Introduction to Artificial Intelligence with Python

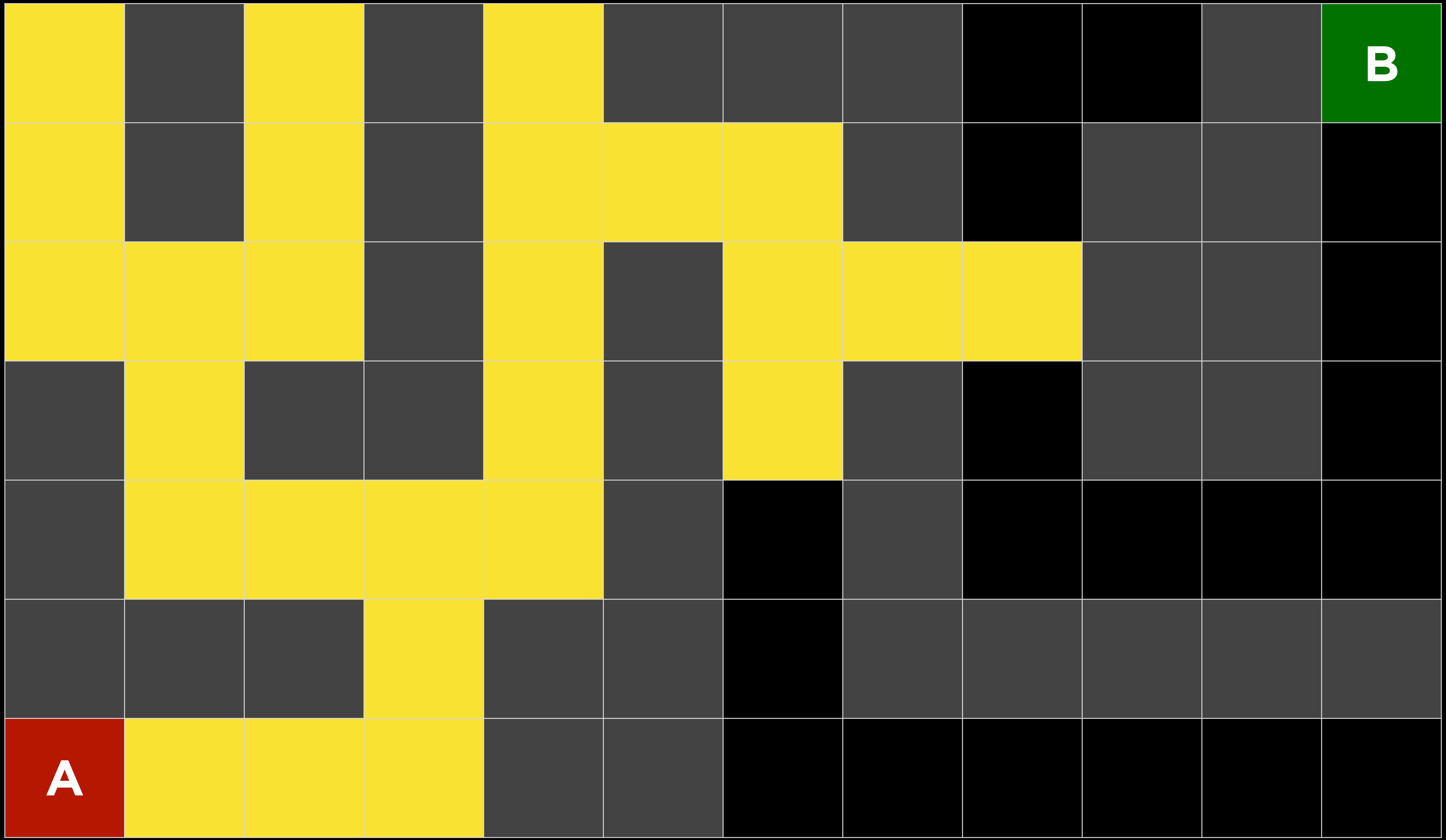
Optimization

optimization

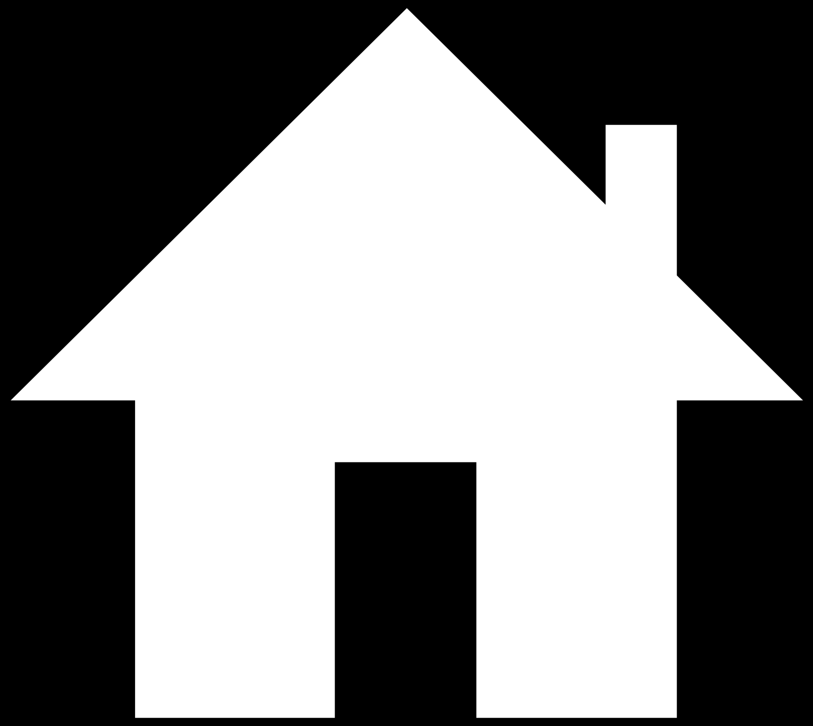
choosing the best option from a set of options

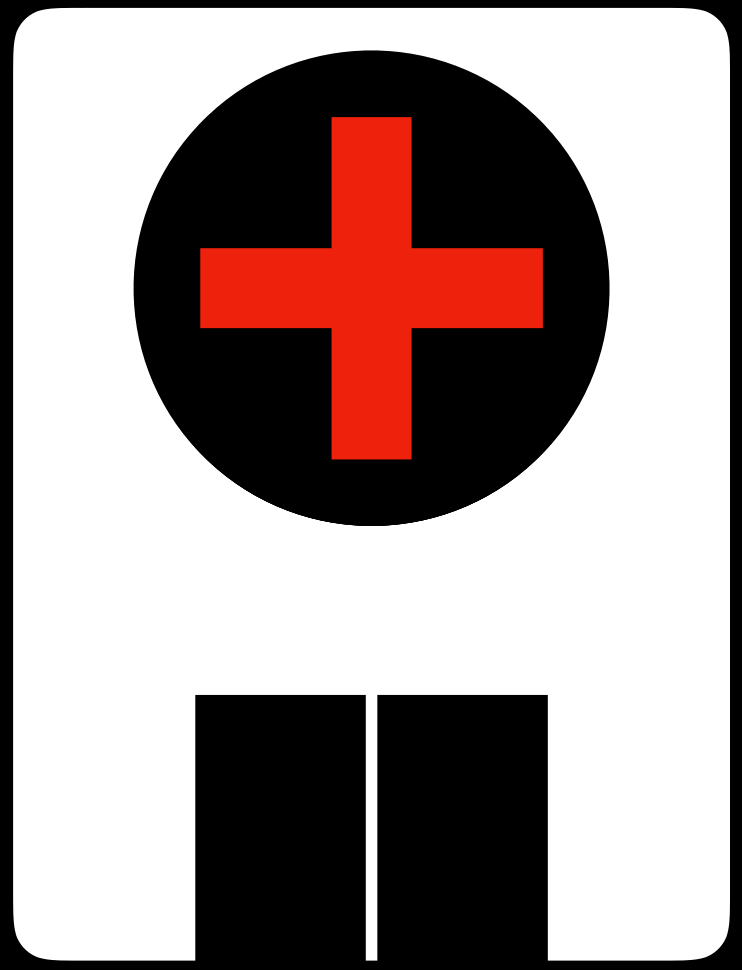
local search

search algorithms that maintain a single node and searches by moving to a neighboring node

