Investment Analysis

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0 Executive Summary

0.1 Data Set

The monthly Vanguard ETF data between May 2018 - Apr 2023 is retrieved from Yahoo Finance page.

0.2 Mutual Fund Descriptions

riskiness of the product and its benefits for diversification

US Stocks. Vanguard Total Stock Market ETF (VTI) Vanguard Total Stock Market Index Fund seek to track the entire U.S. equity market, including small, mid, and large cap growth and value stocks. The key feature of this fund is broad diversification.

Municipal Bonds. Vanguard Tax-Exempt Bond Index ETF (VTEB) Vanguard Tax-Exempt Bond ETF tracks the S&P National AMT-Free Municipal Bond Index and measures the performance of the U.S. municipal bond market.

Foreign Developed Stocks. Vanguard FTSE Developed Markets ETF (VEA) Vanguard FTSE Developed Markets ETF measures the investment return of stocks issued by companies and markets located in Canada, Europe and the Pacific region.

Emerging market stocks. Vanguard FTSE Emerging Markets ETF (VWO) Vanguard FTSE Emerging Markets ETF tracks the performance of the FTSE Emerging Markets All Cap China A Inclusion Index, including 3,550 common stocks of large-, mid-, and small-cap companies located in emerging markets around the world.

Dividend Growth Stocks. Vanguard Dividend Appreciation ETF (VIG) Vanguard Dividend Appreciation ETF seeks to track the investment performance of the S&P U.S. Dividend Growers Index, including common stocks of companies that have a record of increasing dividends over time.

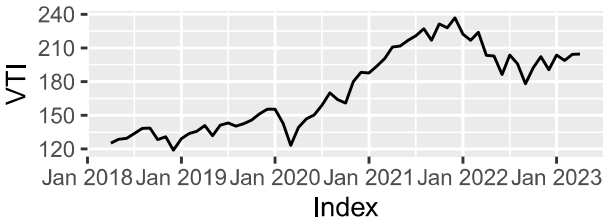
0.3 Main Findings

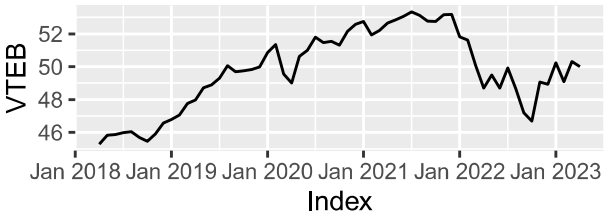
- The mutual funds that index groups of countries all had prices and return drop dramatically around early 2020. Price grew and peacked on mid 2021 and dropped in mid 2022. VTEB had only slight drop in price in early 2020 and decreased significantly in mid 2021.
- None of the six mutual funds have returns that are normally distributed and rolling analysis suggest that expected returns and standard deviations are not stationary.
- VIG has the highest and VWO has the lowest average return. VTI has the highest and VTEB has the lowest standard deviation. Expected returns are not measured as precisely as standard deviations
- The funds shows the relation of risk-return tradeoff. VEA and VWO performs very poorly they have lower return and/or higher risk than the bond funds.
- The Sharpe Ratio measures excess return per unit of risk. The VIG has the highest Sharpe Ratio and VWO has the lowest and negative sharpe ratio.
- Stock index funds (VIG, VWO, VEA, VIG) have strong positive linear relationship with each other. Stock index funds and bond funds have slight positive corrleations. US stock index funds, VTI and VIG has the strongest positive linear relatinoship.
- None of the correlations are negative, but since none of the funds are perfectly correlated, diversification possible. However diversification will be difficulst as mutual funds cannot be shorted.
- VWO has the highest VaR and VTEB has the lowest VaR. The magnitude of standard error is small compared to the estimate and the range of confidence interval is small enough to conclude that the VaR estimate is precise.
- The global minimum portfolio without short sale has a higher standard deviation and lower expected return than the global minimum portfolio with short sales. The potential loss is greater with a global minimum variance portfolio without short sale.
- The tangency portfolio with no shorting has a lower expected return, higher standard deviation, and lower Sharpe Ratio than the tangency with shorting.
- When the risk free asset is introduced, the target portfoilio return can be achieved with a lower variance. Accordingly, the value-at-risk is lower in magnitude when holding a riskless asset.
- In equal weight portfolio, the contributions to risk of all stocks differ. VTI contributes to the risk the most and VTEB contributes the least.

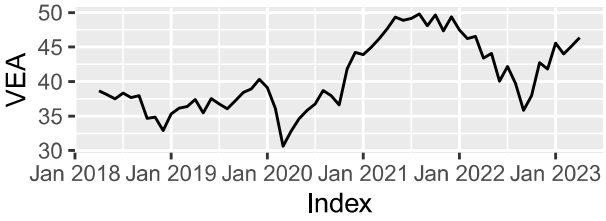
1 Return Calculation and Sample Statistics

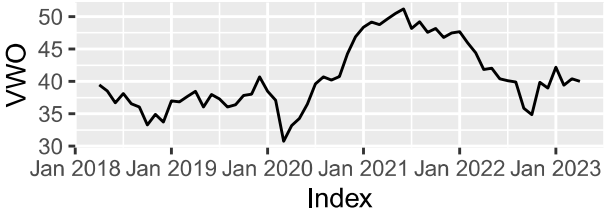
1.1 Time Trends of Mutual Fund Prices and Returns

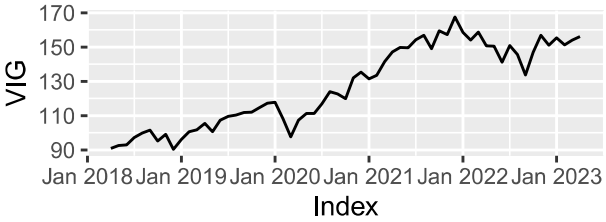
Price plots:



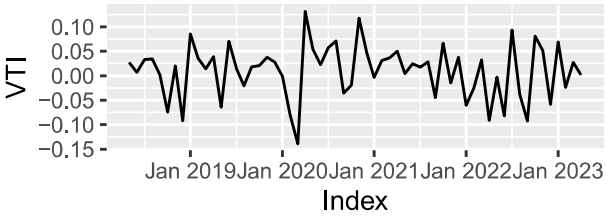


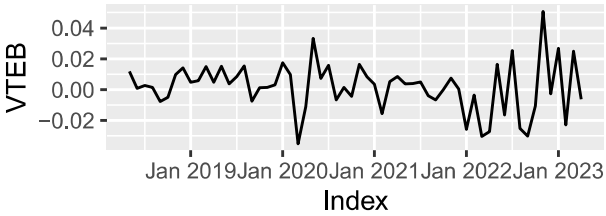


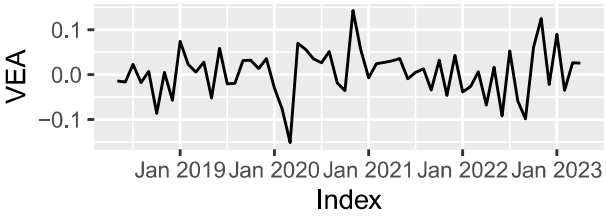


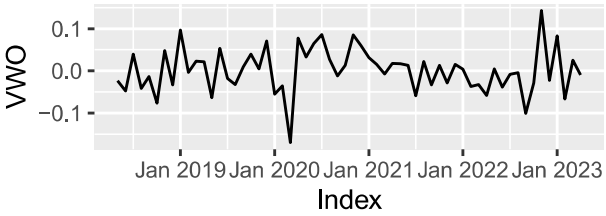


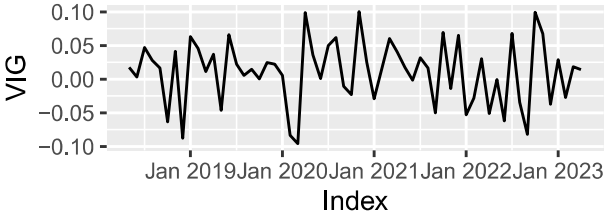
Return Plots:











