Set up Solution Procedure

```
(* Initialize *)
ClearAll["Global`*"]


(* Function of Two Variables *)
mm = nn = 50;
mMax = nMax = 100;
wFun[m_, n_] := ToExpression["w" <> ToString[m] <> ToString[n]];


(* Finite Difference Definitions *)
Dm1[fun_, m_, n_] := (1 / (2 * lambda)) * (fun[m + 1, n] - fun[m - 1, n]);
Dn1[fun_, m_, n_] := (1 / (2 * lambda)) * (fun[m, n + 1] - fun[m, n - 1]);

Dm2[fun_, m_, n_] := (1 / (lambda ^ 2)) * (fun[m - 1, n] - 2 * fun[m, n] + fun[m + 1, n]);
Dn2[fun_, m_, n_] := (1 / (lambda ^ 2)) * (fun[m, n - 1] - 2 * fun[m, n] + fun[m, n + 1]);

Dm3[fun_, m_, n_] :=
  (1 / (2 * lambda ^ 3)) * (-fun[m - 2, n] + 2 * fun[m - 1, n] - 2 * fun[m + 1, n] + fun[m + 2, n]);
Dn3[fun_, m_, n_] := (1 / (2 * lambda ^ 3)) *
    (-fun[m, n - 2] + 2 * fun[m, n - 1] - 2 * fun[m, n + 1] + fun[m, n + 2]);

Dm4[fun_, m_, n_] := (1 / (lambda ^ 4)) *
    (fun[m - 2, n] - 4 * fun[m - 1, n] + 6 * fun[m, n] - 4 * fun[m + 1, n] + fun[m + 2, n]);
Dn4[fun_, m_, n_] := (1 / (lambda ^ 4)) *
    (fun[m, n - 2] - 4 * fun[m, n - 1] + 6 * fun[m, n] - 4 * fun[m, n + 1] + fun[m, n + 2]);

Dm1n1[fun_, m_, n_] := (1 / (2 * lambda)) * (Dm1[fun, m, n + 1] - Dm1[fun, m, n - 1]);

Dm2n2[fun_, m_, n_] := (1 / (lambda ^ 4)) *
    (4 * fun[m, n] - 2 * (fun[m + 1, n] + fun[m - 1, n] + fun[m, n + 1] + fun[m, n - 1]) +
      fun[m + 1, n + 1] + fun[m + 1, n - 1] + fun[m - 1, n + 1] + fun[m - 1, n - 1]);

Dm2n1[fun_, m_, n_] := (1 / (2 * lambda)) * (Dm2[fun, m, n + 1] - Dm2[fun, m, n - 1]);
Dm1n2[fun_, m_, n_] := (1 / (2 * lambda)) * (Dn2[fun, m + 1, n] - Dn2[fun, m - 1, n]);
```

```
In[1482]:= (*Define scaled expressions which can serve for boundary conditions, and are assumed ==0 *)
        GDE = Simplify[(lambda ^ 4) * (Dm4[wFun, mm, nn] + 2 * Dm2n2[wFun, mm, nn] + Dn4[wFun, mm, nn]) - e1];
        (* e1 == (lambda^4)*pz/DD] *)
        mx[fun_, m_, n_] := Simplify[-DD * (Dm2[fun, m, n] + nu * Dn2[fun, m, n]) * (lambda ^ 2) / DD];
        my[fun_, m_, n_] := Simplify[-DD * (Dn2[fun, m, n] + nu * Dm2[fun, m, n]) * (lambda ^ 2) / DD];
        mxy[fun_, m_, n_] := Simplify[(nu - 1) * DD * Dm1n1[fun, m, n] * (4 * lambda ^ 2) / DD];
        nux[fun_, m_, n_] :=
          Simplify[-DD * (Dm3[wFun, m, n] + (2 - nu) * Dm1n2[wFun, m, n]) * (2 * lambda ^ 3) / DD];
        nuy[fun_, m_, n_] := Simplify[
          -DD * (Dn3[wFun, m, n] + (2 - nu) * Dm2n1[wFun, m, n]) * (2 * lambda ^ 3) / DD];

        dn = dt = lambda * Sqrt[2];
        mn[fun_, m_, n_] := Simplify[
          (-DD * (dn ^ 2 * nu * (fun[m - 1, n + 1] - 2 fun[m, n] + fun[m + 1, n - 1]) + dt ^ 2 * (fun[m - 1,
                  n - 1] - 2 * fun[m, n] + fun[m + 1, n + 1])) / (dn ^ 2 * dt ^ 2)) * 2 * lambda ^ 2 / DD];
        nun[fun_, m_, n_] := Simplify[(DD * (dn ^ 2 * (2 - nu) * (fun[m - 2, n] - 2 fun[m - 1, n - 1] +
                  fun[m, n - 2] - fun[m, n + 2] + 2 * fun[m + 1, n + 1] - fun[m + 2, n]) + dt ^ 2 *
                (fun[m - 2, n - 2] - 2 * fun[m - 1, n - 1] + 2 * fun[m + 1, n + 1] - fun[m + 2, n + 2])) /
              (2 * dn ^ 3 * dt ^ 2)) * 4 * Sqrt[2] * lambda ^ 3 / DD];