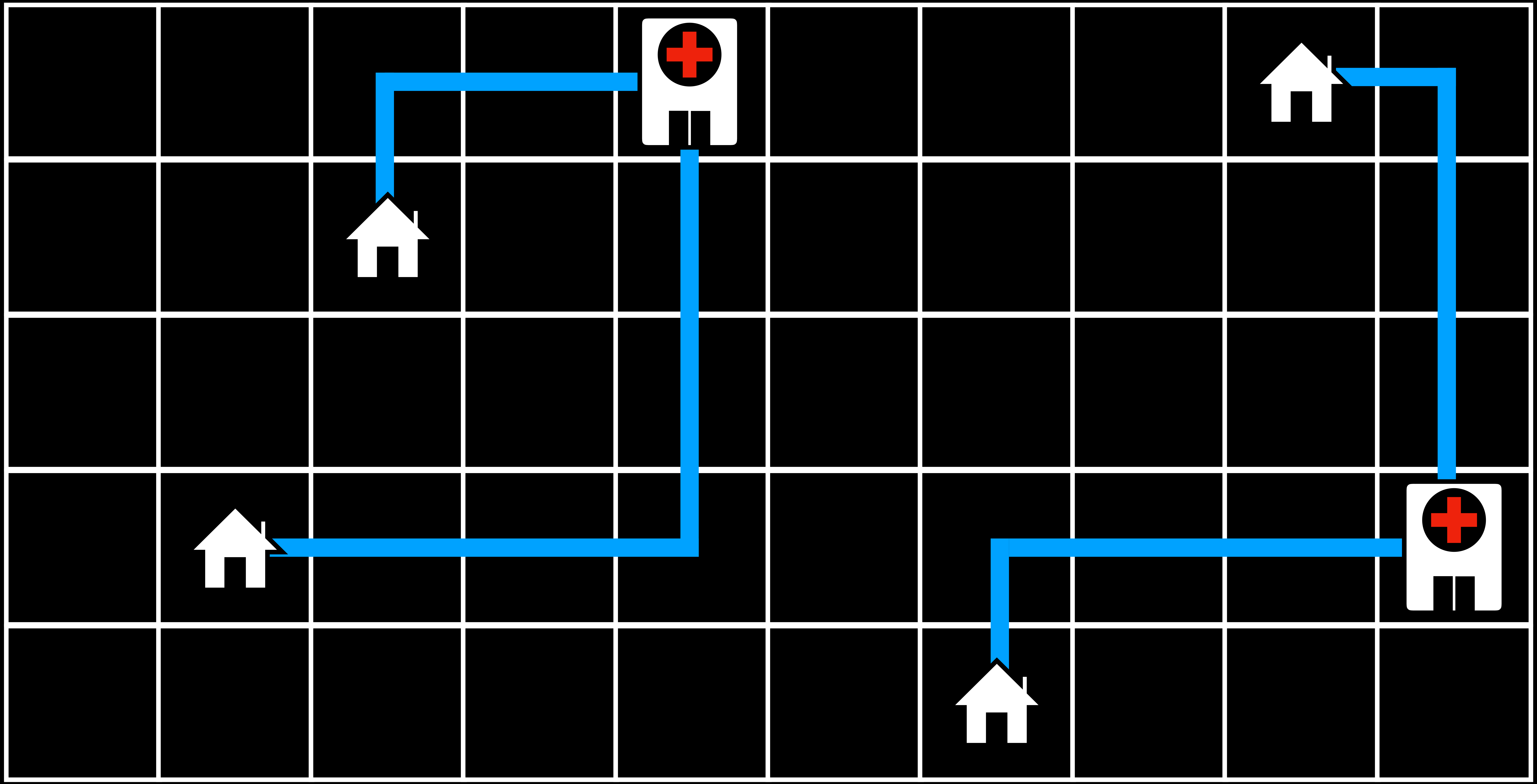

B
A


B





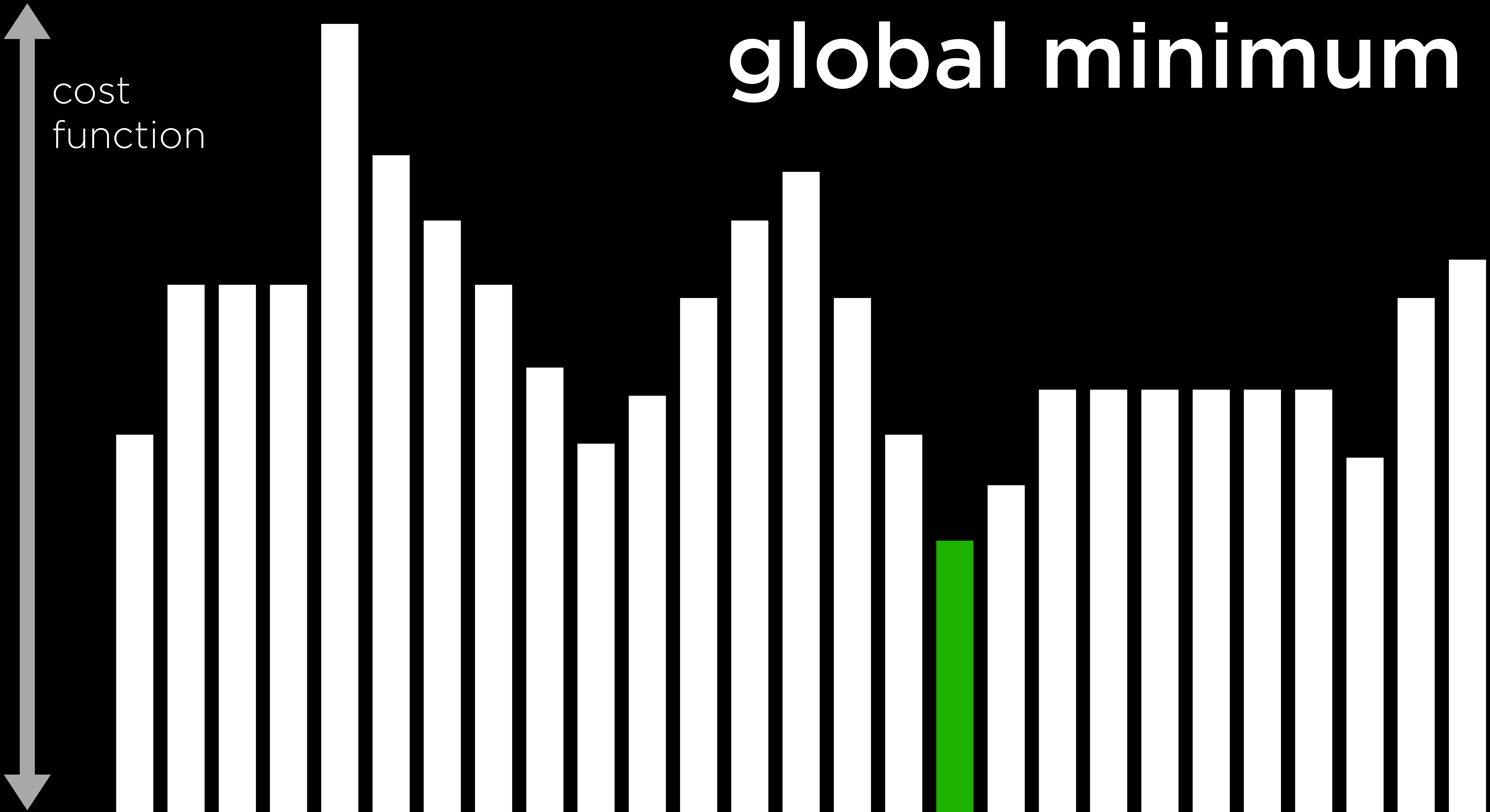
Cost: 17



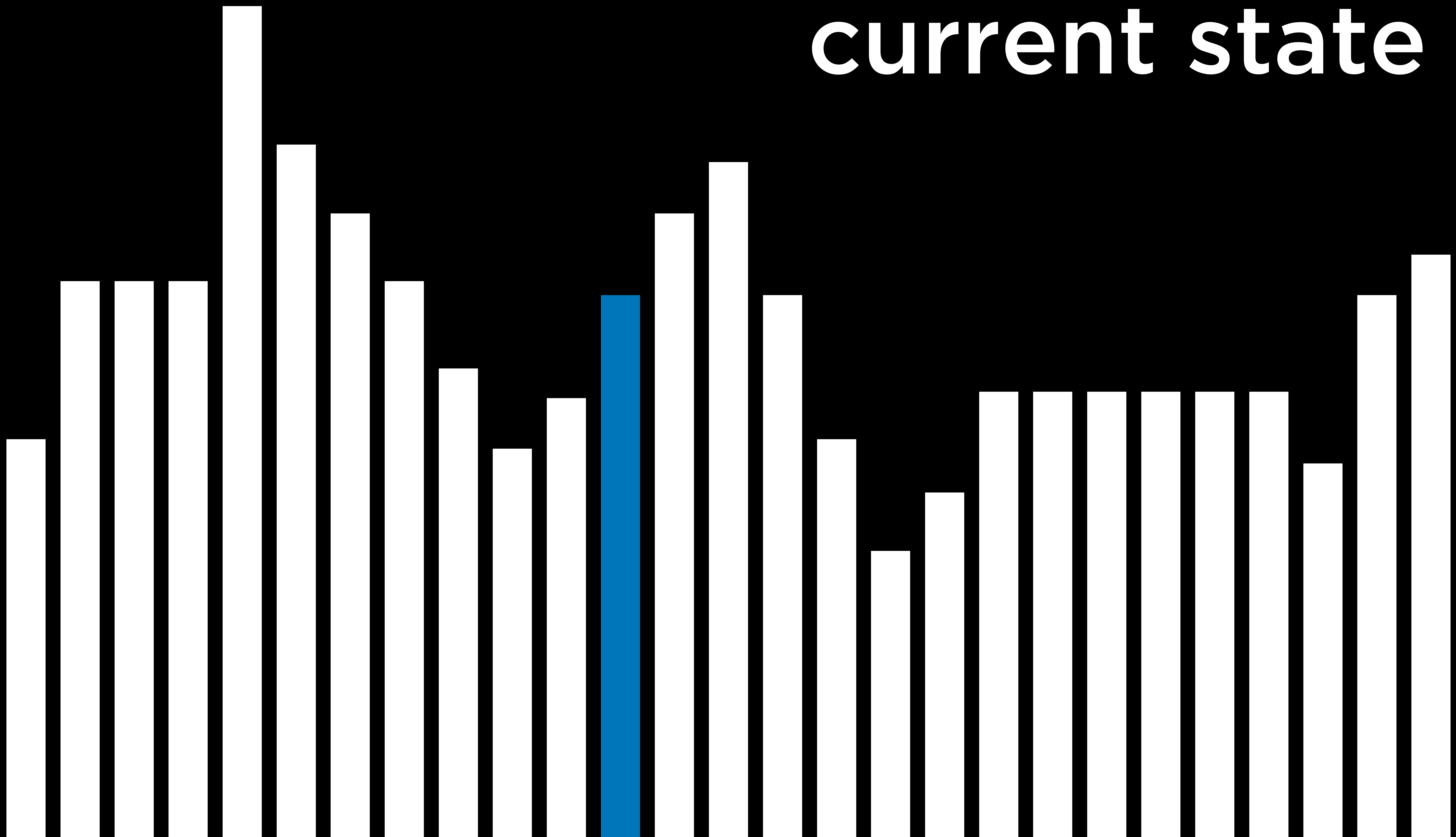
state-space landscape

global maximum

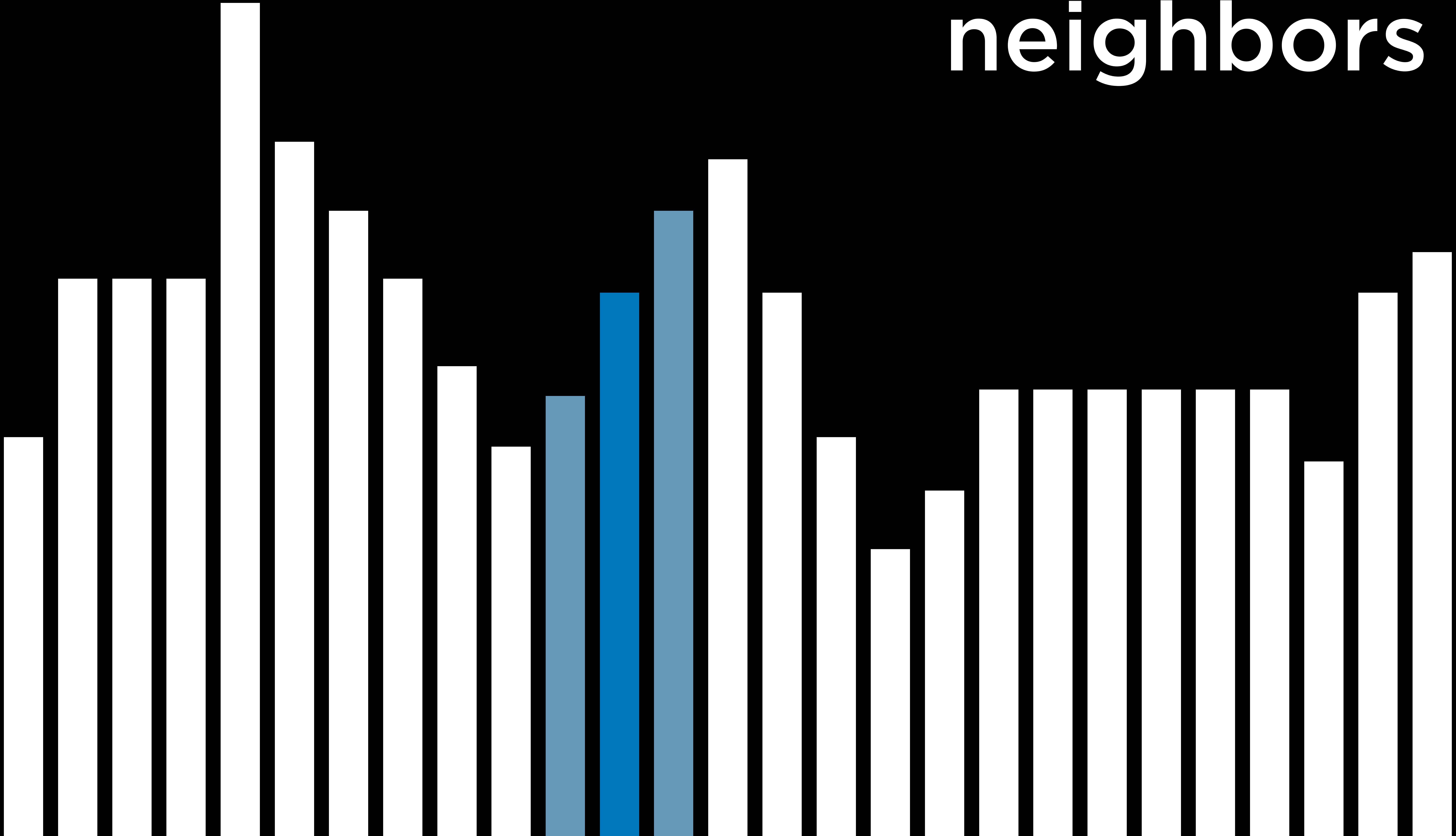
global minimum



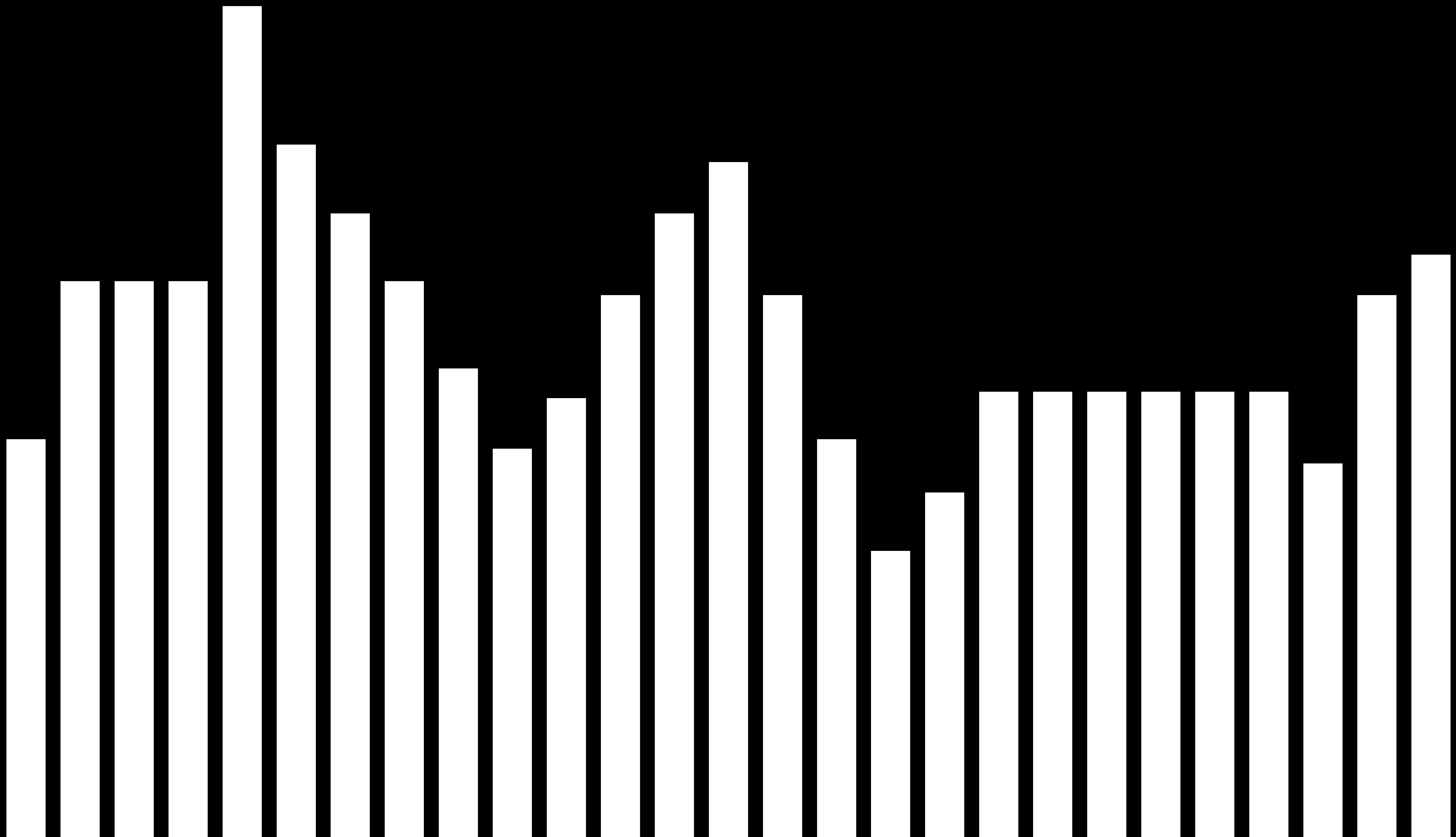
current state

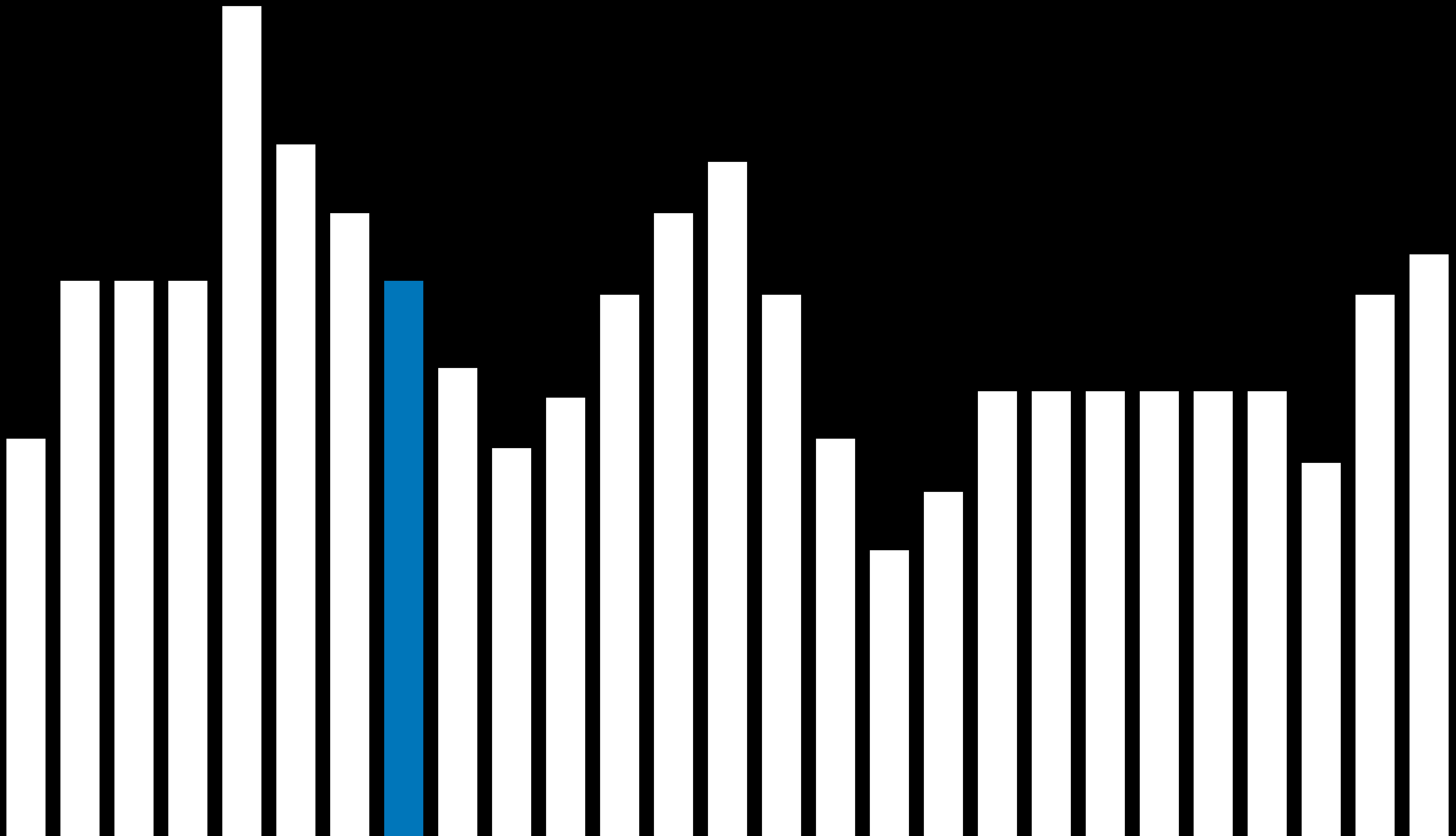


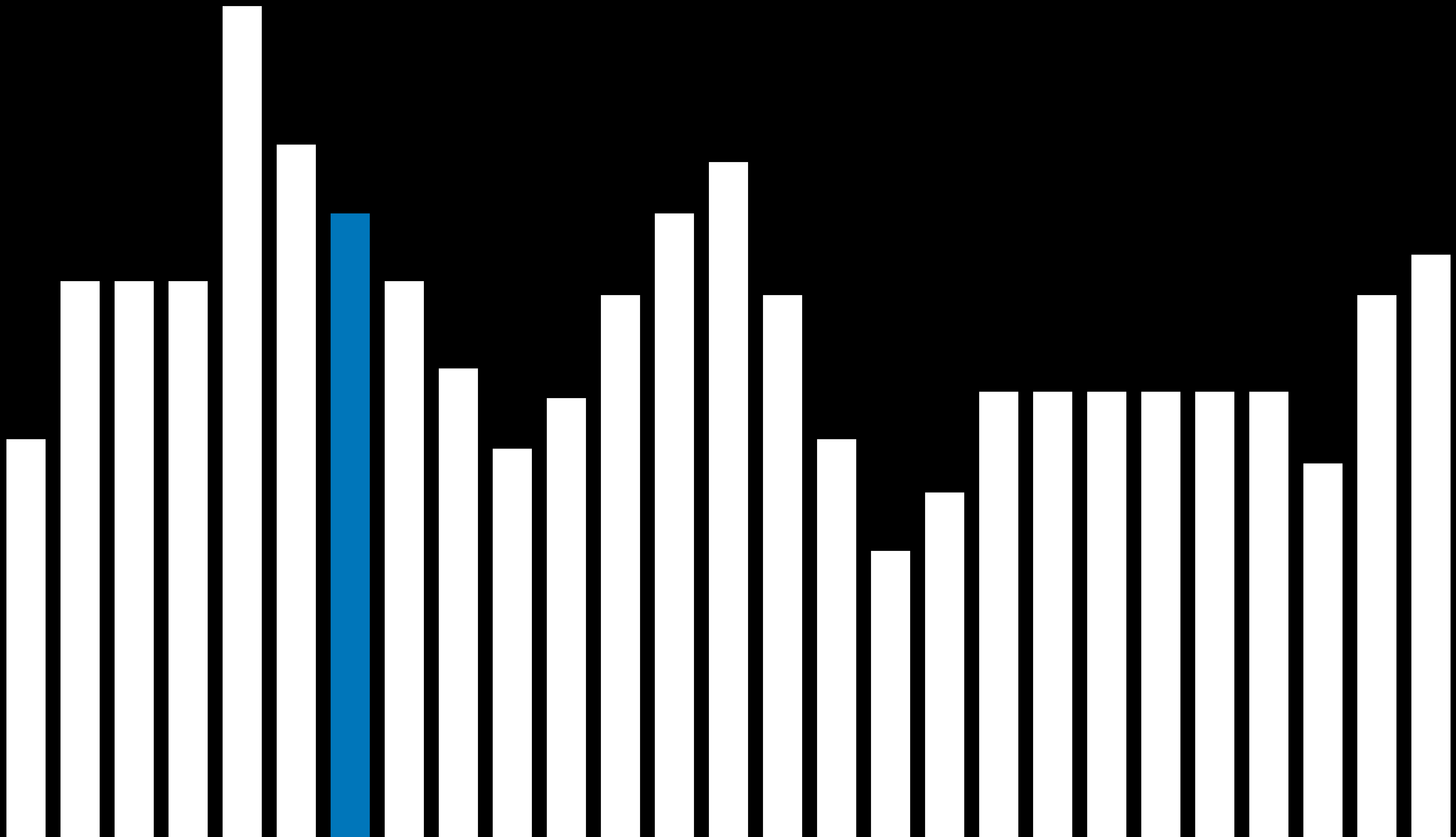
neighbors

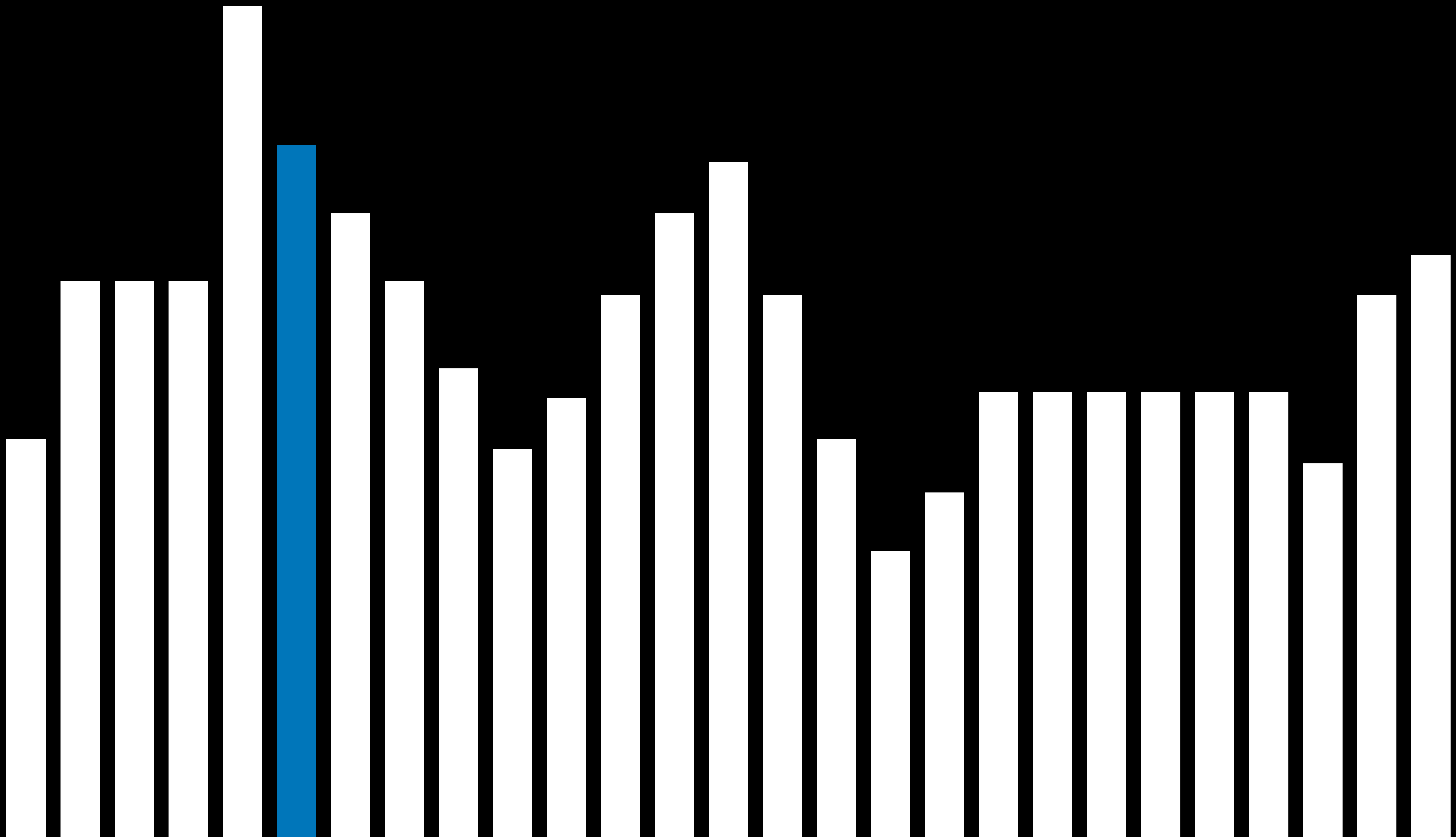


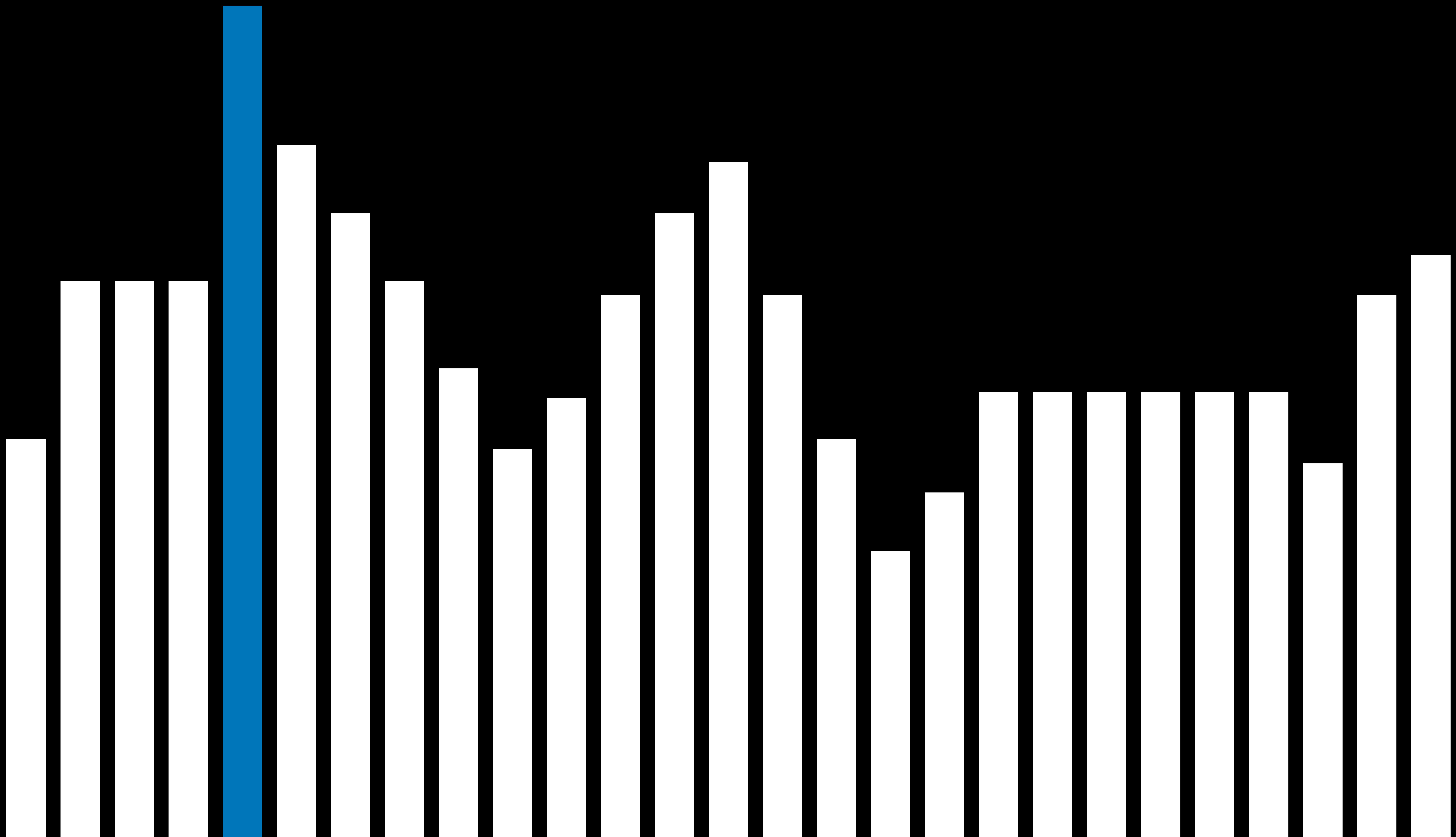
Hill Climbing

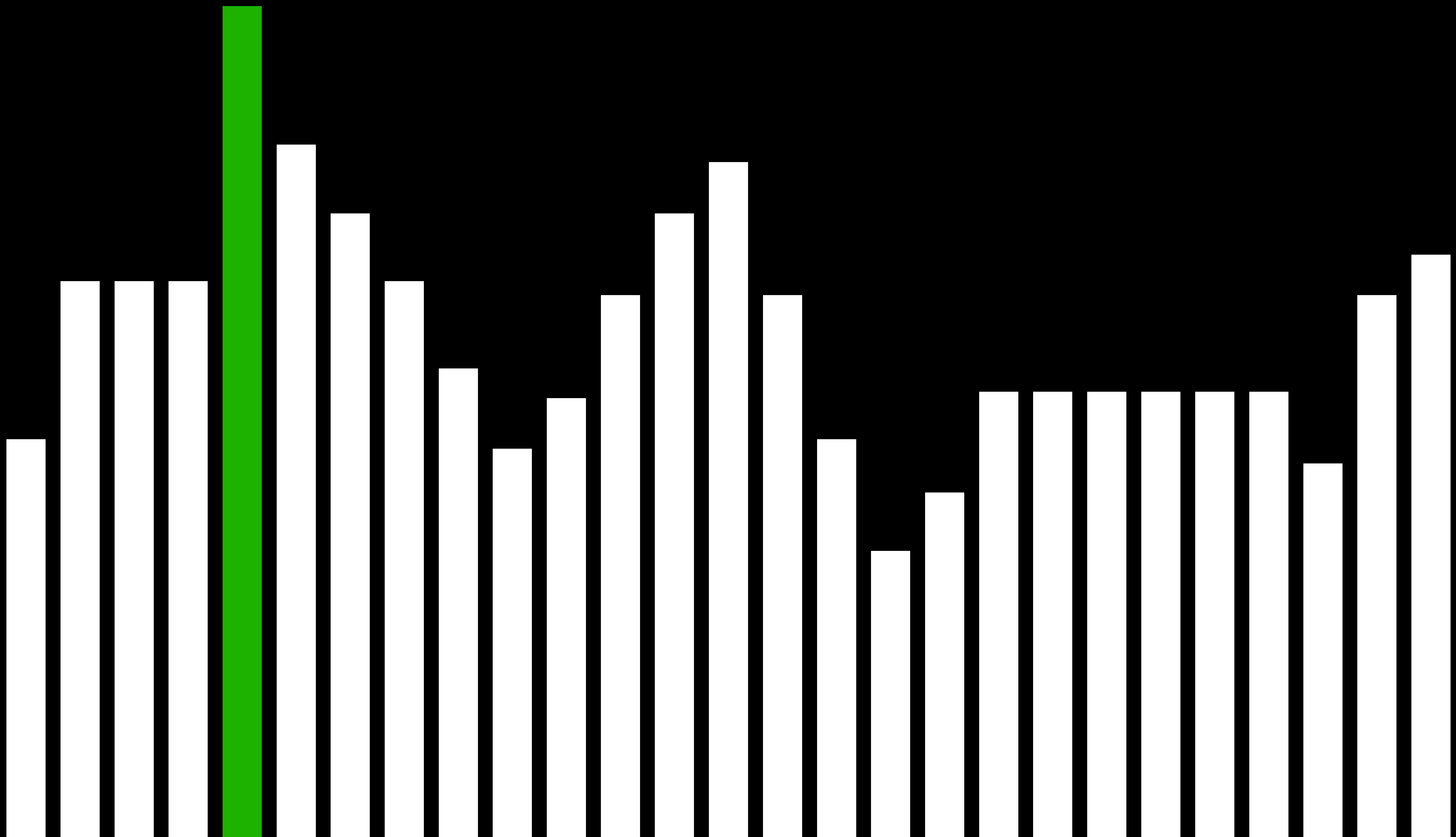


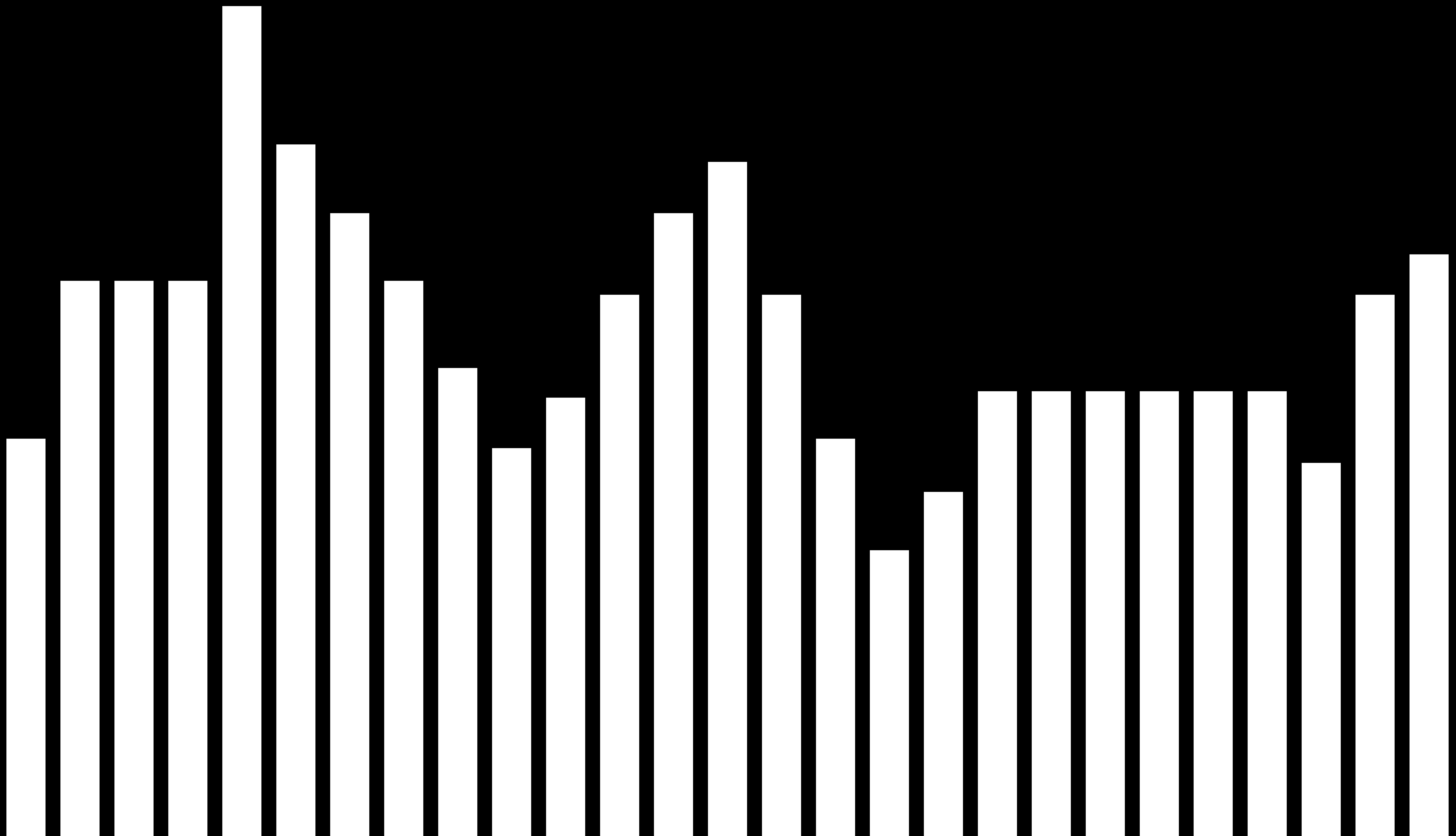


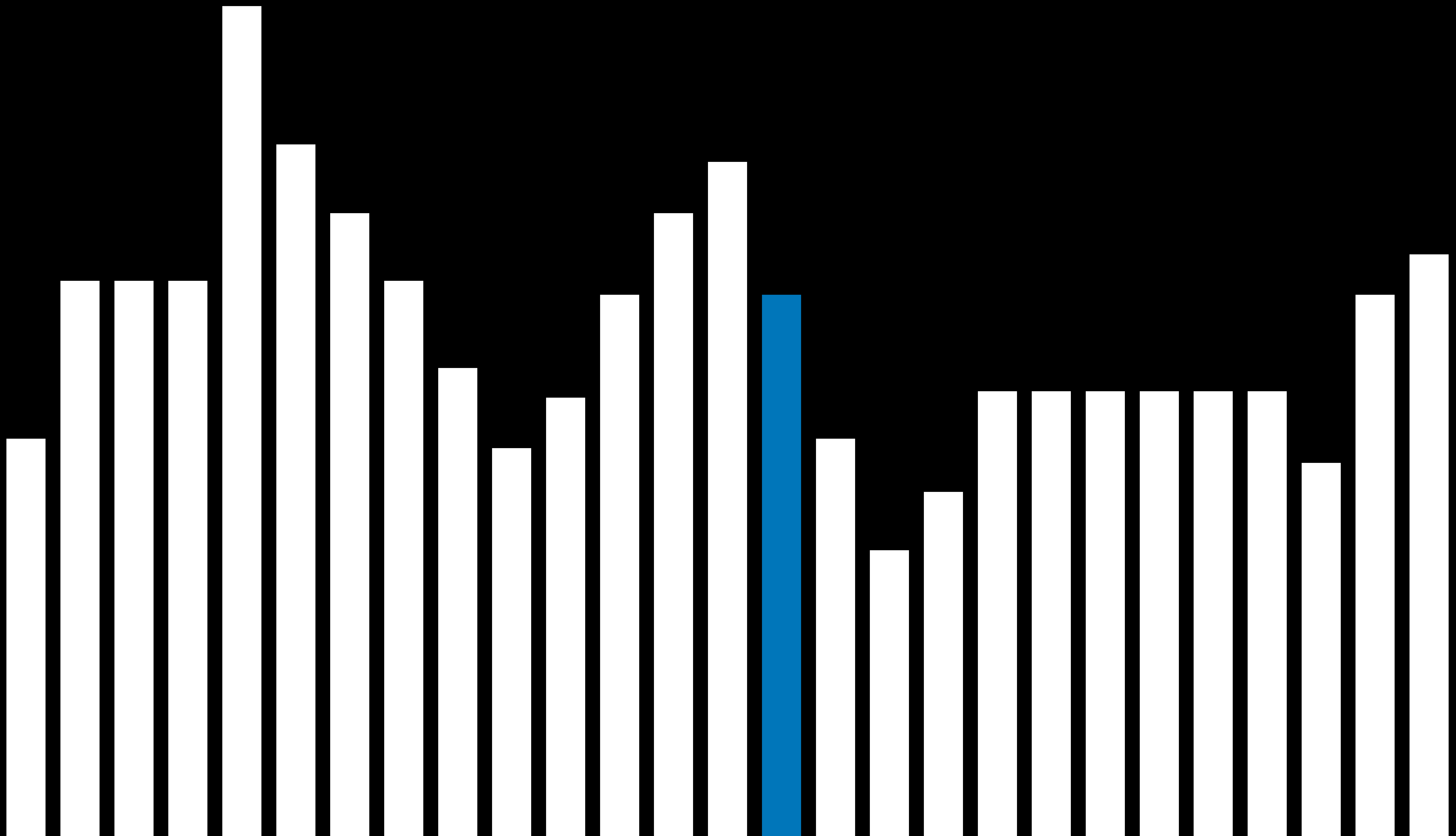


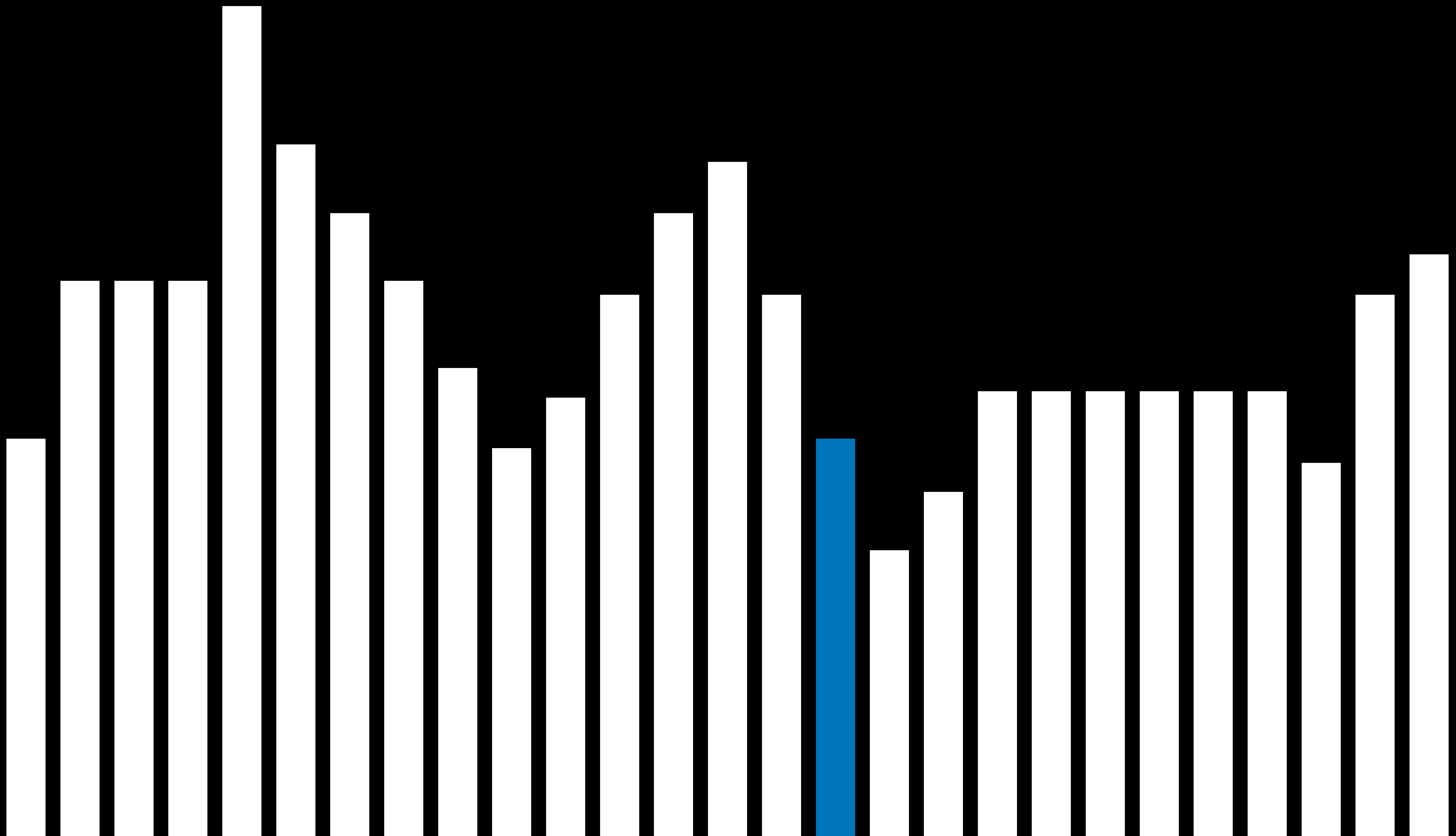


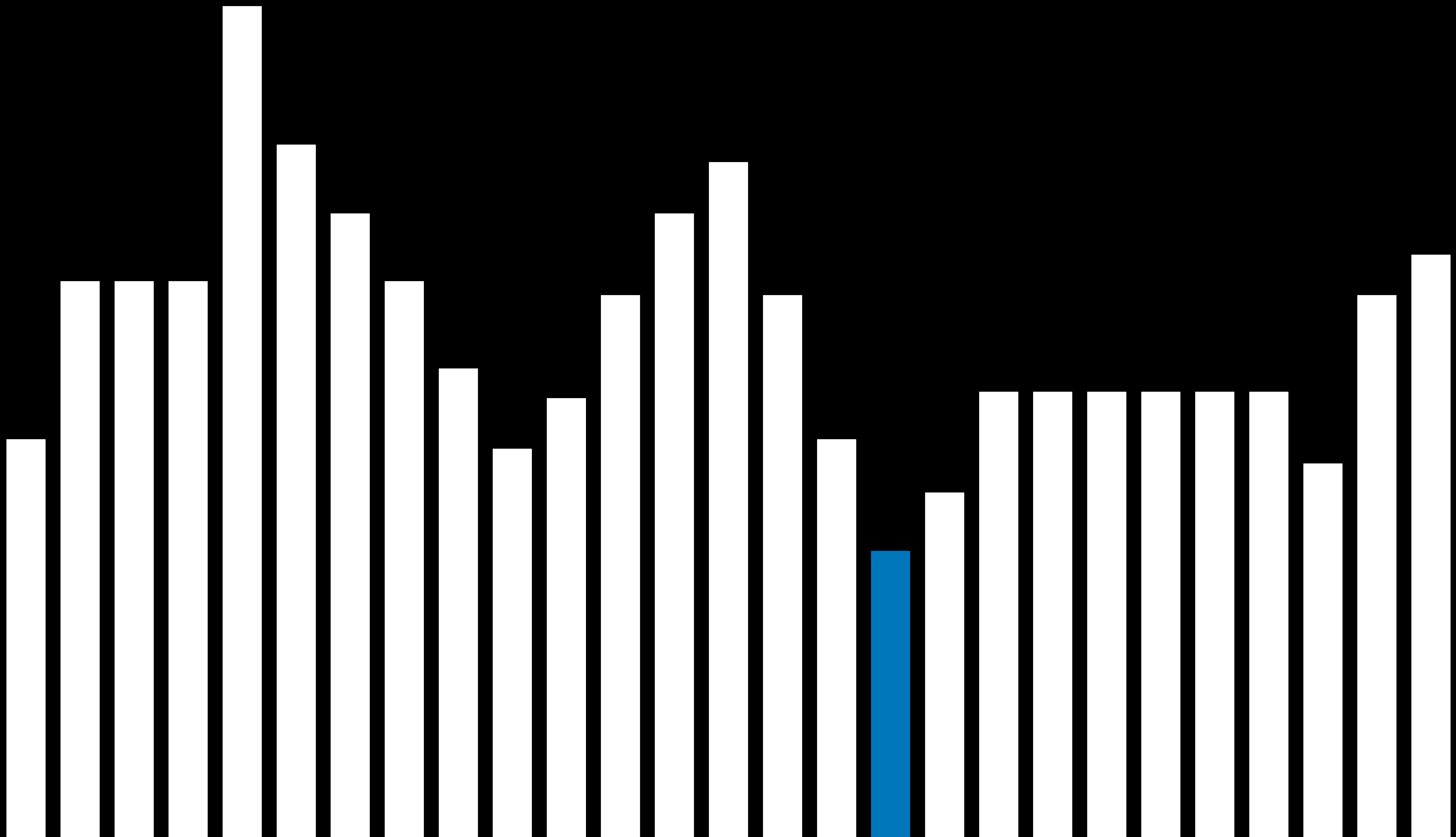


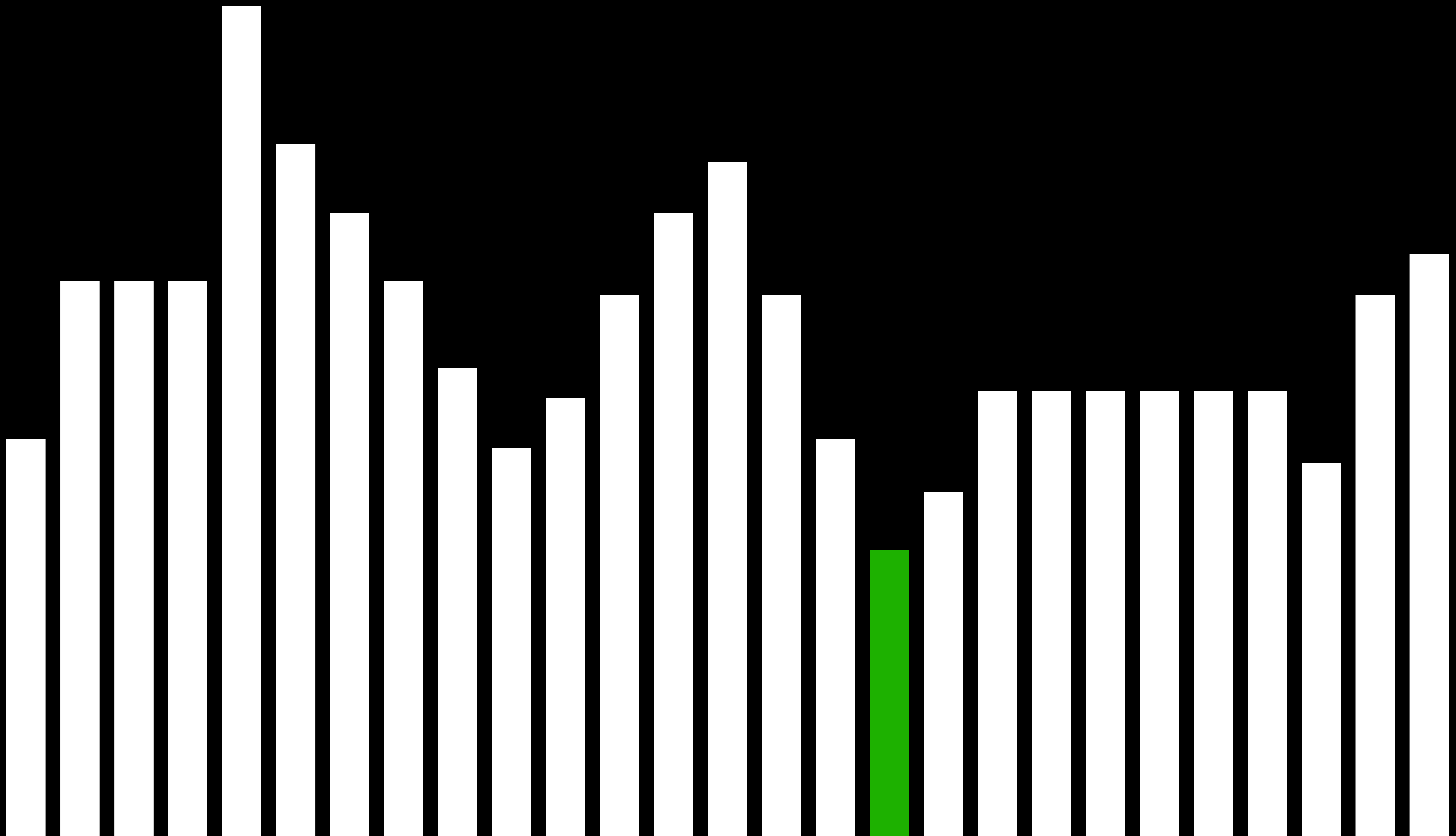








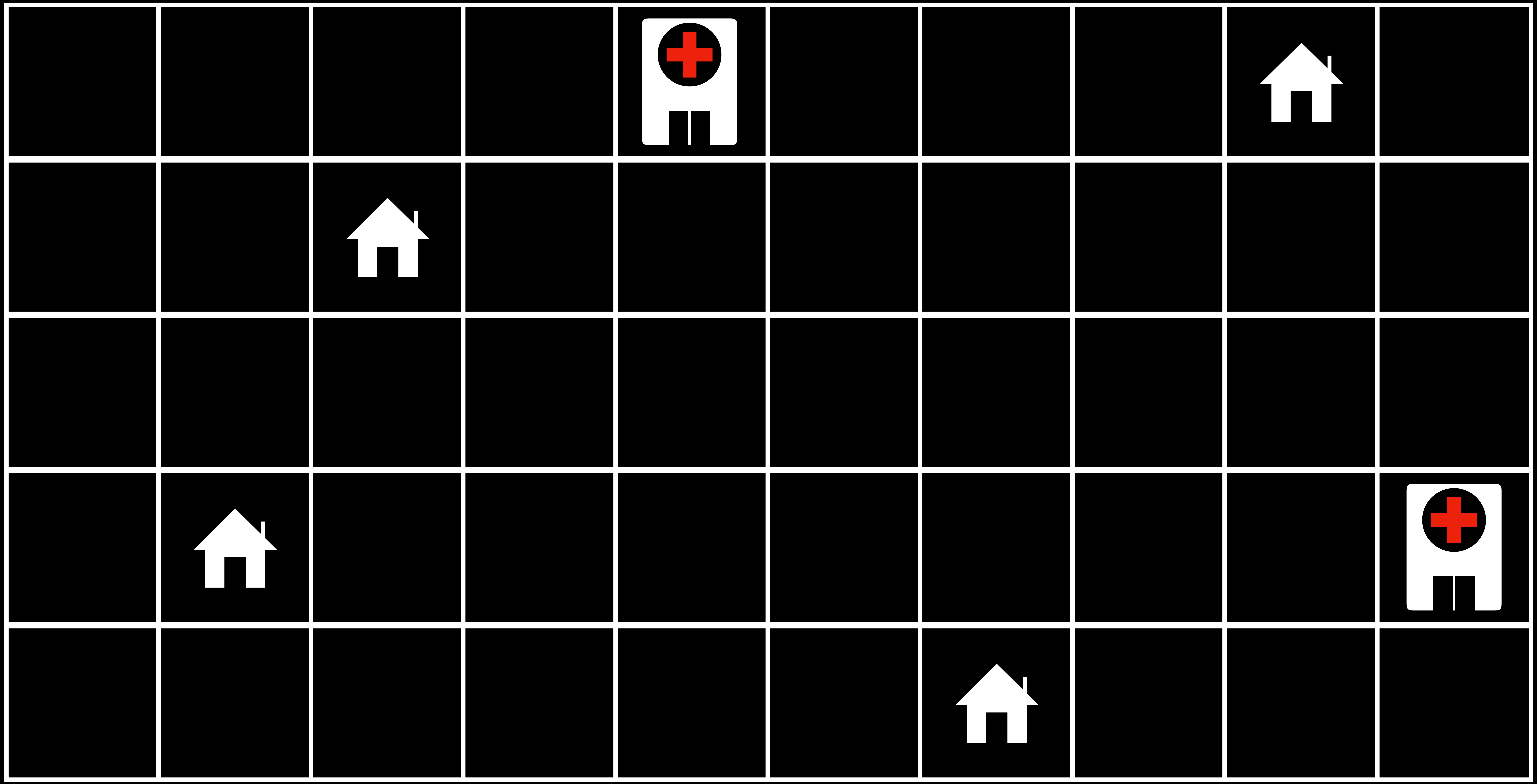


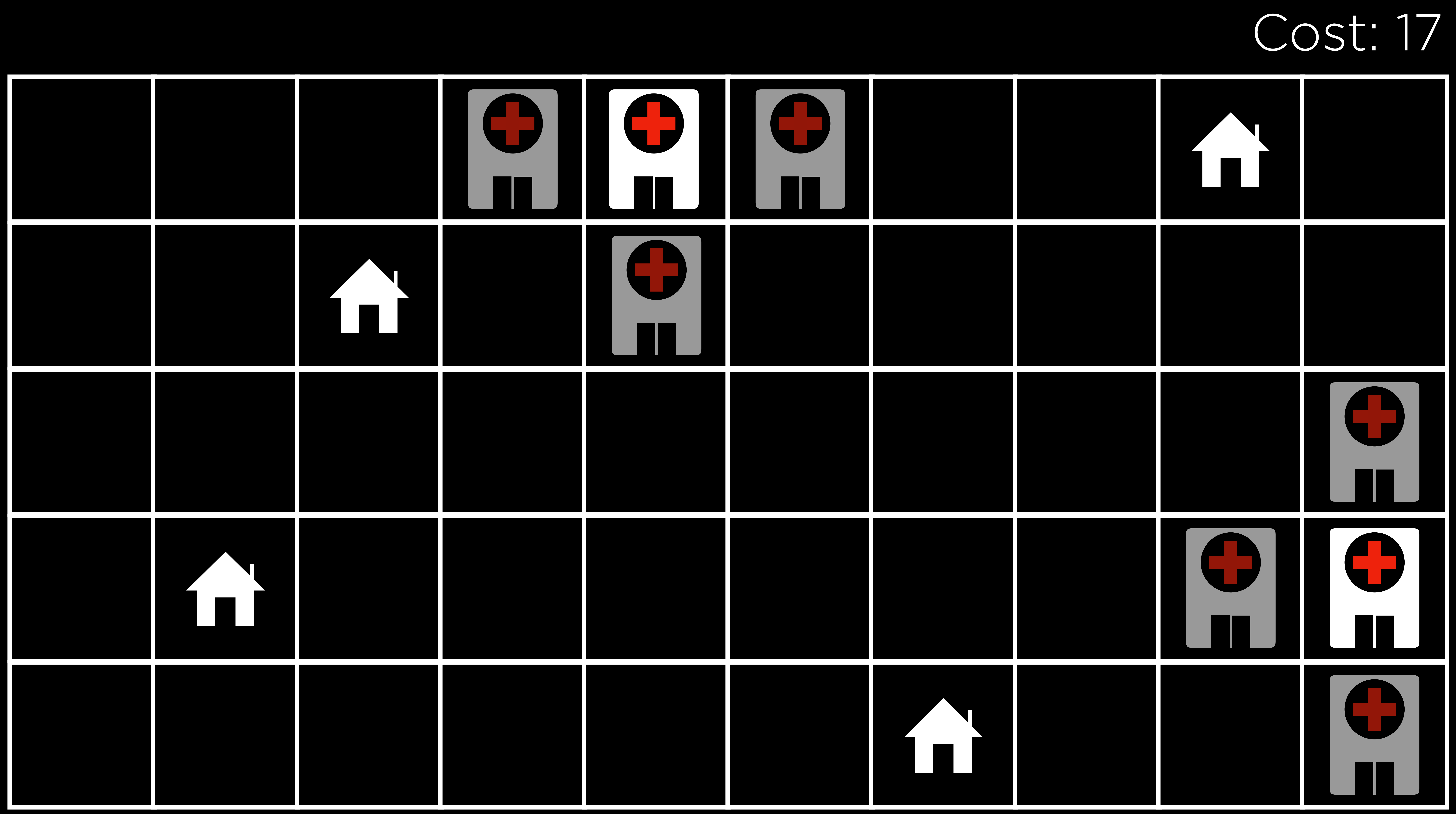


Hill Climbing

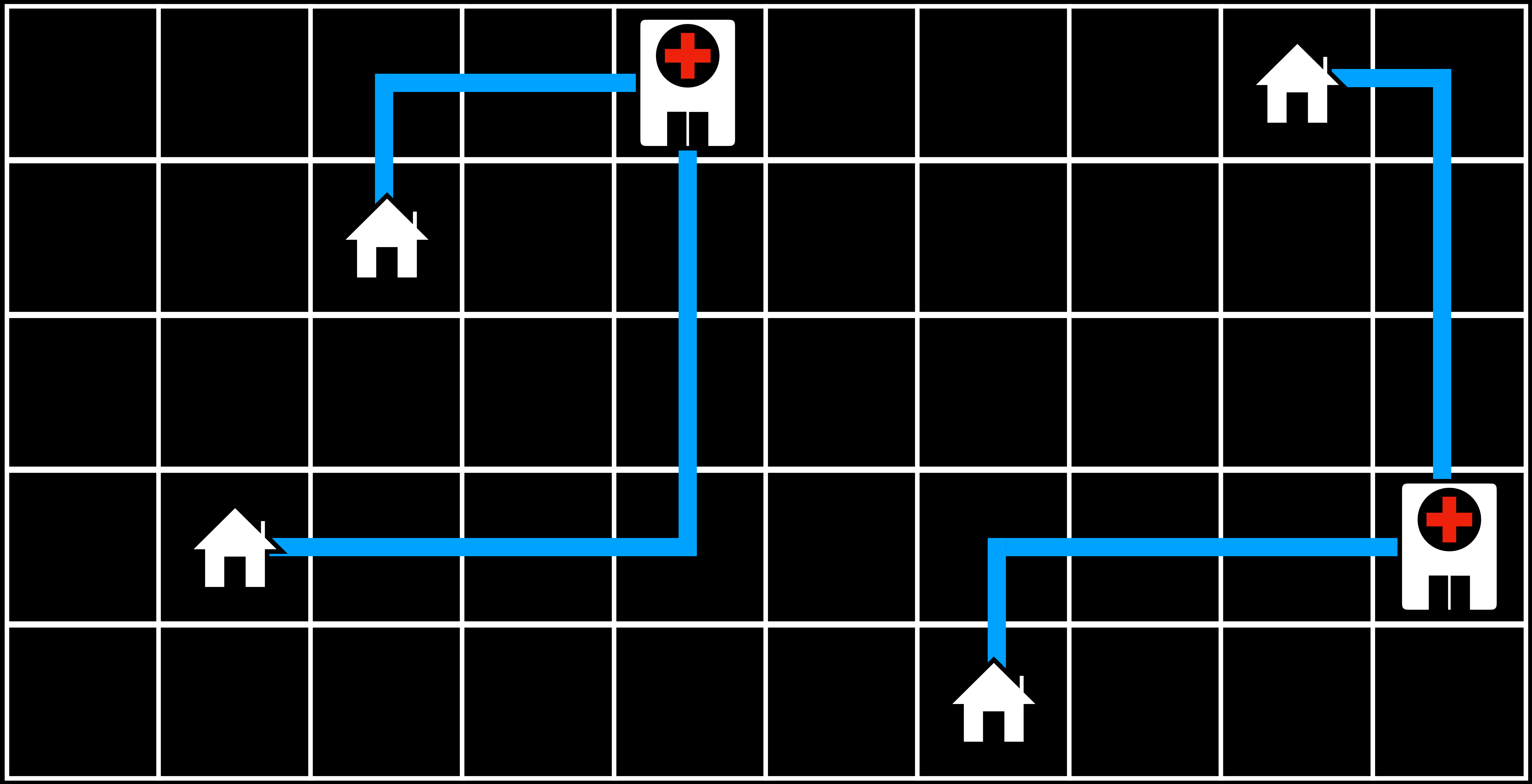
```
function HILL-CLIMB(problem):
    current = initial state of problem
    repeat:
        neighbor = highest valued neighbor of current
        if neighbor not better than current:
            return current
        current = neighbor
```

Cost: 17

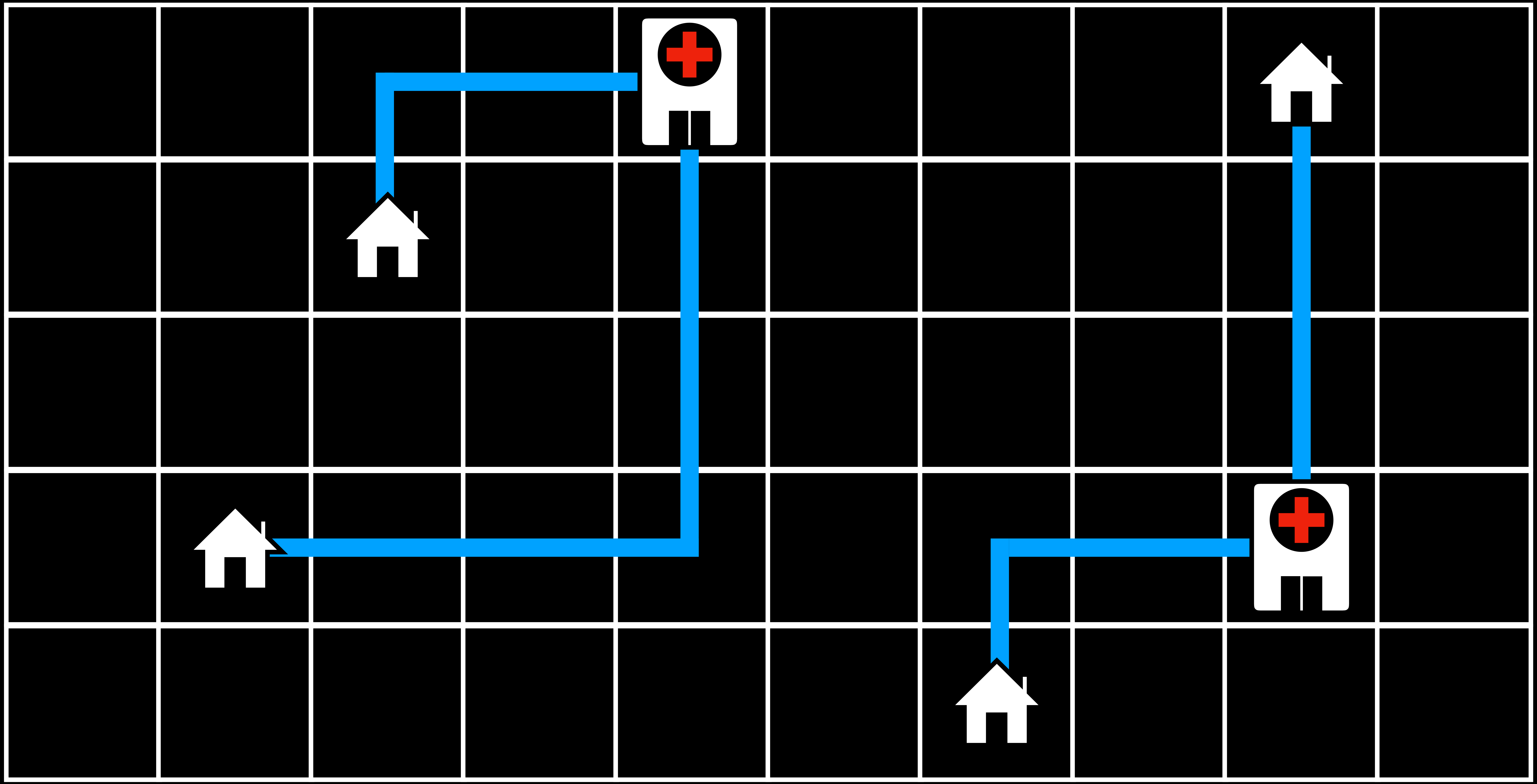



Cost: 17

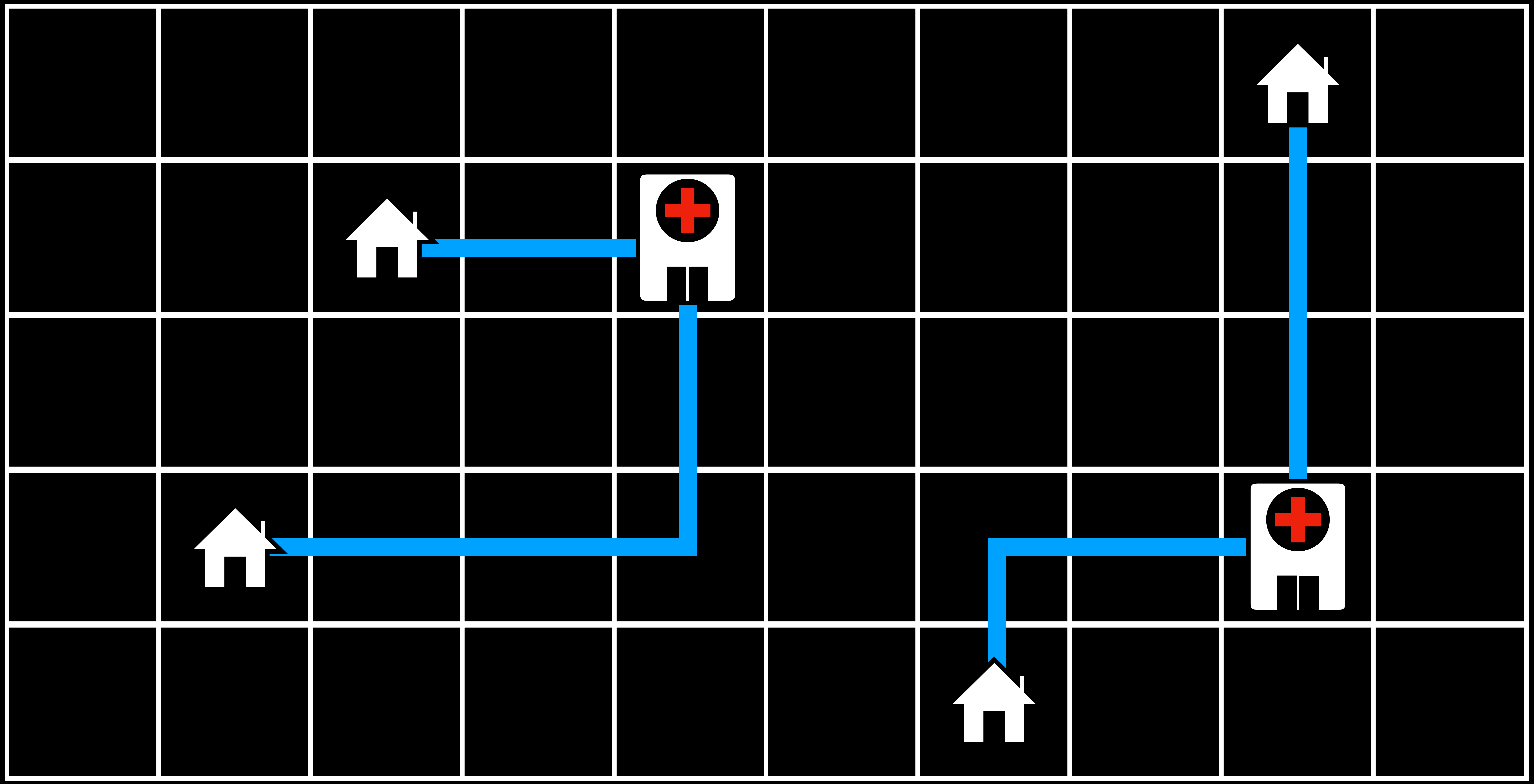
Cost: 17



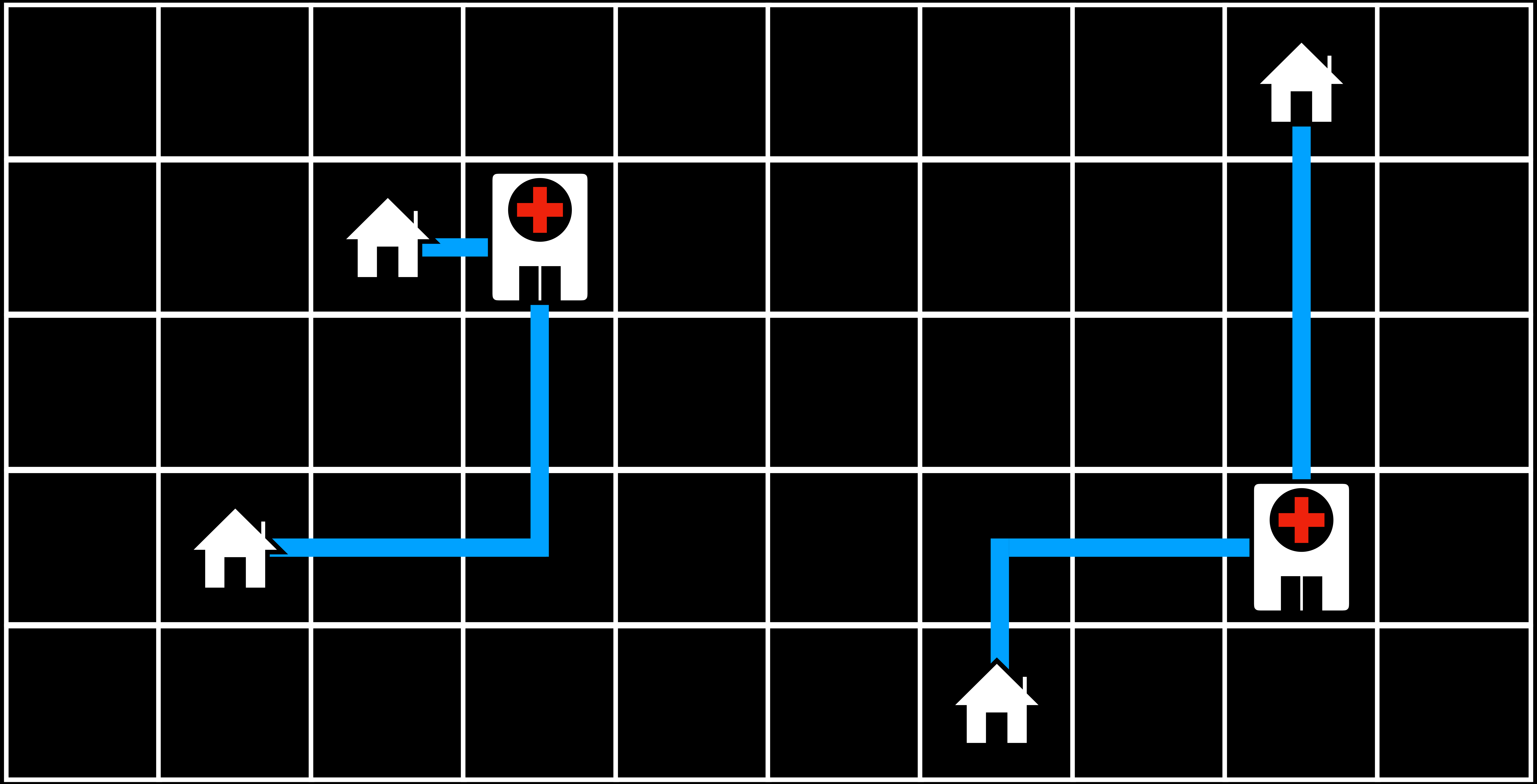
Cost: 15



Cost: 13

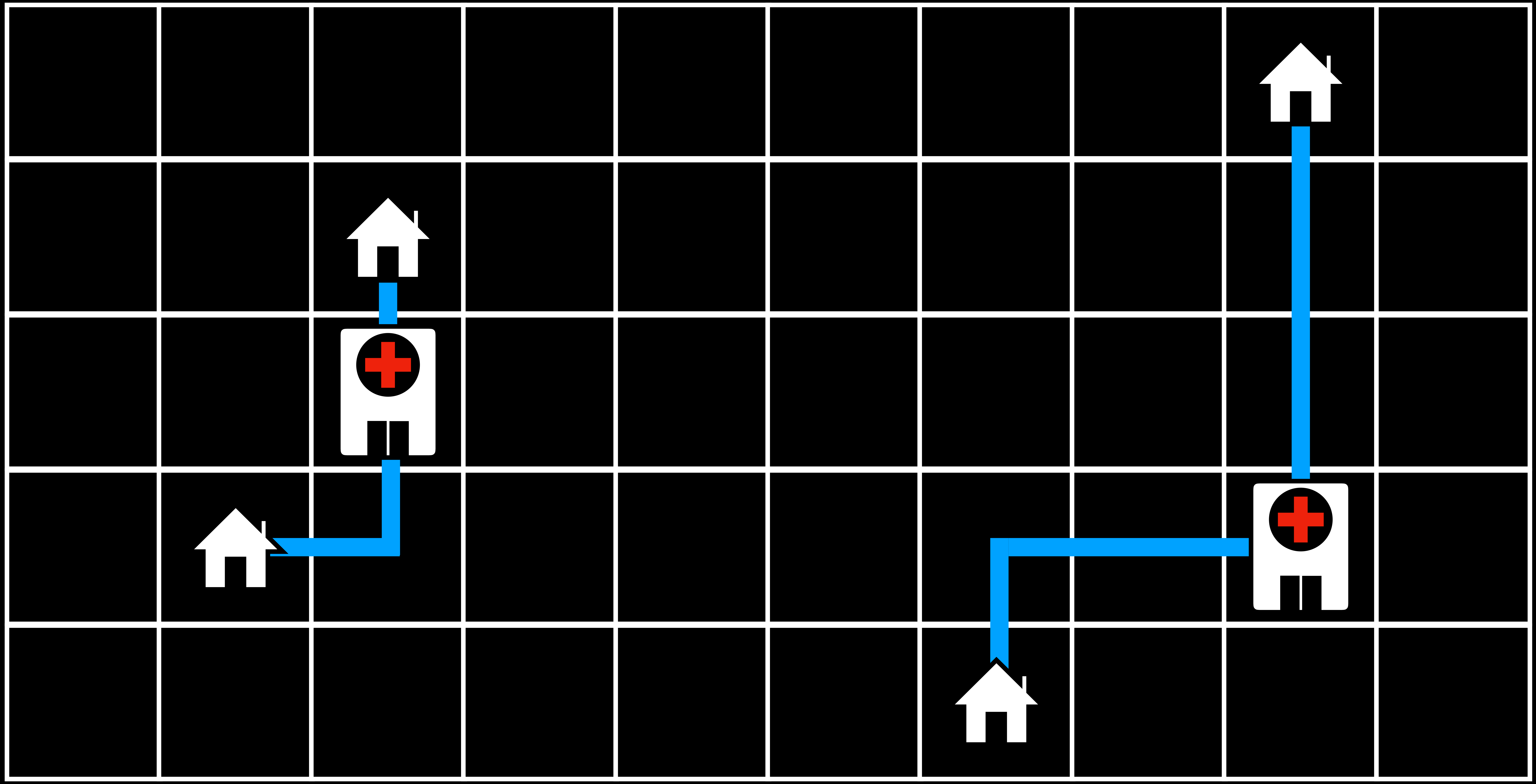


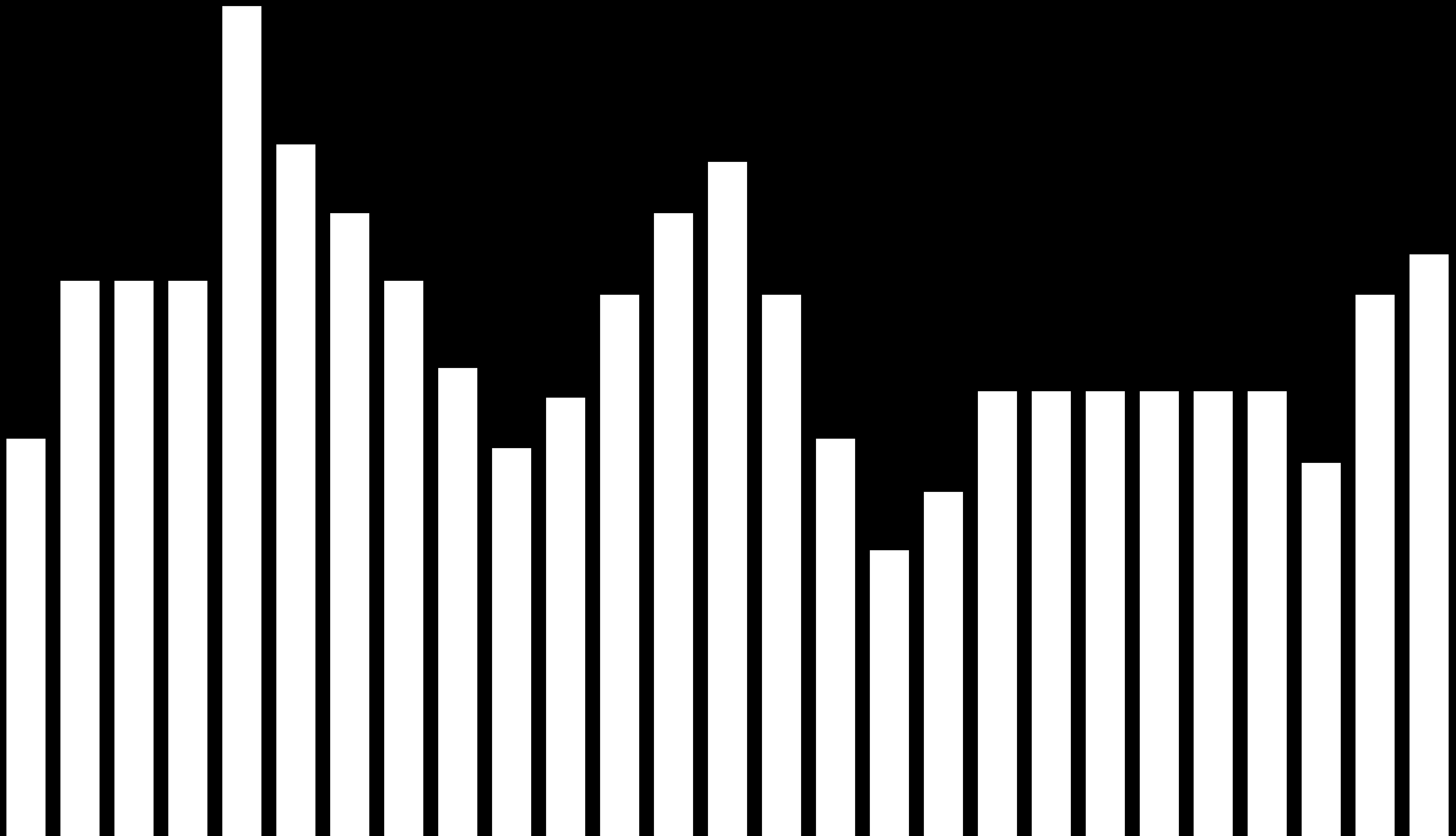
Cost: 11



since no neighbor has lover cost, algorithm exits at this local minimum global minimum