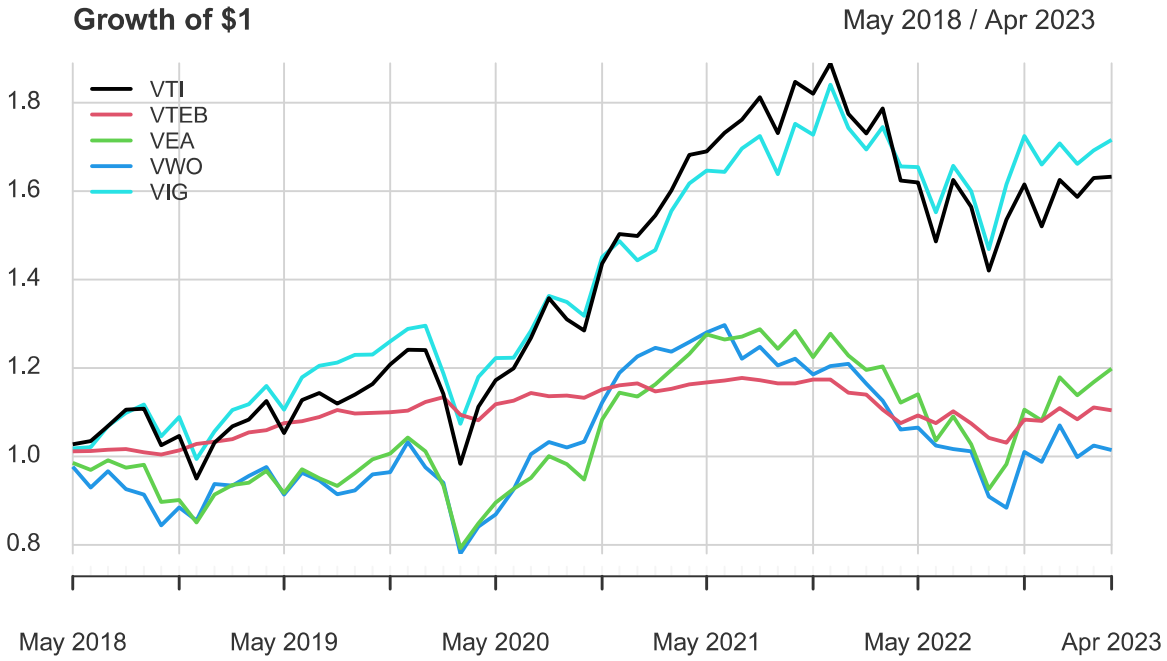
The mutual funds that replicate indexes of different countries like the US Stocks (VTI), foreign developed stock index (VEA), emerging markets fund (VWO) have prices and continuously compounded returns that seem to move together. For example, price dipped in March 2020, continue to grow until and peaked at December 2021, decrease until June 2022, then increase until Apr 2023. The continuously compounded return for these three funds fluctuated around their respective means until January 2020, when the returns dropped dramatically. After, the returns fluctuated around their respective means again.

VTI and VIG are moving most similarly in both prices and returns. VIG have lower price than VTI and VIG dipped less than VTI in March 2020.

Municiple bond, VTEB doing its own thing. The equity curve of VTEB is smoother than any of other stocks. VTEB have smaller range of return volatility. Overall, the price is moving similary with other stocks, but dipped less in March 2020.

The dip in returns and prices around March 2020 is likely to be due to the spread of COVID-19 and pandemic. The drop in returns and prices until December 2022 is likely to be due to aggressive interest rate hikes to curb inflation, fear of recession, and the Ukraine war.

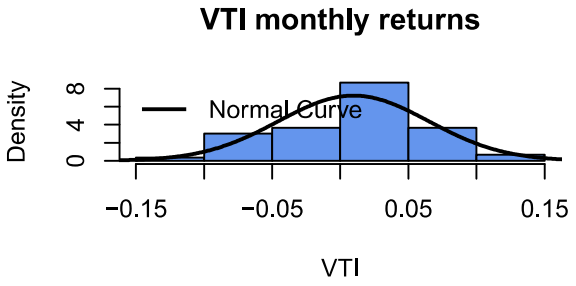
1.2 Equity curve

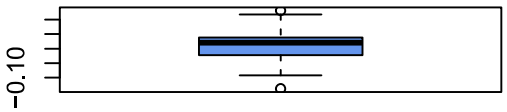


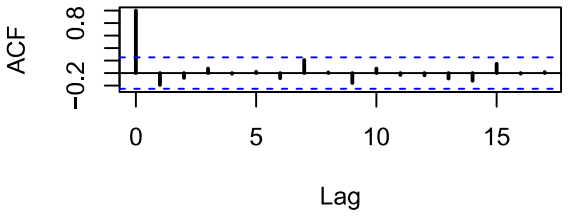
VIG gives the highest future value and VTI comes close. I am not surprised since these index tracks the US stock market, which is the biggest market in the world.

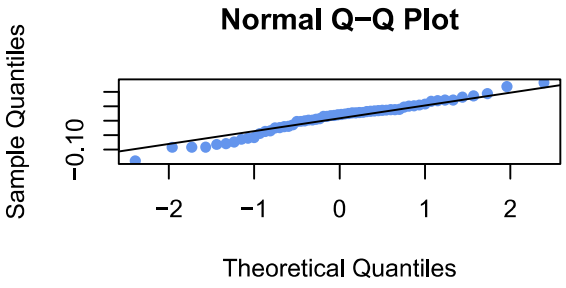
1.3 Normality of Returns

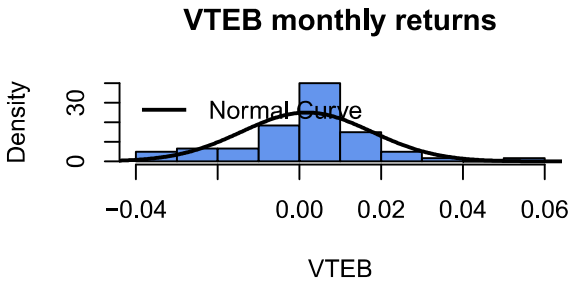
Here are four panel diagnostic plots for each asset containing histograms, smoothed density plots, boxplots and 99-plots, which are used to determine if these returns have normal disturbutions.

