### **Stress Cube Diagram**

#### Input stress tensor (generalized plane stress)

```
ClearAll["Global`*"]

sigma = \{\{7, 4, 0\}, \{4, 2, 0\}, \{0, 0, -1\}\};

sigma = \{\{-4, 4, 0\}, \{4, 2, 0\}, \{0, 0, -1\}\};

sigma = \{\{4, 5, 0\}, \{5, -3, 0\}, \{0, 0, 8\}\};

sigma = \{\{-40, 0, 0\}, \{0, -10, 30\}, \{0, 30, -20\}\};

sigma = \{\{-40, 0, 30\}, \{0, -10, 0\}, \{30, 0, -20\}\};

sigma = \{\{4, 5, 0\}, \{5, -3, 0\}, \{0, 0, 8\}\};

sigma = \{\{4, 5, 0\}, \{5, -3, 0\}, \{0, 0, 8\}\};

sigma // MatrixForm

\begin{pmatrix} 4 & 5 & 0 \\ 5 & -3 & 0 \\ 0 & 0 & 8 \end{pmatrix}
```

#### Determine which plane shear stress is in

```
If[sigma[[1, 2]] # 0, plane = "xy"]
If[sigma[[1, 3]] # 0, plane = "xz"]
If[sigma[[2, 3]] # 0, plane = "yz"]
xy
```

Find points of interest; dependent on which plane the \

#### shear stress is in

\

center, and radius\

#### Find principle stresses and (smallest) angle to \

#### principle plane

```
This computes principal normal stresses

If[plane == "xy",
    {sigmaP1 = center + radius,
        sigmaP2 = center - radius,
        sigmaP3 = sigma[[3, 3]]}];

If[plane == "xz",
    {sigmaP1 = center + radius,
        sigmaP2 = sigma[[2, 2]],
        sigmaP3 = center - radius}];

If[plane == "yz",
    {sigmaP1 = sigma[[1, 1]],
        sigmaP2 = center + radius,
        sigmaP3 = center - radius}];

N[{sigmaP1, sigmaP2, sigmaP3}]

{6.60328, -5.60328, 8.}
```

#### Find (smallest) angle to principle planes

```
minzero[x_] := {min = Max[Abs[x]];
     For [n = 1, n \le Length[x], n++,
      If [Abs[x[[n]]] < Abs[min], min = x[[n]]]
      ];
    min};
givenpoints
center
thetaP1 = (1/2) * ArcTan[(givenpoints[[1, 1]] - center), givenpoints[[1, 2]]]
thetaP2 = (1/2) * ArcTan[(givenpoints[[2, 1]] - center), givenpoints[[2, 2]]]
\{\{4, -5\}, \{-3, 5\}, \{8, 0\}\}
-\frac{1}{2} ArcTan\left[\frac{10}{7}\right]
\frac{1}{2}\left(\pi - \operatorname{ArcTan}\left[\frac{10}{7}\right]\right)
Display output
Compute display parameters
\
Displaycenter is the center of the viewing area; displaydim is a vector \
containing half the display width, half the display height; displayrange is \
formatted {{xmin, xmax},{ymin, ymax}};
Displayelement is a small length characteristic of the size of the display \
(this helps with scaling the graphics appropriately); numtics is the number \
of ticks to show along each axis;
Tickspacingx is the spacing between ticks based on numticks and the display \
```

