1) Prove rigorously that $A - B = A \cap B^c$

Preposition

i) $A - B \subseteq A \cap B^c$

ii) $A \cap B^c \subseteq A - B$

Preposition 1

$A - B \subseteq A \cap B^c$

Let $x \in A - B$

$x \in A$ and $x \notin B$

$x \in A \cap x \notin B$

$x \in A \cap x \in B^c$

$x \in A \cap B^c$

$\therefore A - B \subseteq A \cap B^c$

Impact

Simplified

Preposition 2:

$A \cap B^c \subseteq A - B$

let $x \in A \cap B^c$

$x \in A \cap x \in B^c$

$x \in A \cap x \notin B$

$x \in A$ and $x \notin B$

$x \in A - B$

$\therefore A \cap B^c \subseteq A - B$

Hence $A - B = A \cap B^c$

2) $(A \cup B)^c = A^c \cap B^c$

Preposition

i) $(A \cup B)^c \subseteq A^c \cap B^c$

ii) $A^c \cap B^c \subseteq (A \cup B)^c$

Preposition 1

$(A \cup B)^c \subseteq A^c \cap B^c$

let $x \in (A \cup B)^c$

$x \notin (A \cup B)$

$x \notin A \vee x \notin B$

$x \in A^c \wedge x \in B^c$

$x \in A^c \cap B^c$

$\therefore (A \cup B)^c \subseteq A^c \cap B^c$

Impact

Simplified

Preposition 2

$A^c \cap B^c \subseteq (A \cup B)^c$

$x \in A^c \cap B^c$

$x \in A^c \wedge x \in B^c$

$x \notin A \vee x \notin B$

$x \notin (A \cup B)$

~~$x \notin (A \cup B)^c$~~

$x \in (A \cup B)^c$

$\therefore A^c \cap B^c \subseteq (A \cup B)^c$

Hence $(A \cup B)^c = A^c \cap B^c$

3) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

<u>preposition</u>

i) $(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$

ii) $(A \cup C) \cap (B \cup C) \subseteq (A \cap B) \cup C$

<u>Preposition 1</u>

$(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$

let $x \in (A \cap B) \cup C$

$x \in (A \cap B) \vee x \in C$

$x \in A \cap x \in B \vee x \in C$

$x \in A \vee x \in C \cap x \in B \vee x \in C$

$x \in (A \cup C) \cap x \in (B \cup C)$

$x \in (A \cup C) \cap (B \cup C)$

$\therefore (A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$

Wattang

<u>Preposition 2</u> $(A \cup C) \cap (B \cup C) \subseteq (A \cap B) \cup C$

$(A \cup C) \cap (B \cup C)$

$x \in (A \cup C) \cap (B \cup C)$

$x \in (A \cup C) \cap x \in (B \cup C)$

$x \in A \vee x \in C \cap x \in B \vee x \in C$

$x \in A \cap x \in B \vee x \in C$

$x \in (A \cap B) \vee x \in C$

$x \in (A \cap B) \cup C$

$\therefore (A \cup C) \cap (B \cup C) \subseteq (A \cap B) \cup C$

Hence $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Longest Simplified

Impact Simplified

4) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

preposition

i) $(A \cup B) \cap C \subseteq (A \cap C) \cup (B \cap C)$

ii) $(A \cap C) \cup (B \cap C) \subseteq (A \cup B) \cap C$

Preposition 1

$(A \cup B) \cap C \subseteq (A \cap C) \cup (B \cap C)$

$x \in (A \cup B) \cap C$

$x \in (A \cup B) \cap x \in C$

$x \in A \cup x \in B \cap x \in C$

$x \in A \cap x \in C \cup x \in B \cap x \in C$

$x \in (A \cap C) \cup x \in (B \cap C)$

$x \in (A \cap C) \cup (B \cap C)$

$\therefore (A \cup B) \cap C \subseteq (A \cap C) \cup (B \cap C)$

preposition 2

$(A \cap C) \cup (B \cap C) \subseteq (A \cup B) \cap C$

$x \in (A \cap C) \cup (B \cap C)$

$x \in (A \cap C) \cup x \in (B \cap C)$

$x \in A \cap x \in C \cup x \in B \cap x \in C$

$x \in A \cup x \in B \cap x \in C$

$x \in (A \cup B) \cap x \in C$

$x \in (A \cup B) \cap C$

$\therefore (A \cap C) \cup (B \cap C) \subseteq (A \cup B) \cap C$

Hence, $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

Watimagbo