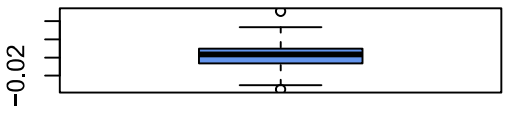

VTI monthly returns
Density
0 4 8
Normal Curve
−0.15
−0.05
0.05
0.15
VTI

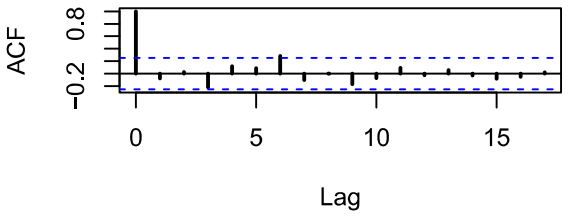

−0.10

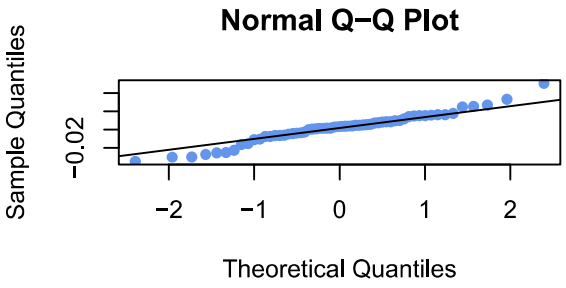

ACF
−0.2 0.8
0
5
10
15
Lag

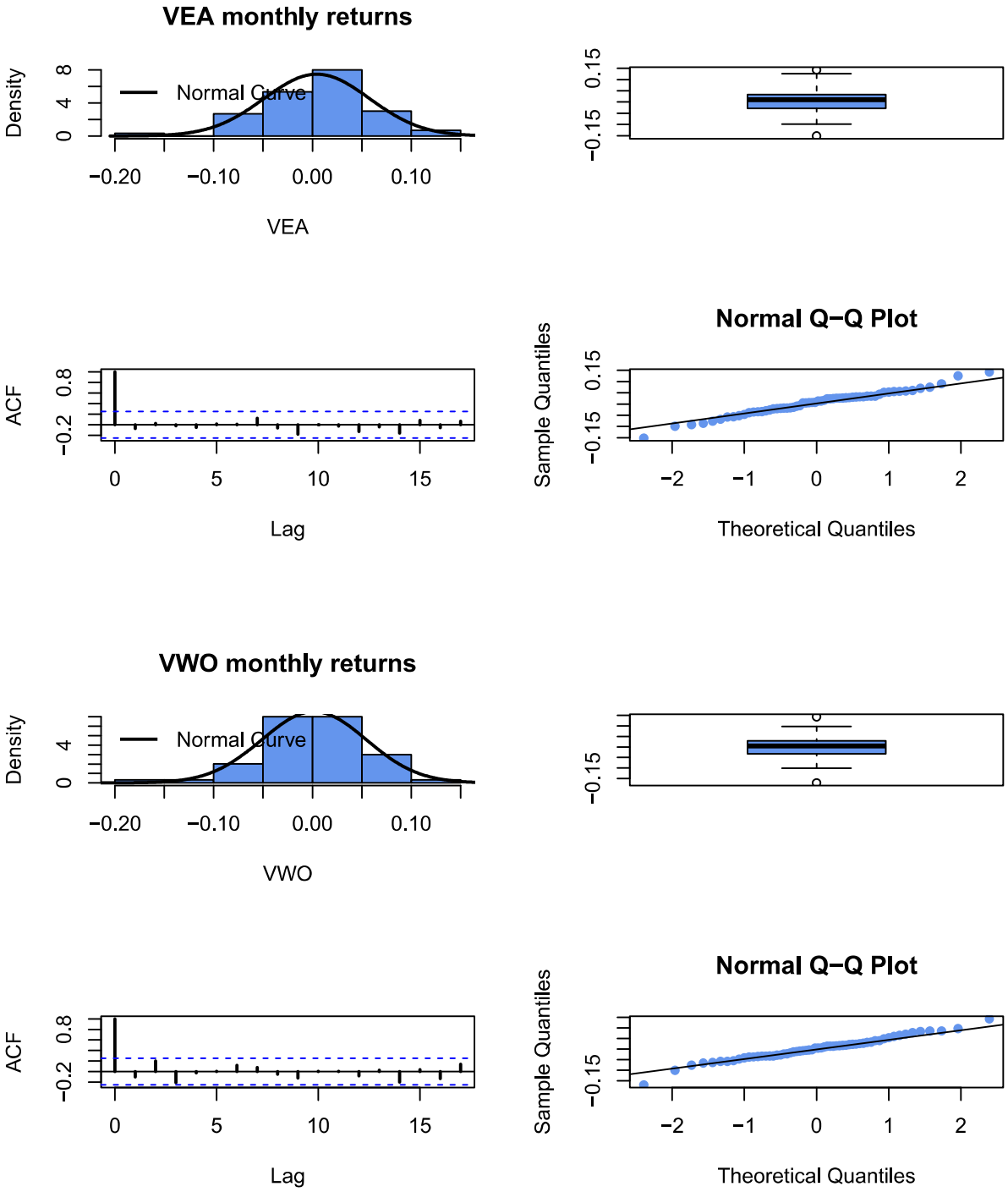

Normal Q−Q Plot
Sample Quantiles
−0.10
−2
−1
0
1
2
Theoretical Quantiles

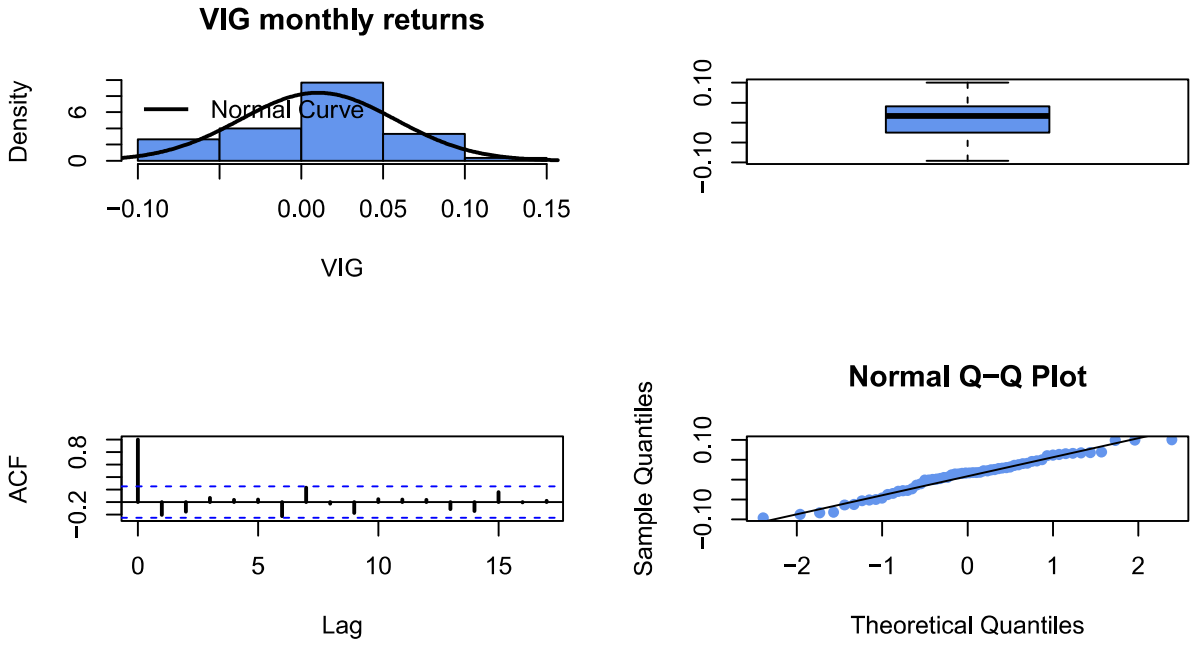

VTEB monthly returns
Density
0 30
Normal Curve
−0.04
0.00
0.02
0.04
0.06
VTEB


−0.02


ACF
−0.2 0.8
0
5
10
15
Lag


Normal Q−Q Plot
Sample Quantiles
−0.02
−2
−1
0
1
2
Theoretical Quantiles


VEA monthly returns
Density
Normal Curve
VEA
ACF
Lag
Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
VWO monthly returns
Density
Normal Curve
VWO
ACF
Lag
Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles



VTI Approximately normal with slight thicker tails (i.e. extreme values are more common than normal distribution) Autocorrelation decreases to 0 as lag increases.

VTEB Approximately normmal with thicker tails. Autocorrelation decreases to 0 as lag increases.

VEA Approximately normmal with thicker tails. Autocorrelation decreases to 0 as lag increases.

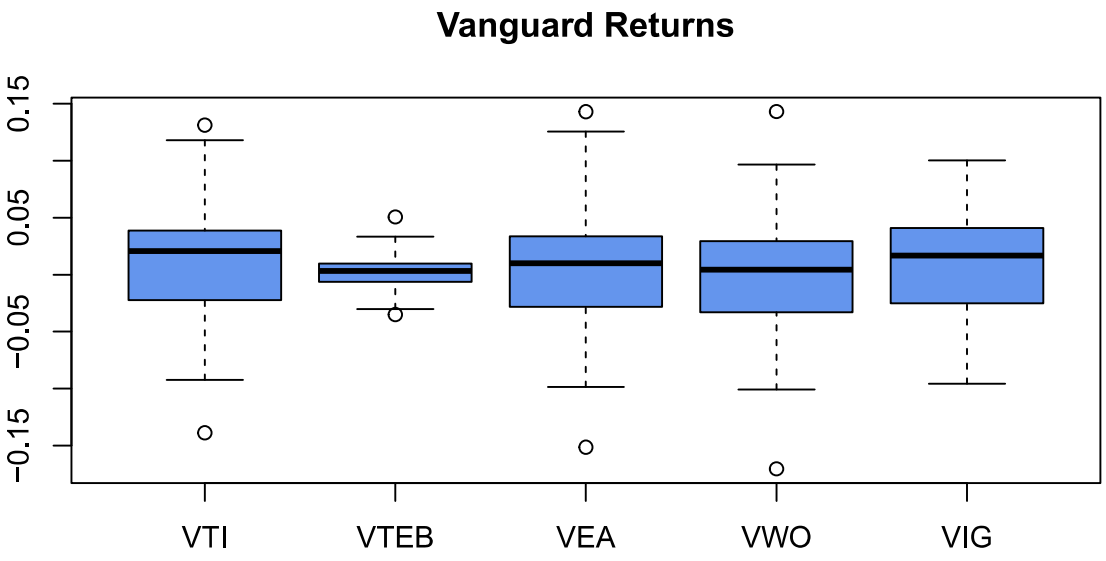
VWO Approximately normmal. Autocorrelation decreases to non 0 as lag increases.

VIG Approximately normmal. Autocorrelation decreases to 0.

Every stock except VIG have outliers.

There is an evidence of linear time dependence for all stocks.

Here is a group boxplot:



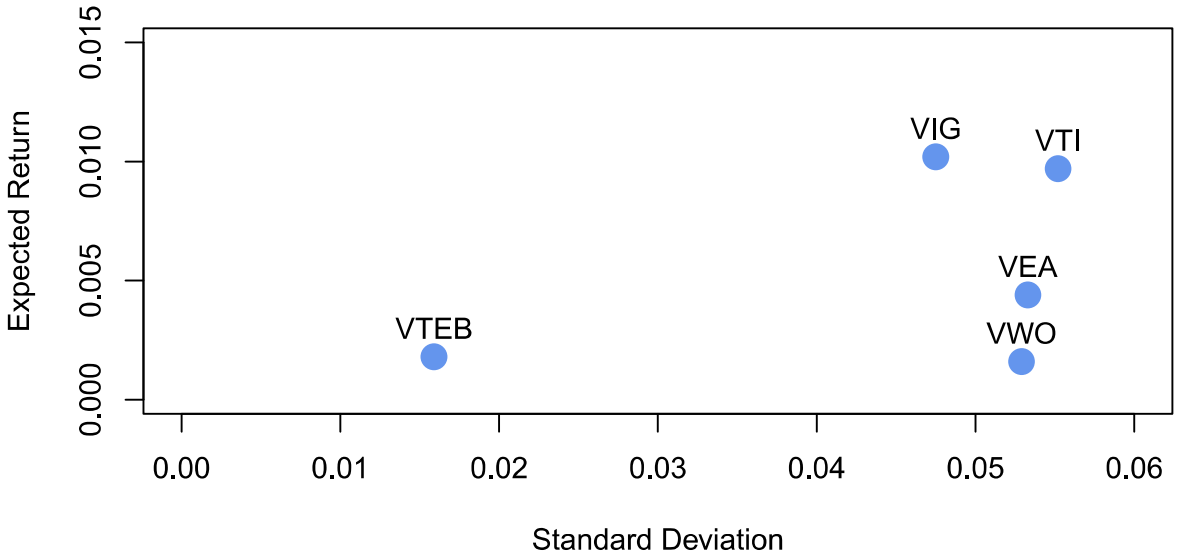
All stocks have similar median return, but VTI and VIG has the highest returns. VTEB have smallest volatility and VEA and VTI have the biggest volatility. All stocks except VIG have some outliers.