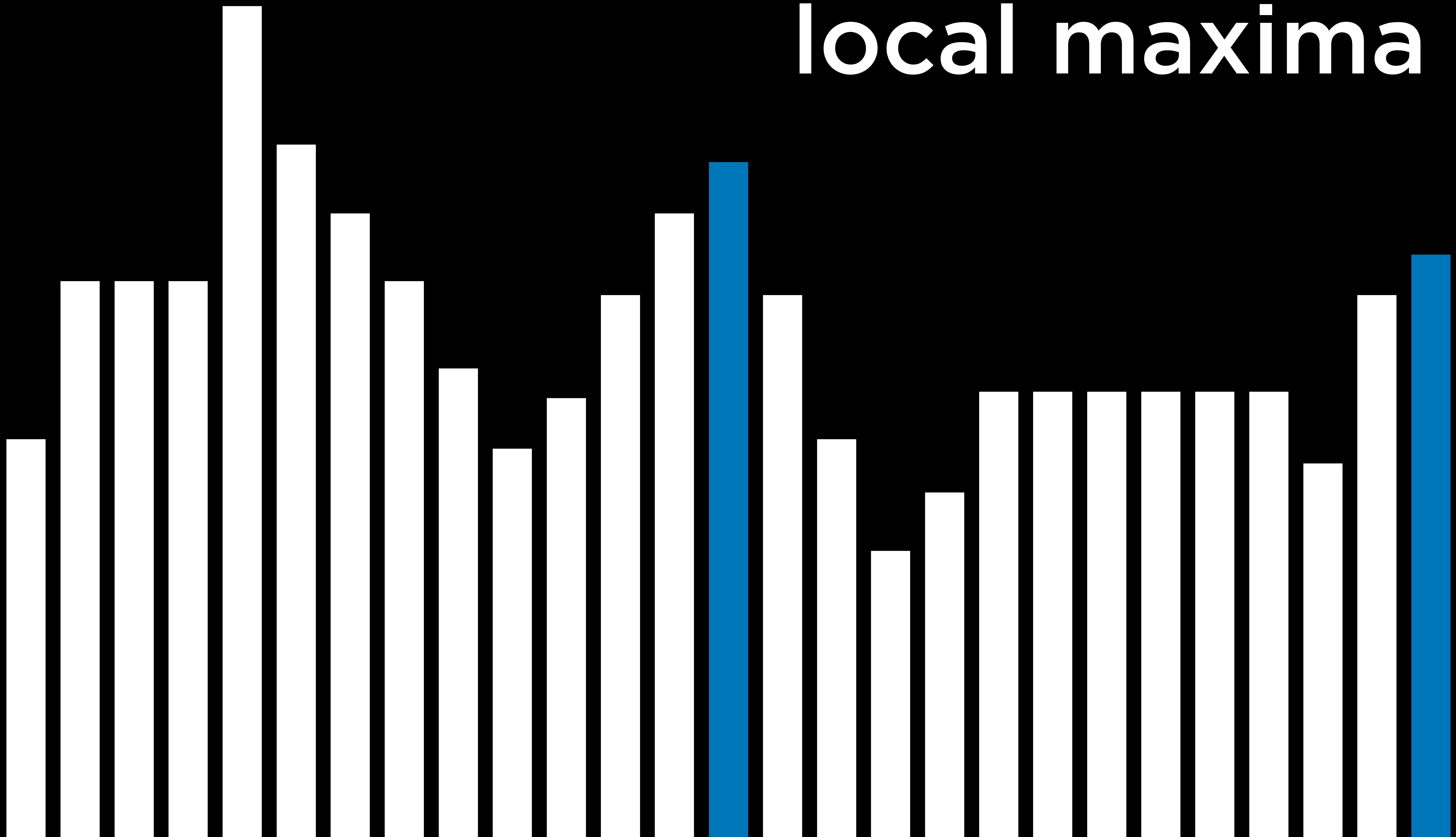
Cost: 9

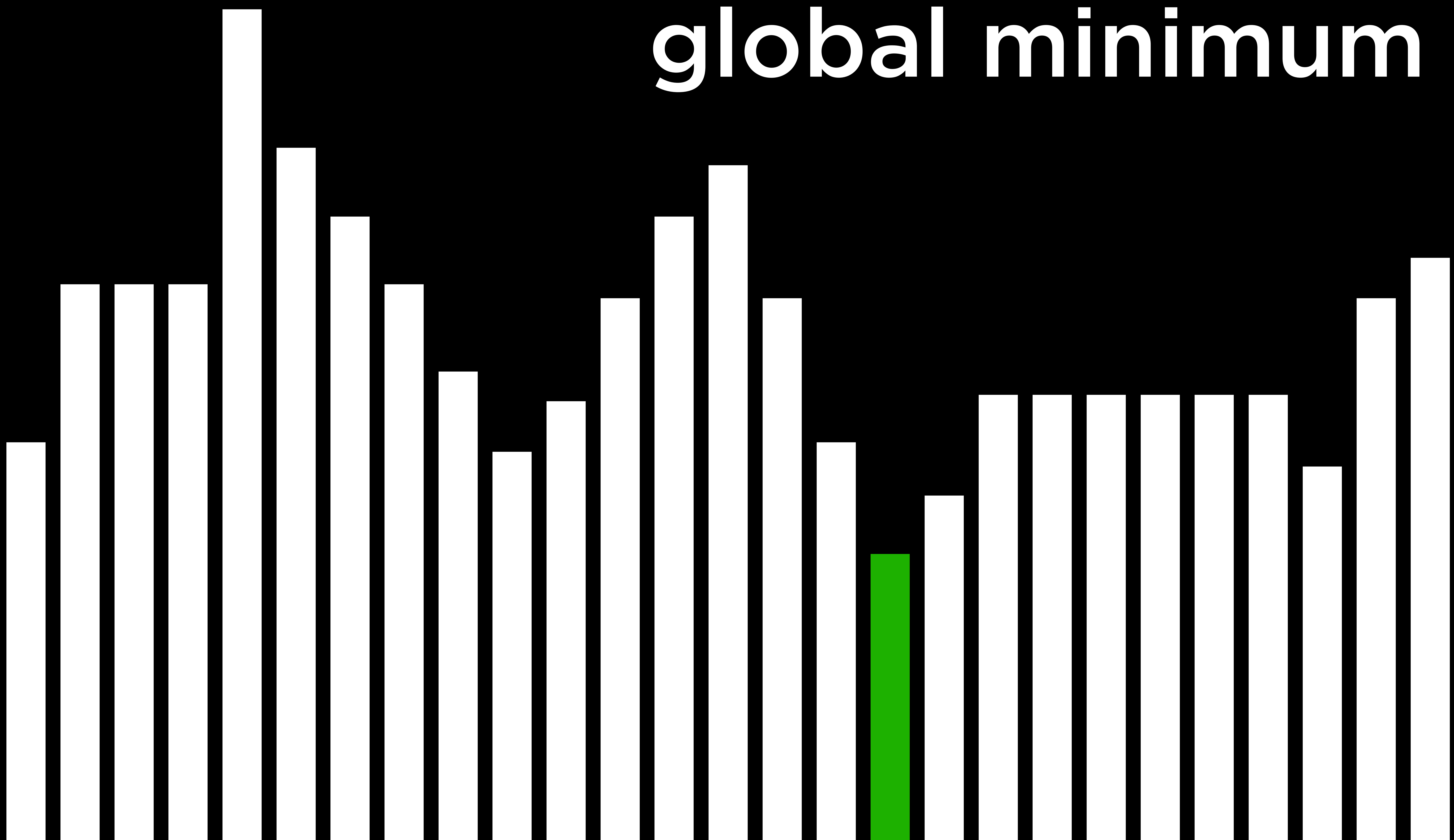




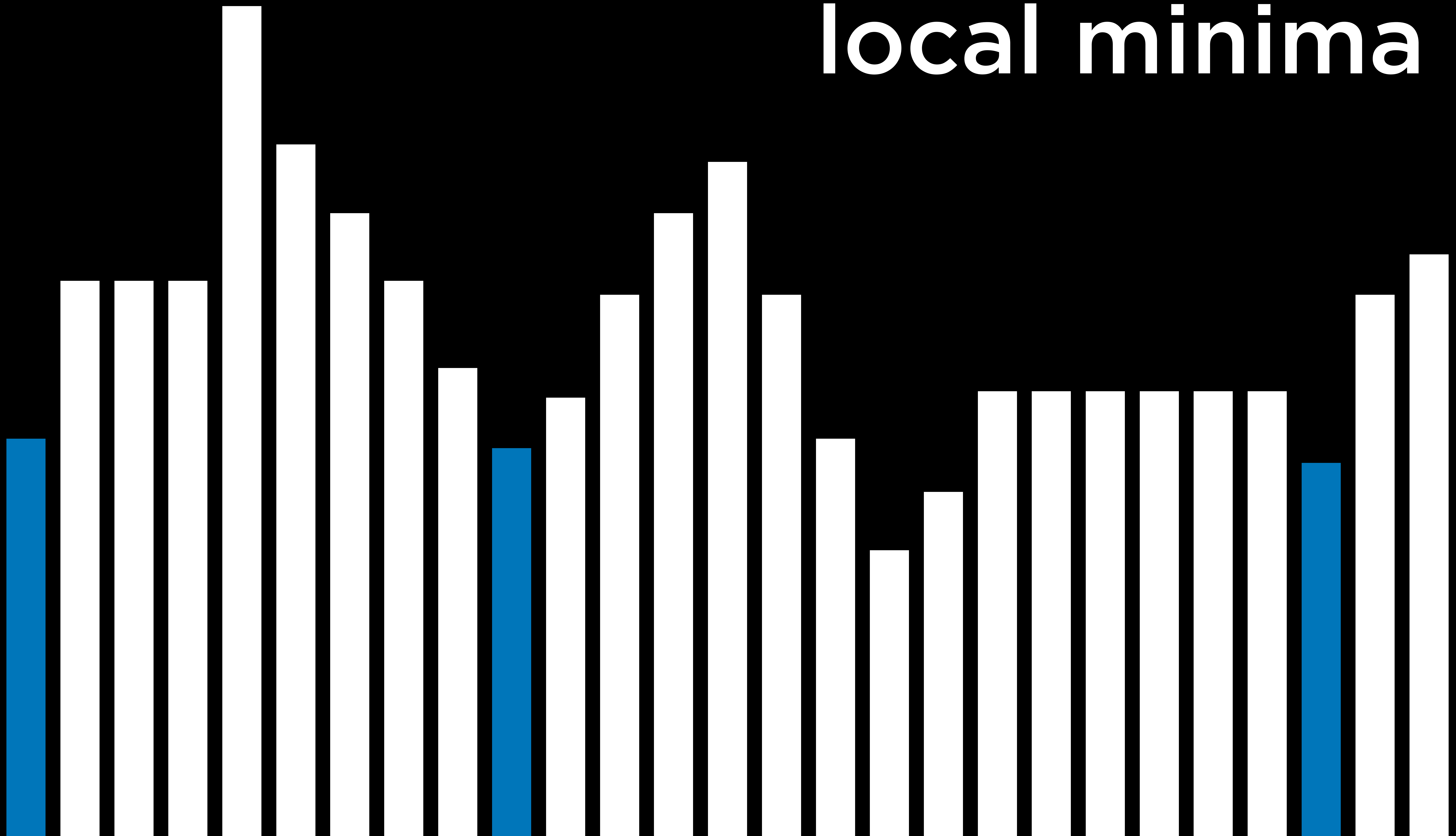
global maximum

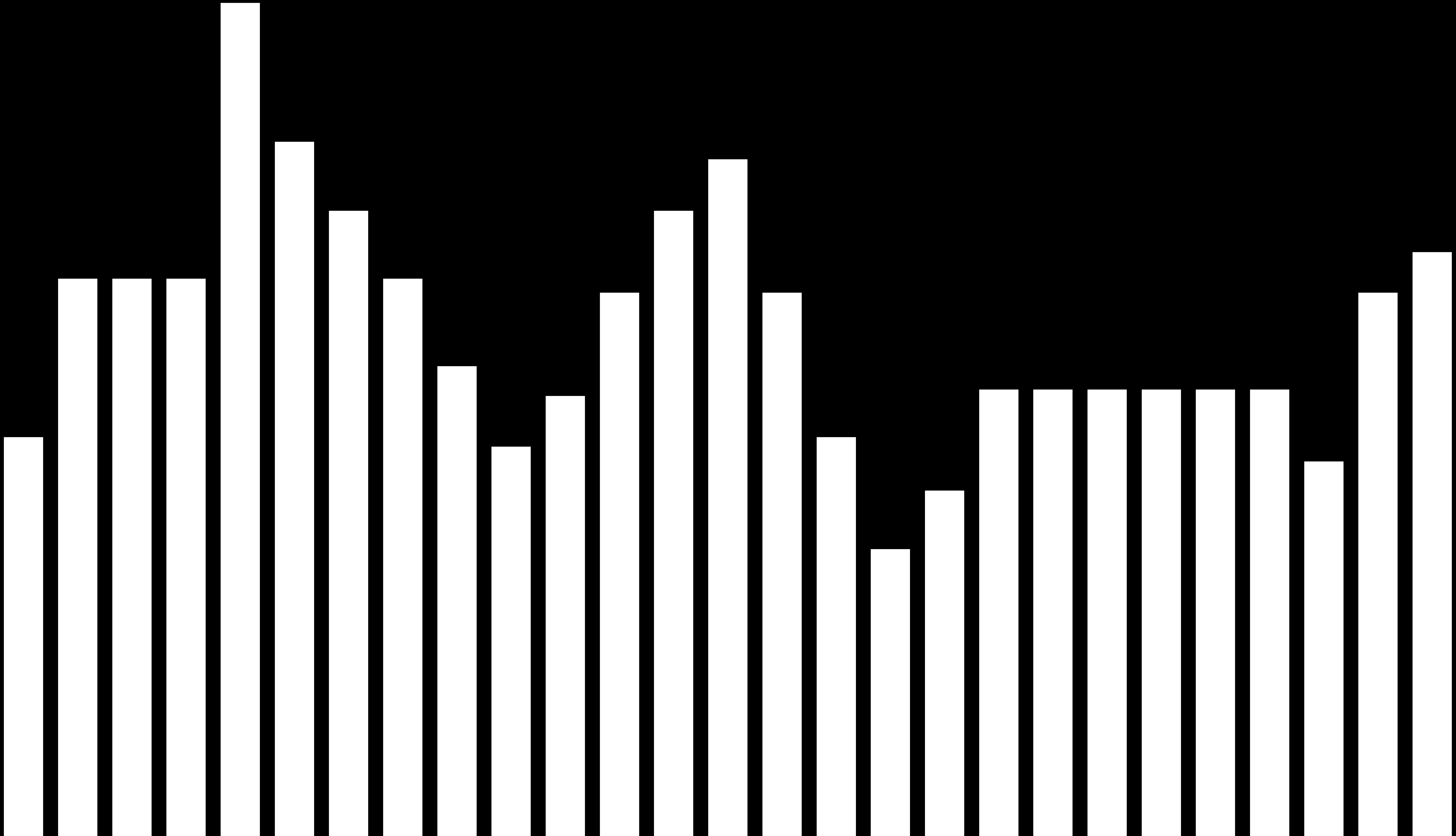
local maxima

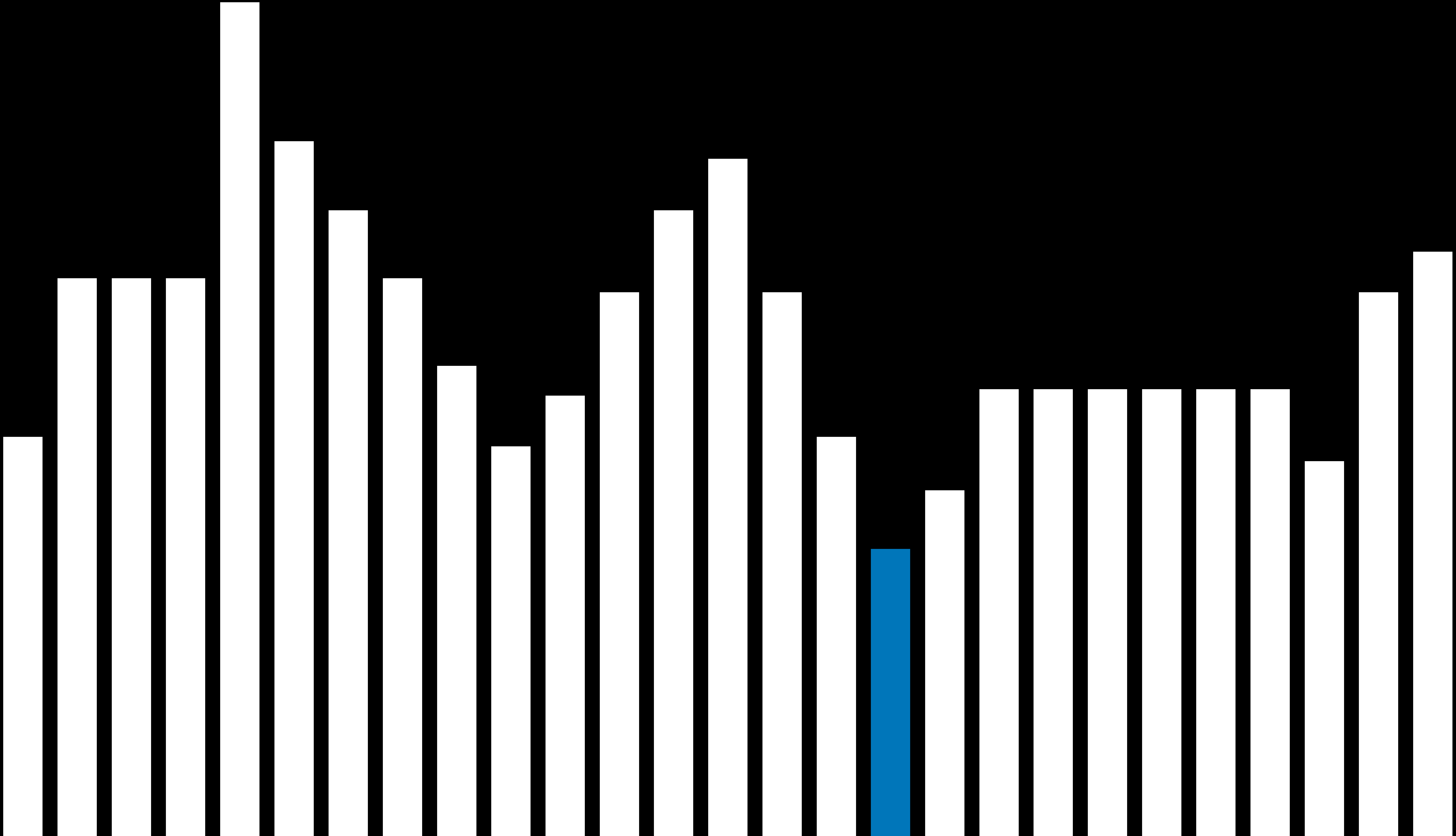


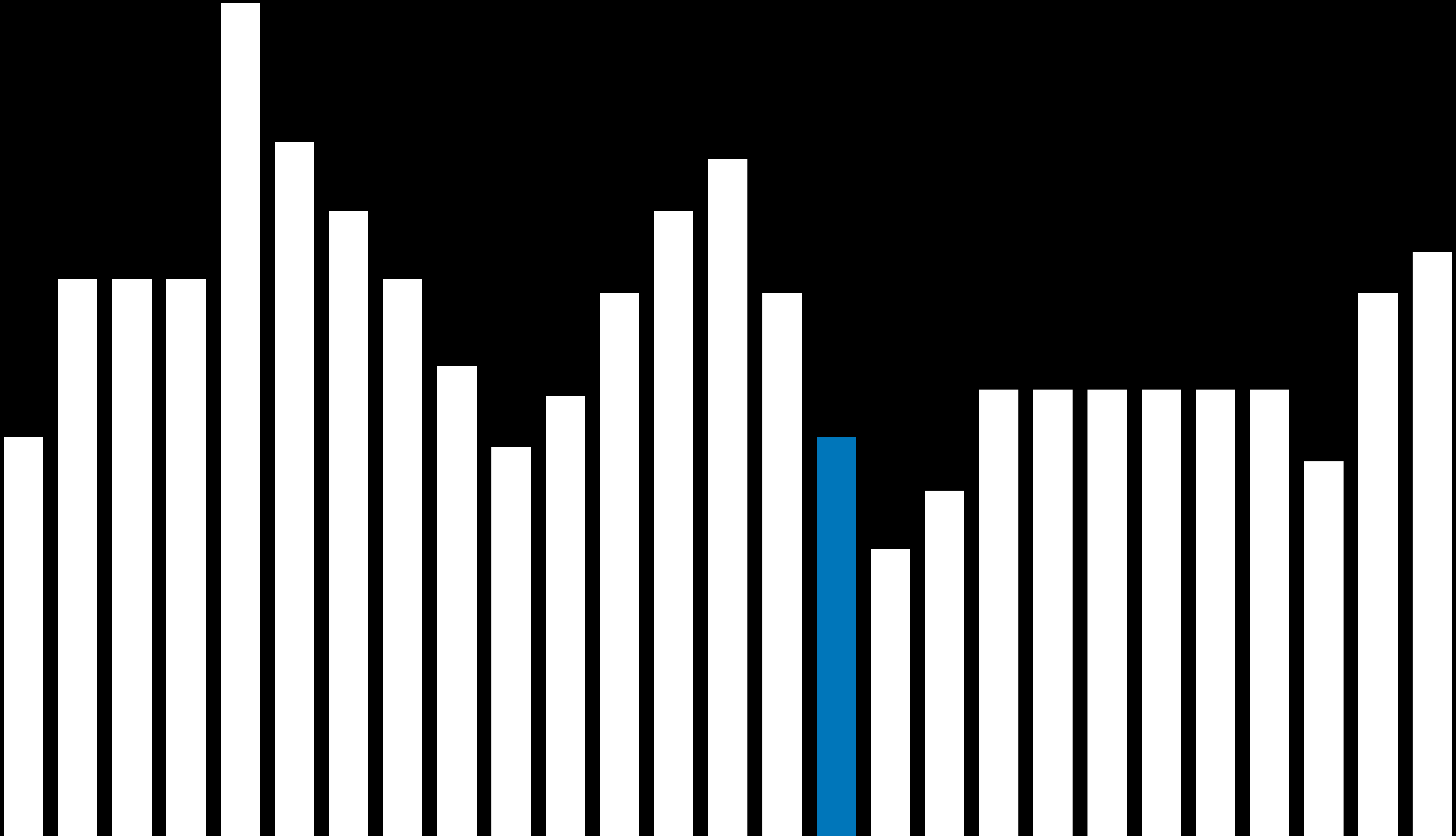

global minimum

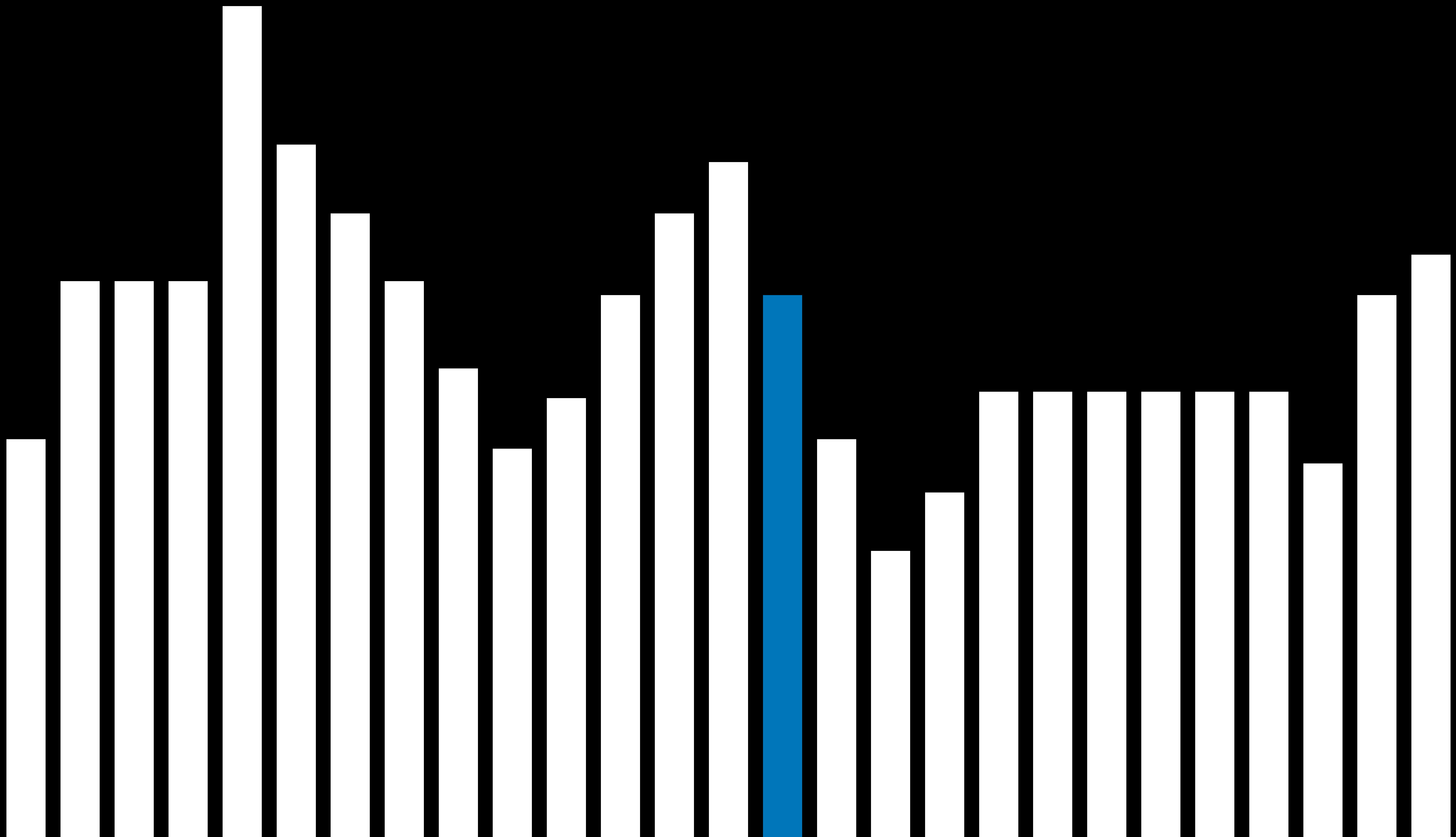
local minima

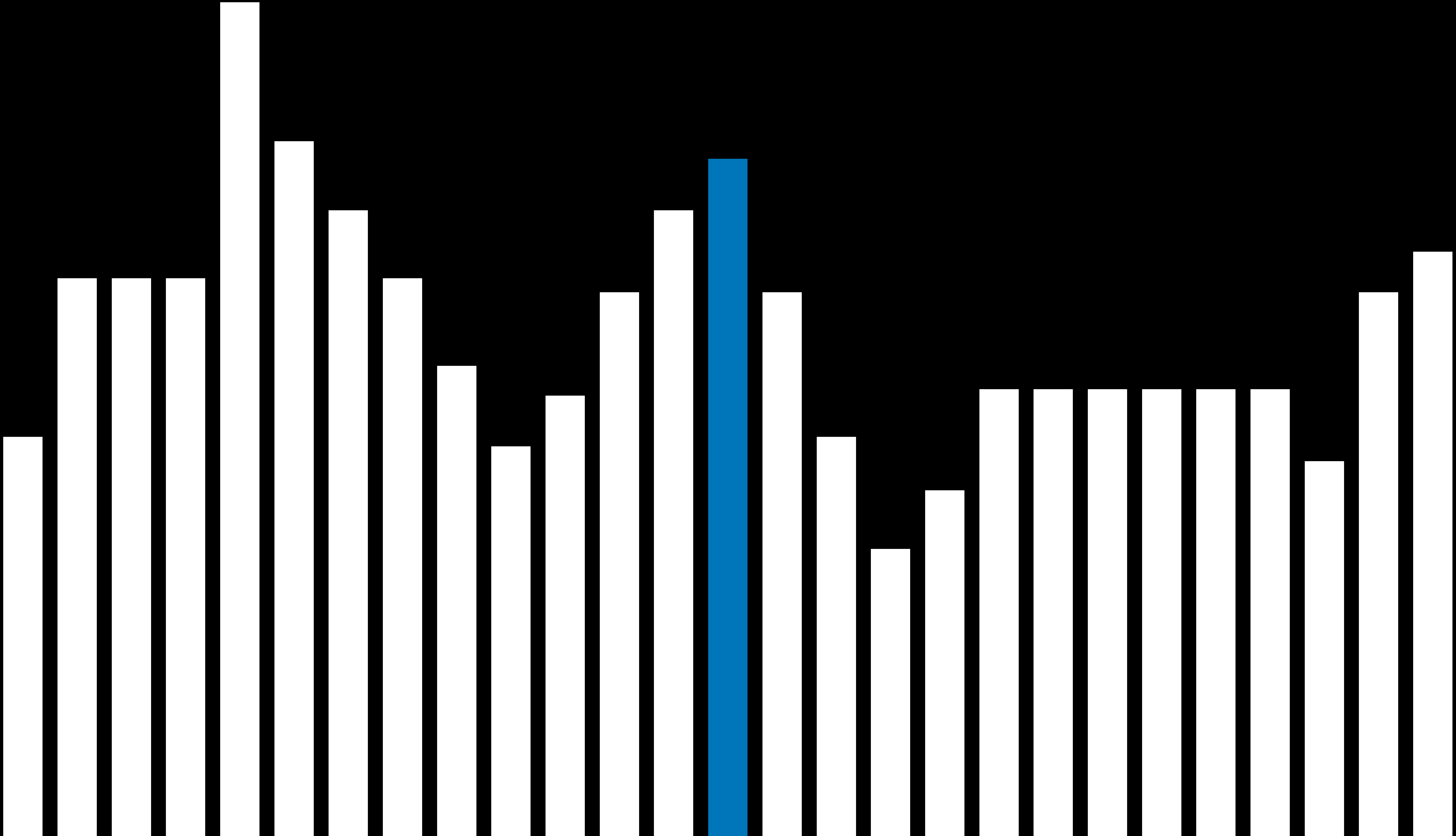




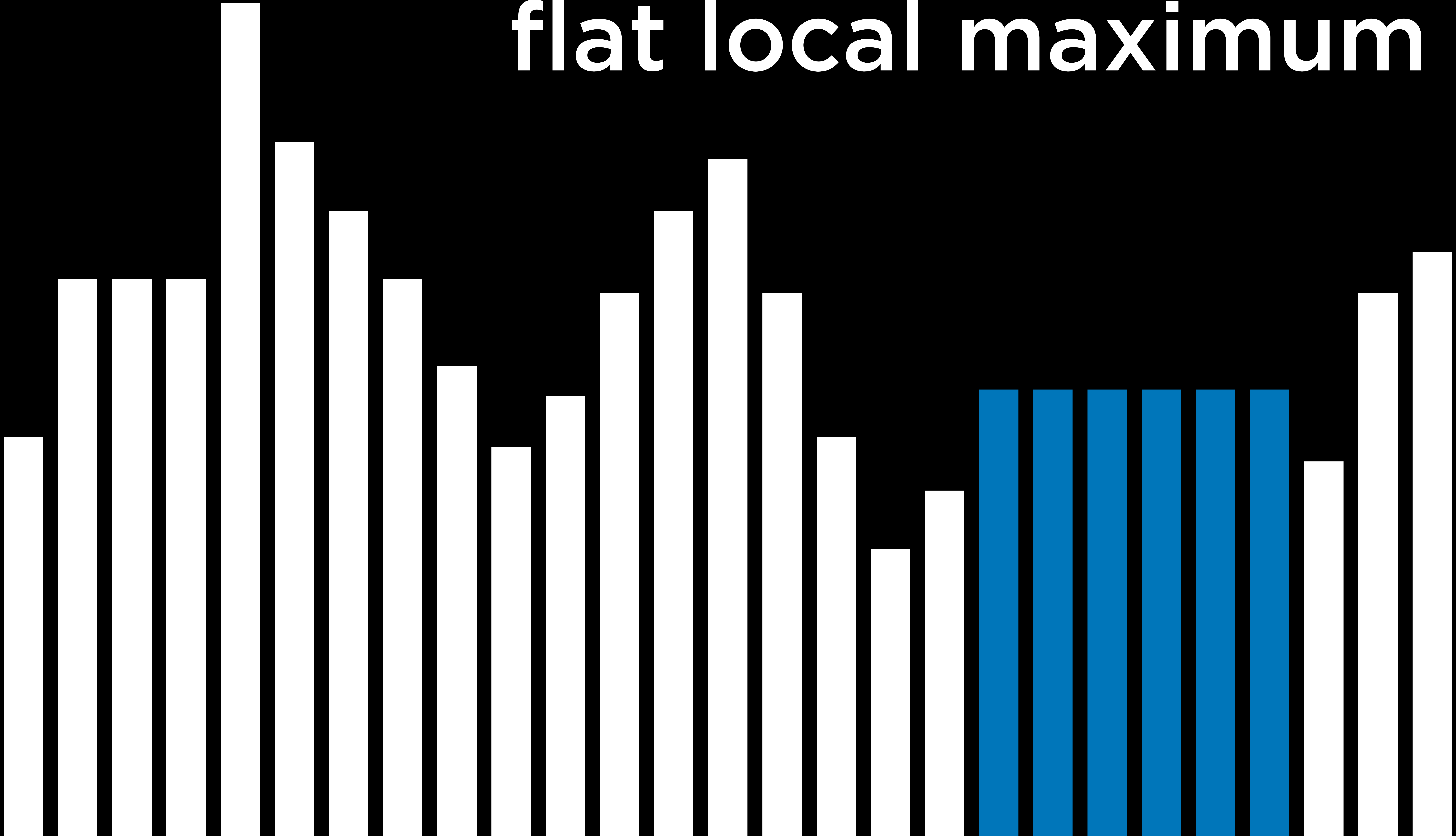




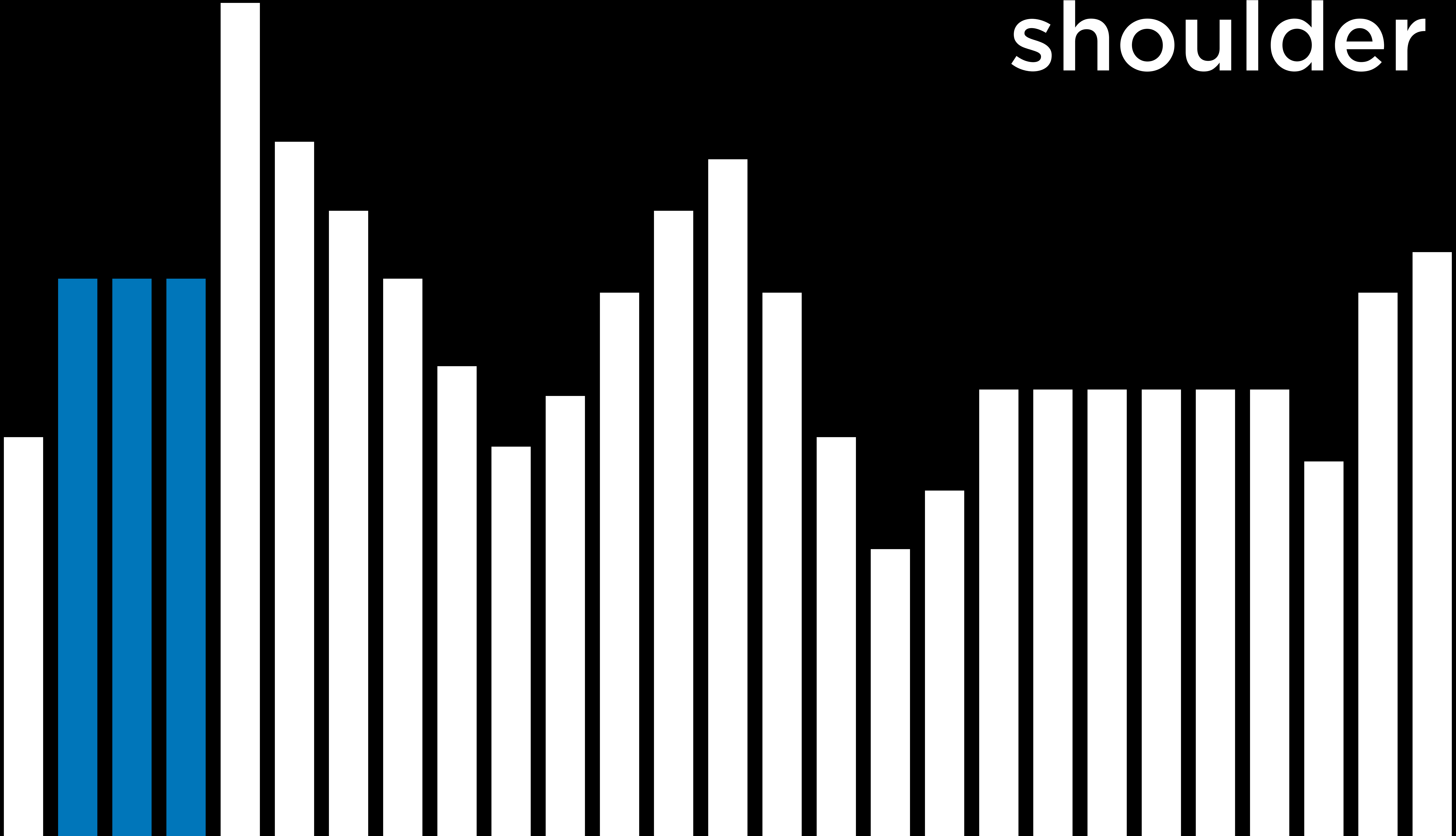




flat local maximum



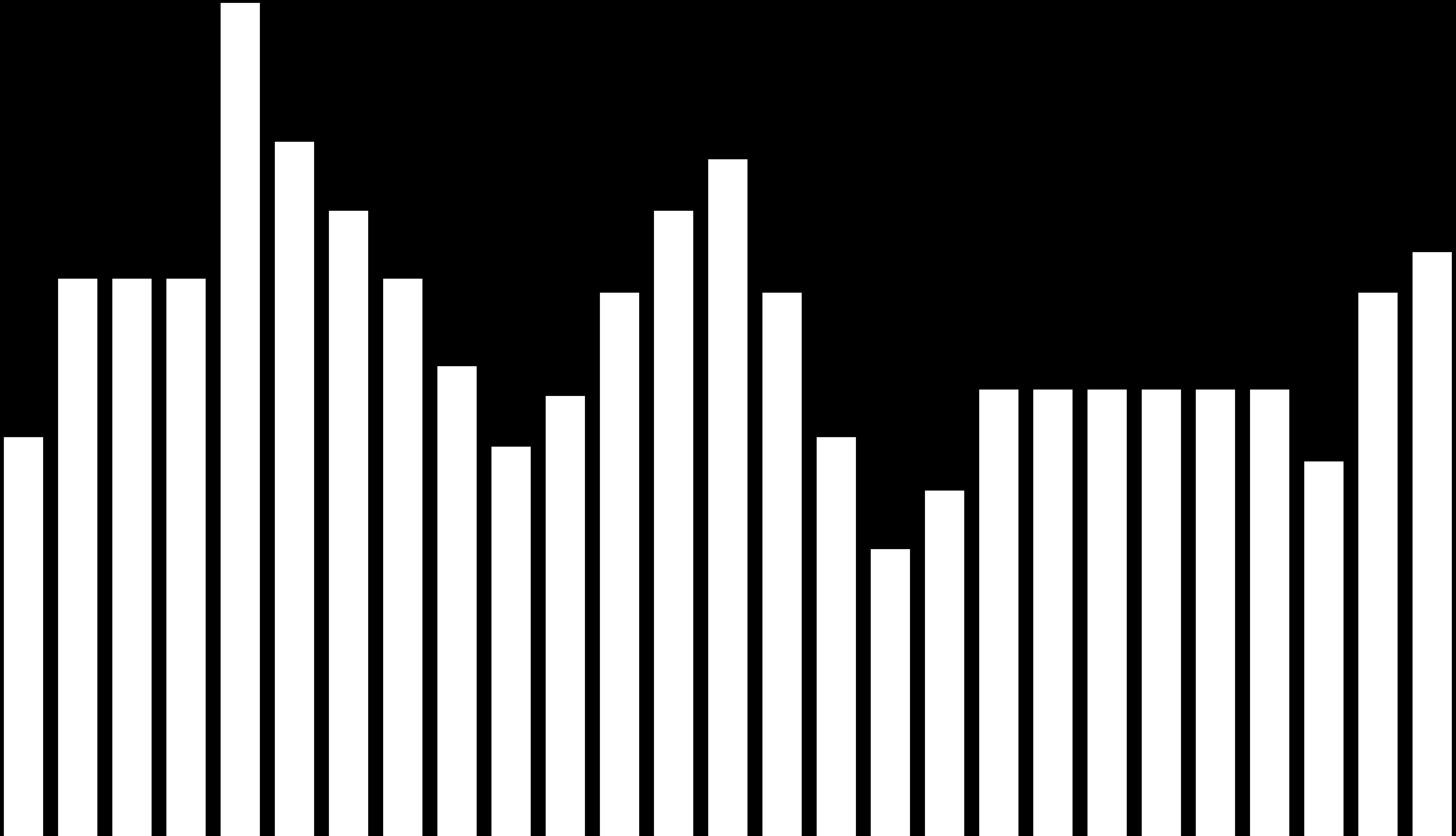
shoulder

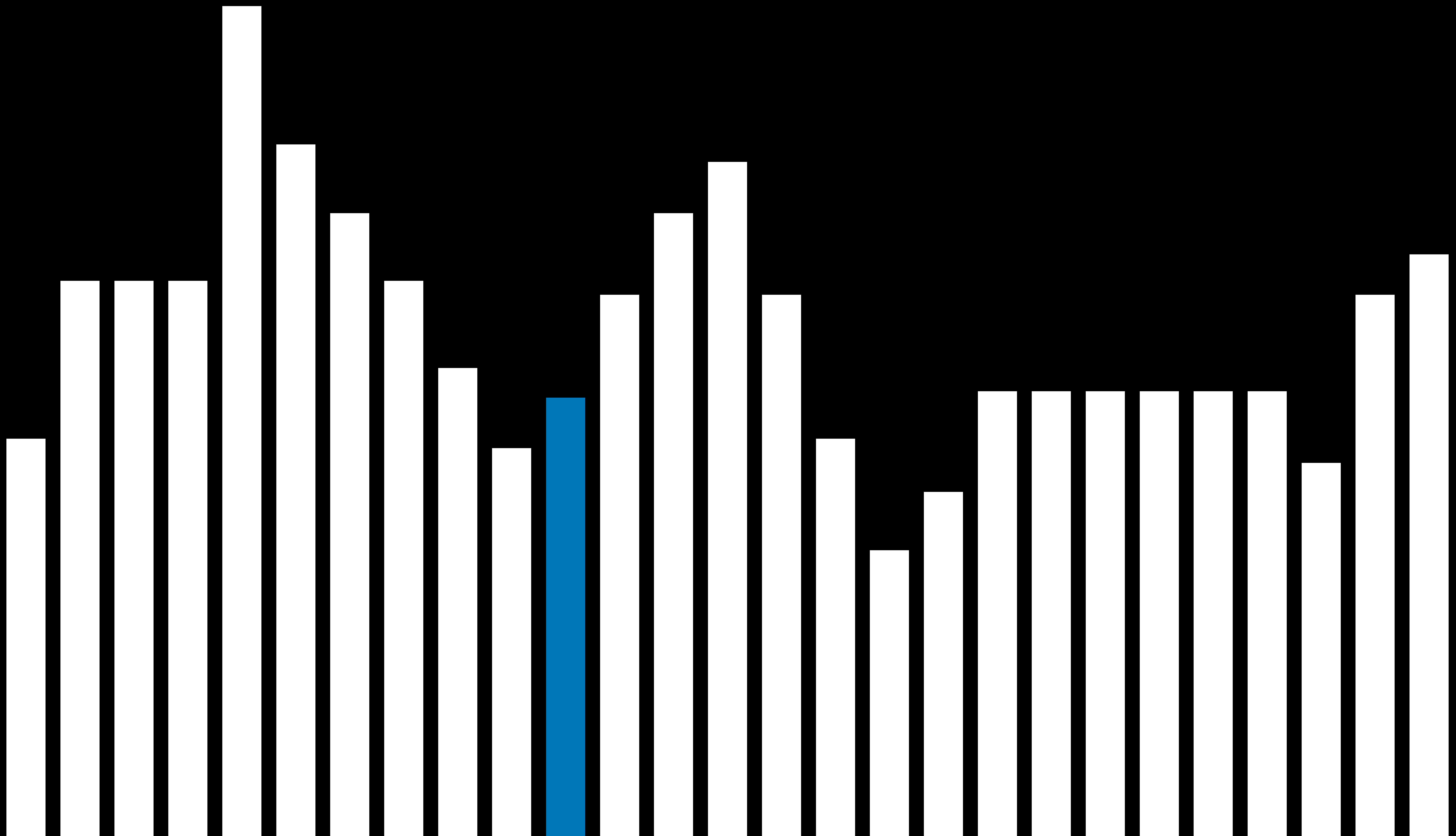


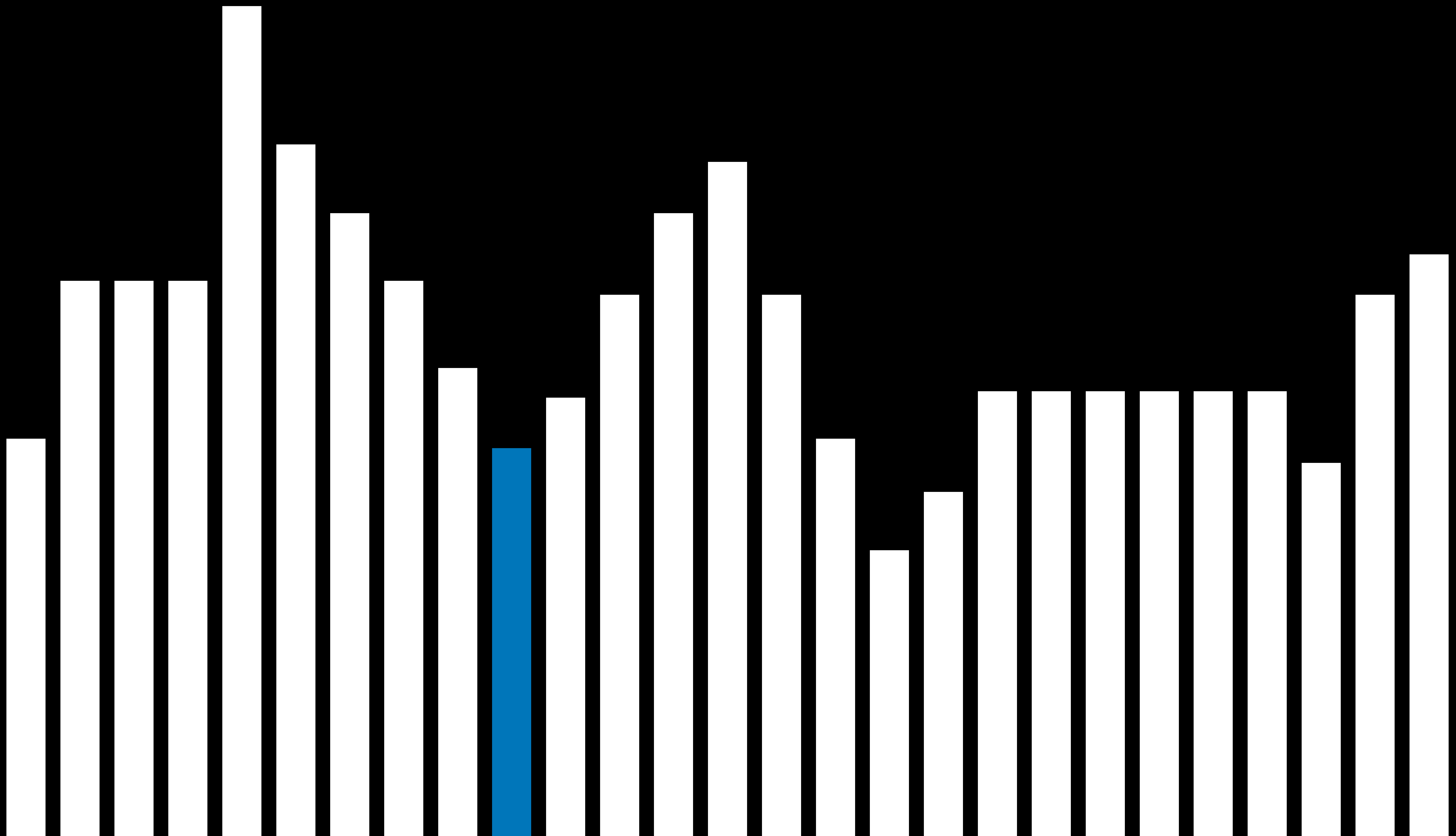
Hill Climbing Variants

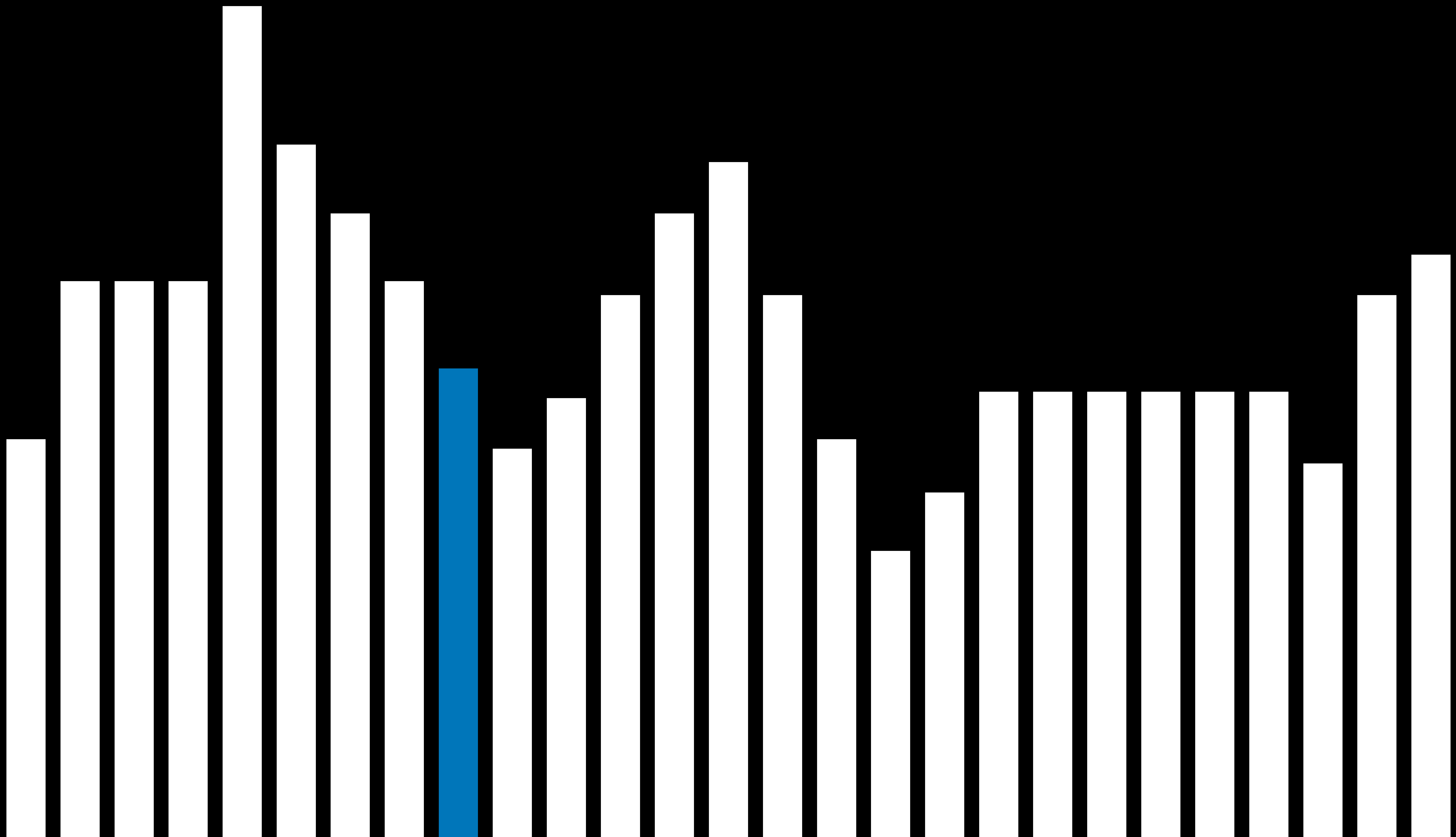
Variant	Definition
steepest-ascent	choose the highest-valued neighbor
stochastic	choose randomly from higher-valued neighbors
first-choice	choose the first higher-valued neighbor
random-restart	conduct hill climbing multiple times
local beam search	chooses the *k* highest-valued neighbors

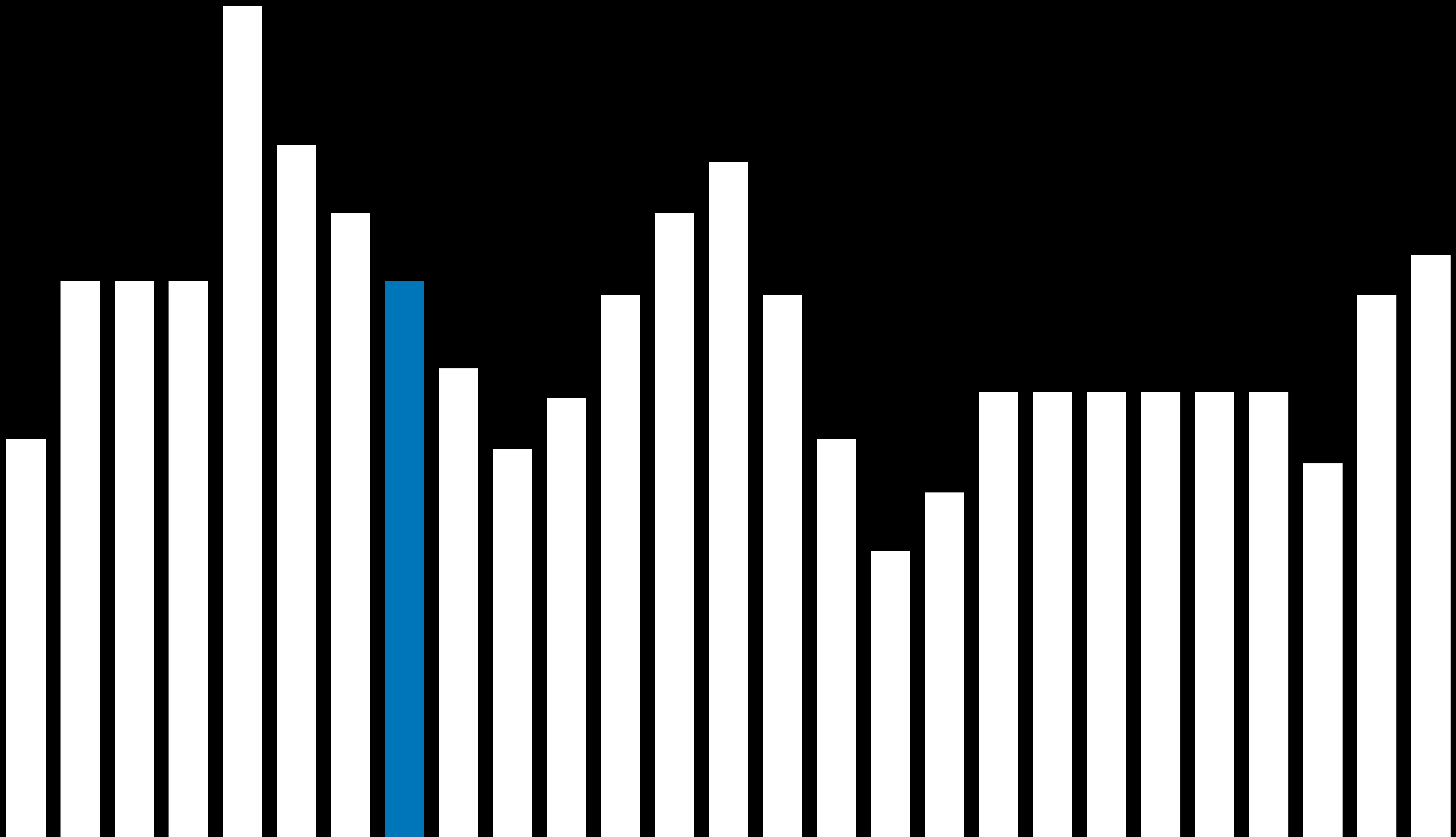
Simulated Annealing

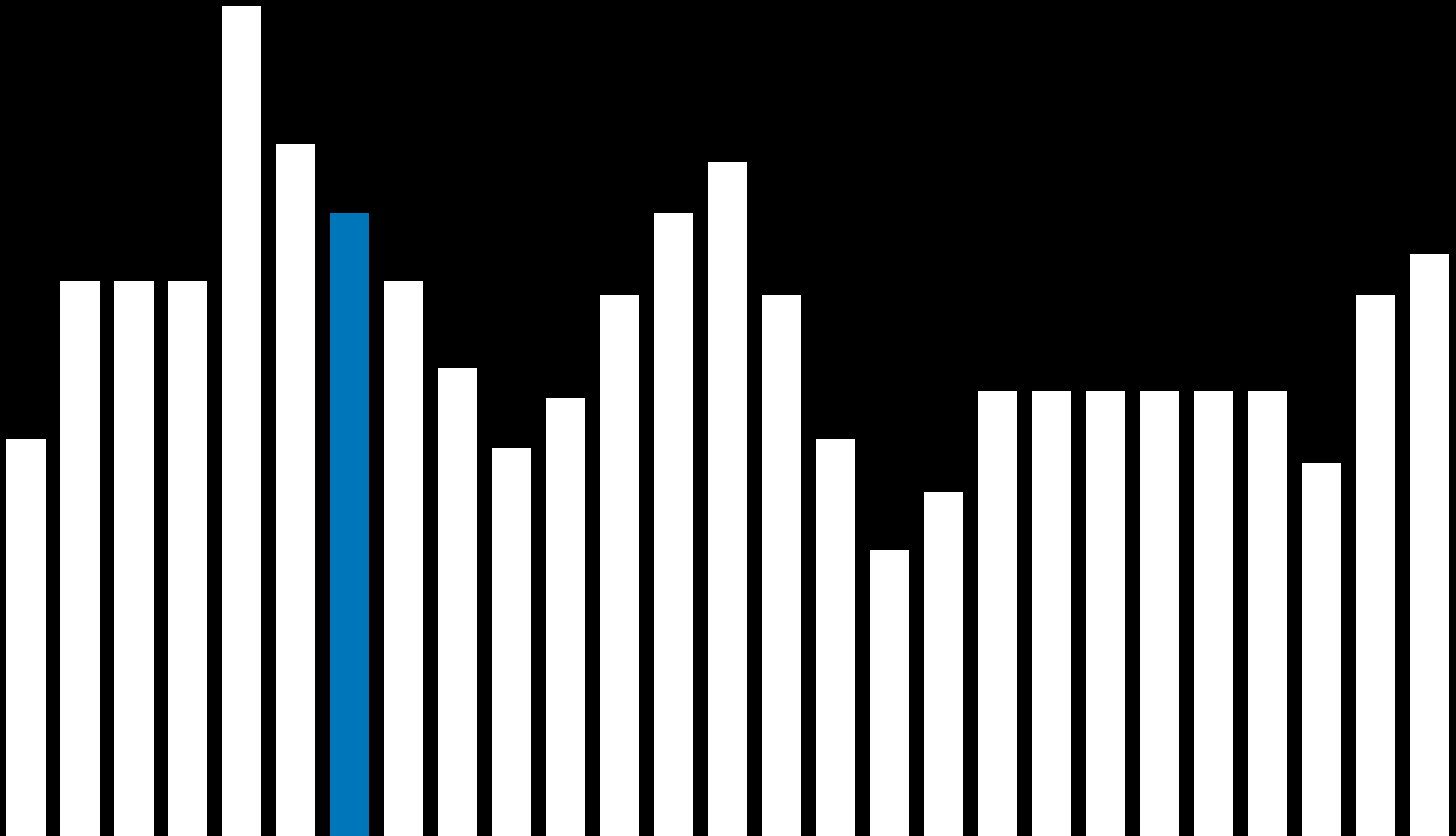


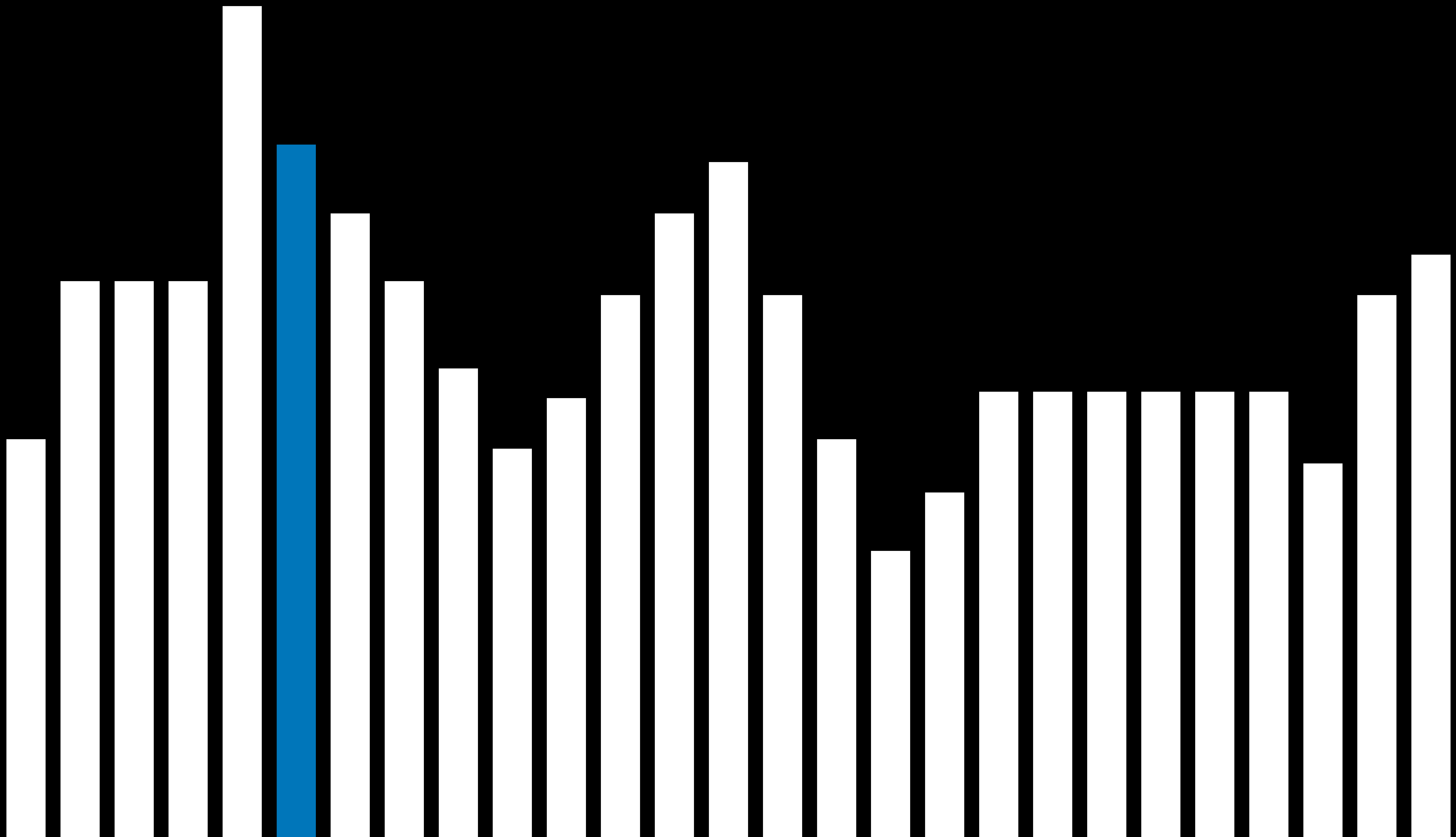


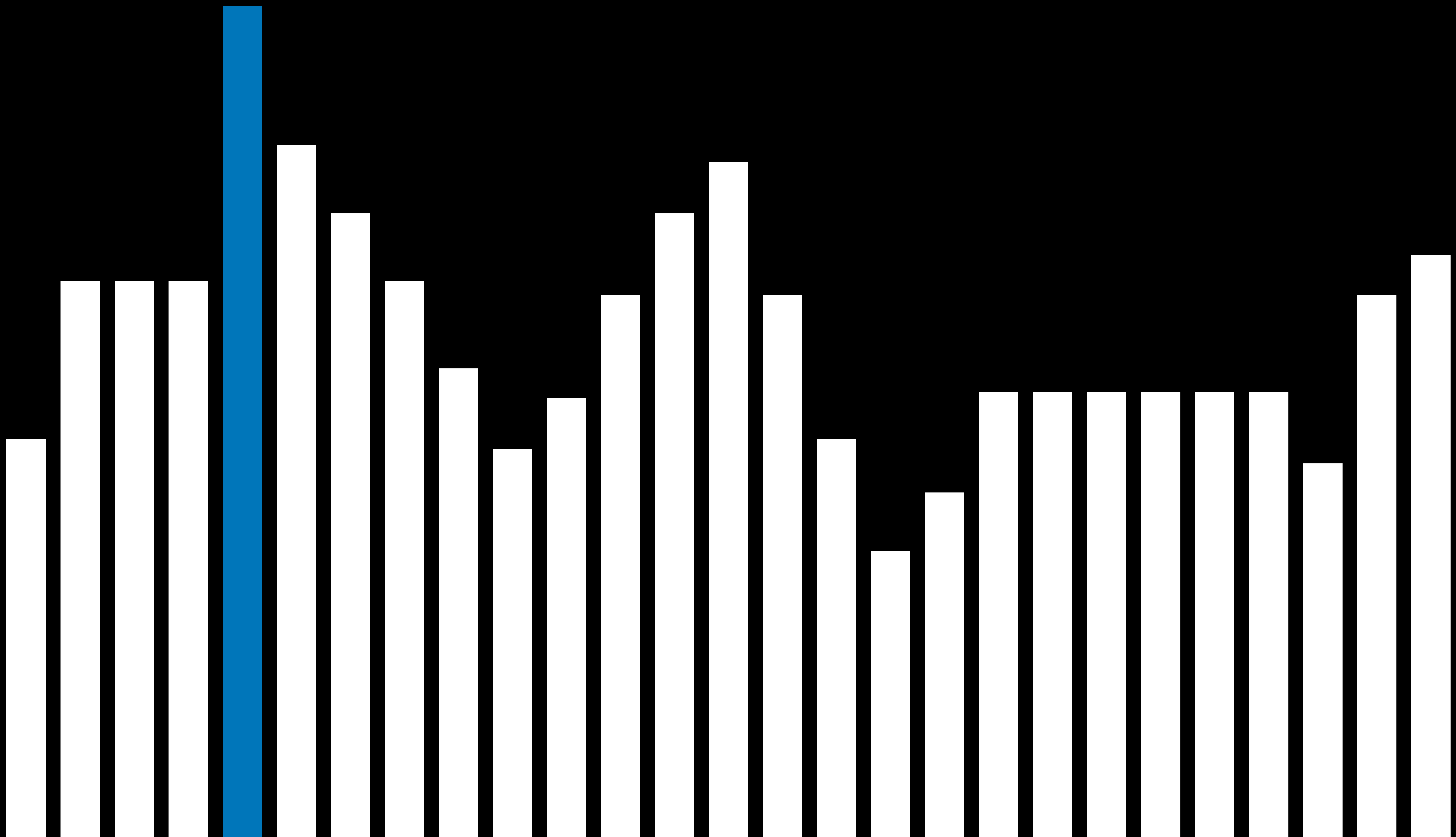












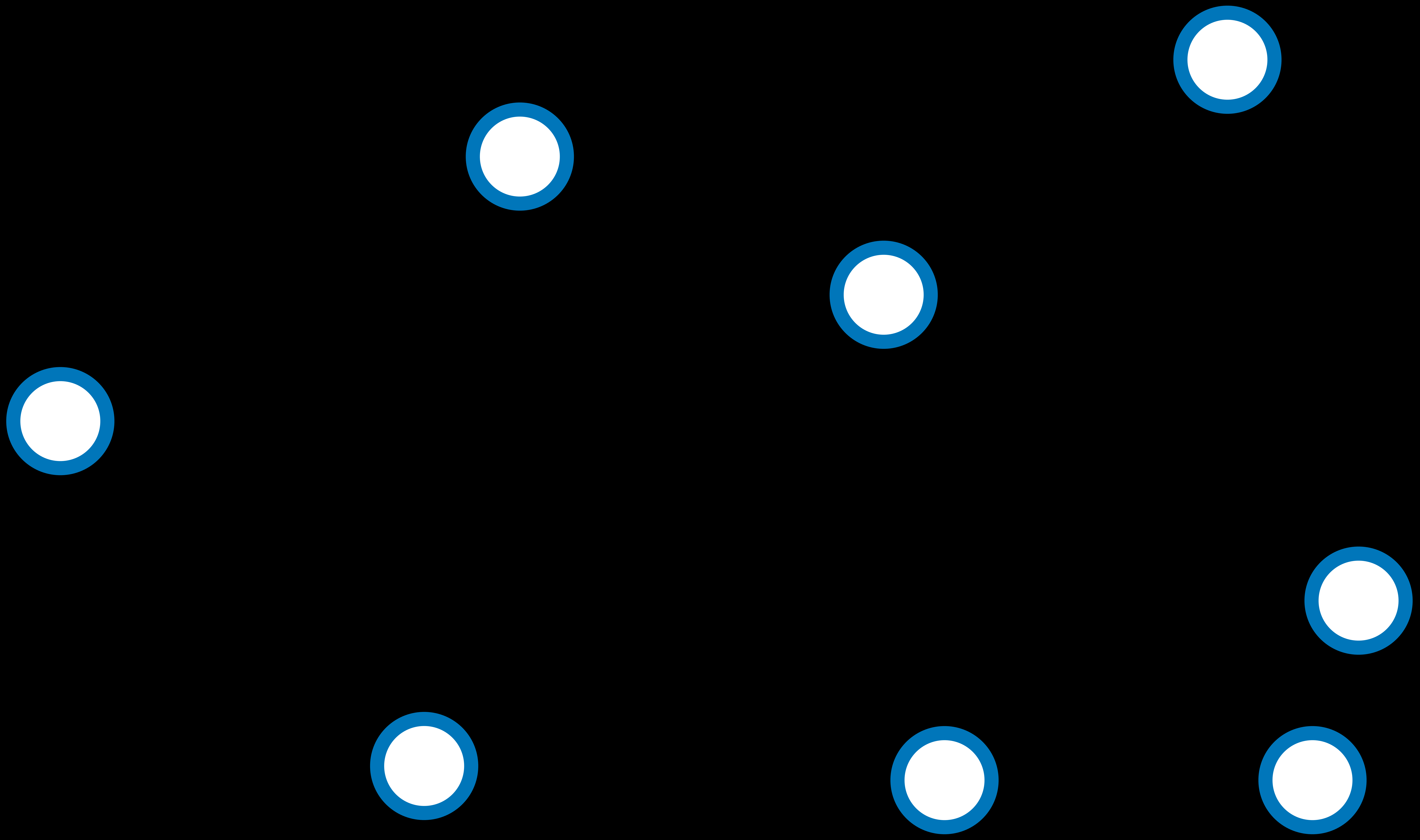
Simulated Annealing

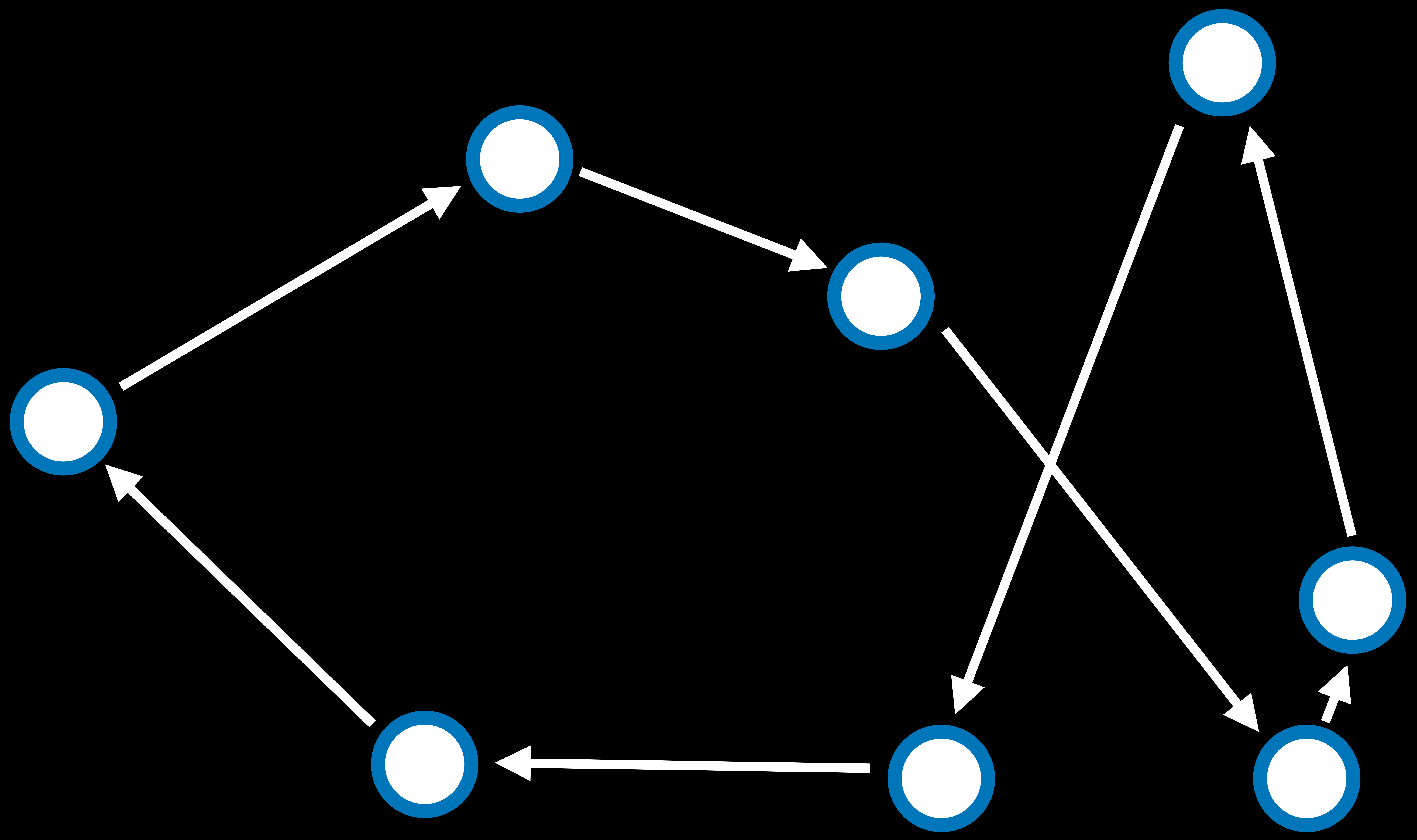
- Early on, higher "temperature": more likely to accept neighbors that are worse than current state
- Later on, lower "temperature": less likely to accept neighbors that are worse than current state

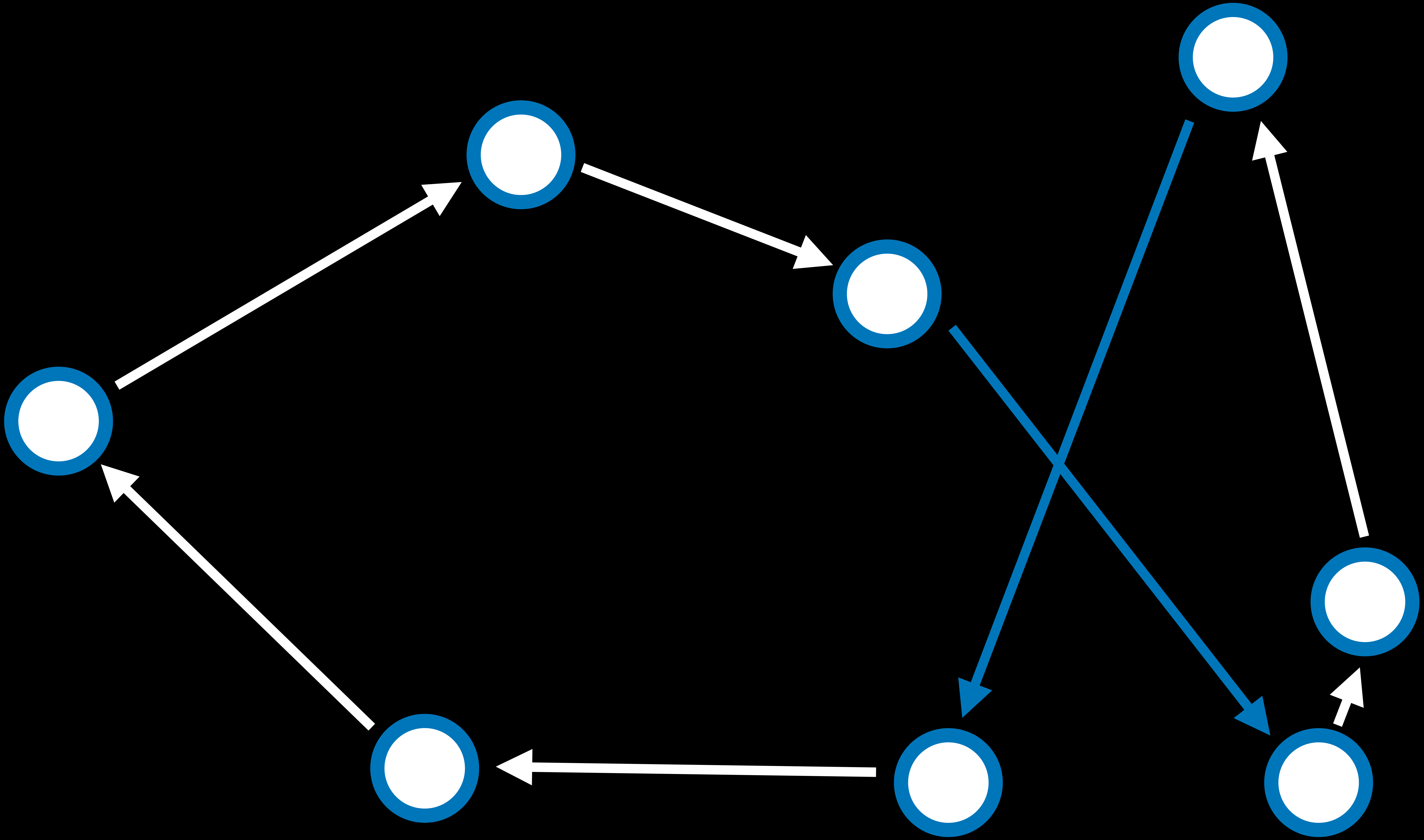
Simulated Annealing

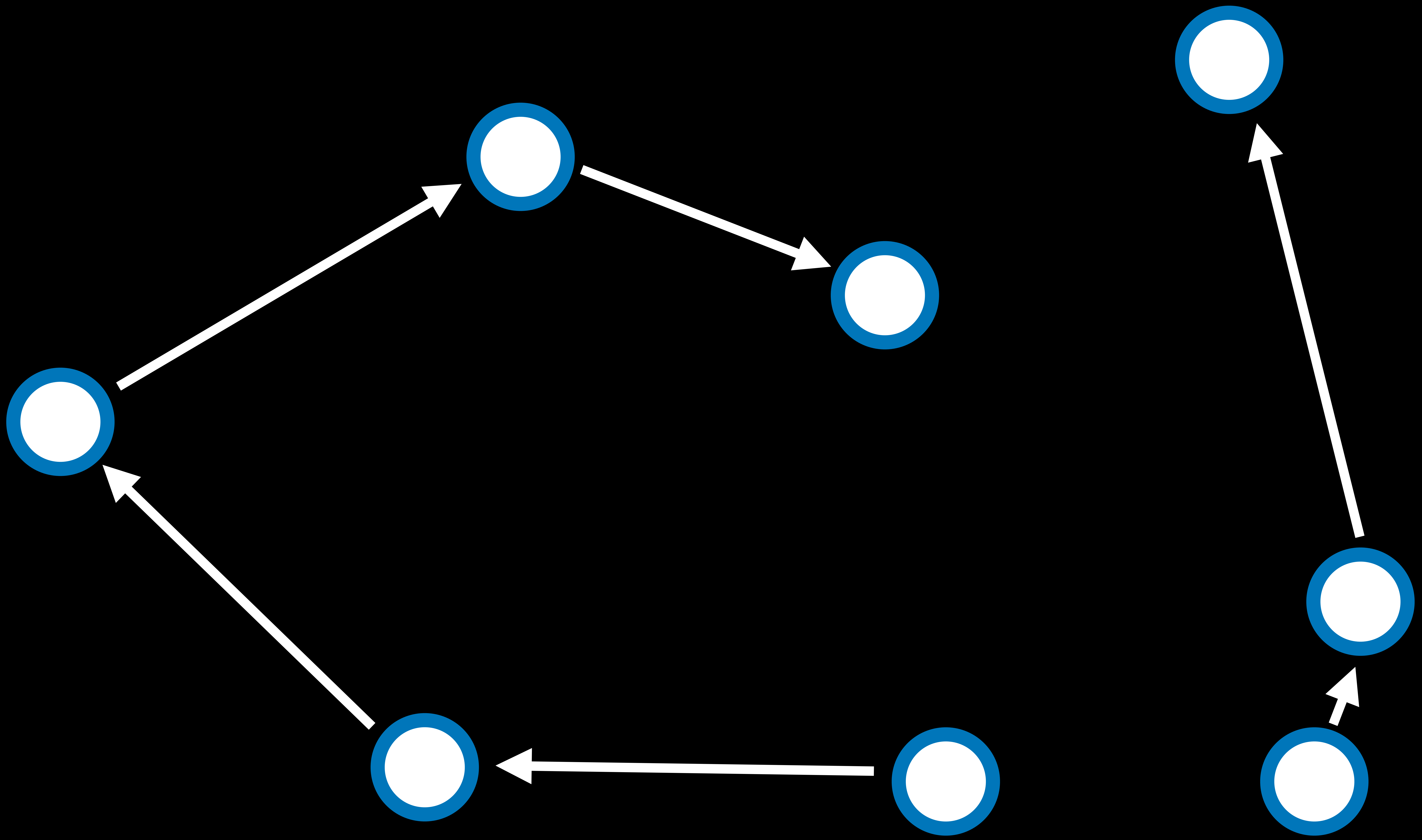
function SIMULATED-ANNEALING(*problem*, *max*):
    current = initial state of *problem*
    for *t* = 1 to *max*:
        T = TEMPERATURE(*t*)
        neighbor = random neighbor of *current*
        ΔE = how much better *neighbor* is than *current*
        if $\Delta E > 0$:
            current = *neighbor*
        with probability $e^{\Delta E/T}$ set *current* = *neighbor*
    return *current*

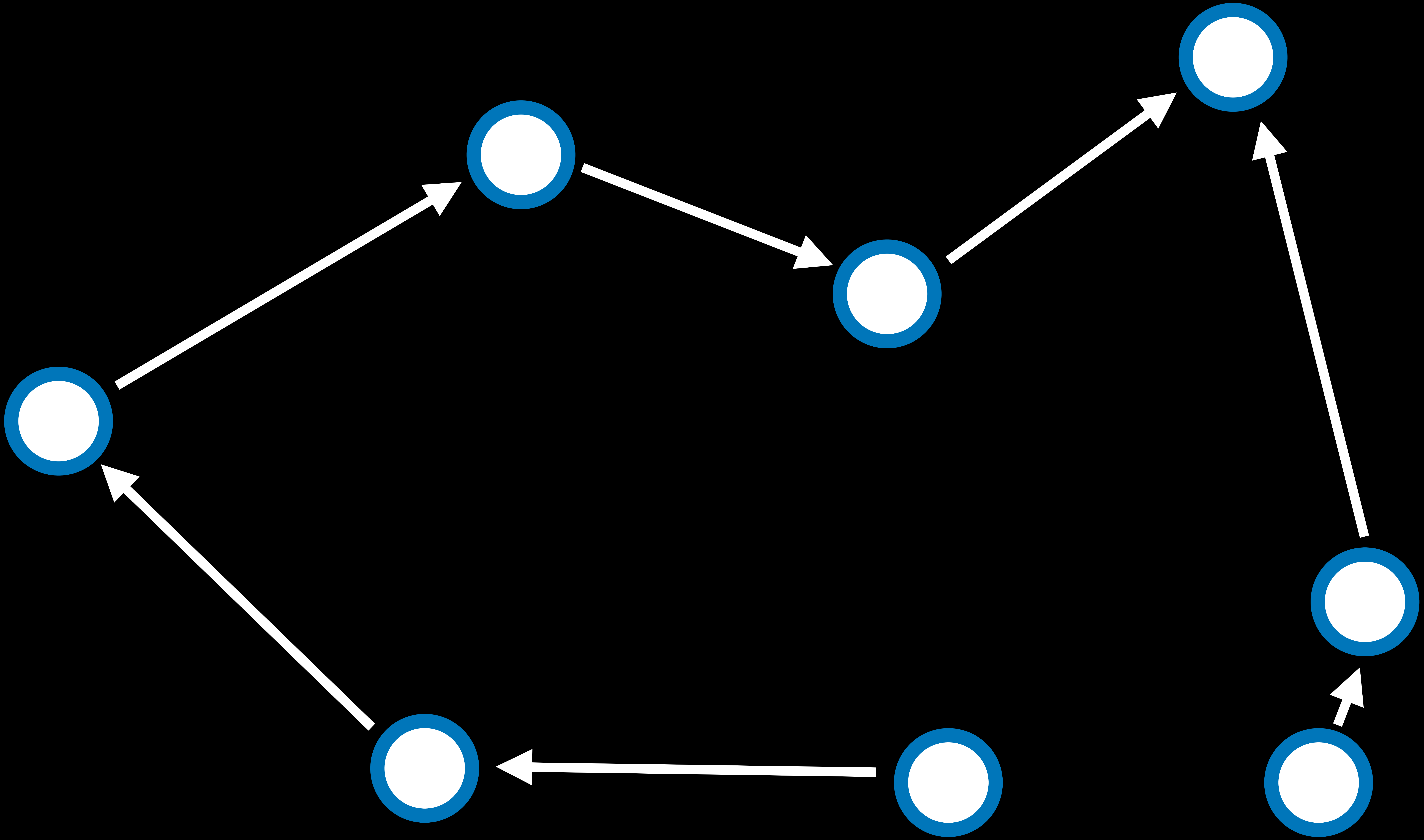
Traveling Salesman Problem

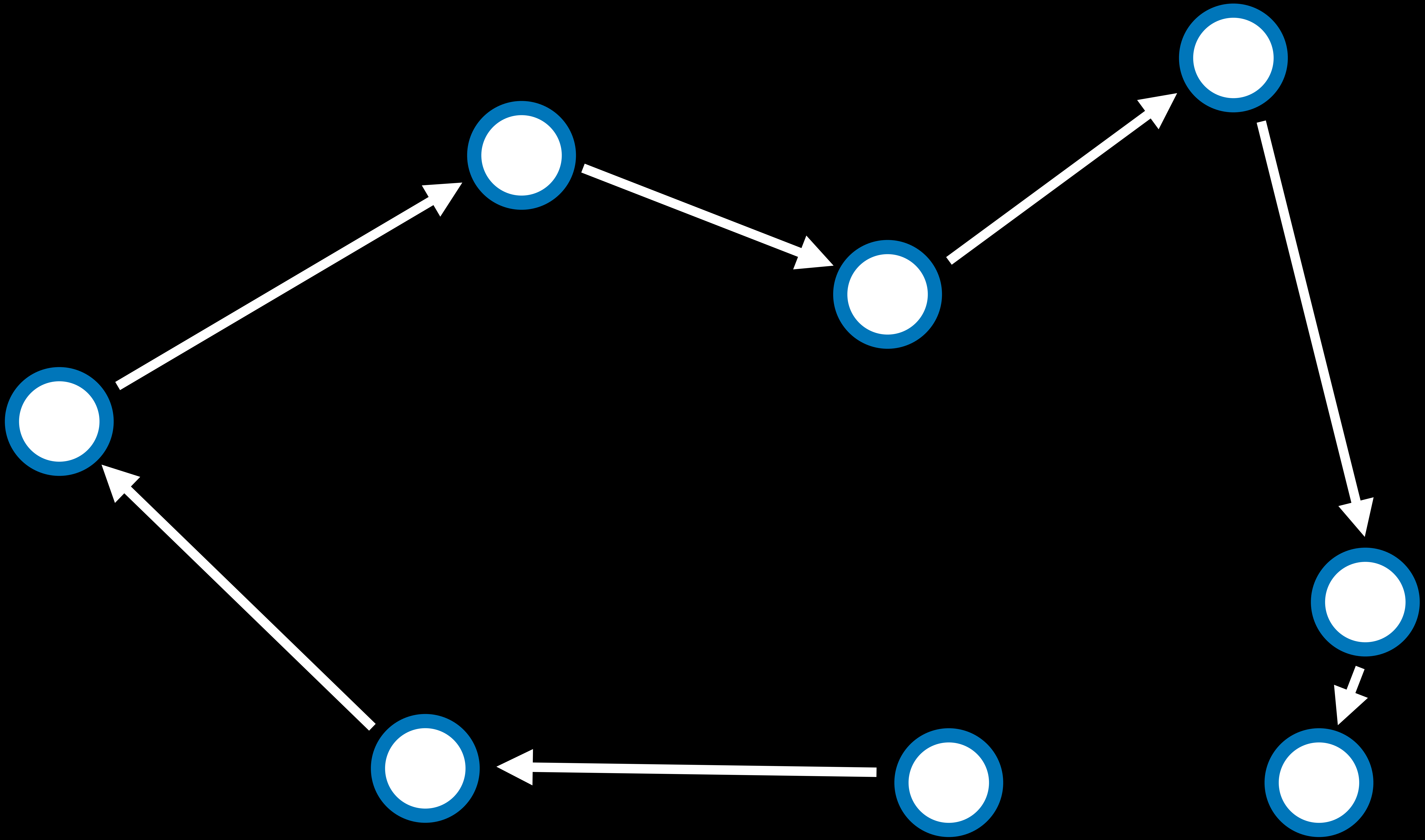


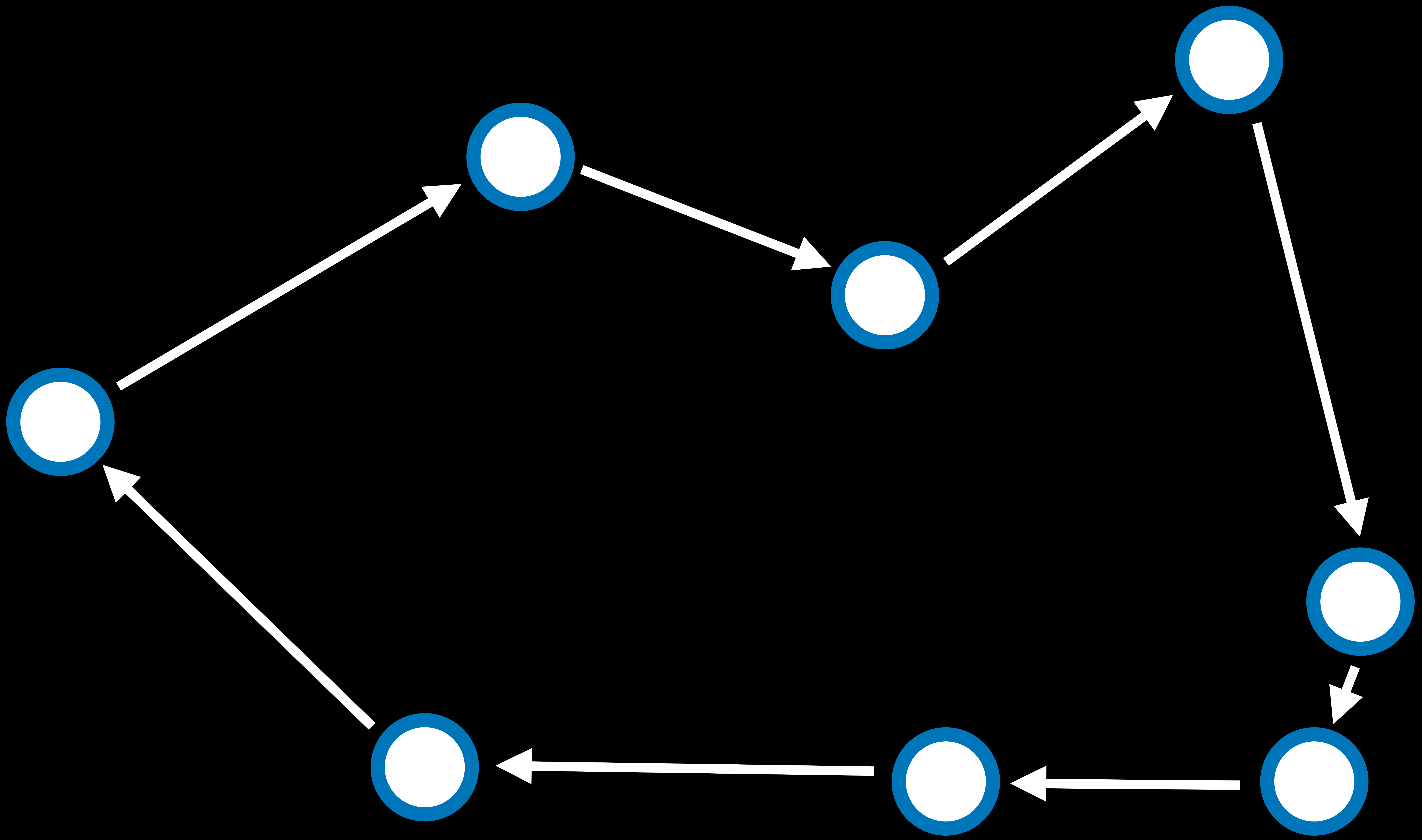












Linear Programming

Linear Programming

- Minimize a cost function $c_1x_1 + c_2x_2 + ... + c_nx_n$
- With constraints of form $a_1x_1 + a_2x_2 + ... + a_nx_n \leq b$
  or of form $a_1x_1 + a_2x_2 + ... + a_nx_n = b$
- With bounds for each variable $l_i \leq x_i \leq u_i$

Linear Programming Example

- Two machines X_1 and X_2. X_1 costs \$50/hour to run, X_2 costs \$80/hour to run. Goal is to minimize cost.
- X_1 requires 5 units of labor per hour. X_2 requires 2 units of labor per hour. Total of 20 units of labor to spend.
- X_1 produces 10 units of output per hour. X_2 produces 12 units of output per hour. Company needs 90 units of output.

