

1. Obtain  $\lim_{x \rightarrow +\infty} \frac{6+3x-x^3}{2x^3+2x^2+9}$ . (a) 3 (b) 2 (c)  $-\frac{1}{2}$  (d)  $-\frac{1}{3}$

2. Determine  $\lim_{x \rightarrow +\infty} \frac{2x^2+2x+6}{3x^3+2x+9}$ . (a) 0 (b) 1 (c)  $\frac{2}{3}$  (d)  $\frac{3}{2}$

3. If  $\sin\theta = \frac{a}{b}$ , where  $\theta$  is an acute angle express  $\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}$  in terms of  $a$  and  $b$ . (a)  $\frac{b}{a^2(b-a)}$  (b)  $\frac{b^4}{a^2(b^2-a^2)}$  (c)  $\frac{a^2}{b-a}$  (d)  $\frac{ba}{a^2}$

4. State  $\frac{\sin\frac{\pi}{3}+\cos\frac{\pi}{3}}{\tan\frac{\pi}{3}}$  in surd form. (a) 3 (b) 7 (c)  $\frac{3+\sqrt{3}}{6}$  (d)  $\frac{3-\sqrt{3}}{6}$

5. Given that  $h(x) = \frac{2x-1}{3-x}$ , find the largest domain of  $h^{-1}(x)$ . (a)  $\mathbb{R} \setminus \{3\}$  (b)  $\mathbb{R} \setminus \{-2\}$  (c)  $\mathbb{R} \setminus \{2\}$  (d)  $\mathbb{R} \setminus \{-3\}$

6. Simplify  $\cos 4\theta$  (a)  $8\cos\theta - 8\sin\theta + 2$  (b)  $8\cos^4\theta - 8\cos^2\theta + 1$  (c)  $3\cos^4\theta - 2\sin^3\theta$  (d)  $\cos\theta + \sin^2\theta$

7. If  $f(x) = 3x^2 - 2$  and  $g(x) = x + 2$ , find  $x$  such that  $f(g(x)) = g(f(x))$ . (a)  $-\frac{3}{4}$  (b)  $\frac{3}{4}$  (c)  $-\frac{4}{3}$  (d)  $\frac{4}{3}$

8. Given  $f(x) = \frac{x^2}{x^2+2}$  and  $g(x) = \sqrt{x-2}$ ,  $x \geq 2$  find the value of  $x$  for which  $f(g(x))$  is not defined. (a) 0 (b) 1 (c) 2 (d) 3

9. Express  $240^\circ$  in radians as a multiple of  $\pi$ . (a)  $3\pi$  (b)  $\frac{3\pi}{2}$  (c)  $\frac{\pi}{8}$  (d)  $\frac{4\pi}{3}$

10. Find the largest domain of  $f(x) = \sqrt{9-x^2}$ . (a)  $x \geq 3$  (b)  $-3 \leq x \leq 3$  (c)  $x \geq -3$  (d)  $x \leq -3$  or  $x \geq 3$

11. If  $\tan\theta = \frac{a}{b}$ , where  $\theta$  is an acute angle find  $\frac{a\cos\theta - b\sin\theta}{a\cos\theta + b\sin\theta}$ . (a) 0 (b) 5 (c) 6 (d) 1

12. Let the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = \begin{cases} 4x+7, & \text{if } x > 3 \\ x^2 - 3, & \text{if } -2 \leq x \leq 3 \\ 2x+3, & \text{if } x < -2 \end{cases}$ . find  $f(-1)$ .  
 (a) 2 (b) 1 (c) -1 (d) -2

13. Simplify  $\frac{\sec\theta}{\cos\theta} - \frac{\tan\theta}{\cot\theta}$ . (a) 1 (b) 7 (c) 6 (d) 4

14. Given that  $\tan\theta = \frac{2t}{1+t^2}$ , and that  $\theta$  is an acute angle, express  $\sin\theta$  in terms of  $t$ . (a)  $5t$  (b)  $\frac{2t}{1+t^2}$  (c)  $2t - 4$  (d)  $\frac{t}{2}$

15.  $2\cos 5\theta \sin 2\theta$  equals (a)  $\sin 2\theta - \cos\theta$  (b)  $\sin 5\theta + 3\sin\theta$  (c)  $\sin 7\theta - \sin 3\theta$  (d)  $\sin\theta + \dots$  30

16. Which of the following is true about the mappings  $f$  and  $g$  ( $f \neq g$ ). (a)  $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$   
 (b)  $(f \circ g)^{-1} = f^{-1} \circ g^{-1}$  (c)  $(f \circ g)^{-1} = f \circ g^{-1}$  (d)  $(f \circ g)^{-1} = f^{-1} \circ g$

D'OLA @ FUNA ABSU 001

-5/6

Find the range of  $f(x) = x^2 + 2$ . (a)  $\{x \in \mathbb{R} : 0 \leq x\}$  (b)  $\{x \in \mathbb{R} : x > 2\}$   
(c)  $\{x \in \mathbb{R} : x < 2\}$  (d)  $\{x \in \mathbb{R} : x \geq 2\}$