1.4 Univariate sample statistics

	VTI	VTEB	VEA	VWO	VIG
Observations	60.0000	60.0000	60.0000	60.0000	60.0000
NAs	0.0000	0.0000	0.0000	0.0000	0.0000
Minimum	-0.1389	-0.0350	-0.1515	-0.1705	-0.0959
Quartile 1	-0.0216	-0.0063	-0.0274	-0.0328	-0.0239
Median	0.0206	0.0033	0.0101	0.0044	0.0168
Arithmetic Mean	0.0097	0.0018	0.0044	0.0016	0.0102
Geometric Mean	0.0082	0.0017	0.0030	0.0002	0.0090
Quartile 3	0.0383	0.0097	0.0330	0.0284	0.0406
Maximum	0.1313	0.0507	0.1430	0.1430	0.1004
SE Mean	0.0071	0.0021	0.0069	0.0068	0.0061
LCL Mean (0.95)	-0.0045	-0.0023	-0.0093	-0.0120	-0.0021
UCL Mean (0.95)	0.0240	0.0059	0.0182	0.0153	0.0224
Variance	0.0030	0.0003	0.0028	0.0028	0.0023
Stdev	0.0552	0.0159	0.0533	0.0529	0.0475
Skewness	-0.4270	0.0000	-0.2014	-0.1601	-0.3240
Kurtosis	0.0173	0.8478	0.6616	1.0953	-0.3855

VIG has the highest average return and VWO has the lowest average return. VTI has largest standard deviation and VTEB has the smallest standard deviation.

Risk return plot:



The risk and return is positively correlated. In other words, stocks with higher returns have higher risk. VIG has slightly higher expected return than VTI, but has smaller risk. VWO and VEA have similar risk, but VEA has higher expected return.

1.5 Standard errors and confidence intervals

```
##            VTI     VTEB      VEA      VWO      VIG
## SE     0.00713  0.00206  0.00689  0.00683  0.00613
## Mean   0.00972  0.00177  0.00444  0.00162  0.01015
## Lower -0.00454 -0.00234 -0.00935 -0.01204 -0.00210
## Upper  0.02399  0.00589  0.01822  0.01528  0.02241
```

```
##           VTI    VTEB     VEA     VWO     VIG
## SE    0.00504 0.00145 0.00487 0.00483 0.00433
## Mean  0.05522 0.01594 0.05335 0.05287 0.04745
## Lower 0.04513 0.01303 0.04360 0.04321 0.03878
## Upper 0.06531 0.01885 0.06309 0.06252 0.05612
```

Comparing SE to the mean, the estimate of mean is not very precise. For example, the return of VWO can be anywhere between losing 1.2% to 1.5% with 95% probability. The standard error is about the same magnitude as the estimate, which is not very precise. Standard deviation, however, is estimated precisely. The standard error is about 1/10 of the estimate.

1.6 Estimated means and Standard deviations

From the previous section, it is shown that the estimated standard deviations is more precise than estimated means

1.7 Sharpe's Ratio

```
##         Sharpe    SE LCL (0.95) UCL (0.95)
## VTI   0.145795 0.141    -0.1394      0.414
```

```
## VTEB  0.006571 0.132    -0.2512      0.264
## VEA   0.051830 0.127    -0.1965      0.302
## VWO  -0.000916 0.133    -0.2659      0.256
## VIG   0.178776 0.137    -0.0971      0.440
```

VIG has the highest slope. The slopes are not estimated precisely. The magnitude of SE is similar or greater than the slope estimate.

1.8 Annualizing monthly sample statistics

Mean, Variance, and Standard Deviation:

```
##                    VTI    VTEB    VEA    VWO   VIG
## muhat.annual    0.1167 0.02130 0.0532 0.0195 0.122
## sig2hat.annual 0.0366 0.00305 0.0342 0.0335 0.027
## sighat.annual   0.1913 0.05521 0.1848 0.1831 0.164
```

VIG has the highest and VWO has the lowest average return. VTI has the highest and VTEB has the lowest standard deviation.

Correlation Matrix:

```
##        VTI  VTEB   VEA   VWO   VIG
## VTI  1.000 0.426 0.904 0.721 0.953
## VTEB 0.426 1.000 0.533 0.525 0.391
## VEA  0.904 0.533 1.000 0.834 0.877
## VWO  0.721 0.525 0.834 1.000 0.642
## VIG  0.953 0.391 0.877 0.642 1.000
```

Covariance Matrix:

```
##         VTI    VTEB     VEA     VWO     VIG
## VTI  0.0366 0.00450 0.03197 0.02525 0.02996
## VTEB 0.0045 0.00305 0.00544 0.00531 0.00354
## VEA  0.0320 0.00544 0.03415 0.02822 0.02665
## VWO  0.0252 0.00531 0.02822 0.03354 0.01932
## VIG  0.0300 0.00354 0.02665 0.01932 0.02702
```

The numbers seem reasonable.

Annualized Sharpe ratios:

```
##           VTI
## VTI   0.50526
## VTEB  0.02349
## VEA   0.17976
## VWO  -0.00295
## VIG   0.61954
```

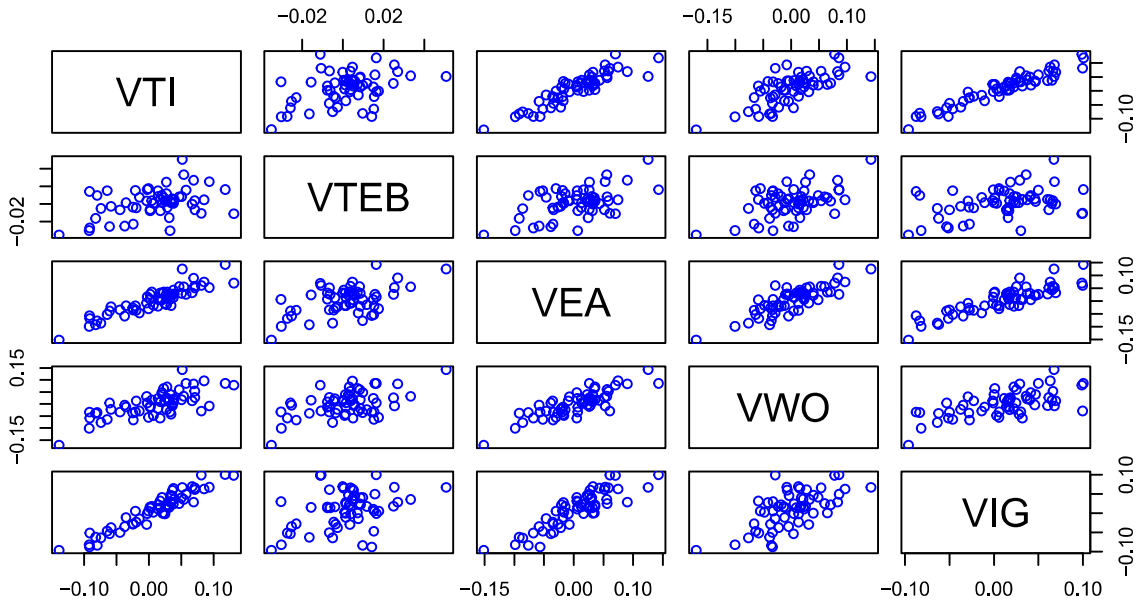
The annual Sharpe ratios are in the same ranking as the monthly Sharpe ratios

```
##  VTI VTEB  VEA  VWO  VIG
## 1.58 1.11 1.27 1.10 1.61
```

Assuming the average annual return every year for 5 years, $1 would each grow to $ 1.58, $ 1.11, $ 1.27, $ 1.10, $ 1.61 after 5 years, in the order of VTI, VTEB, VEA, VWO, and VIG.

