MTS 105

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Set Theory Simplified

## Set theory

→ Definition: Set can be defined as the collection of objects or things that is well defined.

→ It can also be defined as the arrangement of objects, alphabets, numbers etc in a curly bracket {}

Examples of set includes:
- Collection of students in FUNAAB
- Letters of the alphabet
- Collection of positive numbers
- Even numbers.

Note: that capital letters denotes set while small letters denotes the members of the set.

E.g $A = \{a, b, c, d\}$

Definitions of terms use in set theory

1) Members of a set: It simply means the member of a given set belongs to or does not belong to the real set.

For example

If $B = \{a, b, c, d, e\}$; we can say that

$\boxed{a \in B}$: is the notation for 'a is a member of B' or 'a belongs to set B'.

$\boxed{g \notin B}$: is the notation for 'g is not a member of B' or 'g does not belong to set B'.

2) Undefined set: This is a set where all the members of the set are being listed. Examples include $\{2, 4\}$, $\{2, 4, 5, 6\}$, $\{a, e, i, o, u\}$

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3) Defined Set: This is a set where the members of the set are not being listed. Examples include

$A = \{x : x^2 = 4\}$

$B = \{x ; x \in N, 1 \leq x \leq 7\}$

$C = \{x : x^2 - 6x + 8 = 0\}$

4) Finite Set: It is a set whose members are countable i.e one knows the beginning and the ending. → CBT

For example:

- Even numbers between 2 to 10
- Positive integers between 1 to 100
- Days in a week
- Months in a year
- Colleges in FUNAAB

CBT Question

When the number of elements in a given set is finite, the set is called a Finite set

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5) Infinite Set: It is a set whose members are uncountable i.e One knows the beginning but does not know the end.

For examples

- Even numbers
- Positive integers
- Natural numbers
- Rational numbers
- Complex numbers

6) Natural numbers (N): These are Counting numbers that is positive whole numbers excluding 0. They lie between 1 to $\infty$.

$N = \{x : x$ is a natural or counting number, $1, 2, 3, 4, \ldots\}$

For example:

Determine $E = \{x \in N, x^3 - 7x + 6 = 0 \text{ and } 3x = 9\}$

Factorizing

$x^3 - 7x + 6 = 0, \quad x = 1, 2, -3$

$3x = 9 \quad , \quad x = 3$

Note; and when use in A set means union. → CBT

$\{-3, 1, 2, 3\}$

So Since $-3$ is not a natural number

Therefore $E = \{1, 2, 3\}$

7) Integers ($Z$): These are positive and negative whole numbers

$Z = \{x \in Z, \ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Note

$Z^+ = \{x : x \in Z : x \geq 0\}$

$Z^- = \{x ; x \in Z : x \leq 0\}$

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For example:

i) Determine element of $S = \{x \in Z, 2x^3 - 3x^2 - 5x + 6 = 0\}$

Factorizing

$2x^3 - 3x^2 - 5x + 6 = 0, \quad x = 1, 2, -3/2$ → CBT

So Since $-3/2$ is not a integer

Therefore $S = \{1, 2\}$

ii) $-1 \leq x \leq 4$, is a set containing

Answer: $\{-1, 0, \ldots, 4\}$ or $\{-1, 0, 1, 2, 3, 4\}$ → CBT

8) Rational number ($Q$): These can be defined as fractional numbers.

$Q = \{x : x \in Q ; x = a/b ; \text{ where } a, b \in Z, b \neq 0\}$

Note

$Q^+ = \{x \in Q : x \geq 0\}$

$Q^- = \{x \in Q : x \leq 0\}$

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9) Irrational number: These can be defined as a countious number. Examples include pie, $\sqrt{2}$ etc

10) Real number (R): Real number is the combination of any number, both positive and negative whole number or fractions but not a Complex number.
Examples $\{\ldots -3, 0, 1, \frac{1}{2}, \sqrt{2}, \ldots\}$
$R = \{x : x \text{ is a real number}\}$

Titilayo

For example

i) Determine $V = \{x \in R, x^2 = 16 \text{ and } 3x = 9\}$
factorizing
$x^2 = 16$, $x = 4, -4$ → CBT
$3x = 9$, $x = 3$
Note; and when use in A set means Union
Therefore $V = \{-4, 3, 4\}$

ii) Given a set $S = \{x : x \in R, x^2 + 1 = 0\}$ then members of S are
Factorizing → CBT
$x^2 + 1 = 0$, $x = \pm i$
Therefore, $S = \{\}$