Determine  $\tan\theta$  if  $3\sec^2\theta = \text{constant}$  (a)  $3\pi/4$  (b)  $3\pi/2$

If  $\alpha$  and  $\beta$  are two acute angles such that  $\sin\alpha = \frac{3}{5}$  and  $\cos\beta = \frac{4}{5}$ , by using the trigonometrical table, evaluate  $\tan(\alpha - \beta)$ . (a)  $\frac{1}{2}$  (b)  $\frac{3}{2}$  (c)  $\frac{5}{2}$  (d)  $\frac{7}{2}$

A function which is both one-to-one and onto is called (a) injective (b) surjective (c) bijective (d) map

S/N	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

S/N	A	B	C	D
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

## UNIVERSITY OF AGRICULTURE ABEOKUTA

Department of Mathematics

2008/2009 Second Semester Examinations

MTS 102: Calculus

Answer All Questions by shading your answers in the objective answer sheet provided

Time Allowed: 2 Hours. Type: B

A curve  $y$  is given by

$$y = \frac{1}{\sqrt{4x - 3}}$$

D'OLA @ FUNAABSU 001

Now answer the next 3 questions

1. Find the relationship between
- $y$
- ,
- $\frac{dy}{dx}$
- and
- $\frac{d^2y}{dx^2}$
- .

(A)  $y(4x - 3)\frac{d^2y}{dx^2} + [(4x - 3)\frac{dy}{dx} + 8y]\frac{dy}{dx} = 0$  (B)  $(4x - 3)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 16y = 0$

(C)  $x\frac{d^2y}{dx^2} + (4x - 3)\frac{dy}{dx} + 8y = 0$  (D)  $\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^2 + 16y = 0$

2. What is the value of

$$\frac{\frac{d^2y}{dx^2}}{\sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3}}$$

when  $x = 1$ ? (A) 0.4 (B) 1.4 (C) 2.4 (D) ~2.4

3. Find the value of
- $p^2 + ab + q^2 - cd$
- if
- $ay + bx = p$
- and
- $cy + dx = q$
- are the equations of the tangent and the normal to the curve respectively at the point (1,1). (A) -10 (B) 10 (C) -14 (D) 14

4. If
- $y = (3x^2 - 4)^3$
- and
- $y$
- is increased by 0.5% when
- $x = 2$
- , find the approximate percentage change in
- $y$
- . (A) 4% (B) 4.5% (C) 5% (D) 5.5%

5. Let
- $p = kv^{4/3}$
- where
- $k$
- is a constant. What approximate percentage increase in
- $v$
- will cause a 4% increase in
- $p$
- ? (A) 1% (B) 2% (C) 3% (D) 4%

6. The volume of a cube is increasing at the rate of
- $2 \text{ cm}^3 \text{s}^{-1}$
- . Find the rate in
- $\text{cm s}^{-1}$
- of the change of the side of the base when its length is 3 cm. (A)
- $2/27$
- (B)
- $2/7$
- (C)
- $2/9$
- (D)
- $2/3$

7. The area of a circle is increasing at the rate of
- $3 \text{ cm}^2 \text{s}^{-1}$
- . Find the rate in
- $\text{cm s}^{-1}$
- of change of the circumference when the radius is 2 cm. (A)
- $2/3$
- (B)
- $3/2$
- (C)
- $1/3$
- (D) 2

8. In the formula
- $T = 2\pi\sqrt{\frac{l}{g}}$
- , an error of 1% is incurred in the measurement of
- $T$
- . What will be the percentage error in the calculated value of
- $g$
- ? (A) 1% (B) 2% (C) 3% (D) 4%

9. Let
- $yx^\alpha = \beta$
- where
- $\alpha$
- and
- $\beta$
- are constants. If
- $\delta y$
- and
- $\delta x$
- are corresponding small changes in
- $y$
- and
- $x$
- respectively, find an expression for
- $\frac{\delta y}{y}$
- in terms of
- $\alpha$
- ,
- $\delta x$
- and
- $x$
- . (A)
- $\alpha\frac{x}{\delta x}$
- (B)
- $-\frac{1}{\alpha}\frac{\delta x}{x}$
- (C)
- $-\alpha\frac{\delta x}{x}$
- (D)
- $\alpha\frac{\delta x}{x}$

10. If
- $y = \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$
- , find
- $\frac{dy}{dx}$
- . (A)
- $\frac{2}{1+x^2}$
- (B)
- $\frac{-\delta x}{(1+x^2)^2}$
- (C)
- $\frac{8x}{1+x^2}$
- (D)
- $\frac{-2}{1+x^2}$

11. A particle moving in a straight line covered a distance
- $x$
- after time
- $t$
- . If
- $v$
- is the velocity of the particle and
- $a$
- is its acceleration, which of the following is (are) true of the motion of the particle?