1.9 Scatter plots and covariance

Pairwise scatterplots:



All stocks are positively related. VTEB seems to be least correlated with any stocks; the dots are farthest away from a linear line. VTI and VIG has the most linear relationship.

```
##          VTI     VTEB     VEA     VWO     VIG
## VTI   0.0366 0.00450 0.03197 0.02525 0.02996
## VTEB  0.0045 0.00305 0.00544 0.00531 0.00354
## VEA   0.0320 0.00544 0.03415 0.02822 0.02665
## VWO   0.0252 0.00531 0.02822 0.03354 0.01932
## VIG   0.0300 0.00354 0.02665 0.01932 0.02702
```

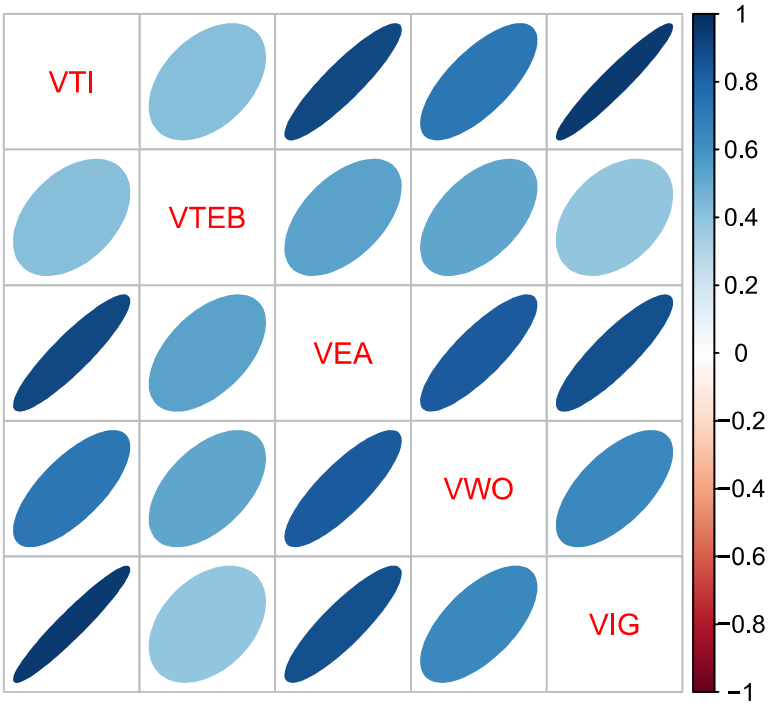
All covariances are positive and the paired values of stocks tend to increase together.

1.10 Scatter plots and correlation

Correlation Matrix:

```
##         VTI  VTEB   VEA   VWO   VIG
## VTI   1.000 0.426 0.904 0.721 0.953
## VTEB  0.426 1.000 0.533 0.525 0.391
## VEA   0.904 0.533 1.000 0.834 0.877
## VWO   0.721 0.525 0.834 1.000 0.642
## VIG   0.953 0.391 0.877 0.642 1.000
```

Correlations plots:



VIG and VTI are most highly correlated with correlation of 0.953. VIG and VTEB are least correlated with correlation of 0.391.

The diversification will reduce risk. The less stocks are correlated, it is easier to diversify risk across the assets. None of the asstes are perfectly correlated (correlation = 1), so there will be at least some diversification.

2. Value-at-Risk (VaR) Calculations

2.1 Bootstrapped Standard Errors and 95% Confidence Intervals for 5% Value at Risk

VaR at 1%:

```
##        VaR   Bias   SE Lower Upper
## VTI  11874 -226.8 1504  8864 14885
## VTEB  3530  -69.8  450  2630  4430
## VEA  11967 -177.7 1502  8963 14972
## VWO  12137 -260.4 1556  9022 15251
## VIG  10023 -163.1 1212  7598 12449
```

VaR at 5%:

```
##       VaR   Bias   SE Lower Upper
## VTI  8111 -117.2 1246  5617 10605
## VTEB 2444  -33.3  341  1761  3127
## VEA  8332  -85.9 1178  5975 10688
## VWO  8534 -164.4 1281  5970 11097
## VIG  6790  -95.3  943  4903  8676
```

VWO has the highest VaR with 1%: VaR = 12137, SE = 1556 and 5%: VaR = 8534, SE = 1281. VTEB has the lowest VaR with 1%: VaR = 3530, SE = 450 and 5%: VaR = 2444, SE = 341. The magnitude of standard error is small compared to the estimate and the range of confidence interval is small enough to conclude that the VaR estimate is precise.

2.2 Epirical Value at Risk

Emprical VaR at 1%

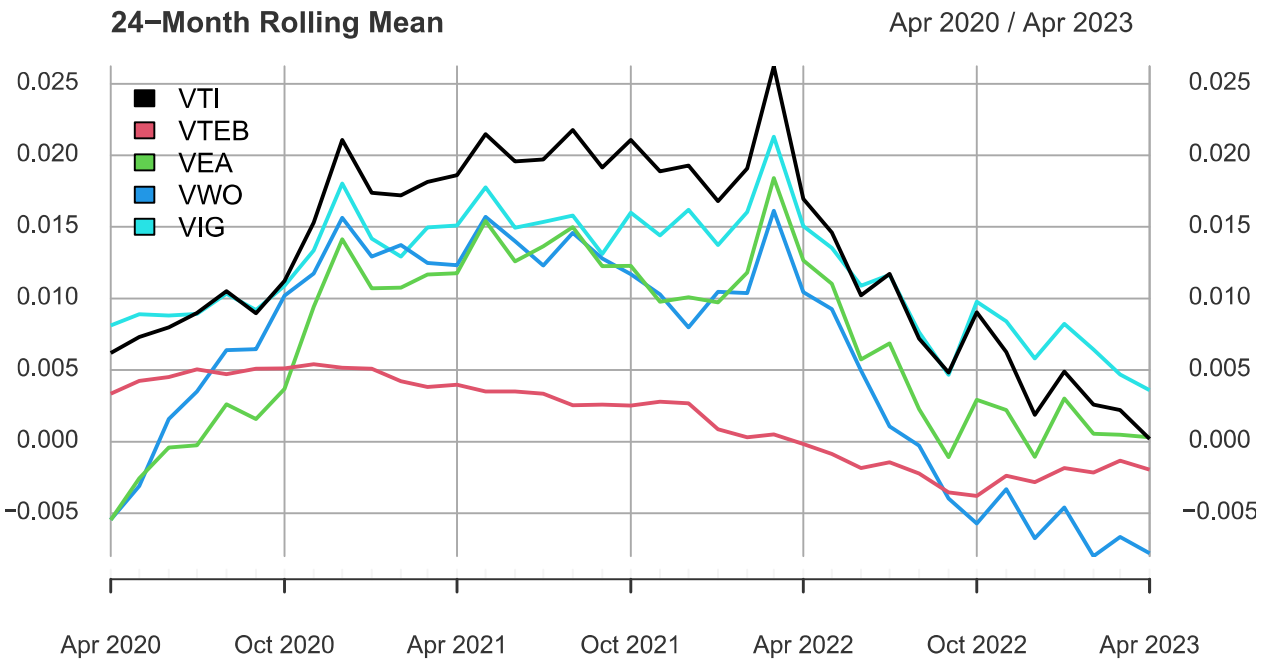
```
##   VTI  VTEB   VEA   VWO   VIG
## 13819  3885 12854 12461 12054
```

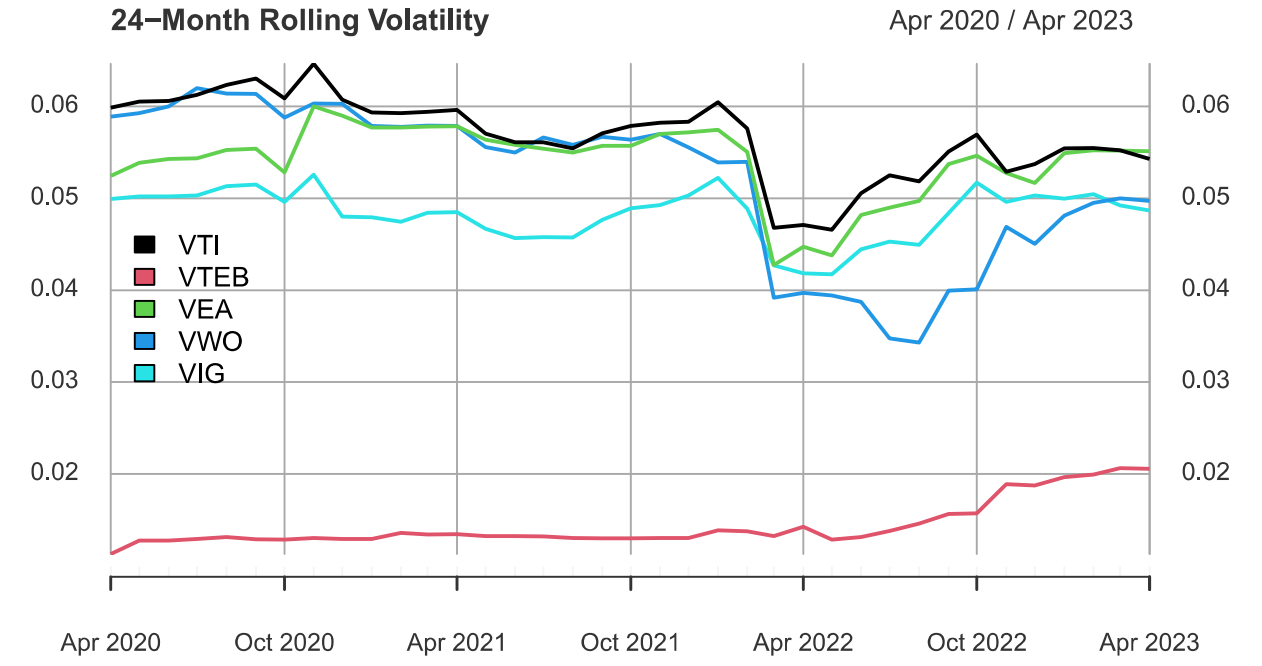
Emprical VaR at 5%

```
##   VTI  VTEB   VEA   VWO   VIG
## 10055  2799  9219  8858  8820
```

In general, Emprical VaR is larger than VaR based on normal distribution. This is expected because the stocks had fatter tails than normal distribution as shown in the section 1.3.

