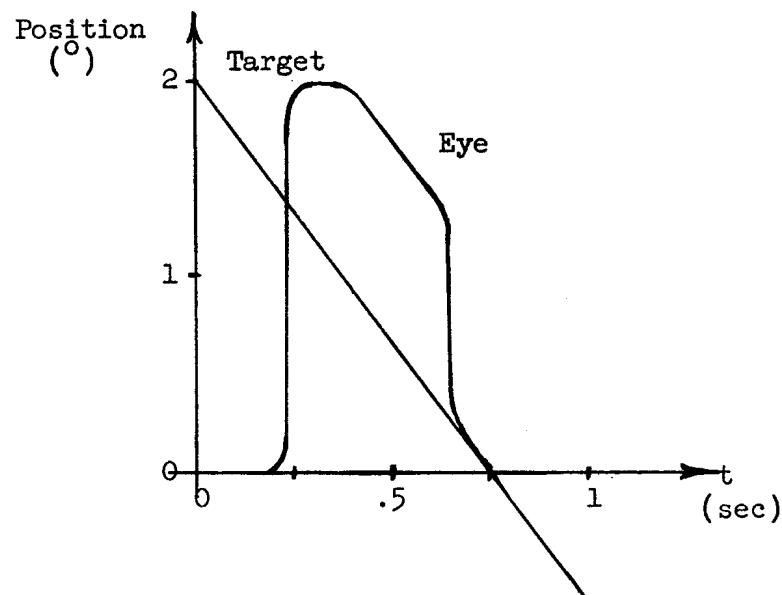
Fig. 2.10 Sketch of step-ramp response (from Young)
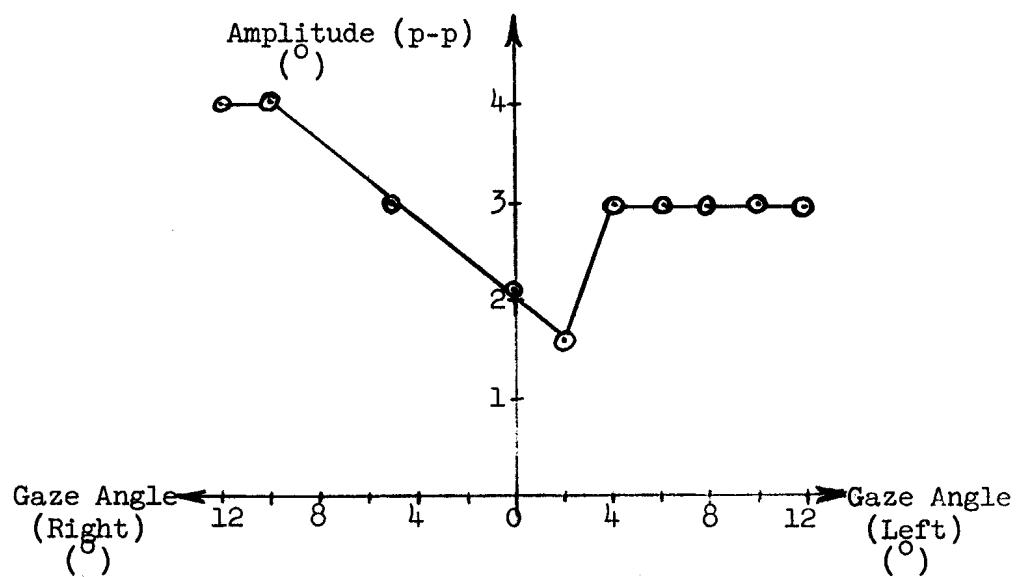
Fig. 2.11 Positional dependence of nystagmus amplitude

CHAPTER III

MODELING THE HORIZONTAL TRACKING SYSTEM

3.1 INTRODUCTION TO MODELING

Obtaining a mathematical model for a dynamic system is an art which combines creativity and application of the scientific method. The model is an abstract entity which provides a representation of a physical phenomenon. One of the more important results of the model is the quantifying of the physiological mechanism by providing the numerical values of the parameters as well as the flow paths involved in the system under study. Obtaining the model usually involves hypothesizing, experimentation and re-hypothesizing until the model developed satisfactorily represents the mechanism within some predetermined tolerances. The only departure from the classic method of science is the extensive use of the computer in hypothesizing. This is due to the complexities involved in describing physiological systems which are highly non-linear. While the model, in its simplest sense, only represents the physiological system on an input-output basis, if it consistently conforms to experimental observations of the biological system under various dynamic situations, it is not too presumptuous to suppose that the internal mechanisms of the model may represent a close approximation to those actually involved in the physiological system. This might lead the biologist to search for the anatomical counterparts of the various functional boxes in the model.

There are two basic approaches to modeling. The first is the writing of equations for known physical processes based on a knowledge of the physical laws governing these processes. An example of this is the construction of a model for the motion of a mass on a spring subject to an initial displacement from the equilibrium position. While this is a very simple example, it should not imply that this first approach is an easy one. Many systems whose physical laws are well known still require a great deal of knowledge and creativity to simplify the configurations so that a model may be postulated. The second approach is the one usually used in modeling the biological system. This is the empirical method which relies on carefully controlled experiments which illustrate cause and effect relationships while excluding non-causal artifacts. Due to our lack of understanding of the details of the biological mechanisms their modeling must necessarily be done by this empirical method. This puts a tremendous burden on the instrumentation necessary to measure the biological responses to various inputs since most biological systems have a very low signal to noise ratio.

3.2 PROPOSED MODELS

A review of the literature reveals several models which have been proposed for the horizontal tracking system. These models reflect the different approaches taken by the investigators involved and a comparison of one with another will serve to point out the relative merits and shortcomings of each.

3.2.1 Saccadic and Smooth Tracking Models

(1) Vossius (7): By assuming that the saccadic mechanism is a proprioceptively controlled movement instead of a ballistic movement,

Vossius proposed the model shown in Fig. 3.1 for the saccadic mechanism. There are two delays in this system. The first is a brain delay of $T_1 = 130$ msec and the second a transport delay of $T_o = 5$ to 10 msec. A constant represents the transfer function of the eye muscles and the eyeball dynamics consist of its inertia and friction with a time constant $t_1 = 15$ msec. The presence of a proprioceptive position feedback and rate feedback from eye position is postulated to explain the existence of various classes of the time course of saccadic movements reported by Vossius.

(2) Fender and Nye (100): The model presented by Fender and Nye is the result of the application of linear systems analysis to the gain versus frequency response curve measured by means of a carefully constructed optical system. It is a highly schematic model which attempts to describe the major flow paths in the system. As can be seen in Fig. 3.2, three feedback paths are postulated. Evidence for the existence of these loops is presented as well as arguments to support the regenerative nature of the oculomotor loop. By comparison of observed phase lag and computed phase lag (from Bode equation) the authors show that the system is non-minimum phase. This fact is coupled with the knowledge that opening the main negative positional feedback loop leaves a system with a gain in excess of unity. The only systems which are non-minimum phase and have an overall gain in excess of unity are those with a regenerative loop. A further anal- ysis of stability criteria identifies the regenerative loop as the oculomotor feedback loop. The role of the first derivative feedback is postulated to be that of damping unstable oscillations. The value
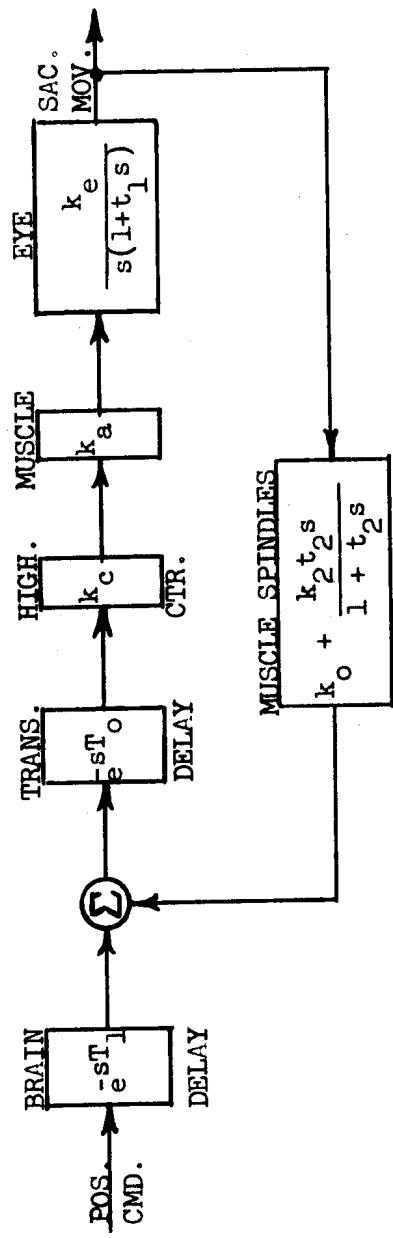
Fig. 3.1 Vossius' model for saccadic movements
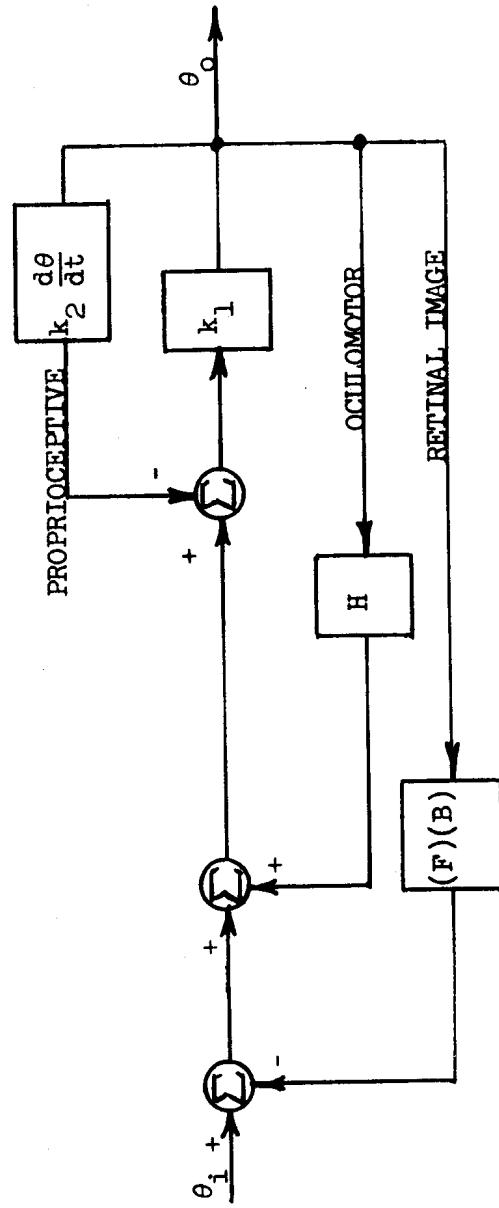
Fig. 3.2 Schematic eye movement control system from Fender and Nye

of k_2 is assumed to vary from small (during saccades) to large (during fixation). A further role postulated for this derivative feedback is that of suppressing visual information during a saccade. The merits of this model in representing the instability of the oculomotor system which causes flicks, drift and tremor are also discussed.

(3) Young (12): A model for the tracking of unpredictable target motion was postulated by Young in 1962. This model was different from any that preceded it because of the assumption that the system is a sampled-data control system instead of a continuous system. Also present are non-linear elements to account for observed physiological data. As can be seen in Fig. 3.3, there are two main forward paths. The upper path is the mechanism for smooth-tracking of moving targets. The velocity of the error is first estimated by taking the difference between the present error and that of the previous sample. This signal then passes into a limiter where it is passed on as received if it falls below a certain rate (25 to 30°/sec) and rejected completely if it is above this value. It has been experimentally determined that the eye will not track smoothly a target moving at a velocity greater than 25 to 30°/sec but will approximate it with saccadic steps. After the first limiter, the signal is integrated to obtain the desired velocity, passed through a saturation non-linearity which insures that the eyes do not move smoothly with a velocity greater than 25 to 30°/sec, and integrated once more to provide a continuous command from the pulsed data. The lower path is the saccadic positioning mode and it consists of a dead-zone of 0.2 to 0.5 degrees followed by a computing delay and an integration to provide the desired position command. It should be

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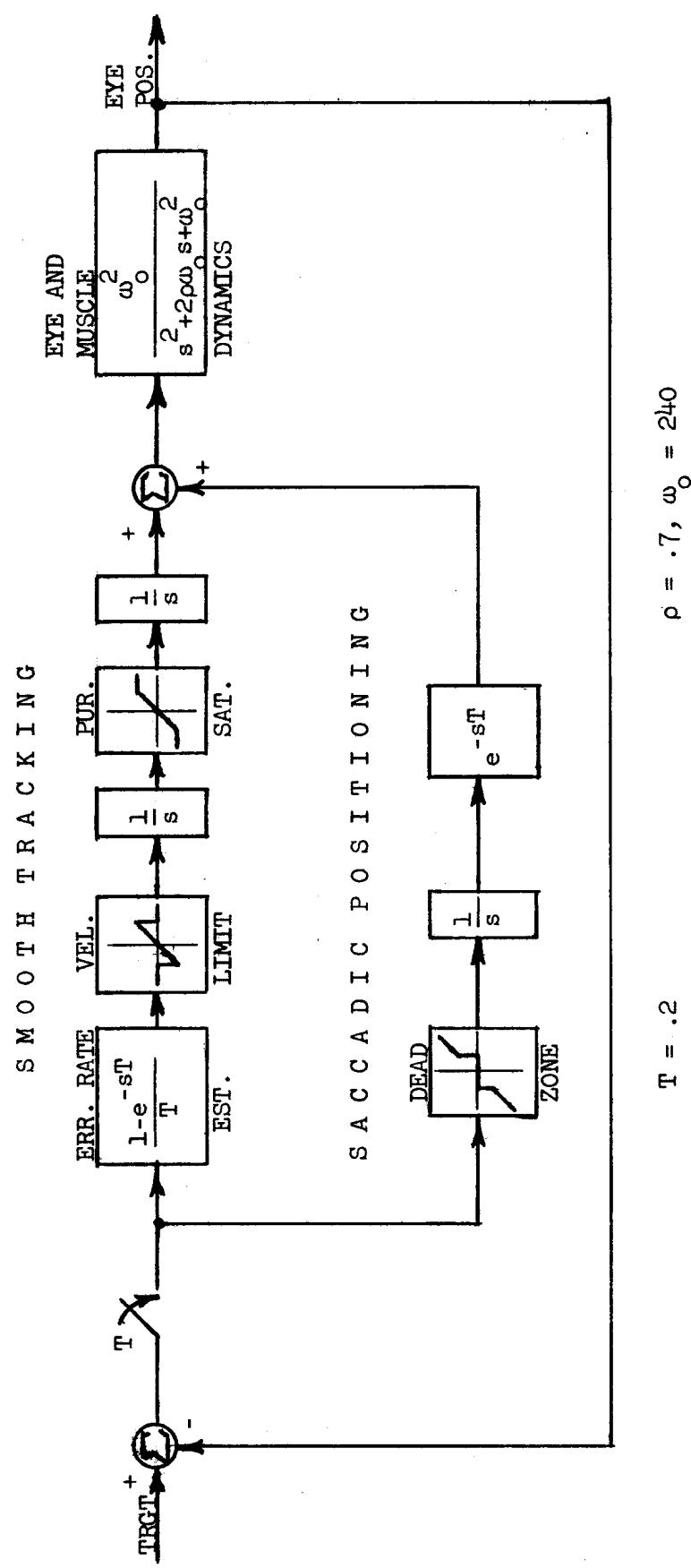
Fig. 3.3 Young's sampled-data model of the eye tracking system

$T = .2$

$\rho = .7$, $\omega_o = 240$

noted that the smooth-tracking path is actually a velocity-segment

approximation mechanism whose output is a series of ramps of different

slopes (if necessary) every sampling period. The eyeball dynamics are

taken from Westheimer's second-order approximation which has a damping

constant of 0.7 and a natural frequency of 240 radians/sec (101).

(4) Pavlidis (102): This model is a modification of Young's

model which is supposed to correct a discrepancy between the observed

saccadic response of the eye and the output which Young's smooth-

tracking system would give to a small step-change in target position.

Fig. 3.4 shows that Pavlidis' model coincides with Young's in both

the saccadic path and the eyeball dynamics. In the smooth-tracking

path the error-rate estimating element is replaced by a differentiator

which is followed by a sampler, a zero-order hold, a limiter and an

integrator. This necessitates the use of two samplers which is in

better agreement with experimental results as shown by Rashbass (9).

(5) Warnmeunde (103): The model shown in Fig. 3.5 was presented

at the 17th Annual Conference of Engineering in Medicine and Biology.

There are several differences between this model and the preceding ones.

The smooth-tracking path is continuous while the saccadic path is

sampled. The eyeball dynamics are based on work by Miller and consist

of a third-order polynomial with positive feedback (104). The smooth-

tracking path is not limited to straight line (velocity) segments and

the method of sampling in the saccadic path is quite unique in that

the dead-zone is present in the controller for the sampler.

3.2.2 Comparative Evaluations

The models presented above represent the thoughts of several

investigators, each of whom used the work of those who preceded him and

$$G(s) = \frac{\omega_o^2}{s^2 + 2\rho\omega_o s + \omega_o^2} \quad , \quad \rho = .7, \quad \omega_o = 240$$

$T_1 = T_2$ or $T_1 \neq T_2$
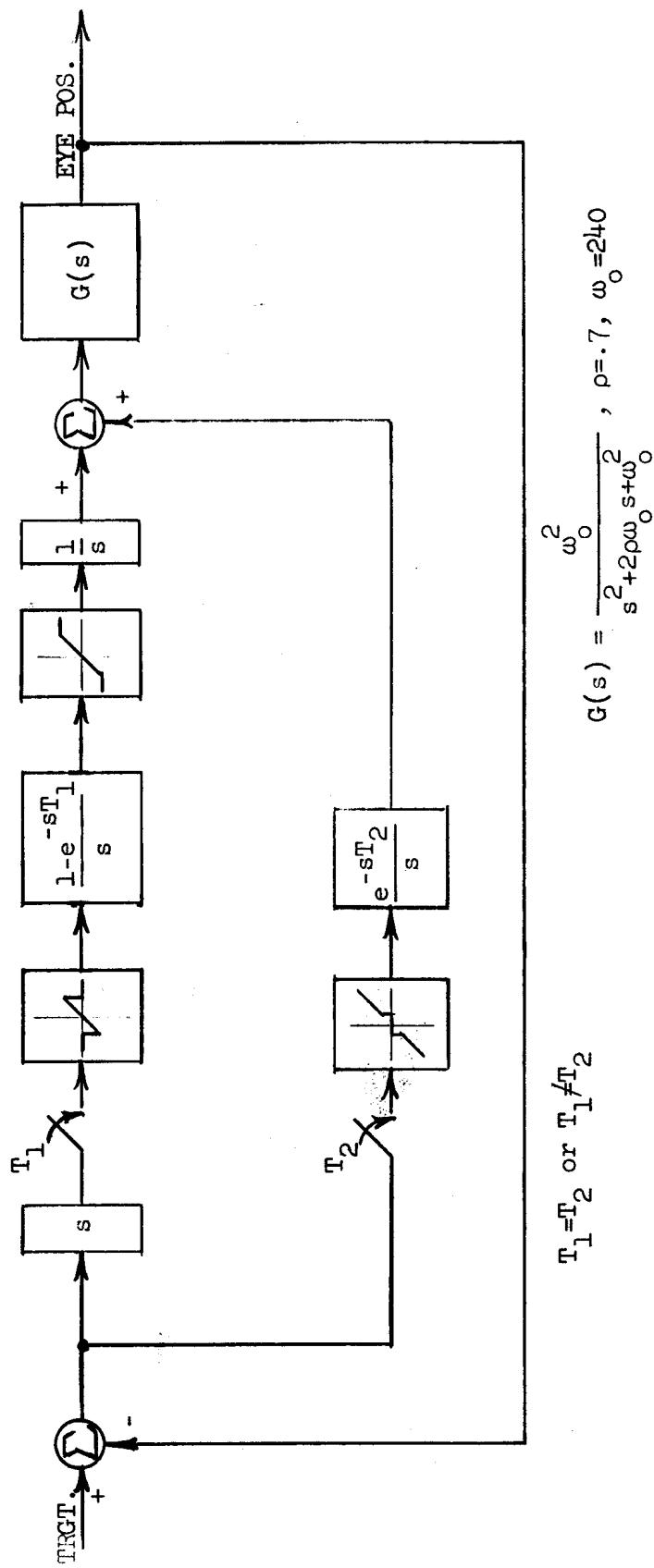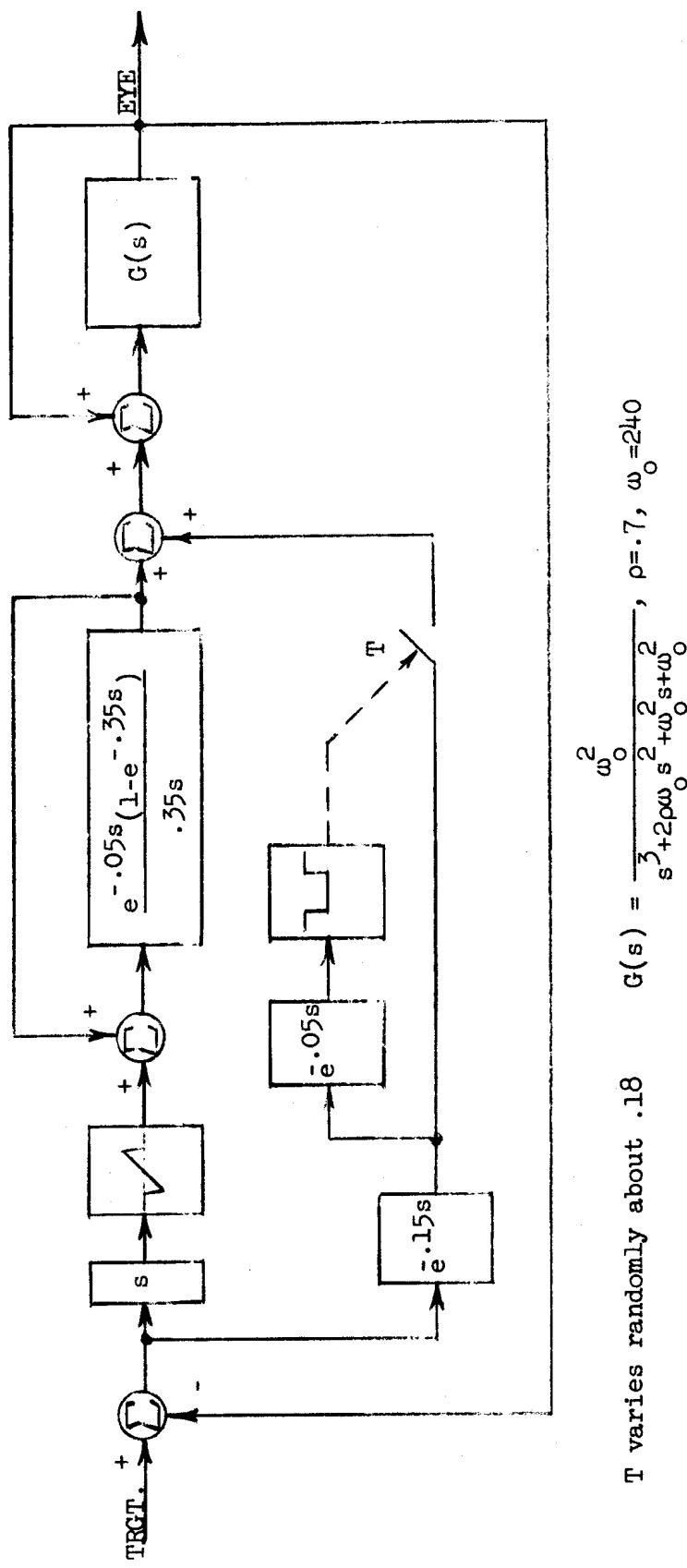
Fig. 3.4 Pavlidis' modified model of the eye tracking system