(I)  $v = \frac{dx}{dt}$

(II)  $a = \frac{dv}{dt}$

(III)  $a = \frac{d^2x}{dt^2}$

(IV)  $a = v\frac{dv}{dx}$

(V)  $x = 0$  when  $a = 0$

(VI)  $x = 0$  when  $v = 0$

- (A) I, II and III only (B) All except IV (C) All except V and VI (D) All

1. If  $x = \sin p$  and  $y = \cos q$ , find the value of  $\frac{y^4 - x^4}{y^2 - x^2}$ . (a)  $1 + \sin^2 p$   
 (d)  $\sin^2 p + \cos^2 q$
2. The secant of an angle is  $-1.0590$  and its cosecant is negative. Find the angle between  $500^\circ$  and  $600^\circ$ . (a)  $539.3^\circ$  (b)  $549.3^\circ$  (c)  $559.3^\circ$  (d)  $599.3^\circ$
3. Given that  $\tan 2x = \frac{3}{4}$ , find the value of  $\tan 3x$ . (a)  $\frac{5}{9}$  (b)  $\frac{7}{9}$  (c)  $\frac{11}{9}$  (d)  $\frac{13}{9}$
4. Simplify  $\left[\frac{1+\sin x}{1-\cos x}\right]\left[\frac{1+\sec x}{1+\csc x}\right]$ . (a)  $\sin x$  (b)  $\cos x$  (c)  $\tan x$  (d)  $\cot x$
5. Find  $x$  given that  $\tan^{-1}(2x+1) - \tan^{-1}(2x-1) = \tan^{-1}\frac{1}{8}$ . (a)  $\pm 1$  (b)  $\pm 2$  (c)  $\pm 3$  (d)  $\pm 4$
6. Find the general solution of the equation  $\cos \frac{x}{5} = \frac{\sqrt{3}}{2}$ . (a)  $n\pi \pm \frac{5\pi}{6}$  (b)  $10n\pi \pm \frac{5\pi}{6}$  (c)  $n\pi$   
 (d)  $10n\pi$
7. Find  $\lim_{x \rightarrow 0} (x^2 + \frac{\cos 5x}{10,000})$ . (a) 0.00010 (b) 0.01000 (c) 0.01000 (d) 0.00001
8. Determine the  $\lim_{x \rightarrow \infty} \frac{4x^2 + 2x - 1}{3x^2 + x - 2}$ . (a)  $\frac{4}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{2}{3}$  (d)  $\frac{3}{5}$  D'OLA @ FUNAABSU 001
9. If  $f(x) = \frac{x+1}{x^2+2x-2}$ , find the values of  $x$  where the function  $f(x)$  will have no images. (a) 1, 2  
 (b) 2, -1 (c) -2, 1 (d) -1, -2
10. If  $\cos x = \frac{15}{17}$ ,  $\sin y = \frac{20}{29}$ , where  $x$  is reflex and  $y$  is obtuse, find the value of  $\sin(x+y)$ .  
 (a)  $\frac{209}{403}$  (b)  $\frac{225}{403}$  (c)  $-\frac{468}{403}$  (d)  $\frac{466}{403}$
11. If  $\cos x = \frac{15}{17}$ ,  $\sin y = \frac{20}{29}$ , where  $x$  is reflex and  $y$  is obtuse, find the value of  $\cos(x+y)$ .  
 (a)  $-\frac{409}{403}$  (b)  $-\frac{475}{403}$  (c)  $-\frac{525}{403}$  (d)  $\frac{475}{403}$
12. If  $\cos x = \frac{19}{17}$ ,  $\sin y = \frac{20}{29}$ , where  $x$  is reflex and  $y$  is obtuse, find the value of  $\cot(x+y)$ .  
 (a)  $\frac{476}{133}$  (b)  $\frac{132}{475}$  (c)  $\frac{475}{133}$  (d)  $-\frac{475}{132}$
13. If  $t = \tan \frac{\pi}{2}$ , express the expression  $\sqrt{\frac{1+\sin x}{1-\sin x}}$  in terms of  $t$ . (a)  $\frac{1+t}{1-t}$  (b)  $\frac{1-t}{1+t}$  (c)  $\frac{2t}{1-t^2}$  (d)  $\frac{2t}{1+t^2}$
14. On what intervals is the function  $f(x) = x^{100} - 2x^{37} + 7$  continuous? (a)  $[-\infty, \infty]$   
 (b)  $(-\infty, \infty)$  (c)  $(-\infty, \infty)$  (d)  $[-\infty, \infty]$
15. Find  $\lim_{x \rightarrow 2} (\frac{3x^2 - 12}{x - 2})$ . (a) 10 (b) 8 (c) 12 (d) 14
16. Find  $\lim_{n \rightarrow \infty} (\frac{3^{n+1} - 5^{n+1}}{3^n - 5^n})$ . (a) 5 (b) 15 (c) 19 (d) 10
17. Find  $\lim_{x \rightarrow 1} (\frac{4x^2 + 3x - 5}{2x^2 + x - 2})$ . (a) 5 (b) 7 (c) 6 (d) 10
18. If  $f(-x) = f(x)$ , then  $f(x)$  is . (a) an even function (b) an odd function  
 (c) a periodic function (d) monotonic function
19. If  $g : y \mapsto \sqrt{y+5}$ . Determine  $f(g(20))$ . (a) 24 (b) 27 (c) 16 (d) 25
20. The function  $f(x) = 2^x$  is . (a) The set of positive integers (b) The set of complex numbers (c) The set of real numbers

