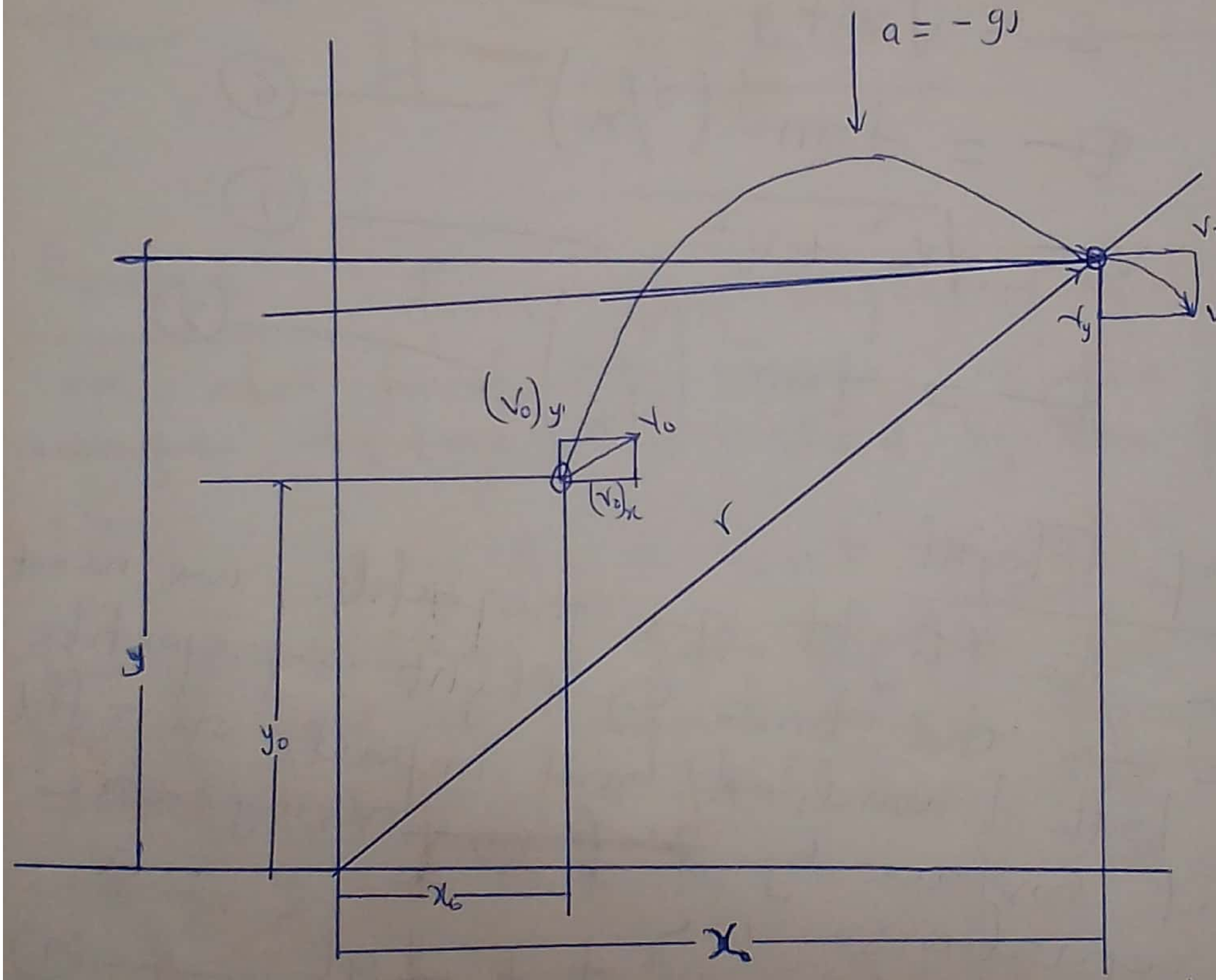
PROJECTILE MOTION

Definition 3.1.1 Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration

KINEMATIC EQUATIONS:

Consider the diagram below



We have $a_x = 0$, the velocity in the horizontal direction remains constant $(v_x = v_{0x})$ and the position in the x direction can be determined by:

$$x = x_0 + v_{0x}(t) \qquad (1)$$

why is a_x equal to zero (assuming movement through the air)?