$$G(s) = \frac{\omega_o^2}{s^3 + 2\rho\omega_o s^2 + \omega_o^2 s + \omega_o^2}, \quad \rho = .7, \quad \omega_o = 240$$

T varies randomly about .18

Fig. 3.5 Warrmeunde's model of the eye tracking system

made modifications in an attempt to better match the physiological
data. Since Vossius' model is contradicted by the existing physiolo-
gical data discussed in the previous chapter, it will not be considered
further. It should be pointed out however, that while eye movements
are not primarily controlled by position feedback proprioception,
there is evidence that such feedback does exist and has some effect on
the system (see section 1.3.3 (101)). Vossius' model is linear and
thus does not represent any of the non-linearities which have been ob-
served in the biological system. The model of Fender and Nye is also
linear and has the same shortcomings. The data gathered by Fender and
Nye are applicable to tracking of predictable targets such as sine
waves and the model they proposed is better suited for this mode than
for the transient target motion with which we are concerned.

Young's model is the first to adequately represent the horizontal
tracking system in the transient mode. As can be seen in Fig. 3.3,
the non-linearities of the physiological system are represented by
piecewise linear approximations in both paths. The major advance made
by Young in modeling this system was representing it by a sampled-
data control system. The following arguments were used by Young to
show the necessity for a sampled system with independent paths for
smooth tracking and saccadic positioning:

a) The response to a narrow pulse change in target position is
a delayed pulse whose width is greater than and independent of the tar-
get pulse width. This type of response cannot be elicited from a
linear, continuous system (even with a transport delay) since super-
position does not hold.

b) The response to a constantly accelerating target (a parabolic input) consists of a series of constant velocity segments changing in slope at some fixed interval of time. This certainly suggests some form of sampling in the biological system.

c) The following gain versus frequency plot of the response to unpredictable target motion was presented in Young's thesis. It was obtained by constructing the target signal with sine waves of various frequencies (non-low-harmonically-related sinusoids) and analyzing the responses for frequency content (see Fig. 3.6). As can be seen from this plot, there is a sharp rise in gain of about 5-6 db at 2-3 Hz. This suggests a great deal of energy at a frequency with a half-period of 100-200 msec. Given an input spectrum $F(s)$, a sampled system would yield an output

$$G(s) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(s + nj\omega_0). \tag{3-1}$$

where T is the sampling period and $\omega_0 = 1/T$. This energy spectrum would have its first peak at $\omega = \omega_0/2$. Thus, if $T = 0.2$ sec, ($\omega_0 = 5$ Hz) the energy peak would occur at 2.5 Hz, which agrees with the experimental data. It was also observed experimentally that the tracking of signals with frequencies greater than 2.5-3 Hz degenerated to such an extent that it could not be considered as tracking at all. This is quite understandable when one remembers the sampling theorem which states that we lose information and are unable to reconstruct the data unless we sample at twice the highest frequency present. Thus, since we are sampling at 5 Hz, we should not be able to track any signals with frequency greater than 2.5 Hz.
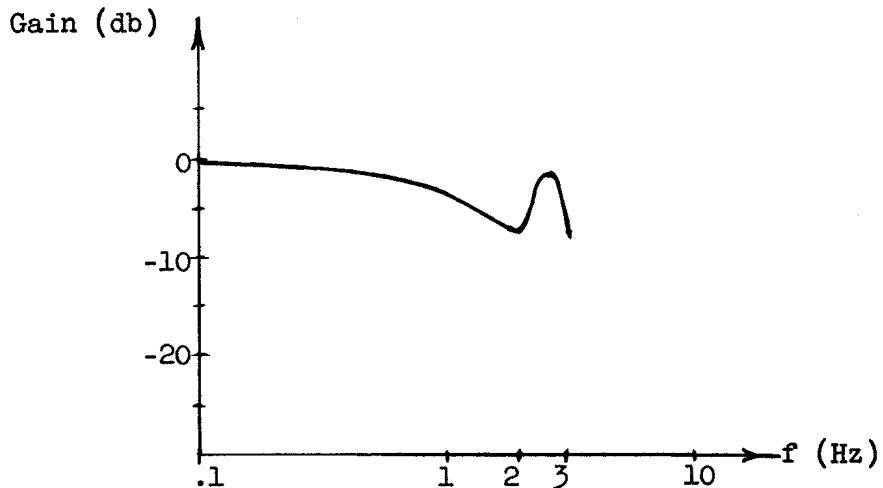
Fig. 3.6 Gain plot for unpredictable target motions (from Young)

d) Another observation in support of a sampled-data system in-
volves the phenomenon of predicting a step change in position. It was
found that if prediction occurred, the eye would move to where the sub-
ject guessed the target would be. This would happen either before
target motion or immediately after it (too soon to be one refractory
period). In the first case the subject's eyes moved to a new position
based solely on a guess and in the second, the guess was made and then,
as a result of the target motion, new visual information was presented
which could have modified the original decision. It was found that
there was no significant difference in the errors of the guess between
the two cases. Thus, the new visual information could not alter the
movement command which was made T seconds before the eye moved. This
is also true of pulse targets where, even though the target has re-
turned from a step change in position, the eye will move to where the
target was and not move back until another interval of T seconds has
elapsed.

The evidence for the independence of the two paths (smooth and
saccadic) comes from the response to a step-ramp target. In this case

the target moves with a step change in position in one direction and at the same time assumes a velocity in the opposite direction. The eye's response to this is a delayed step change in position in the proper direction immediately followed by smooth tracking in the other direction with a velocity equal to that of the target. This is later followed by a corrective step in position to correct for the error due to the initial time lag. Thus, the two paths responded independently, one to the position error and the other to the velocity of the target.

There are however, several areas of disagreement between this model and observed data. The first discrepancy lies in the behavior of the model when presented with a step input. To be consistent with the physiological system, only the saccadic positioning path should respond and the smooth tracking path should remain inactive. This was supposedly assured by the velocity limiter of 30°/sec but if one considers small step inputs of less than or equal to 6° it will be found that, in addition to the saccadic response of a step change in position, the pursuit system will act as follows:
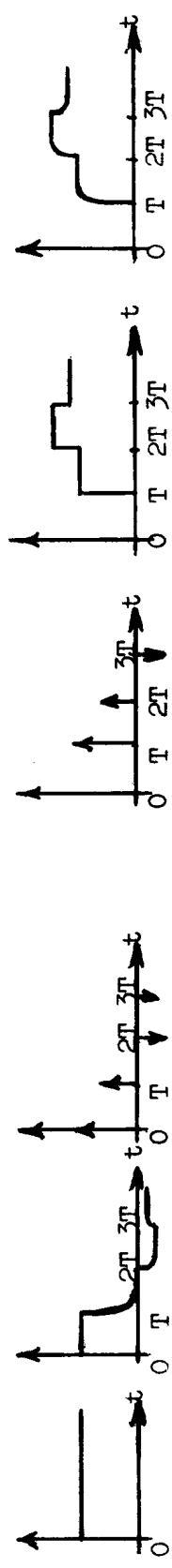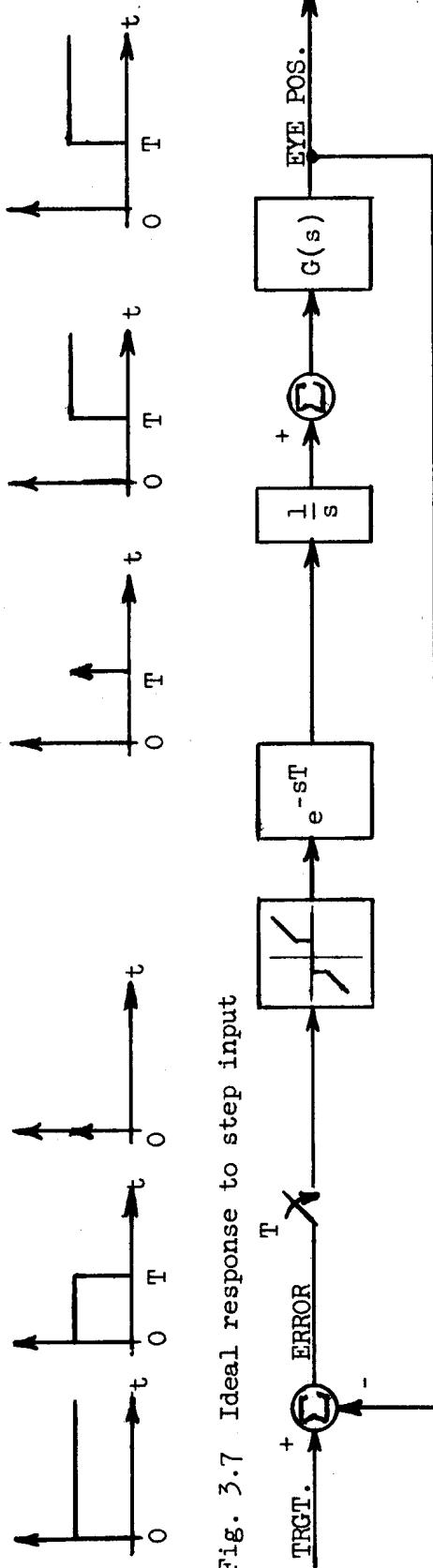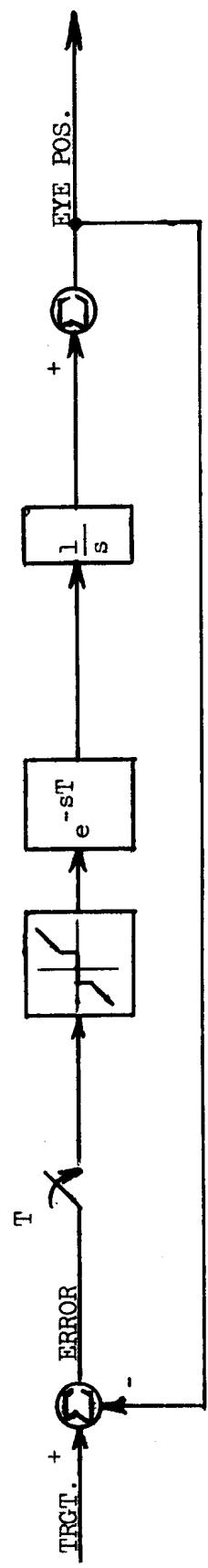
a) The input step will be sampled at t = 0 and at t = 0.2 sec and the difference will be taken between these two values in the error rate estimator (multiplied by $1/T = 5$).

b) This signal (for a 6° step, $6(5) = 30^{\circ}$/sec) will be fed into the velocity limiter and will pass through it and on to the first integrator. Thus, for steps of magnitude equal to 6° or less, the pursuit system will try to follow. This was first pointed out by Pavlidis and his model represents an attempt to correct for this.

The above error in Young's model is a minor one which could be overlooked since for larger steps the proper path is activated. There

is, unfortunately, an error in Young's model which is of much greater

magnitude. In his analysis of the model, Young neglected the effects

of the eyeball dynamics and considered only the two paths up to the

summing junction before the second-order system which is used to simu-

late eye and muscle dynamics. His reasoning was that in the interval

$T = 0.2$ sec, the output of this black box would be within $e^{-33.6}$ of

it's final value. While this is true, it is not sufficient to neglect

the effects of including this subsystem in the overall system. The

second-order dynamics exhibit a finite rise-time which affects the eye

position at a very critical time, when sampling the error.

To better understand Young's analysis and what happens when the

whole system is considered, let us examine the saccadic path when the

target is a step (see Fig. 3.7). As the plots indicate, the error is

sampled at $T = 0$ and this impulse is passed by the dead-zone, delayed

by $T = 0.2$ sec, integrated and this output becomes the eye position

(neglecting effects of eyeball dynamics). The eye position is fed

back, negatively, and the error goes to zero exactly at $t = T$. Thus,

when the sampler again closes at $t = T$, the error is zero, neglecting

the ambiguity of the instantaneous drop in the error signal at the

instant of sampling, and no further command is sent to the eye. This

mechanism is an idealization which is mathematically correct assuming

the value taken at any discontinuities is the final value and not any

value in between the initial and final values.

Now with the aid of Fig. 3.8, let's examine the same system with

the addition of the eyeball dynamics. Here we see the effects of the

finite rise time on the sampled error. At $t = nT$ the eye is always in

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Fig. 3.7 Ideal response to step input

Fig. 3.8 Actual response to step input

transit and thus the system always sees an error and will continue to hunt until the error is small enough for the dead-zone to block the signal to the sampler. This behavior is inconsistent with the observed physiological response. Thus, the model proposed by Young is an excellent mathematical model only under the assumptions of an ideally responding eye (i.e., instant response with zero rise-time) and the value at a discontinuity must always be the final value of the function at that point. Since the aim of this study is to obtain a complete working model for both the normal and nystagmus-inflicted systems, modifications are necessary. Specifically, the system cannot have one sampler for both paths and the sampling period of the saccadic system cannot be the same value as the delay in the saccadic path.

The model proposed by Pavlidis has the same problem as Young's since the saccadic systems are identical. In attempting to prevent small steps from activating the pursuit mode, a differentiator was used in place of Young's error-rate estimator. This indeed does what it was designed to do since the differential of a step will not pass the first limiter. The system will work using the neural-net type differentiator mentioned by Pavlidis because it has an associated delay, but if a differentiating circuit is used the system's ramp response will be a ramp instead of the delayed ramp which is exhibited by the biological system.

Philosophically, Warnmeunde's model is a study in contradiction. He has taken great pains to develop what appears to be a very good

saccadic mechanism and has replaced the second-order approximation to eyeball dynamics with a more sophisticated system developed from the work of Miller. All of this was done to make the model a better representation of the biological system. A look at the pursuit system reveals man's attempt to improve on nature. This pursuit system is much better than the one present in the skulls of mortal men since it can accurately follow velocity changes continuously and thus it can track a moving target more accurately than man's poor velocity-segmented approximations. However, the object in physiological modeling is to synthesize a model that accurately represents the physiological system under study and not to build a better one. Whatever achievement this model represents in the field of control systems is offset by its failure in the field of physiological modeling. The model is presented for its excellent possibilities for further study. A combination of this saccadic pathway and another smooth tracking pathway might yield a very good model.

Recently there have been two other models proposed for the eyeball and muscle dynamics. The first, proposed by Robinson, was the result of a group of controlled experiments which were designed to study saccadic eye movement under abnormal conditions (105). By restraining one eye through a contact lens and measuring the force it applied while the other eye went through a normal saccadic movement he was able to determine that force was not a linear function of eye movement amplitude. Robinson's work was based on that of Alpern (1962) and Bjork (1955) (106,107). Robinson presents evidence that the system is a heavily overdamped one, has no resonant frequency, and is little

affected by eyeball mass. The author of the second study, Cook, went a step further than Robinson by considering the forces generated by the agonist and antagonist muscles instead of just the net forces on the eye (108). An excellent report on both papers as well as a presentation of each can be found in NASA CR-564 (109). Since these models were not used by the author, no further discussion of them will be presented. They do represent, however, significant advances in the modeling of a specific part of the total system under study in this paper. They were not used because the requirements of the model were satisfactorily met by a simpler representation of the extra-ocular system.

3.3 AUTHOR'S MODEL

The model developed for the horizontal tracking system of the normal subject is, for the reasons presented in section 3.2.2, a sampled-data control system. It is based mainly on the work of Young with some significant modification to overcome the shortcomings of his model. It is by no means the "last word" in models of this system for, even as this is being written, many different modifications are under consideration by the author for future work. It does represent the best model to date which has been completely synthesized and tested on a computer by the author and it does fulfill the requirements of duplicating the transient response of the physiological system as measured by the eye movement detection equipment discussed previously. For the following analysis the z-transform used was

$$z = e^{-sT} \qquad (3-2)$$

The transmission z is a delay of T seconds and the region of stability

is outside (not inside) the unit circle in the z-plane. Reference

texts for sampled-data systems analysis and z-transform theory are Kuo

(110), Jury (111), and Ragazzini and Franklin (112). Appendix A con-

tains some notes on the transform-pairs used in this paper.

3.3.1 The Saccadic Positioning Path

This part of the overall system must be able to produce responses

to step and pulse changes in target position which are in agreement

with the physiological responses. Specifically, the step response would

be a delayed step (150-250 msec) with a finite rise-time (5-25 msec)

and the pulse response would be a delayed pulse (150-250 msec) whose

width is independent of the short target pulse (50 msec) and is usually

slightly longer than the delay (200-300 msec). The saccadic path is

shown in Fig. 3.9. As can be seen, the difference between the target

position and the eye position (X_5) is taken and the result represents

the error signal (X_1). The error is fed into a non-linear element

which provides the $\pm 0.25^\circ$ dead-zone which will prevent this model from

responding to saccades below this threshold or to noise. If the error

is large enough it is then sampled by an impulse modulator every 230

msec beginning with the onset of the transient input. The sampled

error (X_2) is then integrated to give a step command (X_3) which is de-

layed 200 msec (X_4) before being sent to the eyeball dynamics, G(S),

which then act on the command to produce the desired eye position (X_5).

Although other models for the extra-ocular system have been presented,

the author feels that for this system, Westheimer's second-order

approximation is more than adequate. The important difference between

this model and that of Young's is the sampling rate of 230 msec as opposed to 200 msec. For a second-order system with $\rho = 0.7$ and $\omega_o = 240$, this 30 msec delay between the instant the position-change command is given to the instant of sampling amounts to more than 5 time constants. This insures that the eye will have assumed its new position before the error is sampled again and that no sampling can occur during a saccade. Fig. 3.10 is the signal flow graph of the system in the linear region of the dead-zone. From this graph the following can be written by inspection:

$$R(s) = \frac{e^{-sT_1} \frac{1}{s} G(s)C(s)^*}{1 + \frac{z^\gamma G(s)^*}{1-z}}, \quad \gamma = T_1/T_2 \tag{3-3}$$

$$= \left(\frac{1-z}{s}\right)\left[\frac{z^\gamma}{1-z + z^\gamma G(s)^*}\right]G(s)C(s)^* \tag{3-4}$$

This represents a zero-order hold with a delay term corresponding to the delay of T_1 msec. Similar analysis of Young's system yields,

$$R(s) = \left(\frac{1-z}{s}\right)\left[\frac{z}{1-z + zG(s)^*}\right] G(s)C(s)^* \tag{3-5}$$

which is a zero-order hold and a delay equal to the sampling interval T. Young's presentation did not include the effects of G(s) although he pointed out that where his analysis showed an abrupt step change in position the actual eye would move as a second-order system with finite rise-time and possible overshoot. To arrive at Young's system equation

TRGT.
(C)

(X_1) T_2 (X_2) (X_3) $\dfrac{1}{s}$ (X_4) e^{-sT_1} $G(s)$ (X_5) EYE POS.