Linear Programming Example

Cost Function: $$50x_1 + 80x_2$$

- X_1 requires 5 units of labor per hour. X_2 requires 2 units of labor per hour. Total of 20 units of labor to spend.
- X_1 produces 10 units of output per hour. X_2 produces 12 units of output per hour. Company needs 90 units of output.

Linear Programming Example

Cost Function: $$50x_1 + 80x_2$$

Constraint: $$5x_1 + 2x_2 \leq 20$$

- X_1 produces 10 units of output per hour. X_2 produces 12 units of output per hour. Company needs 90 units of output.

Linear Programming Example

Cost Function: $50x_1 + 80x_2$

Constraint: $5x_1 + 2x_2 \leq 20$

Constraint: $10x_1 + 12x_2 \geq 90$

Linear Programming Example

Cost Function: $50x_1 + 80x_2$

Constraint: $5x_1 + 2x_2 \leq 20$

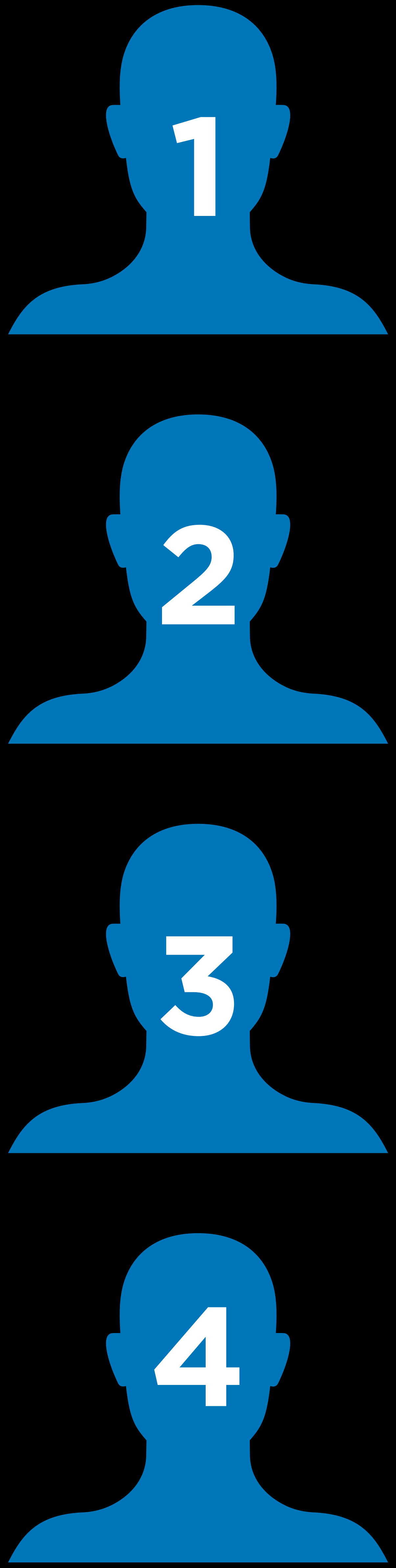
Constraint: $(-10x_1) + (-12x_2) \leq -90$

Linear Programming Algorithms

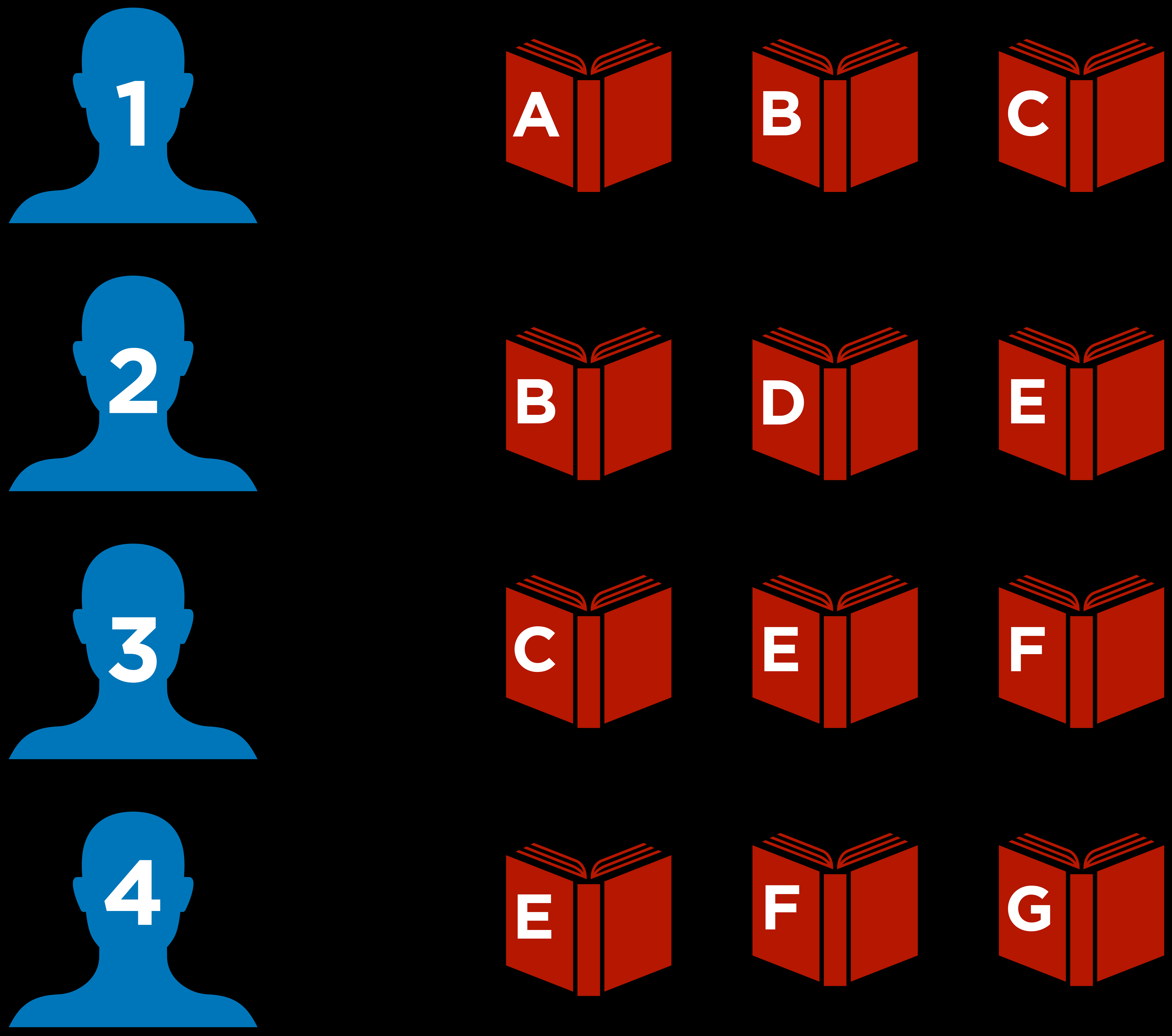
- Simplex
- Interior-Point

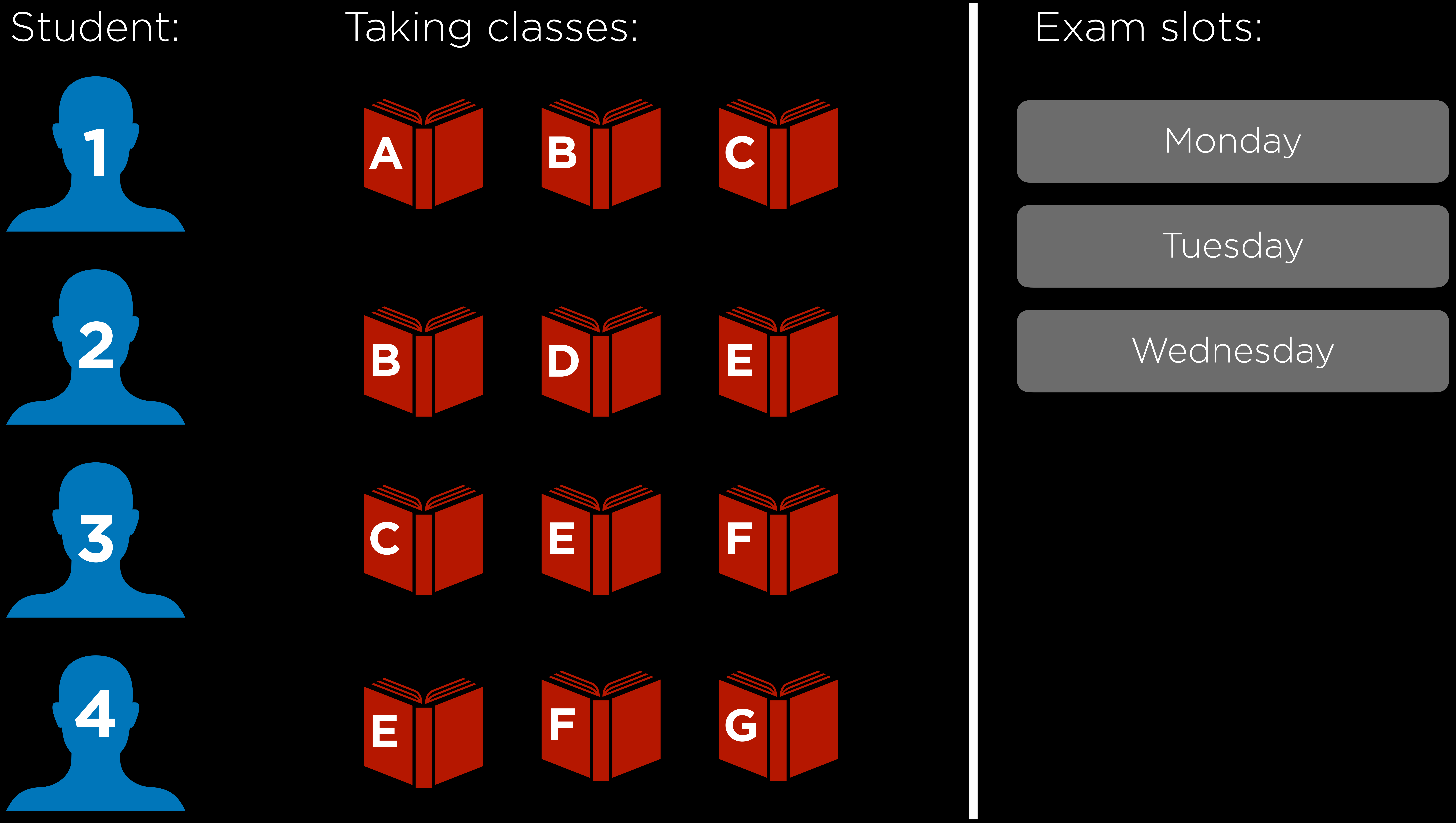
Constraint Satisfaction

Student:



Student: Taking classes:





Student:
Taking classes:
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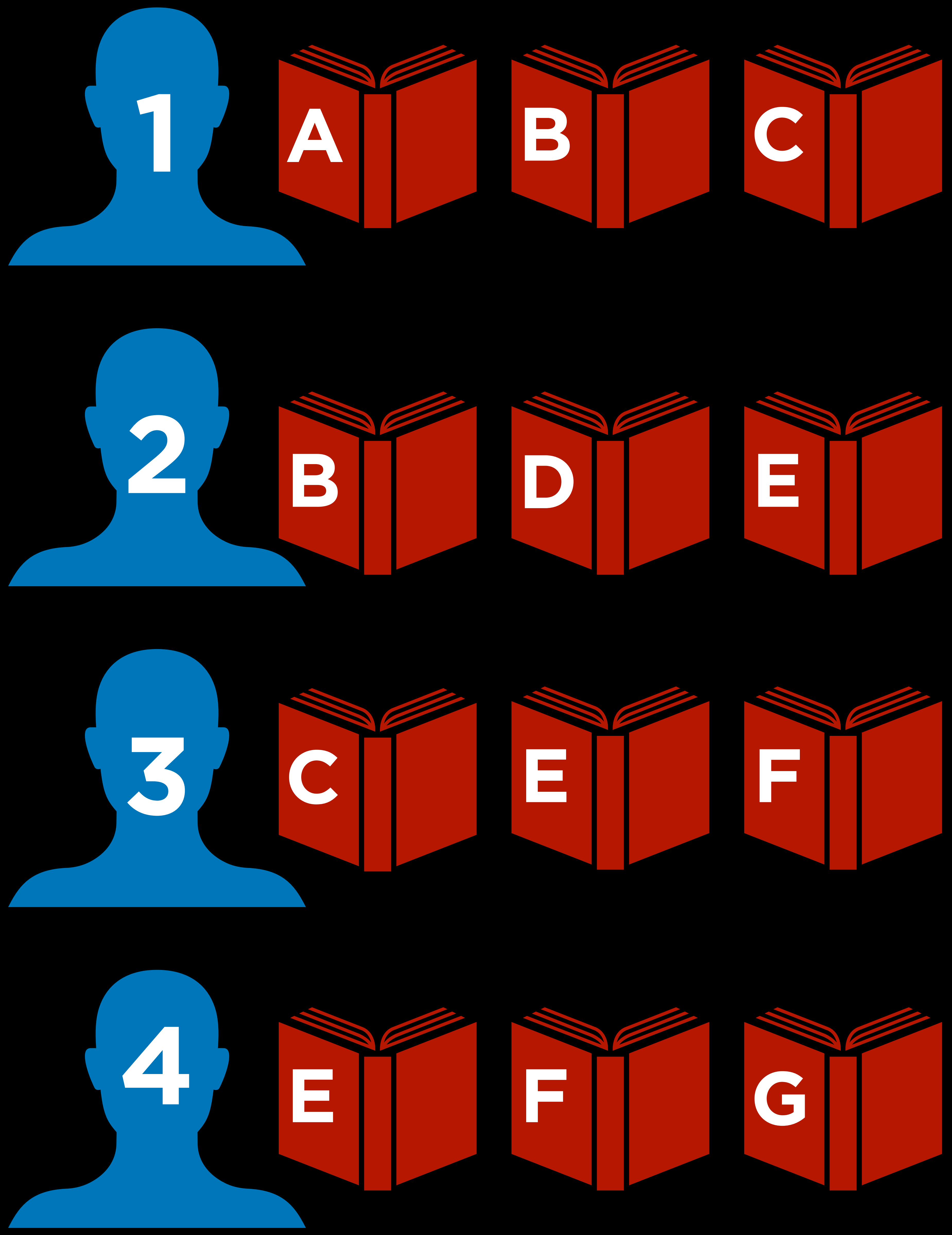
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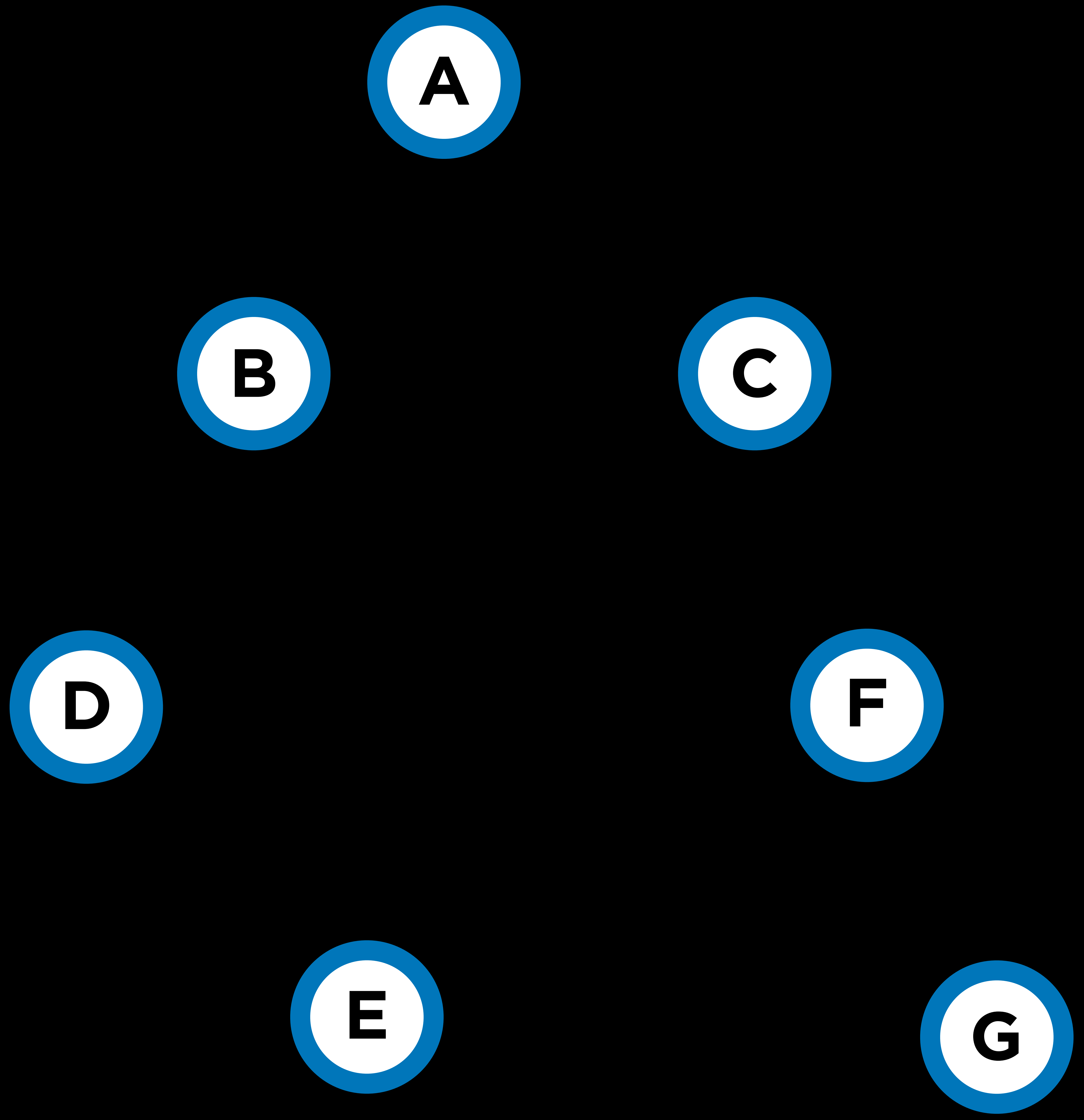
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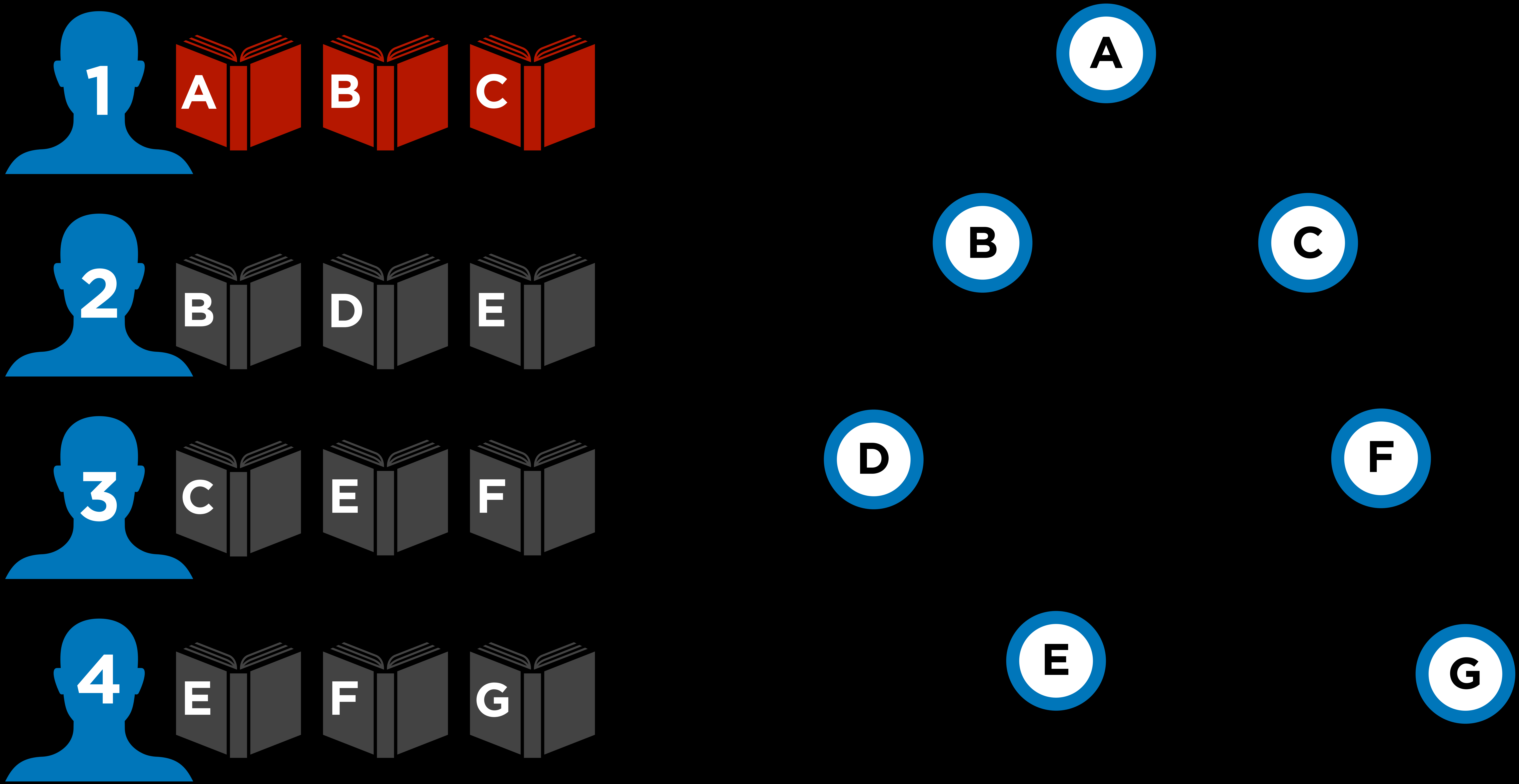
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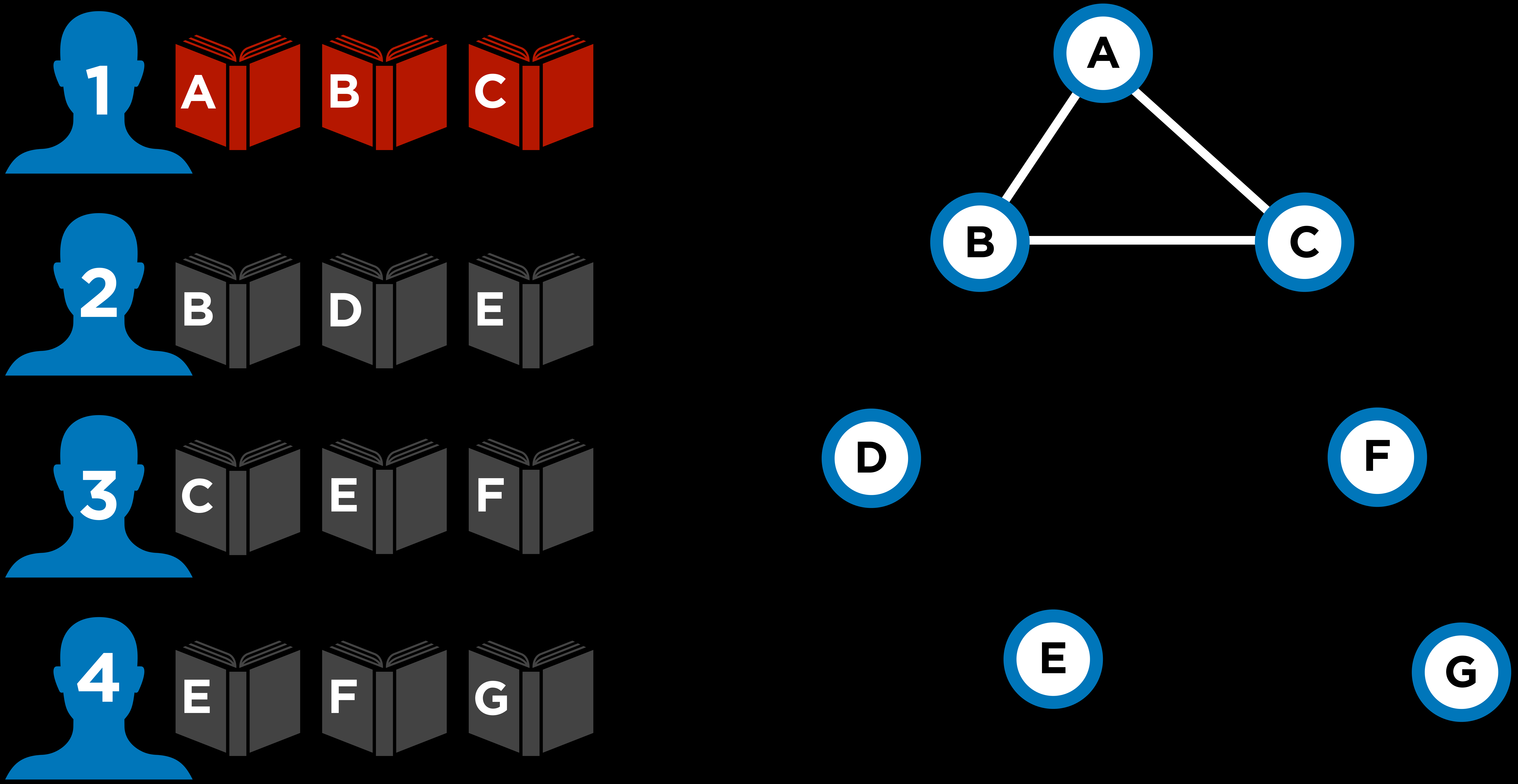
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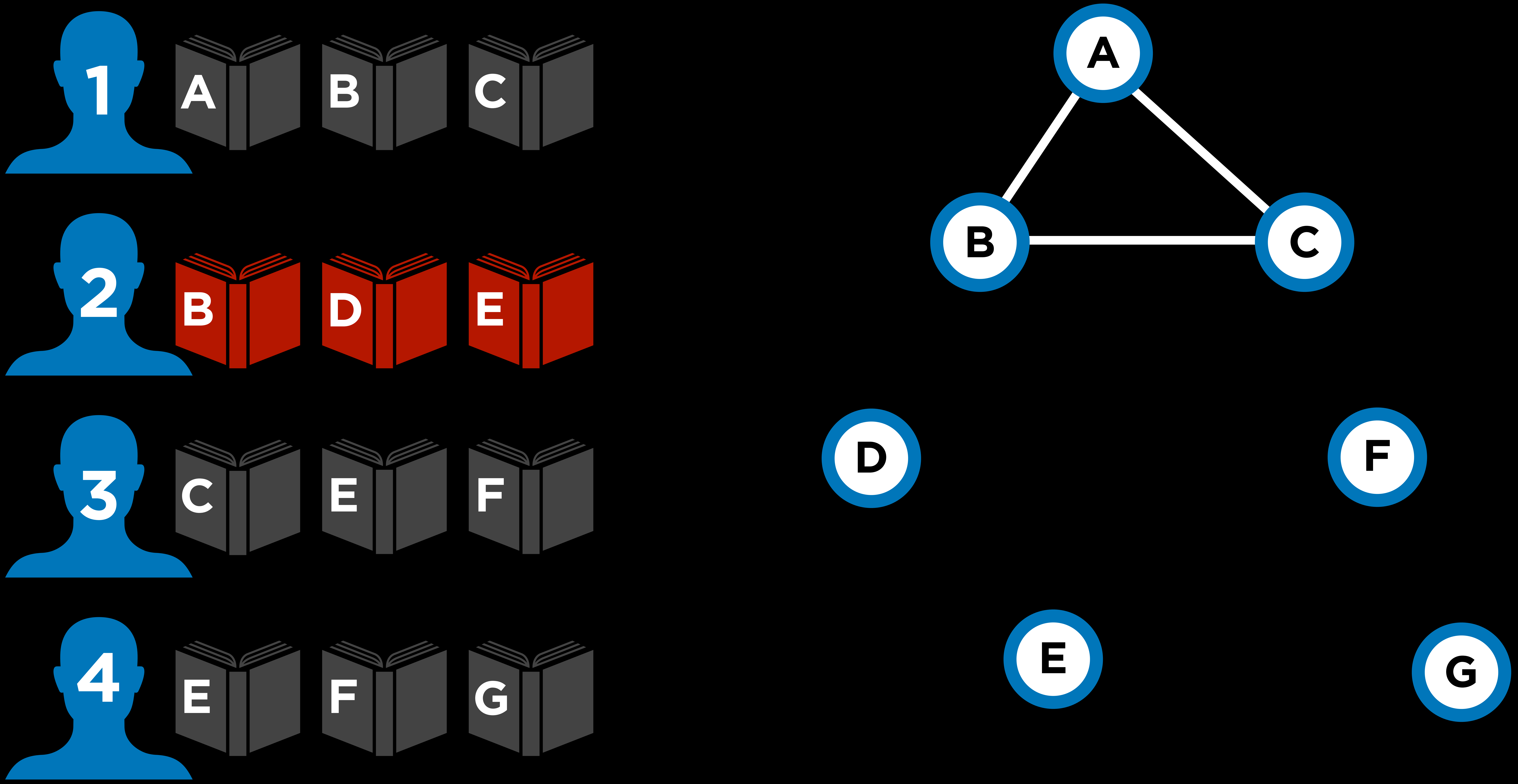
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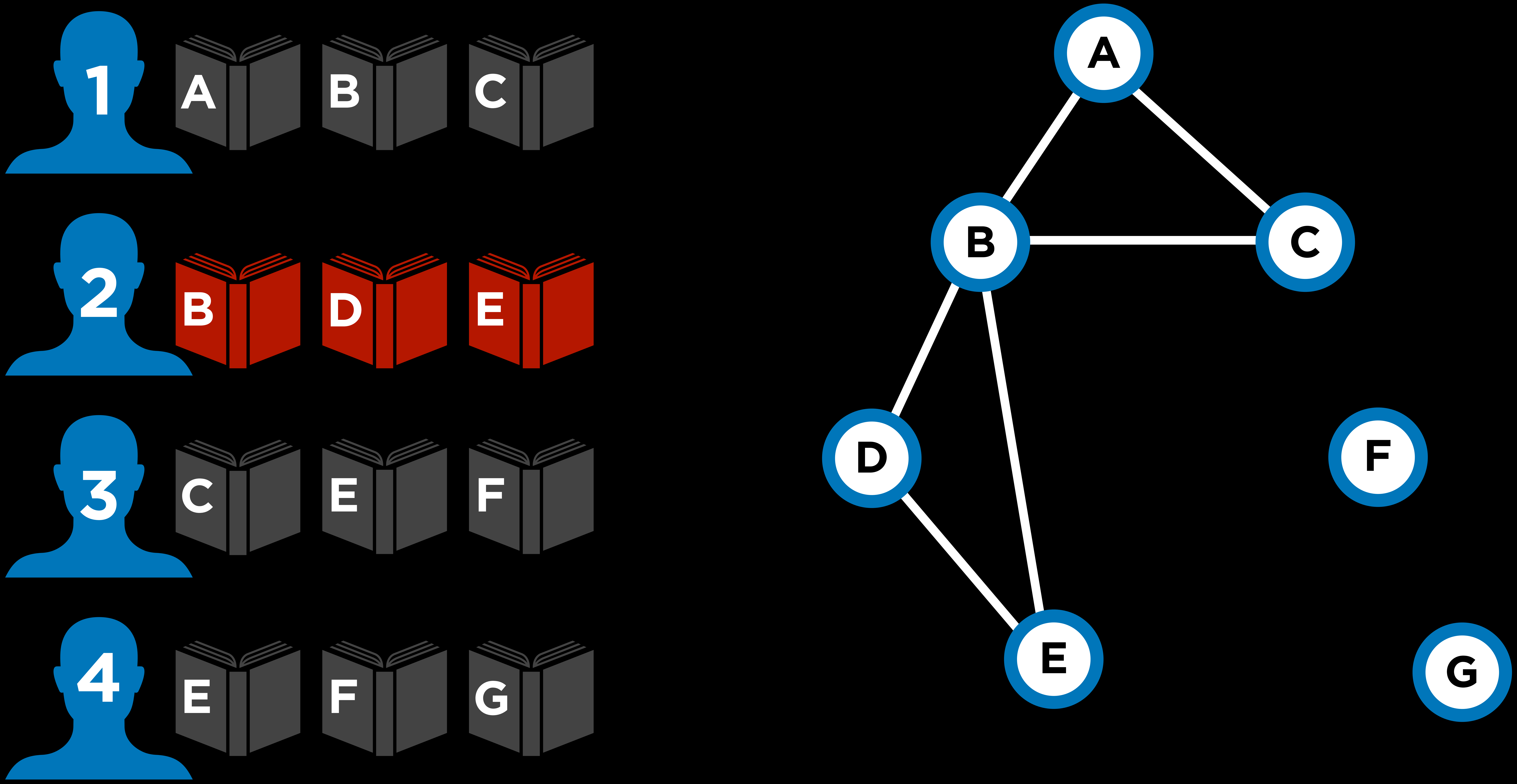
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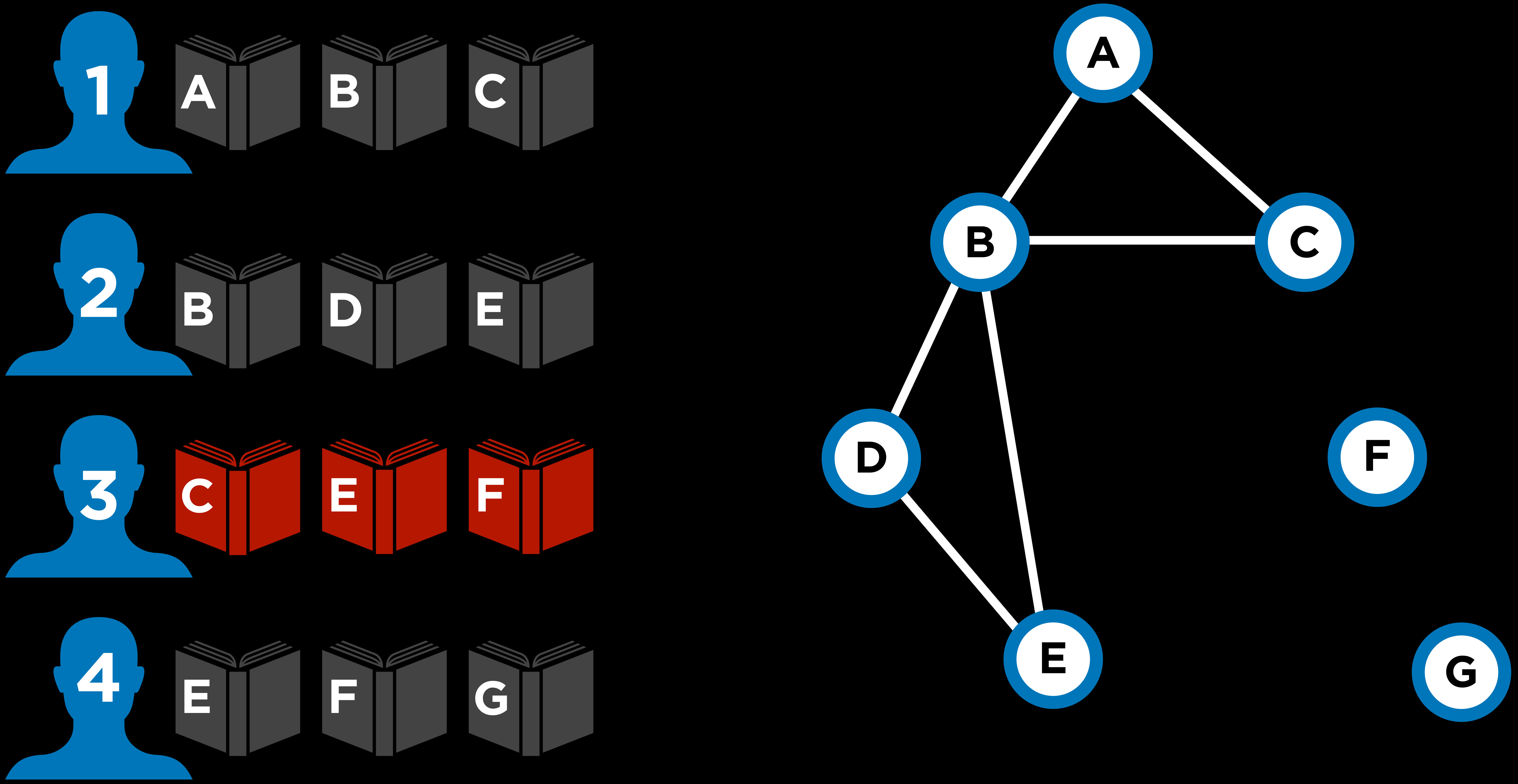

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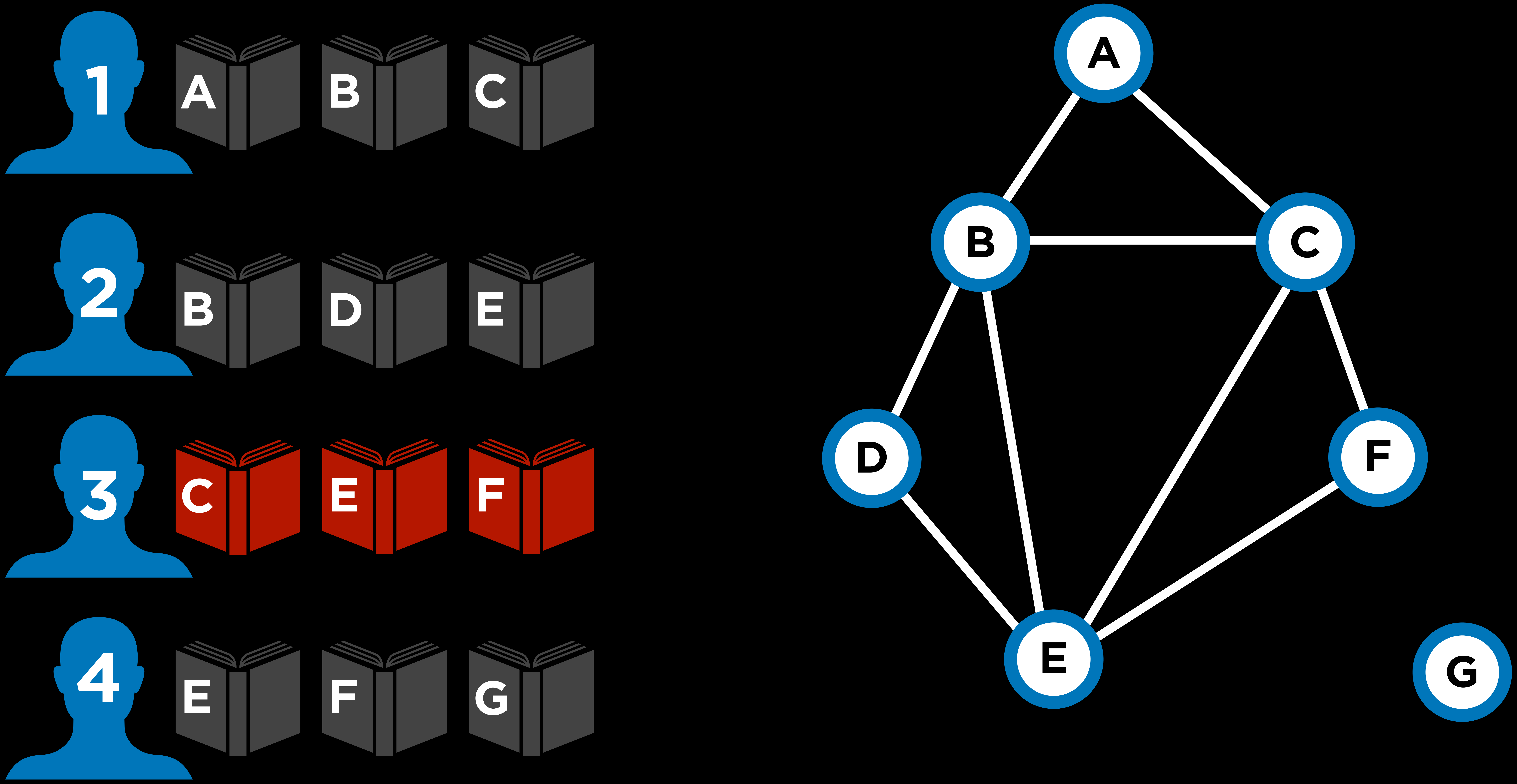
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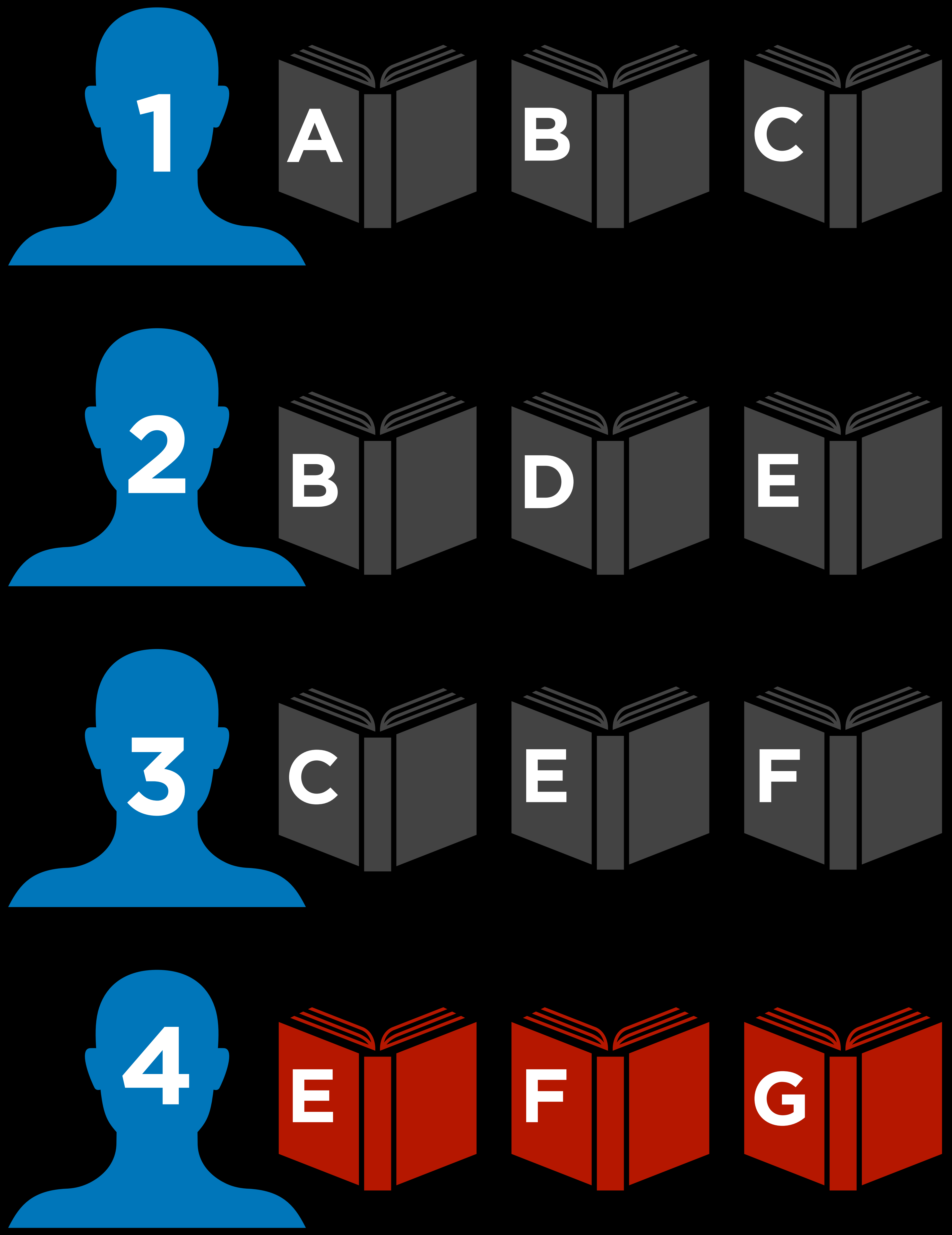
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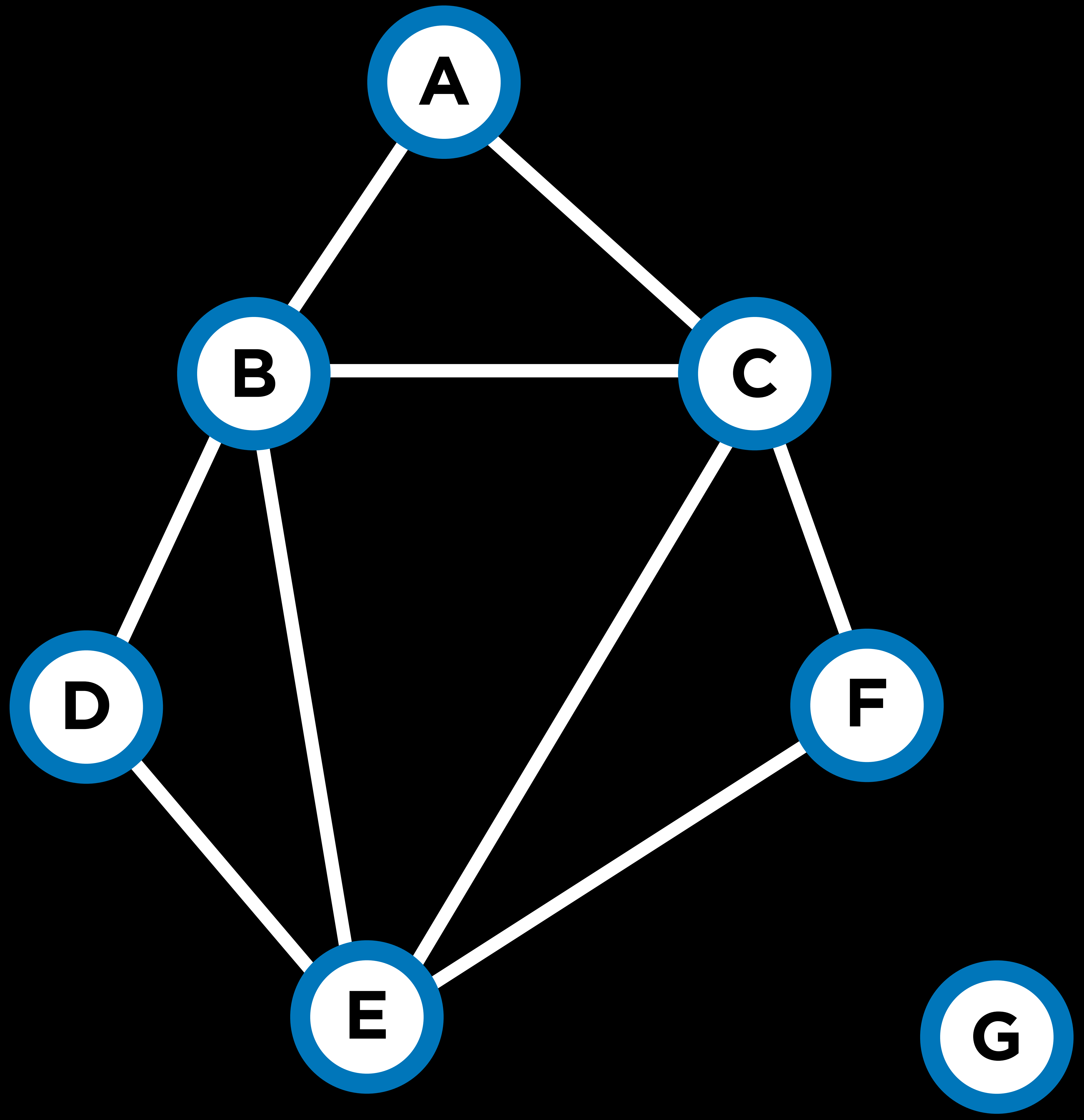
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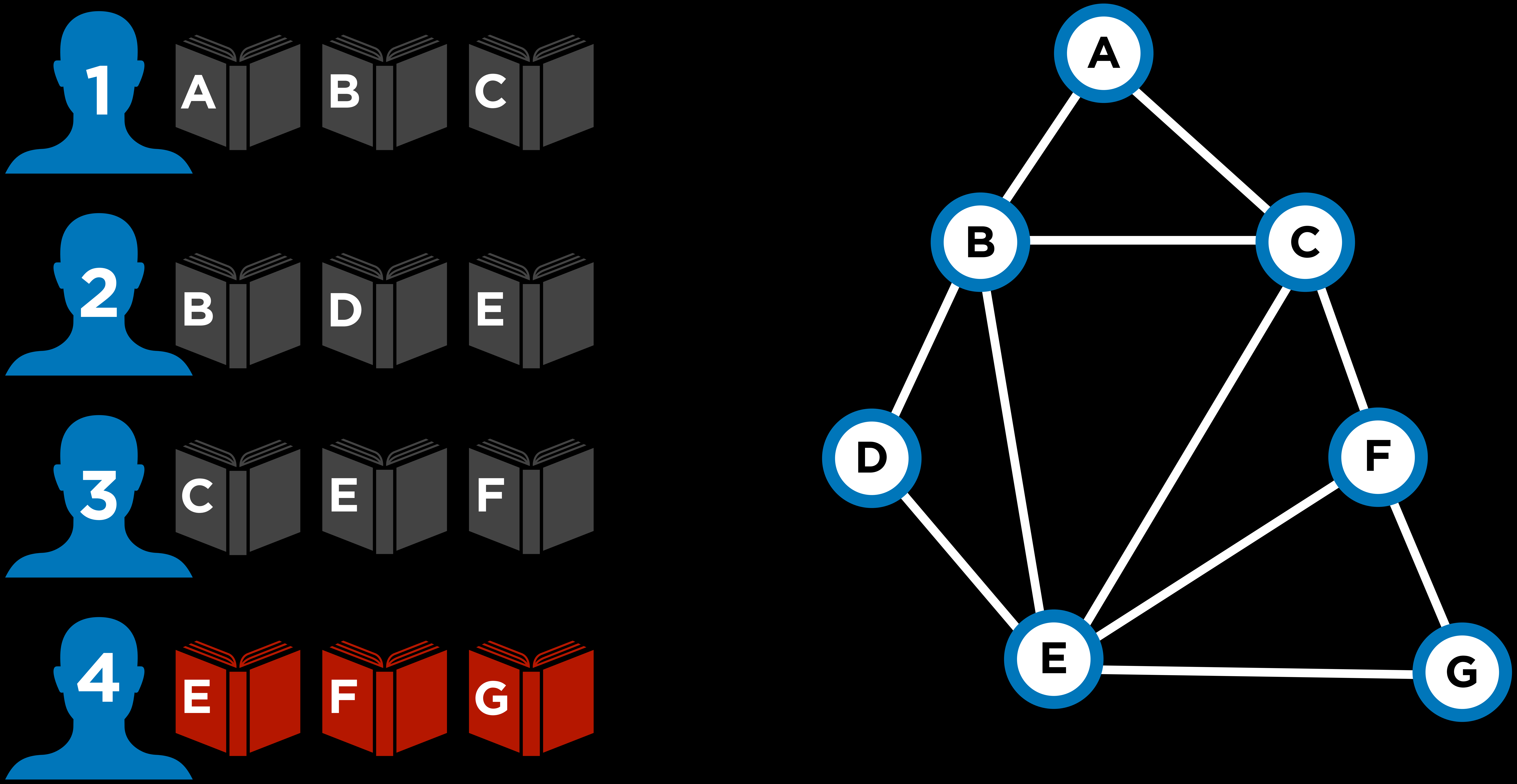

