Mohr's Circle

Input stress tensor (generalized plane stress)

```
ClearAll["Global`*"]
```

```
sigma = {{7, 4, 0}, {4, 2, 0}, {0, 0, -1}};
sigma = {{-4, 4, 0}, {4, 2, 0}, {0, 0, -1}};
sigma = {{4, 5, 0}, {5, -3, 0}, {0, 0, 8}};
sigma = {{-40, 0, 0}, {0, -10, 30}, {0, 30, -20}};
sigma = {{-40, 0, 30}, {0, -10, 0}, {30, 0, -20}};
sigma = N[{{-40, 0, 30}, {0, -10, 0}, {30, 0, -20}}];
sigma = {{4, 5, 0}, {5, -3, 0}, {0, 0, 8}};
sigma // MatrixForm
```

$$\begin{pmatrix} 4 & 5 & 0 \\ 5 & -3 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

Determine which plane shear stress is in

```
If[sigma[[1, 2]] ≠ 0, plane = "xy"]
If[sigma[[1, 3]] ≠ 0, plane = "xz"]
If[sigma[[2, 3]] ≠ 0, plane = "yz"]
```

```
xy
```

Find points of interest; dependent on which plane the \

shear stress is in

\

This assigns values to givenpoints[[]] (several points on the circle), \

center, and radius\

```
If[plane == "xy", {givenpoints = {{sigma[[1, 1]], -sigma[[1, 2]]},
      {sigma[[2, 2]], sigma[[1, 2]]}, {sigma[[3, 3]], 0}}, center = (sigma[[1, 1]] + sigma[[2, 2]]) / 2,
    radius = Sqrt[((sigma[[1, 1]] - sigma[[2, 2]]) / 2) ^ 2 + sigma[[1, 2]] ^ 2]}];
If[plane == "xz", {givenpoints = {{sigma[[1, 1]], sigma[[1, 3]]}, {sigma[[2, 2]], 0},
      {sigma[[3, 3]], -sigma[[1, 3]]}}, center = (sigma[[1, 1]] + sigma[[3, 3]]) / 2,
    radius = Sqrt[((sigma[[1, 1]] - sigma[[3, 3]]) / 2) ^ 2 + sigma[[1, 3]] ^ 2]}];
If[plane == "yz", {givenpoints = {{sigma[[1, 1]], 0}, {sigma[[2, 2]], -sigma[[2, 3]]},
      {sigma[[3, 3]], sigma[[2, 3]]}}, center = (sigma[[2, 2]] + sigma[[3, 3]]) / 2,
    radius = Sqrt[((sigma[[2, 2]] - sigma[[3, 3]]) / 2) ^ 2 + sigma[[2, 3]] ^ 2]}];
```

Find principle stresses and (smallest) angle to \

principle plane

This computes principal normal stresses

```
If[plane == "xy",
  {sigmaP1 = center + radius,
   sigmaP2 = center - radius,
   sigmaP3 = sigma[[3, 3]]}];
If[plane == "xz",
  {sigmaP1 = center + radius,
   sigmaP2 = sigma[[2, 2]],
   sigmaP3 = center - radius}];
If[plane == "yz",
  {sigmaP1 = sigma[[1, 1]],
   sigmaP2 = center + radius,
   sigmaP3 = center - radius}];
N[{sigmaP1, sigmaP2, sigmaP3}]
```

```
{6.60328, -5.60328, 8.}
```

Find (smallest) angle to principle planes

```
minzero[x_] := {min = Max[Abs[x]];
   For[n = 1, n ≤ Length[x], n++,
    If[ Abs[ x[[n]] ] < Abs[min], min = x[[n]] ]
    ];
   min};
givenpoints
center
thetaP1 = (1 / 2) * ArcTan[( givenpoints [[1, 1]] - center ), givenpoints [[1, 2]]]
thetaP2 = (1 / 2) * ArcTan[( givenpoints [[2, 1]] - center ), givenpoints [[2, 2]]]
```

{{4, -5}, {-3, 5}, {8, 0}}

$$\frac{1}{2}$$

$$-\frac{1}{2}\,\text{ArcTan}\left[\frac{10}{7}\right]$$

$$\frac{1}{2}\left(\pi - \text{ArcTan}\left[\frac{10}{7}\right]\right)$$

```
5 / 3.5
```