(R)

$T_1 = .2$
$T_2 = .23$

$$G(s) = \frac{\omega_o^2}{s^2 + 2\rho\omega_o s + \omega_o^2} , \quad \rho = .7, \quad \omega_o = 240$$

Fig. 3.9 Saccadic system

$C(s)$ 1 X_1^+ 1 X_2^+ $1/1-z$ X_3^+ $z(T_1/T_2)$ X_4^+ $G(s)^+$ X_5^+ 1 $R(s)^+$
-1

$C(s)$ 1 X_1 1 X_2 $\dfrac{1}{s}$ X_3 e^{-sT_1} X_4 $G(s)$ X_5 1 $R(s)$
-1
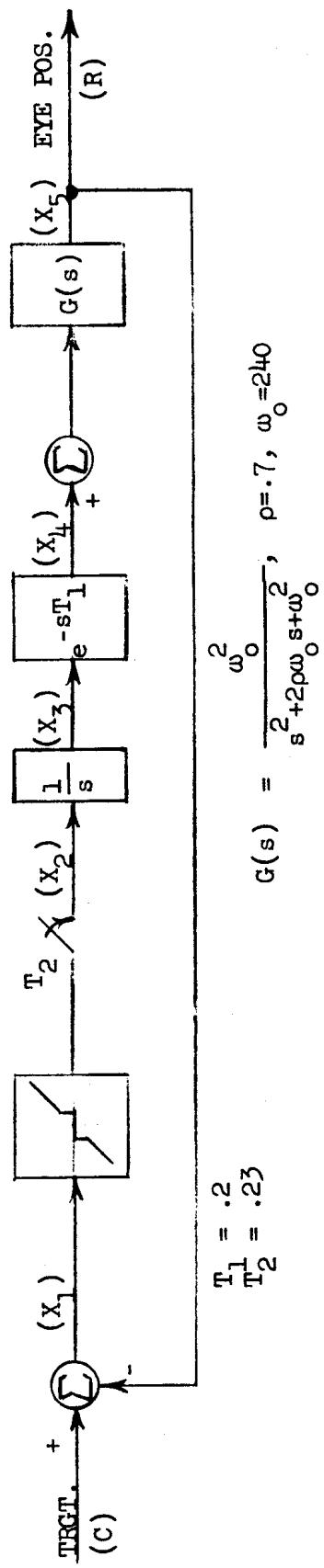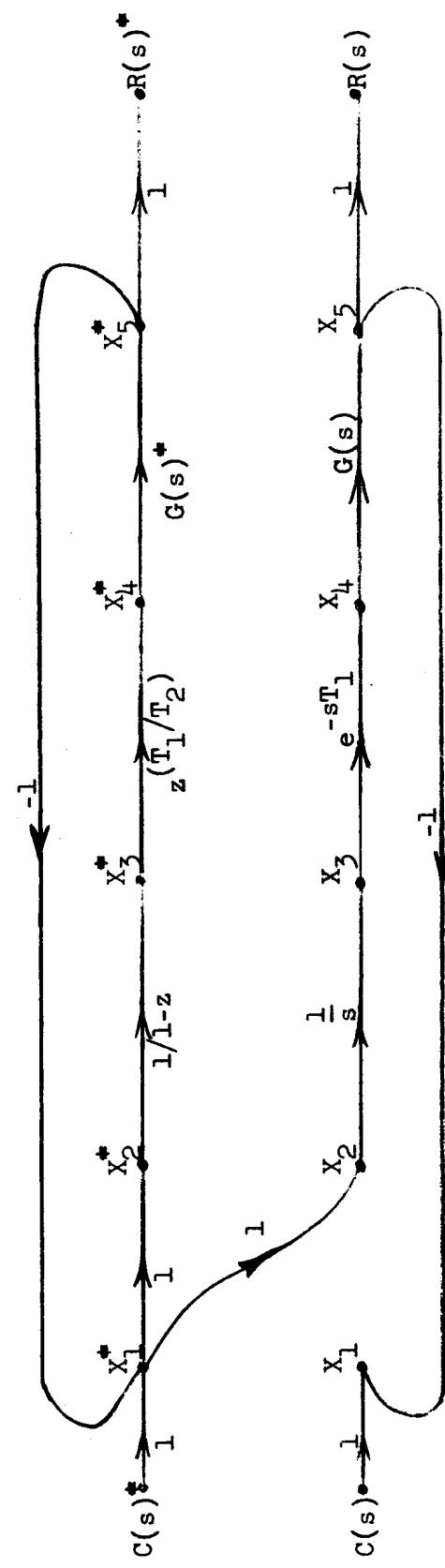
Fig. 3.10 Composite signal flow graph of the saccadic system

set $G(s) = G(s)^* = 1$ in equation (3-5). The step and pulse responses
of this system are shown in Fig. 3.11. As is evident in both cases,
the instant of sampling (T_2) occurs after the eye has assumed its new
position (30 msec after the eye starts to move). This eliminates both
the hunting found in Young's model and the sampling during a saccade;
the latter, a contradiction to physiological evidence which suggests
that vision is suppressed during a saccade (113,114,115,116).

3.3.2 The Smooth Tracking Path

The smooth tracking system must be able to track targets with velo-
cities up to 25-30°/sec, approximate accelerating targets with a series
of constant velocity segments which change in amplitude at a fixed in-
terval and correctly track a moving target regardless of any initial
displacement. The ramp response should be a delayed ramp (150-250
msec) and the velocity segments of the parabolic approximation should
be changed in amplitude at an interval of every 150-250 msec. A model
for this pursuit mode is shown in Fig. 3.12. As in the saccadic system,
the error (Y_1) must pass the dead-zone before it will be acted upon.
If it does, it is first differentiated to obtain the error rate (Y_2)
which is then passed by a limiter if it is less than or equal to
25-30°/sec. This prevents the model from tracking targets with high
velocities, which is consistent with physiological evidence. If passed,
the signal is delayed by 20 msec (Y3) and sampled every 200 msec (Y_4).
This sampled error rate is then integrated and must pass through a
saturation non-linearity. Thus the eye cannot move smoothly with a
velocity greater than 25-30°/sec. If it passes it is integrated once
more to form the command (Y_5) which is sent to the eyeball dynamics,
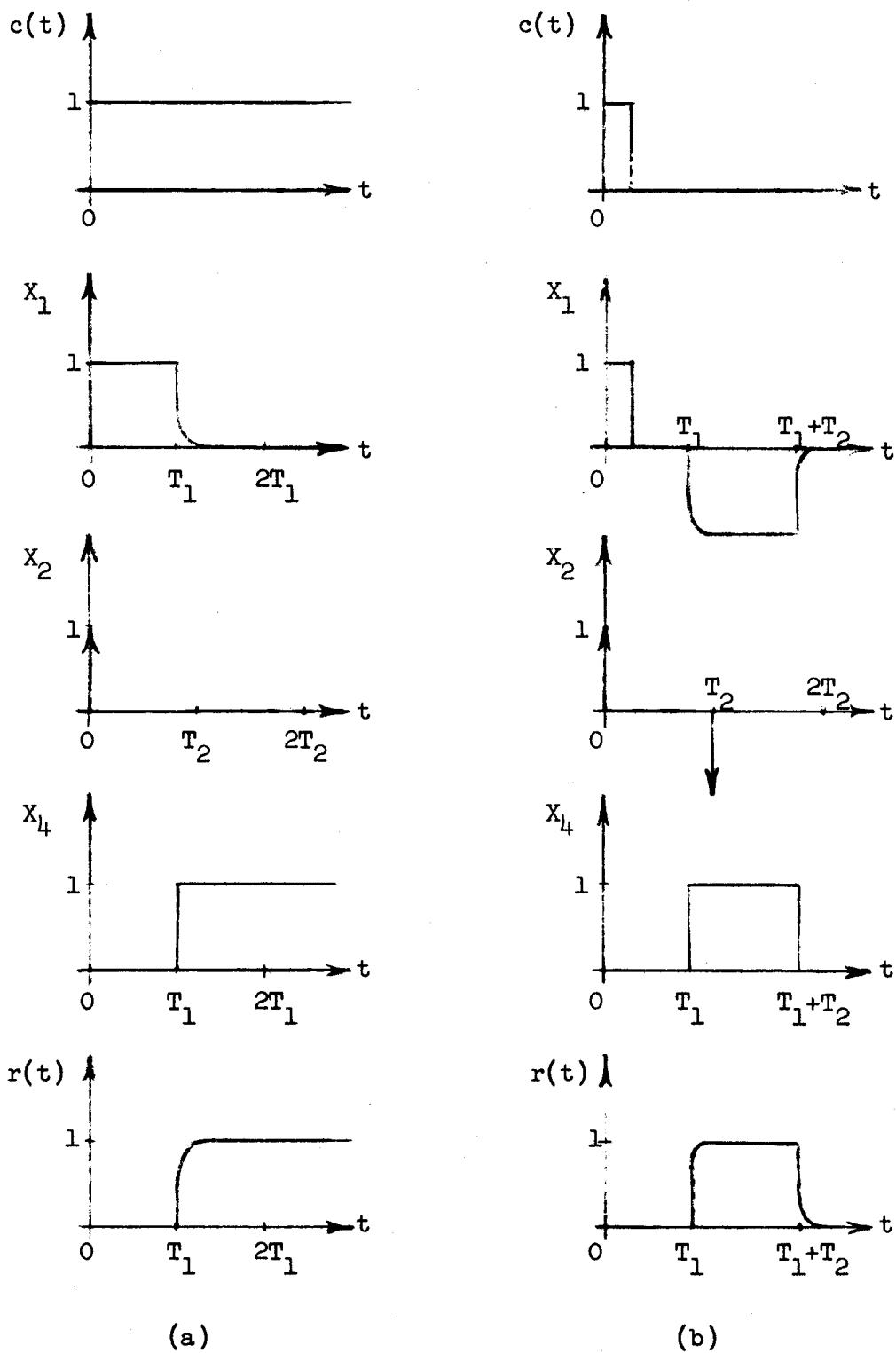
Fig. 3.11   Saccadic system transient responses (a) step response
            (b) pulse response

G(s), which then cause eye motion (Y_6). This system will not respond to any step change in target position no matter how small the amplitude since the differentiator will produce an impulse whose magnitude will pass the first limiter. The 20 msec delay allows the eye to come extremely close to its steady-state value before sampling occurs. For this system the steady-state error to a ramp input of S^o/sec is given by,

$$e(\infty) = \lim_{s \to 0} sE(s) \text{ where,} \qquad (3\text{-}6)$$

$$E(s) = S\left[\frac{1}{s^2} - \frac{\omega_o^2}{s^2(s^2 + 2\rho\omega_o s + \omega_o^2)}\right] \qquad (3\text{-}7)$$

Thus,

$$e(\infty) = \frac{2S\rho}{\omega_o} = .0058 \text{ S.} \qquad (3\text{-}8)$$

For a 10°/sec ramp, this amounts to a steady-state error of 0.058^o. Now, the time from the onset of motion to sampling is equal to 180 msec, which is roughly 30 time constants, so that the error at the instant of sampling is equal to the steady-state error. This is truly negligible. Fig. 3.13 is the signal flow graph of this system in the linear regions of the three non-linearities. The eye position would be given by,

$$R(s) = \frac{z^{0.1}\left(\frac{-\ln z}{T}\right)\left(\frac{1}{s^2}\right) G(s)C(s)^*}{1 - \left(\frac{\ln z}{T}\right)(z^{0.1})\left[\frac{Tz}{(1-z)^2}\right] G(s)^*} \qquad (3\text{-}9)$$

TRGT.
(C)

(Y_1) (Y_2) $\dfrac{e^{-\frac{sT_1}{10}}}{}$ (Y_3) $\dfrac{1}{s}$ (Y_4) $\dfrac{1}{s}$ (Y_5) $G(s)$ (Y_6) EYE POS. (R)

$T_1 = .2$

$$G(s) = \frac{\omega_o^2}{s^2 + 2\rho\omega_o s + \omega_o^2} \quad , \quad \rho = .7, \quad \omega_o = 240$$
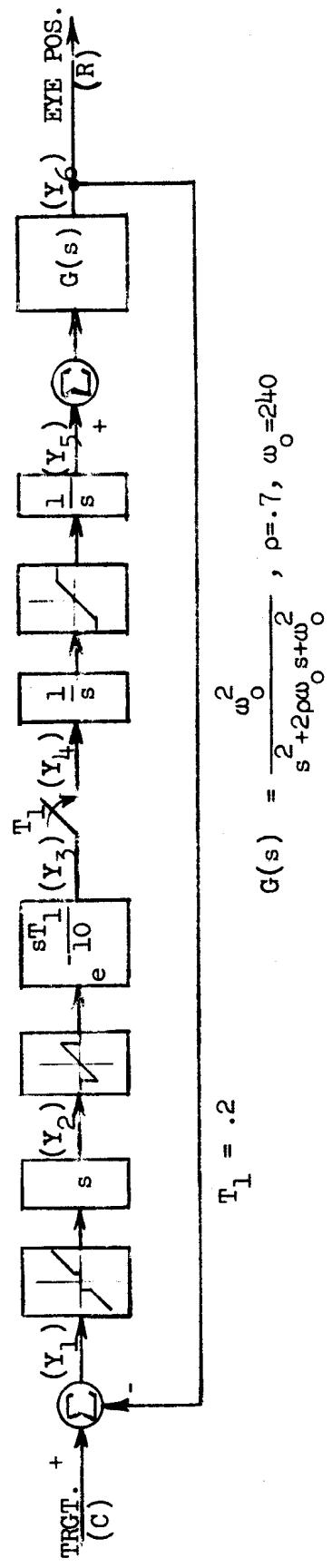
Fig. 3.12 Smooth tracking (pursuit) system

C(s) $\dfrac{-\ln z}{T}$ Y_2^+ $z^{\cdot 1}$ $Y_3^{\#}$ 1 Y_4^+ $\dfrac{Tz}{(1-z)^2}$ Y_5^+ $G(s)$ Y_6^+ 1 R(s)

Y_1^+ -1

C(s) s Y_2 $e^{-.1sT_1}$ Y_3 1 Y_4 $\dfrac{1}{s^2}$ Y_5 $G(s)$ Y_6 1 R(s)

Y_1 -1
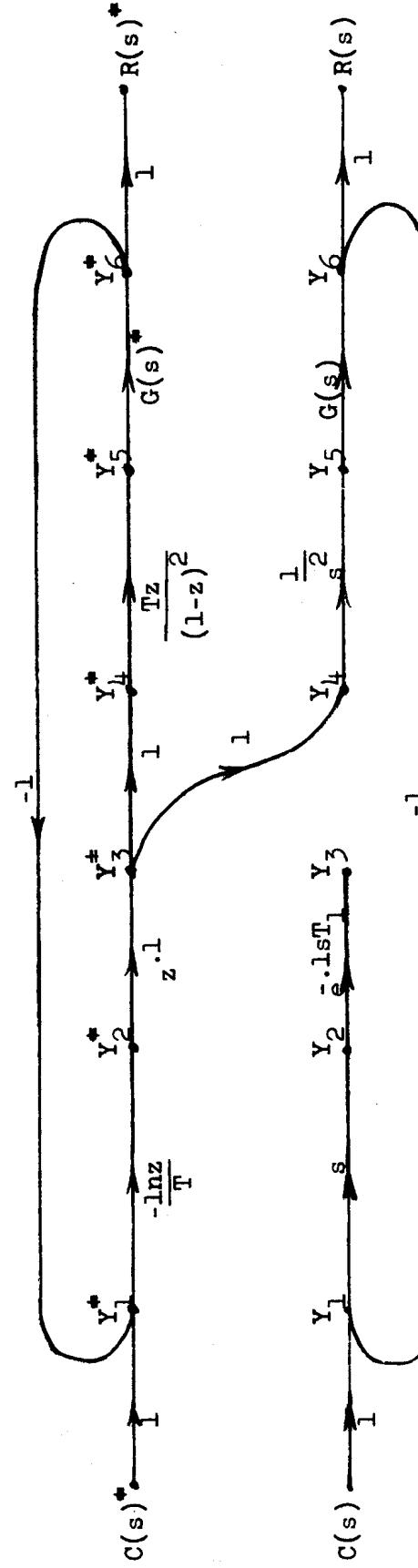
$T = T_1$

Fig. 3.13 Composite signal flow graph of the pursuit system

$$= \left(\frac{1-z}{s}\right)\left(\frac{1}{s}\right)\left(z^{0.1}\right)\left[\frac{\ln z}{(z-1)^2 + z^{1.1}\ln z G(s)^*}\right]\left(\frac{1-z}{T}\right)G(s)C(s)^* \qquad (3\text{-}10)$$
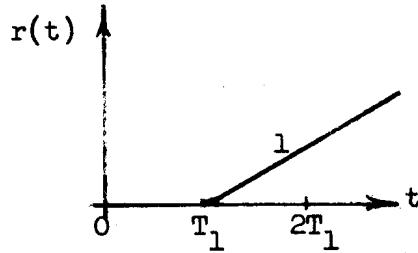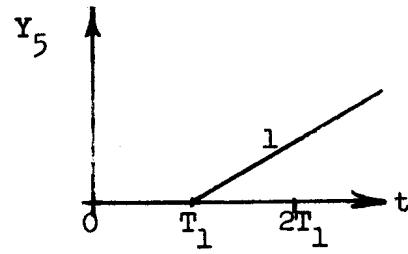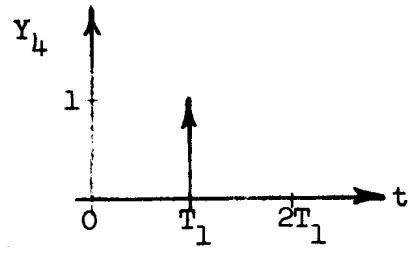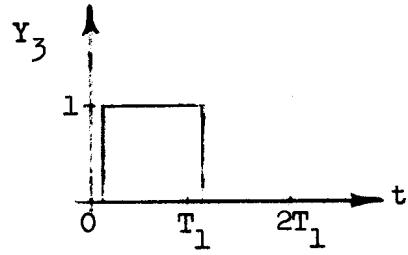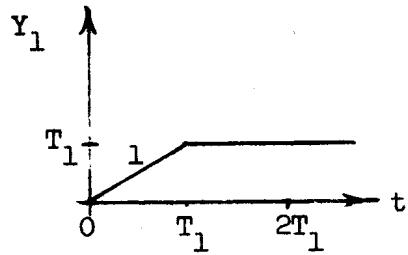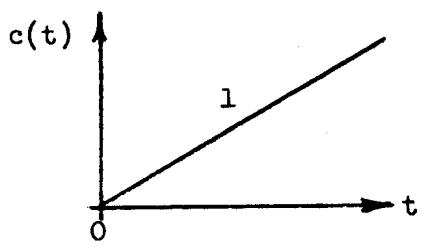
This consists of a zero-order hold, an integrator, a delay term, a term corresponding to the differentiator and a first-difference term. Analysis of Young's system yields,

$$R(s) = \left(\frac{1-z}{s}\right)\left(\frac{1}{s}\right)\left[\frac{1}{1-z-zG(s)^*}\right]\left(\frac{1-z}{T}\right)G(s)C(s)^* \qquad (3\text{-}11)$$
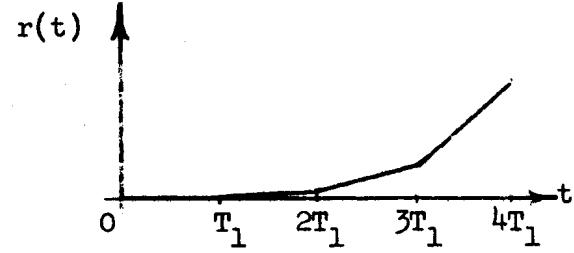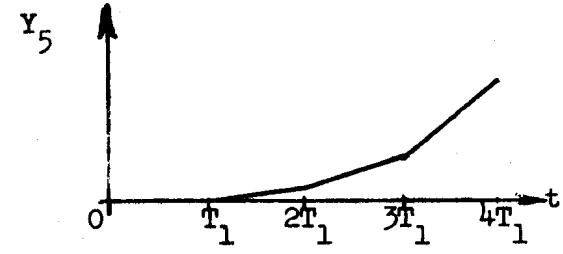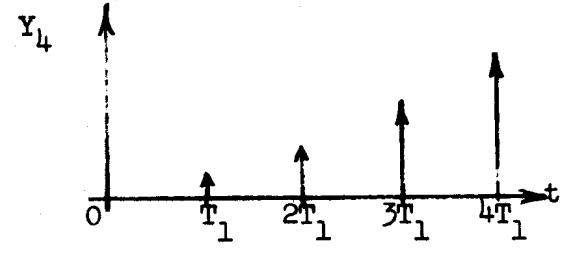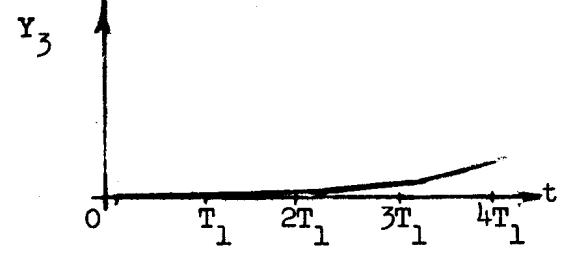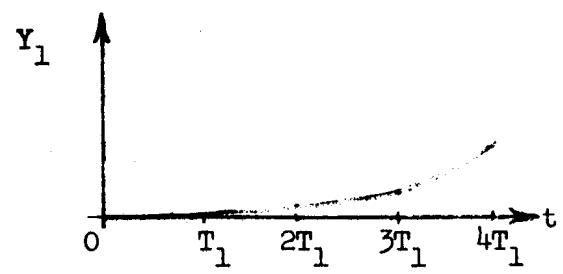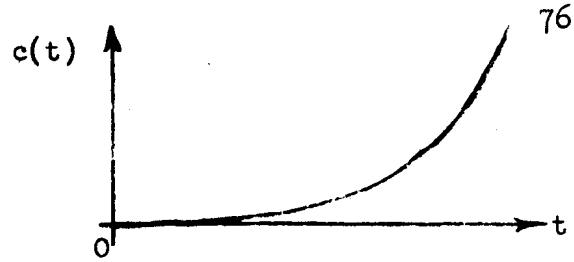
Responses to a ramp and a parabola are shown in Fig. 3.14. Here, as in the saccadic mode, sampling occurs after the eye has had a chance to respond to the previous command. This prevents the inevitable errors associated with sampling at the command instants which would be analogous to sampling during a saccade in the saccadic model. Both will result in responses which differ from the biological responses.