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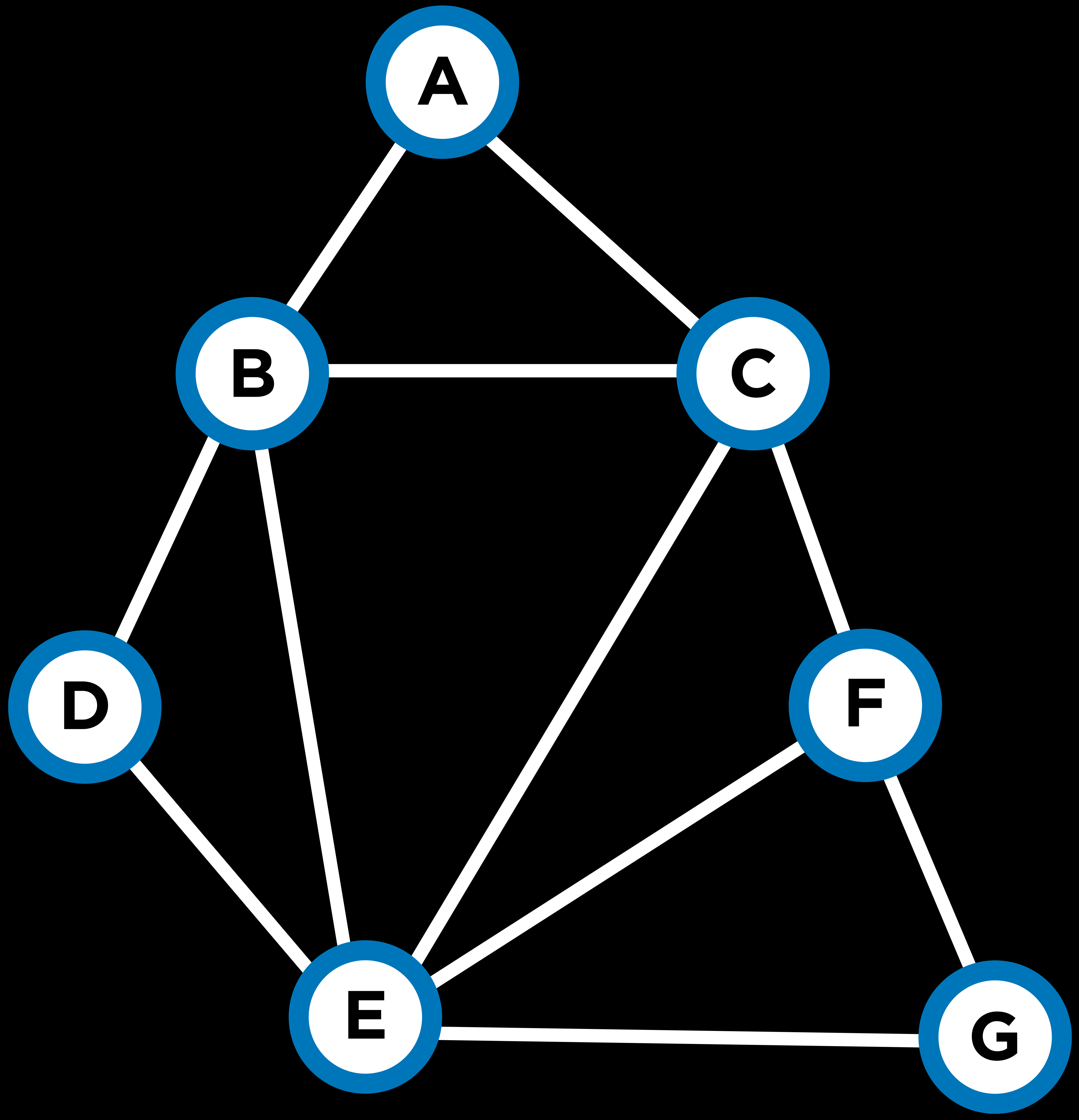
A
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Constraint Satisfaction Problem

- Set of variables $\{X_1, X_2, ..., X_n\}$
- Set of domains for each variable $\{D_1, D_2, ..., D_n\}$
- Set of constraints C

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Variables

$$\{(0, 2), (1, 1), (1, 2), (2, 0), \ldots\}$$

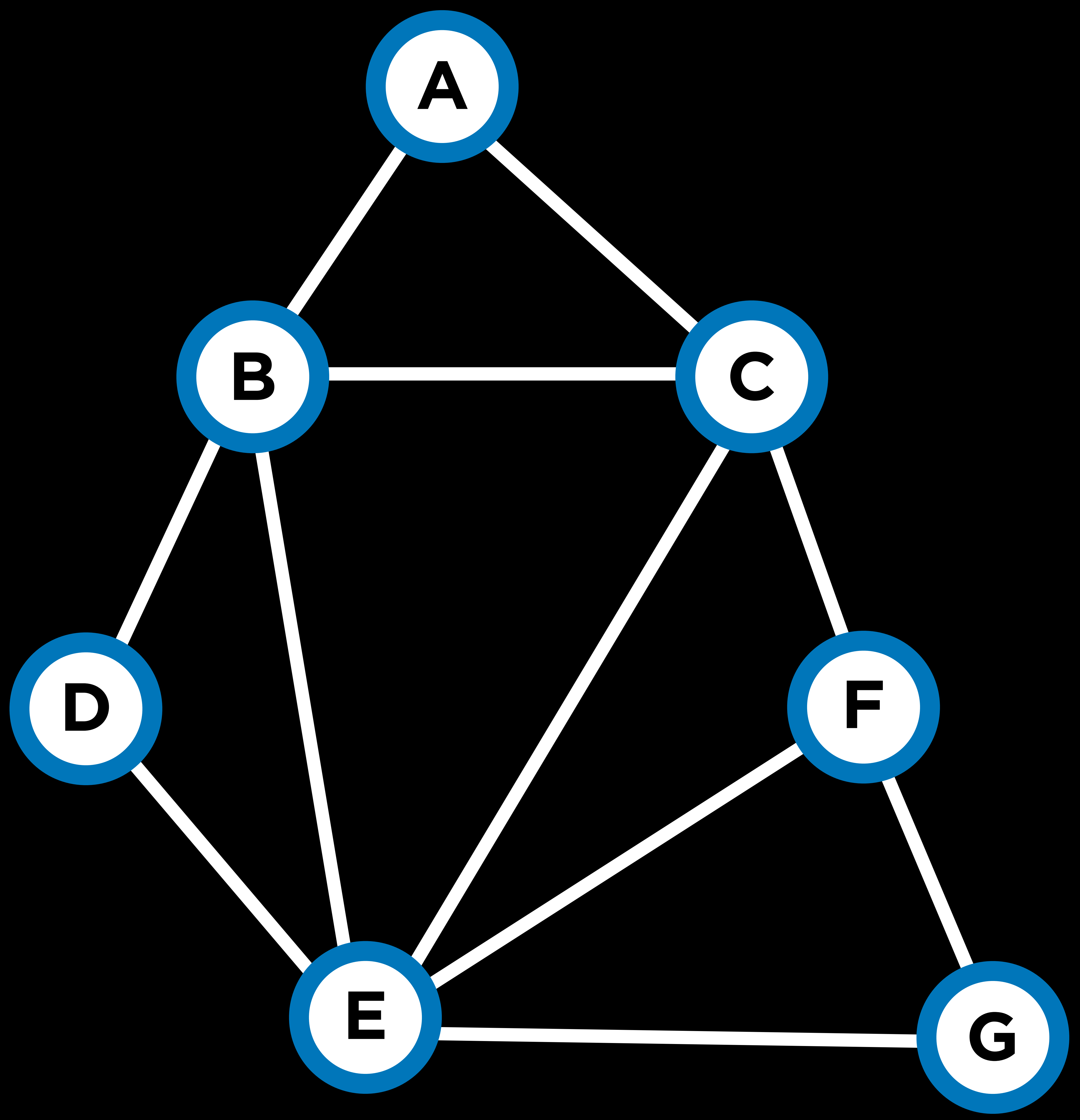
Domains

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

for each variable

Constraints

$$\{(0, 2) \neq (1, 1) \neq (1, 2) \neq (2, 0), \ldots\}$$



Variables

$\{A, B, C, D, E, F, G\}$

Domains

$\{Monday, Tuesday, Wednesday\}$

for each variable

Constraints

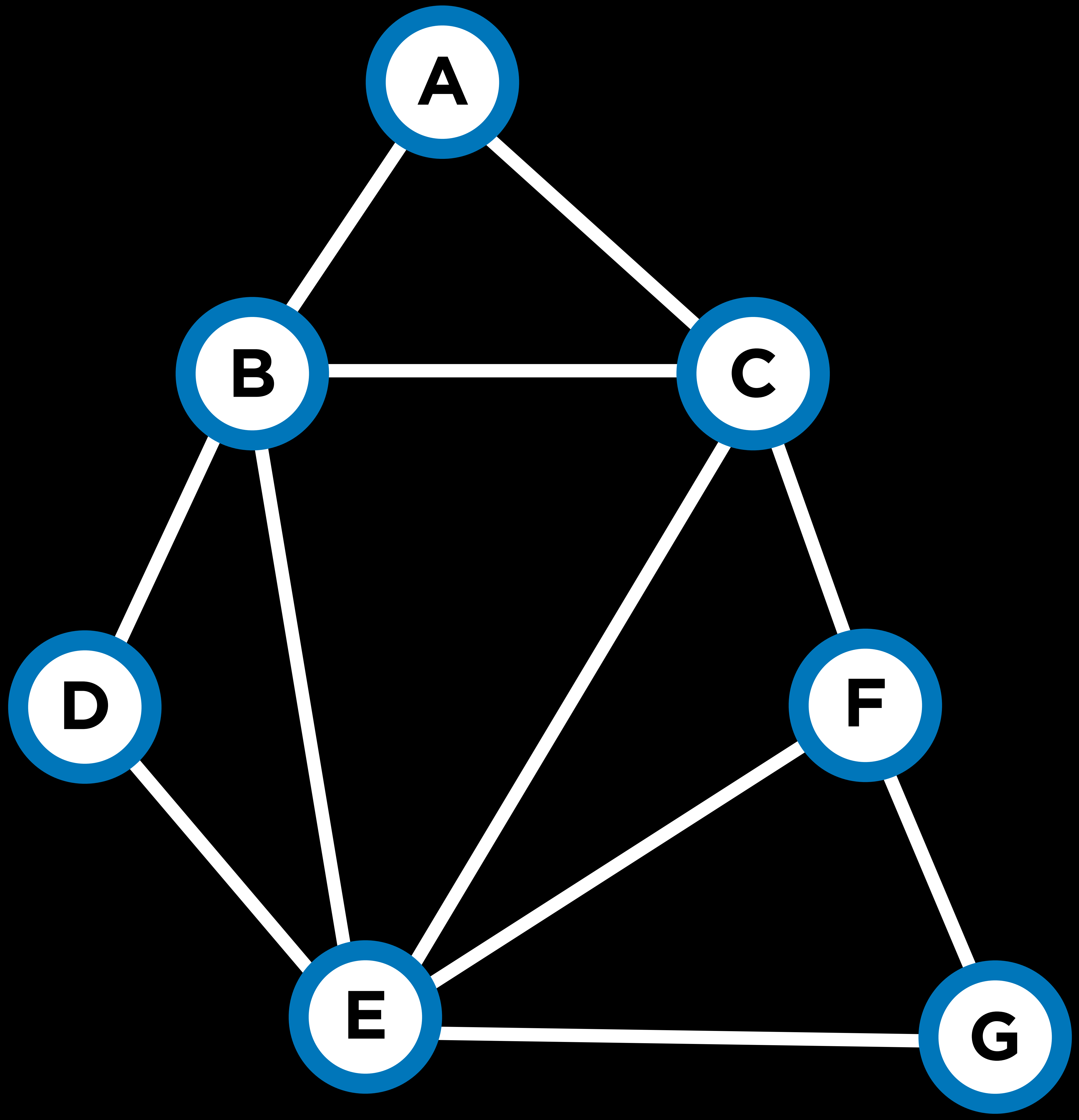
$\{A \neq B, A \neq C, B \neq C, B \neq D, B \neq E, C \neq E,$
$C \neq F, D \neq E, E \neq F, E \neq G, F \neq G\}$

hard constraints

constraints that must be satisfied in a correct solution

soft constraints

constraints that express some notion of which solutions are preferred over others


A
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unary constraint

constraint involving only one variable

unary constraint

$$\{A \neq Monday\}$$

binary constraint

constraint involving two variables

binary constraint

$\{A \neq B\}$

node consistency

when all the values in a variable's domain satisfy the variable's unary constraints

$\{Mon, Tue, Wed\}$ $\{Mon, Tue, Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Mon, Tue, Wed\}$ $\{Mon, Tue, Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Mon, Tue, Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Mon, Tue, Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Mon, Wed\}$

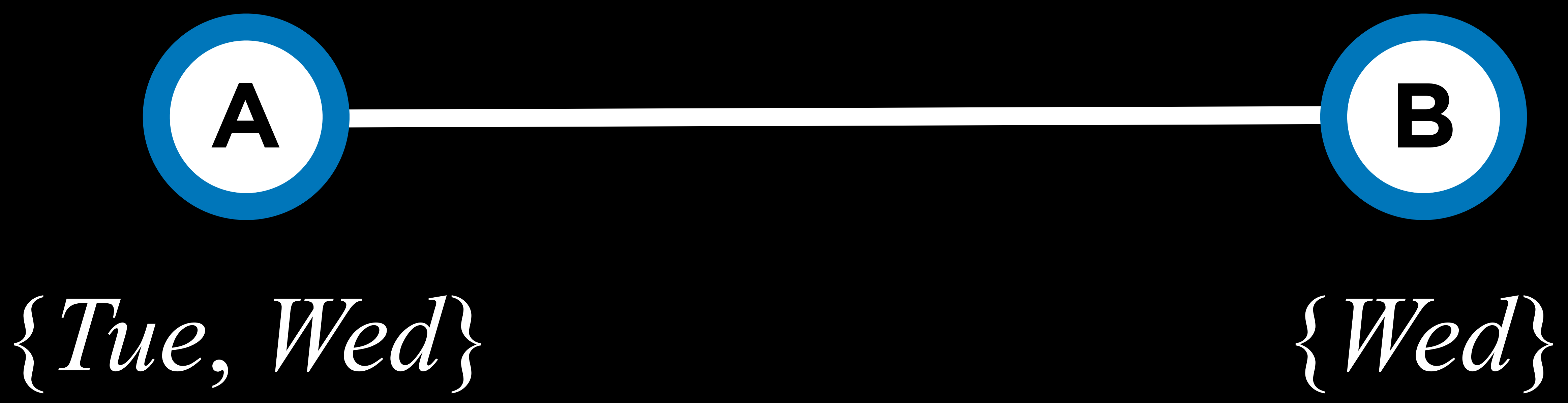
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Mon, Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$



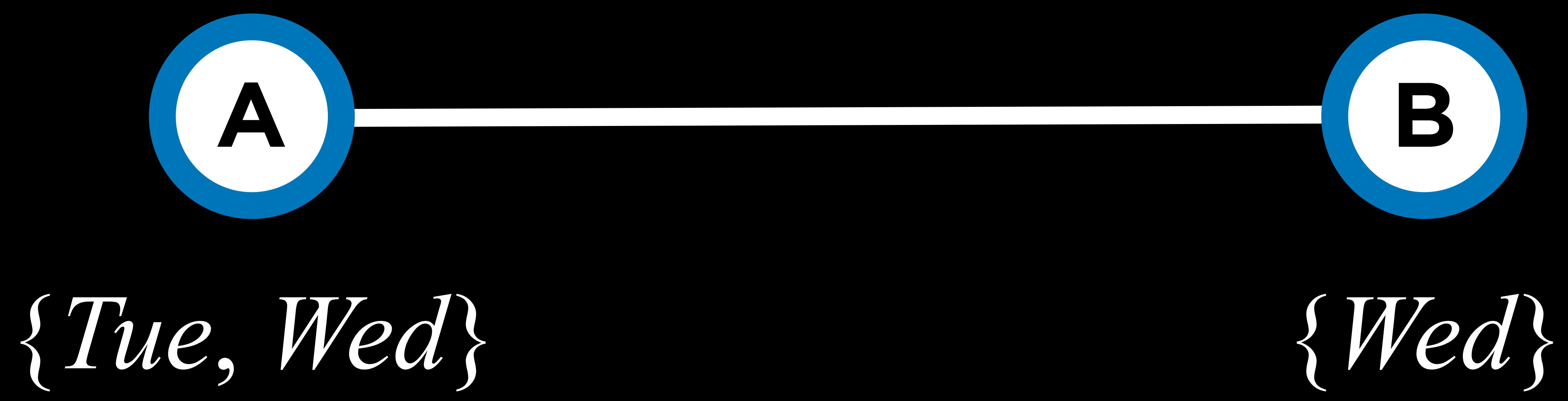
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

arc consistency

when all the values in a variable's domain satisfy the variable's binary constraints

arc consistency

To make X arc-consistent with respect to Y, remove elements from X's domain until every choice for X has a possible choice for Y



$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Wed\}$

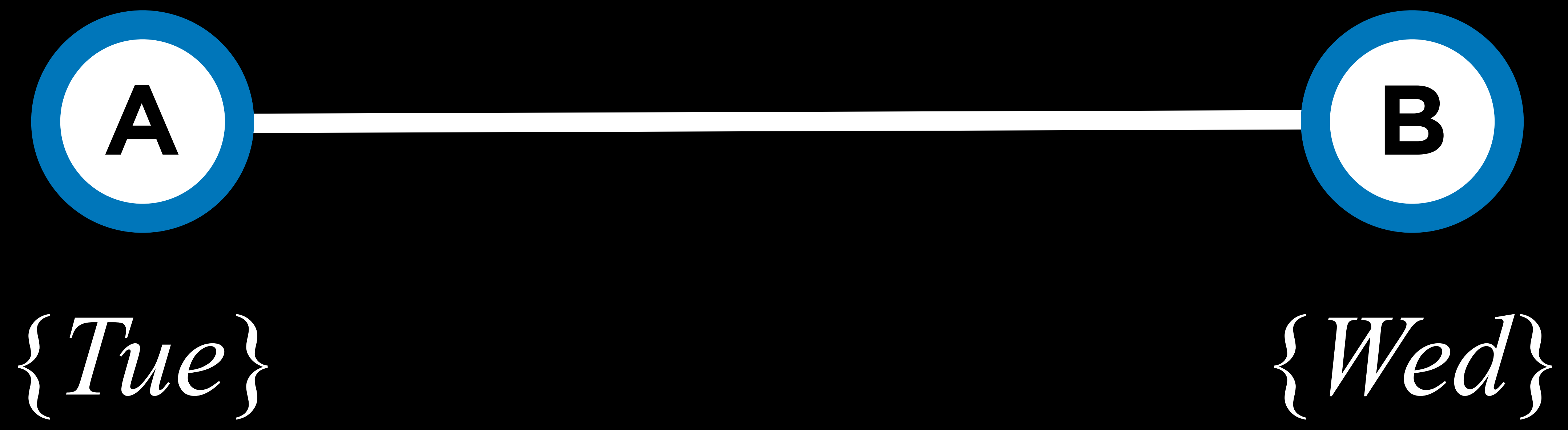
if A is Tue, then B is Wed.
however if A is Wed, then B has no option that doesn't violate arc constraint
A = Wed = B violates constraint. we must remove Wed from A's domain

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue\}$ $\{Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

Resolving node and arc consistency, we have solved the problem



$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

Arc Consistency

constraint satisfaction problem

function REVISE(*csp*, *X*, *Y*):
    revised = *false*
    for *x* in *X.domain*:
        if no *y* in *Y.domain* satisfies constraint for (*X*, *Y*):
            delete *x* from *X.domain*
            revised = *true*
    return *revised*

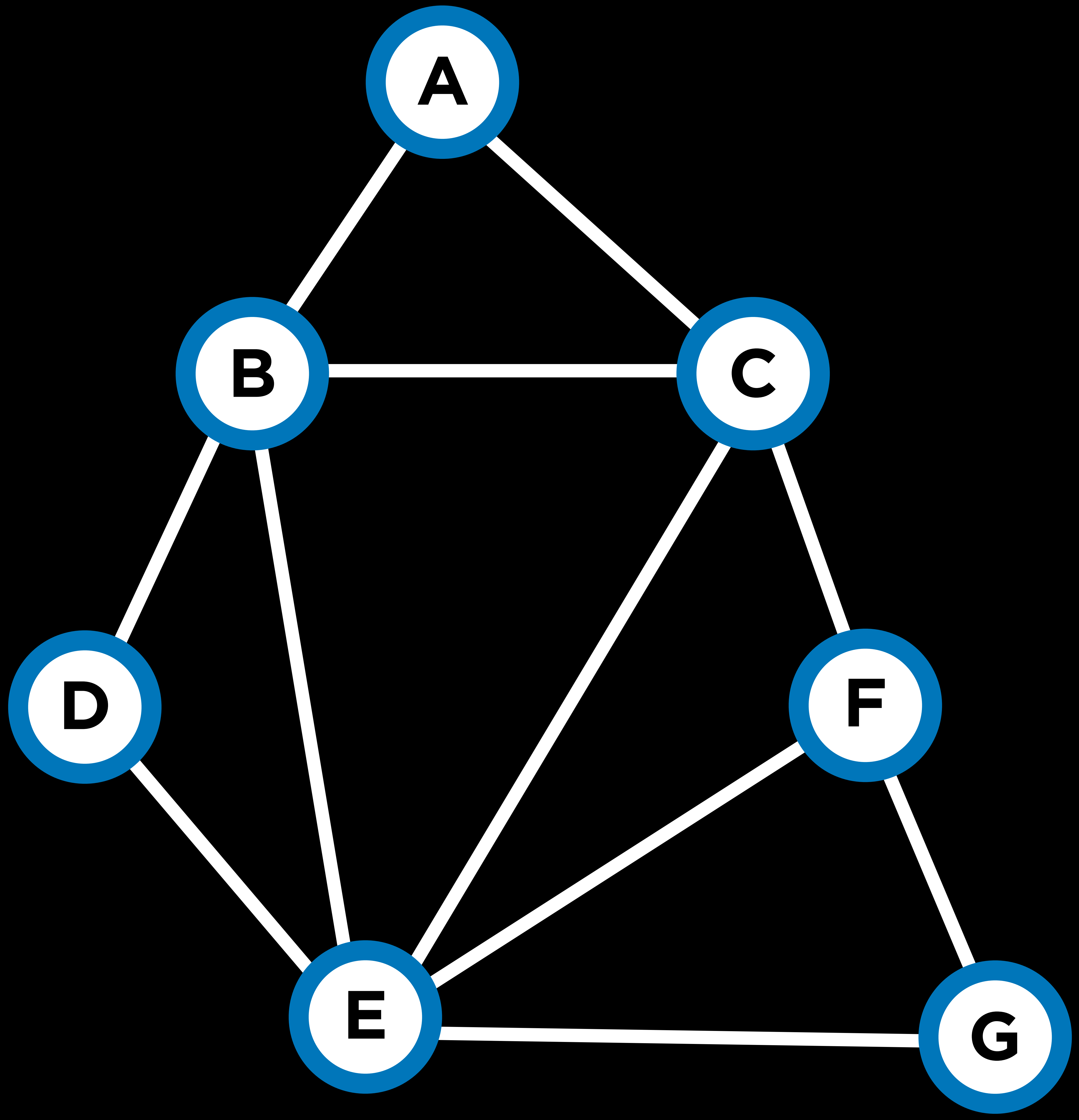
remove conflicting values of x so X is arc consistent wrt Y

we must do integrity check in case some other arc consistency depended on x

Arc Consistency

function AC-3(*csp*):
  queue = all arcs in *csp* ie queue up all edges
  while *queue* non-empty:
    (*X*, *Y*) = DEQUEUE(*queue*) pop edge
    if REVISE(*csp*, *X*, *Y*):
      if size of *X.domain* == 0:
        return *false* no possible solution exists
      for each *Z* in *X.neighbors* - {*Y*}:
        ENQUEUE(*queue*, (*Z*, *X*))
  return *true*

except Y since it is currently being handled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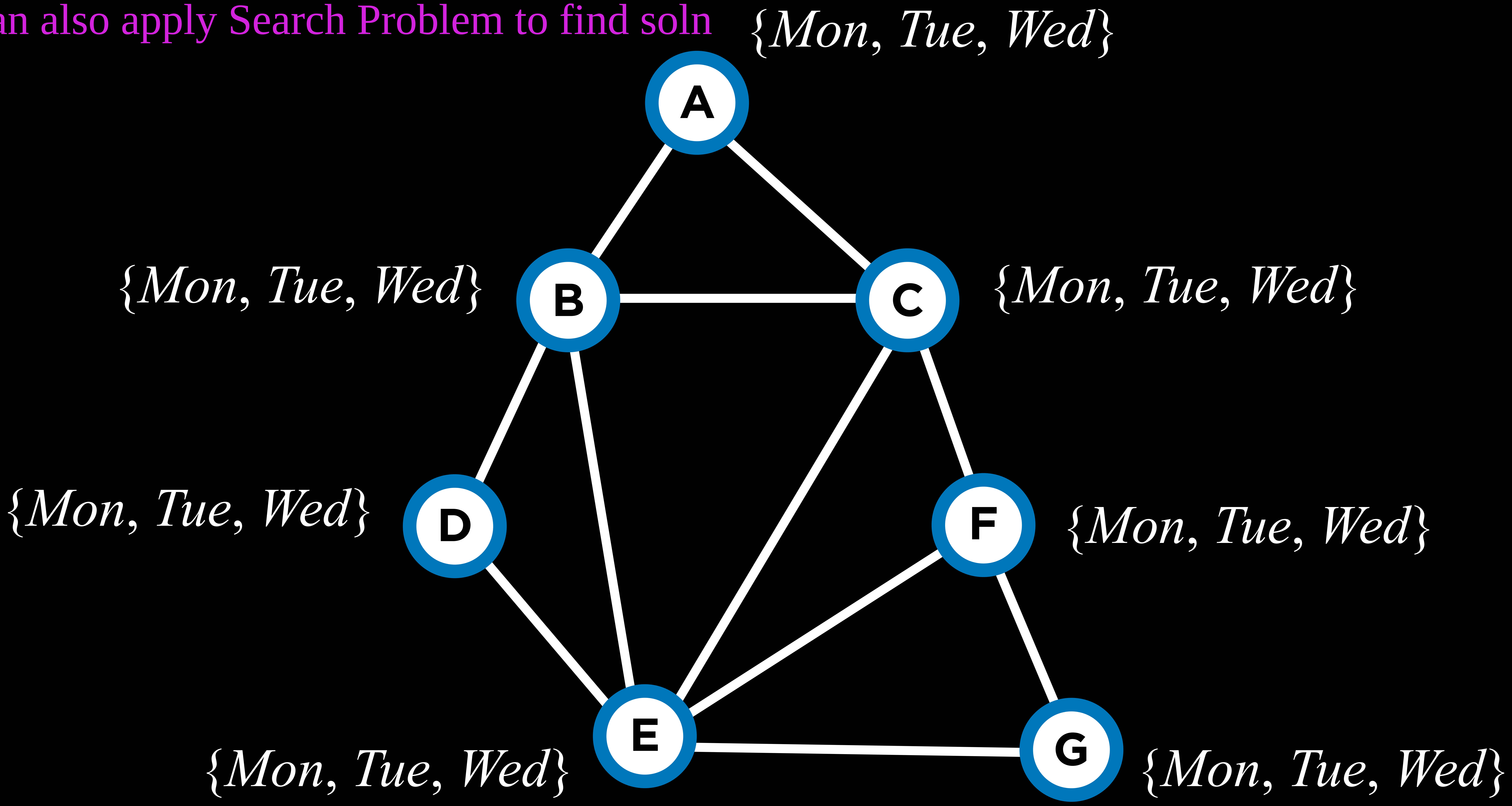
this problem is already Arc Consistent
we can also apply Search Problem to find soln



Search Problems

- initial state
- actions
- transition model
- goal test
- path cost function

CSPs as Search Problems

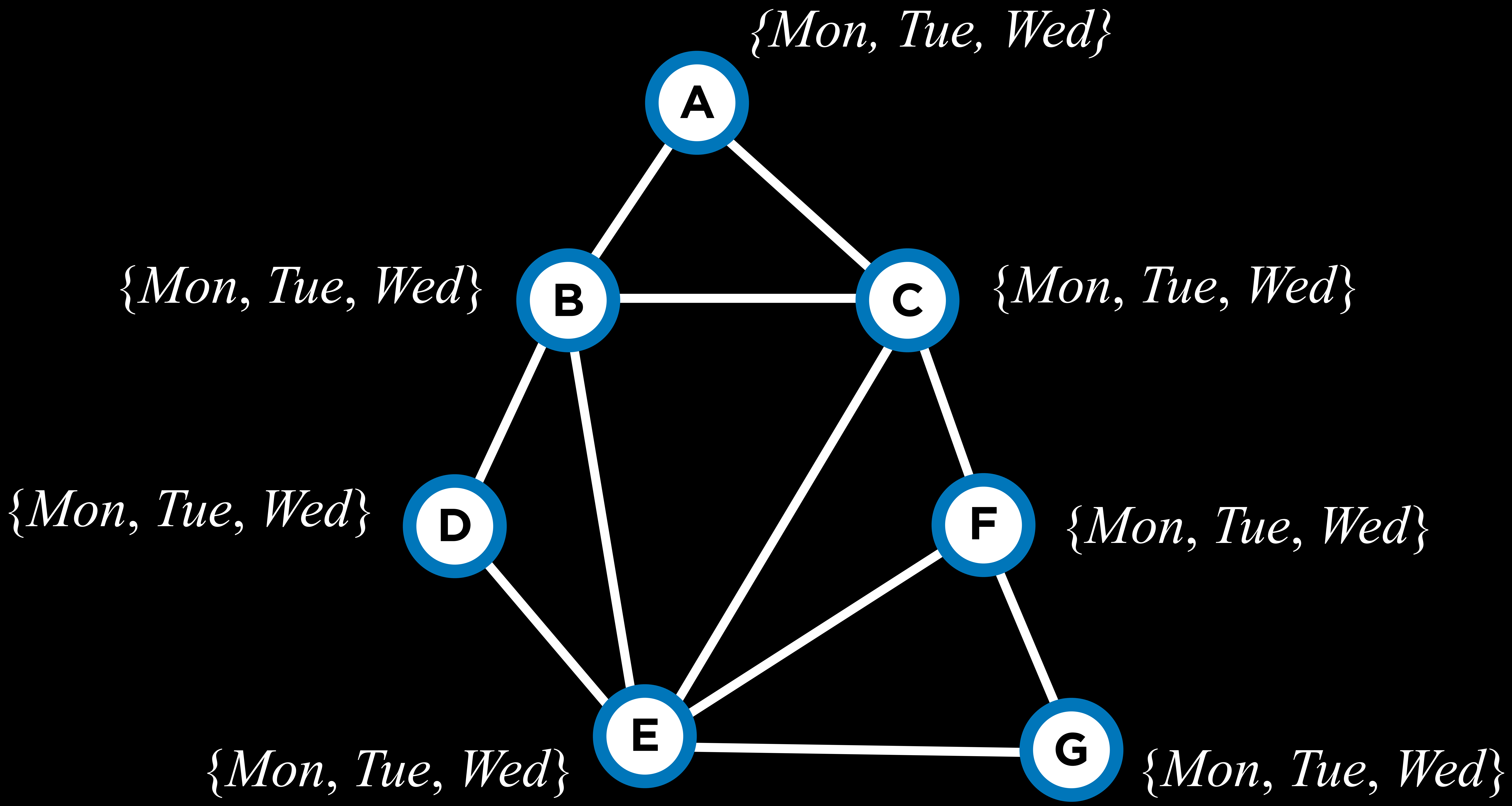
- initial state: empty assignment (no variables)
- actions: add a $\{variable = value\}$ to assignment
- transition model: shows how adding an assignment changes the assignment
- goal test: check if all variables assigned and constraints all satisfied
- path cost function: all paths have same cost

if we ever get stuck, ie no forward progress can be made
we backtrack and try something else

Backtracking Search

Backtracking Search

```
function BACKTRACK(assignment, csp):
    if assignment complete: return assignment  base
    var = SELECT-UNASSIGNED-VAR(assignment, csp)
    for value in DOMAIN-VALUES(var, assignment, csp):
        if value consistent with assignment:
            add {var = value} to assignment
            result = BACKTRACK(assignment, csp)  recursion for consistency
            if result ≠ failure: return result  accept consistent result
        remove {var = value} from assignment  dead-end. must backtrack
    return failure
```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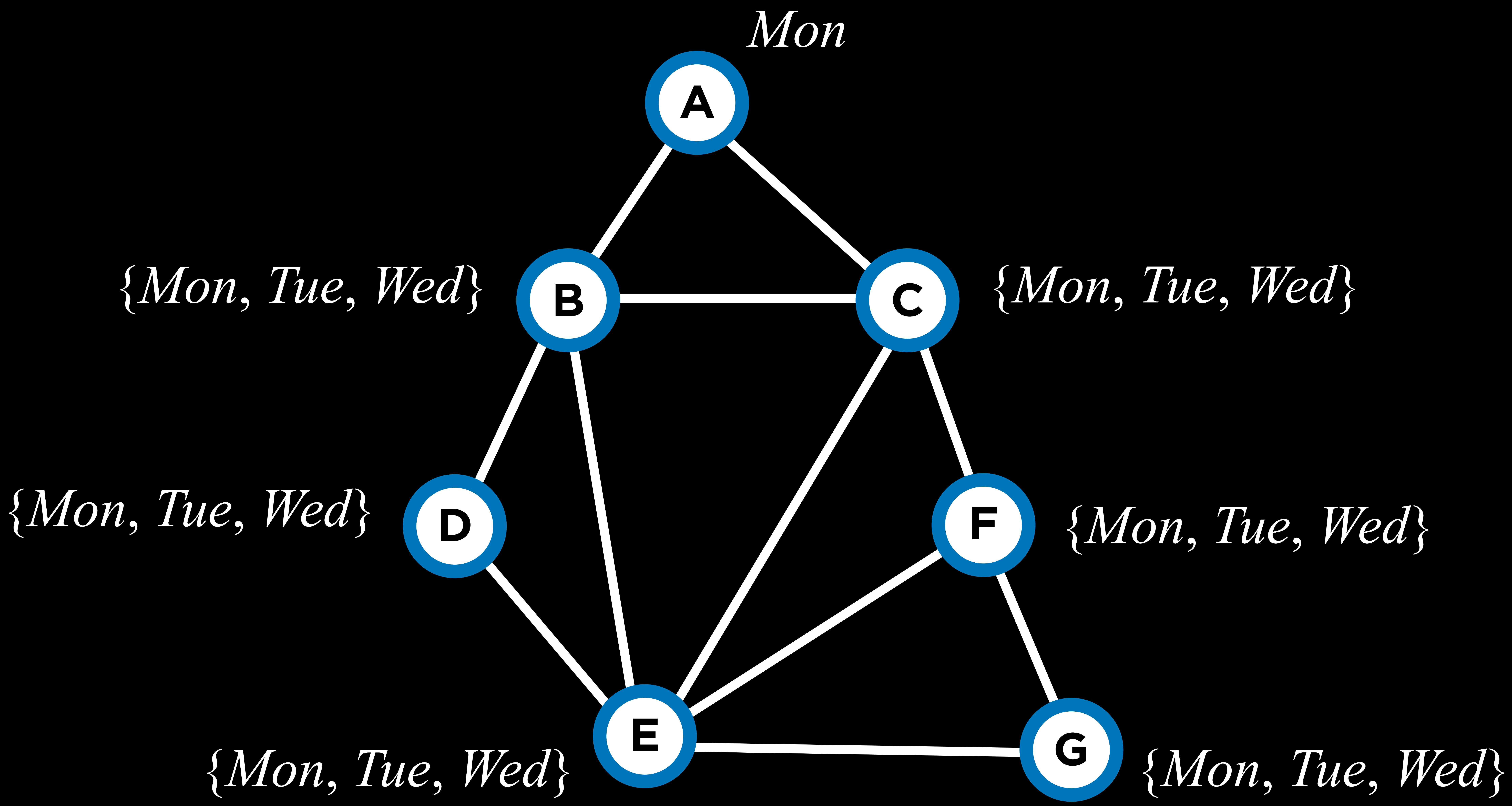
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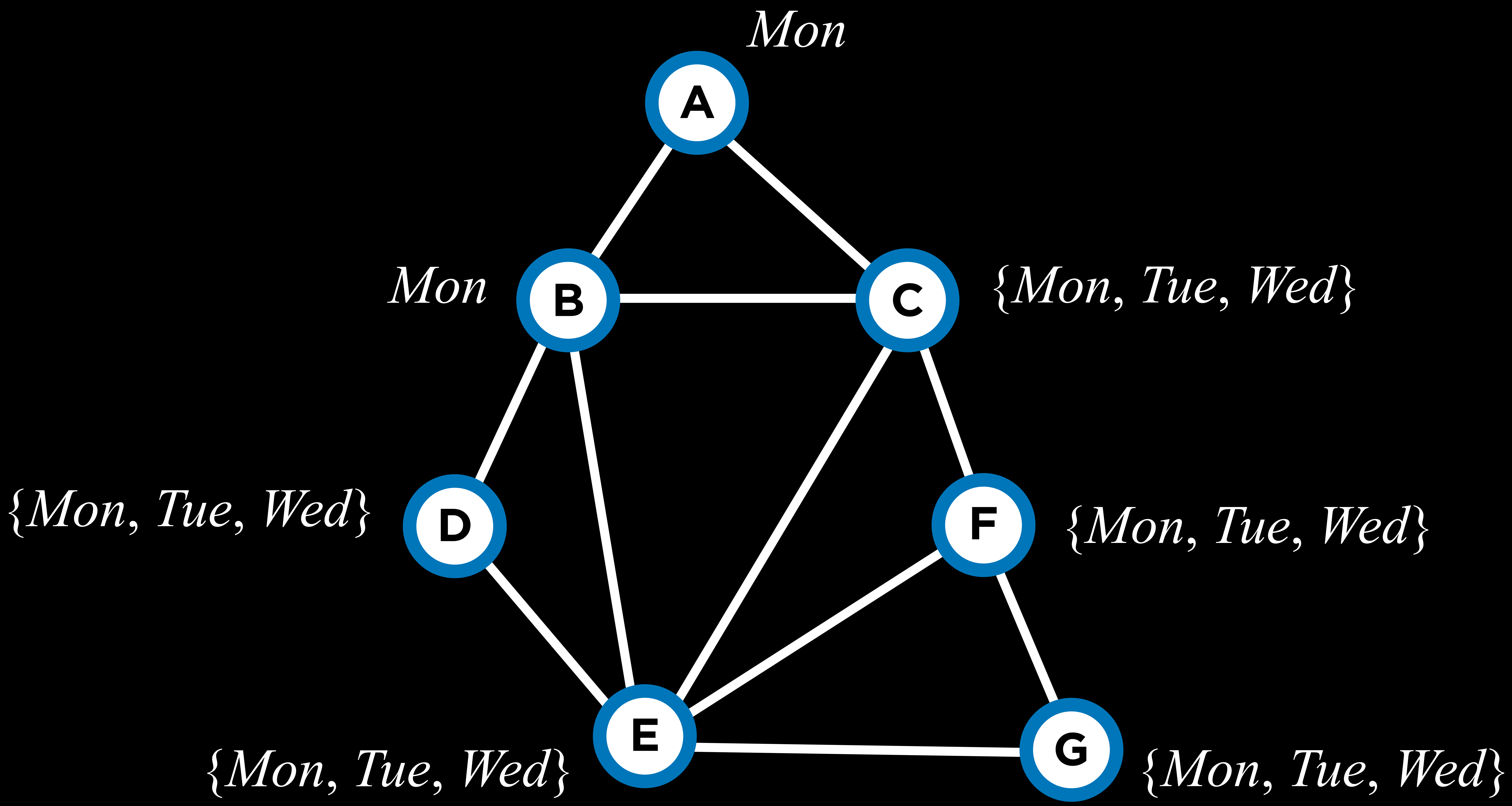
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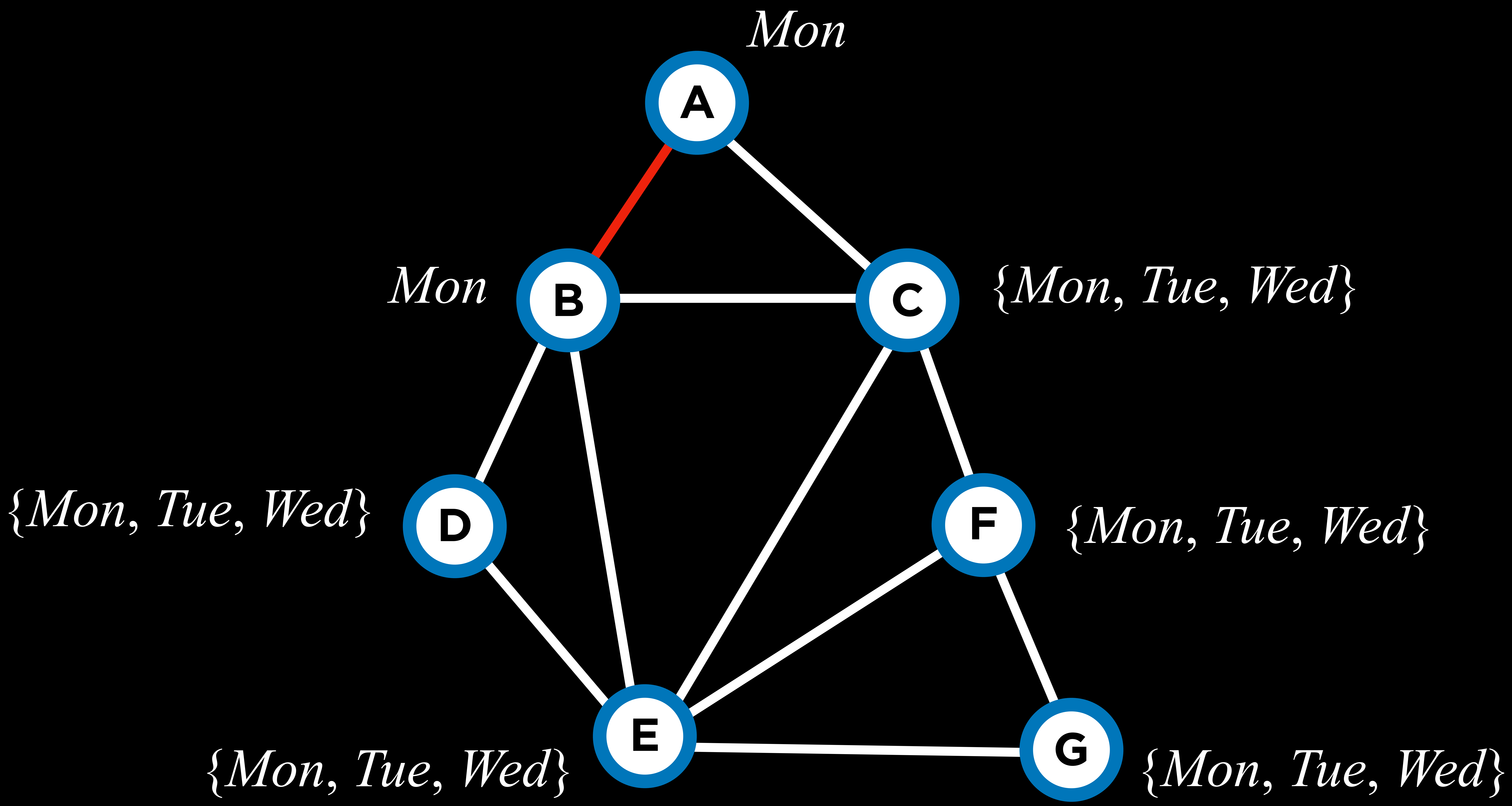
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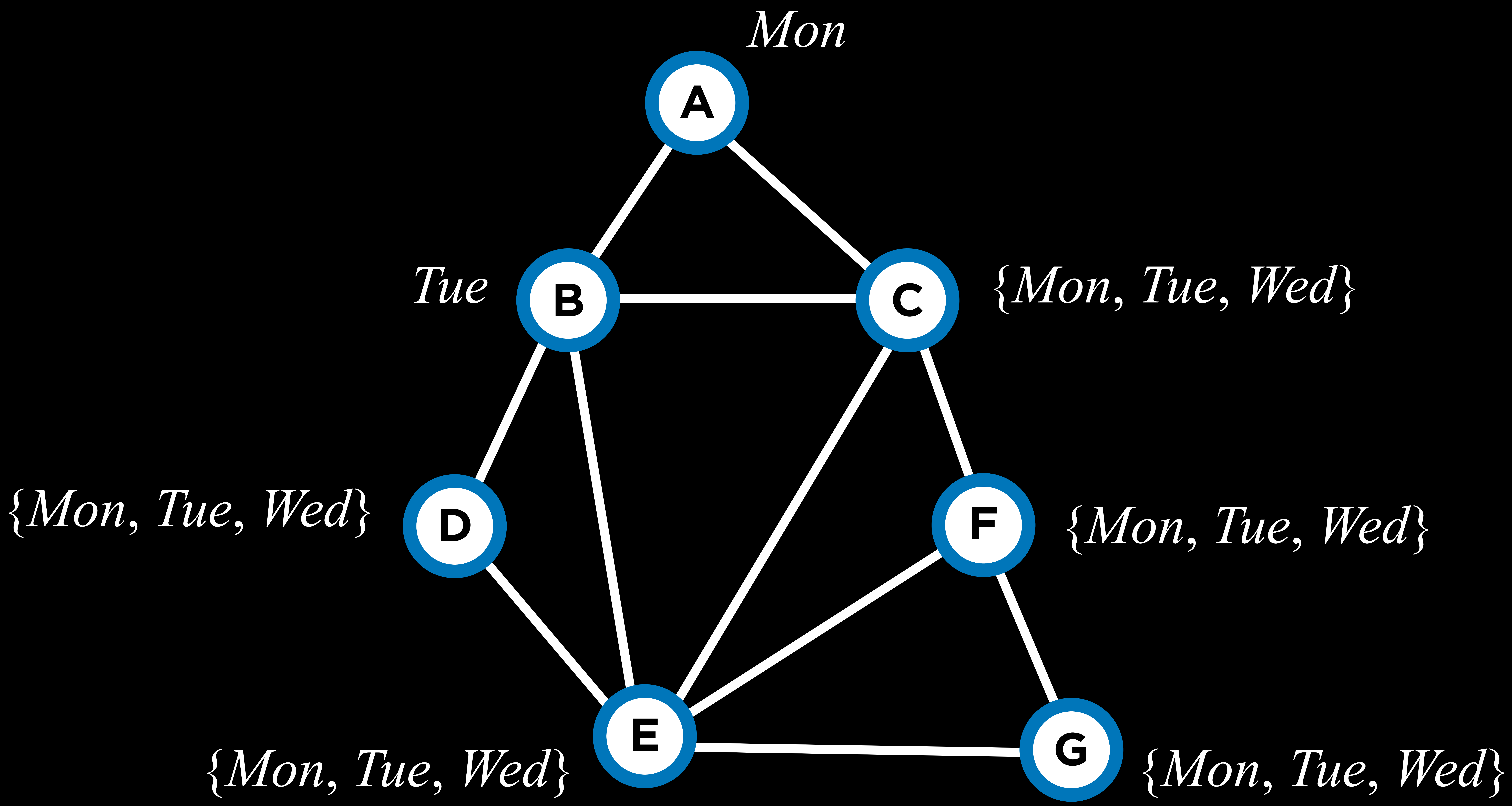
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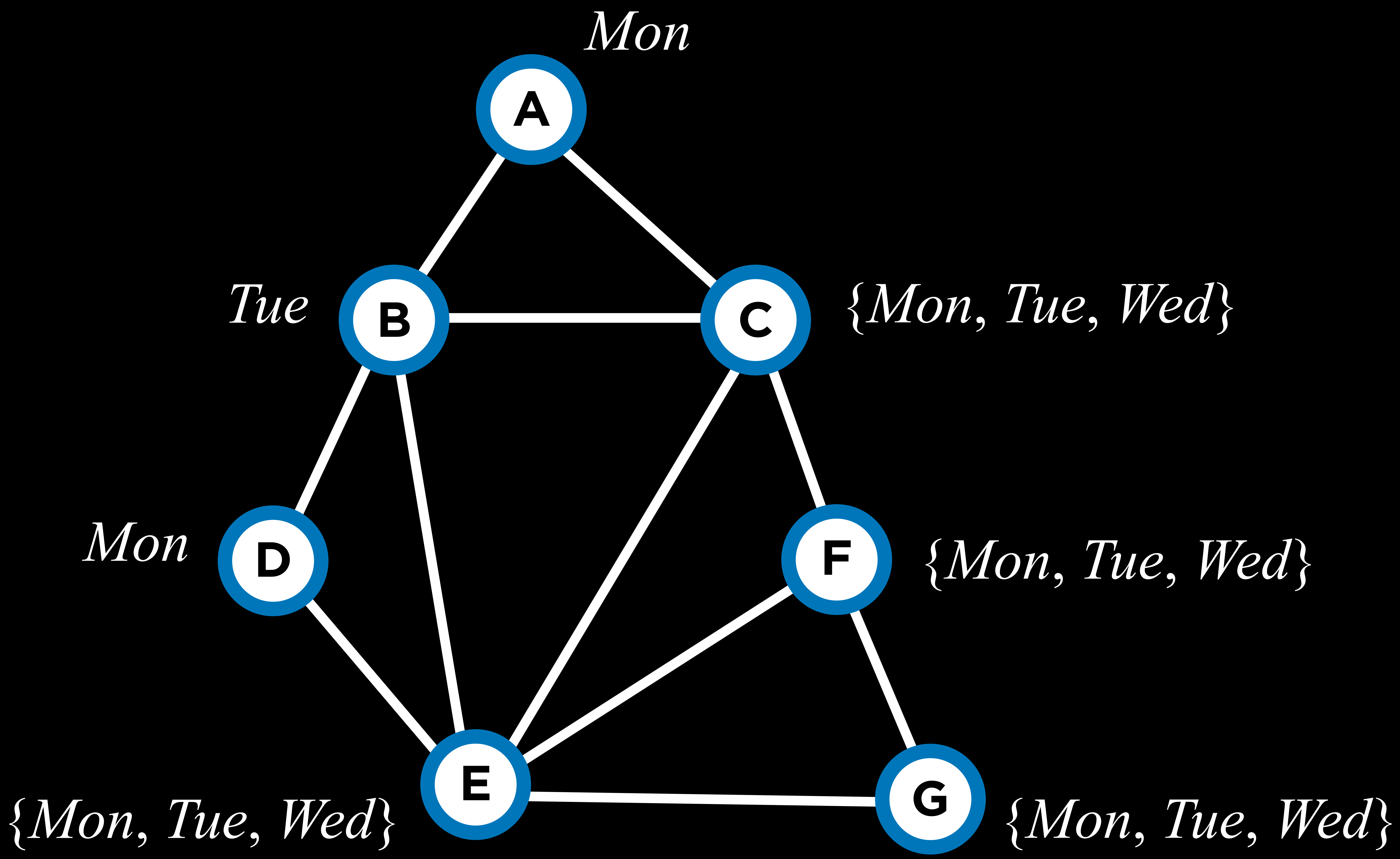
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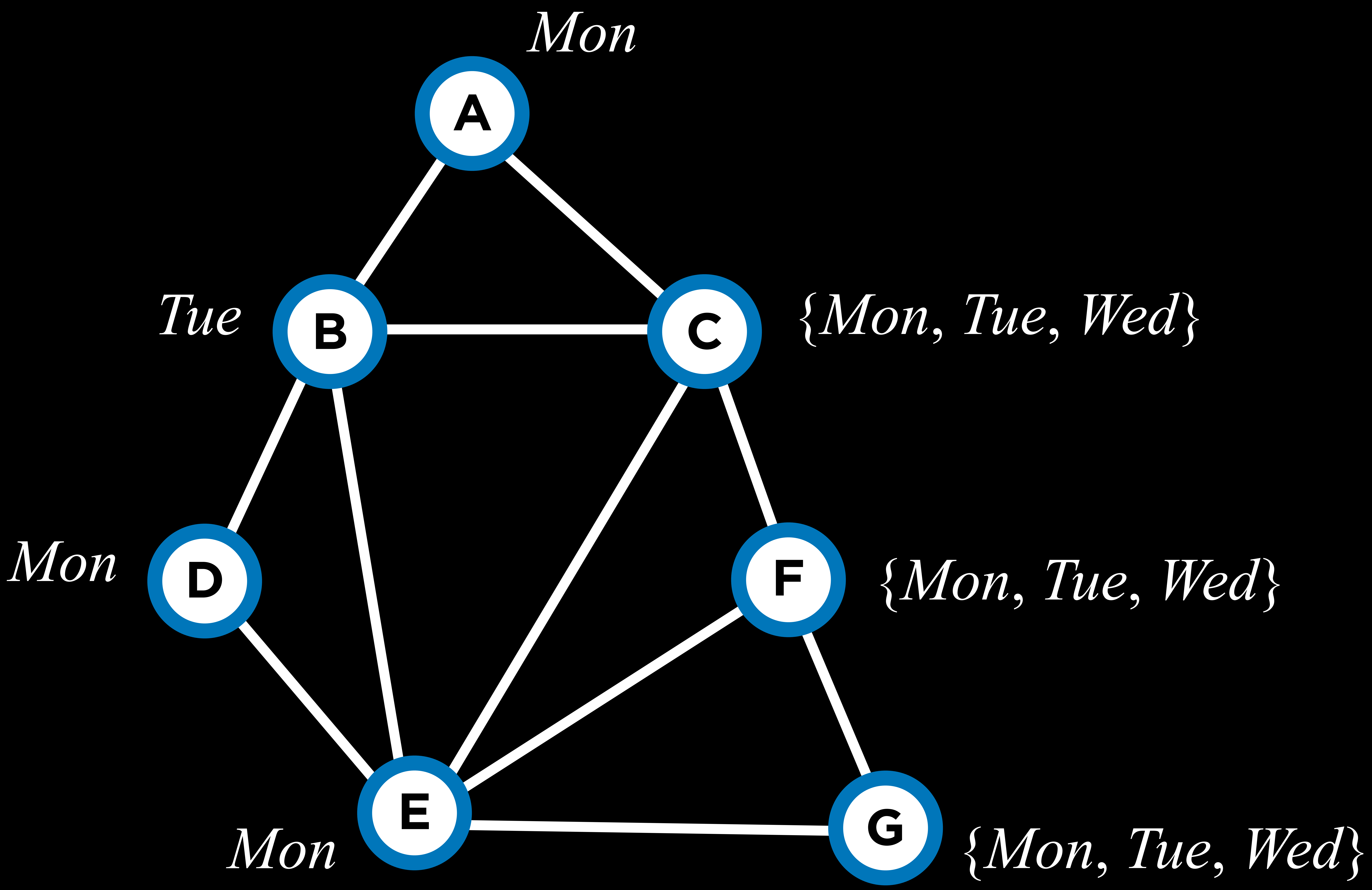