In[1490]:= (*Define function to eliminate variables*)
        fun1 = Function[{expr, fdeMask},

          (*Initialize*)
          nExpressions = Length[expr];
          w = Table[wFun[mm + dm, nn + dn], {dm, -3, 3, 1}, {dn, -3, 3, 1}];

          zeroMask = Abs[fdeMask - 1];

          centeredFde = {{0, 0, 0, 0, 0, 0, 0},
            {0, 0, 0, 1, 0, 0, 0},
            {0, 0, 1, 1, 1, 0, 0},
            {0, 1, 1, 1, 1, 1, 0},
            {0, 0, 1, 1, 1, 0, 0},
            {0, 0, 0, 1, 0, 0, 0},
            {0, 0, 0, 0, 0, 0, 0}};

          eliminationMask = Table[zeroMask[[m, n]] * centeredFde[[m, n]], {m, 1, 7}, {n, 1, 7}];

          nPointsEliminate = Sum[eliminationMask[[m, n]], {m, 1, 7}, {n, 1, 7}];

          eliminationVariables = Table[0, {ii, 1, nPointsEliminate}];
          i = 1;
          For[m = 1, m ≤ 7, m++,
           For[n = 1, n ≤ 7, n++,
             If[eliminationMask[[m, n]] ≠ 0,
              eliminationVariables[[i++]] = w[[m, n]]
             ];
            ];
          ];

          Print[{"Variable elimination:", MatrixForm[eliminationMask]}];
```

```
(*Try all combinations*)
For[i = 0, i < 2 ^ nExpressions, i ++, {

  (*Form mask*)
  exprMask = PadLeft[IntegerDigits[i, 2], nExpressions];

  (*Continue*)
  If[Mod[i, 10 000] == 0, Print[i]];
  If[Sum[exprMask[[j]], {j, 1, nExpressions}] ≠ nPointsEliminate, Continue[]];

  (*Attempt some combination of available expressions*)
  exprI = expr * exprMask /. nu → 0.3;

  (*Solve, eliminating variables which represent out-of-bounds points*)
  eqtn = Join[Table[exprI[[i]] == 0, {i, 1, Length[exprI]}], {GDE == 0}];
  soltn = Eliminate[eqtn, eliminationVariables];

  (*Ignore case of no solution*)
  If[Length[soltn] < 1, Continue[]];

  (*Rearrange GDE to form stencil*)
  lhs = soltn[[1]] - soltn[[2]];
  c1 = Coefficient[lhs, e1];
  If[c1 == 0, Continue[]];
  soltn = Simplify[(lhs / -c1) + e1] == e1;

  (*Extract coefficients for finite difference method*)
  table1 = Table[Coefficient[soltn[[1]], w[[m, n]]], {m, 1, 7}, {n, 1, 7}];

  (*Determine if successful derivation, and continue if not*)
  sum1 = Sum[Abs[(table1[[m, n]] * zeroMask[[m, n]]) /. {nu → 0.3}], {m, 1, 7}, {n, 1, 7}];
  If[sum1 > 0.0000001, Continue[]];

  (*If good result, calculate analytical solution*)
  exprI = expr * exprMask;

  (*Solve, eliminating variables which represent out-of-bounds points*)
  eqtn = Join[Table[exprI[[i]] == 0, {i, 1, Length[exprI]}], {GDE == 0}];
  soltn = Eliminate[eqtn, eliminationVariables];
  lhs = soltn[[1]] - soltn[[2]];
  c1 = Coefficient[lhs, e1];
  If[c1 == 0, Continue[]];
  soltn = Simplify[(lhs / -c1) + e1] == e1;
  table2 = Table[Coefficient[soltn[[1]], w[[m, n]]], {m, 1, 7}, {n, 1, 7}];

  (*Ignore trivial solution*)
  sum2 = Sum[Abs[table2[[m, n]] /. {nu → 0.3}], {m, 1, 7}, {n, 1, 7}];
  If[Abs[sum2] < 0.0000001, Continue[]];

  (*Show solution*)
  Print[{"Successful", exprMask, MatrixForm[table2 /. nu → 0.3], MatrixForm[Simplify[table2]]}]

}];
```

```
"End of calculations."
];
```