3.3.3 Complete Model for Horizontal Tracking System

The model for the whole system involves merely combining the two subsystems discussed above. Unfortunately, this simple little maneuver makes an exact analysis of the system an extremely long and difficult job. Fig. 3.15 shows the complete system for the normal subject. The description of the system merely consists of the two previous descriptions of the subsystems. The dead-zone and retinal feedback paths are now common to both paths. The preliminary flow graph for this multi-rate sampled-data system is shown in Fig. 3.16. The presence of two samplers of different sampling rates calls for an additional step in the analysis leading towards the composite flow graph. In order to

Fig. 3.14   Pursuit system transient responses (a) ramp response
(b) parabolic response

SMOOTH TRACKING

SACCADIC POSITIONING

TRGT. +

DEAD ZONE

s

VEL. LIMIT

$e^{\frac{sT_1}{10}}$

T_1

$\frac{1}{s}$

PUR. SAT.

$\frac{1}{s}$

+

$\sum$

+

EYE AND MUSCLE G(s) DYNAMICS

EYE POS.

T_2

$\frac{1}{s}$

e^{-sT_1}

$-$

$-$

$$G(s) = \frac{\omega_o^2}{s^2 + 2\rho\omega_o s + \omega_o^2} \quad , \quad \rho = .7 \text{ and } \omega_o = 240$$

$T_1 = .20$
$T_2 = .23$
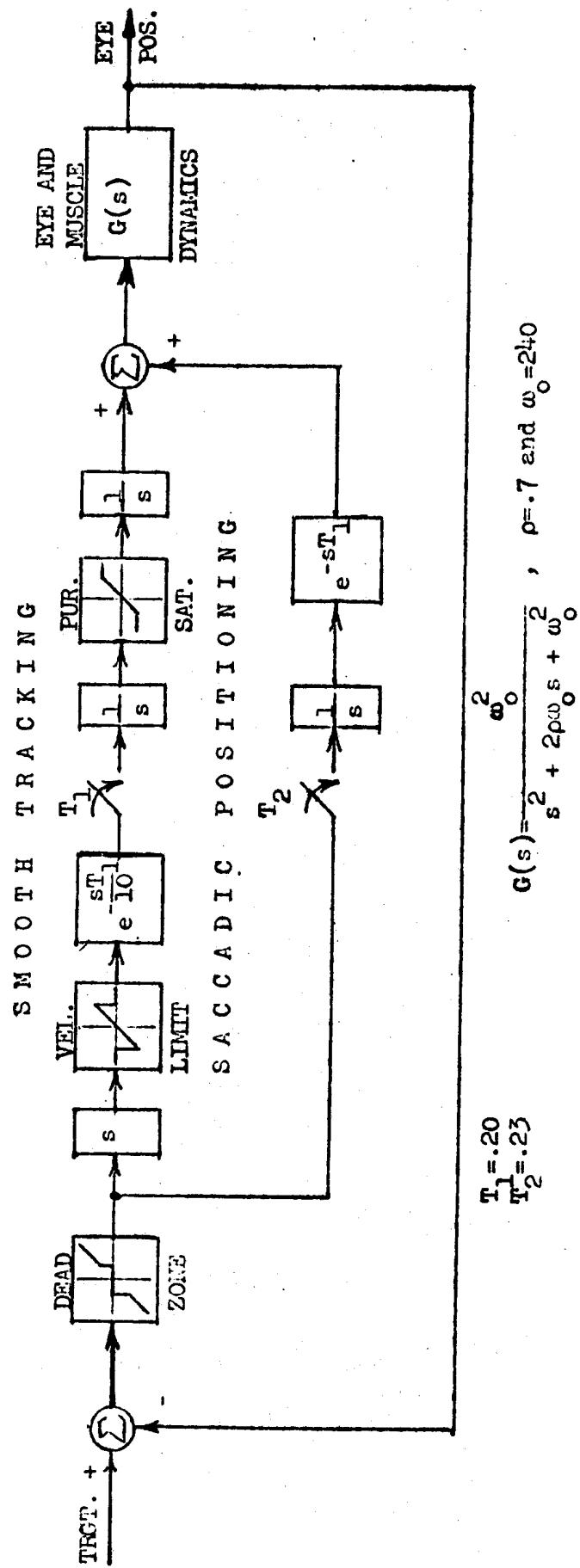
Fig. 3.15 Sampled-data model for normal system

convert this to a system in which all the samplers share a common sampling rate T, integers N_1 and N_2 must be found such that,

$$T = N_1 T_1 = N_2 T_2 \qquad\qquad (3\text{-}12)$$

For this case $N_1 = 23$ and $N_2 = 20$. The next step involves replacing samplers T_1 and T_2 in the flow graph by 23 and 20 parallel samplers respectively. Each sampler has a time advance preceding it and a time delay following it. These are referred to as "skip samplers". Fig. 3.17 shows the 23 skip samplers which would replace sampler T_1 in the new flow graph. Sampler T_2 would be replaced by 20 such samplers. Analysis of this system now proceeds according to the standard practices for obtaining the sampled flow graph and then the composite graph from which the transfer function is written. For sampling rates related by low integers such as 2 or 3, this analysis isn't too cumbersome, but for the system in question where the integers are 23 and 20, the task is prohibitive; especially due to the probable complexity of the final transfer function which will result. In light of the above, the author feels that a better understanding of the system can be obtained by considering its responses to several standard transient inputs.

(1) Step Input: Since the pursuit system does not respond to a step input, the total system response is identical to that of the saccadic path alone as described in section 3.3.1 and illustrated in Fig. 3.11.

(2) Pulse Input: Here again, the analysis is identical to that in section 3.3.1.
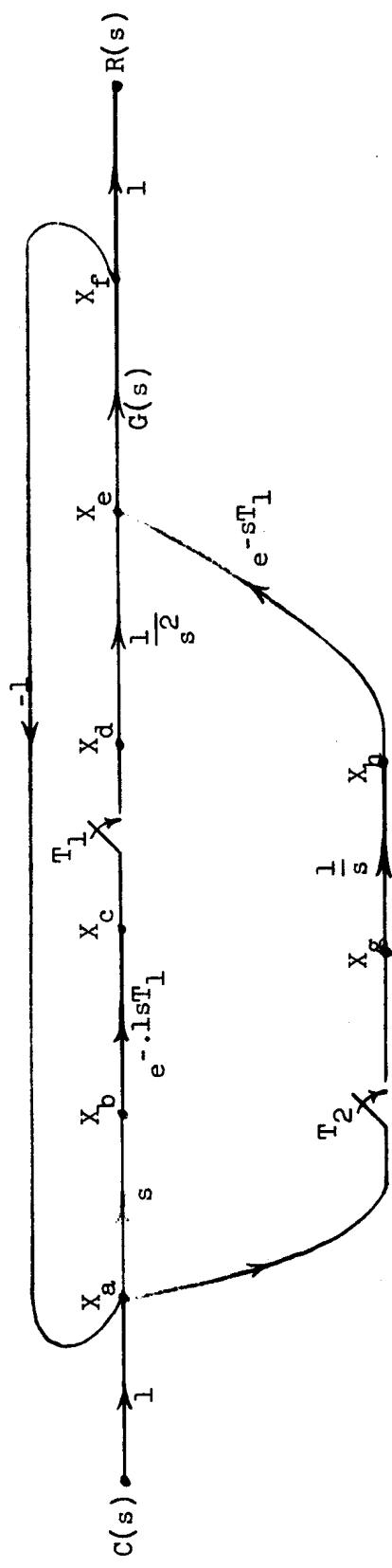
Fig. 3.16 Preliminary flow graph for complete system
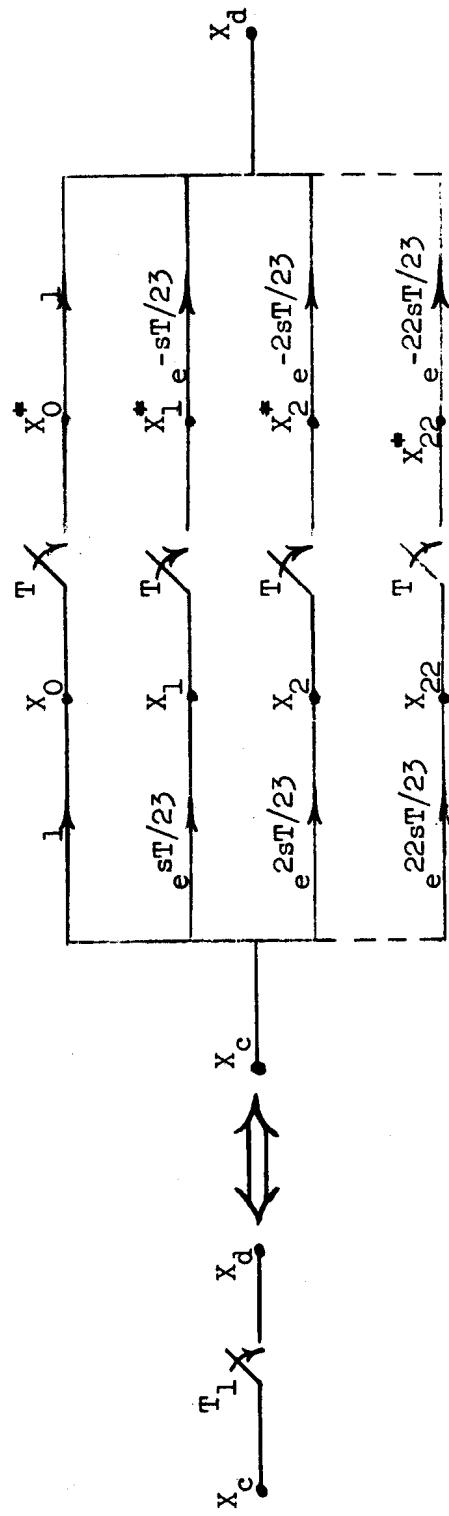
Fig. 3.17 Sampler T_1 replaced by "skip samplers"

(3) Ramp Input: The ramp response of the system is illustrated in Fig. 3.18. For identification of the functions plotted refer to Fig's. 3.9 and 3.12. We observe that the response consists of a ramp delayed by T_1 sec followed by a corrective saccade at $T_1 + T_2$ sec which corrects the position error due to the delay in the pursuit mode.

(4) Parabolic Input: Fig. 3.19 shows the response to a con- stantly accelerating target. This can be looked at as an addition of several ramp responses. Note that this velocity-segment approximation to the accelerating target always has a position error.

(5) Step Ramp Input: This input, where the target is initially displaced in one direction and immediately moves with a constant veloc- ity in the other, serves to demonstrate the independence of the two paths. The response of the pursuit path is unaffected by the pres- ence of the step change in position in the direction opposite to the initial velocity of the target (see Fig. 3.20).

Comparison of these predicted responses of the model with the observed physiological responses of the normal subject affirms the validity of this model.
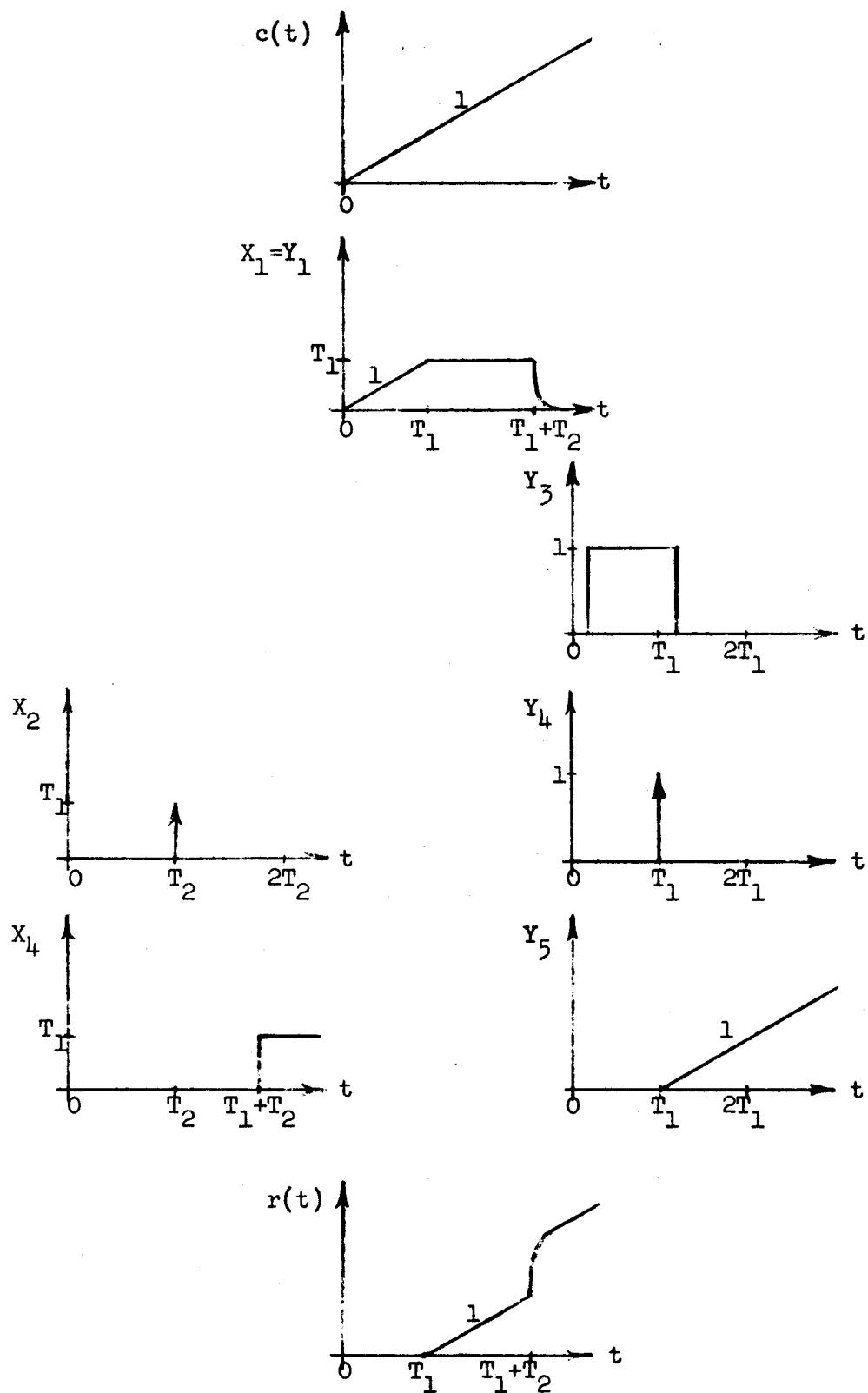
Fig. 3.18 Ramp response of the complete model
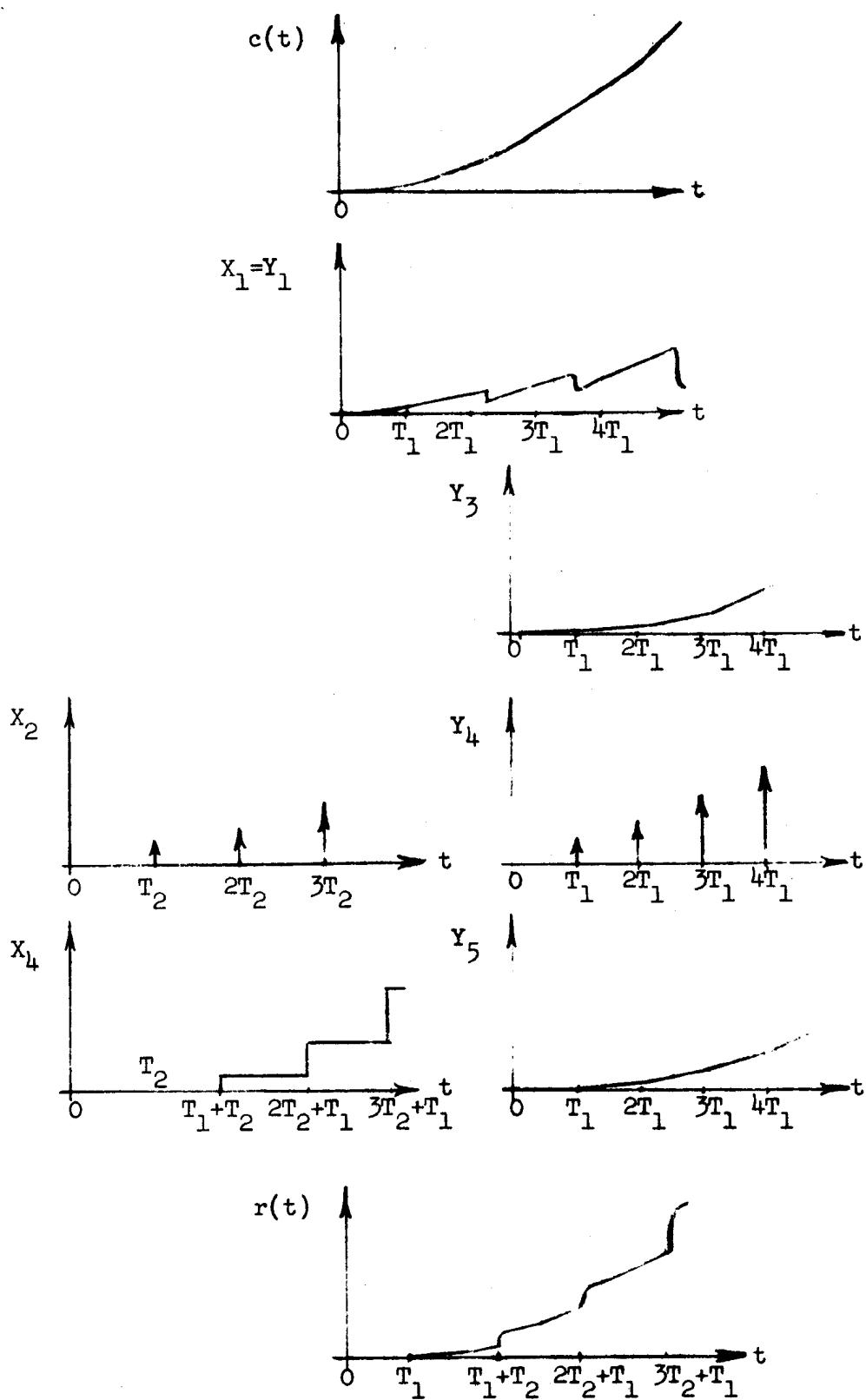
Fig. 3.19 Parabolic response of the complete model
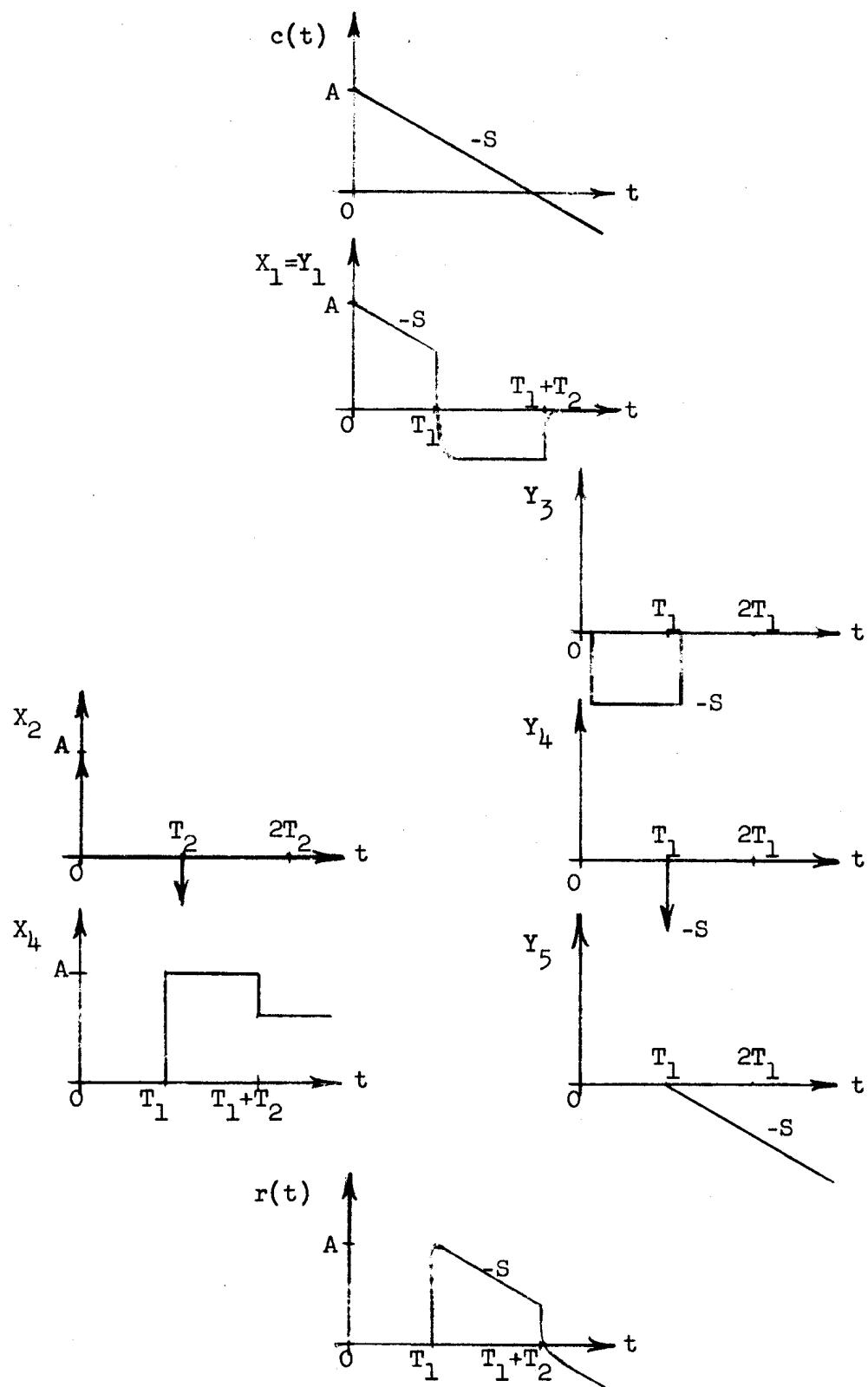
Fig. 3.20 Step-ramp response of the complete model