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## Practice questions

1) Given that $E = \{$positive integers$\}$, identify which of the following sets is equal to $\{2, 4\}$
$A = \{$Even numbers less than 6$\}$
$B = \{x : x < 5\}$
$C = \{x : (x-2)(x-4)(x+2) = 0\}$

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Answer:
$A = \{$Even numbers less than 6$\}$
$A = \{2, 4\}$
Therefore, $A = \{2, 4\}$

 

$B = \{x : x < 5\}$
$B = \{1, 2, 3, 4\}$
Therefore, $B \neq \{2, 4\}$

$C = \{x : (x-2)(x-4)(x+2) = 0\}$
$C = \{2, 4, -2\}$
Therefore, $C \neq \{2, 4\}$

2) List the elements of the following set if set:
$N = \{1, 2, 3, \ldots\}$

a) $A = \{x : x \in N, 3 < x < 7\}$
$A = \{\text{Natural numbers}; 4, 5, 6\}$
$A = \{4, 5, 6\}$

b) $B = \{x : x \in N, x \text{ is even}, x < 15\}$
$B = \{\text{Natural numbers}, x \text{ is even}; 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$
$B = \{2, 4, 6, 8, 10, 12, 14\}$

c) $D = \{x : x \in N, 4 + x = 3\}$
$D = \{\text{Natural numbers}, x = 3 - 4\}$
$D = \{\text{Natural numbers}, x = -1\}$
Therefore, $D = \{\}$

3) Let Q be rational numbers and Z the set of integers. Write out the elements of

a) $A = \{x : x \in Q, 3 < x < 9\}$
$A = \{x : x \in \text{Rational}, 3 < x < 9\}$
$A = \{3.01, \ldots, 8.99\}$

b) $B = \{x : x \in Q, x^2 + 1 = 10\}$
$B = \{x : x \in \text{Rational}, x^2 = 9\}$
$B = \{x : x \in \text{Rational}, x = \pm 3\}$
$B = \{x : x \in \text{Rational}, +3, -3\}$
$B = \{+3, -3\}$

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c) $C = \{x : x \in \mathbb{Z}, x^2+1 = 10\}$
$C = \{x : x \in \text{Integer}, x^2 = 9\}$
$C = \{x : x \in \text{Integer}, x = \pm 3\}$
$C = \{x : x \in \text{Integer}, +3, -3\}$
$C = \{+3, -3\}$

D) $D = \{x : x \in \mathbb{Z}, x \text{ is odd}, -5 < x < 5\}$
$D = \{x : x \in \text{Integer}, x \text{ is odd}, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$
$D = \{-3, -1, 1, 3\}$

4) List the elements of the following sets:
a) $A = \{x : x$ is a vowel, is not 'a' or 'i'$\}$
$A = \{x : a, e, i, o, u$; is not 'a' or 'i'$\}$
$A = \{e, o, u\}$

b) $B = \{x : x$ is a name of a state in Nigeria, $x$ begins with the letter O$\}$
$B = \{$Ogun, Osun, Oyo, Ondo$\}$

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c) $C = \{x : x \in R, -5 < x < 5\}$
$C = \{-4.9, -4.8, \ldots, 4.9\}$

D) $D = \{x : x \in N, x$ is a multiple of $3\}$
$D = \{3, 6, 9, \ldots\}$

E) $E = \{x : x$ is a citizen of Nigeria, $x$ is a teenager$\}$
$E = \{$Teenagers in Nigeria$\}$

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Further example

Note $N \subseteq Z \subseteq Q \subseteq R \subseteq C$

Answer appropriately; True or False:

1) $Z^+ \subseteq Q^+$

$Z^+ = \{x : x \in Z; x \geq 0$, All set of +ve integers$\}$

$Q^+ = \{x : x \in Q; x \geq 0$, All set of all +ve fractions$\}$

Since all positive integers are also positive fractions

Hence $Z^+ \subseteq Q^+$ True

2) $Z^+ \subseteq Q$

$Z^+ = \{x : x \in Z; x \geq 0$, All set of +ve integers$\}$

$Q = \{x : x \in Q;$ -ve and +ve fractions$\}$

Since all positive integers are fractional numbers e.g. $3/1$, $4/1$

Hence $Z^+ \subseteq Q$ True

3) $Q^+ \subseteq R$

$Q^+ = \{x : x \in Q, x \geq 0$, set of +ve fractions$\}$

$R = \{x ; x \in R$, set of +ve and -ve fractions and integers$\}$

Since all positive fractions are also part of real numbers

Hence $Q^+ \subseteq R$ Impact Simplified

4) $R^+ \subseteq Q$

$R^+ = \{x ; x \in R ; x \geq 0$, +ve fractions and integers$\}$

$Q = \{x ; x \in Q ;$ set of all +ve & -ve fractions$\}$

Since positive fractions and integers are still an element of positive and negative fractions in Q

Hence $R^+ \subseteq Q$

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5) $Q^+ \cap R^+ = Q^+$

$Q^+ = \{x; x \in Q; x \geq 0$; All set of positive fractions$\}$

$R^+ = \{x; x \in R; x \geq 0$; Set of positive fractions and integers$\}$

Therefore $Q^+ \cap R^+ = Q^+$ because all positive fractions are also positive real numbers

True

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6) $Z^+ \cup R^+ = R^+$

$Z^+ = \{x; x \in Z; x \geq 0$; All positive integers$\}$

$R^+ = \{x; x \in R, x \geq 0$; All positive fractions and integers$\}$

Therefore $Z^+ \cup R^+ = R^+$; because $Z^+$ is contained in $R^+$

True

7) $R^+ \cap C = R^+$

$R^+ = \{x; x \in R, x \geq 0$; All positive fractions and integers$\}$

$C = \{$Set of complex numbers i.e $a+bi\}$

Therefore $R^+ \cap C = R^+$, because real numbers are said to be a subset of complex numbers

True.

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8) $C \cup R = R$

$C = \{$set of complex numbers i.e $a+bi\}$

$R = \{$Set of positive fractions and integers$\}$

Therefore $C \cup R \neq R$; but $C \cup R = C$; Since real numbers are said to be a subset of complex numbers

False

9) $Q^+ \cap Z = Z$

$Q^+ = \{$Set of positive fractions$\}$

$Z = \{$Set of positive and negative integers$\}$

Therefore $Q^+ \cap Z = Z^+ \neq Z$

$Q^+ \cap Z \neq Z$

False

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10) $\mathbb{Z} \cup Q = \mathbb{Z}$

$\mathbb{Z}$ = {set of positive and negative integers}

$Q$ = {set of positive and negative fractions}

$\mathbb{Z} \cup Q$ = {positive and negative integers and fractions} = $R$

Hence $\mathbb{Z} \cup Q = R \neq \mathbb{Z}$ — False

## Continuation of Terms

11) Cardinality of a set, Order of a set, $|A|$, $n(A)$: It can be defined as the number of elements in a given set.

Example

1) If $A = \{1, 2, 3, 4, 5\}$, therefore the $n(A)$ or $|A| = 5$ → CBT

2) Find the cardinality of $A = \{5, 4, 7, 3, 1, 0\}$. → CBT

Cardinality of $A = 6$

12) Subset ($\subset$): Set A is said to be a subset of set B if the same element in set A are in set B leaving no remainder

For Example

$A = \{2, 4\}$ and $B = \{x : x^2 - 6x + 8 = 0\}$

factorizing

$$x^2 - 4x - 2x + 8 = 0$$

$$x(x-4) - 2(x-4) = 0$$

$$(x-2)(x-4) = 0$$

$$x = 2, 4$$

Therefore, $A = \{2, 4\}$ and $B = \{2, 4\}$, hence A is said to be a subset of B.

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13) Proper Subset ($\subseteq$): Q is said to be a proper subset of P if there is at least one member of P which is not a member of Q. That is, common elements but leaving a remainder in the other set.

For example
If P = {a, b, c, d, e, f} and Q = {c, d, e}, therefore Q is said to be a proper subset of P.

14) Power of a set, Boolean algebra, derived set: It simply means in how many ways can a given set exist or how many subsets are there in a given set. Note that the formula for a derived set is $2^n$, where n is the number of elements in a set.