Generate Finite Difference Expressions

In[1491]:= `(* Center point *)`

```
expr = {};
fdeMask = {{0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 1, 0, 0, 0},
    {0, 0, 1, 1, 1, 0, 0},
    {0, 1, 1, 1, 1, 1, 0},
    {0, 0, 1, 1, 1, 0, 0},
    {0, 0, 0, 1, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0}};
fun1[expr, fdeMask];
```

$$\left\{ \text{Variable elimination:,} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

0

$$\left\{ \text{Successful,} \{\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -8 & 2 & 0 & 0 \\ 0 & 1 & -8 & 20 & -8 & 1 & 0 \\ 0 & 0 & 2 & -8 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -8 & 2 & 0 & 0 \\ 0 & 1 & -8 & 20 & -8 & 1 & 0 \\ 0 & 0 & 2 & -8 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

In[1494]:= `(* Free Edge0 *)`

```
expr = {mx[wFun, mm, nn],
    nux[wFun, mm, nn],
    mx[wFun, mm, nn - 1],
    mx[wFun, mm, nn + 1]};
fdeMask = {{1, 1, 1, 1, 1, 1, 1},
    {1, 1, 1, 1, 1, 1, 1},
    {1, 1, 1, 1, 1, 1, 1},
    {1, 1, 1, 1, 1, 1, 1},
    {0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0}};
fun1[expr, fdeMask];
```

$$\left\{\text{Variable elimination:,}\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

0

$$\left\{\text{Successful},\ \{1, 1, 1, 1\},\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3.4 & -10.8 & 3.4 & 0 & 0 \\ 0 & 0.91 & -6.44 & 13.06 & -6.44 & 0.91 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 - 2\,\text{nu} & 4\,(-3 + \text{nu}) & 4 - 2\,\text{nu} & 0 & 0 \\ 0 & 1 - \text{nu}^2 & 4\,\left(-2 + \text{nu} + \text{nu}^2\right) & 16 - 8\,\text{nu} - 6\,\text{nu}^2 & 4\,\left(-2 + \text{nu} + \text{nu}^2\right) & 1 - \text{nu}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

```
In[1497]:= (*Free Edge0, fewer points available *)
expr = {mx[wFun, mm, nn],
    nux[wFun, mm, nn],
    mx[wFun, mm, nn - 1],
    mx[wFun, mm, nn + 1],
    my[wFun, mm, nn + 1],
    my[wFun, mm, nn - 1]};
fdeMask = {{1, 1, 1, 1, 1, 0, 0},
    {1, 1, 1, 1, 1, 0, 0},
    {1, 1, 1, 1, 1, 0, 0},
    {0, 0, 1, 1, 1, 0, 0},
    {0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0}};
fun1[expr, fdeMask];
```

$$\left\{\text{Variable elimination:,} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