range\

```
displaycenter = {(Min[sigmaP1, sigmaP2, sigmaP3] + Max[sigmaP1, sigmaP2, sigmaP3])/2, 0};
displaydim =
  1.5 * .5 * {Max[sigmaP1, sigmaP2, sigmaP3] - Min[sigmaP1, sigmaP2, sigmaP3], 2 * radius};
displayrange = {{displaycenter [[1]] - displaydim [[1]], displaycenter [[1]] + displaydim [[1]]},
    {displaycenter [[2]] - displaydim [[2]], displaycenter [[2]] + displaydim [[2]]}};
displayelement = .1 * ((displaydim [[1]] + displaydim [[2]]) / 2);
numticks = 10;
tickspacingx = Abs[displayrange[[1, 2]] - displayrange[[1, 1]]]/numticks;
tickspacingy = Abs[displayrange [[2, 2]] - displayrange [[2, 1]]]/numticks;
Circles
\
First the circle for the faces of the unrotated stress cube is determined. \
Then the other two circles are found such that they intersect the remaining \
principal stresses. The circle information is put in a generic circledata \
array, to be used to generate the graphics primitives.\
If[plane == "xy",
  circledata = {{center, 0, radius}, {(sigmaP2 + sigmaP3)/2, 0, Abs[sigmaP2 - sigmaP3]/2},
     {(sigmaP1 + sigmaP3)/2, 0, Abs[sigmaP1 - sigmaP3]/2}}];
If[plane == "xz",
  circledata = {{center, 0, radius}, {(sigmaP1 + sigmaP2)/2, 0, Abs[sigmaP1 - sigmaP2]/2},
     {(sigmaP2 + sigmaP3)/2, 0, Abs[sigmaP2 - sigmaP3]/2}}];
If[plane == "yz",
  circledata = {{center, 0, radius}, {(sigmaP1 + sigmaP2)/2, 0, Abs[sigmaP1 - sigmaP2]/2},
     {(sigmaP1 + sigmaP3)/2, 0, Abs[sigmaP1 - sigmaP3]/2}}];
\
The largest circle is determined. circledata[[largestcircle]] is then the row \
containing enough information to draw the largest circle.\
```

```
largestcircle = 1;
rowmax = Dimensions[circledata, 1];
For[row = 1, row \leq rowmax[[1]], row++,
  If[circledata[[row, 3]] > circledata[[largestcircle , 3]], largestcircle = row]
 }
1
Finally the graphics objects are created, all three circles in one foul \
swoop. Since all graphics objects in the show command follow the formatting \
rules set by the first, and the circle object is always shown in the graphics \
output, it was chosen to contain all plot formatting information, such as \
axis labels and tick marks.\
circle1 := Graphics[{Circle[{circledata[[1, 1]], circledata[[1, 2]]}, circledata[[1, 3]]],
     Circle[{circledata[[2, 1]], circledata[[2, 2]]}, circledata[[2, 3]]],
     Circle[{circledata[[3, 1]], circledata[[3, 2]]}, circledata[[3, 3]]]},
    Axes → True,
    AxesLabel \rightarrow {Subscript[\sigma, nn], Subscript[\sigma, ns]},
    LabelStyle → Directive[Large, Bold],
    Ticks \rightarrow { Floor[Table[tickspacingx * Floor[.5 * n] * (-1) ^ Floor[1.5 * n],
         {n, 0, Ceiling[(2 * Max[Abs[ displayrange [[1]] ]] + 1) / tickspacingx ]}]],
      Floor[Table[tickspacingy * Floor[.5 * n] * (-1) ^ Floor[1.5 * n],
         {n, 0, Ceiling[(2 * Max[Abs[displayrange [[2]]]] + 1) / tickspacingy ]}]]},
    TicksStyle → Directive[Small, Italic],
    PlotRange → displayrange,
    AspectRatio → 1];
Lines
This creates the line graphics object which draws a line from the appropriate \
points related to the faces of the unrotated stress cube.\
```