- Determine the range of the real valued function  $f(x) = \frac{3}{4x^2+1}$
- A.  $\mathbb{R} - \{-\frac{1}{4}\}$  B.  $\mathbb{R} - \{0\}$  C.  $\mathbb{R} - \{\frac{1}{4}\}$  D.  $\{0\}$
37. If a side of a square increases by 0.5 percent, find the approximate percentage in the area
- A. 1 percent B. 2 percent C. 0.5 percent D. 5 percent
38. Find the rate at which the volume of a spherical balloon is increasing if the surface area is increasing at the rate of  $6\text{cm}^2\text{s}^{-1}$  when the radius of the spherical balloon is 5cm.
- A.  $10\text{cm}^2\text{s}^{-1}$  B.  $15\text{cm}^2\text{s}^{-1}$  C.  $30\text{cm}^2\text{s}^{-1}$  D.  $20\text{cm}^2\text{s}^{-1}$
39. The motion of a particle along a straight line is specified by the equation  $s = 4t^4 - 3t^3$ , find the acceleration after 5 seconds.
- A.  $1111\text{ms}^{-2}$  B.  $378\text{ms}^{-2}$  C.  $351\text{ms}^{-2}$  D.  $1110\text{ms}^{-2}$
40. Evaluate  $\int \frac{x^2 + ax + 2}{x^2 + x + 1} dx$
- A.  $x + \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) + c$  B.  $x - \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) + c$  C.  $x - \frac{\sqrt{3}}{2} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) + c$  D.  $x + \frac{\sqrt{3}}{2} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) + c$
41. Evaluate  $\int_0^1 \frac{dx}{(2-x)(1+x)}$
- A.  $\frac{1}{3} \log_e 2$  B.  $\log_e 4$  C.  $\frac{1}{3} \log_e 3$  D.  $\frac{1}{3} \log_e 4$
42. Solve  $\frac{dy}{dx} = \frac{2}{x} + \frac{1}{x^2}$  where  $y = -1$  when  $x = 1$ .
- A.  $y = \log_e x^2 + \frac{1}{x} - 2$  B.  $y = 2 \log_e x + \frac{1}{x} - 1$  C.  $y = 2 \log_e x + \frac{1}{x}$  D.  $y = \log_e x^2 - \frac{1}{x} + 2$
43. If  $y = 3t^3 + 2t^2 - 7t + 3$ , find  $\frac{dy}{dt}$  at  $t = -1$ .
- A. -1 B. 1 C. -2 D. 2
44. Find the point  $(x, y)$  on the Euclidean plane where the curve  $y = 2x^2 - 2x + 3$  has 2 as gradient.
- A. (1, 3) B. (2, 7) C. (0, 3) D. (3, 15)
45. Differentiate  $\frac{5x^3 - 5x^2 + 1}{3x^2}$  with respect to  $x$ .
- A.  $2 + \frac{2}{3x^2}$  B.  $2 + \frac{1}{6x}$  C.  $2 - \frac{3}{3x^2}$  D.  $2 - \frac{1}{6x}$
46.  $\frac{d}{dx} \cos(3x^2 - 2x)$  is equal to
- A.  $-\sin(6x - 2)$  B.  $-\sin(3x^2 - 2x)$  C.  $(6x - 2)\sin(3x^2 - 2x)$  D.  $-(6x - 2)\sin(3x^2 - 2x)$
47. If  $y = x \sin x$ , find  $\frac{d^2y}{dx^2}$ .
- A.  $2 \cos x - x \sin x$  B.  $\sin x + x \sin x$  C.  $\sin x - x \cos x$  D.  $x \sin x - 2 \cos x$
48. Obtain a maximum value of the function  $x^3 - 12x + 1$ .
- A. -5 B. -2 C. 2 D. 27
49. If  $x$  is an obtuse angle and  $\tan 2x = \frac{5}{12}$ , what is the value of  $\cos 4x$ ?
- (A)  $-\frac{119}{169}$  (B)  $\frac{119}{169}$  (C)  $-\frac{24}{25}$  (D)  $-\frac{12}{13}$
50. Given that  $\tan(x+y) = 1$  and  $\tan(x-y) = \frac{1}{7}$ , what is the value of  $(\tan x, \tan y)$  if  $x$  and  $y$  are acute?
- (A)  $(1/2, 1/3)$  (B)  $(2, 3)$  (C)  $(1/3, 1/2)$  (D)  $(3, 2)$
51. If  $x$  is an acute angle such that  $\sin x = 3/5$  and  $y$  is an obtuse angle such that  $\cos y = -5/13$ , find the value of  $\sin y + \sin x \cos(x+y) + \cos x \sin(x-y)$ .
- (A)  $31/65$  (B)  $-24/25$  (C)  $144/169$  (D)  $-24/65$
52. Find the value of  $\sin 345^\circ$  in surd form
- (A)  $1/4(\sqrt{2} - \sqrt{3})$  (B)  $1/4(\sqrt{2} - \sqrt{5})$  (C)  $1/4(\sqrt{2} - \sqrt{6})$  (D)  $1/4(\sqrt{6} - \sqrt{2})$
53. In radian measure,  $720^\circ$  is equivalent to
- (A)  $\pi$  (B)  $2\pi$  (C)  $3\pi$  (D)  $4\pi$
- D'OLA @ FUNAABSU 001
1. 100% + 100% = 1