0

$$\left\{\text{Successful, } \{1, 1, 1, 1, 1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3.4 & -10.8 & 3.4 & 0 & 0 \\ 0 & 0 & -4.62 & 11.24 & -4.62 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4-2\,\text{nu} & 4\,(-3+\text{nu}) & 4-2\,\text{nu} & 0 & 0 \\ 0 & 0 & 2\,\left(-3+2\,\text{nu}+\text{nu}^2\right) & 14-8\,\text{nu}-4\,\text{nu}^2 & 2\,\left(-3+2\,\text{nu}+\text{nu}^2\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

In[1500]:=
```
(*Free Edge1*)
expr = {mx[wFun, mm + 1, nn]};
fdeMask = {{1, 1, 1, 1, 1, 1, 1},
   {1, 1, 1, 1, 1, 1, 1},
   {1, 1, 1, 1, 1, 1, 1},
   {1, 1, 1, 1, 1, 1, 1},
   {1, 1, 1, 1, 1, 1, 1},
   {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}};
fun1[expr, fdeMask];
```

$$\left\{\text{Variable elimination:,} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

0

$$\left\{\text{Successful, } \{1\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -8 & 2 & 0 & 0 \\ 0 & 1 & -8 & 19 & -8 & 1 & 0 \\ 0 & 0 & 1.7 & -5.4 & 1.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -8 & 2 & 0 & 0 \\ 0 & 1 & -8 & 19 & -8 & 1 & 0 \\ 0 & 0 & 2-\text{nu} & 2\,(-3+\text{nu}) & 2-\text{nu} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

In[1503]:= **(*Free Corner0 *)**
expr = {mx[wFun, mm, nn],
  mx[wFun, mm, nn - 1],
  my[wFun, mm, nn],
  my[wFun, mm - 1, nn],
  mxy[wFun, mm, nn],
  nux[wFun, mm, nn],
  nuy[wFun, mm, nn]};
fdeMask = {{1, 1, 1, 1, 0, 0, 0},
  {1, 1, 1, 1, 0, 0, 0},
  {1, 1, 1, 1, 0, 0, 0},
  {1, 1, 1, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0}};
fun1[expr, fdeMask];

$$\left\{\text{Variable elimination:}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

0

$$\left\{\text{Successful}, \{1, 1, 1, 1, 1, 1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.82 & 0 & 0 & 0 \\ 0 & 0 & 5.6 & -9.24 & 0 & 0 & 0 \\ 0 & 1.82 & -9.24 & 9.24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right.,$$

$$\left.\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 - 2\,\text{nu}^2 & 0 & 0 & 0 \\ 0 & 0 & 8 - 8\,\text{nu} & 4\,(-1 + \text{nu})\,(3 + \text{nu}) & 0 & 0 & 0 \\ 0 & 2 - 2\,\text{nu}^2 & 4\,(-1 + \text{nu})\,(3 + \text{nu}) & -4\,(-1 + \text{nu})\,(3 + \text{nu}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

In[1506]:= **(*Free Corner1_1 *)**
expr = {mx[wFun, mm + 1, nn],
  my[wFun, mm, nn + 1]};
fdeMask = {{1, 1, 1, 1, 1, 0, 0},
  {1, 1, 1, 1, 1, 0, 0},
  {1, 1, 1, 1, 1, 0, 0},
  {1, 1, 1, 1, 1, 0, 0},
  {1, 1, 1, 1, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0}};
fun1[expr, fdeMask];

$$\left\{\text{Variable elimination:}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