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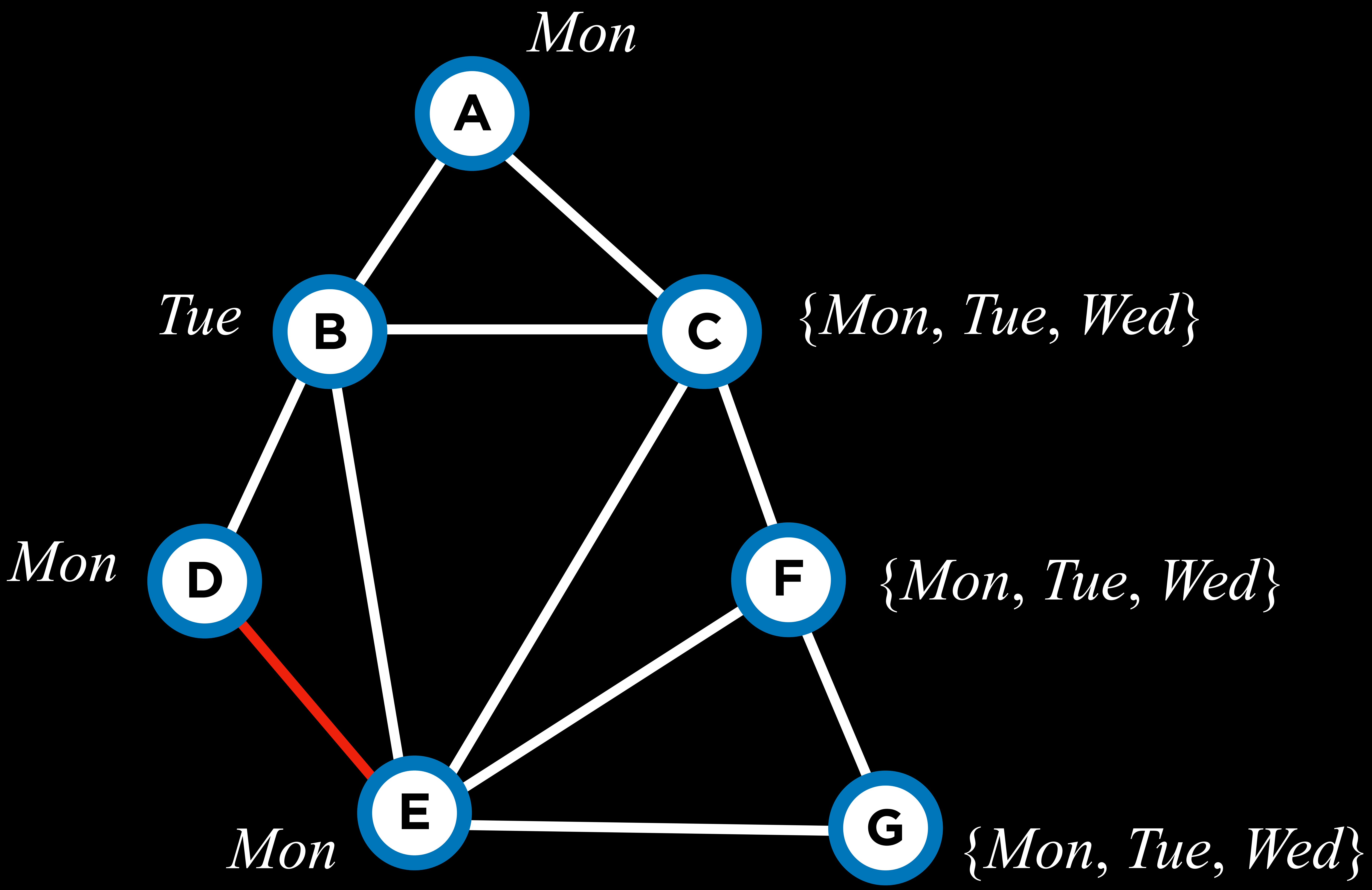
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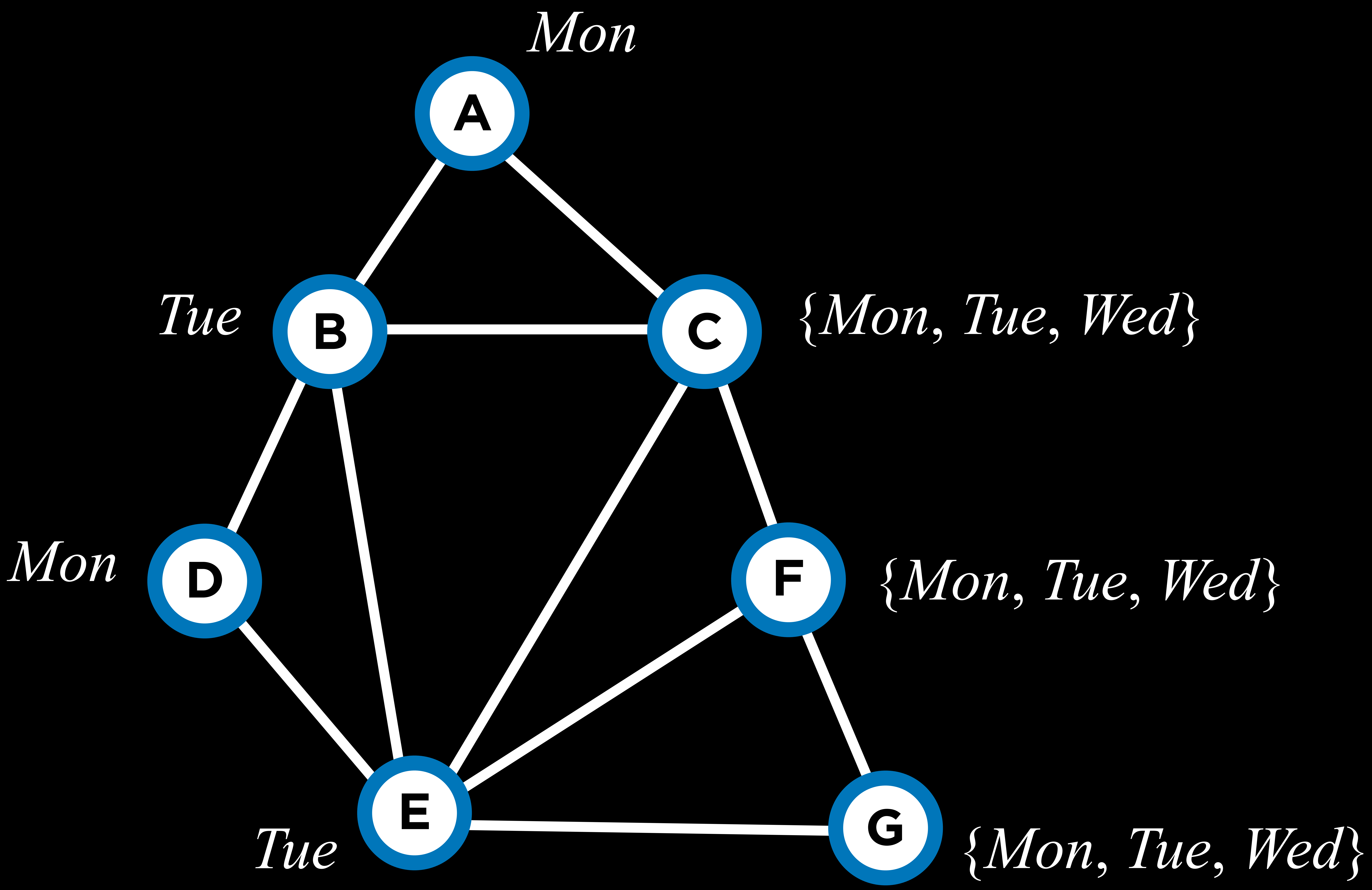
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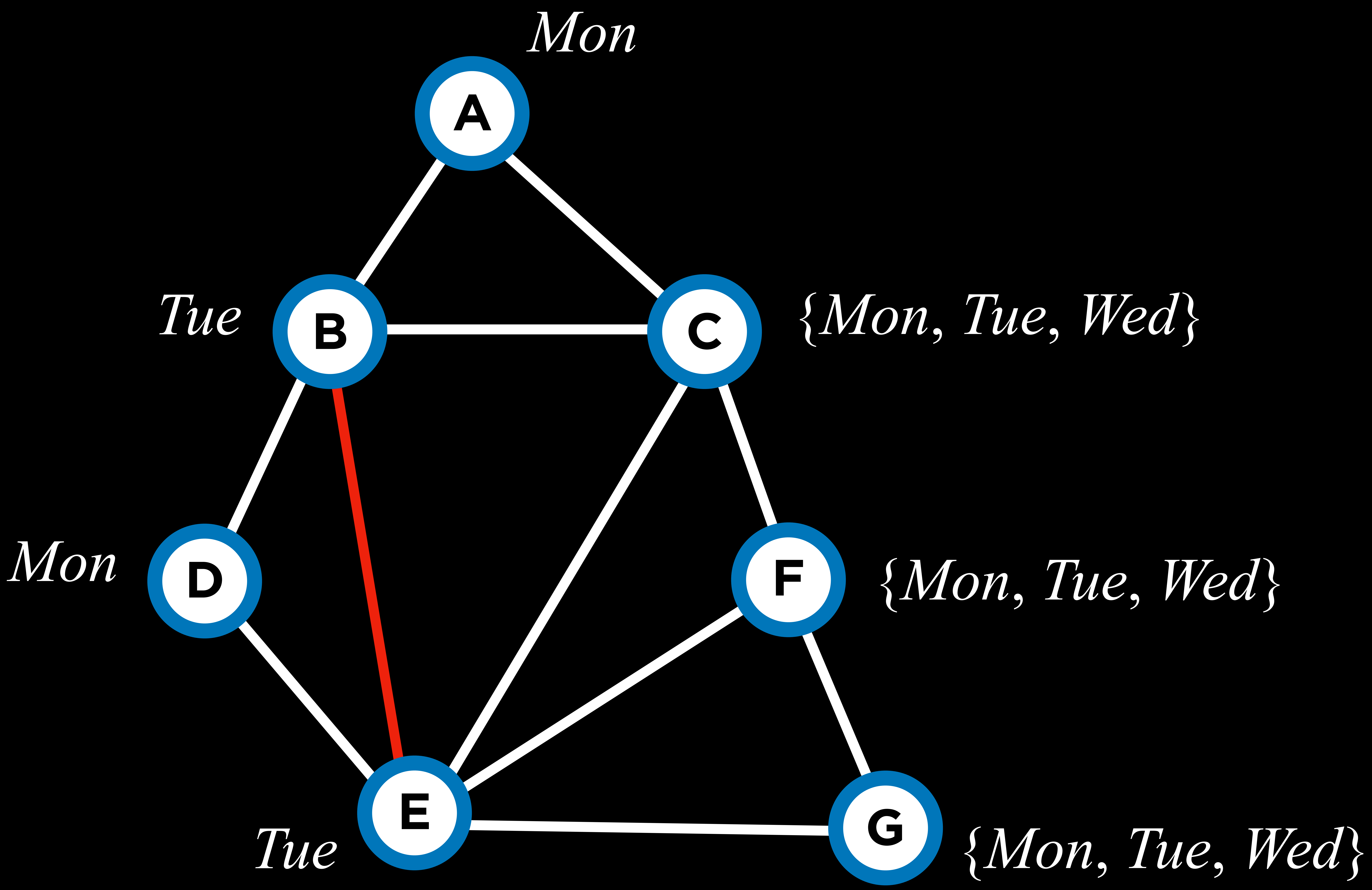

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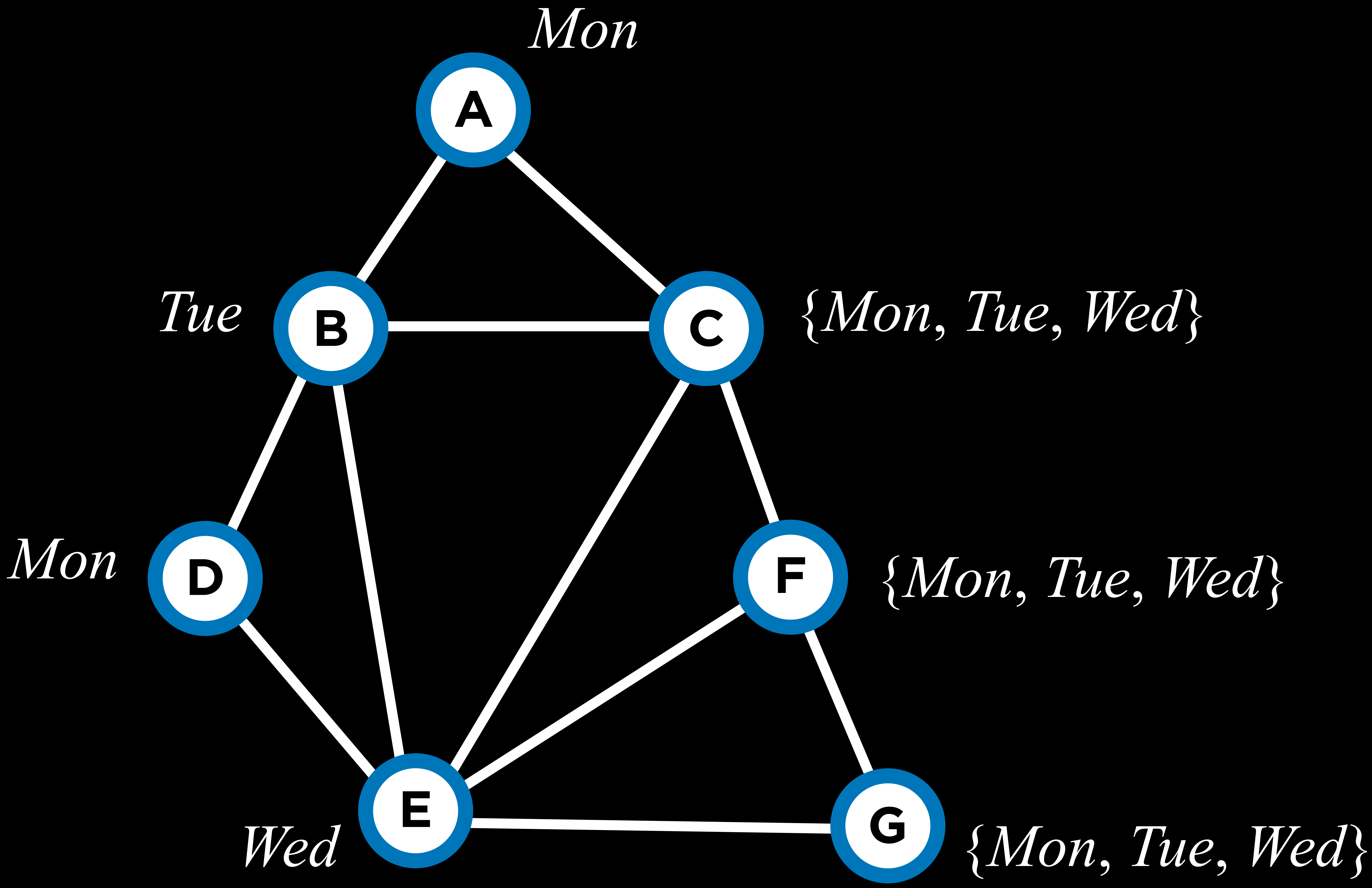
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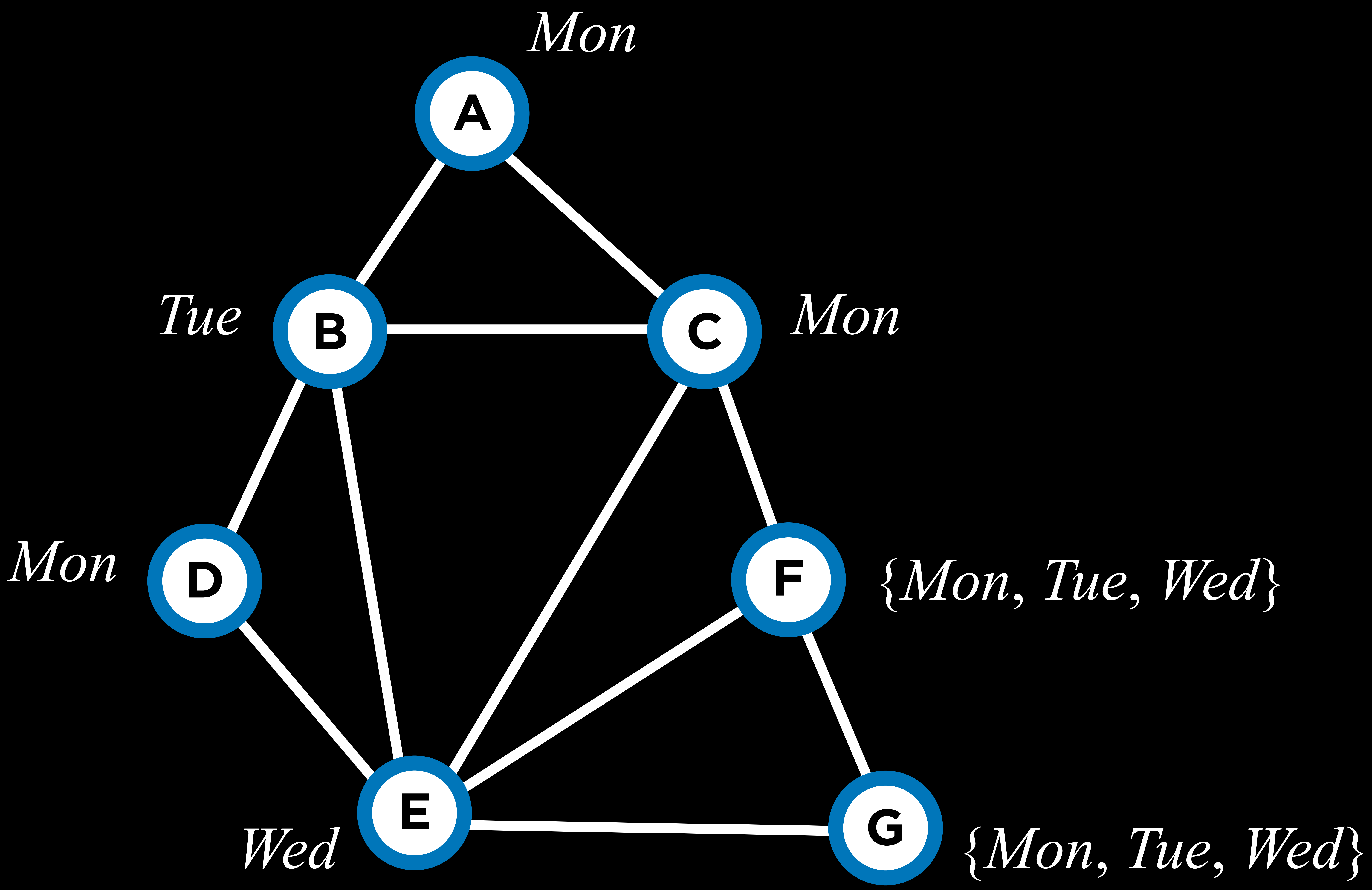
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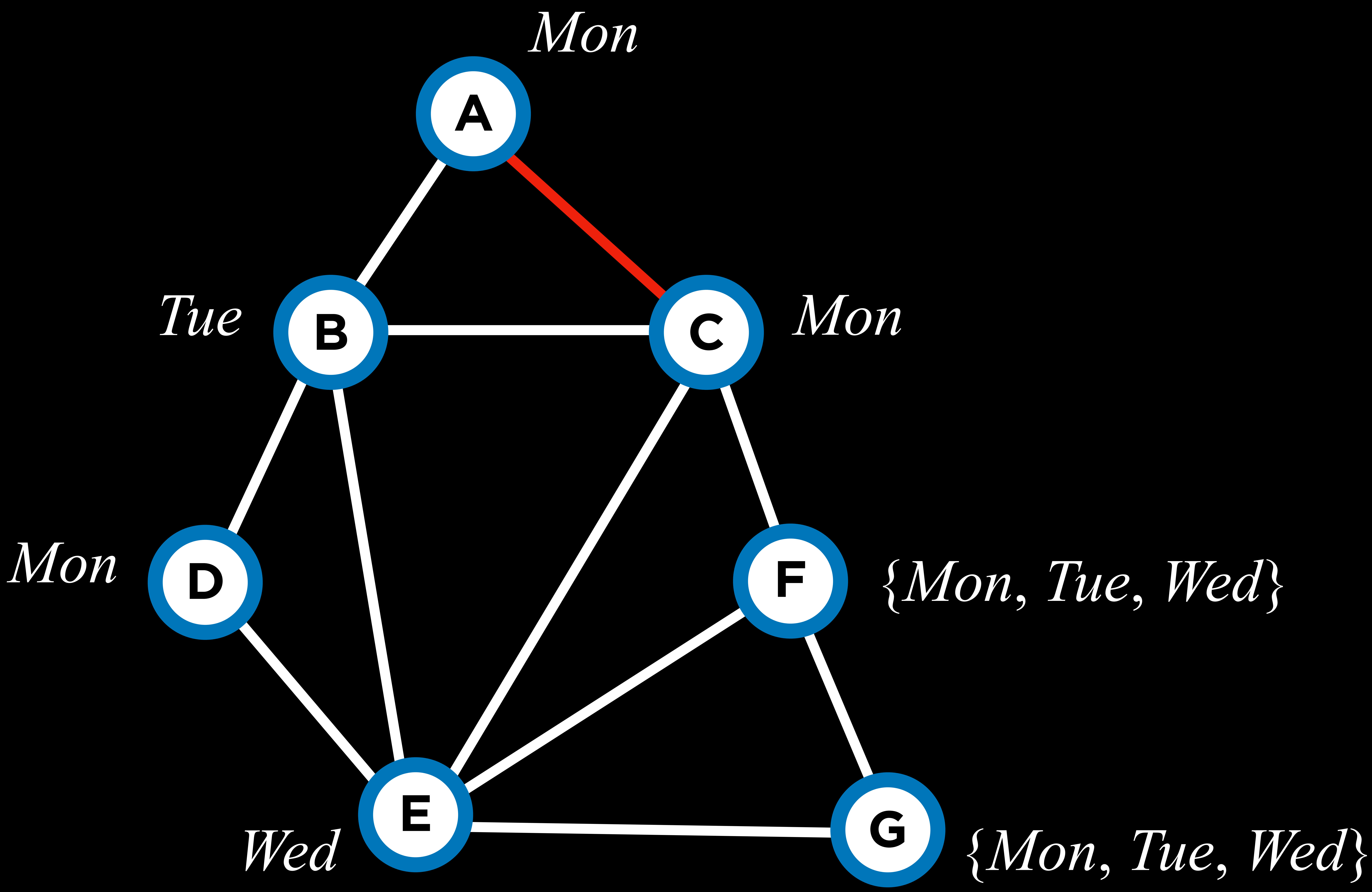

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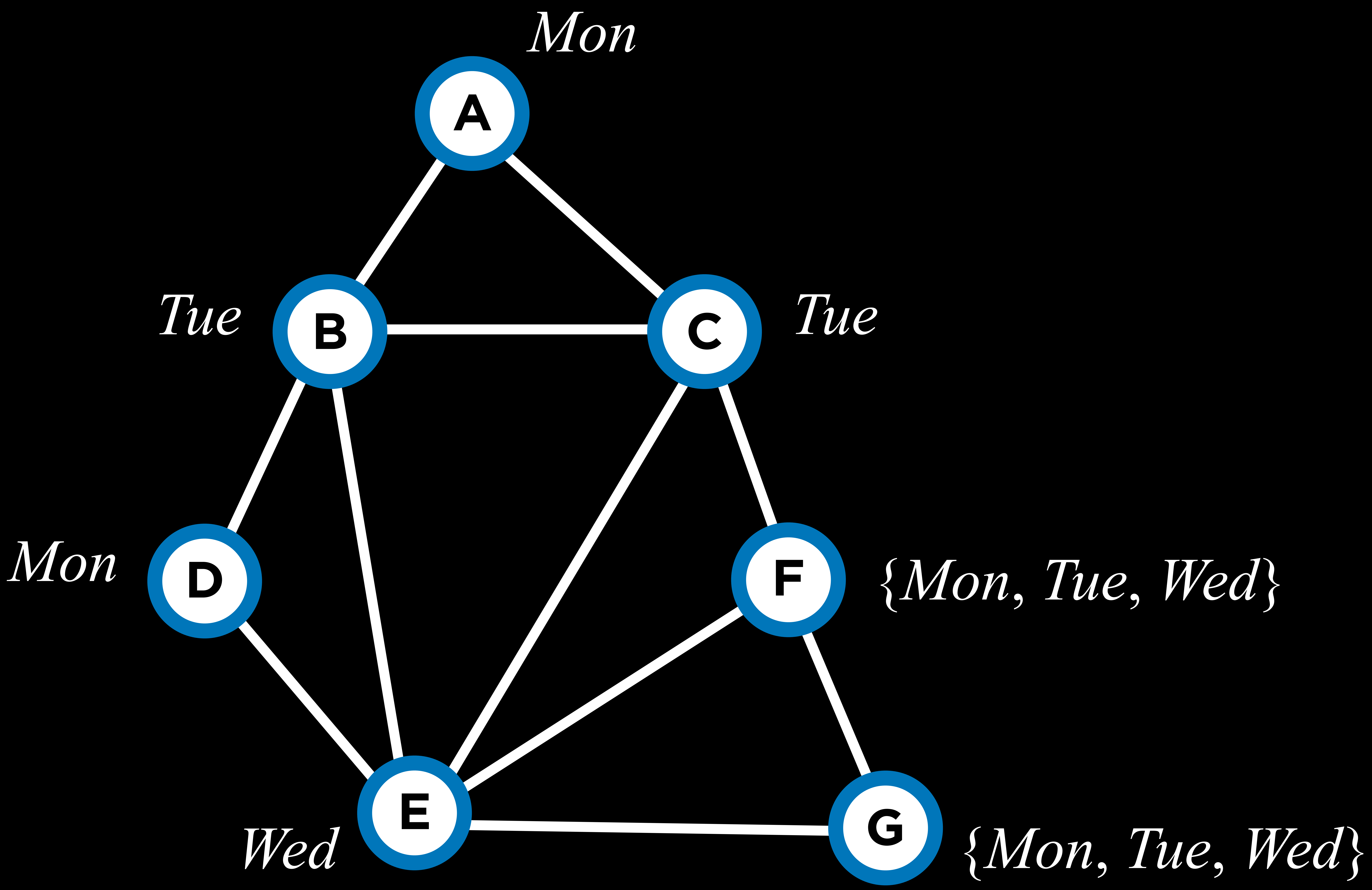
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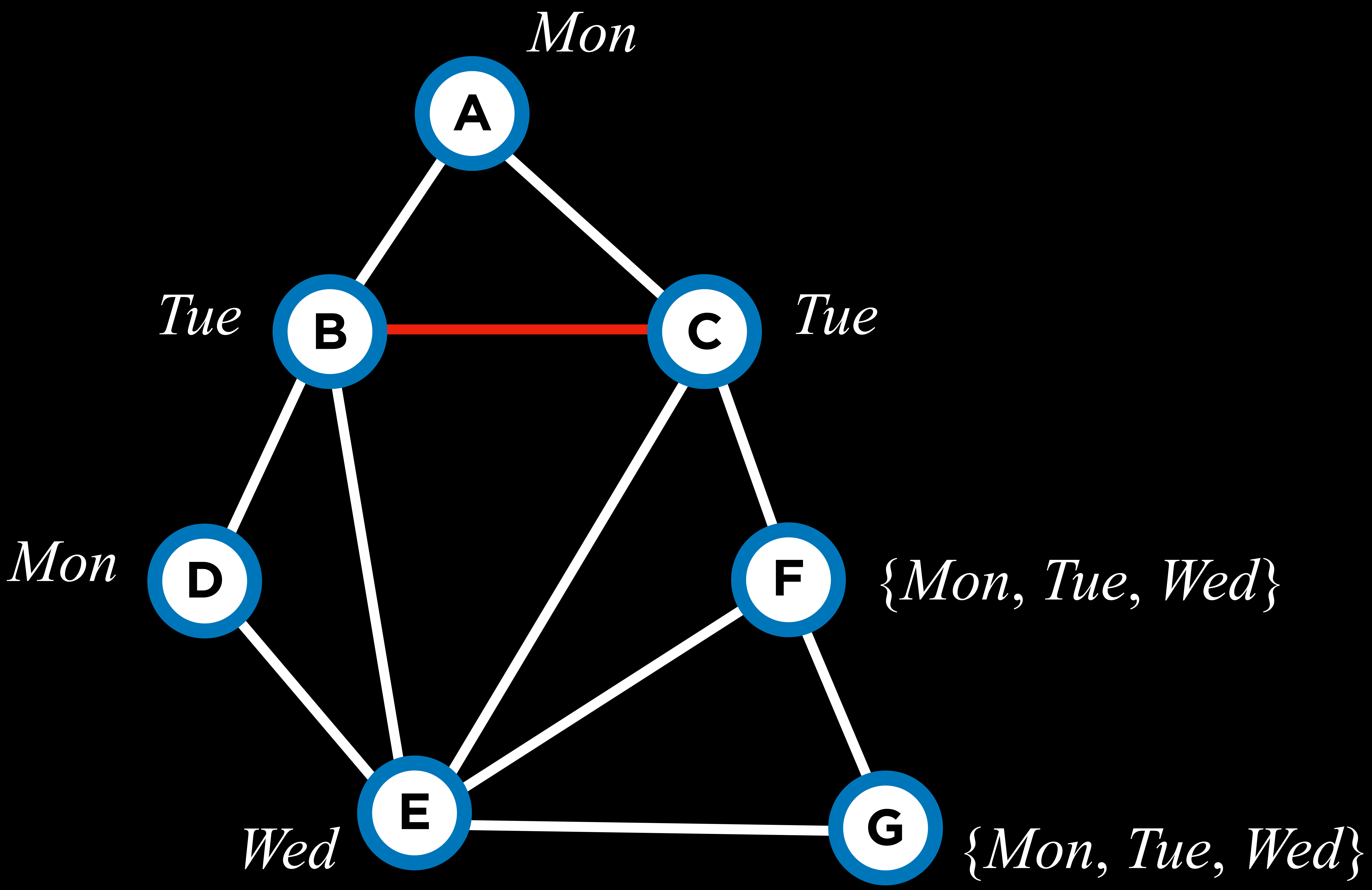

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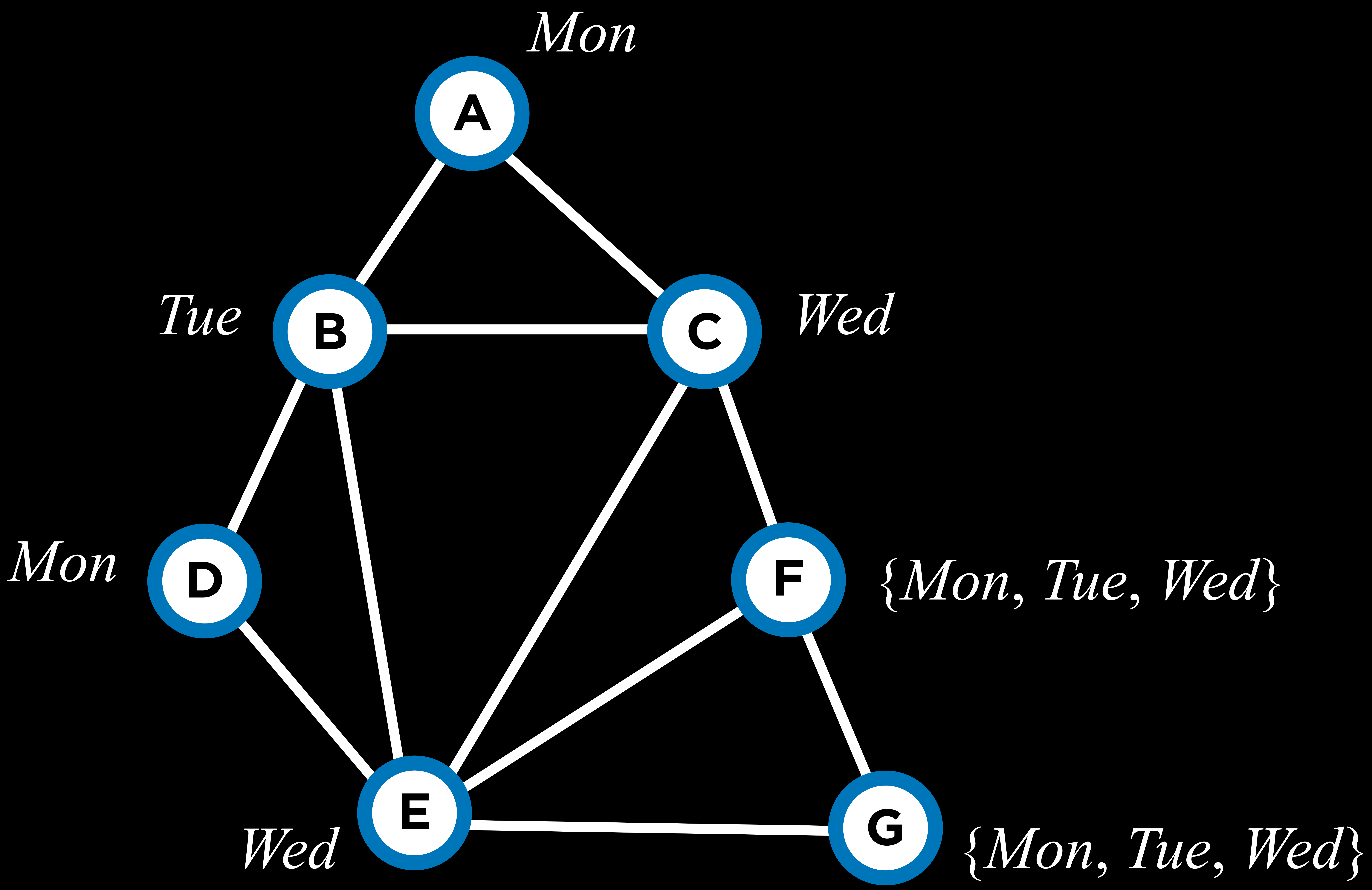
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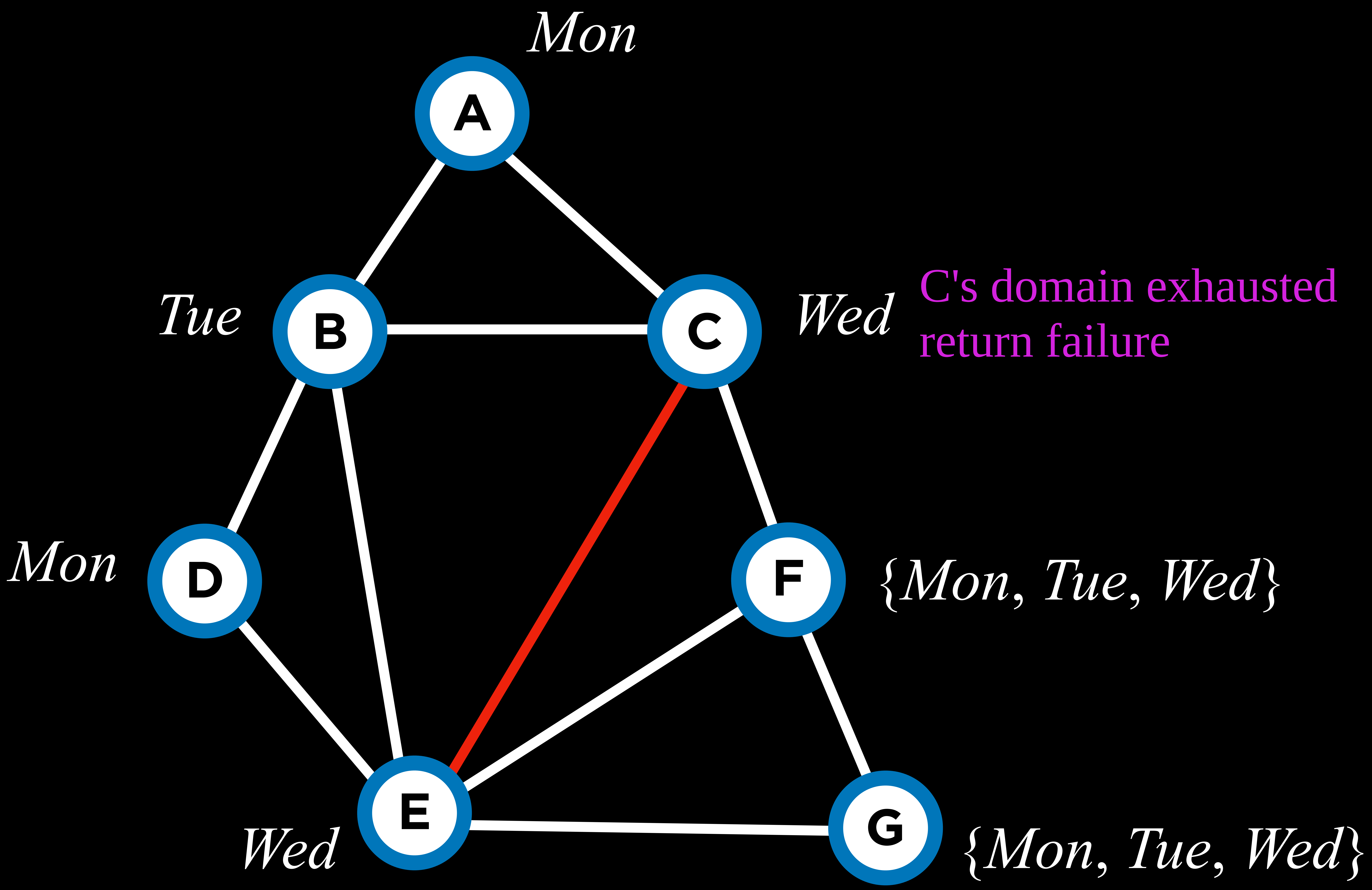
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C's domain exhausted
return failure
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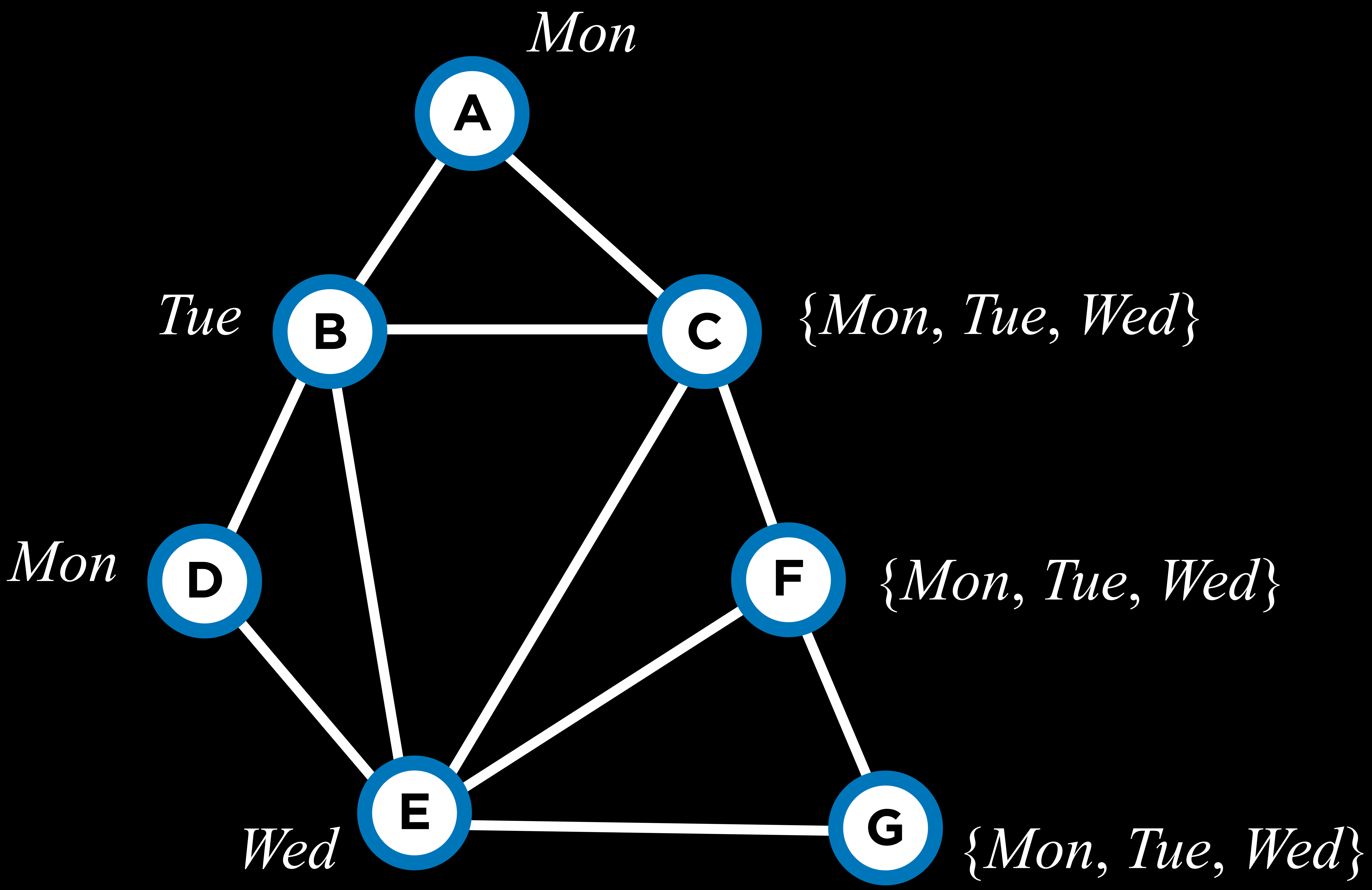
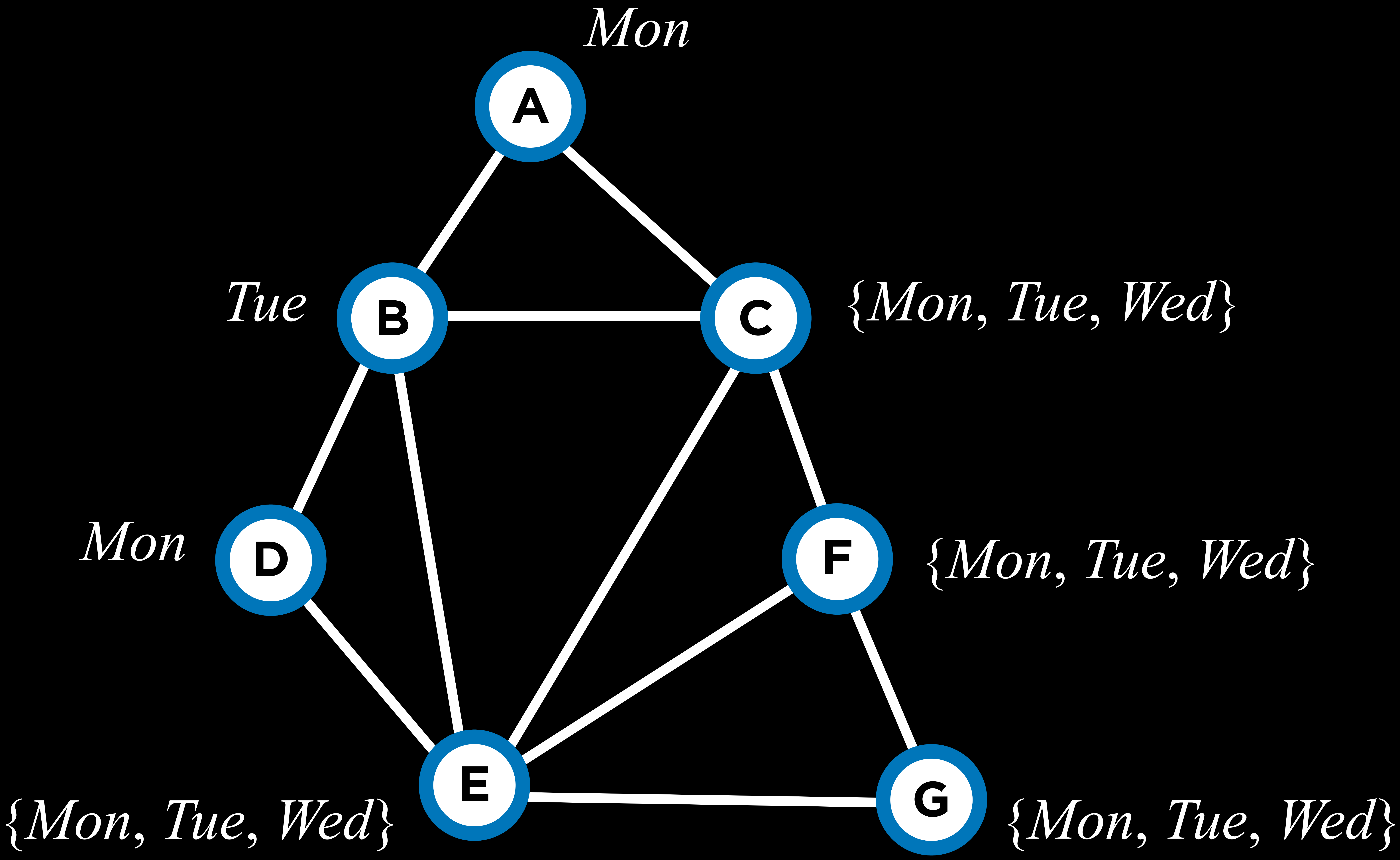


E's domain exhausted
return failure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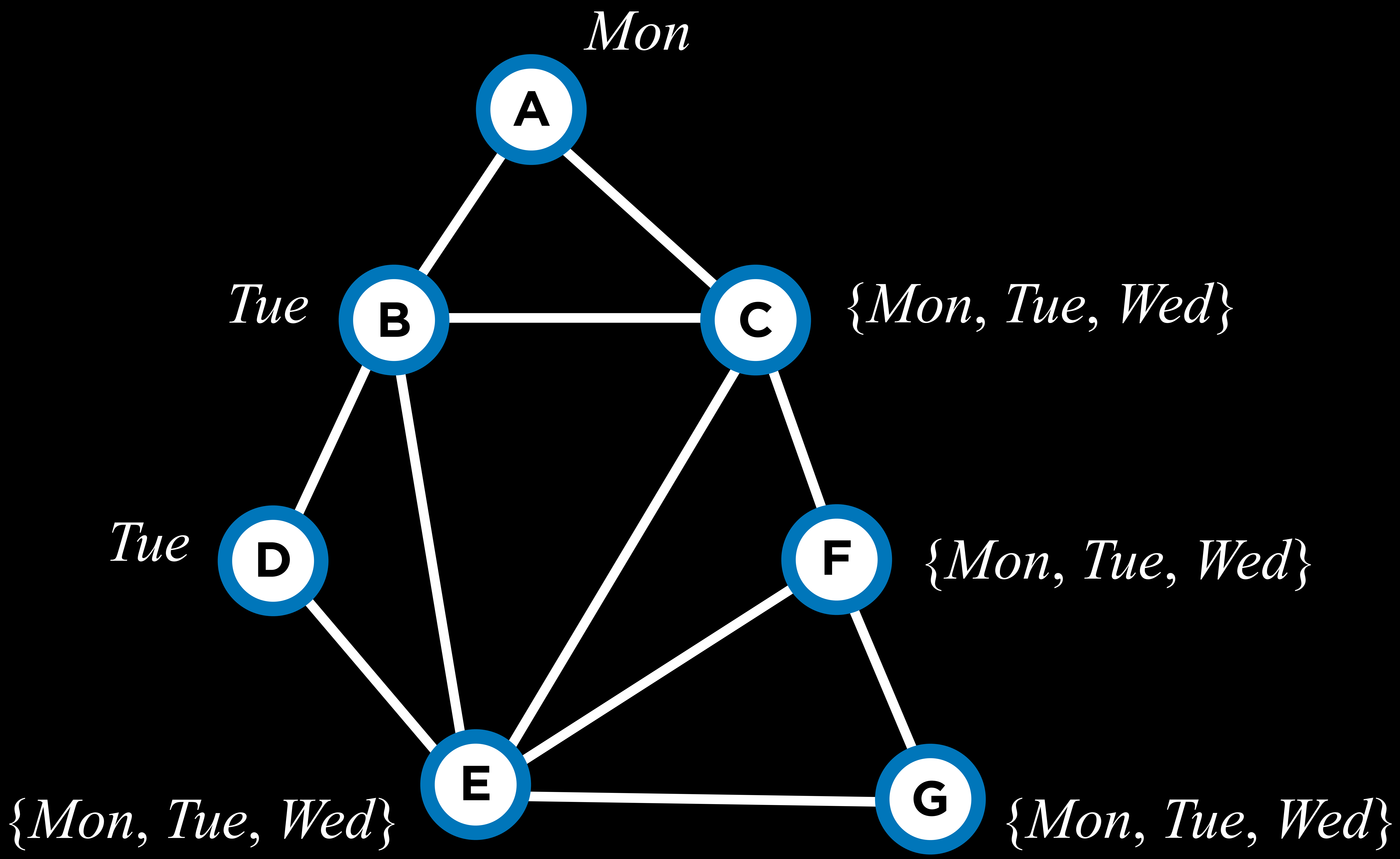
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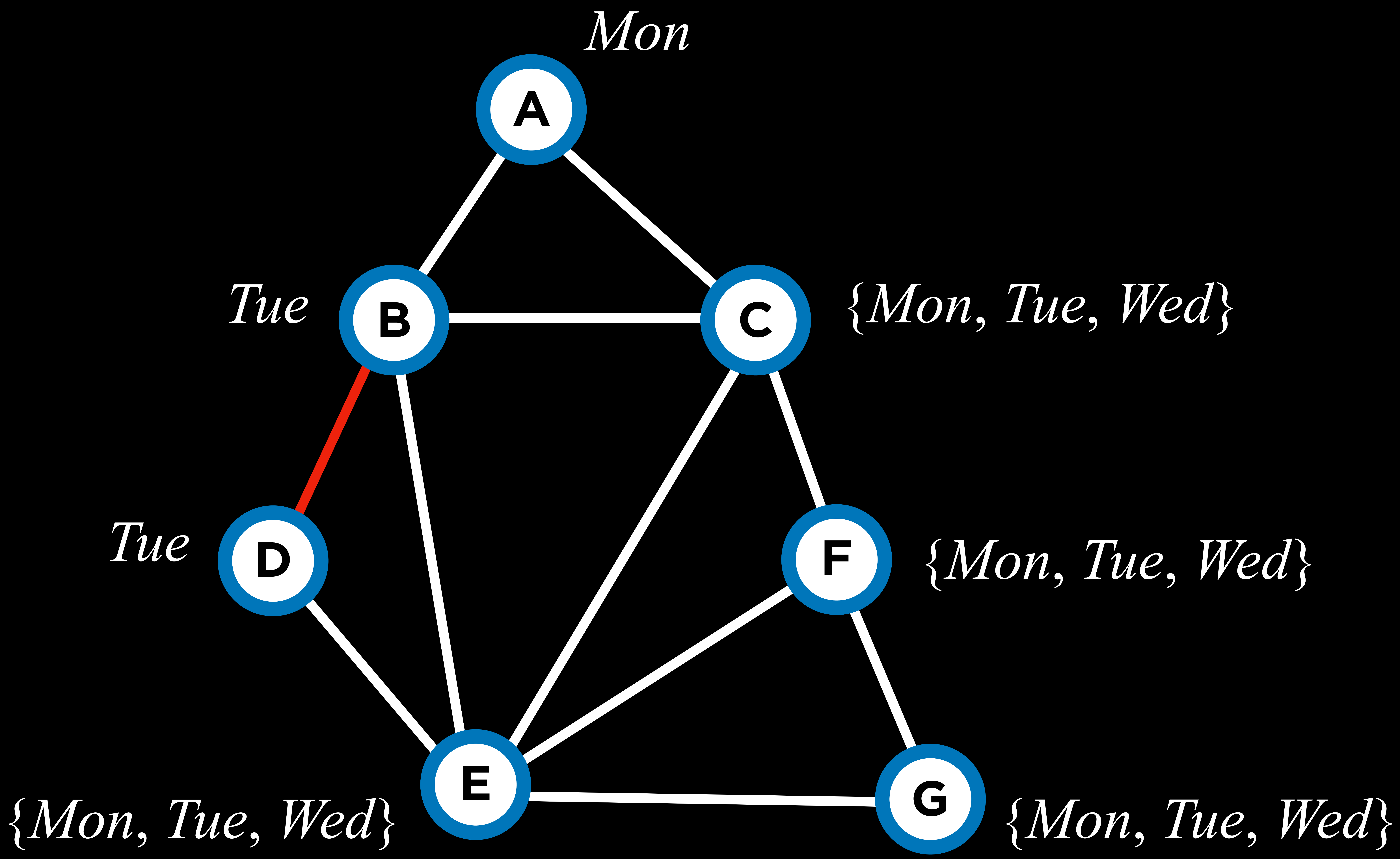
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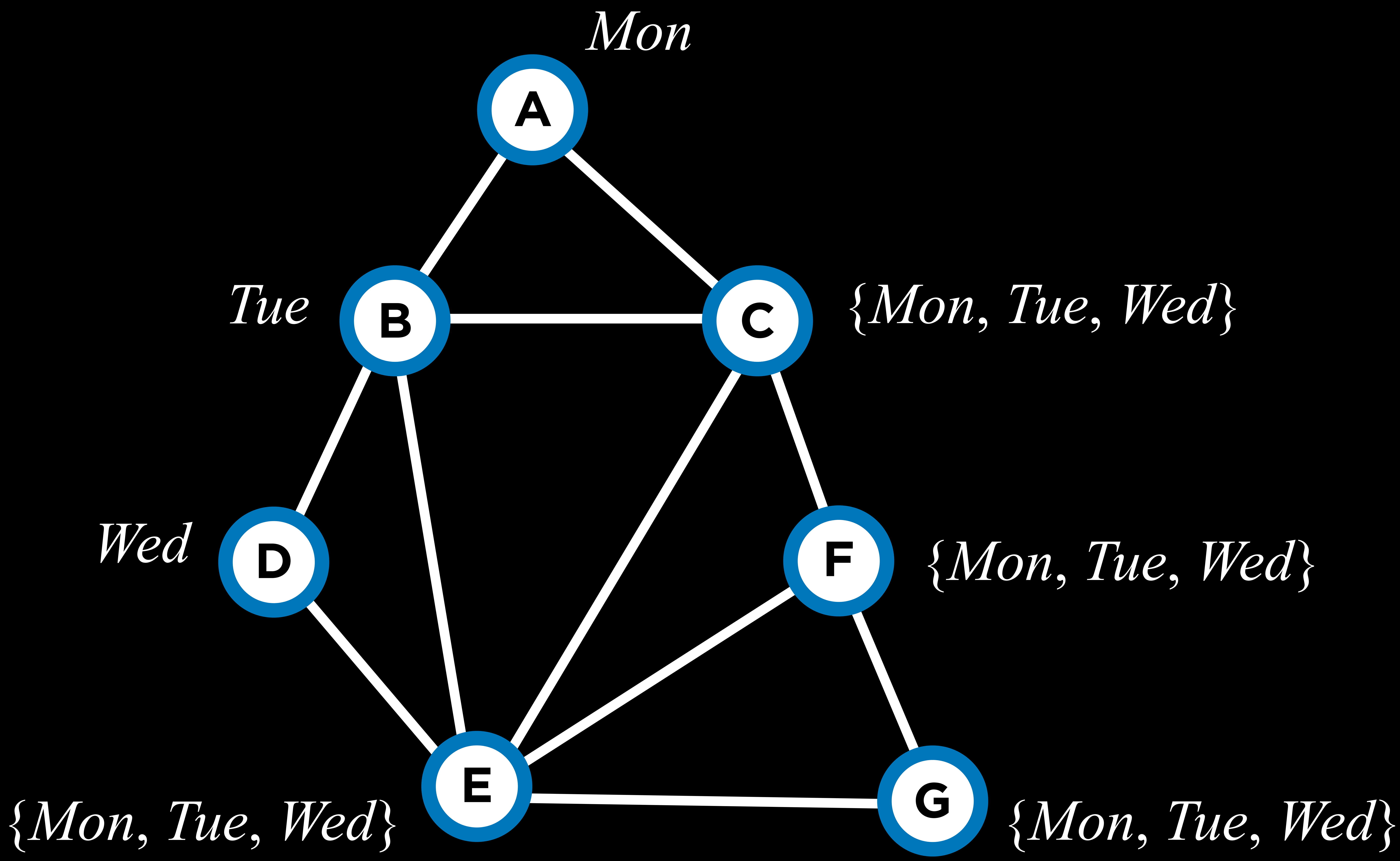
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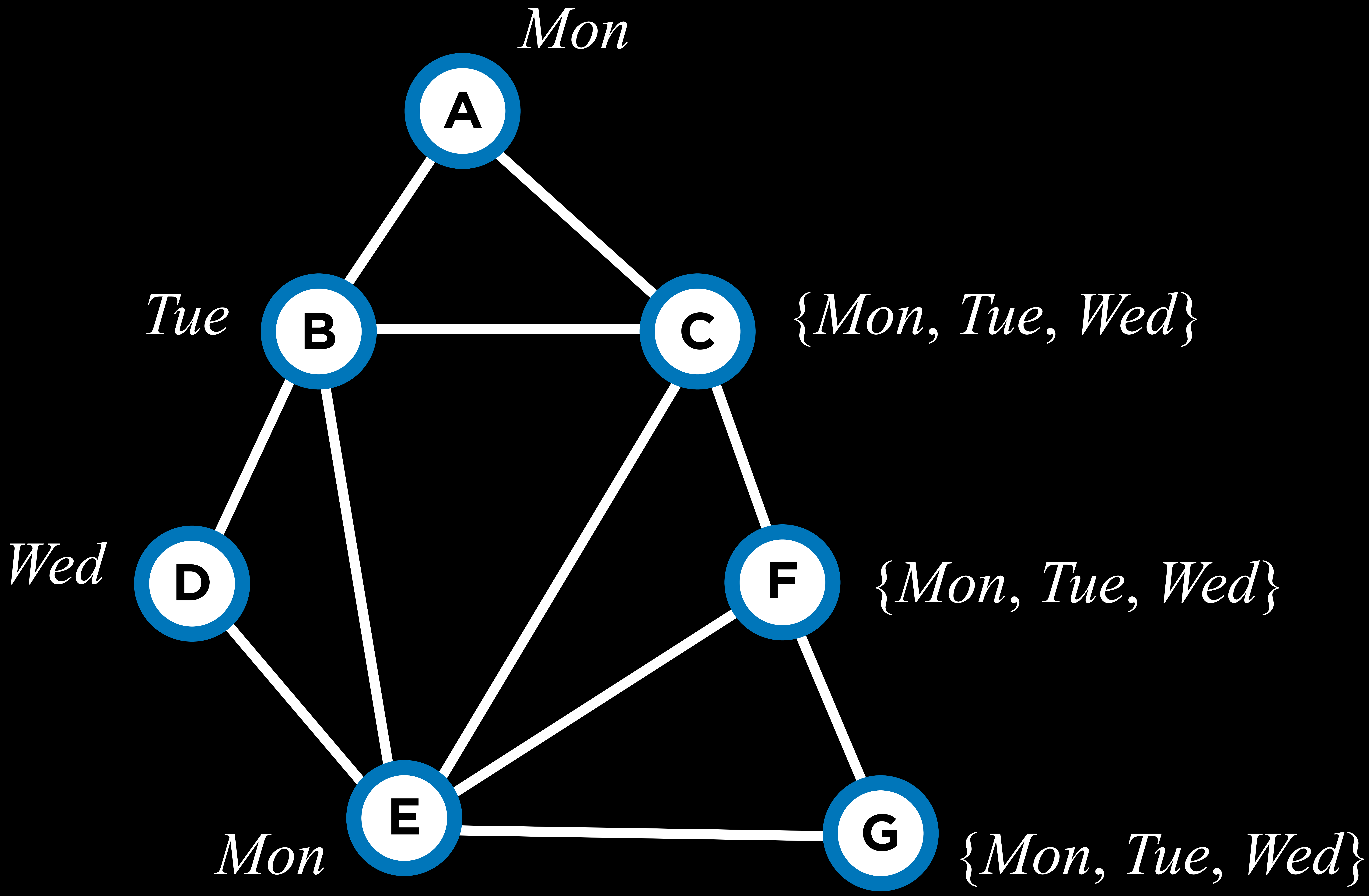
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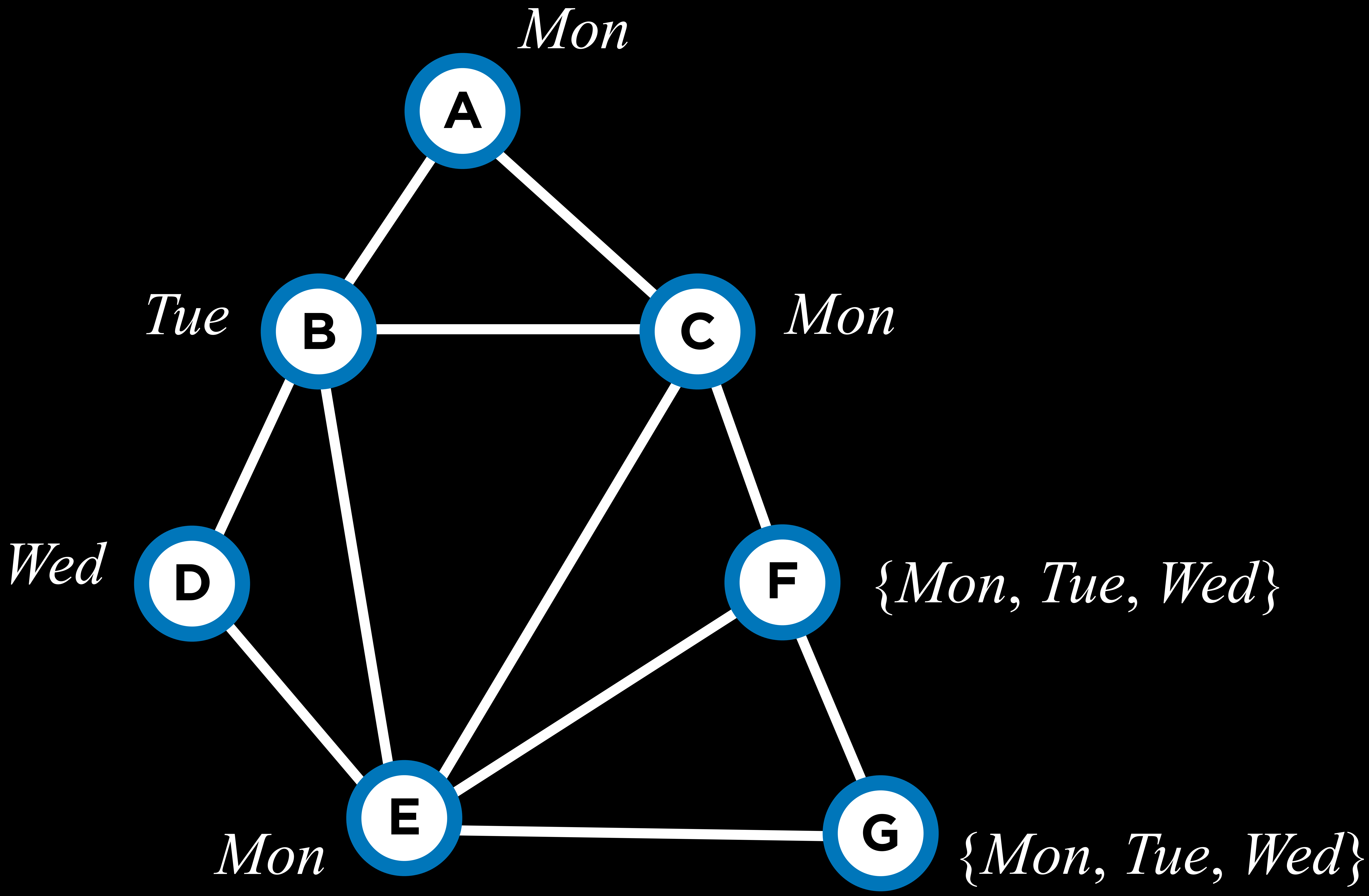
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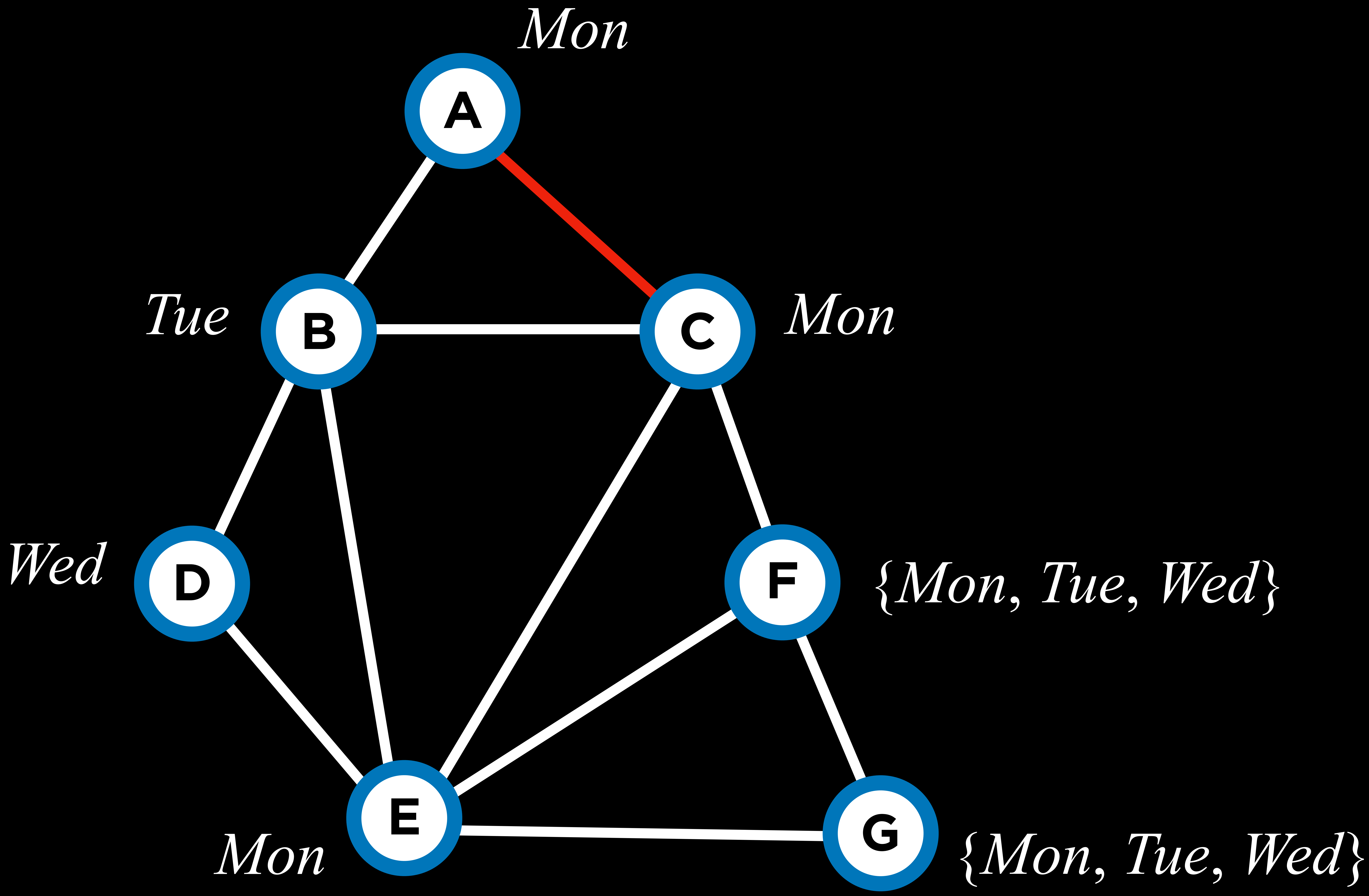
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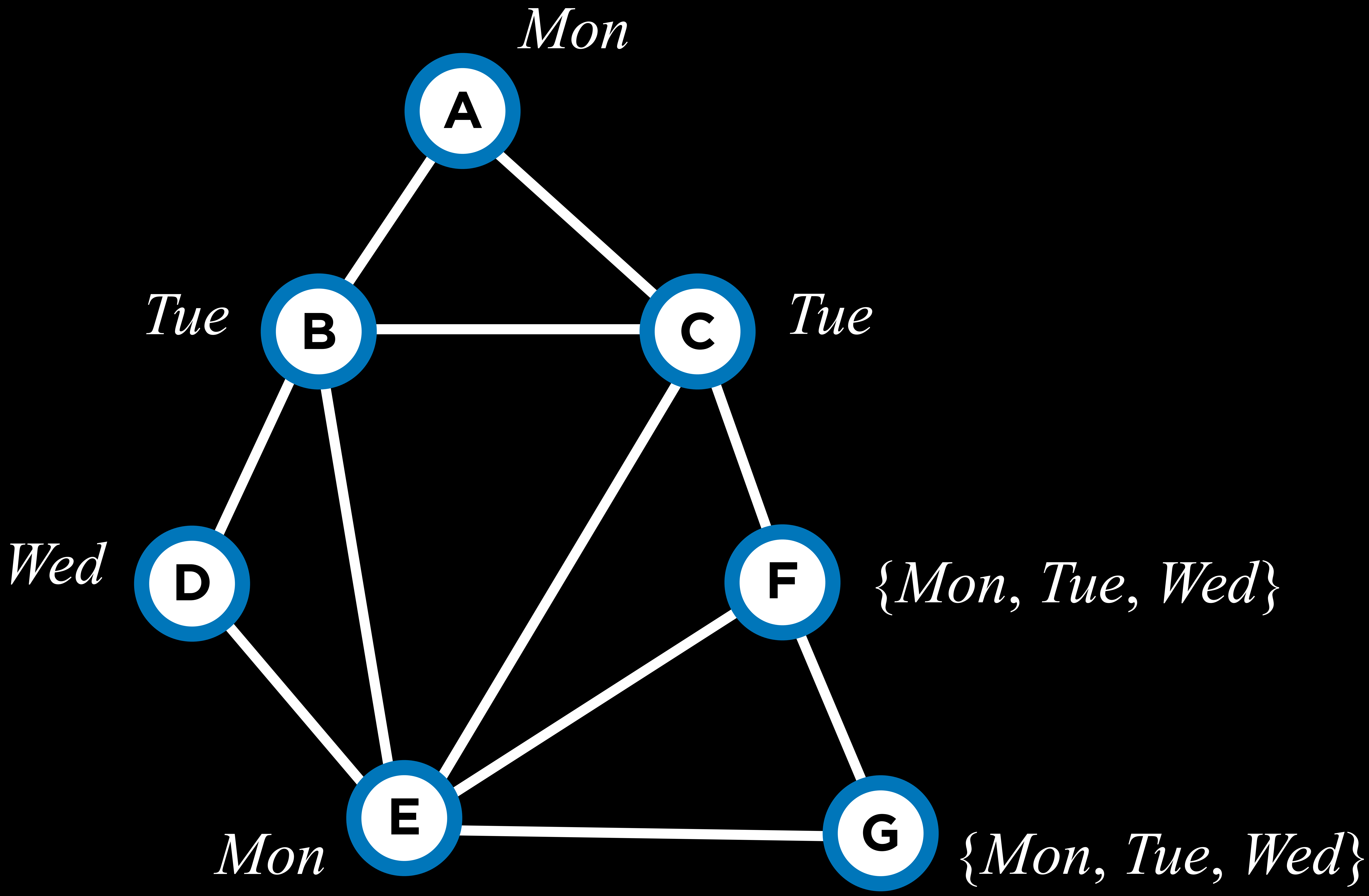
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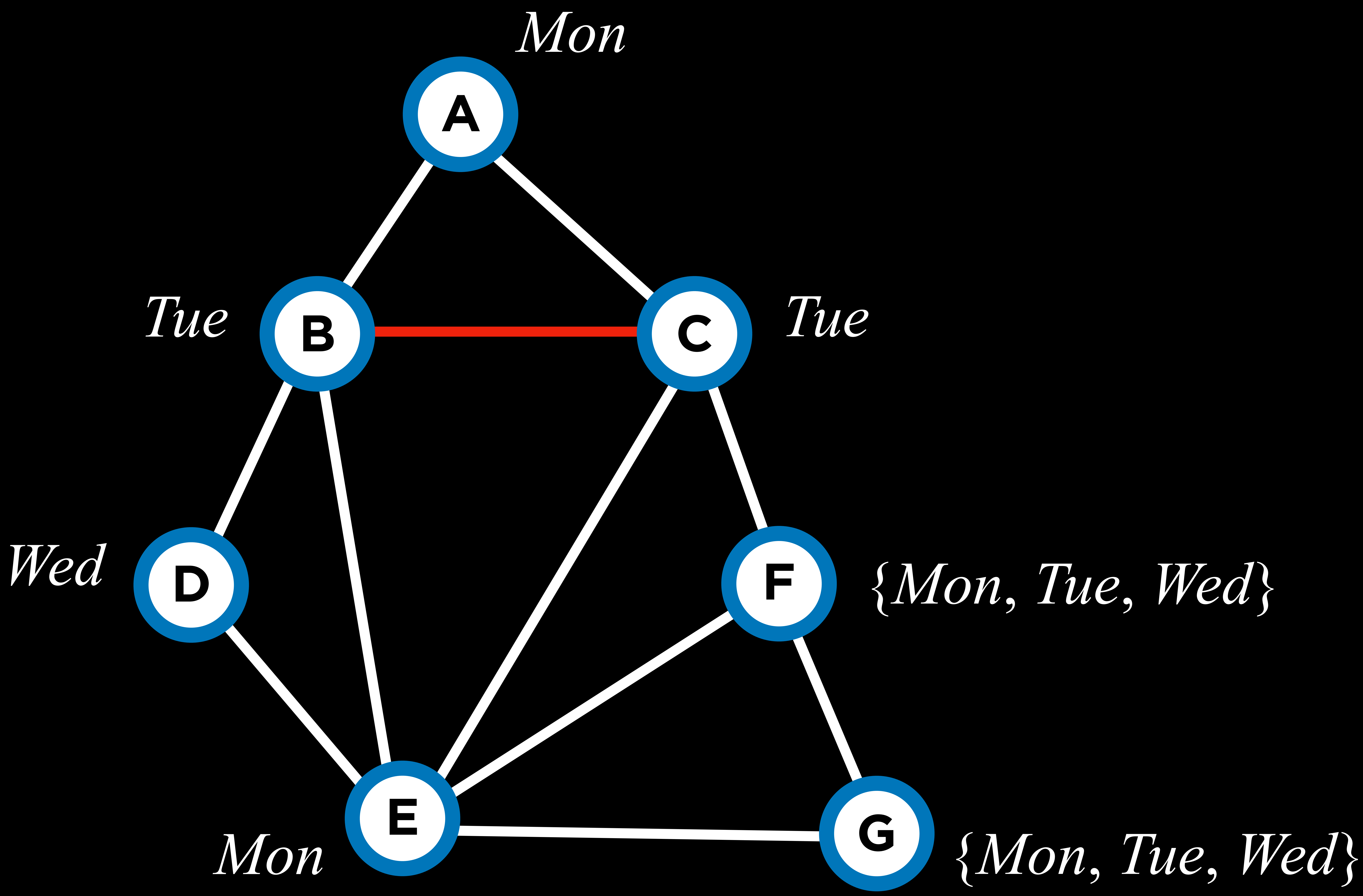