12. Determine the point of discontinuity in  $f(x) = \frac{x+3}{x^2 + 6x + 9}$

- (a) 3    (b) -3    (c) 0    (d) none

13. Evaluate the integral  $\int (x^3 - 2)dx$

- (a)  $\frac{x^4}{4} - 2x + c$     (b)  $\frac{x^4}{4} + 2x + c$     (c)  $-\frac{x^4}{4} - 2x + c$     (d)  $\frac{x^4}{4} - 2x - c$

14. Evaluate the integral  $\int (3x+2)^4 dx$

- (a)  $\frac{(3x-2)^5}{15} + c$     (b)  $\frac{(3x-2)^5}{5} + c$     (c)  $\frac{(3x+2)^5}{15} + c$     (d)  $\frac{(3x+2)^5}{5} + c$

15. Determine the area bounded by the curve  $y = e^x$ , at  $x = -2$  and  $x = 3$

- (a) 18.95 units    (b) -18.95 units    (c) 19.95 units    (d) -19.95 units

16. Evaluate  $\lim_{x \rightarrow 2} \frac{3x^2 - 12}{x-2}$

- (a) 12    (b) -12    (c) -6    (d) 6

17. Find  $\lim_{x \rightarrow \infty} \frac{4x^2 + 2x - 1}{3x^2 + x - 2}$

- (a)  $\frac{3}{4}$     (b)  $-\frac{4}{3}$     (c)  $\frac{3}{4}$     (d)  $\sqrt{3}$

18. Find  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 - 4}$

- (a) -2    (b) -3    (c) 4    (d) 1

19. Let the mappings f and g on the set of real numbers be defined by  $f(x) = 2x^2 - 3$  and  $g(x) = x + 1$ . Find gof (a)  $2x^2 + 4x - 1$     (b)  $2x^2 - 4x + 1$     (c)  $2x^2 - 2$     (d)  $2x^2 + 2$

20. Given that  $y = x^2 \cos 2x$ , find the values of  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ , when  $x = \frac{\pi}{2}$

- (a)  $-\pi, \pi^2 - 2$     (b)  $-\pi, \pi^2 + 2$     (c)  $\pi, \pi^2 - 2$     (d)  $\pi, 2$     (e)  $\pi, \pi^2 + 2$

D'OLA @ FUNAABSU OOI

NAME----- MATRIC/NO----- DEPT-----

1	A	B	C	D	11	A	B	C	D
2	A	B	C	D	12	A	B	C	D
3	A	B	C	D	13	A	B	C	D
4	A	B	C	D	14	A	B	C	D
5	A	B	C	D	15	A	B	C	D
6	A	B	C	D	16	A	B	C	D
7	A	B	C	D	17	A	B	C	D
8	A	B	C	D	18	A	B	C	D
9	A	B	C	D	19	A	B	C	D
10	A	B	C	D	20	A	B	C	D

44. If  $f$  and  $g$  are functions defined over the set of real numbers by  $f(x) = x + 2$  and  $g(x) = 2x^2 + 1$  respectively. The value of  $x$  for which  $f \circ g = g \circ f$  is  
 A.  $\frac{-3}{4}$  B.  $\frac{3}{4}$  C.  $\frac{-7}{4}$  D.  $\frac{7}{4}$
45. Determine the constants in the function  $f(x) = ax^3 + bx^2$   $a$  and  $b$  are constants, so that  $f'(1) = 5$  and  $f''(2) = 32$ . Write your answer as (a,b).  
 A. (-3,2) B. (-2,-3) C. (3,-2) D. (-2,3).
46. The function  $f(x) = ax^2 + bx + c$  is defined on the set of real numbers is such that  $f(0) = -4$ ,  $f(1) = 1$  and  $f(-1) = -5$ ,  $a$ ,  $b$  and  $c$  are respectively  
 A. -4,3,2 B. 2,-3,-4 C. 3,2,-4 D. 2,3,-4.

Use this information to answer the next three questions.

Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  be define as  $f(x) = \sqrt{4 - x^2}$  and  $g(x) = \sqrt{x - 1}$ , then

47. Domain of  $f$  is  
 A.  $\{x \in \mathbb{R} : -2 \leq x < \infty\}$  B.  $\{x \in \mathbb{R} : 1 \leq x < \infty\}$  C.  $\{x \in \mathbb{R} : -2 \leq x \leq 4\}$  D.  
 $\{x \in \mathbb{R} : -2 \leq x \leq 2\}$

D'OLA @ FUNAABSU001

48. Domain of  $g$  is  
 A.  $\{x \in \mathbb{R} : 1 \leq x < \infty\}$  B.  $\{x \in \mathbb{R} : -\infty \leq x < \infty\}$  C.  $\{x \in \mathbb{R} : 1 \leq x \leq 2\}$  D.  
 $\{x \in \mathbb{R} : -1 \leq x \leq 1\}$

49. Domain of  $f + g$  is  
 A.  $\{x \in \mathbb{R} : -1 \leq x \leq 1\}$  B.  $\{x \in \mathbb{R} : -2 \leq x \leq 2\}$  C.  $\{x \in \mathbb{R} : 1 \leq x \leq 2\}$  D.  
 $\{x \in \mathbb{R} : 1 \leq x < \infty\}$   
 $\int_1 \frac{dx}{\sqrt{4-x^2}}$  is A.  $\frac{\pi}{3}$  B.  $\frac{2\pi}{3}$  C.  $\frac{4\pi}{3}$  D.  $\frac{3\pi}{2}$

1. Determine the constants in the function  $f(x) = ax^3 + bx^2$ , if  $a$  and  $b$  are constants, so that  $f(3) = 5$  and  $f'(2) = 2$ . Write your answer as (a,b).
- (a) (-3,2) (b) (-2,3) (c) (3,-2) (d) (-2,3) (e) (2,3)
2. Find  $\frac{dy}{dx}$  for  $y = \frac{1-x}{1+x}$ .
- (a)  $\frac{1}{1+x^2}$  (b)  $\frac{-1}{1+x^2}$  (c)  $\frac{1}{1+x^2}$  (d)  $\frac{-1}{1+x^2}$
3. Find  $\frac{dy}{dx}$  if  $y = \tan^{-1}\left(\frac{x}{x+1}\right)$ .
- (a)  $\frac{1}{1+x^2}$  (b)  $\frac{1}{1-x^2}$  (c)  $\frac{1}{1+x^2}$  (d)  $\frac{1}{1-x^2}$
4. If  $f(x) = 2x + 4$  and  $g(x) = 1$ , then  $(f \circ g)(x)$  and  $(f \circ g)(x)$  are respectively.
- (a) 6, 1 (b) 1, 6 (c)  $2x+4$  (d)  $2x^2+4$
5. If  $y = 0^n$ , where  $n$  is a positive integer, then  $0^n \frac{dy}{dx}$  is
- (a)  $ny$  (b)  $n(n-1)y$  (c)  $ny^2$  (d)  $n(n+1)y^2$
6. If  $y = \tan \theta$ , then  $\frac{dy}{dx}$  is.
- (a)  $2y(1-y^2)$  (b)  $y(2y-1)$  (c)  $1+y^2$  (d)  $2y^2+1$
7. If  $f(x) = \sqrt{4-x^2}$  and  $g(x) = \sqrt{x-1}$ , then Domain of  $f+g$  is
- (a) [-1,1] (b) [-2,2] (c) [1,2] (d) [1,  $\infty$ )
8. Find  $\frac{dy}{dx}$  if  $y=f(x)=6u+8$  and  $u=g(x)=-5+10$ .
- (a) -30 (b) -3 (c) 30 (d) - $\frac{1}{5}$
9.  $\int 7e^{-7x} dx$  is
- (a)  $e^{-7x} + C$  (b)  $-e^{-7x} + C$  (c)  $\frac{e^{-7x}}{7} + C$  (d)  $-\frac{e^{-7x}}{7} + C$
10.  $\int \sin^2 x dx$
- (a)  $\frac{x}{2} - \frac{\sin 2x}{4} + C$  (b)  $\frac{1}{2}(x + \sin 2x) + C$  (c)  $\frac{1}{2}(x - \frac{\sin 2x}{2}) + C$  (d)  $\frac{1}{2}(x - \frac{\sin 2x}{6}) + C$
11. Find  $\int \frac{3}{(x+1)(x-2)} dx$
- (a)  $3\ln\left(\frac{x+1}{x-2}\right) + C$  (b)  $\frac{3}{2}\ln\left(\frac{x-1}{x+2}\right) + C$  (c)  $9\ln\left(\frac{x+2}{x-1}\right) + C$  (d)  $3\ln\left(\frac{x+2}{x-1}\right) + C$
12. If  $f(x) = \frac{1}{x^2}$ , then  $f\left(\frac{2x}{x+1}\right)$  is
- (a)  $\frac{1}{x^2}$  (b)  $\frac{1}{x^2}$  (c)  $\frac{1}{x^2}$  (d)  $\frac{1}{x^2}$
13. The equation of the line with slope  $\frac{3}{2}$ , through the point (-3,1) is
- (a)  $3y - 2x - 6 = 0$  (b)  $3y - 2x + 6 = 0$  (c)  $3y + 2x - 6 = 0$  (d)  $-3y - 2x + 6 = 0$
14. Find the equation of a circle passing the points (0,0), (3,1) and (5,5)
- (a)  $x^2 + y^2 + 10y = 0$  (b)  $x^2 + y^2 - 10y = 0$  (c)  $x^2 - y^2 - 10y = 0$  (d)  $x^2 - y^2 + 10y = 0$
15. Find the area bounded by the curve with the parametric equations  $x = at^2$  and  $y = 2at$  with the ordinates at  $t = 0, t = 2$ .
- (a)  $\frac{28a^2}{9}$  (b)  $\frac{18a^2}{9}$  (c)  $\frac{28a^2}{3}$  (d)  $\frac{28a^2}{3}$
16. The gradient at the point (1,1) of the tangent to the curve  $x^2 + y^2 - 3x + 4y - 3 = 0$
- (a) 5 (b) 6 (c)  $\frac{1}{6}$  (d)  $\frac{3}{4}$

