1.0 Linear Momentum and Collisions

<u>Defn</u> 1.1 The linear momentum p of an object is equal to the product of the mass m and velocity v.

$$\vec{p} = m\vec{v} \quad \text{——— } (1)$$

or In Component form: $\vec{p} = p_x\vec{x} + p_y\vec{y} =$
$mv_x\hat{x} + mv_y\hat{y}$.

By Newton's second law, which can be written as
$$\vec{F} = \frac{\text{change in momentum}}{\text{time interval}} = \frac{\Delta\vec{p}}{\Delta t}$$

then
$$v_f = v_i + at = v_i + a\Delta t$$
$$a\Delta t = v_f - v_i = \Delta v$$
$$a = \frac{\Delta v}{\Delta t}$$

So that
$$\Delta p = m\Delta v \Rightarrow \Delta v = \frac{\Delta p}{m}$$

Since $a = \frac{\Delta v}{\Delta t} = \frac{\Delta p/m}{\Delta t} = \frac{\Delta p}{m\Delta t}$

So that
$$\frac{\Delta P}{\Delta t} = ma = F. \quad \text{——— } (2)$$

We have

$$\vec{F}\Delta t = \Delta \vec{p} = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i) \quad \text{——— (B)}$$

where $\vec{F}\Delta t$ is called the impulse of the force, with $\vec{F}$ as the average force during a collision.

Example 1.1.2 (a) Show that the kinetic energy of a particle of mass m is related to the magnitude of the momentum p of that particle by

$$K.E = \frac{p^2}{2m}$$

Solution

recall that $K.E = \frac{1}{2}mv^2$

since $p = mv$, then

$$v^2 = \frac{2KE}{m}, \quad v^2 = \frac{p^2}{m^2} \text{ from } v = \frac{p}{m}$$

$$\frac{2KE}{m} = \frac{p^2}{m^2}$$

$$K.E = \frac{mp^2}{2m^2} = \frac{p^2}{2m}$$

(5) Suppose an object is moving so that K.E = 150J and the absolute value of its momentum is 30. Kg m/s. What is the mass of the object and at what velocity Is it travelling?

Solution

Since $\underline{2K \cdot E} = mv^2$, $p = mv$

$m = \frac{2KE}{v^2}$, $m = \frac{p}{v}$

by equating m $\Rightarrow$ $\frac{p}{v} = \frac{2KE}{v^2}$

$v = \frac{2KE}{p} = \frac{2(150J)}{30.0 kg m/s}$

$v = \frac{300. kg m^2/s^2}{30.0 kg m/s} = 10.0 m/s$

Since $m = \frac{p}{v} = \frac{30. kg m/s}{10. m/s} = 3.00 kg$

Example 1.1.3 A tennis player receives a shot with the ball (0.0600kg) travelling horizontally at 50.0 m/s and returns the shot with the ball traveling horizontally at 40.0 m/s in the opposite direction

(a) What is the impulse delivered to the ball by the racquet? (b) What work does the racquet do on the ball?

Solution (a)

Assume the ball is initially in the $-x$ direction away from the net. then $v_i = -50.0\ m/s$ and $v_f = +40.0\ m/s$. Using

$$I\,\Delta t = \Delta P = m(v_f - v_i)$$

$$= 0.0600\left[40.0\ m/s - (-50.0\ m/s)\right]$$

$$= 5.40\ kg\ m/s$$

(b) The work is just the change of kinetic energy as given by

$$W = \Delta KE = \frac{1}{2} m(v_f^2 - v_i^2)$$

$$= \frac{0.0600}{2}\left[40^2 - (+50)^2\right]$$

$$= -27.0\ J$$

where the negative sign means that the racquet is supplying work to the ball.

Scanned with CamScanner

1.2 Conservation of Linear Momentum

Let $\vec{F}_1 \Delta t = m_1 \vec{v}_{1f} - m_1 \vec{v}_{i1}$ —— (1)

$\vec{F}_2 \Delta t = m_2 \vec{v}_{2f} - m_2 \vec{v}_{2i}$ —— (2)

By Newton's 3rd law

$$\vec{F}_1 = -\vec{F}_2$$

$$\vec{F}_1 \Delta t = -\vec{F}_2 \Delta t$$

$$m_1 \vec{v}_{1f} - m_1 \vec{v}_{i1} = -\left(m_2 \vec{v}_{2f} - m_2 \vec{v}_{2i}\right)$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad \text{—— (3)}$$

Equation (3) is the Conservation of linear momentum.

Example 1.2.1 An 80.0 kg astronaut is working on the engines of her spaceship, which is drifting through space with a constant velocity. The astronaut, wishing to get a better view of the universe, pushes against the ship and later find herself 30.0 m behind the ship and moving so slowly that she can be considered at rest with respect to the ship. Without a thruster, the only way to return to the ship is to throw a 0.500 kg wrench with a speed of 20.0 m/s in the opposite direction from the ship. How long will it take to get back to the ship once the wrench thrown?