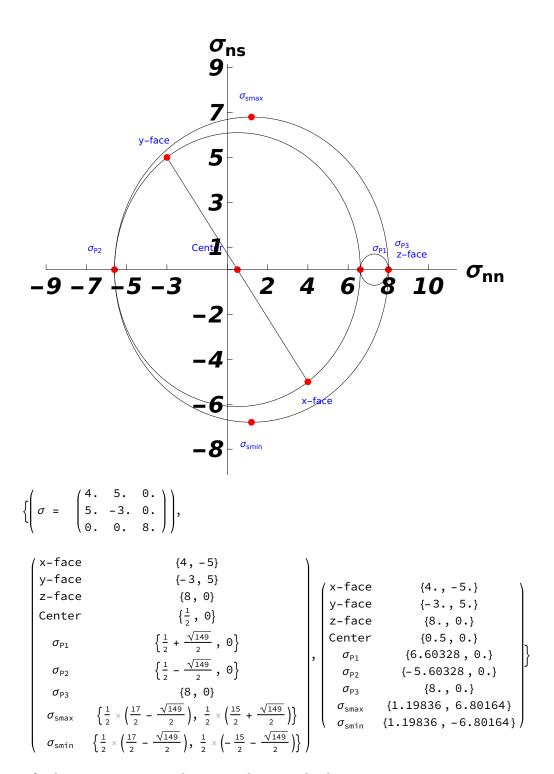
```
If[plane == "xy",
  line1 := Graphics [Line[{givenpoints [[1]], givenpoints [[2]]}]]];
If[plane == "xz",
  line1 := Graphics[Line[{givenpoints[[1]], givenpoints[[3]]}]]];
If[plane == "yz",
  line1 := Graphics[Line[{givenpoints[[2]], givenpoints[[3]]}]]];
Points
\
Pointsdata is an array of two-element vectors representing points to be \
displayed. The order of the points in this array is identical to the order in \
which they are assigned annotation elements.\
pointsdata = { givenpoints [[1]], givenpoints [[2]],
    givenpoints[[3]], {center, 0}, {sigmaP1, 0}, {sigmaP2, 0}, {sigmaP3, 0},
   {circledata[[largestcircle, 1]], circledata[[largestcircle, 3]]},
   {circledata[[largestcircle, 1]], -circledata[[largestcircle, 3]]}};
\
A graphics object to generate a plot of all points is next created.\
points1 := Graphics[
   {PointSize[Large], Red, Point[pointsdata]}
  ];
Annotation
Create r1, a set of vectors pointing from the origin to the points to be \
annotated\
```

```
r1 = N[pointsdata - ConstantArray [displaycenter, Dimensions [pointsdata, 1]]];
dim = Dimensions[r1, 1];
r1norm = N[Table[{Norm[r1[[i]]], Norm[ r1[[i]]]}, {i, dim[[1]]}]];
Create r2, a set of vectors pointing from the points to be annotated out a \
small distance away from the points\
r2 = ConstantArray [{0, 0}, dim[[1]]];
For[row = 1, row \leq dim[[1]], row++,
 If[r1norm[[row, 1]] # 0, r2[[row]] = displayelement *r1[[row]]/r1norm[[row]]]
]
Create annotationdata, the array of points at which annotations will \
eventually be placed. The points in annotationdata are slightly offset from \
those in pointsdata, so that the annotations do not overlap the points.\
annotationdata = ConstantArray [displaycenter , Dimensions [pointsdata , 1]] + r1 + r2;
\
Debug annotationdata; move annotation elements that are near axes away from \
those axes\
```

```
dim = Dimensions [annotationdata , 1];
For[row = 1, row \leq dim[[1]], row++, {
    If[Abs[annotationdata [[row, 1]]] < displayelement ,</pre>
     { annotationdata [[row, 1]] =
        displayelement *(2*Ceiling[(Sign[annotationdata[[row, 1]]]+1)/2]-1),
      r2[[row]] = annotationdata [[row]] - pointsdata [[row]] }
   ],
    If[Abs[annotationdata [[row, 2]]] < displayelement ,</pre>
     { annotationdata [[row, 2]] =
       displayelement *(2*Ceiling[(Sign[annotationdata[[row, 2]]]+1)/2]-1),
      r2[[row]] = annotationdata [[row]] - pointsdata [[row]] }
   1
  }];
\
Determine which points in annotationdata are "duplicates", where a \
"duplicate" pair of pionts are two points separated by no more than the \
length displayelement.\
duplicates = {};
temparray = annotationdata;
dim = Dimensions [temparray, 1];
For[row1 = 1, row1 \leq dim[[1]] - 1, row1++,
 {temp = {row1},
  For[row2 = row1 + 1, row2 \leq dim[[1]], row2 ++,
    If[EuclideanDistance [temparray[[row2]], temparray[[row1]]]≤ displayelement &&
      temparray[[row2]] # {},