Since the positive y-axis is directed upward, $a_y = -g$. Application of the constant acceleration equation yields

$$V_y = V_{oy} - g(t) \quad \text{—— (2)}$$

$$y = y_o + V_{oy}(t) - \frac{1}{2}g(t)^2 \quad \text{—— (3)}$$

$$V_y^2 = V_{oy}^2 - 2g(y - y_o) \quad \text{—— (4)}$$

$$S = \sqrt{x^2 + y^2} \quad \text{—— (5)}$$

$$\theta = \tan^{-1}(y/x) \quad \text{—— (6)}$$

$$V = \sqrt{V_x^2 + V_y^2} \quad \text{—— (7)}$$

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) \quad \text{—— (8)}$$

Time of Flight

While the flight of a projectile, we need ~~to~~ solve for the time of flight of projectile that is both launched and impacts on a flat horizontal surface by ~~performing~~ using the kinetic equations. i.e

$$y - y_o = V_{oy}t + \frac{1}{2}gt^2 \quad \text{—— (9)}$$

Solving for t, we have

$$T = \frac{2V_o \sin\theta}{g} \quad \text{—— (10)}$$

Note that $V_x = V_o \cos\theta$ $\quad x = (V_o \cos\theta)t$

$V_y = V_o \sin\theta$ $\quad y = (V_o \sin\theta)t$

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Maximum height: the maximum height of trajectory of a projectile can be found from the kinematic equations. i.e

$$v^2 = v_0^2 + 2a\Delta x$$

so that if $v_y = 0$, then

$$0 = v_0^2 + 2a\Delta x$$

then

$$H = \frac{v^2 \sin^2\theta}{2g} \quad \text{———} \quad (11)$$

Range: From the trajectory equation, we can also find the range, or the horizontal ~~range~~ distance travelled by the projectile. i.e

$$x - x_0 = v_{0x}t + \frac{1}{2}gt^2$$

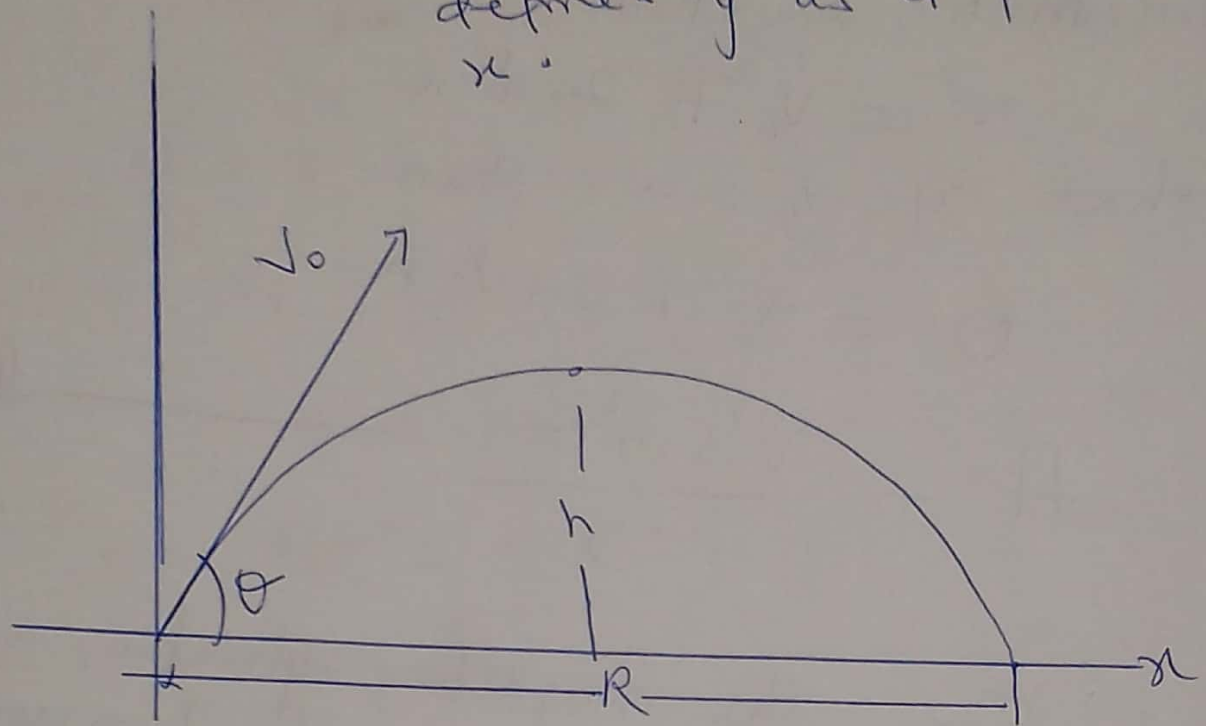
the position y is zero for both the launch point and the impact point since we are considering only a flat horizontal surface. setting $y = 0$, then $x = 0$, so that