CHAPTER IV

COMPUTER REALIZATION

4.1 SYSTEM USED

The computer used to synthesize the model for the horizontal tracking system was a Burroughs B-5500. The programming was done via the ANALOG-ALGOL system. This is a method of programming, on a digital computer, analog processes which are described by integro-differential equations. Analog-Algol is designed so that a person with no experience on a digital computer can use it to solve complex problems. The commands used are standard English words whose mathematical meaning is common knowledge to engineers and mathematicians. Using Analog-Algol one can go directly from a block diagram to the program. The order of operations, as stated on the program, is not important since the program will run correctly regardless of order. Thus, this system places a powerful tool, namely a large digital computer, within the reach of most engineers and scientists.

4.2 PROGRAM FOR THE NORMAL TRACKING SYSTEM

The model for the normal tracking system as shown in Fig. 3.15 was programmed first. Only after this model was successfully synthesized was a model postulated for the system with nystagmus. Successful synthesis entails the synthesis of a model whose responses, as printed out by the computer, agree with those of the normal biological system when subjected to the same inputs. The actual mechanics of programming the computer to carry out the operations indicated in Fig. 3.15 will

not be discussed here except to give the reader some insight into how

it is done by the following brief description of the commands which

would accomplish the first operation. This first operation is simply

obtaining the error signal from the target and eye position. The com-

mand for this would be,

$$\text{SUM 2(TRGT,1,EYE,-1,ERR)};\qquad\qquad\qquad\text{(4-1)}$$

This tells the computer to sum two variables, which are the target

(multiplied by 1) and the eye (multiplied by -1) and the result shall

be called the error. This simplicity is carried through most of the

operations so that one just starts at the input of a block diagram and

works his way to the output by writing commands, in abbreviated English

terms, for each operation. Appendix B contains a copy of the final

program which is a dual-mode program capable of representing either the

normal system or the system with nystagmus. Between the commands

"BEGIN" and "BEGINANALOG" the variables are defined, operating instruc-

tions are given, and the break points of non-linearities are given.

After this, the forcing functions are given and the sampler defined.

After the command "ENDFORCING;" the operations are listed in the

manner just shown. At the very end is the command "ENDANALOG;".

4.2.1 The Non-Linearities

As can be seen in Fig. 3.15, there are several non-linear devices.

These were programmed into the system as follows:

(1) Dead-Zone: When the input to the dead-zone is between -0.25

and +0.25 the output is zero. Any input less than -0.26 or greater than

+0.26 will produce an output equal to the input. The inputs falling in

the transition regions (-0.25 to -0.26 and +0.25 to +0.26) yield outputs

which vary linearly from 0 at $\pm$0.25 to $\pm$0.26 at $\pm$0.26 (see Fig. 4.1).

(2) Velocity Limit: This non-linearity will pass on a one-to-one

basis any velocity between -25 and +25 degrees per second and will pro-

vide a zero output for any input less than -26°/sec or greater than

$+26^{\circ}$/sec. Again, the transition zones yield outputs which vary linearly

between the two extremes (see Fig. 4.2).

(3) Pursuit Saturation: This is a simple saturation character-

istic where any input between -25°/sec and $+25^{\circ}$/sec is passed as received

but any input less than -25°/sec or greater than $+25^{\circ}$/sec will produce

-25°/sec or $+25^{\circ}$/sec outputs respectively (see Fig. 4.3).

4.2.2 Sampling

The sampling operation, which mathematically is multiplication by

an impulse train, was accomplished on the computer by the use of a

square pulse whose height was 1/H and whose width was H, where H was a

variable which could be set to some arbitrarily small number. This

pulse of unit area is equal to an impulse in the limit as H approaches

zero. For this problem a value of 0.01 was satisfactory. In the sac-

cadic path the sampling and the integration were combined into a sub-

routine which performed these operations by another method. A discussion

of how this was done within the digital computer, or how any of the

other operations were carried out for that matter, is something which

is not within the scope of this paper.

4.2.3 Equivalence to Mathematical Model

The construction of the non-linearities was carried out in the

above-described manner to eliminate the possibility of an oscillatory

Output   Input

25

25

-25

-25

Output   Input

(25,25)

26

-26

(-25,-25)

Output   Input

(.26,.26)

.25

-.25

(-.26,
-.26)
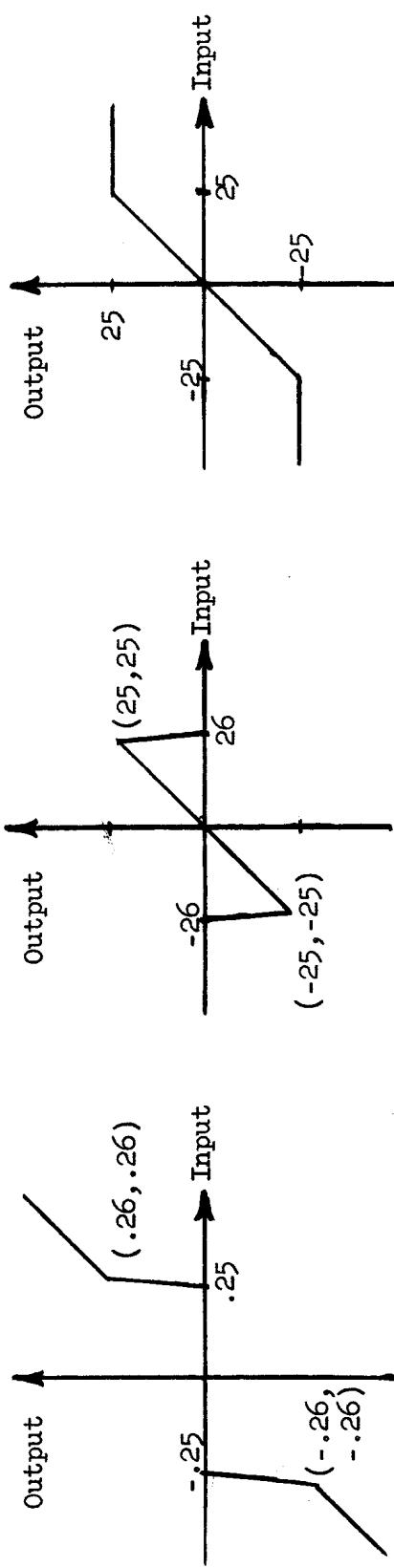
Fig. 4.3 Pursuit saturation

Fig. 4.2 Velocity limit

Fig. 4.1 Dead-zone

condition which could have developed if they contained the infinite slopes present in the mathematical model. Actually, they represent a closer approximation to the physiological system since it is very unlikely that one would find a step discontinuity in a natural system.

The unit area pulse approximation to an impulse function is a common method of achieving this ideal function, whose height is infinite, width is zero, and area is equal to one. Obviously the area of the pulse is equal to one for all H greater than zero. The only other criterion that must be met is that it must function in the system as an impulse would. More simply, its duration must be short in comparison to the sampling rate. The choice of H = 0.01 satisfies this since T = 0.20. Had this proven unsatisfactory, H could have been reduced to a much smaller value, limited only by the computer's ability to handle the large pulse.

4.2.4 Alternate Program

Since the program used contained a differentiator and many systems cannot handle this noisy operation, the following changes can be made and an equivalent system will result:

(1) Instead of a target input, feed in the derivative of the target.

(2) Instead of feeding back eye position feed back the derivative of eye position. This is available since the realization of the eye dynamics consists of two integrations so that the feedback can come from the point just before the final integration.

(3) Now, feed the difference of these two signals,

$$\frac{d}{dt} \text{ (TRGT)} - \frac{d}{dt} \text{ (EYE POS.)} = \frac{d}{dt} \text{ (ERROR)} \qquad (4\text{-}2)$$

to the smooth tracking path, which now begins with the first limiter.

(4) Into the saccadic path, feed the derivative of the error and integrate it before sampling it.

Thus, without a differentiator, we now have a system which will yield the same eye position output to various target inputs as the system which was used. The saccadic system will have position error as its input and the smooth tracking system will have error rate. All that need be done is to use the derivative (calculated) of the target of interest as the input (see Fig. 4.4). The computer used had a differentiator available so that this alternate method was not employed.

4.3 INPUTS

The inputs to the computer were: (a) a pulse of variable height (S) and width (ST); (b) a step change of variable height (S); (c) a ramp of variable slope (SL); (d) a parabola of variable acceleration (SL); and (e) a step-ramp of variable height (S) and slope (SL). The values for S, ST and SL are punched in on a data card which is read by the computer along with other variables.

4.4 NYSTAGMUS

Once the above program was completed so that the system responded correctly to the given transient inputs, the model was re-examined to determine the modifications necessary to enable it to reproduce nystagmus responses to these inputs.
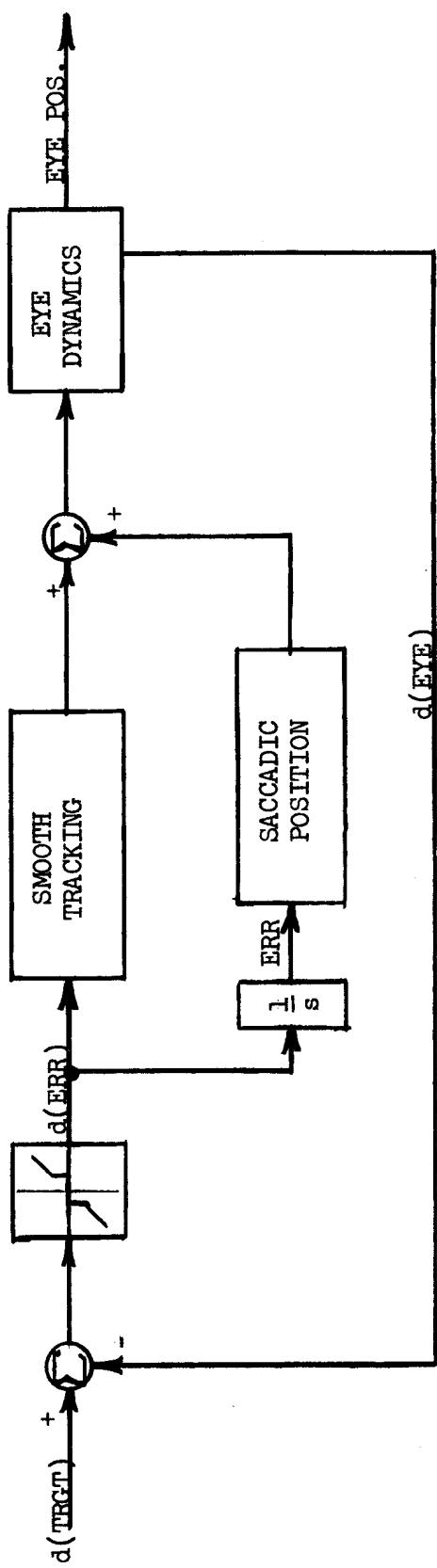
Fig. 4.4 Alternate program

4.4.1 <u>Physiology</u>

Before proceeding with the actual modeling it would be advantageous to review the known facts about congenital, horizontal, fixation nystagmus which exhibits an atypical jerk waveshape to the left and has a saddle in the slow phase. Introductory remarks in Chapter I discussed the type and genetic history of the particular nystagmus of interest and in Chapter II some comments and observations were presented based on the observed waveshape. In order to obtain a clear picture of the particular affliction studied (as present in the subject used) a review of the pertinent literature with a discussion of the case of interest will be presented.

Briefly, the heredity appears to be either recessive sex-linked which affects only males and is transmitted by females or simple recessive. Although simple recessive is very rare, the presence of a consanguineous marriage in the parentage is indicative of such transmission. If it is of the sex-linked, recessive type one would expect no appearance in the subject's offspring and possible appearances in the male children of the subject's daughters. This may not show up for several generations. There have been many family histories traced which illustrate this type of transmission (73,117,118,119,120,121). Nystagmus occurs in males at the rate of 1/5,032 and in females, 1/10,596 (72). Some common charactistics of congenital nystagmus are: (a) it persists throughout life; (b) head movements, which may be present, are compensatory in nature; (c) it is always bilateral; (d) the eye is anatomically normal; (e) the waveshape may change from pendular to jerky as eye position is varied; (f) it is mainly horizontal; (g) visual acuity

is surprisingly good (20/30 to 20/50); (h) the subject does not see movement of stationary objects; (i) visual acuity may improve with age; (j) head shaking and/or nystagmus magnitude is aggravated by excitement or attempts to see more clearly; (k) many cases show spontaneous nystagmus with eyes closed; (l) in the dark, a non-visual stimulus may induce nystagmus; (m) a rare form exhibits a jerk with an atypical slow phase containing a saddle-like form (notch) or even a rise; (n) the minimal position is often at some angle to the straight-ahead position and the maximum position may not be at the terminal lateral position; (o) it often disappears with lid closure; and (p) the frequency may be from 2-5 Hz (27,31,30,122,123). The subject in question would be well described by statements (a) through (j) and (l) through (p). His vision is 20/40. Other findings which have been reported include: (a) the quick component is in the direction of gaze; (b) some subjects have a hypoactive labyrinth; and (c) nystagmus is worse on one side than the other (28). The subject tested had a quick component which was always to the left, his labyrinths appear to be normal (clinical tests revealed no abnormality) and his nystagmus is worse when the gaze is to the right.

The exact mechanisms involved in congenital fixation nystagmus are still a mystery. However, evidence has been presented to support the conclusion that it is a brain stem mechanism (48,31,124,125,126). Studies have also been made to show that the slow and quick phases are due to separate mechanisms but they have not been conclusive (129). In support of the midbrain theory are various reports of the effects of barbiturates (26,128,129,130). It has been found that intra-venous administration of any of several barbiturates (amobarbital, phenobarbital

or pentobarbital) caused nystagmus to decrease temporarily and visual
acuity increase in subjects with nystagmus and when given to normal sub-
jects it caused nystagmus. These substances act on the brain stem (131).

Since the position of minimum nystagmus is often an eccentric one,
an operation has been devised and performed which rotates the eyes
toward the position of greater nystagmus enough to reset the minimum
position to be straight-ahead. This is done by re-positioning the
points of attachment of the horizontal recti muscles (72,132).
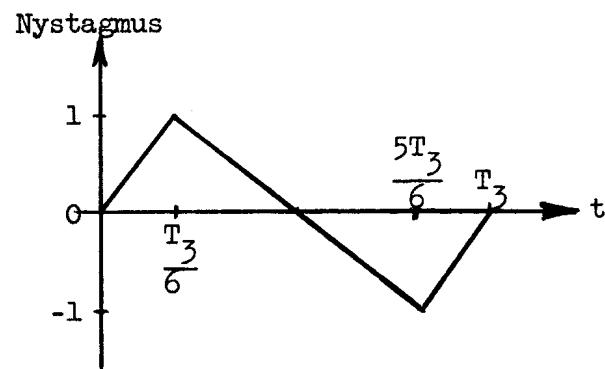
4.4.2 Modeling Nystagmus

The first item of concern in the modeling of the system with nys-
tagmus is the waveshape which will be considered an acceptable repre-
sentation of the actual motion. As the data of Chapter II show, the
subject had an atypical jerk-type nystagmus with a saddle-like form in
the slow phase. The frequency varied about 3 Hz. For the model this
will be represented by a simple asymmetrical triangular waveshape. The
slope of the quick component will be twice that of the slow component
(consistent with the data) and the amplitude must be made to vary with
eye position as shown in Fig. 2.11. Since the notch does not always
appear and its position varied it will not be part of the simulation.
This will not affect one of the main purposes of the model, which is to
demonstrate the functional independence of the nystagmus mechanism and
the tracking mechanism. The literature cited in section 4.4.1 contained
the commonly accepted theory that nystagmus was independent of the nor-
mal movements of the eyes. The data presented in Chapter II substanti-
ates this statement since the responses to the various transient inputs
were equivalent for both subjects with the exception of the 3 Hz

oscillation superimposed on the response of the subject with nystagmus.
If the waveshape shown in Fig. 4.5 is used to simulate nystagmus and it
is multiplied by the output of the non-linearity shown in Fig. 4.6
(where the input is eye position) the result will be a 3.3 Hz sawtooth
whose magnitude varies with eye position in a manner conforming to the
physiological data and whose quick component is twice as fast as its
slow component.

4.5 COMPLETE DUAL-MODE PROGRAM

To incorporate the above nystagmus generator into the model, this
variable-magnitude waveshape must be fed into the input of the eye
dynamics and also to some point in the system before the error signal
is fed into the two forward paths. This must be done to prevent the
model from acting on this oscillation. The subject does not misinter-
pret this apparent target motion as real target motion (in fact, he is
not even conscious of it) so the model must also act as though it were
not present. This is done by adding the same motion (slightly delayed)
to the error signal (which carries the negative of this motion) and
thereby canceling it out. The final dual-mode model is shown in Fig.
4.7. The value of N is punched in on a data card as well as the value
of GAIN. If N = 1, we have a nystagmus model. If N is any other number
the model represents the normal system. The value of GAIN is nominally
1 but it can be changed to a higher number to represent an excited or
psychologically anxious subject or to a number less than one to simulate
an unusually calm subject or the effects of various doses of barbitu-
rates. The program shown in Appendix B is the program for this dual-
mode model. With the use of Fig. 4.8, which has all the dummy variables

$$T_3 = .30$$

Fig. 4.5 Nystagmus waveshape
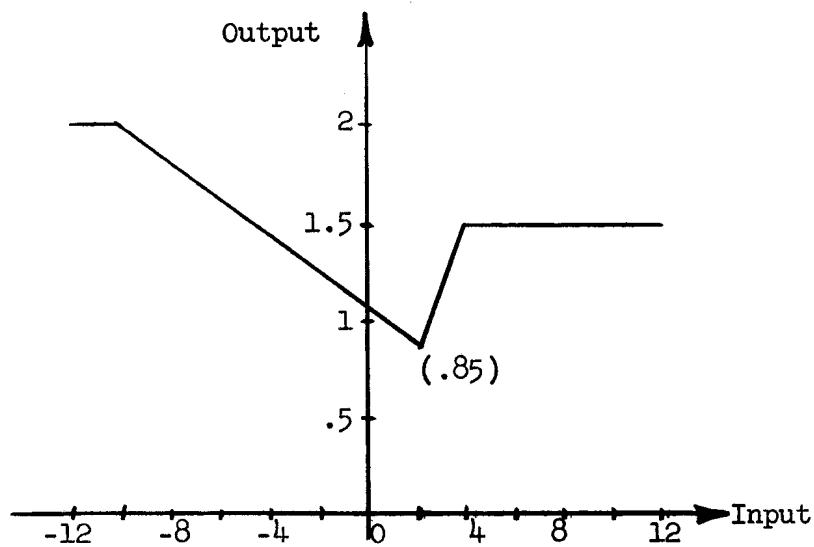
Fig. 4.6 Position non-linearity

96

EYE POS.