     {temp = Append[temp, row2], temparray[[row2]] = {}}
   1
  ],
  If[Length[temp] > 1, duplicates = Append[duplicates, temp]]}
1
\
Step through the rows of the duplicates[[]] array, and separate all points \
that are considered "overlapping". All r2 vectors associated with the \
```

```
points in the rows of duplicates[[]] are rotated to generate an angular \setminus
spread.\
rmatrix[theta_] = {{Cos[theta], Sin[theta]}, {-Sin[theta], Cos[theta]}};
dim = Dimensions[duplicates, 1];
For[row = 1, row \leq dim[[1]], row++,
 {numcollisions = Length[duplicates[[row]]],
  If[numcollisions > 1,
   {spread =
      (Pi/6)*(Range[0, numcollisions -1]-N[Median[Range[0, numcollisions -1]]]),
     For[col = 1, col ≤ Length[duplicates[[row]]], col++,
      r2[[duplicates[[row, col]]]] = r2[[duplicates[[row, col]]]].rmatrix[spread[[col]]]
     ]}
  ]}
]
annotationdata = ConstantArray [displaycenter , Dimensions [pointsdata , 1]] + r1 + r2;
Create graphics objects that will draw annotation elements at the points \
contained within annotationdata\
```

```
annotation1 := Graphics[Text[Style["x-face", Larger, Blue], annotationdata [[1]]]];
annotation2 := Graphics[Text[Style["y-face", Larger, Blue], annotationdata [[2]]]];
annotation3 := Graphics[Text[Style["z-face", Larger, Blue], annotationdata[[3]]]];
annotation4 := Graphics[Text[Style["Center", Larger, Blue], annotationdata[[4]]]];
annotation5 :=
  Graphics [Text[Style[Subscript[\sigma, P1], Larger, Blue], annotationdata [[5]]]];
annotation6 := Graphics[Text[Style[Subscript[σ, P2], Larger, Blue],
     annotationdata [[6]]];
annotation7 := Graphics[Text[Style[Subscript[\sigma, P3], Larger, Blue],
     annotationdata [[7]] ]];
annotation8 := Graphics [Text[Style[Subscript[\sigma, smax], Larger, Blue],
     annotationdata [[8]]];
annotation9 := Graphics[Text[Style[Subscript[\sigma, smin], Larger, Blue],
     annotationdata [[9]]];
Display
Show graphical output, followed by the values of the points.
Show[circle1, line1, points1, annotation1, annotation2, annotation3,
 annotation4, annotation5, annotation6, annotation7, annotation8, annotation9]
\{\{\{"\sigma = ", N[sigma] // MatrixForm\}\} // MatrixForm,
 {{"x-face", pointsdata[[1]]},
    {"y-face", pointsdata [[2]]},
    {"z-face", pointsdata[[3]]},
   {"Center", pointsdata[[4]]},
    {Subscript [\sigma, P1], pointsdata [[5]]},
    {Subscript [\sigma, P2], pointsdata [[6]]},
   {Subscript [\sigma, P3], pointsdata [[7]]},
    {Subscript [\sigma, smax], pointsdata [[8]]},
    {Subscript [\sigma, smin], pointsdata [[9]]}
  } // MatrixForm , N[{{"x-face", pointsdata[[1]]},
     {"y-face", pointsdata[[2]]},
     {"z-face", pointsdata[[3]]},
     {"Center", pointsdata[[4]]},
     {Subscript [\sigma, P1], pointsdata [[5]]},
     {Subscript [\sigma, P2], pointsdata [[6]]},
     {Subscript[\sigma, P3], pointsdata[[7]]},
     {Subscript [\sigma, smax], pointsdata [[8]]},
     {Subscript [\sigma, smin], pointsdata [[9]]}
   }] // MatrixForm }
```