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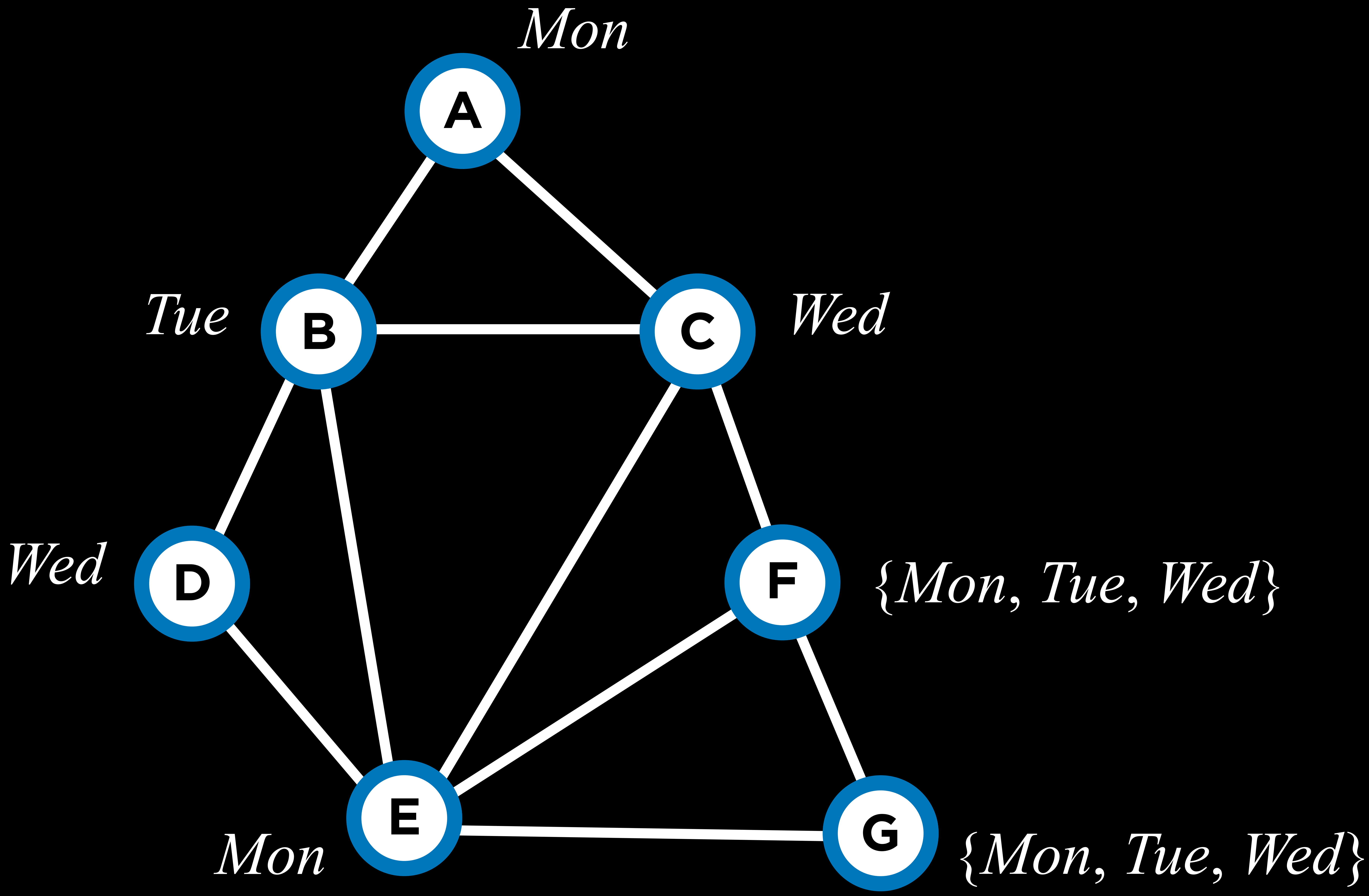
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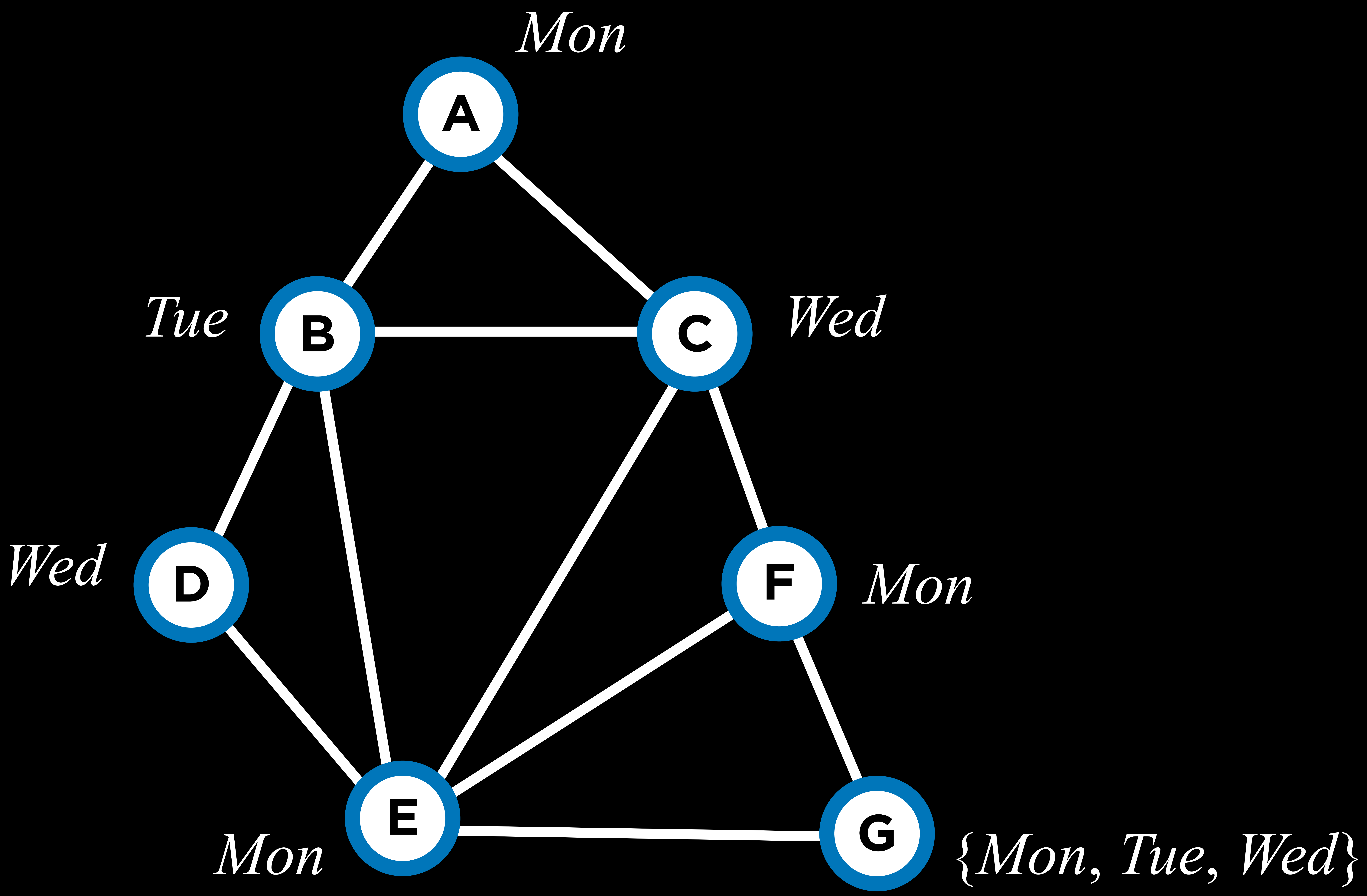
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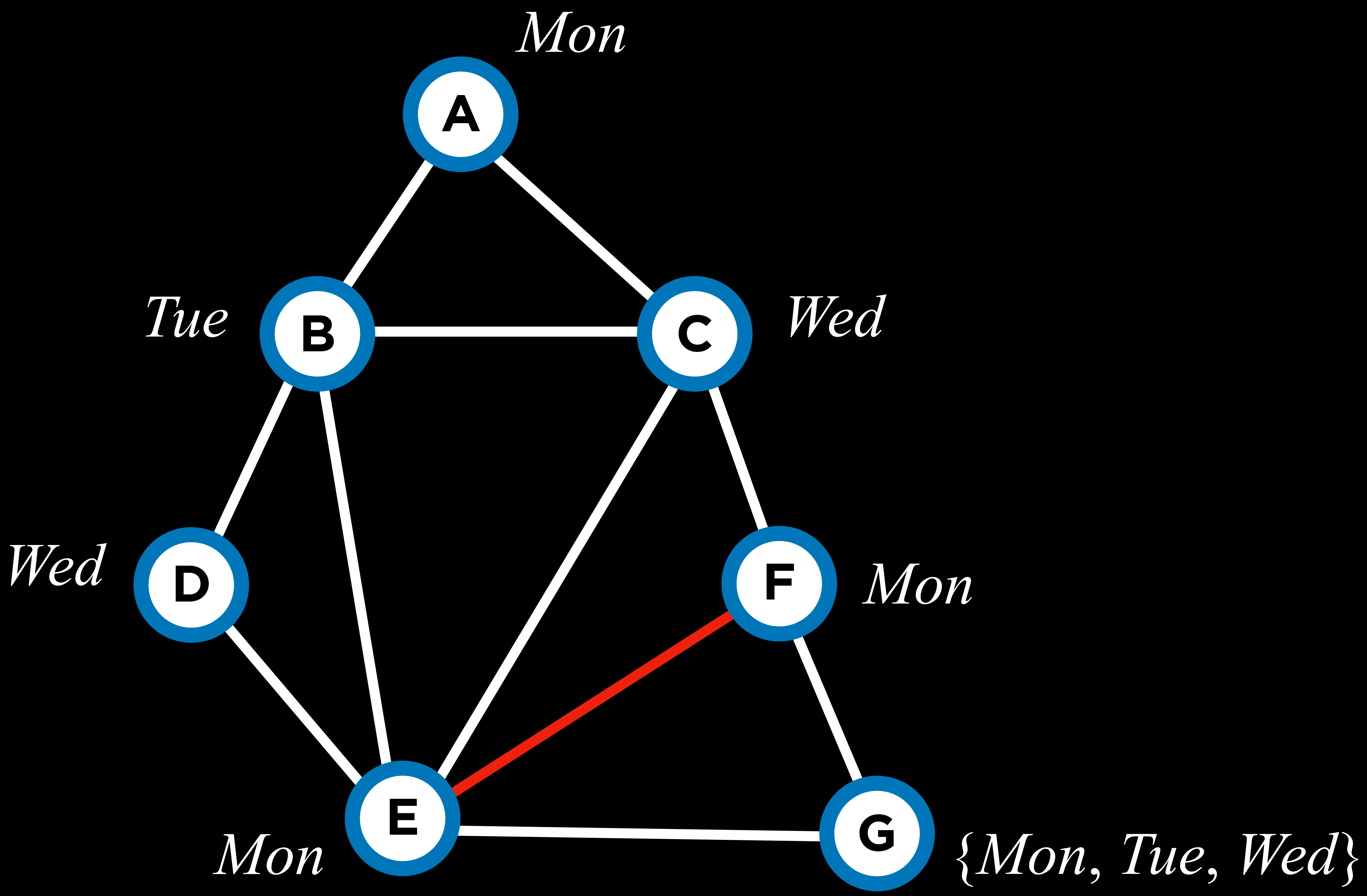

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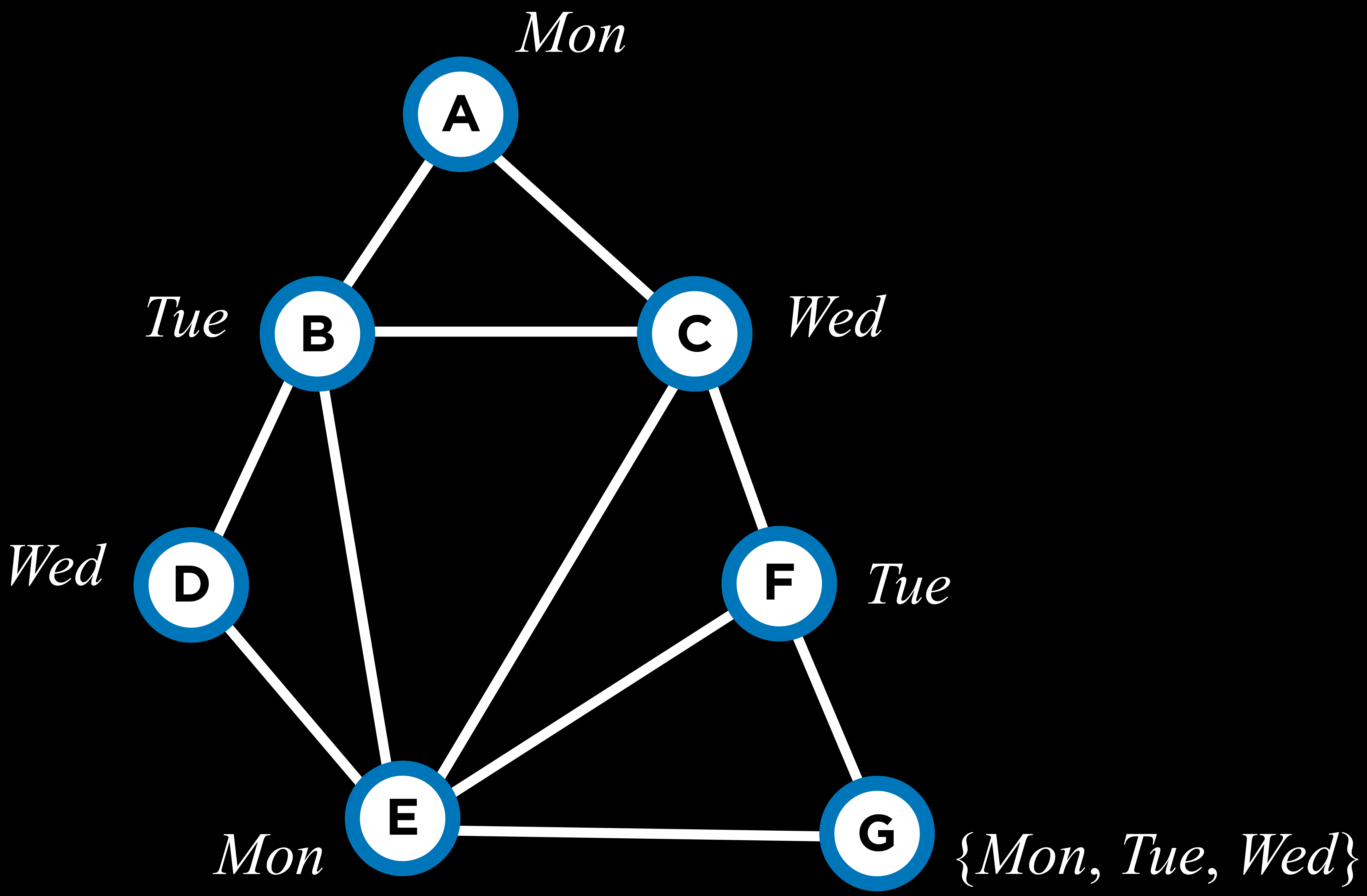
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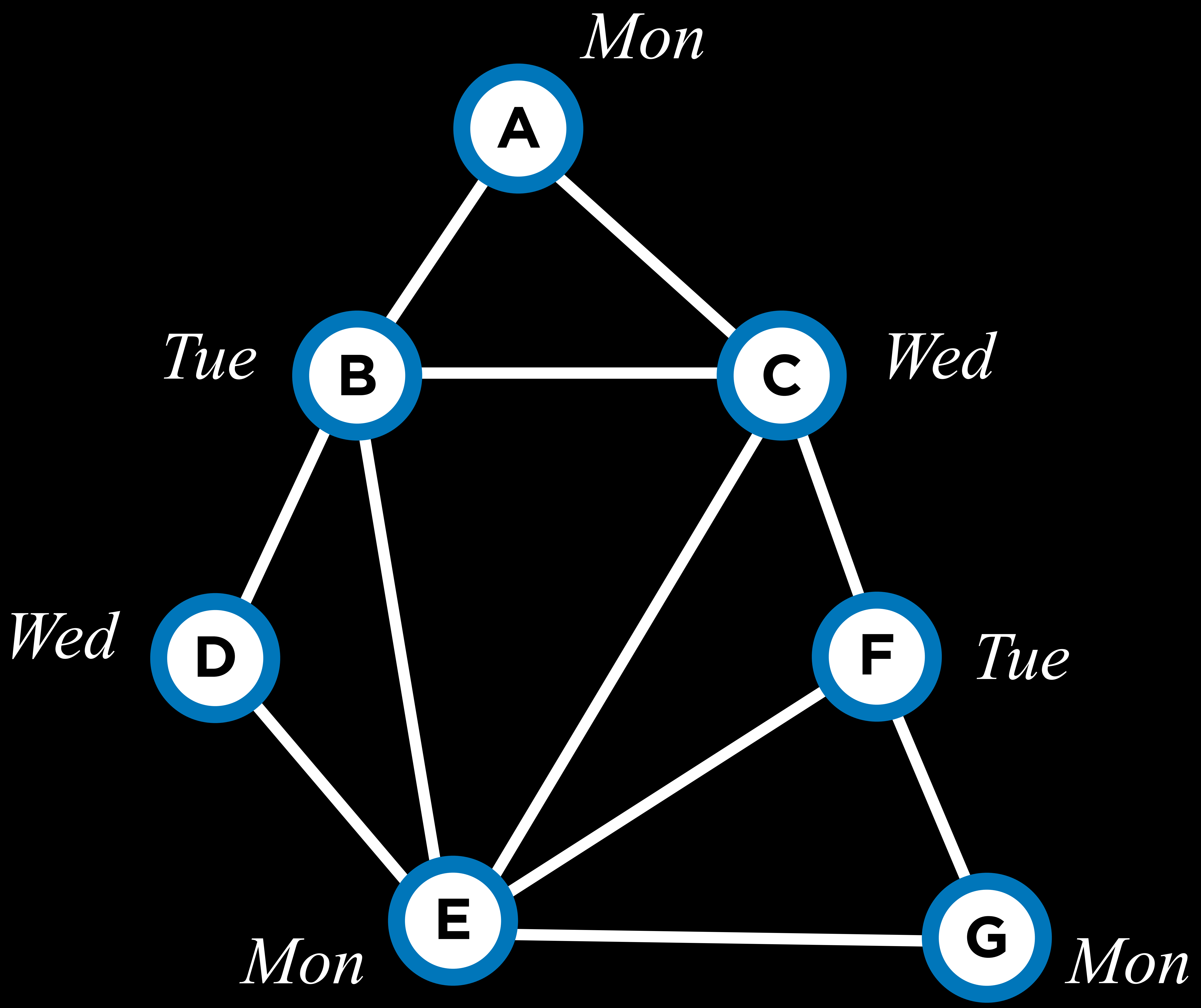
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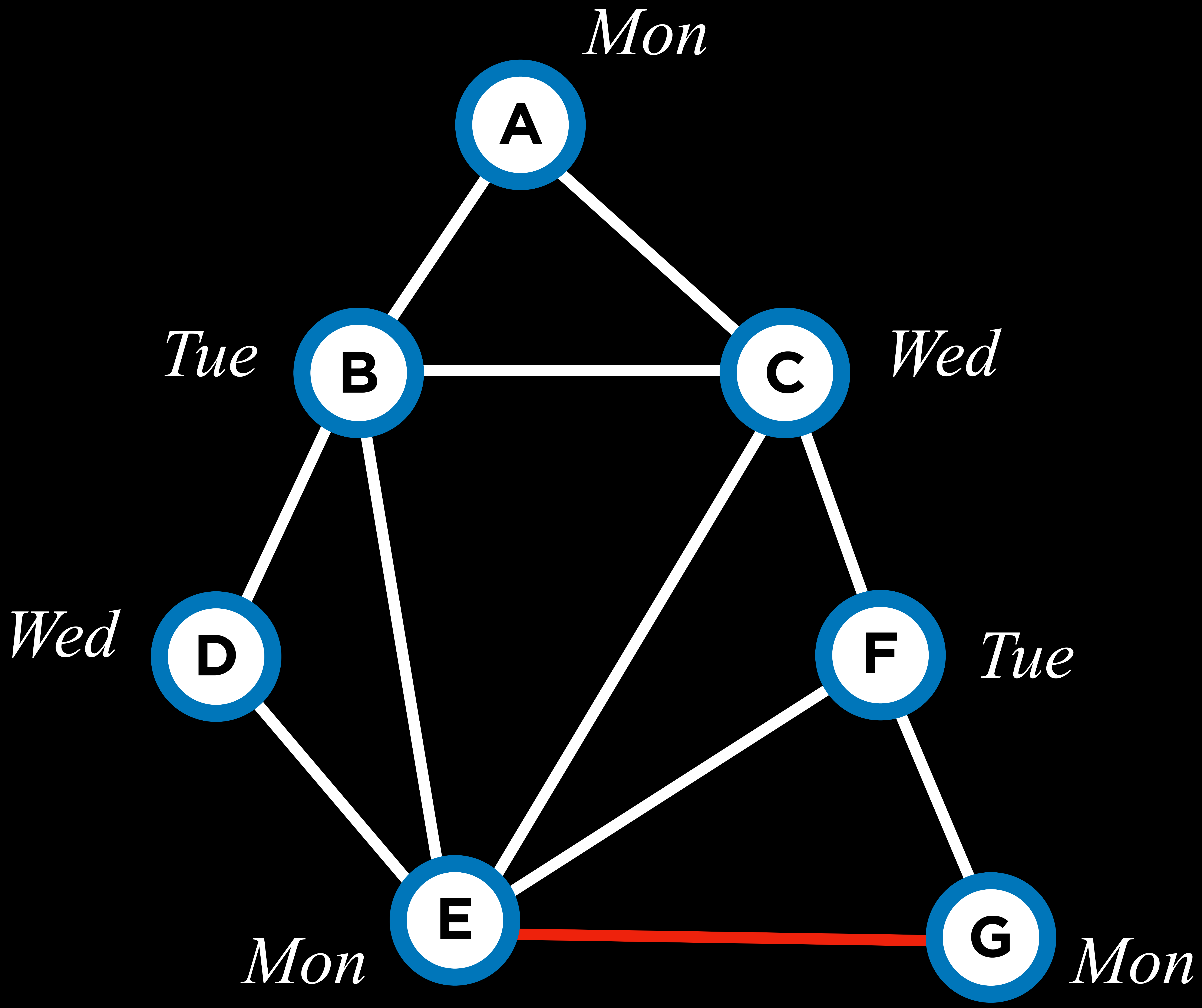
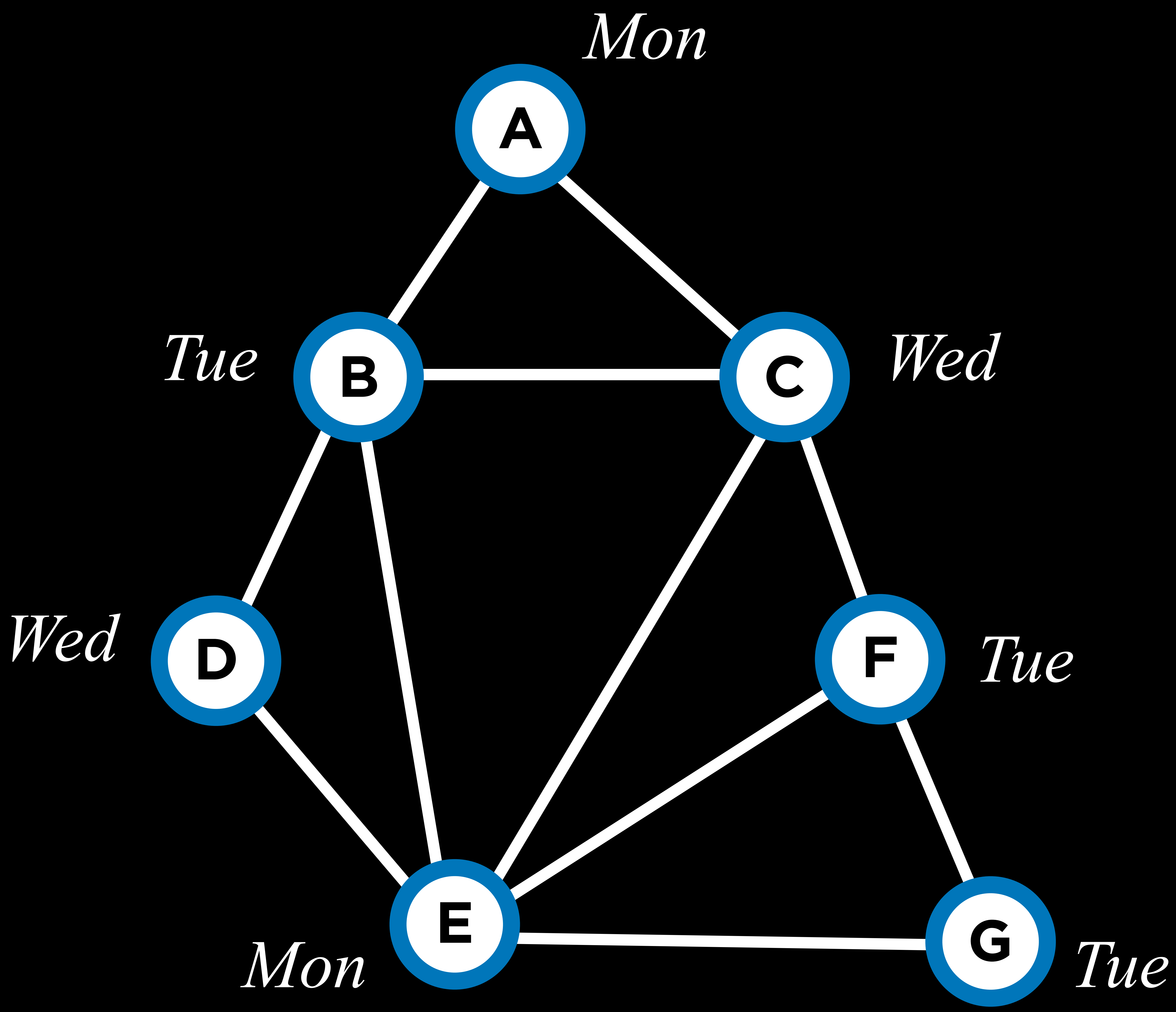
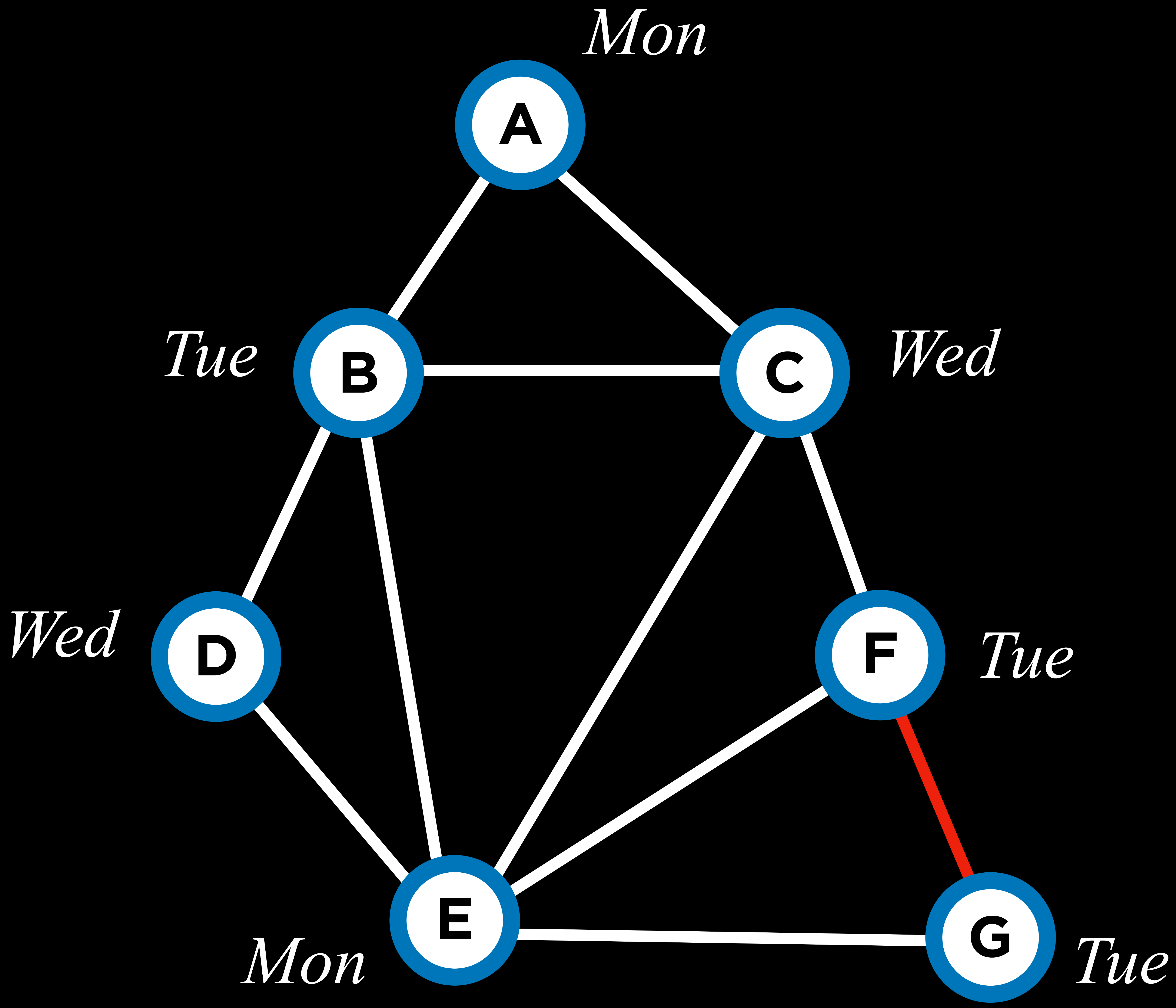
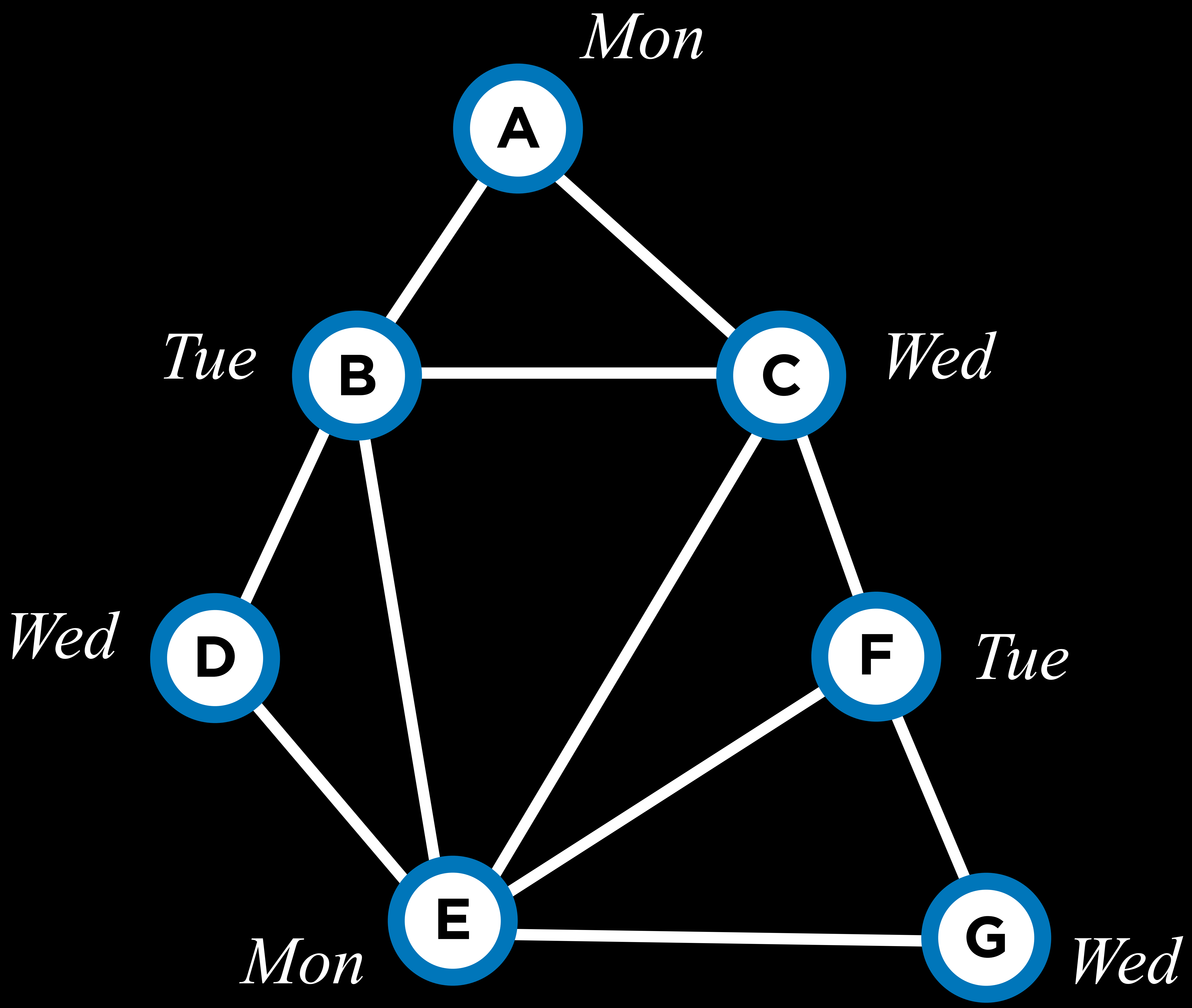
D
F
Tue
E
Mon
G
Mon

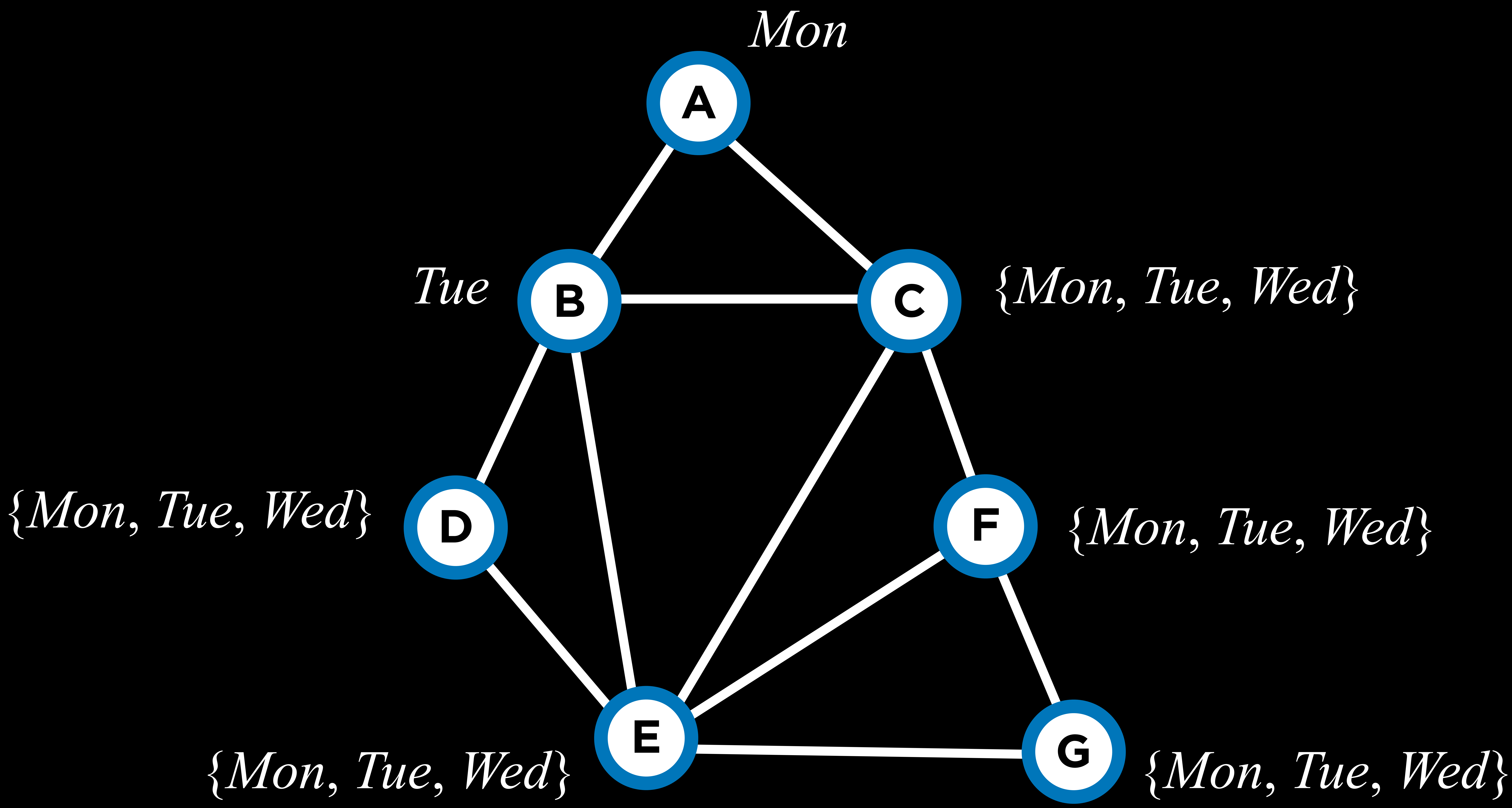

Mon
A
Tue
B
C
Wed
Wed
D
F
Tue
Mon
E
G
Mon


Mon
A
Tue
B
C
Wed
Wed
D
F
Tue
E
Mon
G
Tue

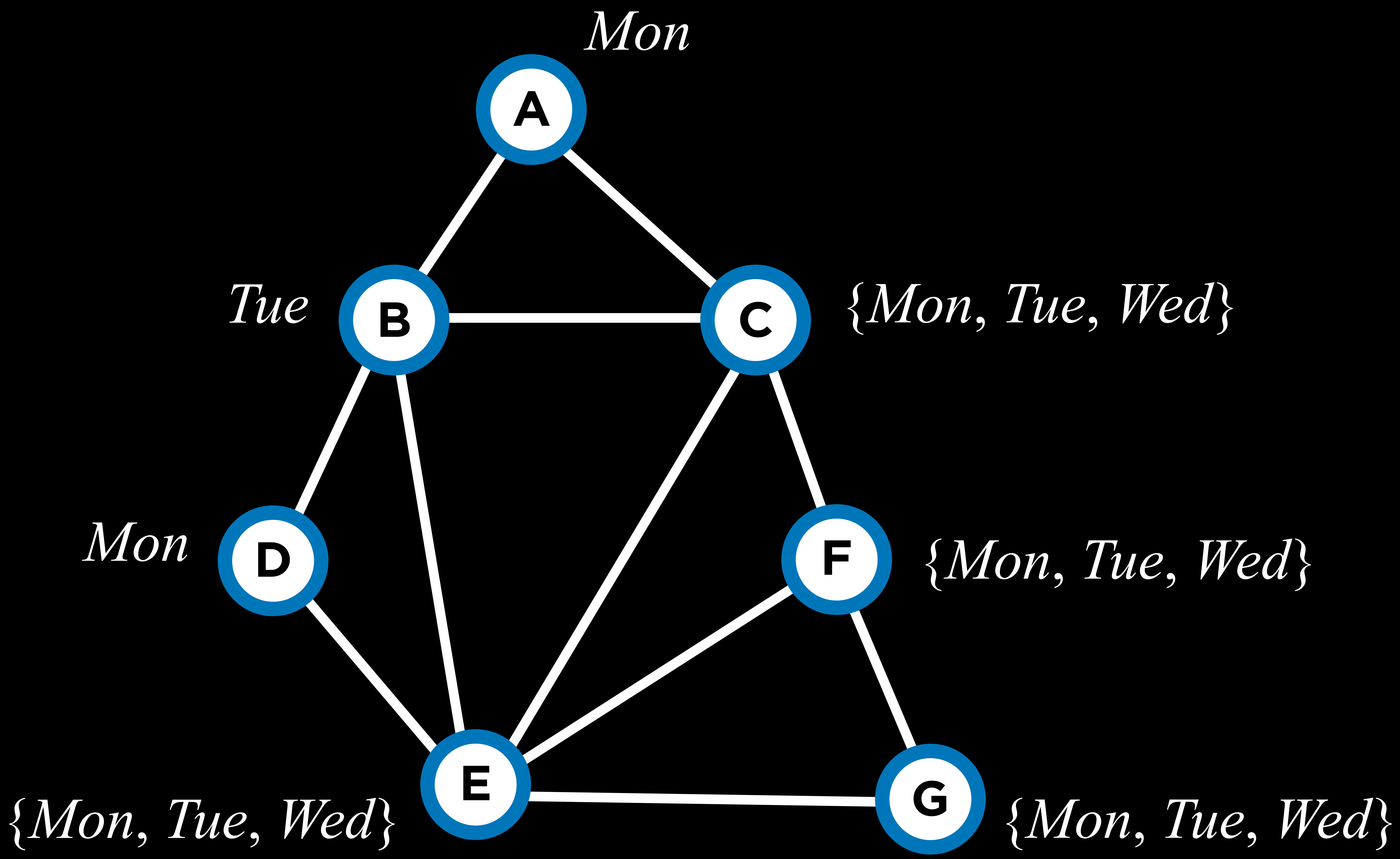

Mon
A
Tue
B
C
Wed
Wed
D
F
Tue
E
Mon
G
Tue


Mon
A
Tue
B
C
Wed
Wed
D
F
Tue
E
Mon
G
Wed

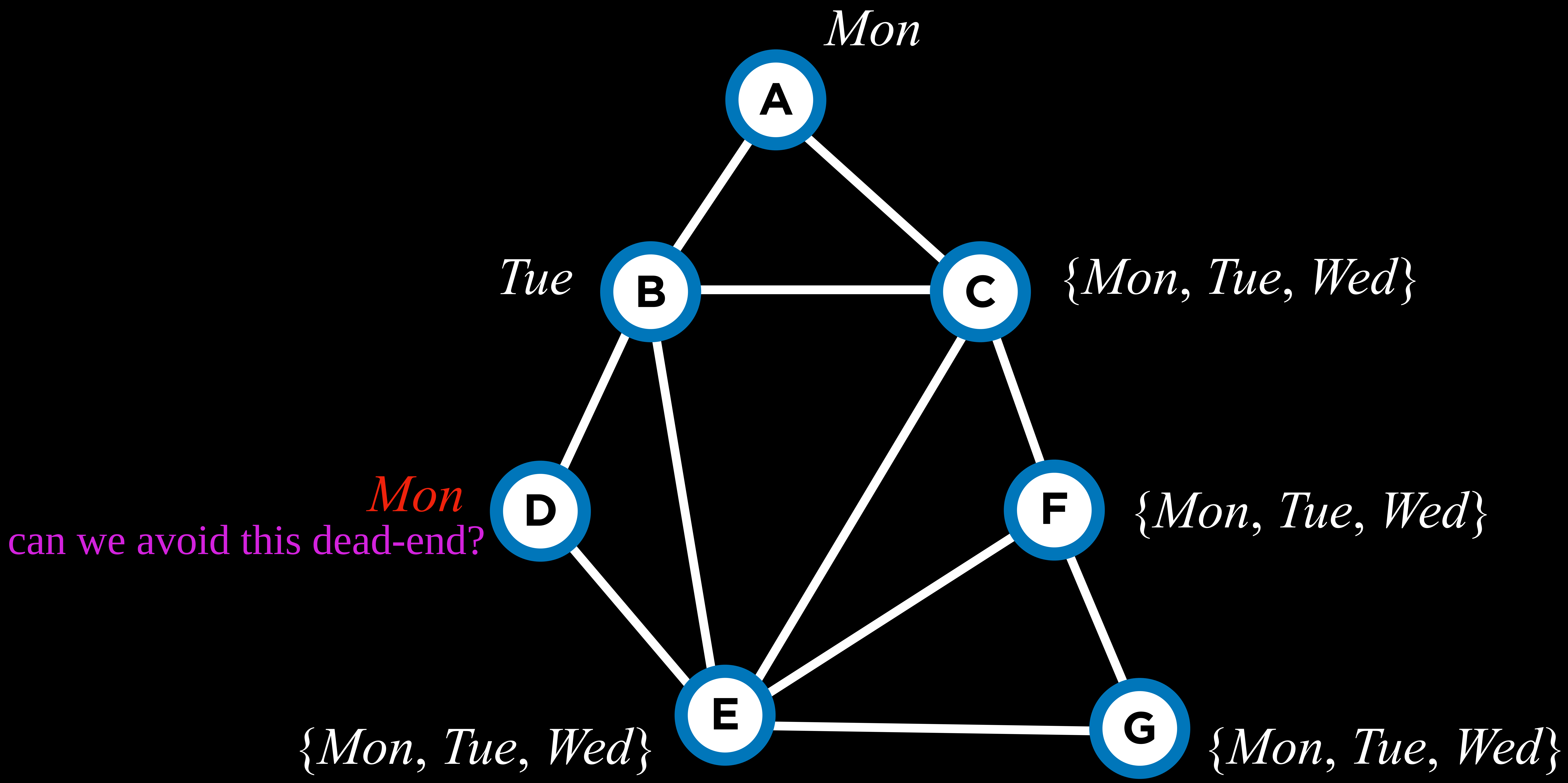
Inference



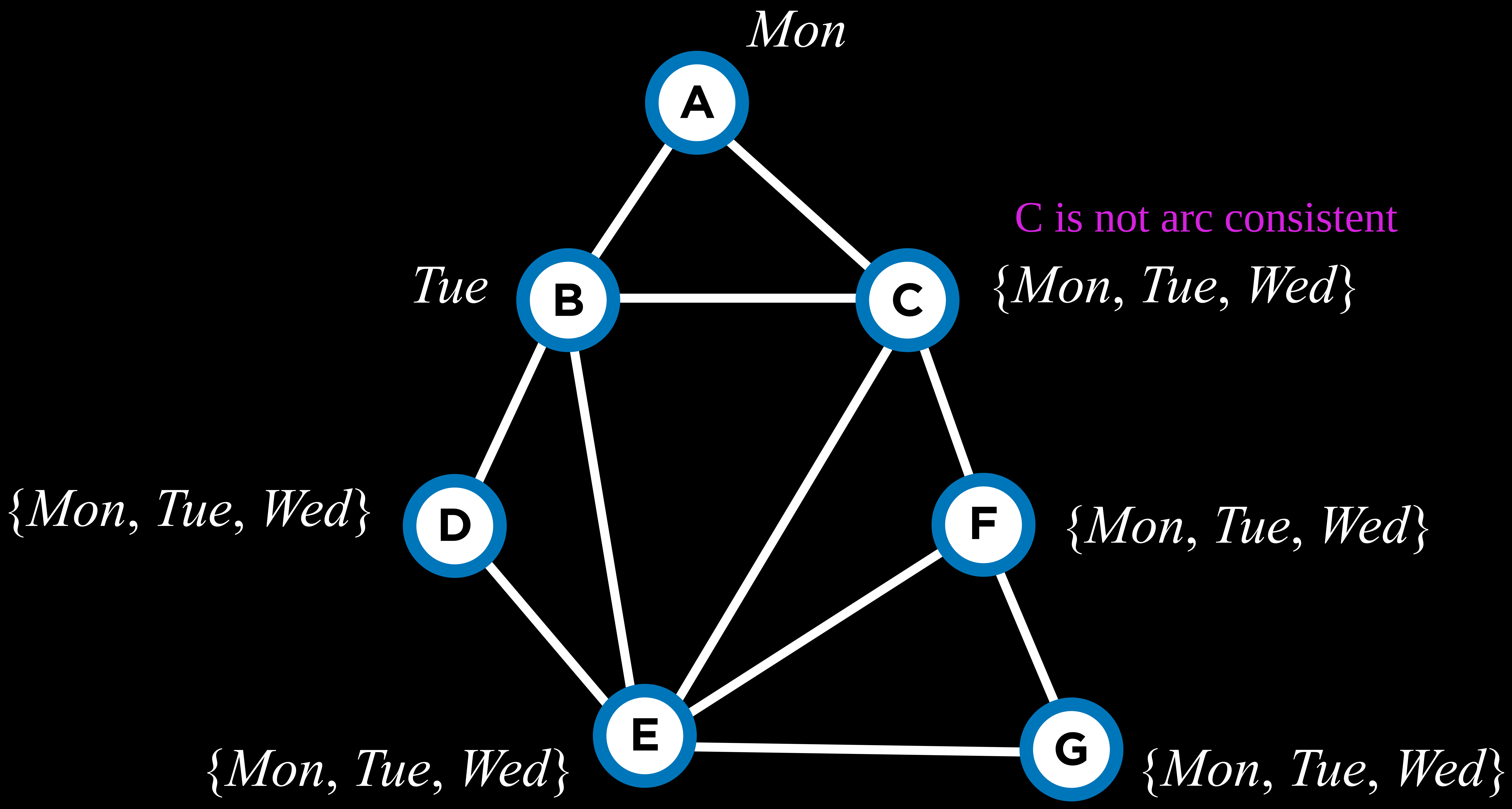
Mon
A
Tue
B
C
{Mon, Tue, Wed}
{Mon, Tue, Wed}
D
F
{Mon, Tue, Wed}
{Mon, Tue, Wed}
E
G
{Mon, Tue, Wed}