POS. N-L

EYE AND MUSCLE DYNAMICS $G(s)$

GAIN

NYSTAGMUS T_3

N=1

$e^{-\frac{sT_1}{20}}$

S M O O T H   T R A C K I N G

PUR. SAT.

$\frac{1}{s}$

$\frac{1}{s}$

T_1

$e^{-\frac{sT_1}{10}}$

VEL. LIMIT

s

DEAD ZONE

S A C C A D I C   P O S I T I O N I N G

T_2

$\frac{1}{s}$

e^{-sT_1}

TRGT.

$$G(s) = \frac{\omega_o^2}{s^2 + 2\rho\omega_o s + \omega_o^2} \quad , \quad \rho = .7 \text{ and } \omega_o = 240$$

$T_1 = .20$
$T_2 = .23$
$T_3 = .30$
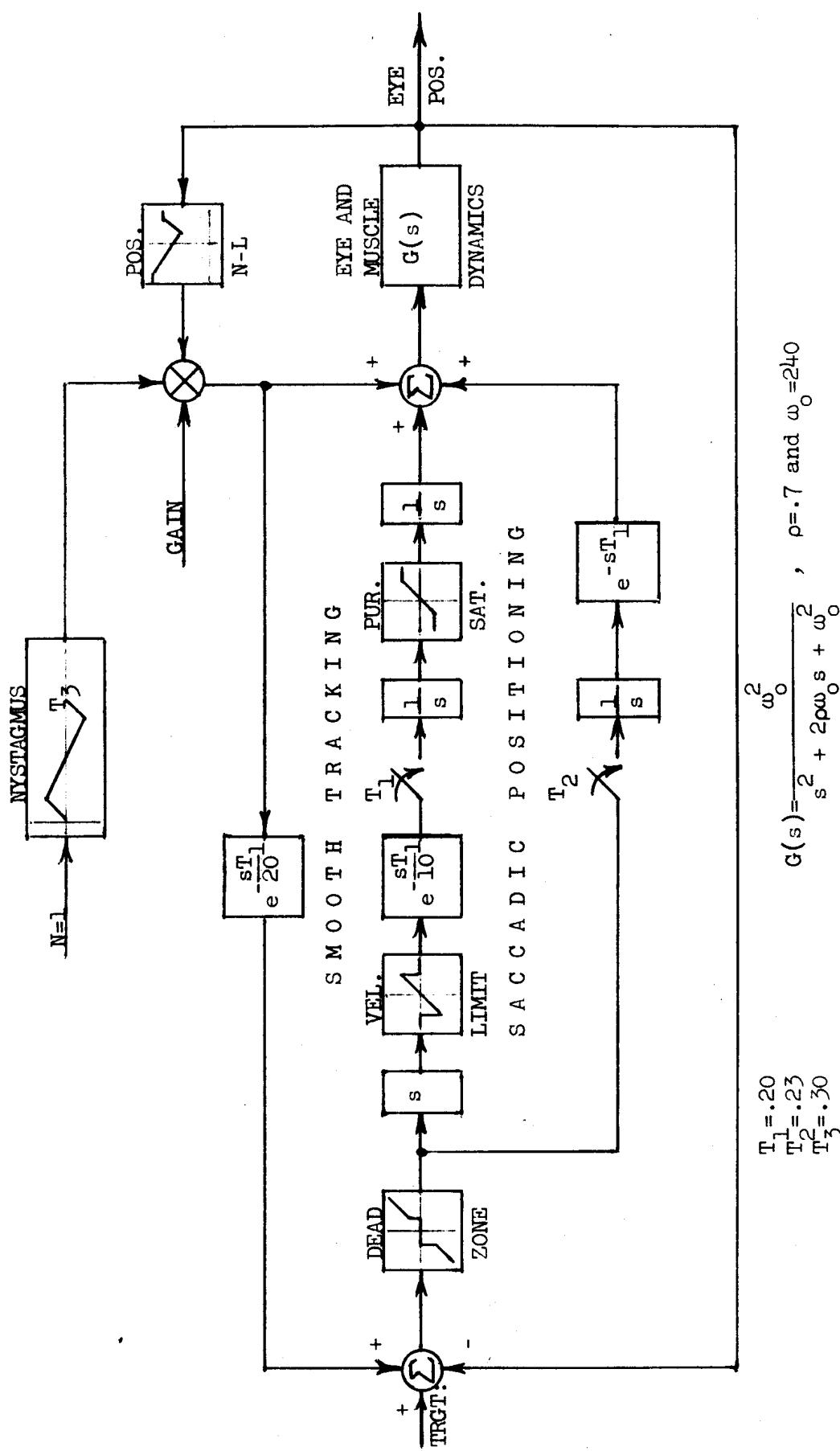
Fig. 4.7 Complete dual-mode model

used in the computer program labeled, the reader should have very little
trouble tracing the operations step by step.
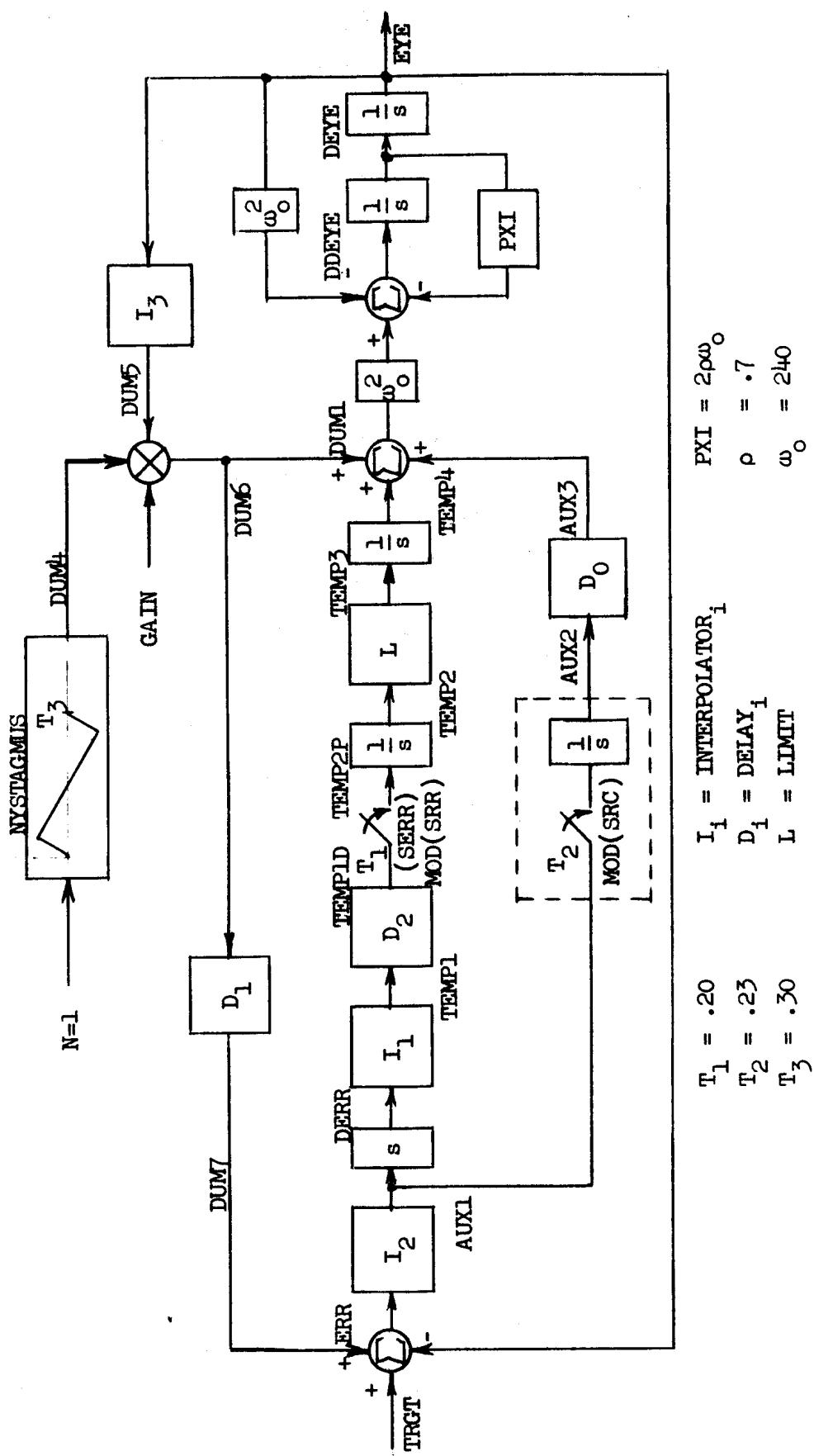
Fig. 4.8 Dual-mode program (with variables)

CHAPTER V

RESULTS

5.1 DATA FROM COMPUTER

Since the Burroughs B-5500 is a digital computer, the output

resulting from a particular "run" consists of a sheet containing a

numerical printout of all the variables of interest at discreet time

points. These time points are determined by programming in the "step

size" that the computer is to take in solving the problem. For the

dual-mode program, the variables printed out included the target

(TRGT) and the (EYE) as well as nine other variables of interest (see

lines 00006700 and 00006800 in the program shown in Appendix B).

From this digital data, plots can be made of the variables as a

function of time by fitting a curve to the data points. The choice

of inputs is controlled by the value of variable FF punched into a

data card (see lines 00002600 to 00003500 in the program).

5.1.1 Pulse Responses

By setting FF = 0 a pulse input is selected. Fig. 5.1 shows the

responses to a pulse of height S = 10° and width ST = 0.05 sec. The

normal response to this narrow pulse is a 10° pulse which begins

200 msec after the initial target motion. At t = 430 msec the system

returns the eye to the zero position. The rise-time exhibited by

the model is 15 msec which can be varied by programming in new values

of ρ. In the nystagmus mode the model reproduces this delayed and

widened pulse along with the simulated nystagmus. The magnitude of
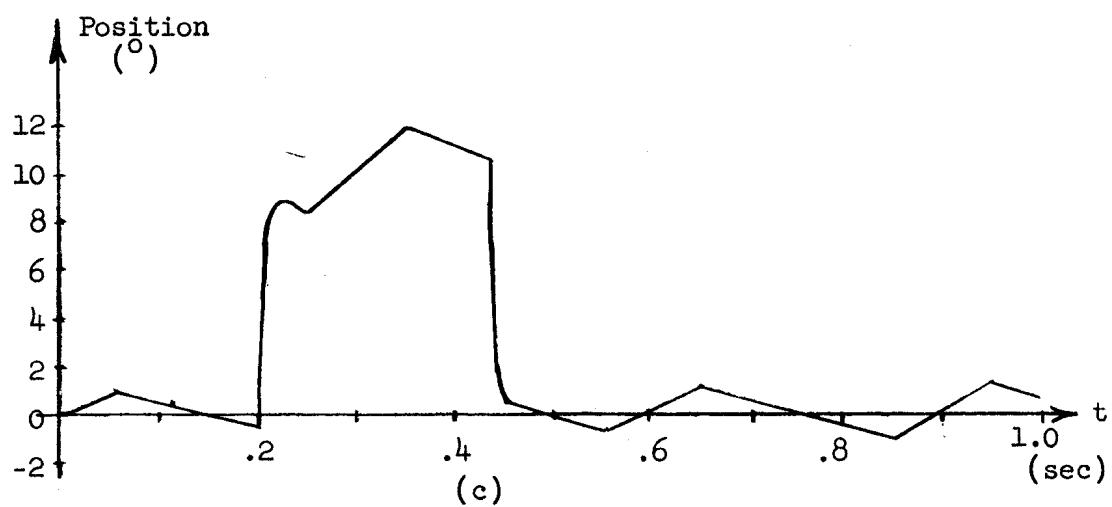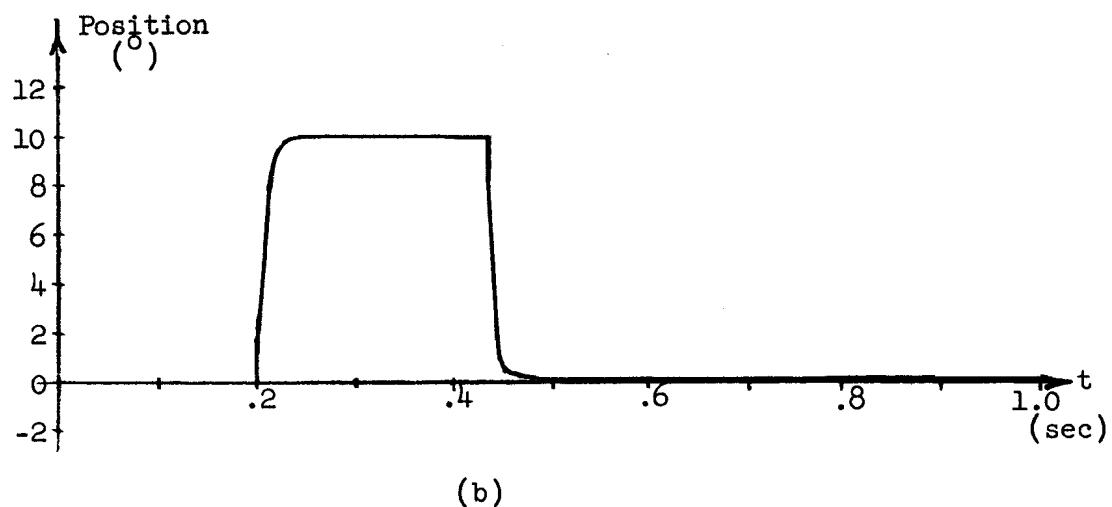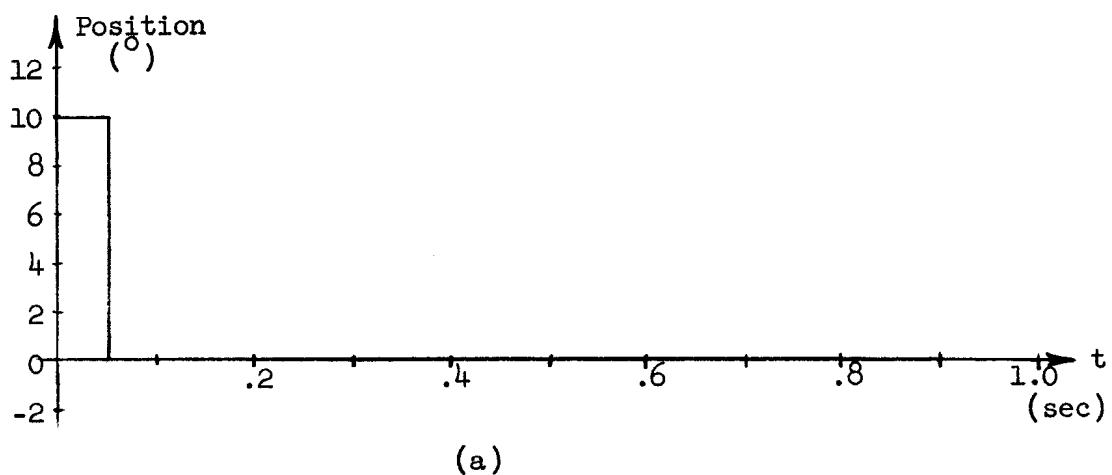
Fig. 5.1 Dual-mode pulse response (a) target (b) normal (c) nystagmus

the waveshape is a function of position as can be seen in Fig. 5.1 (c).
The independence of the positioning mechanism and the nystagmus simu-
lation can be seen by comparing the responses of the model in both modes.

5.1.2 Step Responses

When FF = 1 a step of magnitude S is the target. Fig. 5.2 shows
the responses to a 10° step change in target position. These responses
are again delayed changes in eye position. The eye then remains at
the 10° position or, in the nystagmus mode, varies about it.

5.1.3 Ramp Responses

At FF = 2 the target is a constant velocity (SL) starting at
t = 0. The values of SL are 10 and 20°/sec for Figures 5.3 and 5.4
respectively. The normal response to a ramp is a ramp of the same
magnitude, delayed by 200 msec, which is followed at t = 430 msec, by
a corrective step to bring the eye on target. The nystagmus mode
results in a response which has the same basic components plus the
nystagmus waveshape. This results in an oscillation about the line
described by the target motion.