3 Rolling Sample Statistics





The returns are covariance stationary if means, variances, covariances, and correlations are constant over time for all assets. Clearly, the mean and volatility are not constant over time, so they are not covariance stationary.

4 Portfolio Theory

4.1 Expected return and volatility

```
##     Portfolio Expected return Volatility VTI.x VTEB.x VEA.x VWO.x
## 1 EqualWeight           0.006      0.041   0.2    0.2   0.2   0.2
##   VIG.x
## 1   0.2
```

Annualized sharpe ratios:

```
##      VTI     VTEB      VEA      VWO      VIG     P.EQ
##  0.50526  0.02349  0.17976 -0.00295  0.61954  0.33052
```

```
##      VTI VTEB   VEA   VWO   VIG  P.eq
## 1% 13819 3885 12854 12461 12054 10001
## 5% 13819 3885 12854 12461 12054  7233
```

The sarpe ratio of equal weight portfolio is higher than VTEB, BEA, and VWO, but lower than VTI and VIG. In other words, The risk-adjusted returns of the equal weight portfolio is higher than higher than VTEB, BEA, and VWO, but lower than VTI and VIG.

In one month investment horizon, equal weight portfolio is likely to lose more money than VTEB, but less than all other individual stocks.

4.2 Global minimum variance portfolio

```
##              Portfolio Expected return Volatility  VTI.x VTEB.x
## 1          EqualWeight           0.006      0.041    0.2    0.2
## 2 MinVariance w/ short           0.003      0.014 -0.108  1.025
##    VEA.x VWO.x VIG.x
## 1    0.2   0.2   0.2
## 2 -0.249 0.029 0.304
```

```
##      VTI     VTEB      VEA      VWO      VIG     P.EQ  P.min.s
##  0.50526  0.02349  0.17976 -0.00295  0.61954  0.33052  0.26954
```

There are two negative weights. The negative weights suggest that VTI and VEA will be short saled in a portfolio with least volatility.

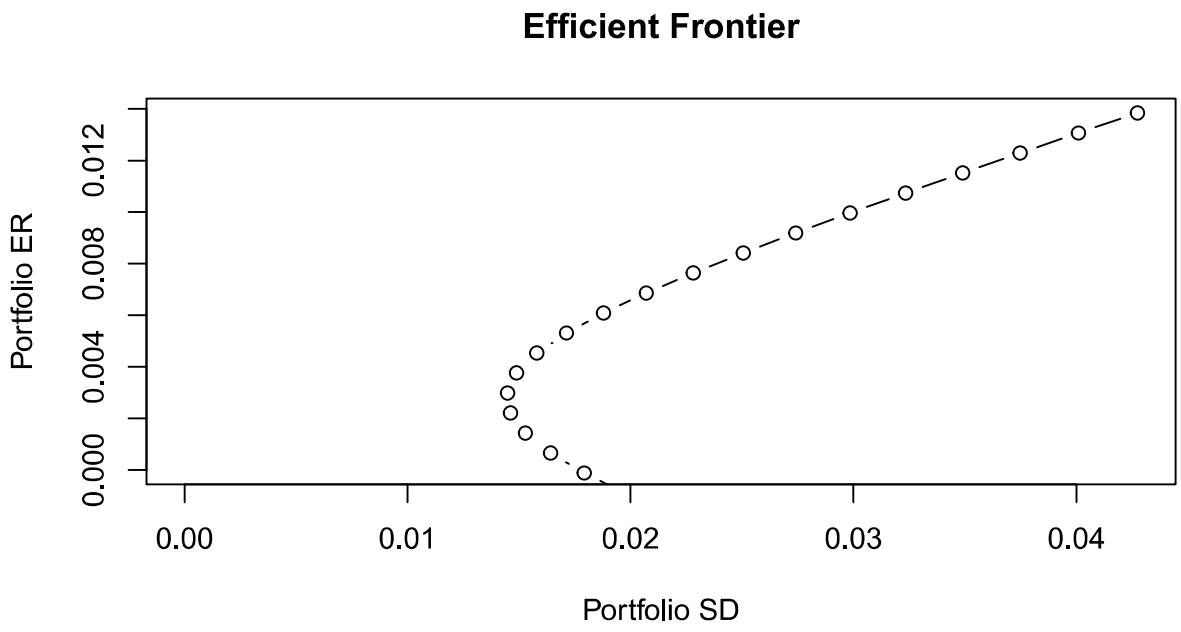
The sarpe ratio of equal weight portfolio is higher than VTEB, BEA, and VWO, but lower than VTI and VIG. In other words, The risk-adjusted returns of the equal weight portfolio is higher than higher than VTEB, BEA, and VWO, but lower than VTI and VIG.

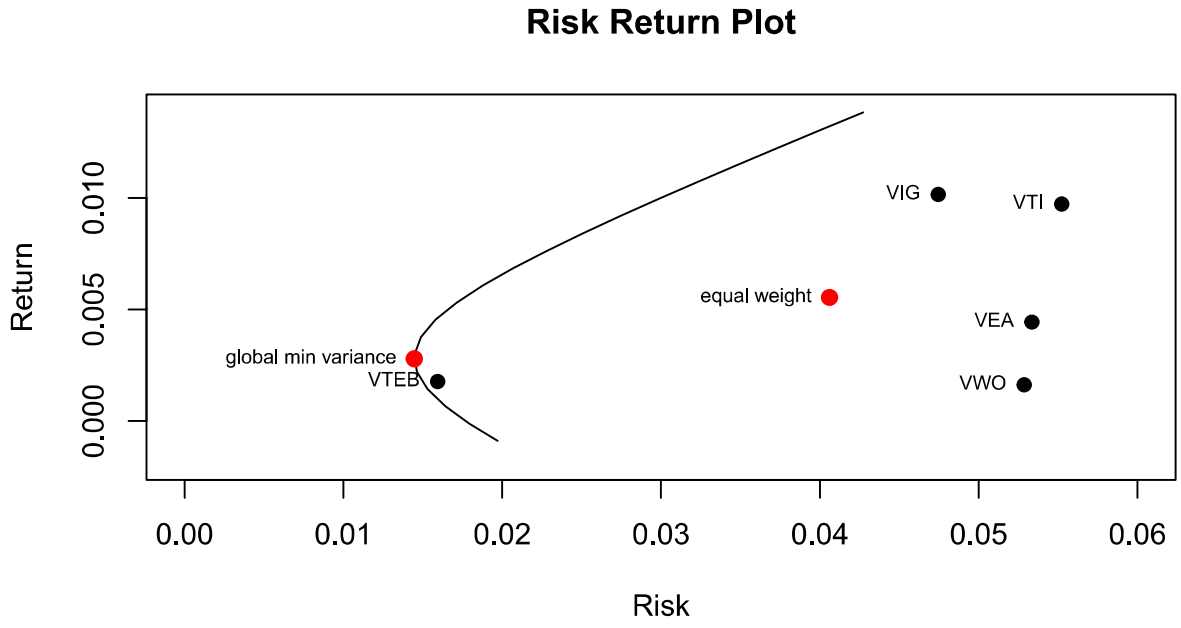
4.3 Global minimum variance portfolio Value-at-Risk

```
##      VTI VTEB   VEA   VWO   VIG  P.eq P.min.s
## 1% 13819 3885 12854 12461 12054 10001    3644
## 5% 13819 3885 12854 12461 12054  7233    2659
```

In one month investment horizon, equal weight portfolio is likely to lose less amount of money than all other individual stocks.

