

# Formulas

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## 1. M1

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$$\text{Performance} = \frac{1}{\text{execution time}}$$

$$\text{Execution Time} = \text{IC} \times \text{CPI} \times T$$

$$\text{Speedup} = \frac{\text{Perf}_A}{\text{Perf}_B} = \frac{\text{Time}_B}{\text{Time}_A} = \frac{T_{\text{original}}}{T_{\text{enhanced}}} = \frac{1}{(1-E)+E/S}$$

- $E$  is the fraction of the process that got speed up by a factor of  $S$
- $\text{Speedup}_{\text{max}} = \frac{T_s + n \times T_p}{T_s + T_p}$  for Gustafson's Law

$$\text{Dynamic Power } P_{\text{dyn}} = ACV^2 f$$

$C$ : total load capacitance in the circuit

$f$ : clock frequency

Reduction of frequency reduces  $P_{\text{dyn}}$ , but can degrade performance

$V$ : operating voltage (also called  $V_{dd}$ )

Reduction of voltage reduces power consumption significantly

$A$ : switching activity factor: the fraction of transistors switch during a clock cycle (in average).

Can be reduced by turning-off the unused resources/components

$$\text{Static / Leakage Power } P_{st} = VI_{leak}$$

$$\text{Maximum operating frequency } f_{\text{max}} \propto [V - V_{th}]^2 / V$$

## 2. M4

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$$\text{Steady State CPI} = \frac{\text{number of instructions} + \text{number of stalls}}{\text{number of instructions}}$$

## 3. M5

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- Size of main memory =  $2^n$  number of bits for addressing
  - e.g. main memory size = 2GB, number of bits for addressing = 31

$$\text{cache size} = (1 \text{ bit} + t \text{ bits} + \text{BS bytes}) \times E \times S$$

- Can also be Block size  $\times$  bytes used for tag, LRU, dirty bits (number of bits / 8)  $\times$  no. of blocks
- $t$  = tag, BS = blocksize, E = cache lines in 1 set, S = number of sets
- Number of bits in `offset` is given by number of bytes in 1 block (if byte addressable)

- Number of bits in **set** is given by size of cache / size of set
- Number of bits in **tag** is the remaining bits
- **Other special stuff**
  - Number of bits in a cache line = "#tag + #valid + #data" (data bits = block size)
  - Cache Size = "#cachelines x #bits in cacheline"
  - Cache Overhead = Cache Size / Data size in cache (excludes tag and valid bits)
- Number of sets = number of blocks / set associativity (2 if 2-way set associative)
- Execution time =  $IC \times CPI \times \text{Cycle time}$ 
  - If cache hit costs are included,  $CPI = CPI_{ideal} + \text{Memory-stall cycles}$
  - Memory-stall cycles = memory accesses / program  $\times$  miss rate  $\times$  miss penalty
- A processor with a  $CPI_{ideal}$  of 2, a 100 cycle miss penalty,
- 36% load/store instr's, and 2% IM and 4% DM miss rates

Memory-stall cycles = Memory stall cycles in IM + memory stall cycles in DM

Memory stall cycles in IM = 2% (miss rate of IM)  $\times$  100 (Miss penalty) = 2  
(All instructions has to be fetched from IM)

Memory stall cycles in DM =  
36% (LW/SW)  $\times$  4% (miss rate)  $\times$  100 (miss penalty) = 1.44

So  $CPI_{stalls} = 2 + 2 + 1.44 = 5.44$  (more than twice the  $CPI_{ideal}$  !)

- What if the  $CPI_{ideal}$  is reduced to 1?  
For ideal  $CPI = 1$ , then  $CPI_{stall} = 4.44$  and the amount of execution time spent on memory stalls would have risen from  $3.44/5.44 = 63\%$  to  $3.44/4.44 = 77\%$
- For our example,  $CPI_{ideal}$  of 2,
  - 100 cycle miss penalty (to main memory)
  - 25 cycle miss penalty (to Unified L2 cache)
  - 36% load/stores,
  - 2% miss-rate for L1 I-Mem and 4% miss-rate for L1 D-Mem
  - 0.5% Unified L2 cache miss rate.

Note: every instruction will access the instruction memory for instruction fetch (100%)  
only load/store instructions will access the data memory (36%)

$$\begin{aligned}
 CPI &= CPI_{ideal} + CPI_{L1Inst\_miss} + CPI_{L1data\_miss} + CPI_{L2Inst\_miss} + CPI_{L2data\_miss} \\
 &= CPI_{ideal} + CPI_{L2\_hit\_inst} + CPI_{L2\_hit\_data} + CPI_{MM\_inst} + CPI_{MM\_data} \\
 &= 2 + 2\% \times 25 + 36\% \times 4\% \times 25 + 2\% \times 0.5\% \times 100 + 36\% \times 4\% \times 0.5\% \times 100 \\
 &= 2 + 0.5 + 0.36 + 0.01 + 0.0072 \\
 &= 2.8772 \text{ cycles/instr. (as compared to 5.44 (no L2cache))}
 \end{aligned}$$

## Average Memory Access Time (AMAT)

- Average time required to access memory considering both hits and misses
  - $\text{Time for hit} + \text{Miss Rate} \times \text{Miss Penalty}$