5.1.4 Parabolic Responses

By setting FF = 3 a target with an acceleration equal to SL is
generated. Fig. 5.5 shows the responses to a parabola where SL = 10.
Inspection of Fig. 5.5 (b) reveals the following: (a) the model
responds with a ramp starting at t = 200 msec; (b) at t = 400 msec
the ramp is replaced by a ramp of greater magnitude since the target is
constantly accelerating; (c) at t = 430 msec the first corrective
saccade occurs and the eye continues at the velocity assumed at t = 400
msec; (d) at t = 600 msec the velocity is again changed to a higher
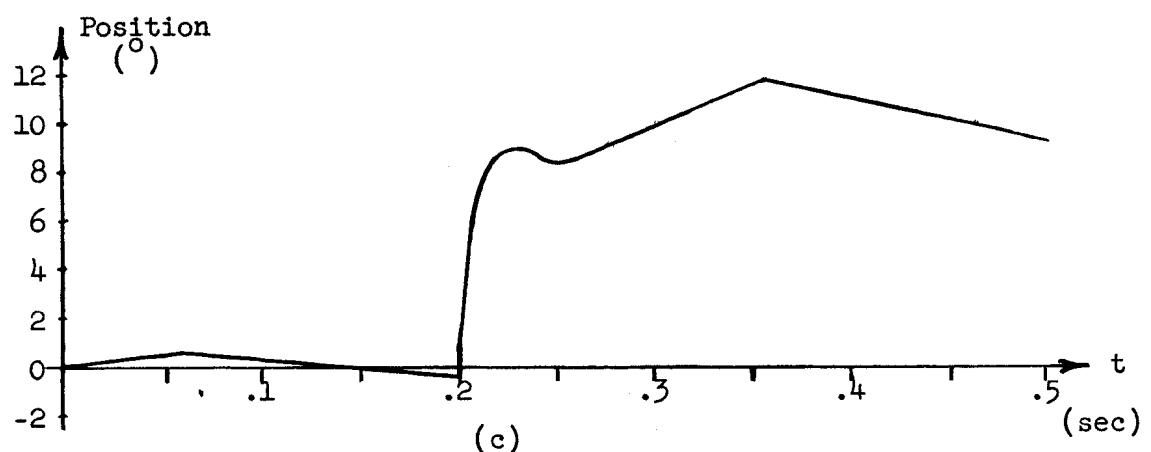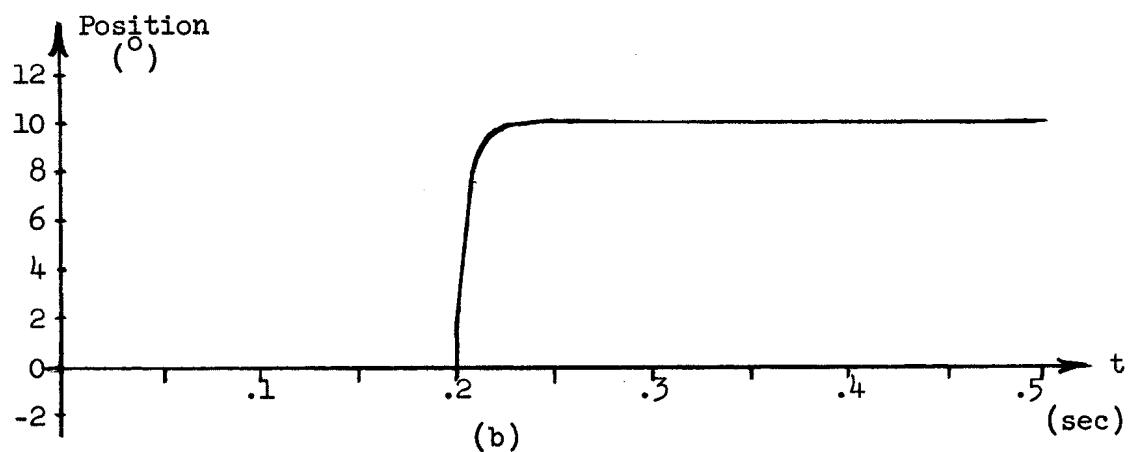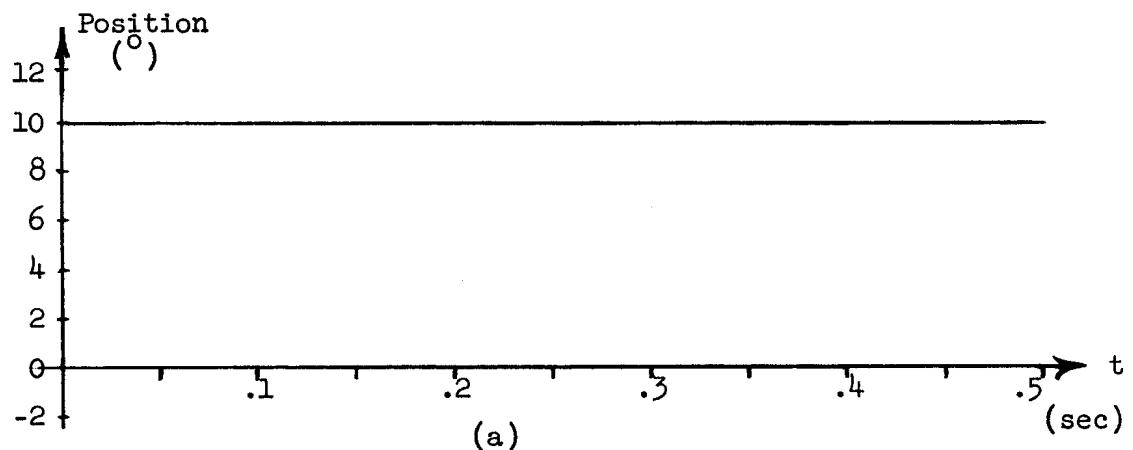
Fig. 5.2 Dual-mode step response (a) target (b) normal (c) nystagmus
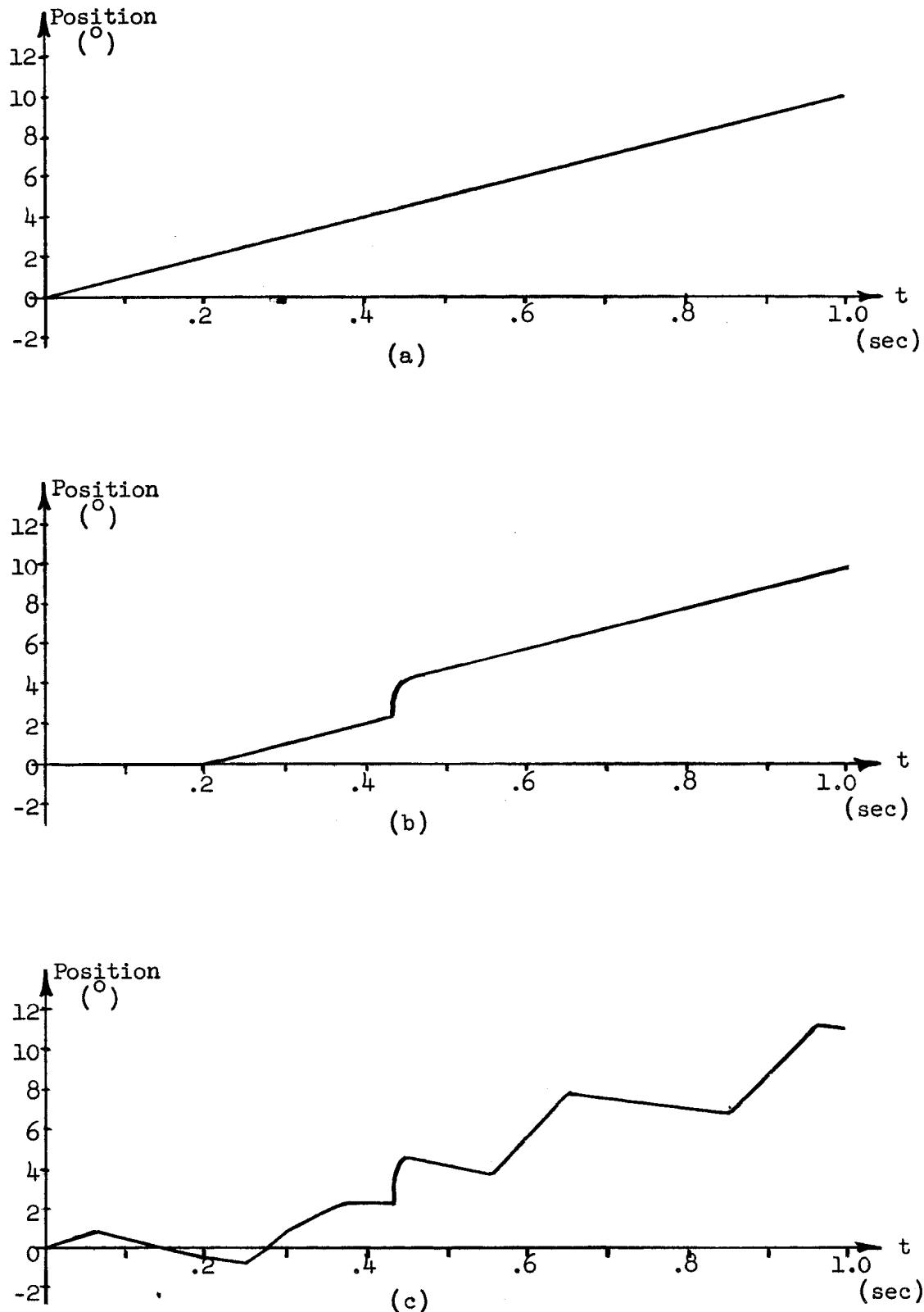
Fig. 5.3 Dual-mode ramp response (10°/sec) (a) target (b) normal
(c) nystagmus
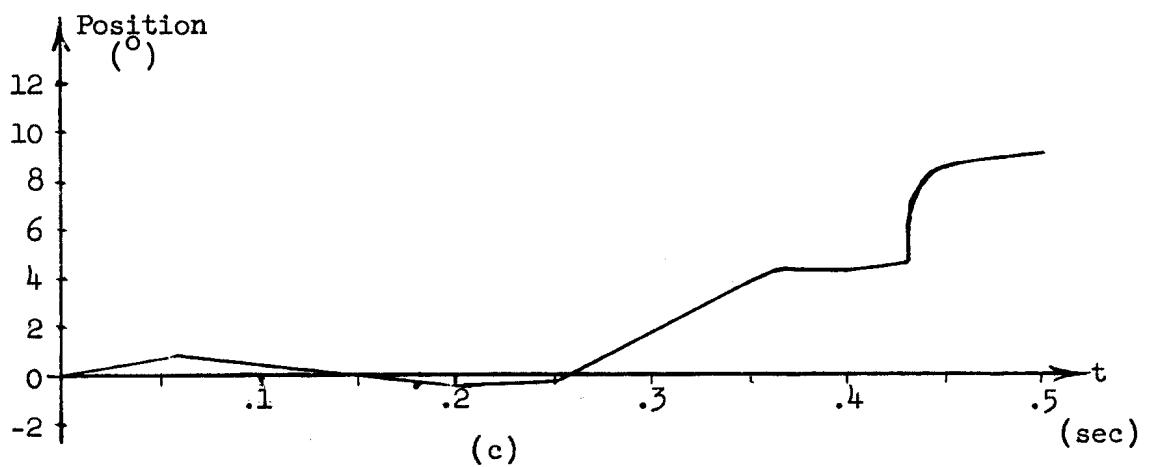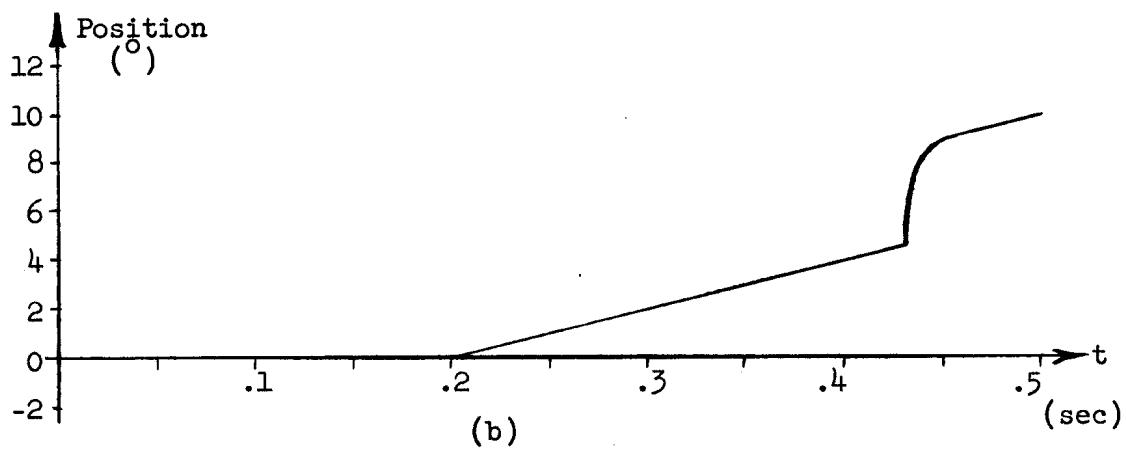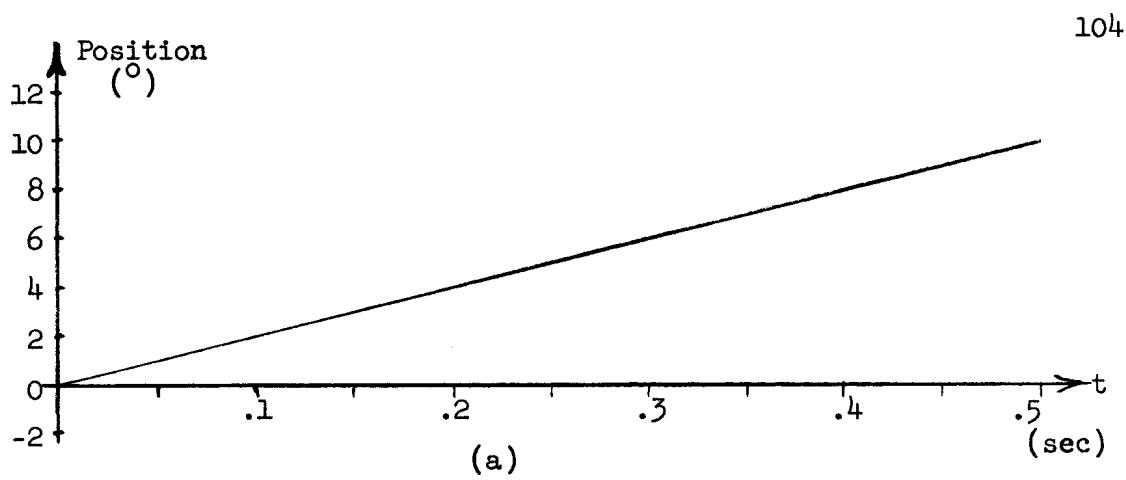
Fig. 5.4 Dual-mode ramp response (20°/sec) (a) target (b) normal
(c) nystagmus
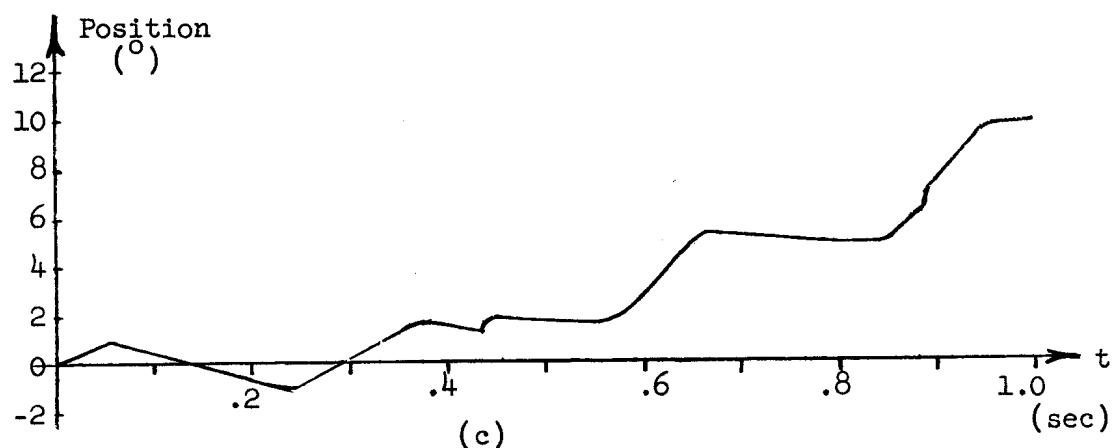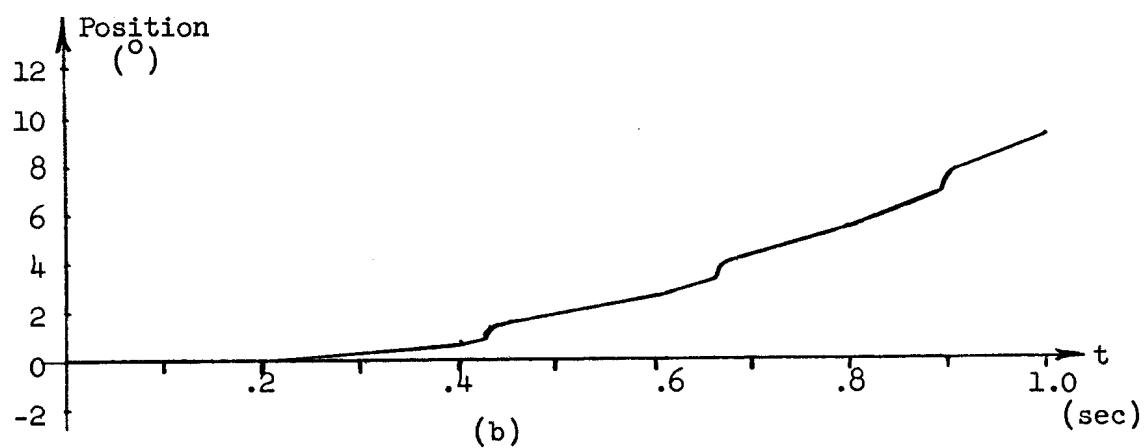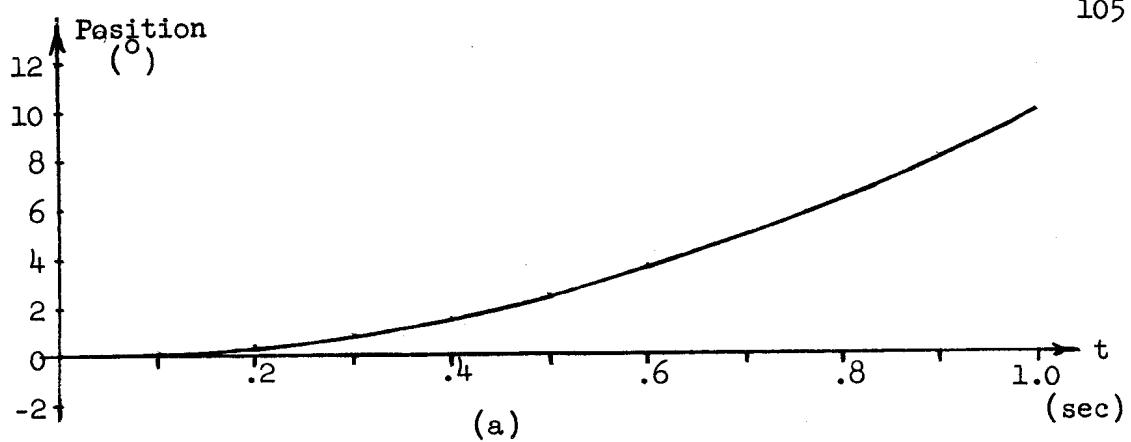
Fig. 5.5 Dual-mode parabolic response (a) target (b) normal
         (c) nystagmus

value and (e) at t = 660 msec the second saccade occurs and the eye con-
tinues with the velocity assumed at t = 600 msec. This continues as
long as tracking is required of the model. The nystagmus response,
shown in Fig. 5.5 (c) consists of exactly the same components but the
oscillation masks them quite well. Two of the three corrective saccades
are visible at 430 and 890 msec. Since the computer output consisted
of other variables in addition to target and eye position, the equiv-
alence of the two responses was verified. Note that the magnitude of
the nystagmus grows as the eye deviates from the straight-ahead position.

5.1.5 Step-Ramp Responses

FF = 4 provides a step-ramp input with the magnitude of the step
equal to S and the slope of the ramp equal to SL. Fig. 5.6 shows the
responses to a target where S = 10° and SL = 20°/sec. Both responses
demonstrate the independence of the two forward paths in the model.
Since the response to a step-ramp is merely the superposition of a step
response and a ramp response no additional descriptive comments will be
made. It should be noted that the return saccade in the nystagmus
response was not large enough and that additional saccades were neces-
sary, at t = 660 msec and t = 890 msec, to bring the eye on target.
This is consistent with observed physiological responses.

5.2 DISCUSSION OF RESULTS

The data presented in section 5.1 conform to the theoretical
responses presented in Chapter III which, in turn, were calculated
from a model designed to reproduce the physiological responses that
appear in Chapter II. This series of steps has led to the creation
of a computer program which will simulate either a normal person's
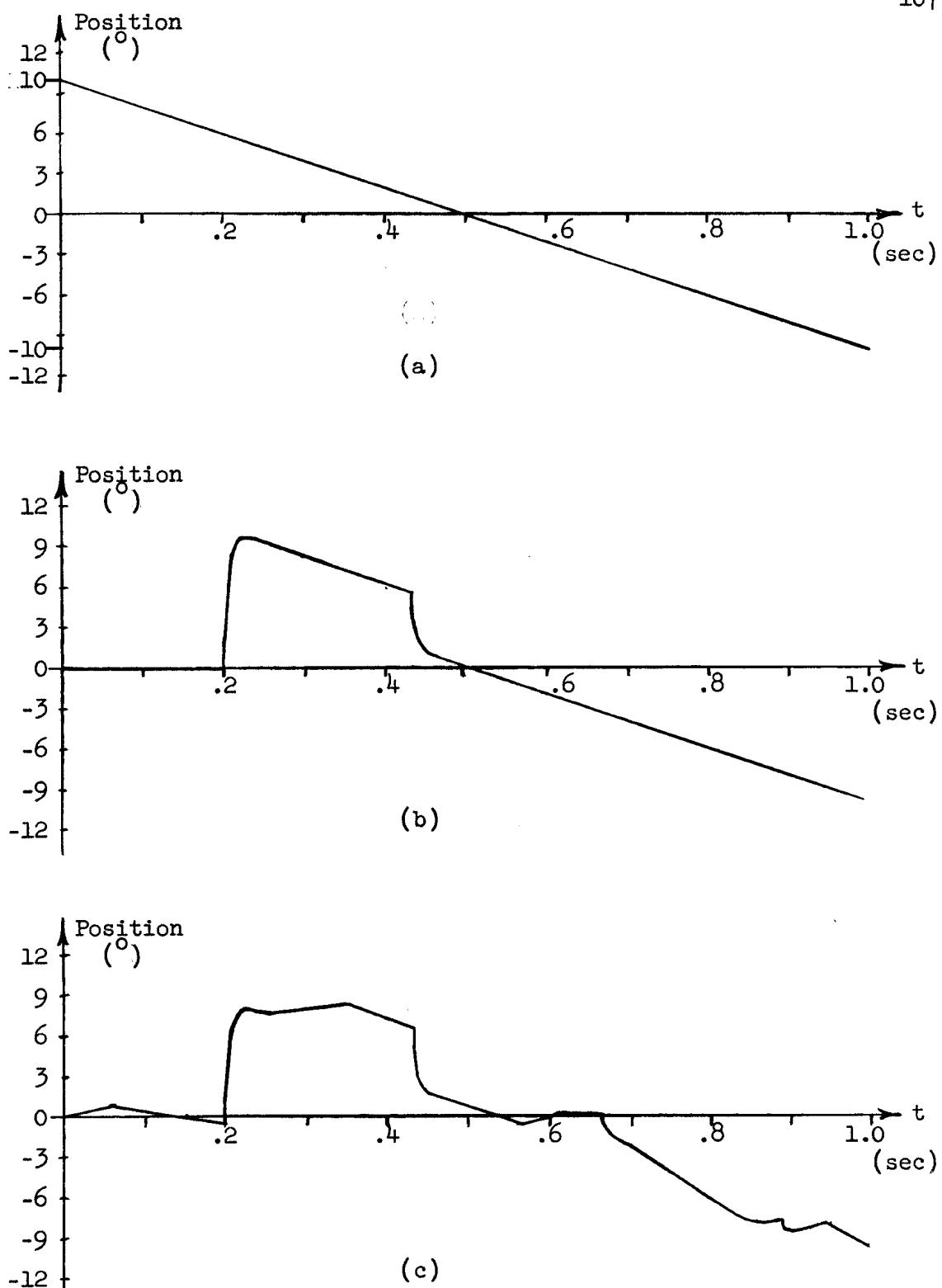
Fig. 5.6 Dual-mode step-ramp response (a) target (b) normal
(c) nystagmus

responses to a transient input or those of a person with nystagmus.
Provision is made in the program to vary the parameters of each input
or to provide additional inputs. Also, several parameters of the model
are variable so that the condition of the subject may be altered to
observe the effects on the response to a particular input. A method was
also presented which would yield the same model but would do so without
the use of a differentiator, thereby extending the availability of this
model.