$$x - x_0 = v_{0x}t$$

Using the Identity $2\sin\theta\cos\theta = \sin 2\theta$ such that $x = R$, then

$$R = \frac{v_0 \sin 2\theta}{2g} \quad \text{———} \quad (12)$$

Example 3.2 Given V_0 and θ from the diagram below: Find the equation that defines y as a function of x.



Solution: using $V_x = V_0 \cos\theta$ and $V_y = V_0 \sin\theta$

we have $x = (V_0 \cos\theta)t$. or $t = \dfrac{x}{V_0 \cos\theta}$

$$y = (V_0 \sin\theta)t - \frac{1}{2} g (t)^2$$

substituting for t

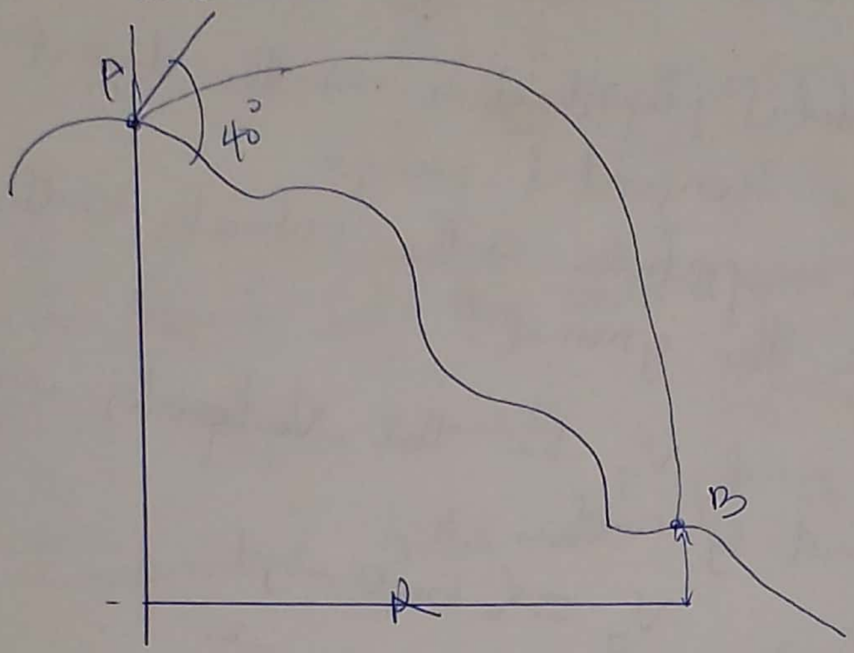
$$y = (V_0 \sin\theta)\left(\frac{x}{V_0 \cos\theta}\right) - \frac{g}{2}\left(\frac{x}{V_0 \cos\theta}\right)^2$$

By simplification,

$$y = x\tan\theta - \left(\frac{gx^2}{2V_0^2}\right)(1 + \tan^2\theta) \qquad (13)$$

The above equation (13) is called the "path equation" which describes the path of a particle in projectile motion.