For example:

1) Consider the set A = {1, 2, 3}
$2^n = 2^3 = 8$ ways → CBT
Power set of A = The following sets can be derived from the set = { }, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}

Note: The set {1, 2, 3} is not a proper subset of A but a subsets; where as all others including { } are proper subsets of A.

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2) Find the power set of A = {a, b, c}
$2^n = 2^3 = 8$ ways → CBT
Power set of A = { }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}

3) Find the power set of S = {3, 4}
$2^n = 2^2 = 4$ ways → CBT
Power set of S = { }, {3}, {4}, {3, 4}

4) Find the power set of S = {a, {b, c}}
$2^n = 2^2 = 4$ ways → CBT
Power set of S = { }, {a}, {b, c}, {a, {b, c}}

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5) Find the power set P(A) of A = {1,2,3,4,5} → CBT
Answer: Power set of A = $2^n = 2^5 = 32$

6) How many subsets will a set containing 5 elements have? → CBT
$= 2^n = 2^5 = 32$

7) Empty set {} is contained in any set. → CBT

8) How many subset does the set A = {a,b,c,d,e} have? → CBT
Answer = $2^n = 2^5 = 32$

9) The set of all subsets of a set X is called the power of set X. → CBT

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10) Consider the following set:
A = {a}, B = {a, c, b}, C = {c, a}, D = {c, b, a}
E = {{b}, {}}
i) which of them is a subset of X = {a, b, c}?
ii) which is a proper subsets of X?
Answer: (B and D) are Subset of X while (A, C, E) are proper subsets of X.

15) Equality or equity of sets: Two set X and Y are said to be equal if and only if X ⊂ Y and Y ⊂ X. That is, if every elements in X are in Y and viceversa.
For example:
If X = {1, 2, 3} and Y = {3, 1, 2}, then we can say that X ⊂ Y and Y ⊂ X, therefore X = Y CBT

Set A is said to be equal to set B if A ⊂ B and B ⊂ A, which now makes A = B CBT

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16) Equivalent of a set: Two sets A and B are said to be equivalent if they have the same cardinality or number of a set i.e. $n(A) = n(B)$, and note that it is not ~~ness~~ neassary that they have the same elements or they are a subset of each other.

T. Titians

For example
1) If $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ then A is equivalent to B since $n(A) = n(B)$ → CBT

2) state whether or not the following are equals
1) $A = \{a, b, d, c\}$ and $B = \{e, d, a, c\}$
$A \not\subset B$ and $B \not\subset A$ therefore $A \neq B$
But A is equivalent to B since $n(A) = n(B)$ → CBT

2) $F = \{1, 2, 5\}$ and $H = \{5, 1, 2\}$
F is equal to H since $F \subset H$ and $H \subset F$, which makes $F = H$

17) Singleton set: It is a set which has only one member as its element. Example: {3}, {FUNDAMB}, {Watimagbo}.

18) Universal set: It is a subset of a larger set. or a set that contains all the given set. Examples include FUNABITE, positive numbers, whole numbers etc.
~~For example~~

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19) Null set or empty set or void set: It simply means a set that has no member or element. It is denoted as {} or $\emptyset \neq \{0\}$

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2a) Zero set: It is a set whose only element is 0, that is {0}.

For example

1) Consider the words

i.) Empty

ii.) Void

iii.) Zero

iv.) Null

Which of these word is different from the other and why?

Answer: Other set has no element but zero set has an element which is 0.

2) Use the sets below to answer the questions that follows: $X = \{x : x^2 = 9, 2x = 4\}$, $Y = \{x : x \neq x\}$, $Z = \{x : x + 8 = 8\}$

a) Is X an empty set?

b) Is Y an empty set?

c) Is Z an empty set?

Note that (,) = intersection

$X = \{x : x^2 = 9, 2x = 4$

$x = \pm 3, x = 2$

$X = \{\}$, Since there is no common element

$Y = \{x : x \neq x\}$

Since $x$ is not equal to $x$, therefore $Y = \{\}$

$Z = \{x : x + 8 = 8\}$

$x = 8 - 8$

$x = 0$

Therefore $Z = \{0\}$

Expect More Simplified material

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