Solution $U_{wf} = -20.0$ m/s $\quad V_{af} = ?$

$m_w = 0.500$ kg $\quad m_a = 80.1$ kg

Wrench + astronaut initial mom = Wrench + astronaut final mo

$$P_{wi} + P_{ai} = P_{wf} + P_{af}$$

$$P_{ai} = 0, \quad \text{since } V_{ai} = 0$$

$$P_{wi} = 0 \quad \text{since } U_{wi} = 0$$

By momentum; then

$$0 = m_w V_{wf} + m_a V_{af}$$

$$V_{af} = -\frac{m_w}{m_a} V_{wf} = -\frac{0.500}{80.1} = (-20.0 \text{ m/s})$$

$$= 0.125 \text{ m/s}$$

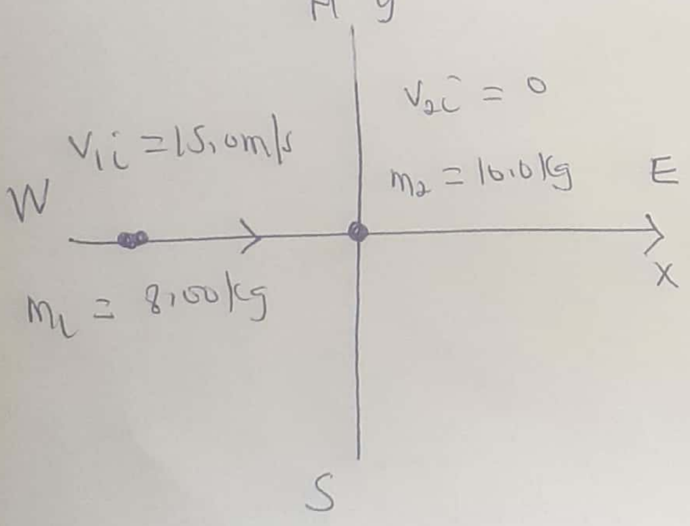
Since the velocity of the astronaut will be constant once the wrench is thrown $a = 0$, so $V_{af} = x_a / t$

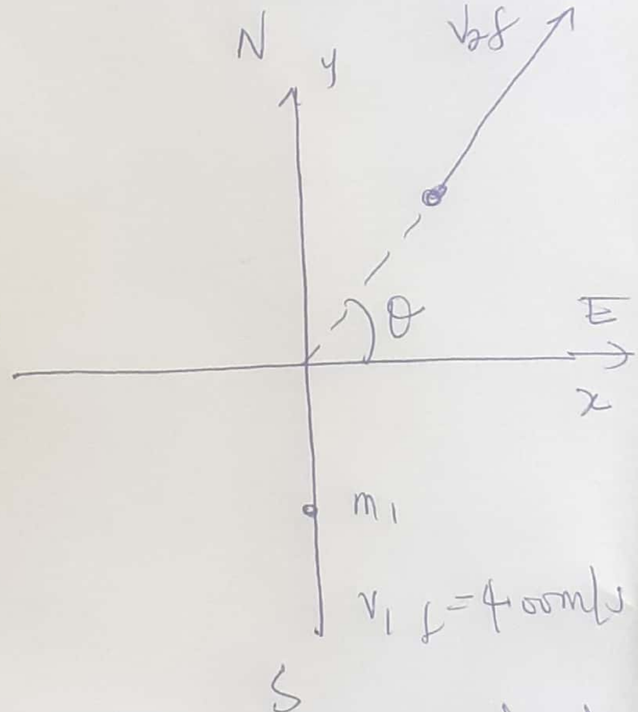
$$t = \frac{x_a}{V_{af}} = \frac{30.0 \text{ m}}{0.125 \text{ m/s}} = 240 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}}$$

$$= 4.00 \text{ m}$$

Scanned with CamScanner

Example 1.2.3 An 8.00 kg object moving east at 15.0 m/s on a frictionless horizontal surface collides with a 10 kg object that is initially at rest. After the collision, the 8.00 kg object moves south at 4.00 m/s (a) What is the velocity of the 10 kg object after the collision? (b) What percentage of the initial kinetic energy is lost in the collision?





By Conservation of linear momentum in the x-direction

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1ix} + 0 = 0 + m_2 v_{2ix}$$

$$v_{2fx} = \frac{m_1 v_{1ix}}{m_2}$$

$$= \left(\frac{8}{10}\right)(15.0) = 12.0 \text{ m/s}$$

Scanned with CamScanner

Conservation of linear momentum in the y direction

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

$$0 + 0 = m_1 v_{1fy} + m_2 v_{2fy}$$

$$-m_2 v_{2fy} = m_1 v_{1fy}$$

$$v_{2fy} = \frac{-m_1}{m_2} v_{1fy}$$

$$= -\left(\frac{8}{10}\right)(-4.0\ m/s)$$

$$= 3.20\ m/s$$

the magnitude of the final velocity of object 2 is then found from the pythagoras theorem

$$v_{2f} = \sqrt{v_{2fx}^2 + v_{2fy}^2}$$

$$= \sqrt{12^2 + 3.2^2} = 12.4 m/s$$

the angle θ can now easily be found with

$$\tan\theta = \frac{v_{2fy}}{v_{2fx}} = \tan^{-1}\left(\frac{3.20}{12.0}\right) = 14.9^0$$

$$\theta = \tan^{-1}\left(\frac{v_{2fy}}{v_{2fx}}\right)$$

Thus $v_{2f} = 12.4\,m/s$ at $14.9°$ N of E

(b) The percentage of kinetic energy lost is given by

$$\frac{KE_{loss}}{KE_i} = \frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i}$$

$$KE_i = \frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2}(8.0)(15.0) = 900J$$

$$KE_f = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$= \frac{1}{2} \times 8 \cdot (-4)^2 + \frac{1}{2}(10)(12.4)^2$$

$$= 640J + 770J = 834J.$$

Hence, the percentage lost is

$$\frac{KE_{loss}}{KE_i} = 1 - \frac{834J}{900J} = 1 - 0.928$$

$$= 0.072$$

or 7.2% of the original kinetic energy is lost in the collision.