Example 3.3 The below diagram shows a snowmobile going 15m/s at point A. Find the horizontal distance (R) it travels and the time in the air.



Solution from the equation for the horizontal motion

$$X_B = X_A + V_{Ax} t_{AB}, \quad V_{Ax} = V_{0x} \cos\theta \text{ m/s}$$
$$= 15 \cos 40° \quad (1)$$

~~Vertical motion~~.

Vertical motion equation:

$$y_B = y_A + V_{Ay} t_{AB} - \frac{1}{2} g t_{AB}^2 \quad (3) \qquad V_{Ay} = 15 \sin 40° \text{ m/s} \quad (2)$$

Note that

$$X_B = R, \quad x_A = 0, \quad y_B = (-3/4) R. \quad (4)$$

and $y_A = 0$. Solving (3) & (4), we have

$R \approx 42.8$m and $t_{AB} = 3.72$s

Example 3.4 An object is launched at a velocity of 20 m/s in a direction making an angle of 25° upward with the horizontal.

(i) What is the maximum height reached by the object?

(ii) What is the total flight time of the object?

(iii) What is the horizontal range

(iv) What is the magnitude of the velocity of the object just before it hits the ground?

Soln Let V_x and V_y be the velocity with respect to x and y. then

$$V_x = V_0 \cos\theta, \quad V_y = V_0 \sin\theta - gt.$$

$V_0 = 20\,m/s$, $\theta = 25°$, $g = 9.8\,ms^{-2}$.

So that $V_y = V_0 \sin\theta - gt = 0$.

$$t = \frac{V_0 \sin\theta}{g} = \frac{20 \sin 25°}{9.8} = 0.86 \text{ secs}$$

To find the maximum height.

$$S = V \sin\theta\, t - \frac{1}{2}gt^2$$

$$= 20 \sin 25° (0.86) - \frac{1}{2} 9.8 (0.86)^2 = 3.64m$$

(ii) At $t = t_1$ and $t = t_2$, $y = 0$. Hence

$$V_0 \sin\theta t - \frac{1}{2}gt^2 = 0$$

$$t = t_1 = 0 \quad \text{and} \quad t = t_2 = \frac{2 V_0 \sin\theta}{g}$$

$$\text{Time of flight} = t_2 - t_1 = \frac{2(20)\sin\theta}{g}$$

$$= \frac{2 \times 20 \sin 25°}{9.8} = 1.72 \text{ secs}$$

(vii) The horizontal Range is

$$R = \frac{2V_0 \cos\theta V_0 \sin\theta}{g} = \frac{V_0^2 \sin 2\theta}{g}$$

$$= \frac{20^2 \times \sin 2(25)}{9.8} = \underline{\underline{31.26m}}$$

(viii) The object hits the ground at $t = t_2 = \frac{2V_0 \sin\theta}{g}$.

So that $V_x = V_0 \cos\theta$, $V_y = V_0 \sin\theta - gt$.