1.42857

```
ArcTan[7, 10]
```

$$\text{ArcTan}\left[\frac{10}{7}\right]$$

```
N[10 / 7]
```

1.42857

Display output

Compute display parameters

\

Displaycenter is the center of the viewing area; displaydim is a vector \

containing half the display width, half the display height; displayrange is \

formatted {{xmin, xmax},{ymin, ymax}};

Displayelement is a small length characteristic of the size of the display \

(this helps with scaling the graphics appropriately); numtics is the number \

of ticks to show along each axis;

Tickspacingx is the spacing between ticks based on numticks and the display \

range\

```
displaycenter = {(Min[sigmaP1, sigmaP2, sigmaP3] + Max[sigmaP1, sigmaP2, sigmaP3]) / 2, 0};
displaydim =
  1.5 * .5 * {Max[sigmaP1, sigmaP2, sigmaP3] - Min[sigmaP1, sigmaP2, sigmaP3], 2 * radius};
displayrange = {{displaycenter [[1]] - displaydim [[1]], displaycenter [[1]] + displaydim [[1]]},
   {displaycenter [[2]] - displaydim [[2]], displaycenter [[2]] + displaydim [[2]]}};
displayelement = .1 * ((displaydim [[1]] + displaydim [[2]]) / 2);

numticks = 10;
tickspacingx = Abs[ displayrange [[1, 2]] - displayrange [[1, 1]] ] / numticks ;
tickspacingy = Abs[ displayrange [[2, 2]] - displayrange [[2, 1]] ] / numticks ;
```

Circles

\

First the circle for the faces of the unrotated stress cube is determined. \

Then the other two circles are found such that they intersect the remaining \

principal stresses. The circle information is put in a generic circledata \

array, to be used to generate the graphics primitives.\

```
If[plane == "xy",
  circledata = {{center, 0, radius}, {(sigmaP2 + sigmaP3) / 2, 0, Abs[sigmaP2 - sigmaP3] / 2},
    {(sigmaP1 + sigmaP3) / 2, 0, Abs[sigmaP1 - sigmaP3] / 2}}];
If[plane == "xz",
  circledata = {{center, 0, radius}, {(sigmaP1 + sigmaP2) / 2, 0, Abs[sigmaP1 - sigmaP2] / 2},
    {(sigmaP2 + sigmaP3) / 2, 0, Abs[sigmaP2 - sigmaP3] / 2}}];
If[plane == "yz",
  circledata = {{center, 0, radius}, {(sigmaP1 + sigmaP2) / 2, 0, Abs[sigmaP1 - sigmaP2] / 2},
    {(sigmaP1 + sigmaP3) / 2, 0, Abs[sigmaP1 - sigmaP3] / 2}}];
```

\

The largest circle is determined. circledata[[largestcircle]] is then the row \

containing enough information to draw the largest circle.\

```
largestcircle = 1;
rowmax = Dimensions[circledata , 1];
For[row = 1, row ≤ rowmax[[1]], row++,
 {
  If[circledata[[row, 3]] > circledata[[largestcircle , 3]], largestcircle = row]
 }
]
```

\

Finally the graphics objects are created, all three circles in one foul \

swoop. Since all graphics objects in the show command follow the formatting \

rules set by the first, and the circle object is always shown in the graphics \

output, it was chosen to contain all plot formatting information, such as \

axis labels and tick marks.\

```
circle1 := Graphics[{Circle[{circledata[[1, 1]], circledata[[1, 2]]}, circledata[[1, 3]]],
     Circle[{circledata[[2, 1]], circledata[[2, 2]]}, circledata[[2, 3]]],
     Circle[{circledata[[3, 1]], circledata[[3, 2]]}, circledata[[3, 3]]]},
    Axes → True,
    AxesLabel → {Subscript[σ, nn], Subscript[σ, ns]},
    LabelStyle → Directive[Large, Bold],
    Ticks → { Floor[ Table[tickspacingx * Floor[.5 * n] * (-1) ^ Floor[1.5 * n],
         {n, 0, Ceiling[(2 * Max[Abs[ displayrange [[1]] ]] + 1) / tickspacingx ]}] ],
      Floor[ Table[tickspacingy * Floor[.5 * n] * (-1) ^ Floor[1.5 * n],
         {n, 0, Ceiling[(2 * Max[Abs[ displayrange [[2]] ]] + 1) / tickspacingy ]}] ] },
    TicksStyle → Directive[Small, Italic],
    PlotRange → displayrange ,
    AspectRatio → 1];
```