Display parameters that may be tweaked

```
Clear["Global`*"]
dx = 1;
lennn = .8;
offsetnn = .1;
lenns = .9;
offsetns = .2;
colnn = {0, 0, 0, 1};
colyz = \{1, 0, 0, 1\};
colxz = {0, 1, 0, 1};
colxy = {0, 0, 1, 1};
(*colyz={1,0,0,1};
colxz={1,.8,0,1};
colxy={1,0,.8,1};*)
Define transformaion matrices
\
The rotation and reflection matrices that compose the transformation matrices \
assume a right-handed coordinate system, with counter-clockwise rotation \
positive, as dictated by the right-hand rule.
Reflection matrices reflect along their axis; RF1 reflects along the x-axis \
(across the yz plane), and so forth.
Transformation matrices transform point sets between indicated faces; TX2Y \
```

transforms points from X-face to Y-face.\

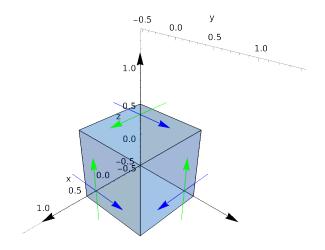
```
R1[theta_] = \{\{1, 0, 0\},\
    {0, Cos[theta], -Sin[theta]},
    {0, Sin[theta], Cos[theta]}};
R2[theta_] = {{Cos[theta], 0, Sin[theta]},
    {0, 1, 0},
    {-Sin[theta], 0, Cos[theta]}};
R3[theta_] = {{Cos[theta], -Sin[theta], 0},
    {Sin[theta], Cos[theta], 0},
    {0, 0, 1}};
R1[\theta] // MatrixForm;
R2[θ] // MatrixForm;
R3[\theta] // MatrixForm;
\mathsf{RF1} = \{\{-1,\ 0,\ 0\},\ \{0,\ 1,\ 0\},\ \{0,\ 0,\ 1\}\};
RF2 = \{\{1, 0, 0\}, \{0, -1, 0\}, \{0, 0, 1\}\};
RF3 = \{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, -1\}\};
TX2Y = RF1.R3[Pi/2];
TX2Z = RF1.R2[-Pi/2];
TY2Z = RF2.R1[Pi/2];
```

Generate sets of points necessary for display

```
(*xx,xz,xy*)
vectorscube = \{\{(offsetnn + 1/2) * dx, 0, 0\}, \{(lennn + offsetnn + 1/2) * dx, 0, 0\}\}
    \{(offsetns + 1/2) * dx, 0, -lenns * dx/2\}, \{(offsetns + 1/2) * dx, 0, lenns * dx/2\}\},
    \{(offsetns + 1/2) * dx, -lenns * dx/2, 0\}, \{(offsetns + 1/2) * dx, lenns * dx/2, 0\}\}\};
vectorstemp1 = {{TX2Y.vectorscube [[1, 1]], TX2Y.vectorscube [[1, 2]]},
    {TX2Y.vectorscube [[2, 1]], TX2Y.vectorscube [[2, 2]]},
    {TX2Y.vectorscube [[3, 1]], TX2Y.vectorscube [[3, 2]]}};
vectorstemp2 = {{TY2Z.vectorstemp1 [[1, 1]], TY2Z.vectorstemp1 [[1, 2]]},
    {TY2Z.vectorstemp1 [[2, 1]], TY2Z.vectorstemp1 [[2, 2]]},
   {TY2Z.vectorstemp1 [[3, 1]], TY2Z.vectorstemp1 [[3, 2]]}};
vectorstemp3 = {{TX2Z.vectorscube [[1, 1]], TX2Z.vectorscube [[1, 2]]},
    {TX2Z.vectorscube [[2, 1]], TX2Z.vectorscube [[2, 2]]},
   {TX2Z.vectorscube [[3, 1]], TX2Z.vectorscube [[3, 2]]}};
(*RX2Yxx=R3[-Pi/2];
RX2Yxz=R3[-Pi/2];
RX2Yxy=R3[-Pi/2].R1[-Pi];
\label{eq:constant} \textit{vectorscube} \ [[1,1]], RX2Yxx \ . \textit{vectorscube} \ [[1,2]]\}, \\
  {RX2Yxz.vectorscube [[2,1]], RX2Yxz.vectorscube [[2,2]]},
  {RX2Yxy.vectorscube [[3,1]],RX2Yxy.vectorscube [[3,2]]}};
RX2Zxx=R2[-Pi/2];
RX2Zxz=R2[-Pi/2].R1[-Pi];
RX2Zxy=R3[-Pi/2].R1[-Pi];
vectorstemp1 ={{RX2Yxx.vectorscube[[1,1]],RX2Yxx.vectorscube[[1,2]]},
  {RX2Yxz.vectorscube [[2,1]], RX2Yxz.vectorscube [[2,2]]},
  {RX2Yxy .vectorscube [[3,1]], RX2Yxy .vectorscube [[3,2]]}};*)
```