D'OLA @ FUNAABSU 001

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

17. Given a function  $f(x)$ , where  $\frac{df}{dx} = 0$  at the point  $(x_0, y_0)$  and  $\frac{d^2f}{dx^2} < 0$  at  $(x_0, y_0)$ , then the point  $(x_0, y_0)$  is a

- (a) minima      (b) maxima      (c) inflection      (d) saddle

18. If  $y = 3x^4 + 3x^2 - 2x + 1/10$ , find  $\frac{dy}{dx}$  at  $x = 1$

$$9x^3 + 4x - 7 \\ 9 - 4 - 7 = -2$$

- (a) -1      (b) 1      (c) -2      (d) 2

19. Find the equation of the curve whose gradient is  $1 - 2x^2$  and which passes through the point  $x = 0, y = 1$

- (a).  $y = x + \frac{2}{3}x^3$       (b).  $y = 1 + x - \frac{2}{3}x^3$       (c).  $y = 1 + x + x^3$       (d).  $y = x - \frac{2}{3}x^3 - 1$

20. Given the parametric equations of the curve with  $x = 3t^2$ ,  $y = 3t - t^3$ . Find the volume generated when the plane is bounded by the curve with ordinates at  $t = 0$  and  $t = 2$ .

- (a). 15.6 units<sup>3</sup>      (b). 1.56 units<sup>3</sup>      (c). -156 units<sup>3</sup>      (d). 156 units<sup>3</sup>

21. The distance between the centres of the circles  $x^2 + y^2 + 8x + 10y - 4 = 0$  and  $x^2 + y^2 - 2x - 4 = 0$  is

- (a).  $\sqrt{45}$       (b).  $\sqrt{50}$       (c). 45      (d). 5

22. Evaluate  $\int (2x+1)^{\frac{1}{2}} dx$

- (a).  $\frac{1}{3}(2x+1)^{\frac{3}{2}} + C$       (b).  $\frac{1}{4}(2x+1)^{\frac{5}{2}} + C$       (c).  $\frac{2}{3}(2x+1)^{\frac{5}{2}} + C$       (d).  $\frac{1}{3}(2x+1)^{\frac{3}{2}} + C$

23.  $\int_{\pi/4}^{\pi/2} \frac{dx}{1-\cos x}$  is      (a).  $\frac{\pi}{4}$       (b).  $\frac{\pi}{3}$       (c).  $\frac{4\pi}{9}$       (d).  $\frac{5\pi}{6}$

24. Evaluate  $\int \frac{\cos x}{1-\sin x} dx$       (a).  $-\log_e \cos x + C$       (b).  $\log_e(1-\sin x) + C$       (c).  $-\log_e(1-\sin x) + C$       (d).  $\log_e \cos x + C$

25.  $\lim_{x \rightarrow 0} \frac{\sin x - \sin 2x}{x - \tan x}$  is      (a). 1      (b). 0      (c). 2      (d).  $\infty$

26. Simplify  $\frac{\sin 3\theta}{\cos 3\theta - \cos 5\theta}$       (a).  $\sin 3\theta$       (b).  $\cos 3\theta$       (c).  $\tan 3\theta$       (d).  $\sec 3\theta$

~~Simplifying  
Cos + Cos  
= 2Cos 30 Cos 20  
= Cos 30 + Cos 20~~

27.  $\int \frac{dx}{x^2 + 3x + 6}$  is      (a).  $\frac{1}{2} \ln(x^2 + 2x)$       (b).  $\ln(x^2 + 3x^2 + 6)$       (c).  $\frac{2}{3} \ln(x^2 + 3x^2 + 6)$       (d).