0

$$\left\{\text{Successful}, \{1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -8 & 1.7 & 0 & 0 \\ 0 & 1 & -8 & 18 & -5.4 & 0 & 0 \\ 0 & 0 & 1.7 & -5.4 & 1.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -8 & 2-\text{nu} & 0 & 0 \\ 0 & 1 & -8 & 18 & 2\,(-3+\text{nu}) & 0 & 0 \\ 0 & 0 & 2-\text{nu} & 2\,(-3+\text{nu}) & 2-2\,\text{nu} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

```
In[1509]:=  (* 45-Degree  Free  Edge0 *)
        expr = {mn[wFun, mm, nn],
           mn[wFun, mm + 1, nn - 1],
           mn[wFun, mm - 1, nn + 1],
           mn[wFun, mm - 1, nn],
           mn[wFun, mm, nn - 1]};
        fdeMask = {
           {1, 1, 1, 1, 1, 1, 1},
           {1, 1, 1, 1, 1, 1, 0},
           {1, 1, 1, 1, 1, 0, 0},
           {1, 1, 1, 1, 0, 0, 0},
           {1, 1, 1, 0, 0, 0, 0},
           {1, 1, 0, 0, 0, 0, 0},
           {1, 0, 0, 0, 0, 0, 0}};
        fun1[expr, fdeMask];
```

$$\left\{ \text{Variable elimination:}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

0

$$\left\{ \text{Successful}, \{1, 1, 1, 1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 2.4 & -0.3 & 0 \\ 0 & 8 & 0 & -26.4 & 4 & 0 & 0 \\ 0 & 0 & -26.4 & 24.6 & 0 & 0 & 0 \\ 0 & 2.4 & 4 & 0 & 0 & 0 & 0 \\ 0 & -0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 8\,nu & -nu & 0 \\ 0 & 8 & 0 & -8\,(3+nu) & 4 & 0 & 0 \\ 0 & 0 & -8\,(3+nu) & 2\,(12+nu) & 0 & 0 & 0 \\ 0 & 8\,nu & 4 & 0 & 0 & 0 & 0 \\ 0 & -nu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

```
In[1512]:= (*45-Degree Free Edge .5*)
expr = {mn[wFun, mm + 1, nn - 1],
   mn[wFun, mm, nn],
   mn[wFun, mm - 1, nn + 1]};
fdeMask = {
   {1, 1, 1, 1, 1, 1, 1},
   {1, 1, 1, 1, 1, 1, 1},
   {1, 1, 1, 1, 1, 1, 0},
   {1, 1, 1, 1, 1, 0, 0},
   {1, 1, 1, 1, 0, 0, 0},
   {1, 1, 1, 0, 0, 0, 0},
   {1, 1, 0, 0, 0, 0, 0}};
fun1[expr, fdeMask];
```

$$\left\{ \text{Variable elimination:}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

0

$$\left\{ \text{Successful}, \{1, 1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.3 & 0 \\ 0 & 0 & 0 & -8 & 4 & 0 & 0 \\ 0 & 0 & -8 & 24.6 & -8 & 0 & 0 \\ 0 & 0 & 4 & -8 & 0 & 0 & 0 \\ 0 & -0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -nu & 0 \\ 0 & 0 & 0 & -8 & 4 & 0 & 0 \\ 0 & 0 & -8 & 2\,(12+nu) & -8 & 0 & 0 \\ 0 & 0 & 4 & -8 & 0 & 0 & 0 \\ 0 & -nu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

In[1515]:= `(*135-Degree Free-Free Corner0*)`
```
expr = {mn[wFun, mm - 1, nn + 1],
   mn[wFun, mm, nn],
   mn[wFun, mm - 1, nn],
   nux[wFun, mm, nn],
   mx[wFun, mm, nn - 1],
   mx[wFun, mm, nn]};
fdeMask = {
   {1, 1, 1, 1, 1, 1, 1},
   {1, 1, 1, 1, 1, 1, 0},
   {1, 1, 1, 1, 1, 0, 0},
   {1, 1, 1, 1, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}};
fun1[expr, fdeMask];
```

$$\left\{\text{Variable elimination:}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