4.4 Efficient portfolio frontier (with short sales)






```
##                   Portfolio Expected return Volatility  VTI.x VTEB.x
## 1               EqualWeight           0.006      0.041    0.2    0.2
## 2       MinVariance w/ short           0.003      0.014 -0.108  1.025
## 3 Efficient w/ target return           0.006      0.018 -0.029  0.901
```

```
##    VEA.x VWO.x VIG.x
## 1    0.2   0.2   0.2
## 2 -0.249 0.029 0.304
## 3  -0.56 0.032 0.656
```

The efficient portfolio with the same return as equal weight portfolio has volatility of 0.018, compared to the volatility of the equal weight portfolio, which is 0.041. The volatility of the efficient portfolio is smaller.

```
##                    Portfolio Expected return Volatility  VTI.x VTEB.x
## 1                EqualWeight           0.006      0.041    0.2    0.2
## 2       MinVariance w/ short           0.003      0.014 -0.108  1.025
## 3 Efficient w/ target return           0.006      0.018 -0.029  0.901
## 4     Efficient w/ target sd           0.014      0.041  0.138  0.925
##    VEA.x VWO.x VIG.x
## 1    0.2   0.2   0.2
## 2 -0.249 0.029 0.304
## 3  -0.56 0.032 0.656
## 4 -1.442 0.051  1.66
```

The expected return of the equal weight portfolio is 0.006. In constrast, the expected return of the efficient portfolio with the same volatility as the equal weight portfolio is 0.014.

4.5 Tangency Portfolio (with short sales)

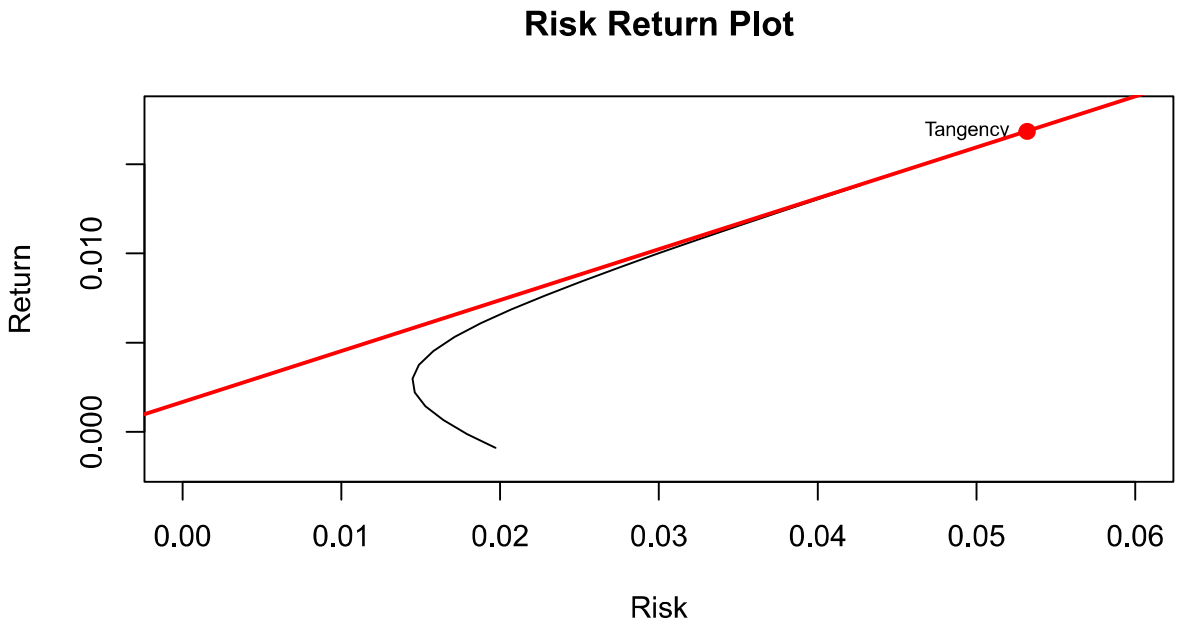
```
##                    Portfolio Expected return Volatility  VTI.x VTEB.x
## 1                EqualWeight           0.006      0.041    0.2    0.2
## 2       MinVariance w/ short           0.003      0.014 -0.108  1.025
## 3 Efficient w/ target return           0.006      0.018 -0.029  0.901
## 4     Efficient w/ target sd           0.014      0.041  0.138  0.925
## 5          Tangency w/ short           0.017      0.053  0.297  0.393
##    VEA.x VWO.x VIG.x
## 1    0.2   0.2   0.2
## 2 -0.249 0.029 0.304
## 3  -0.56 0.032 0.656
## 4 -1.442 0.051  1.66
## 5 -1.839 0.047 2.102
```

Sharpe Ratios:

```
##      VTI     VTEB      VEA      VWO      VIG     P.EQ  P.min.s
##  0.50526  0.02349  0.17976 -0.00295  0.61954  0.33052  0.26954
##  P.tan.s
##  0.98849
```

VaR:

```
##      VTI VTEB   VEA   VWO   VIG  P.eq P.min.s P.tan.s
## 1% 13819 3885 12854 12461 12054 10001    3644   14060
## 5% 13819 3885 12854 12461 12054  7233    2659   10434
```



In the tangency portfolio, VEA has a negative weight. The expected return is 1.7% and volatility is 5.3% The sharpe ratio of this tangency portfolio is higher than any individual stocks.

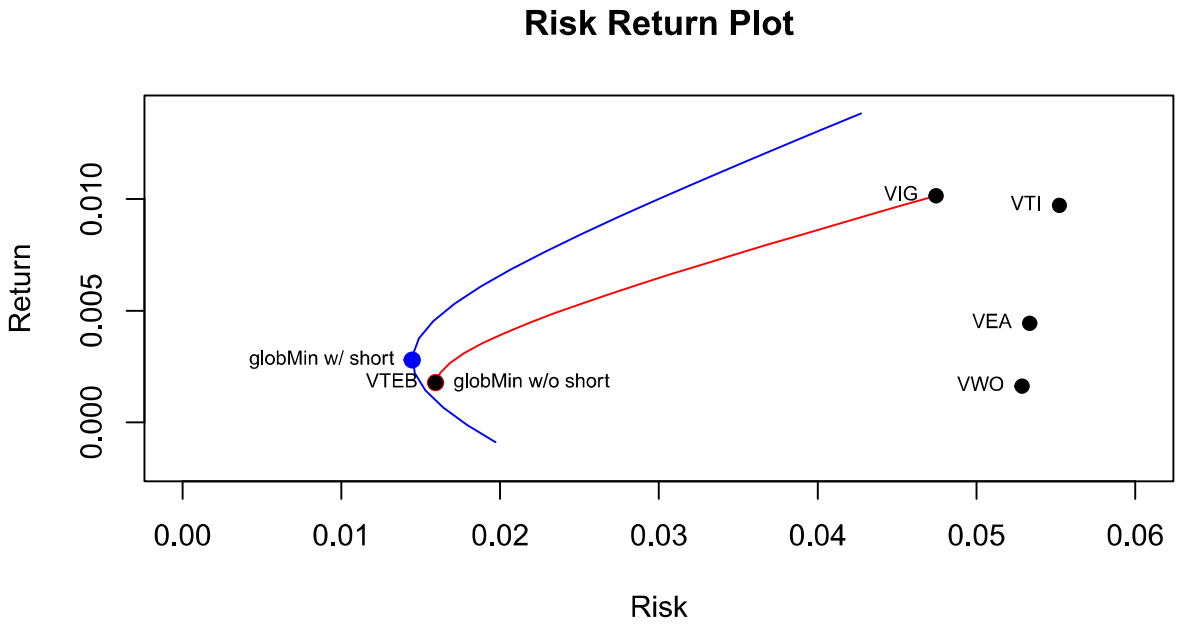
4.6 Efficient portfolio frontier (without short sales)

```
##                   Portfolio Expected return Volatility  VTI.x VTEB.x
## 1               EqualWeight           0.006      0.041    0.2    0.2
## 2       MinVariance w/ short           0.003      0.014 -0.108  1.025
## 3 Efficient w/ target return           0.006      0.018 -0.029  0.901
## 4     Efficient w/ target sd           0.014      0.041  0.138  0.925
## 5          Tangency w/ short           0.017      0.053  0.297  0.393
## 6      MinVariance w/o short           0.002      0.016      0      1
##    VEA.x VWO.x VIG.x
## 1    0.2   0.2   0.2
## 2 -0.249 0.029 0.304
## 3  -0.56 0.032 0.656
## 4 -1.442 0.051  1.66
## 5 -1.839 0.047 2.102
## 6      0     0     0
```

```
##      VTI     VTEB      VEA      VWO      VIG     P.EQ  P.min.s
##  0.50526  0.02349  0.17976 -0.00295  0.61954  0.33052  0.26954
##  P.tan.s    P.min
##  0.98849  0.02349
```

```
##      VTI VTEB   VEA   VWO   VIG  P.eq P.min.s P.tan.s P.min
## 1% 13819 3885 12854 12461 12054 10001    3644   14060  3885
## 5% 13819 3885 12854 12461 12054  7233    2659   10434  2799
```

In global minimum variance portfolio without shorting, 100% of assets are invested into VTEB. Compared to the global minimum variance with shorting, this portfolio has lower expected return, higher volatility, and larger annual value-at-risk. Overall, the sharpe ratio is lower than the portfolio with shorting.