Lines

\

This creates the line graphics object which draws a line from the appropriate \

points related to the faces of the unrotated stress cube.\

```
If[plane == "xy",
  line1 := Graphics[Line[{givenpoints[[1]], givenpoints[[2]]}]]];
If[plane == "xz",
  line1 := Graphics[Line[{givenpoints[[1]], givenpoints[[3]]}]]];
If[plane == "yz",
  line1 := Graphics[Line[{givenpoints[[2]], givenpoints[[3]]}]]];
```

Points

\

Pointsdata is an array of two-element vectors representing points to be \

displayed. The order of the points in this array is identical to the order in \

which they are assigned annotation elements.\

```
pointsdata = {givenpoints[[1]], givenpoints[[2]],
   givenpoints[[3]], {center, 0}, {sigmaP1, 0}, {sigmaP2, 0}, {sigmaP3, 0},
   {circledata[[largestcircle, 1]], circledata[[largestcircle, 3]]},
   {circledata[[largestcircle, 1]], -circledata[[largestcircle, 3]]}};
```

\

A graphics object to generate a plot of all points is next created.\

```
points1 := Graphics[
   {PointSize[Large], Red, Point[pointsdata]}
  ];
```

Annotation

\

Create r1, a set of vectors pointing from the origin to the points to be \

annotated\

```
r1 = N[ pointsdata - ConstantArray [displaycenter , Dimensions [pointsdata , 1]] ];
dim = Dimensions [r1 , 1];
r1norm = N[ Table[ {Norm[ r1[[i]] ], Norm[ r1[[i]] ]}, { i , dim[[1]] } ] ];
```

\

Create r2, a set of vectors pointing from the points to be annotated out a \

small distance away from the points\

```
r2 = ConstantArray [{0 , 0}, dim[[1]]];
For[row = 1, row ≤ dim[[1]], row ++,
 If[ r1norm[[row , 1]] ≠ 0, r2[[row]] = displayelement * r1[[row]] / r1norm[[row]] ]
]
```

\

Create annotationdata, the array of points at which annotations will \

eventually be placed. The points in annotationdata are slightly offset from \

those in pointsdata, so that the annotations do not overlap the points.\

```
annotationdata = ConstantArray [displaycenter , Dimensions [pointsdata , 1]] + r1 + r2 ;
```

\

Debug annotationdata; move annotation elements that are near axes away from \

those axes\

```
dim = Dimensions [annotationdata , 1];
For[row = 1, row ≤ dim[[1]], row ++, {
   If[ Abs[annotationdata [[row, 1]] ] < displayelement ,
    { annotationdata [[row, 1]] =
       displayelement * (2 * Ceiling [(Sign[annotationdata [[row, 1]]] + 1) / 2] - 1),
      r2[[row]] = annotationdata [[row]] - pointsdata [[row]] }
   ],
   If[ Abs[annotationdata [[row, 2]] ] < displayelement ,
    { annotationdata [[row, 2]] =
       displayelement * (2 * Ceiling [(Sign[annotationdata [[row, 2]]] + 1) / 2] - 1),
      r2[[row]] = annotationdata [[row]] - pointsdata [[row]] }
   ]
  }];
```

\

Determine which points in annotationdata are "duplicates", where a \

"duplicate" pair of pionts are two points separated by no more than the \

length displayelement.\

```
duplicates = {};
temparray = annotationdata ;
dim = Dimensions [temparray , 1];
For[row1 = 1, row1 ≤ dim[[1]] - 1, row1 ++,
 {temp = {row1},
  For[row2 = row1 + 1, row2 ≤ dim[[1]], row2 ++,
   If[EuclideanDistance [ temparray [[row2]], temparray [[row1]] ] ≤ displayelement &&
      temparray [[row2]] ≠ {},
    {temp = Append[temp , row2], temparray [[row2]] = {}}
   ]
  ],
  If[ Length[temp] > 1, duplicates = Append[duplicates , temp] ]}
]
```

\

Step through the rows of the duplicates[[]] array, and separate all points \

that are considered "overlapping". All r2 vectors associated with the \

points in the rows of duplicates[[]] are rotated to generate an angular \

spread.\

```
rmatrix[theta_] = {{Cos[theta], Sin[theta]}, {-Sin[theta], Cos[theta]}};
dim = Dimensions[duplicates, 1];
For[row = 1, row ≤ dim[[1]], row++,
 {numcollisions = Length[duplicates[[row]]],
  If[numcollisions > 1,
   {spread =
     (Pi / 6) * (Range[0, numcollisions - 1] - N[Median[Range[0, numcollisions - 1]]]),
    For[col = 1, col ≤ Length[duplicates[[row]]], col++,
     r2[[duplicates[[row, col]]]] = r2[[duplicates[[row, col]]]].rmatrix[spread[[col]]]
    ]}
  ]}
]

annotationdata = ConstantArray[displaycenter, Dimensions[pointsdata, 1]] + r1 + r2;
```