0

$$\left\{\text{Successful}, \{1, 1, 1, 1, 1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7.22 & 1 & 2.166 & -0.3 & 0 \\ 0 & 0 & 3.19 & -29.572 & 6.21 & 0 & 0 \\ 0 & 0.937 & -4.508 & 13.657 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2\left(-4 + nu + nu^2\right) & 1 & -2\,nu\left(-4 + nu + nu^2\right) & -nu & 0 \\ 0 & 0 & 4 - 3\,nu + nu^2 & 4\left(-7 - 2\,nu + 2\,nu^2 + nu^3\right) & 6 + nu - nu^2 & 0 & 0 \\ 0 & 1 - nu^2 + nu^3 & -4\left(2 - 3\,nu + nu^3\right) & 16 - 7\,nu - 3\,nu^2 + nu^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

In[1518]:= **(∗135-Degree Free-Free Corner0_1∗)**
expr = {mx[wFun, mm, nn],
   mx[wFun, mm, nn - 1],
   mx[wFun, mm, nn + 1],
   nux[wFun, mm, nn],
   mn[wFun, mm - 1, nn + 1]};
fdeMask = {
   {1, 1, 1, 1, 1, 1, 1},
   {1, 1, 1, 1, 1, 1, 1},
   {1, 1, 1, 1, 1, 1, 0},
   {1, 1, 1, 1, 1, 0, 0},
   {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0}};
fun1[expr, fdeMask];

$$\left\{ \text{Variable elimination:,} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

0

$$\left\{ \text{Successful,} \{1, 1, 1, 1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.09 & 0 & -0.273 & 0 \\ 0 & 0 & 3.4 & -10.8 & 5.766 & 0 & 0 \\ 0 & 0.91 & -6.44 & 12.787 & -6.44 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 + nu^2 & 0 & nu\,(-1 + nu^2) & 0 \\ 0 & 0 & 4 - 2\,nu & 4\,(-3 + nu) & -2\,(-3 + nu^2 + nu^3) & 0 & 0 \\ 0 & 1 - nu^2 & 4\,(-2 + nu + nu^2) & 16 - 9\,nu - 6\,nu^2 + nu^3 & 4\,(-2 + nu + nu^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

In[1521]:= **(∗Flat Stepped Free Corner0∗)**
(∗ --> Let this be picked up by the 135-Degree Free-Free Corner0∗)
fdeMask = {
   {1, 1, 1, 1, 0, 0, 0},
   {1, 1, 1, 1, 0, 0, 0},
   {1, 1, 1, 1, 0, 0, 0},
   {1, 1, 1, 1, 0, 0, 0},
   {1, 1, 1, 0, 0, 0, 0},
   {1, 1, 1, 0, 0, 0, 0},
   {1, 1, 1, 0, 0, 0, 0}};

In[1522]:= **(* Concave Corner0, Free – Free *)**
expr = {mn[wFun, mm, nn],
   my[wFun, mm, nn + 1]};
fdeMask = {
   {1, 1, 1, 1, 1, 0, 0},
   {1, 1, 1, 1, 1, 0, 0},
   {1, 1, 1, 1, 1, 0, 0},
   {1, 1, 1, 1, 1, 0, 0},
   {1, 1, 1, 1, 0, 0, 0},
   {1, 1, 1, 1, 0, 0, 0},
   {1, 1, 1, 1, 0, 0, 0}};
fun1[expr, fdeMask];

$$\left\{\text{Variable elimination:}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$

0

$$\left\{\text{Successful}, \{1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & -8 & 1.19 & 0 & 0 \\ 0 & 1 & -8 & 23.42 & -5.4 & 0 & 0 \\ 0 & 0 & 1.49 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & nu & -8 & 2 - 3\,nu + nu^2 & 0 & 0 \\ 0 & 1 & -8 & 23 + 2\,nu - 2\,nu^2 & 2\,(-3 + nu) & 0 & 0 \\ 0 & 0 & 2 - 2\,nu + nu^2 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\right\}$$