# Perform all computations to get values and matrices Draw local graphics elements

```
Graphics3D [{{RGBColor[0, .5, 1, .2], Opacity[.2],
    Cuboid [{-dx/2, -dx/2, -dx/2}, {dx/2, dx/2, dx/2}],
  {RGBColor[colnn], Arrow[vectorscube [[1]]]},
  {RGBColor[colxz], Arrow[vectorscube [[2]]]},
  {RGBColor[colxy], Arrow[vectorscube [[3]]]},
  {RGBColor[colnn], Arrow[vectorstemp1[[1]]]},
  {RGBColor[colxz], Arrow[vectorstemp1 [[2]]]},
  {RGBColor[colxy], Arrow[vectorstemp1 [[3]]]},
  {RGBColor[colnn], Arrow[vectorstemp2 [[1]]]},
  {RGBColor[colxz], Arrow[vectorstemp2 [[2]]]},
  {RGBColor[colxy], Arrow[vectorstemp2 [[3]]]},
  {RGBColor[colnn], Arrow[vectorstemp3 [[1]]]},
  {RGBColor[colxz], Arrow[vectorstemp3 [[2]]]},
  {RGBColor[colxy], Arrow[vectorstemp3 [[3]]]}
 },
 Boxed → False,
 Axes → True,
 AxesLabel \rightarrow {"x", "y", "z"},
 SphericalRegion → True,
 ViewPoint \rightarrow {1, 1, 1},
 ViewVertical \rightarrow {0, 0, 1},
 PlotRange → All
1
```



#### Draw global graphics elements

#### Define graphics elements for first coordinate system \

#### (C.S.)

#### Stress cube

#### **Stress vectors**

#### Perform first rotation

If[

Decide which plane maximum shear is in

Perform second rotation

# **Editing Notes**

## **Display Organization**

**Graphics Elements - Local C.S.** 

```
Geometry -Stress cube Stresses -sigmaxx, sigmayy, etc. -show \ stresses into/out of page properly in 2D mode View A-A lines Annotation -\theta_P, axes (x,y,z), \theta_s, stresses, "A-A", C.S. title ("View A-A") Axes -(x,y,z), \ (x',y',z'), etc.
```

**Graphics Elements - Global C.S.** 

Positioning lines

-line from ctr. of 1st cube to side o 2nd cube

\ -optional ghost line -arrow indicating angular measurement 1. Generate rotation matrices 2. Generate sets of points  $\setminus$ necessary for display 3. Perform all computations to obtain all values and  $\setminus$ matrices 4. Draw graphics elements local to coordinate system a) Draw all \ geometry (i.e., all three cubes) b) Draw stresses c) Draw axes d) Draw \ view A-A lines (if applicable) e) Draw annotation 5. Draw global graphics \ elements

a) Positioning lines