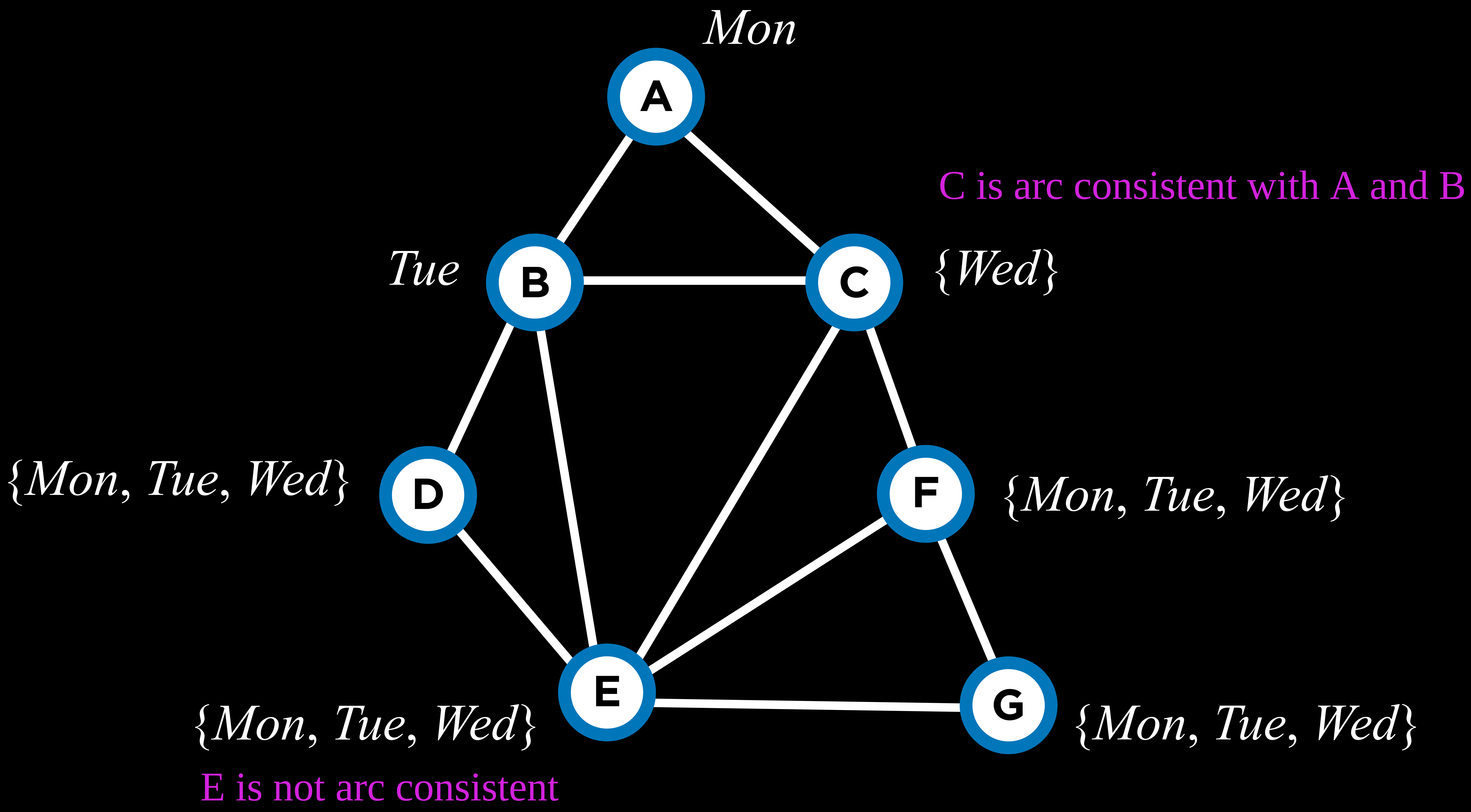


Mon
A
Tue
B
C
{Mon, Tue, Wed}
Mon
D
F
{Mon, Tue, Wed}
E
{Mon, Tue, Wed}
G
{Mon, Tue, Wed}

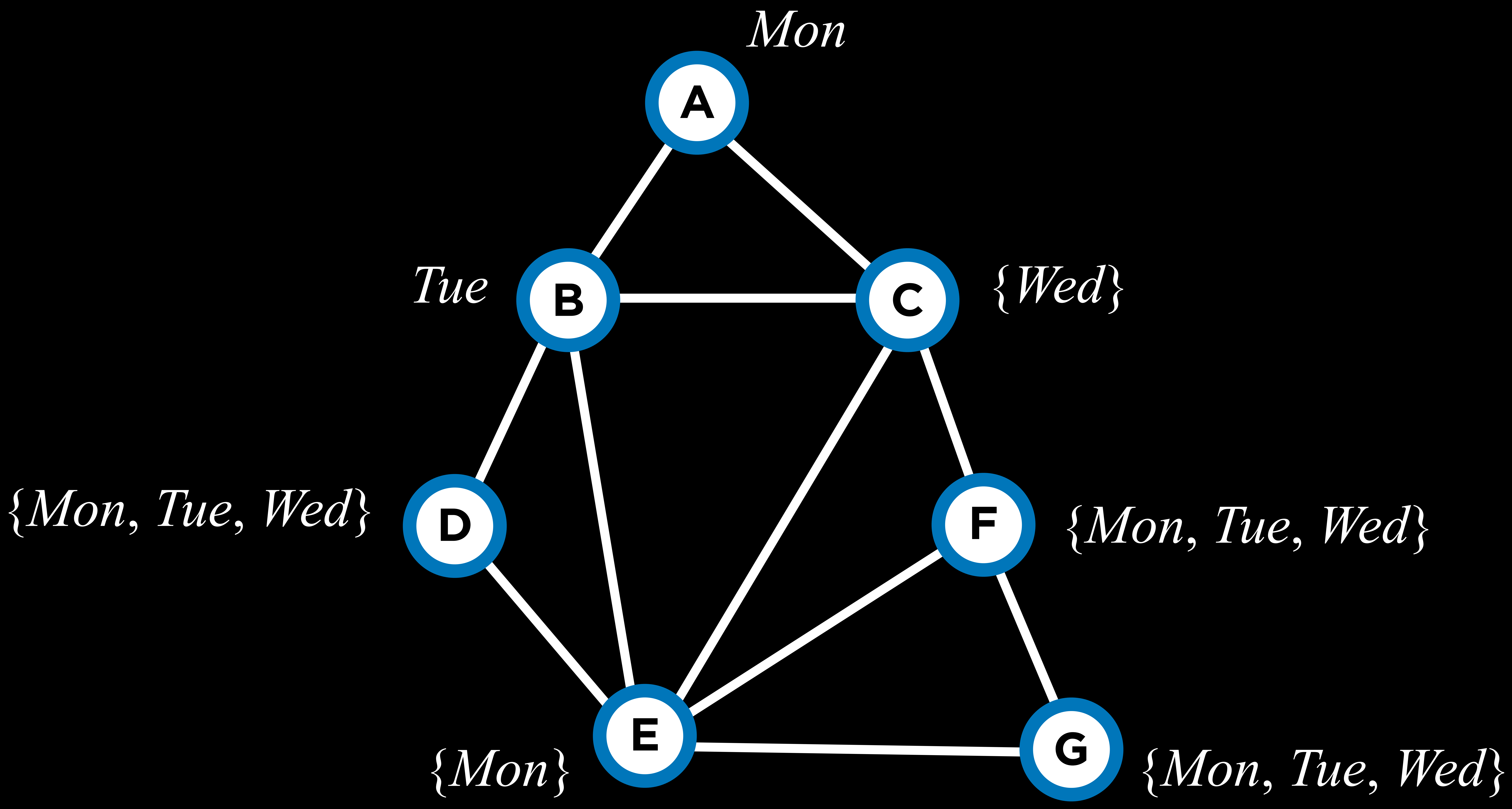


Mon
A
Tue
B
C
{Mon, Tue, Wed}
Mon
D
can we avoid this dead-end?
F
{Mon, Tue, Wed}
E
{Mon, Tue, Wed}
G
{Mon, Tue, Wed}

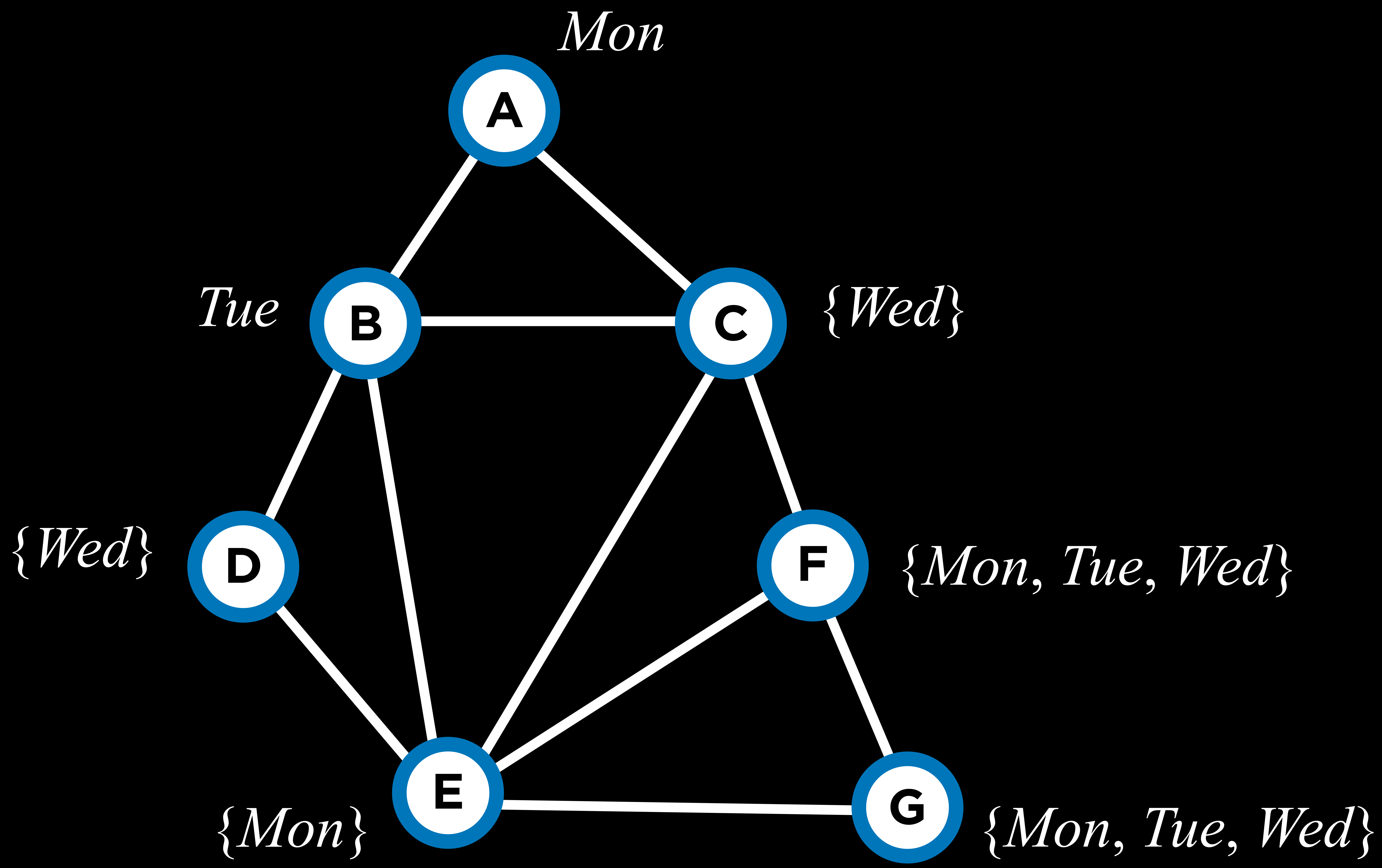

Mon
A
C is not arc consistent
Tue
B
C
{Mon, Tue, Wed}
{Mon, Tue, Wed}
D
F
{Mon, Tue, Wed}
E
G
{Mon, Tue, Wed}
{Mon, Tue, Wed}

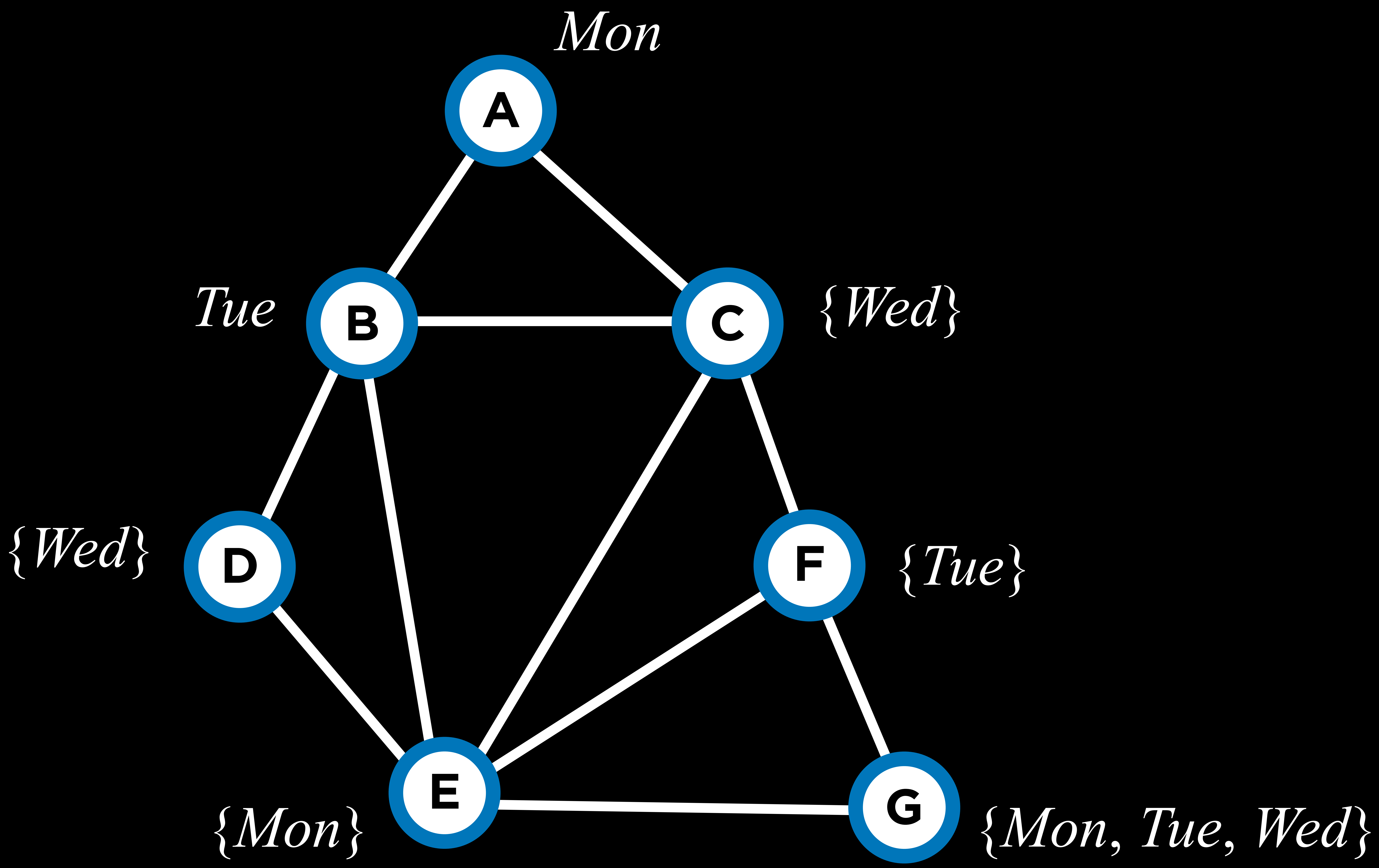


Mon
A
C is arc consistent with A and B
Tue
B
C
{Wed}
{Mon, Tue, Wed}
D
F
{Mon, Tue, Wed}
E
G
{Mon, Tue, Wed}
{Mon, Tue, Wed}
E is not arc consistent



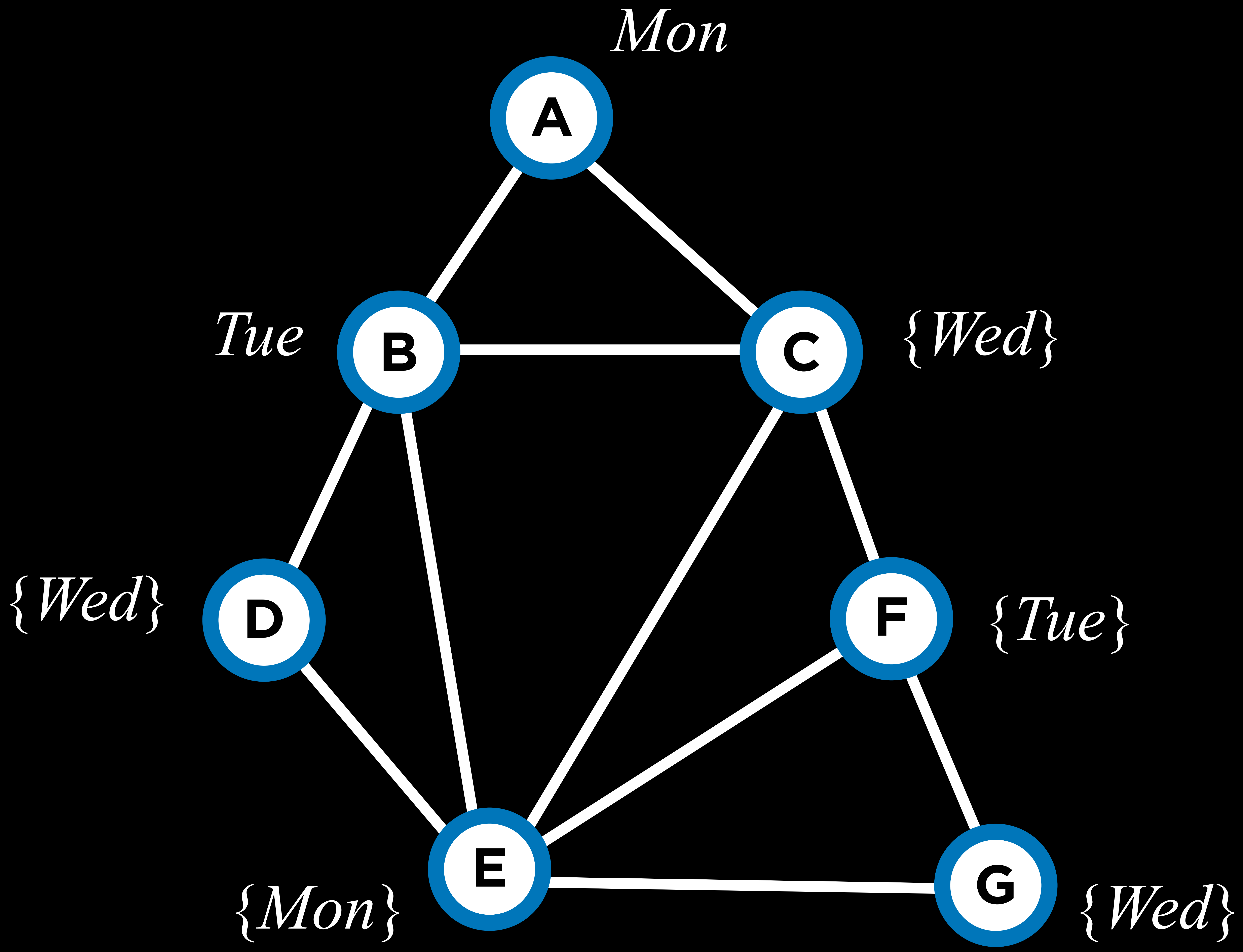
E is arc consistent with B and C

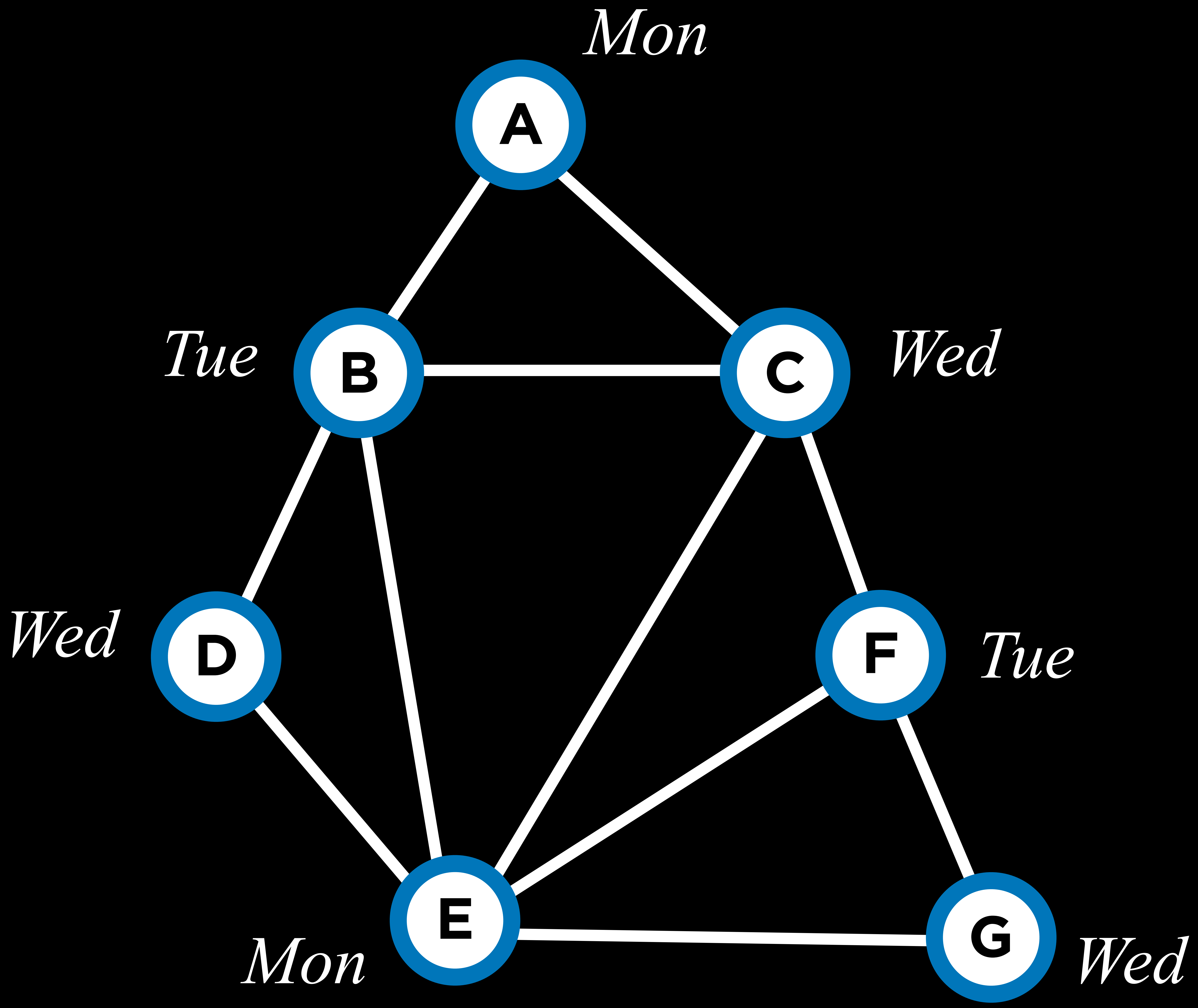

Mon
A
Tue
B
C
{Wed}
{Wed}
D
F
{Mon, Tue, Wed}
{Mon}
E
G
{Mon, Tue, Wed}



Mon
A
Tue
B
C
{Wed}
{Wed}
D
F
{Tue}
{Mon}
E
G
{Mon, Tue, Wed}

from interleaving backtracking with arc consistency, we have solution




Mon
A
Tue
B
C
Wed
Wed
D
F
Tue
E
Mon
G
Wed

maintaining arc-consistency

algorithm for enforcing arc-consistency every time we make a new assignment

maintaining arc-consistency

When we make a new assignment to X, calls $\mathrm{AC\text{-}3}$, starting with a queue of all arcs (Y, X) where Y is a neighbor of X

```
function BACKTRACK(assignment, csp):
    if assignment complete: return assignment
    var = SELECT-UNASSIGNED-VAR(assignment, csp)
    for value in DOMAIN-VALUES(var, assignment, csp):
        if value consistent with assignment:
            add {var = value} to assignment
run arc consistency   inferences = INFERENCE(assignment, csp)
save if valid         if inferences ≠ failure: add inferences to assignment
            result = BACKTRACK(assignment, csp)
            if result ≠ failure: return result
        remove {var = value} and inferences from assignment
    return failure
```

backtrack also needs to remove arc consistency

```
function BACKTRACK(assignment, csp):
    if assignment complete: return assignment
    var = SELECT-UNASSIGNED-VAR(assignment, csp)
    for value in DOMAIN-VALUES(var, assignment, csp):
        if value consistent with assignment:
            add {var = value} to assignment
            inferences = INFERENCE(assignment, csp)
            if inferences ≠ failure: add inferences to assignment
            result = BACKTRACK(assignment, csp)
            if result ≠ failure: return result
        remove {var = value} and inferences from assignment
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```

```
function BACKTRACK(assignment, csp):
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            inferences = INFERENCE(assignment, csp)
            if inferences ≠ failure: add inferences to assignment
            result = BACKTRACK(assignment, csp)
            if result ≠ failure: return result
        remove {var = value} and inferences from assignment
    return failure
```
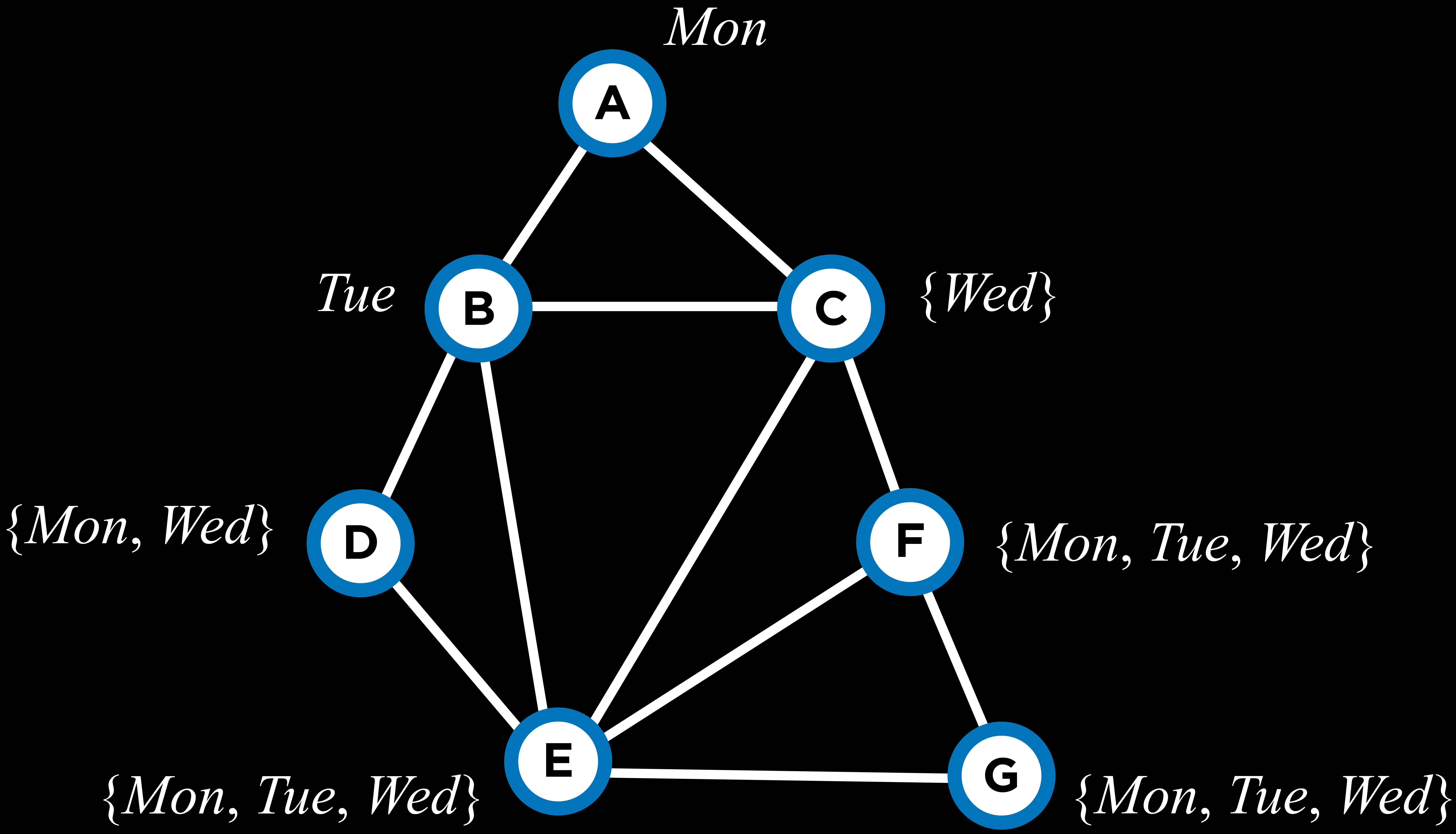
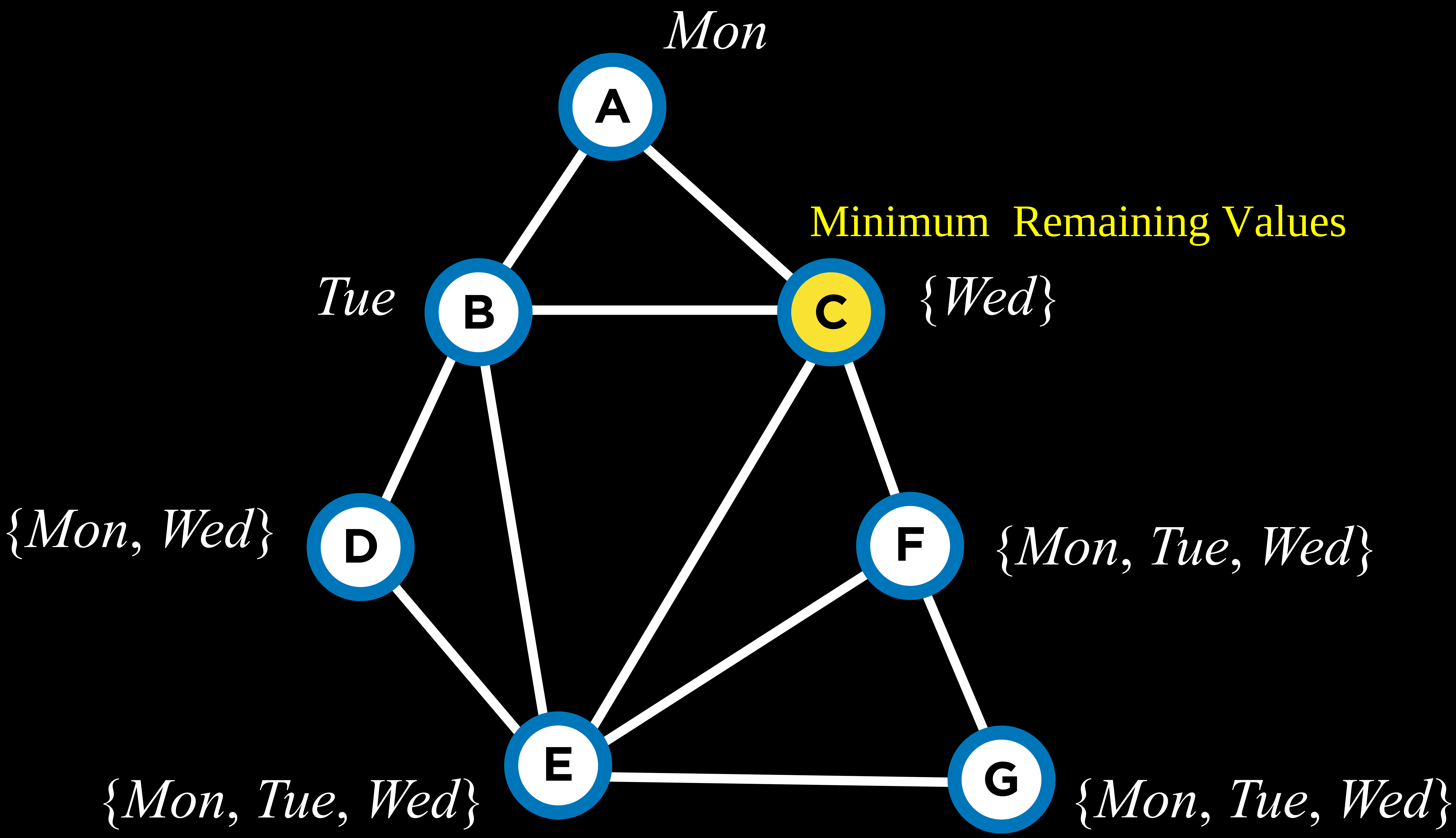

SELECT-UNASSIGNED-VAR

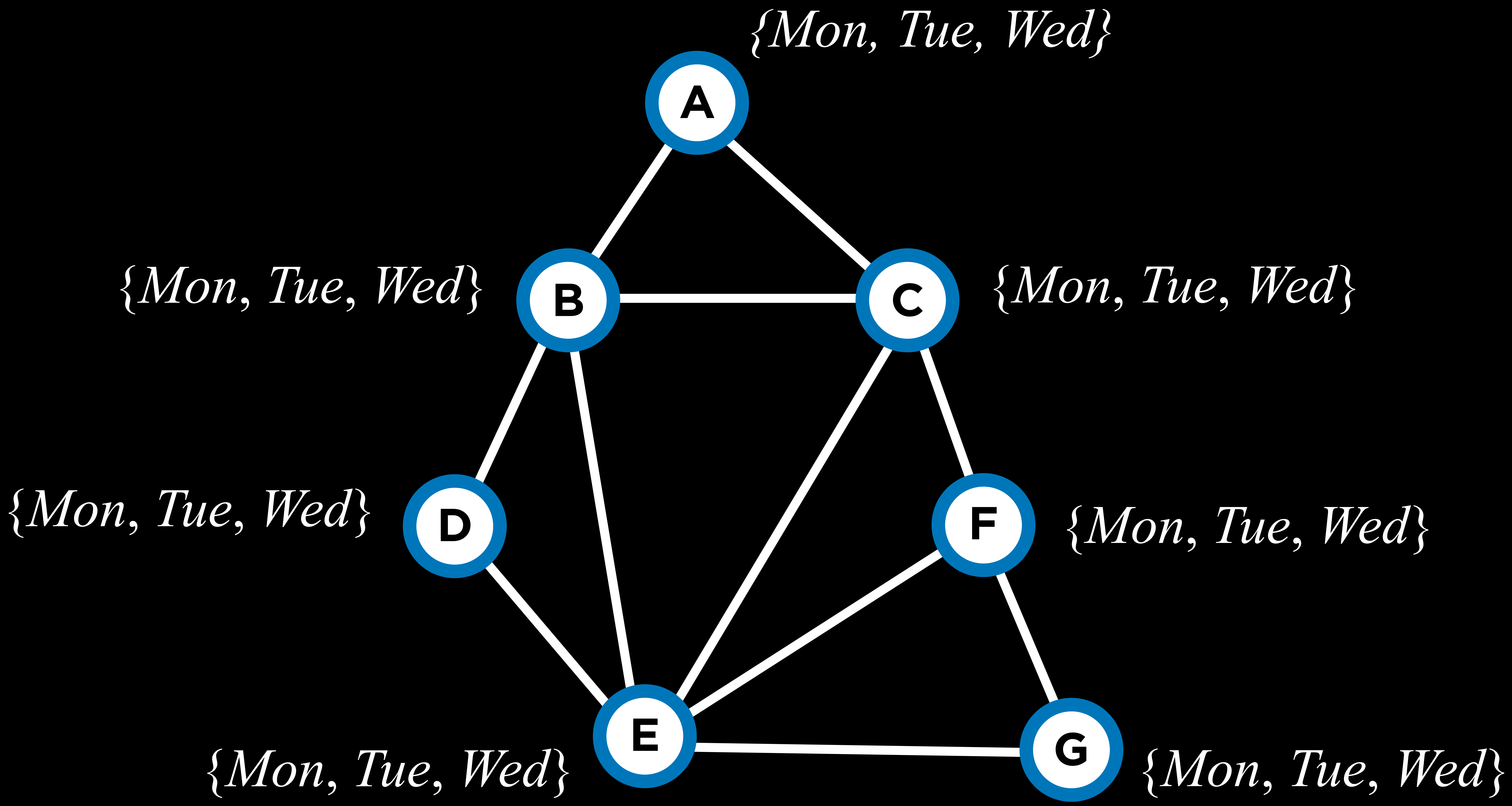
- **minimum remaining values (MRV)** heuristic: select the variable that has the smallest domain
- **degree** heuristic: select the variable that has the highest degree

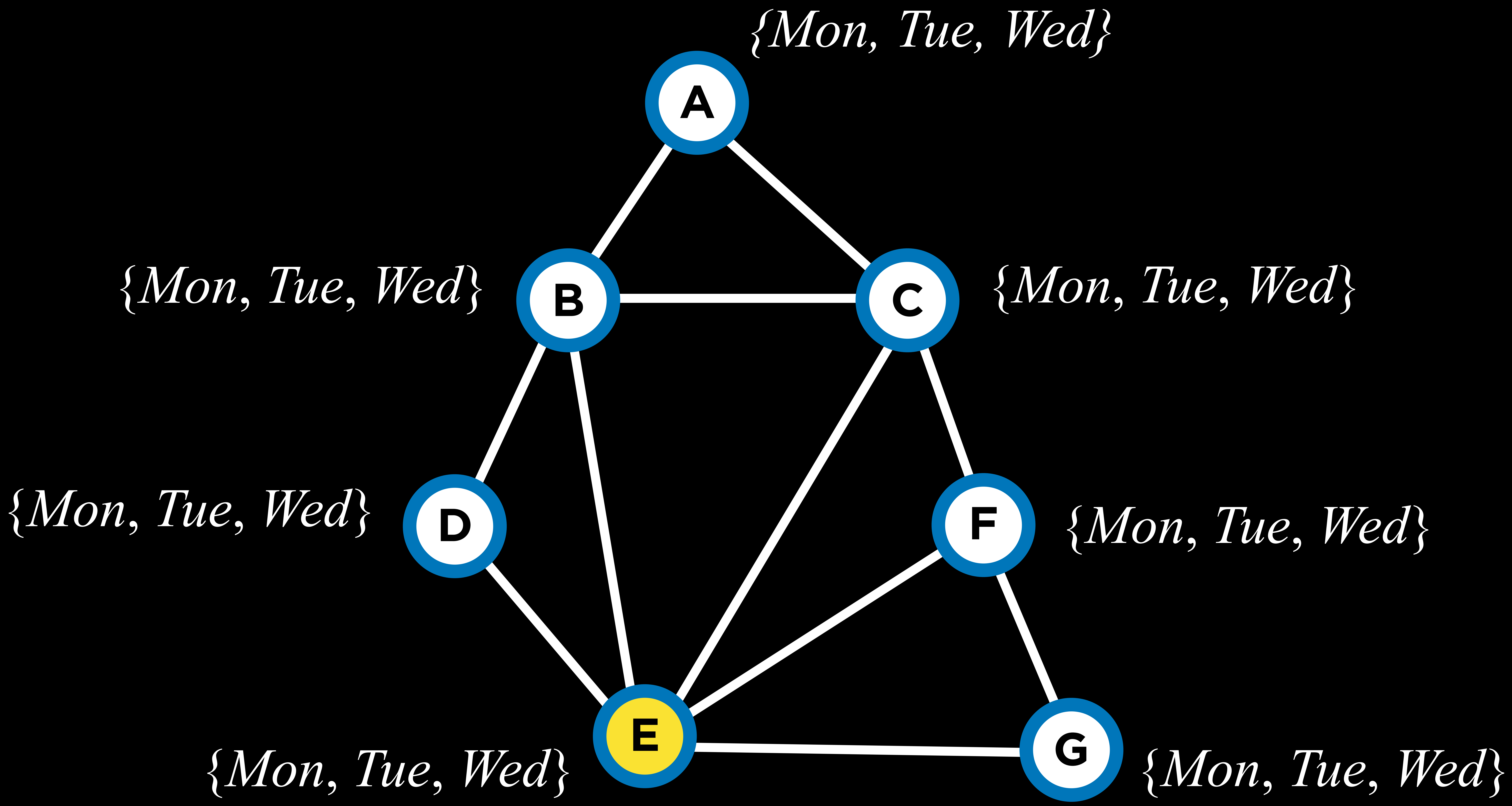
degree is number of connected nodes
assignment here means pruning from all connected node's domain

Optimal practice is to start with highest degree node
while leaving most options for other nodes to reduce dead-ends


Mon
A
Tue
B
C
{Wed}
{Mon, Wed}
D
F
{Mon, Tue, Wed}
{Mon, Tue, Wed}
E
G
{Mon, Tue, Wed}


Mon
A
Minimum Remaining Values
Tue
B
C
{Wed}
{Mon, Wed}
D
F
{Mon, Tue, Wed}
{Mon, Tue, Wed}
E
G
{Mon, Tue, Wed}


{Mon, Tue, Wed}
A
{Mon, Tue, Wed}
B
C
{Mon, Tue, Wed}
{Mon, Tue, Wed}
D
F
{Mon, Tue, Wed}
{Mon, Tue, Wed}
E
G
{Mon, Tue, Wed}



all domains have same size. E has greatest degree 5

```
function BACKTRACK(assignment, csp):
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        if value consistent with assignment:
            add {var = value} to assignment
            inferences = INFERENCE(assignment, csp)
            if inferences ≠ failure: add inferences to assignment
            result = BACKTRACK(assignment, csp)
            if result ≠ failure: return result
        remove {var = value} and inferences from assignment
    return failure
```

```
function BACKTRACK(assignment, csp):
    if assignment complete: return assignment
    var = SELECT-UNASSIGNED-VAR(assignment, csp)
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        if value consistent with assignment:
            add {var = value} to assignment
            inferences = INFERENCE(assignment, csp)
            if inferences ≠ failure: add inferences to assignment
            result = BACKTRACK(assignment, csp)
            if result ≠ failure: return result
        remove {var = value} and inferences from assignment
    return failure
```
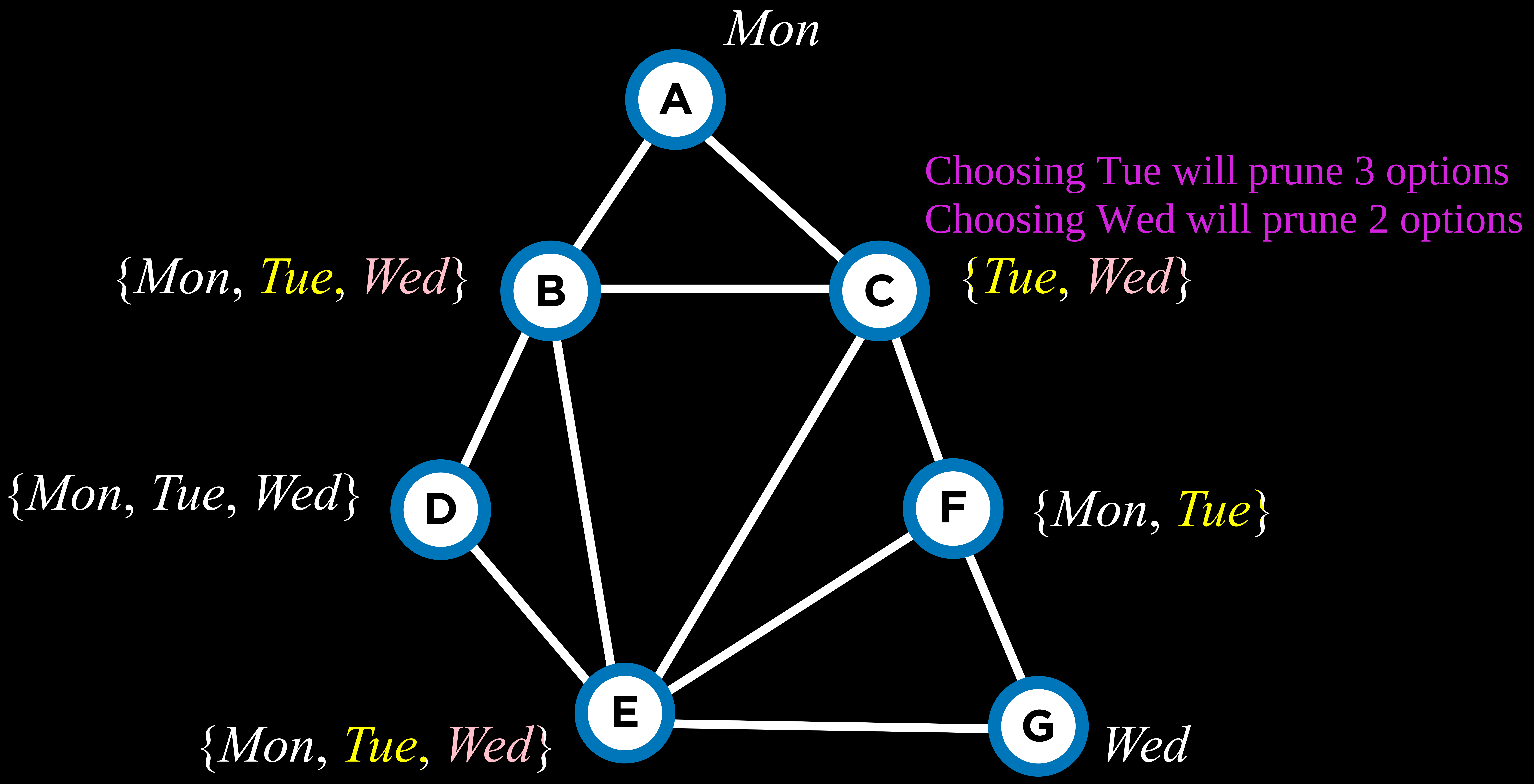
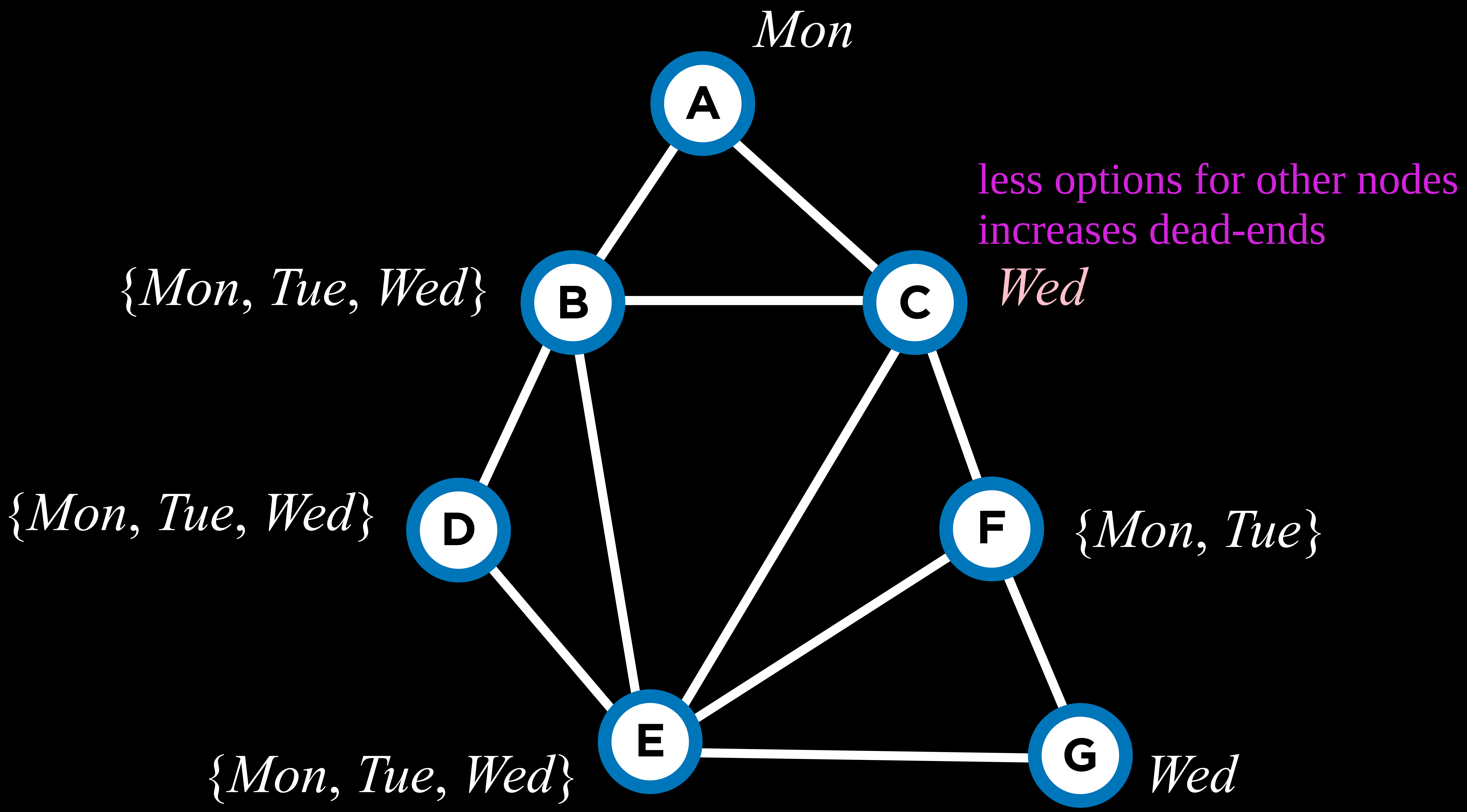
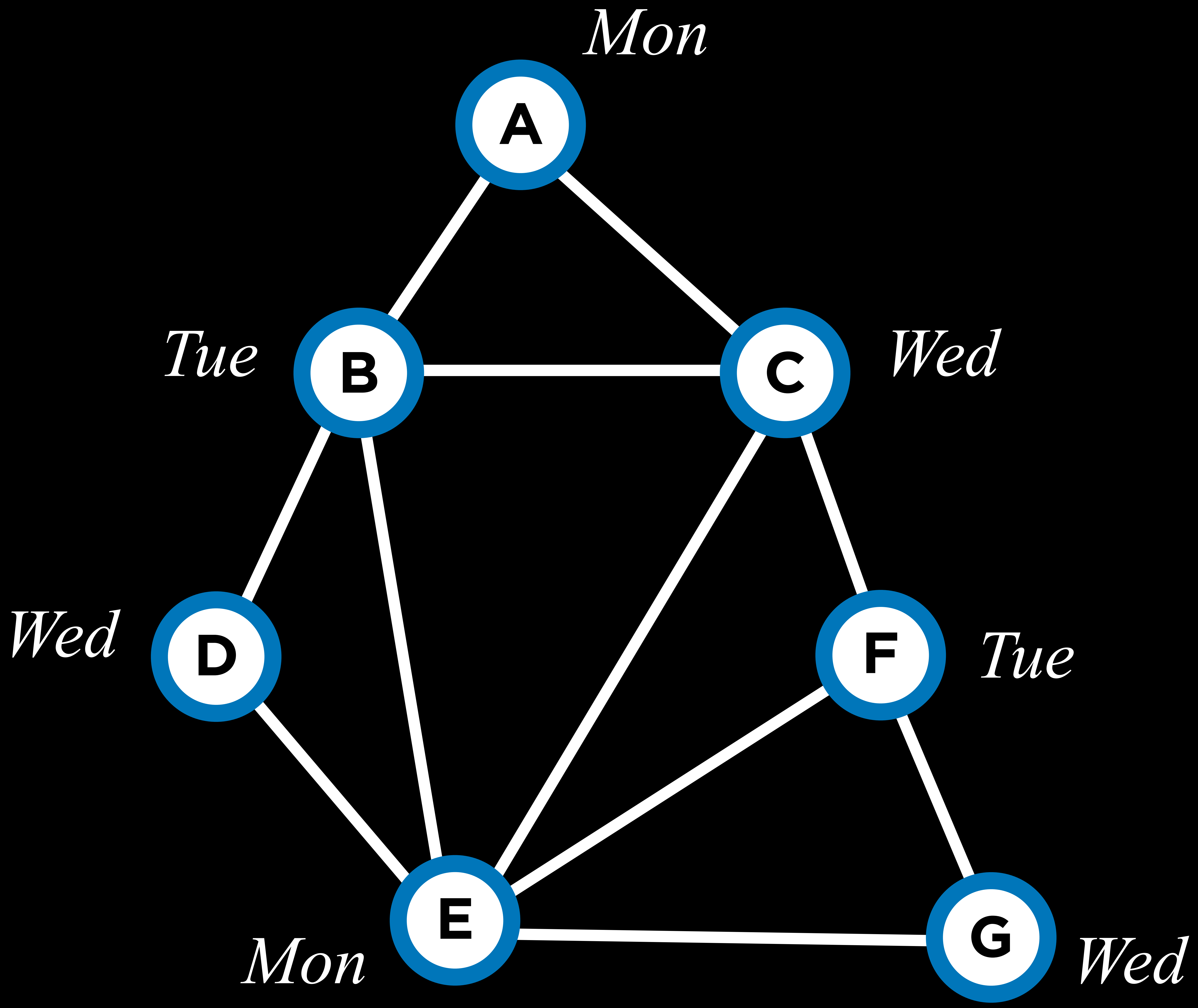

Domain-Values

- **least-constraining values** heuristic: return variables in order by number of choices that are ruled out for neighboring variables
  - try least-constraining values first

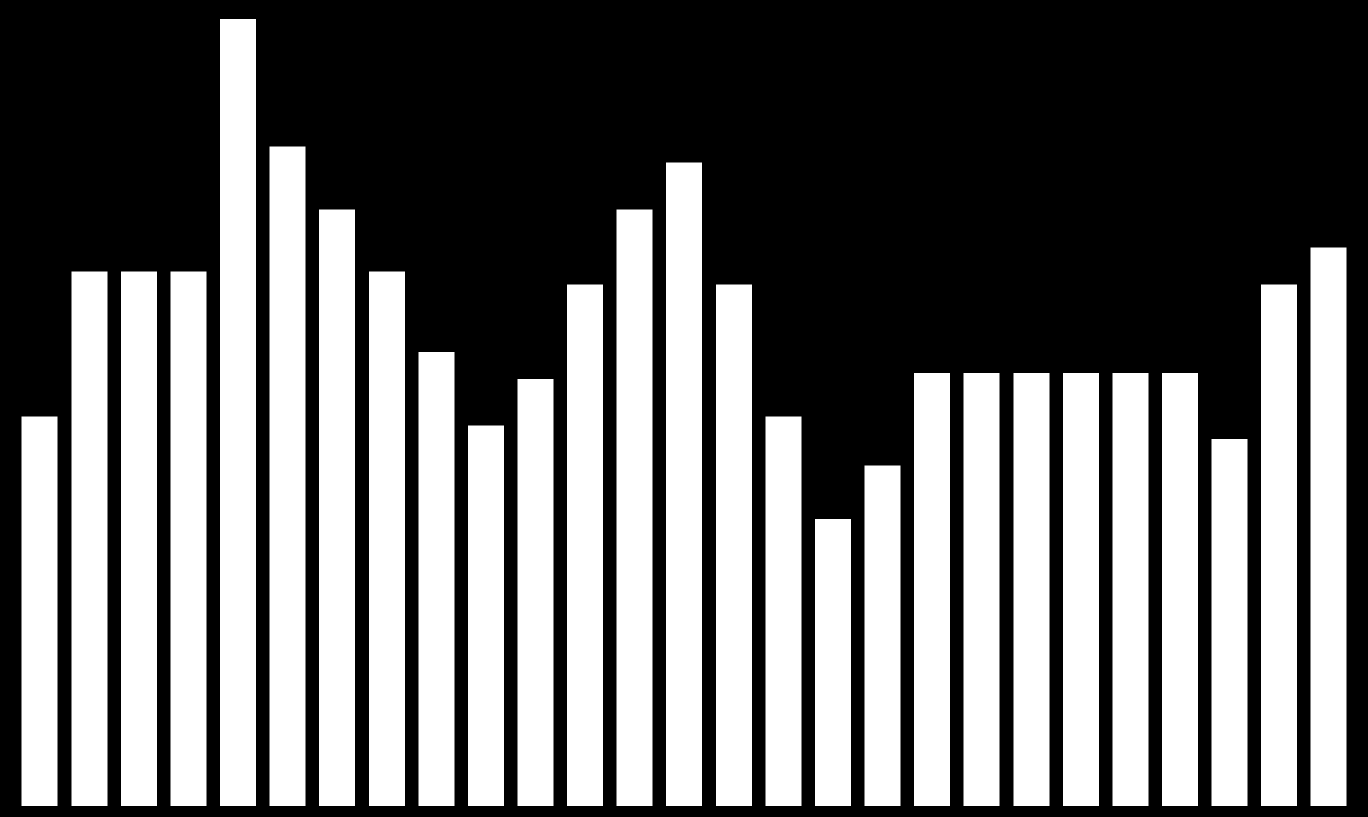


Mon
A
Choosing Tue will prune 3 options
Choosing Wed will prune 2 options
{Mon, Tue, Wed}
B
C
{Tue, Wed}
{Mon, Tue, Wed}
D
F
{Mon, Tue}
{Mon, Tue, Wed}
E
G
Wed


Mon
A
less options for other nodes
increases dead-ends
{Mon, Tue, Wed}
B
C
Wed
{Mon, Tue, Wed}
D
F
{Mon, Tue}
E
G
{Mon, Tue, Wed}
Wed


Mon
A
Tue
B
C
Wed
Wed
D
F
Tue
E
Mon
G
Wed

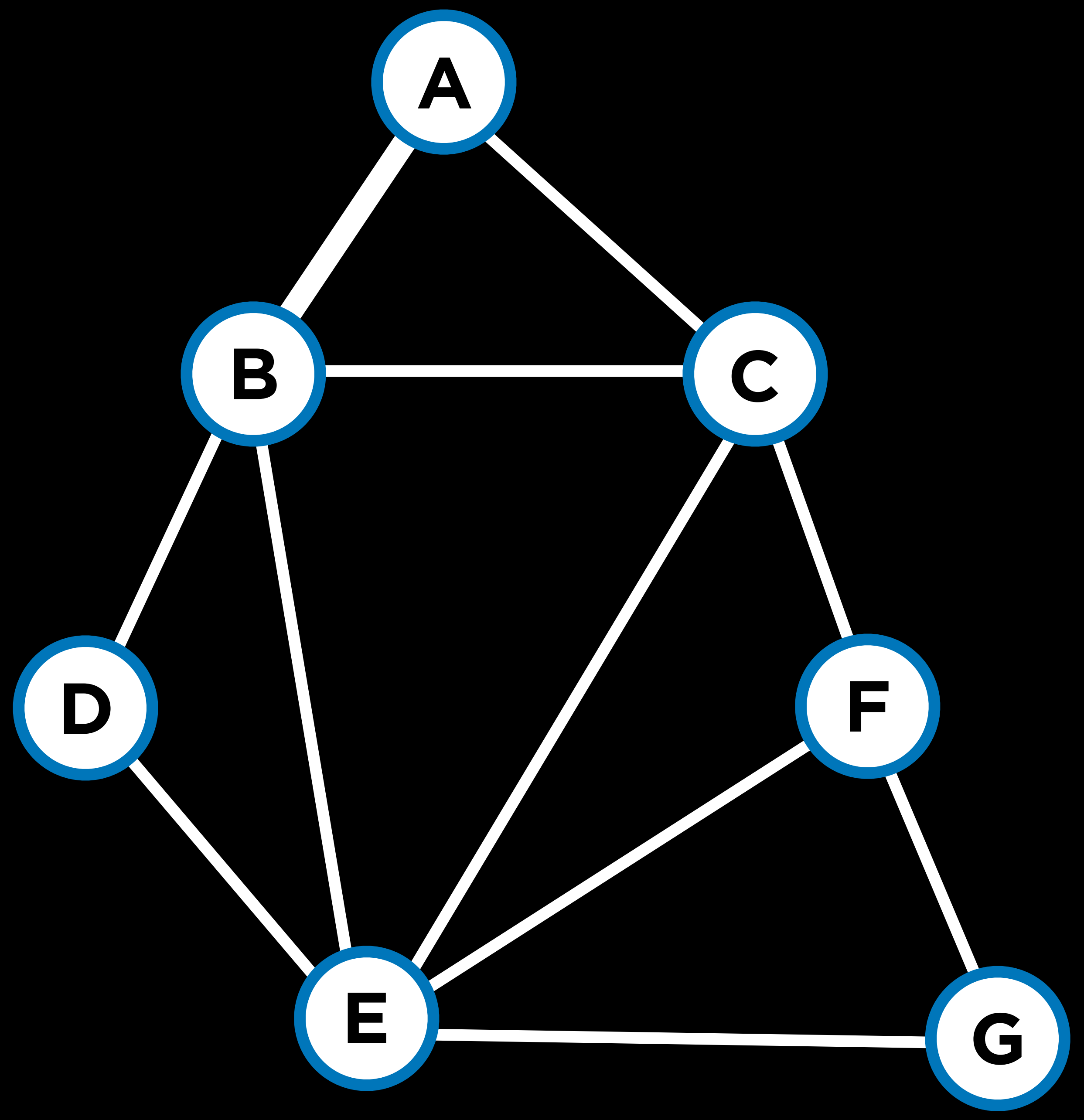
Problem Formulation



Local Search

$$50x_1 + 80x_2$$
$$5x_1 + 2x_2 \leq 20$$
$$(-10x_1) + (-12x_2) \leq -90$$

Linear Programming



Constraint Satisfaction

Optimization

Introduction to

Artificial Intelligence

with Python