\

Create graphics objects that will draw annotation elements at the points \

contained within annotationdata\

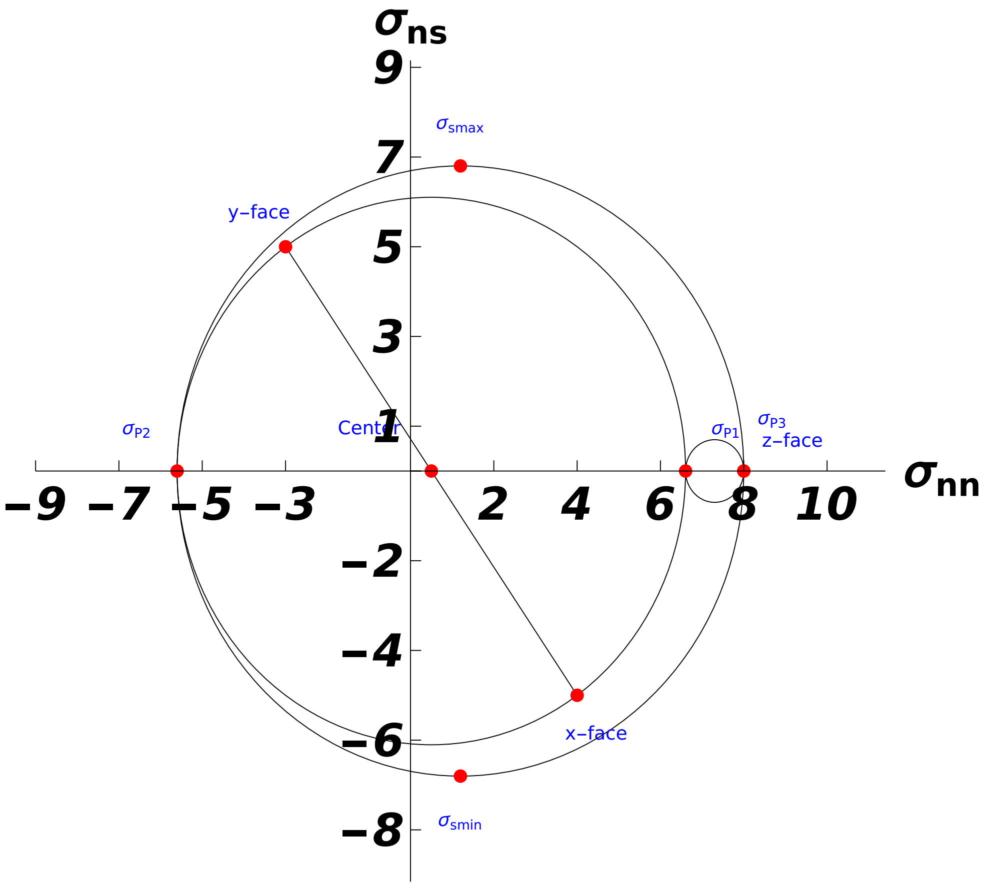
```
annotation1 := Graphics[Text[Style["x-face", Larger, Blue], annotationdata[[1]]]];
annotation2 := Graphics[Text[Style["y-face", Larger, Blue], annotationdata[[2]]]];
annotation3 := Graphics[Text[Style["z-face", Larger, Blue], annotationdata[[3]]]];
annotation4 := Graphics[Text[Style["Center", Larger, Blue], annotationdata[[4]]]];
annotation5 :=
  Graphics[Text[Style[Subscript[σ, P1], Larger, Blue], annotationdata[[5]]]];
annotation6 := Graphics[Text[Style[Subscript[σ, P2], Larger, Blue],
    annotationdata[[6]]]];
annotation7 := Graphics[Text[Style[Subscript[σ, P3], Larger, Blue],
    annotationdata[[7]]]];
annotation8 := Graphics[Text[Style[Subscript[σ, smax], Larger, Blue],
    annotationdata[[8]]]];
annotation9 := Graphics[Text[Style[Subscript[σ, smin], Larger, Blue],
    annotationdata[[9]]]];
```