The component of the velocity at $t = \frac{2V_0 \sin\theta}{g}$

are

$V_x = V_0 \cos\theta = 20\cos 25^\circ$

$V_y = V_0 \sin 25^\circ - g\left(\frac{2 \cdot V_0 \sin 25^\circ}{g}\right) = -V_0 \sin 25^\circ$

The magnitude V of the velocity is given by

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(20\cos 25^\circ)^2 + (-V_0 \sin 25^\circ)^2}$$

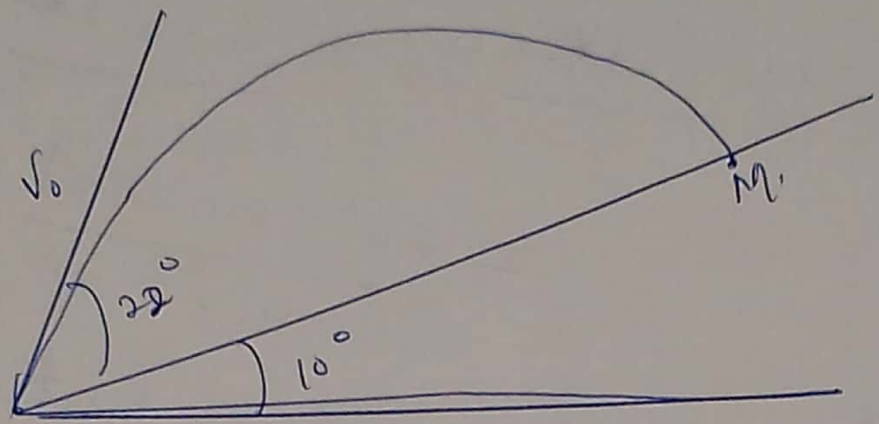
$$= V_0 = 20 m/s$$

Example 3-5 A projectile is launched from the point O at angle of 22° with an initial velocity of 15 m/s up an inclined plane that makes an angle of 10° with the horizontal. The projectile hits the incline plane at point M.

(i) Find the time it takes for the projectile to hit the incline plane

(ii) Find the distance OM.

Solution



The x and y Component of the displacement are given by

$x = V_0 \cos\theta\, t \qquad y = V_0 \sin\theta\, t - \frac{1}{2} g t^2$ — (1)

with $\theta = 22° + 10° = 32°$, $V_0 = 15\,m/s$

The relationship between the coordinate x and y on the incline is given by

$\tan 10° = \frac{y}{x}$ ———— (2)

Substitute x and y by the expression into (2)

$$\tan 10° = \frac{(V_0 \sin\theta - \frac{1}{2} g t^2)}{V \cos\theta\, t}$$

By Simplification to obtain t.

$\frac{1}{2} g t + V_0 \cos\theta \tan 10° - V_0 \sin\theta = 0$

Solve for t.

$$t = \frac{V_0 \sin\theta - V_0 \cos\theta \tan 10°}{(0.2)\, g} = \frac{15 \sin 32° - 15 \cos 32° \tan 10°}{0.5 \times 9.8}$$

$= 1.16\,s.$

(ii)

$$OM = \sqrt{(v_0 \cos \theta t)^2 + (v_0 \sin \theta t - \tfrac{1}{2} g t^2)^2}$$

~~at~~ $t = 1.16$

$$OM = \sqrt{15 \cos 32 \,(1.16)^2 + (15 \sin 32 \,(1.16) - \tfrac{1}{2} 9.8 \,(1.16)^2)^2}$$

$$= \underline{\underline{15m}}$$

Example 3.6 Two balls A and B of masses 100 grams and 300 grams respectively are pushed horizontally from a table of height 3 metres. Ball A is pushed so that its initial velocity is 10m/s and ball B is pushed so that its initial velocity is 15m/s. Find (i) the time it takes each ball to hit the ground.

(ii) What is the difference in the distance between the points of impact of the two balls on the ground?

Sol the two balls are subject to the same gravitational acceleration and therefore will hit the ground at the same time t.

By equation $-3 = -\frac{1}{2} g t^2$

$$t = \sqrt{\frac{3(2)}{9.8}} = 0.785.$$

(i) Horizontal distance X_A of ball A

$X_A = 10m/s \cdot 0.785 = 7.8m$

$X_B = 15 m/s \times 0.78 s = 11.7 m$

Difference in distance X_A & X_B

$|X_B - X_A| = |11.7 - 7.8| = \underline{\underline{3.9 m}}$