The data collected from the subject with nystagmus verify the fact
that this affliction has no effect on the ability to track moving targets.
The responses of the two subjects were equal, both for transients and in
the predictive mode, as evidenced by the gain and phase plots shown in
Chapter II.

A mechanism was developed for nystagmus which is in agreement with
what little is known about the true mechanism. This mechanism, as pro-
grammed into the model, makes use of proprioceptive position feedback at
what would correspond to an unconscious level in the physiological sys-
tem. The position information modifies the amplitude of the nystagmus
waveform which is represented by an asymmetrical triangular wave whose
slope in one direction (left) is twice that in the other (right). The
source of this waveform is a center (unconscious) located in the mid-
brain which is affected by higher centers (conscious) that reflect the
subject's anxiety, tension, fatigue, etc. By varying the value of GAIN
in the model, the effects of these factors can be studied and might lead
to a better understanding of the many interconnecting pathways involved
in this oscillatory condition.

The mechanism proposed for nystagmus appears to conform to the physiological facts. However, the model's validity as an input-output representation is not dependent on such a correspondence. The object of this study was to produce a black box whose transient responses duplicated those of the biological system. The synthesis of a model whose components were functionally equivalent to those of the living system was undertaken in the hope that such a model would contribute more information about the actual mechanisms involved in horizontal nystagmus.

CHAPTER VI

CONCLUSIONS

A dual-mode, sampled-data control system has been synthesized which reproduces the transient responses of both the normal horizontal tracking system and the system with horizontal nystagmus. As stated in the introduction, the object of this study was to gather physiological data from both the normal and nystagmus-afflicted subject, compare these data, postulate a mathematical model which would simulate the responses of both systems to transient inputs and finally, to synthesize this model on a computer and thus, provide a physical vehicle for future studies of both the normal system and of the nystagmus condition itself. The model developed satisfies these requirements as evidenced by the data of Chapter V.

6.1 LIMITATIONS OF THE MODEL

Intelligent use of this model, and most other models, is dependent upon a through understanding of its theory of operation and, more importantly, the basic assumptions that went into its postulation. The first limitation is a result of the attempt to model a living system with an inherently deterministic model. Since the biological system is not a deterministic one, the responses to a given input are not always exactly the same but vary about some typical response both in time course and in magnitude. By constructing a deterministic model to represent a non-deterministic system we have eliminated this random variation about some "normal" response. The computer output to a given input will always be

the same in every detail. This discrepancy does not detract from the model's usefulness in studying the expected responses of either subject to a given target input.

Two other limitations are due to the methods chosen to represent the sampling mechanism. First, the sampling rates were fixed at some nominal values instead of being allowed to vary randomly about some mean value, as they do in the biological system. Should the variations about a norm be desired, the program could be re-written so that the values of T_1 and T_2 were not fixed but varied randomly about some mean values. The second limitation is due to the synchronization of the samplers with the onset of target motion. Physiological evidence points to non-synchronous sampling of the two paths. These two limitations have the effect of yielding normalized responses to given inputs instead of the observed responses where the time course of various components of a response may not always be the same (i.e., one time a saccade may precede a velocity change and another time, for the same input, the velocity change may occur first).

Another limitation is due to the piecewise-linear approximations used for the non-linearities of the system. The effects of these straight lines with well-defined break points are minimized by the many integrators in the system so that this is a minor limitation when one is looking at gross responses to transient inputs.

The piecewise-linear representation of the nystagmus waveform is another place·where a quasi-random variation in waveshape was normalized and represented by a triangular wave of constant shape and frequency. The elimination of the small notch in the slow phase represents a loss

of information, in a sense, since it may be due to factors important in the generation of this oscillation. However, the consideration of the small movements which accompany large responses was not a part of this study and the data taken did not yield enough information to enable the author to attempt to postulate a mechanism for such movements.

The model is also limited, by design, to the tracking of targets whose excursions lie within $\pm 10^{\circ}$. In this region the non-linear effects of movement limitations on the eye are not present.

6.2 FUTURE WORK

In spite of the above assumptions and limitations, the model does adequately represent the systems under study and can be a good foundation for elaboration and modification to make it suitable for future studies. One reason the model does a good job of representing the biological system is the incorporation of different sampling periods for the two paths. Although the periods are fixed and the rates synchronized, the refractory periods of the pursuit system (200 msec) and of the saccadic system (230 msec) compare well with the physiological data in both absolute and relative magnitudes. These data reveal that the refractory period of the pursuit system is slightly less than that of the saccadic system. Maintaining this difference in sampling rates and delaying the exact instant of sampling until a saccade is completed has resulted in a model whose methods as well as results agree with the known physiology.

Further study of the nystagmus waveshape and its relation to eye position (if any) appears to be a promising endeavor. If, indeed, there is a relationship between eye position and the size, shape or position of a notch in the slow phase of the nystagmus, the model could be

programmed to reflect this and it might yield new insight into the factors responsible for the disorder.

In conclusion, the study of an abnormality present in a complex biological control system, by methods previously alien to the medical field, has shed some new light on an old problem and perhaps has brought its solution a step closer.

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APPENDIX A

THE Z-TRANSFORM

This appendix will introduce the z-transform, describe its use
and list several simple transform pairs which were useful in the anal-
ysis presented in Chapter III. The reader who is unfamiliar with this
area is referred to the texts cited in Chapter III for a complete treat-
ment of this method of solution to sampled-data control systems.

When a signal $e(t)$ is sampled by means of an impulse train modu-
lator $\partial_T(t)$, where,

$$\partial_T(t) = \sum_{n=-\infty}^{\infty} \partial(t-nT) \qquad (A-1)$$

the resulting signal $e^*(t)$ is given by,

$$e^*(t) = e(t) \, \partial_T(t) \qquad (A-2)$$

$$= \sum_{n=-\infty}^{\infty} e(nT) \, \partial(t-nT) \qquad (A-3)$$

and, since $e(t) = 0$ for t less than 0,

$$e^*(t) = \sum_{n=0}^{\infty} e(nT) \, \partial(t-nT) \qquad (A-4)$$

the Laplace transform of $e^*(t)$ is

$$e^*(s) = \sum_{n=0}^{\infty} e(nT) \, e^{-nTs} \qquad (A-5)$$

The function, $E^*(s)$, contains the factor e^{-Ts}, which makes it a non-algebraic equation. This introduces difficulties in taking the inverse transform to return to the time domain. To alleviate this problem a change of variables is introduced as follows:

$$z = e^{-Ts} \tag{A-6}$$

or $\qquad s = -\ln z/T \tag{A-7}$

thus, $E^*(s)$ becomes the function of z, $E(z)$. This is called the z-transform of $e(t)$ and is equivalent to the Laplace transform of $e^*(t)$. Substitution of equation (A-6) into equation (A-5) yields,

$$E(z) = \sum_{n=0}^{\infty} e(nT)z^n \tag{A-8}$$

which is a rational function of z making it easy to take the inverse transform and return to a function of time.

A disadvantage of the z-transform is that it contains information about the corresponding time function, $e(t)$, only at the sampling instants nT. Thus, an $e(t)$ has a unique $E(z)$ but for a given $E(z)$ there are an infinite number of functions of time. An example of this would be a sinusoid and a d-c value of zero. If the period of the sinusoid is T and we sampled both time functions with a sampling period of T/2, the resulting $e^*(t)$ would be exactly the same for each and the knowledge of $E(z)$ would not be sufficient to define the corresponding time function. Since the discreet $E(z)$ may represent any continuous function $e(t)$ that has the same values at the sampling

instants as $e^{*}(t)$, we are faced with the problem of loss of information necessary to reconstruct the signal. Looking back at equation (A-5) for $E^{*}(s)$, one can see that it may be re-written as

$$E^{*}(s) = \frac{1}{T} \sum_{k=-\infty}^{\infty} E(s+jk\omega_o), \text{ where } \omega_o = \frac{2\pi}{T} \qquad (A-9)$$

This is a periodic function with a frequency, ω_o, which is the sampling frequency. What this transformation does in the frequency domain is to transform a continuous spectrum into an infinite series where the original spectrum is repeated at intervals of ω_o. This being the case, one can see that the original $E(s)$ must be band-limited with no component higher than $\omega_o/2$ for the information to be recovered. Only under these conditions does $E(z)$ carry all the information present in the Laplace transform, $E(s)$ and, by means of an ideal filter, $e(t)$ can be recovered. Table A.1 shows some of the elementary transform pairs which were useful in the analysis of the model.

TABLE A.1

Transform Pairs		
LAPLACE TRANSFORM $E(s)$	TIME FUNCTION $e(nT)$	Z-TRANSFORM $E(z)$
e^{-kTs}	$\partial(n-k)T$	z^k
$\frac{1}{s}$	1	$\frac{1}{1-z}$
$\frac{1}{s^2}$	nT	$\frac{Tz}{(1-z)^2}$

The other aid to solution of sampled-data systems is the composite
flow graph. This is obtained by combining the continuous flow graph
and the sampled flow graph. Once this is done, the transfer function
may be written by inspection using Mason's gain formula. The mechanics
of this operation are presented very clearly in Kuo's text (110).

APPENDIX B

THE DUAL-MODE PROGRAM

The following pages are copies of the Analog-Algol program used
to simulate the dual-mode model on a Burroughs B-5500 digital computer.
The program appears as it was printed out by the computer. On the
left are the command statements and on the right the card identifi-
cation numbers for the computer.

MONDAY, JUNE 26, 1967   NRL ALGOL VERSION OF 10/20/66

BEGIN

COMMENT A.LAVI - BIOLOGICAL PROGRAM;
ARRAY TRGT,ERR,DERR,TEMP1,TEMP2,TEMP3, TEMP4,SERR,DUM7,AUX1,AUX2,EYE,
DUMA,DUM2,DUM3,AUX3,DUM4,DUM5,DUM6,DUM1,DEYE,DDEYE,TEMP2P,TEMP1D[0:9];
DEFINE ANALOG= TRGT 0 ERR 0 DERR 0 TEMP1 0 TEMP2 0 TEMP3 0 TEMP4 0 SERR
0 TEMP2P 0 TEMP1D 0 DUM2 0 DUM3 0 DUM4 0 DUM5 0 DUM6 0 DUM7
0 AUX3 0 DUM8 0 AUX2 0 AUX1 0 EYE 0 DUM1 0 DEYE 0 DDEYE#;
INTEGER FF,SRR,SRC;
ARRAY X1,Y1,X2,Y2[0:5],X3,Y3[0:4];
REAL S, IC,OMEGA,OMEG2,XI,PXI,ST,SL,N,GAIN;

READ(READER,/,FF,S,ST,SL,SRR,SRC,IC,OMEGA,XI,H,NUMBERSTEPS,N);
PXI + XI × OMEGAx2; OMEG2 + OMEGA*2;
FILL XI[*] WITH -30,-26,-25,-25,26,30;

FILL Y1[*] WITH 0 , 0, -25, 25, 0, 0;

FILL X2[*] WITH -.5,-.26,-.25,.25,.26,.5;

FILL Y2[*] WITH -.5,-.26,0,0,.26,.5;

FILL X3[*] WITH -12,-10,2,4,10;

FILL Y3[*] WITH 2,2,.85,1,5,1,5;

READ(READER,/,MAXIMUMITERATIONS,OPTIONITERATE,GAIN);

WRITE(PRINTER,<
       TIME   TARGET   TEMP2   TEMP4
AUX1   AUX3   EYE">);

TITLEOFFORMATS+TRUE;NUMBERDELAYERS+2;
BEGINANALOG

START OF SEGMENT ********** 11   010
                                  0002
00000000 SC **********
00000200 SC   2; 20512
00000300 SC   2; 20512
00000400 SC   2; 20813
00000500 SC   2; 30310
00000600 SC   2; 30310
00000800 SC   2; 30310
00000900 SC   2; 30310
00001000 SC   2; 30712
00001010 SC   2; 30712
00001020 SC   2; 30712
00001100 SC   2; 30712
00001200 SC   2; 33113
00001300 SC   2; 33413

START OF SEGMENT ********** 0051
0051 IS 0006 LONG, NEXT SEG 0002
0001400 SC   2; 33612
START OF SEGMENT ********** 0052
0052 IS 0006 LONG, NEXT SEG 0002
0001500 SC   2; 33811
START OF SEGMENT ********** 0053
0053 IS 0006 LONG, NEXT SEG 0002
0001600 SC   2; 34010
START OF SEGMENT ********** 0054
0054 IS 0006 LONG, NEXT SEG 0002
0001610 SC   2; 34113
START OF SEGMENT ********** 0055
0055 IS 0005 LONG, NEXT SEG 0002
0001620 SC   2; 34312
START OF SEGMENT ********** 0056
0056 IS 0005 LONG, NEXT SEG 0002
0001700 SC   2; 345;1
0001800 SC   2; 35713
0001900 SC   2; 35713
0002000 SC   57; 35912
0002100 SC   57; 350;2
0057 IS 0021 LONG, NEXT SEG 0002
0002200 SC   2; 36013
0002300 SC   2; 36211
START OF SEGMENT ********** 0058
START OF SEGMENT ********** 0059

129

```
CONSTANT S INTO DUM2 ENDIT;

RAMP SLOPE 1 AT 0 INTO DUM3 ENDIT;

IF FF=0 THEN
CONSTANT(IF TIME≤ST THEN DUM2[1] ELSE 0)INTO TRGT ENDIT ELSE

IF FF=1 THEN
CONSTANT S INTO TRGT ENDIT ELSE

IF FF=2 THEN
RAMP SLOPE SL AT 0 INTO TRGT ENDIT ELSE

IF FF=3 THEN
MULTIPLY(DUM3,DUM3,SL,TRGT) ELSE
IF FF=4 THEN
SUM(DUM2,1,DUM3,SL,TRGT);

CONSTANT(IF TIMEI MOD(SRR)=0 THEN 1/H ELSE 0) INTO SERR ENDIT;

IF N=1 THEN
LINEAR R STEPS 20,.05,-10,.25,20,.35,-10,.55,20,.65,-10,.85,20,.95,-10

INTO DUM4 ENDIT;

ENDFORCING;
SUM3(TRGT,1,EYE,-1,DUM7,1,ERR);
INTERPOLATOR(ERR,X2,Y2,5,2,AUX1);
DIFART(AUX1,1,DERR);
INTERPOLATOR(DERR,X1,Y1,5,1,TEMP1);
DELAY 2,.02,TEMP1 INTO TEMP1D ENDIT;
```

```
0059 IS 0037 LONG, NEXT SEG 0058
START OF SEGMENT ********* 0060
0060 IS 0037 LONG, NEXT SEG 0058
         00002400 SC 58: 5611

START OF SEGMENT ********* 0061
0061 IS 0002 LONG, NEXT SEG 0058
         00002500 SC 58: 5710

START OF SEGMENT ********* 0062
0062 IS 0003 LONG, NEXT SEG 0053
         00002600 SC 58: 5810
         00002700 SC 58: 5813

START OF SEGMENT ********* 0063
0063 IS 0005 LONG, NEXT SEG 0052
         00002800 SC 58: 6010
         00002900 SC 58: 6111

START OF SEGMENT ********* 0064
0064 IS 0002 LONG, NEXT SEG 0058
         00003000 SC 58: 6310
         00003100 SC 58: 6411

START OF SEGMENT ********* 0065
0065 IS 0003 LONG, NEXT SEG 0058
         00003200 SC 58: 6610
         00003300 SC 58: 4711
         00003400 SC 58: 7013
         00003500 SC 58: 7210
         00003600 SC 58: 7513
         00003700 SC 58: 7513

START OF SEGMENT ********* 0066
0066 IS 0005 LONG, NEXT SEG 0058
         00003800 SC 58: 7710
         00003900 SC 58: 7710
         00003910 SC 58: 7710
         00003970 SC 58: 7713

START OF SEGMENT ********* 0067
START OF SEGMENT ********* 0068
0068 IS 0016 LONG, NEXT SEG 0047
0067 IS 0022 LONG, NEXT SEG 0053
         00004100 SC 58: 7010
         00004200 SC 58: 8013
         00004210 SC 58: 8511
         00004300 SC 58: 8011
         00004400 SC 58: 9112
         00004500 SC 58: 9512

START OF SEGMENT ********* 0065
0069 IS 0005 LONG, NEXT SEG 0052
```

```
MULTIPLY(TEMP1D, SERR, 1, TEMP2P)$
INT(TEMP2P,1,0,TEMP2)$
LIMIT(TEMP2,-25,25,TEMP3)$
INT(TEMP3,1,IC,TEMP4)$

IF TIME1 MOD(SRC)=0 THEN
BEGIN
    IF AUX2[0]=0 THEN
        BEGIN
            AUX2[0]+2$
            AUX2[4]+AUX2[1]
        END$
        AUX2[1]+AUX2[4]+AUX1[1]$
    AUX2[2]+AUX2[3]+0$
    END ELSE AUX2[0]+2$
DELAY 0,.20,AUX2 INTO AUX3 ENDIT$


SUM3(AUX3,1,TEMP4,1,DUM6,1,DUM1)$
SUM3(DUM1,OMEG2,DEYE,-PXI,EYE,-OMEG2,DDEYE)$
INT(DDEYE,1,0,DEYE)$
INT(DEYE,1,0,EYE)$
INTERPOLATOR(EYE,X3,Y3,4,3,DUM5)$
MULTIPLY(DUM4,DUM5,GAT,DUM6)$
DELAY 1,.01,DUM6 INTO DUM7 ENDIT$


CONVERGE$



WRITEV TRGT[1],TEMP2[1],TEMP4[1],AUX1[1],AUX3[1],EYE[1],
    ERR[1], DUM5[1], DUM6[1], DUM1[1],AUX2[1] ENDIT$
SETINITIALIZATIONFLAG-NOT INITIALIZATIONFLAG$
ENDANALOG$


WRITEV OMEGA,OMEG2,XI, PXI  ENDIT$ FINISH.

ARCTAN    IS SEGMENT NUMBER 0076, PRT ADDRESS IS 0200
COS                                              0203
                                                 0077
```

```
00004600 SC  5R:  9610
00004700 SC  5R:  9910
00004800 SC  5R: 10112
00004900 SC  5R: 10411
00005000 SC  5R: 10613
00005100 SC  5R: 10613
00005200 SC  5R: 10810
00005300 SC  5R: 10812
00005400 SC  5R: 10912
00005500 SC  5R: 11010
00005600 SC  5R: 11111
00005700 SC  5R: 11210
00005800 SC  5R: 11213
00005900 SC  5R: 11510
00006000 SC  5R: 11711
00006100 SC  5R: 11910
START OF SEGMENT ********** SEG 0070
0070 IS 0005 LONG, NEXT SEG 0058
00006200 SC  5R: 12040
00006300 SC  5R: 12411
00006400 SC  5R: 12910
00006500 SC  5R: 13112
00006510 SC  5R: 13410
00006520 SC  5R: 13810
00006530 SC  5R: 14110
START OF SEGMENT ********** SEG 0071
0071 IS 0005 LONG, NEXT SEG 0058
00006600 SC  5R: 14210
START OF SEGMENT ********** SEG 0072
START OF SEGMENT ********** SEG 0073
0073 IS 0049 LONG, NEXT SEG 0072
0072 IS 0066 LONG, NEXT SEG 0058
START OF SEGMENT ********** SEG 0074
0074 IS 0081 LONG, NEXT SEG 0058
00006700 SC  5R: 14510
00006800 SC  5R: 16410
00006900 SC  5R: 17211
00007000 SC  5R: 17511
START OF SEGMENT ********** SEG 0075
0075 IS 0076 LONG, NEXT SEG 0058
0058 IS 0216 LONG, NEXT SEG 0002
00007100 SC  2: 34210
0002 IS 0385 LONG, NEXT SEG 0001
```

```
EXP             0078    0210
LN              0079    0215
SIN             0080    0204
SQRT            0081    0253
OUTPUT(W)       0082    0032
BLOCK CONTROL   0083    0005
INPUT(W)        0084    0237
X TO THE I      0085    0221
GO TO SOLVER    0086    0214
ALGOL WRITE     0087    0014
ALGOL READ      0088    0015
ALGOL SELECT    0089    0016
```

NUMBER OF ERRORS DETECTED = 000   LAST CARD WITH ERROR HAS SEQ #
PRT SIZE=0318; TOTAL SEGMENT SIZE=03686 WORDS;   DISK STORAGE REQ.=04110 WORDS;   NO. SEGS.=0090.
ESTIMATED CORE STORAGE REQUIREMENT = 07095 WORDS.
21123186 MONDAY, JUNE 26, 1967   PROCESSOR TIME = 25.23 SECONDS   I/O TIME = 44.23 SECONDS