Display

Show graphical output, followed by the values of the points.

```
Show[circle1, line1, points1, annotation1, annotation2, annotation3,
 annotation4, annotation5, annotation6, annotation7, annotation8, annotation9]

{{{"σ = ", N[sigma] // MatrixForm}} // MatrixForm,
 {{"x-face", pointsdata[[1]]},
   {"y-face", pointsdata[[2]]},
   {"z-face", pointsdata[[3]]},
   {"Center", pointsdata[[4]]},
   {Subscript[σ, P1], pointsdata[[5]]},
   {Subscript[σ, P2], pointsdata[[6]]},
   {Subscript[σ, P3], pointsdata[[7]]},
   {Subscript[σ, smax], pointsdata[[8]]},
   {Subscript[σ, smin], pointsdata[[9]]}
  } // MatrixForm, N[{{"x-face", pointsdata[[1]]},
    {"y-face", pointsdata[[2]]},
    {"z-face", pointsdata[[3]]},
    {"Center", pointsdata[[4]]},
    {Subscript[σ, P1], pointsdata[[5]]},
    {Subscript[σ, P2], pointsdata[[6]]},
    {Subscript[σ, P3], pointsdata[[7]]},
    {Subscript[σ, smax], pointsdata[[8]]},
    {Subscript[σ, smin], pointsdata[[9]]}
   }] // MatrixForm}
```



$$\left\{\left(\sigma = \begin{pmatrix} 4. & 5. & 0. \\ 5. & -3. & 0. \\ 0. & 0. & 8. \end{pmatrix}\right),\right.$$

$$\left.\begin{pmatrix} \text{x-face} & \{4, -5\} \\ \text{y-face} & \{-3, 5\} \\ \text{z-face} & \{8, 0\} \\ \text{Center} & \left\{\frac{1}{2}, 0\right\} \\ \sigma_{P1} & \left\{\frac{1}{2} + \frac{\sqrt{149}}{2}, 0\right\} \\ \sigma_{P2} & \left\{\frac{1}{2} - \frac{\sqrt{149}}{2}, 0\right\} \\ \sigma_{P3} & \{8, 0\} \\ \sigma_{smax} & \left\{\frac{1}{2} \times \left(\frac{17}{2} - \frac{\sqrt{149}}{2}\right), \frac{1}{2} \times \left(\frac{15}{2} + \frac{\sqrt{149}}{2}\right)\right\} \\ \sigma_{smin} & \left\{\frac{1}{2} \times \left(\frac{17}{2} - \frac{\sqrt{149}}{2}\right), \frac{1}{2} \times \left(-\frac{15}{2} - \frac{\sqrt{149}}{2}\right)\right\} \end{pmatrix}, \begin{pmatrix} \text{x-face} & \{4., -5.\} \\ \text{y-face} & \{-3., 5.\} \\ \text{z-face} & \{8., 0.\} \\ \text{Center} & \{0.5, 0.\} \\ \sigma_{P1} & \{6.60328, 0.\} \\ \sigma_{P2} & \{-5.60328, 0.\} \\ \sigma_{P3} & \{8., 0.\} \\ \sigma_{smax} & \{1.19836, 6.80164\} \\ \sigma_{smin} & \{1.19836, -6.80164\} \end{pmatrix}\right\}$$

Editing Notes

To-Do List

1. Test different examples; fix any issues

Display Organization

Plotted Elements

Circle

Points

-A,B,C,X,Y

-Points at top and bottom of circle

Lines

\

-Connecting X and Y through C

-Reference lines

-Dimension lines with arrows\

-Axes - so C displacement is obvious

Annotation Elements

Angles

-2Thetap,2Thetas

Distances

-R==radius

-sigma_min, sigma_max, \

tao_max

Point Labels - Display text in the plot, and associate text with \

values in the cmdline

-sigmaP1, "x-face"

-sigmaP2, "y-face"

-sigmaP3, \

"z-face"

-center

Axis labels

Command Line Elements

Angles

-2Thetap,2Thetas

Distances

-radius

-sigma_min, sigma_max, \

tao_max

Point Labels - Display text in the plot, and associate text with \

values in the cmdline

-sigmaP1, "x-face"

-sigmaP2, "y-face"

-sigmaP3, \

"z-face"

-center

Axis labels

Order

1. Plotted Elements

a) Circles

b) Lines

c) Points

2. Annotation \

Elements

a) Axis labels

b) Point labels

c) Distances

d) Angles

3. \

Command Line Elements

a) ?

b) ?