The efficient frontier without short sales are inside the frointer with short sales. All five stocks are within both frontiers. The frontier without short sales traces out the path between stocks with minimum risk given a return.

```
## $short.p
## Call:
## getPortfolio(er = muhat, cov.mat = cov.mat, weights = scaled_w)
##
## Portfolio expected return:     0.00667
## Portfolio standard deviation:  0.02
## Portfolio weights:
##     VTI    VTEB     VEA     VWO     VIG
##  0.0682  0.4554 -0.7101  0.0252  0.8178
##
## $long.p
## Call:
## getPortfolio(er = muhat, cov.mat = cov.mat, weights = scaled_w)
##
## Portfolio expected return:     0.00428
## Portfolio standard deviation:  0.02
## Portfolio weights:
##   VTI  VTEB   VEA   VWO   VIG
## 0.000 0.000 0.000 0.000 0.421
```

When the target volatility is 2%, the approximate cost in lost expected return of investing in a long only efficient portfolio versus a short sale efficient portfolio is 0.239 %.

```
## Call:
## efficient.portfolio(er = muhat, cov.mat = cov.mat, target.return = 0.005,
##     shorts = TRUE)
##
```

```
## Portfolio expected return:     0.005
## Portfolio standard deviation:  0.0166
## Portfolio weights:
##     VTI    VTEB     VEA     VWO     VIG
## -0.0447  0.9256 -0.4986  0.0315  0.5863
```

```
## Call:
## efficient.portfolio(er = muhat, cov.mat = cov.mat, target.return = 0.005,
##     shorts = FALSE)
##
## Portfolio expected return:     0.005
## Portfolio standard deviation:  0.0239
## Portfolio weights:
##   VTI  VTEB   VEA   VWO   VIG
## 0.000 0.615 0.000 0.000 0.385
```

When the target volatility is 5%, the approximate cost in lost expected return of investing is 0.239 %. If the target return is 0.005, the approximate cost in volatility is -0.007.

4.7 Tangency Portfolio (without short sales)

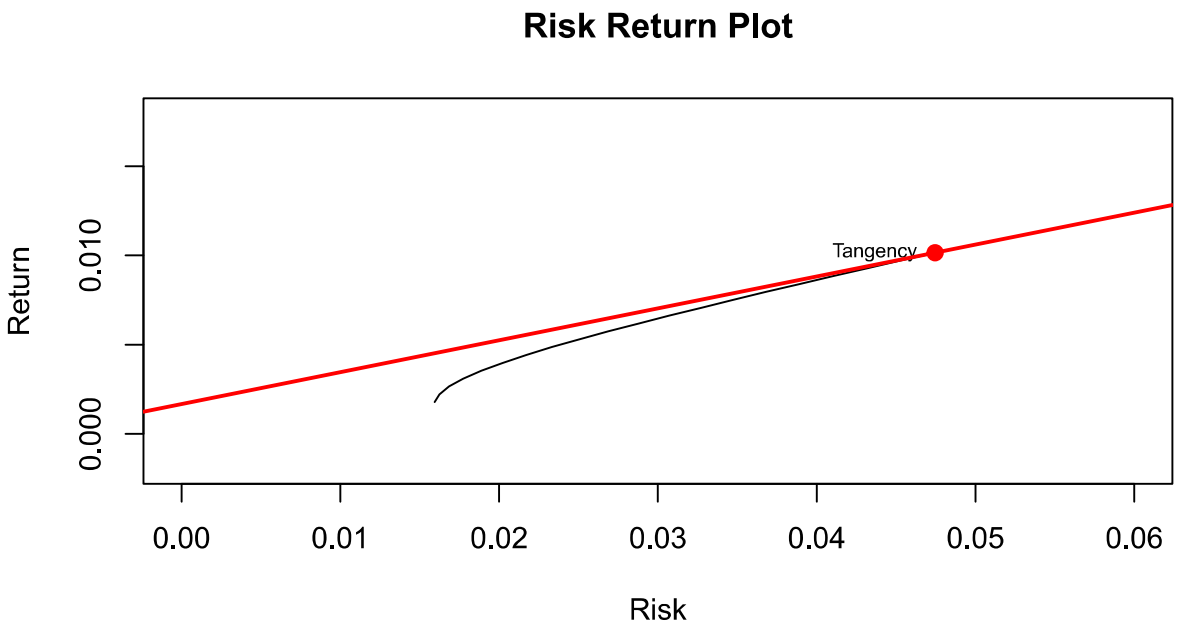
```
##                    Portfolio Expected return Volatility  VTI.x VTEB.x
## 1                EqualWeight           0.006      0.041    0.2    0.2
## 2       MinVariance w/ short           0.003      0.014 -0.108  1.025
## 3 Efficient w/ target return           0.006      0.018 -0.029  0.901
## 4     Efficient w/ target sd           0.014      0.041  0.138  0.925
## 5          Tangency w/ short           0.017      0.053  0.297  0.393
## 6      MinVariance w/o short           0.002      0.016      0      1
## 7         Tangency w/o short            0.01      0.047      0      0
##    VEA.x VWO.x VIG.x
## 1    0.2   0.2   0.2
## 2 -0.249 0.029 0.304
## 3  -0.56 0.032 0.656
## 4 -1.442 0.051  1.66
## 5 -1.839 0.047 2.102
## 6      0     0     0
## 7      0     0     1
```

Sharpe Ratios:

```
##      VTI     VTEB      VEA      VWO      VIG     P.EQ  P.min.s
##  0.50526  0.02349  0.17976 -0.00295  0.61954  0.33052  0.26954
##  P.tan.s    P.min    P.tan
##  0.98849  0.02349  0.61954
```

VaR:

```
##      VTI VTEB   VEA   VWO   VIG  P.eq P.min.s P.tan.s P.min P.tan
## 1% 13819 3885 12854 12461 12054 10001    3644   14060  3885 12054
## 5% 13819 3885 12854 12461 12054  7233    2659   10434  2799  8820
```



The expected return of a long only tangency portfolio is 0.01. The volaility is 0.047 and the variance is 0.002. When no short sales are allowed, 100% of wealth is invested into VIG. The sharpe value is 0.62, which is lower than 0.99 of tangency portfolio with short sales and equals to the sharp ratios of VIG. The VaR is much smaller in absolute value for the global minimum variance portfolio without shorting than with shorting, implying this portfolio has safer in worse-case-scenario than holding the shorting portfolio.

5 Risk Budgeting

5.1 Volatility risk report

```
##   Asset Dollars Invested Allocation Weights     MCR      CR    PCR
## 1   VTI            20000                0.2 0.05265 0.01053 0.2593
## 2  VTEB            20000                0.2 0.00896 0.00179 0.0442
## 3   VEA            20000                0.2 0.05189 0.01038 0.2556
## 4   VWO            20000                0.2 0.04582 0.00916 0.2257
## 5   VIG            20000                0.2 0.04371 0.00874 0.2153
##   Asset correlation Asset beta
## 1             0.953      1.297
## 2             0.562      0.221
## 3             0.973      1.278
## 4             0.867      1.128
## 5             0.921      1.076
```

While the allocation weights are all equal (2 %), the contributions to risk (CR) of all stocks differ. marginal contributions to risk and percentage contributions to risk also differ. There is no equal risk allocation in the equal weight portfolio. VTI contributes to the risk the most and VTEB contributes the least.

6 Asset Allocation

6.1 Weights in efficient portfoilo with short sales

```
## Call:
## efficient.portfolio(er = muhat, cov.mat = cov.mat, target.return = 0.005,
##     shorts = FALSE)
##
## Portfolio expected return:     0.005
## Portfolio standard deviation:  0.0239
## Portfolio weights:
##   VTI  VTEB   VEA   VWO   VIG
## 0.000 0.615 0.000 0.000 0.385
```

```
##    [,1]
## 1% 6052
## 5% 4426
```

In the efficient portfolio with a target expected return of 6% per year (0.5% per month) using only the 5 ETFs and no short sales, 0, 61.504, 0, 0, 38.496 % is invested into VTI, VTEB, VEA, VWO, and VIG. The monthly volatility of this portfolio is 0.024.

VaR at 1 % is $6052 and VaR at 5 % is $4426.

6.2 Weights in portfoilo with target expected return

```
## [1] 0.0117
```

This portfolio invests 0.219 in tangency portfolio and 0.781 in risk free asset. Each 5 ETF is invested 0.065, 0.086, -0.404, 0.01, 0.461. The volatility is 0.012, which is -0.012 less than the efficient portfolio with only 5 ETF. Value-at-risk at 5% is 2419.932 and at 1% is 3215.397.

6.3 Weights in portfoilo with target expected return and without short sales

The maximum annual return possible from all risky assets is 12.184 %, or monthly 1.015 %. Thus, it is possible to achieve the target expected return of 12% per year (1% per month) with long only portfolio.