$\ln(x^2 + 3x^2 + 6)$

28. A particle starts with an initial speed of  $20 \text{ ms}^{-1}$ . Its acceleration at any time  $t$  is  $(18 - 2t) \text{ ms}^{-2}$ . Find the speed at the end of 6 seconds.      (a).  $72 \text{ ms}^{-1}$       (b).  $36 \text{ ms}^{-1}$       (c).  $6 \text{ ms}^{-1}$       (d).  $92 \text{ ms}^{-1}$

29. The expression

$$\frac{\sin x - \sin 2x + \sin 3x}{\cos x + \cos 2x + \cos 3x}$$

DOLA @ FUNA ABSU 001

- (a).  $\cot 2x$       (b).  $\sec 2x$       (c).  $\tan 2x$       (d).  $\tan^2 2x$

30. Integrate  $\sin^3 x \cos^8 x$  with respect to  $x$

- (a).  $\frac{2}{11} \cos^{11} x + \frac{1}{9} \cos^9 x + C$       (b).  $\frac{1}{11} \sin^{11} x - \frac{1}{9} \sin^9 x + C$       (c).  $\frac{1}{11} \sin^{11} x + \frac{1}{9} \sin^9 x + C$       (d).  $\frac{1}{11} \cos^{11} x - \frac{1}{9} \cos^9 x + C$

i. Express in terms of the trigonometrical ratios of acute angles:

- (i.)  $\sin 170^\circ$
- (ii.)  $\cos 164^\circ$
- (iii.)  $\cos 293^\circ$
- (iv.)  $\tan 300^\circ$
- (v)  $\csc 230^\circ$
- (vi)  $\sin(-230^\circ)$
- (vii)  $\cos 200^\circ$
- (viii)  $\tan 143^\circ$
- (ix)  $\sec 152^\circ$
- (x)  $\sin(-50^\circ)$
- (xi)  $\cos(-130^\circ)$
- (xii)  $\tan 300^\circ$
- (xiii)  $\cot 156^\circ$
- (xiv)  $\cos(-20^\circ)$
- (xv)  $\sin 250^\circ$
- (xvi)  $\csc(-53^\circ)$
- (xvii)  $\sin 325^\circ$
- (xviii)  $\cot 200^\circ$
- (xix)  $\sec(-172^\circ)$
- (xx)  $\tan(-140^\circ)$
- (xxi)  $\cot 200^\circ$

ii. If  $\sin(x - \alpha) = \cos(x + \alpha)$ , prove that  $\tan x = 1$

iii. Find the value of  $x$  if  $\sin 3x = \cos(2x - 15^\circ)$ .

iv. If  $\cos A = 4/5$ ,  $\sin B = 8/17$ , find without using tables, the values of  $\cos(A \pm B)$ .

v. Assuming the formulae for  $\sin(A + B)$  and  $\cos(A + B)$ , prove that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

and deduce that if  $\tan \alpha = 1/5$ ,  $\tan \beta = 4/19$  and  $\tan \gamma = 2/5$  then  $\tan(\alpha + \beta + \gamma) = 1$

i. If  $A$ ,  $B$  and  $C$  are angles of a triangle, show that

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

ii. Prove that  $\sin^2 \alpha + \sin^2 \beta + 2\sin \alpha \sin \beta \cos(\alpha + \beta) = \sin^2(\alpha + \beta)$

iii. Show that  $\sin(A + B) \sin(A - B) = \sin^2 A - \sin^2 B$ .

iv. Evaluate  $\sin 15^\circ$  and  $\cos 15^\circ$  in terms of  $\sqrt{2}$ ,  $\sqrt{3}$ .

v. Without using tables, find the value of  $\tan 75^\circ$ .

vi. Show that  $\tan A + \tan B = \frac{\sin(A + B)}{\cos A \cos B}$

vii. Show that,  $(\cos \theta + \cos \phi)^2 + (\sin \theta + \sin \phi)^2 = 2 + 2 \cos(\theta - \phi)$ .

- Show that;

$$i. \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$ii. \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$iii. \frac{\sin \theta + \sin 3\theta}{\cos \theta + \cos 3\theta} = \tan 3\theta$$

$$iv. \text{If } \sin \theta + \cos \phi = a \text{ and } \cos \theta + \cos \phi = b, \text{ then; } \cos^2 \frac{1}{2}(\theta - \phi) = \frac{1}{2}(a^2 + b^2)$$

$$\frac{\sin A \sin 2A}{\sin A \cos 2A} = \tan 5A$$

$$\text{Sin } C = 4 \cos A / 2 \cos B / 2 \cos C / 2 \text{ where } A + B + C = 180^\circ$$

$$\cot C \cot A + \cot A \cot B = 1, \text{ where } A + B + C = 180^\circ$$

$$\text{triangle } ABC: (i) \sin \frac{1}{2}A = \cos \frac{1}{2}(B + C) \quad (ii) \cos \frac{1}{2}B = \sin$$

D'OLA @ FUNA ABSU 001