CALCULUS AND TRIGONOMETRY

MTS 102

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Functions of a real variable and their graphs.

Limits, and ideal of continuity of functions.

Removable discontinuity.

Background Let $D \subseteq \mathbb{R}$.

A function f from D into $\mathbb{R}$ is a rule which associates with each $x \in D$ one and only one $y \in \mathbb{R}$. Notation: $f : D \to \mathbb{R}$.

D is called the **domain** of the function.

If $x \in D$, then the element $y \in \mathbb{R}$ which is associated with x is called the **value of** f **at** x or the **image of** x **under** f. y is denoted by $f(x)$.

If $U \subseteq D$, then

$$f(U) = \{y \in \mathbb{R} \mid y = f(x) \text{ for some } x \in U\}.$$

If $U = D$, then $f(D)$ is called the **range** of f.

Functions are the major tools for describing the real world in mathematical terms.

Definition . A function f is a set of ordered pairs of numbers (x, y) satisfying the property that if (x, y_1), $(x, y_2) \in f$ then $y_1 = y_2$ (that is, no two distinct ordered pairs in f have the same first component).

If f is a function from a set X into a set Y, then we write

$f: X \to Y$ read as "f is a function from set X into set Y.

If $(x, y) \in f$, then we may write

$y = f(x)$ (read as "y equals f of x")

so that y is called the **image** of x under the function f; x is a **pre-image** of y under f. We also say that "y is the **function value** of x under f."

In the function $f : X \to Y$, the set X containing all of the first components of ordered pairs in f is called the **domain** of the function f; the set Y is called the **co-domain** of f. The set of all second components of ordered pairs in f is called that **range** of f, that is,

$$\text{range } f = \{y \in Y : y = f(x) \text{ for some } x \in X\}.$$

Clearly,

$$\text{range } f \subseteq Y.$$

Example 1. Let $f = \{(x, y) : \; y = \sqrt{x-2}\}$.

- The value of y that corresponds to $x = 6$ is $y = \sqrt{6-2} = \sqrt{4} = 2$. Hence, $f(6) = 2$ and $(6, 2) \in f$.
- The domain of f is the set

$$\mathrm{dom}\ f = [2, +\infty)$$

and the range of f is the set

$$\mathrm{range}\ f = \mathbb{R}_{\geq 0} = [0, \infty).$$

Operations on functions

1. **Arithmetic:** $f,\ g : D \to \mathbb{R}$

a. $(f \pm g)(x) = f(x) \pm g(x)$

b. $(f \cdot g)(x) = f(x) \cdot g(x)$

c. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}, \quad g(x) \neq 0$

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2. Composition:

Let $f : D \to \mathbb{R}$ and let $g : E \to \mathbb{R}$.

If $f(D) \subseteq E$, then g **composed with** f is the function $g \circ f : D \to \mathbb{R}$ defined by

$$(g \circ f)(x) = g\left[f(x)\right].$$

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Example

Given that $f(x) = 2x - 1$ and $g(x) = x^2$. Find

(a) $gof(x)$

(b) $fog(x)$

(c) $fog(2)$

(d) $gof(-\frac{1}{2})$

(e) $(g+f)(2)$

(f) $(g-f)(x)$

(g) $\frac{f}{g}(-3)$

(h) $(g \cdot f)(x)$

Solution

(a) $gof(x) = g(f(x))$

$= g(2x - 1) = (2x - 1)^2$

$= 4x^2 - 4x + 1.$

(b) $fog(x) = f(g(x))$

$= f(x^2)$

$= 2(x^2) - 1$

$= 2x^2 - 1.$

(c) Since $fog(x) = 2x^2 - 1$ from (b), then $fog(2) = 2(2^2) - 1$

$= 8 - 1$

$= 7.$

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(d) Also, gof(x) from (a) is $gof(x) = 4x^2 - 4x + 1$.

Then,

$gof(-\frac{1}{2}) = 4(\frac{1}{4}) - 4(-\frac{1}{2}) + 1$

$= 1 + 2 + 1$

$= 4.$

(e) $(g + f)(x) = g(x) + f(x)$

$= x^2 + 2x + 1.$

(g) $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$

$= \frac{2x-1}{x^2}.$

Then,

$\frac{f}{g}(-3) = \frac{2(-3)-1}{(-3)^2}$

$= \frac{-7}{9}.$

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The Elementary Functions

1. Polynomial functions:

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_o$$

where n is a nonnegative integer,

$a_n, \ldots a_1, a_o \in \mathbb{R},\ a_n \neq 0.$

2. Rational functions:

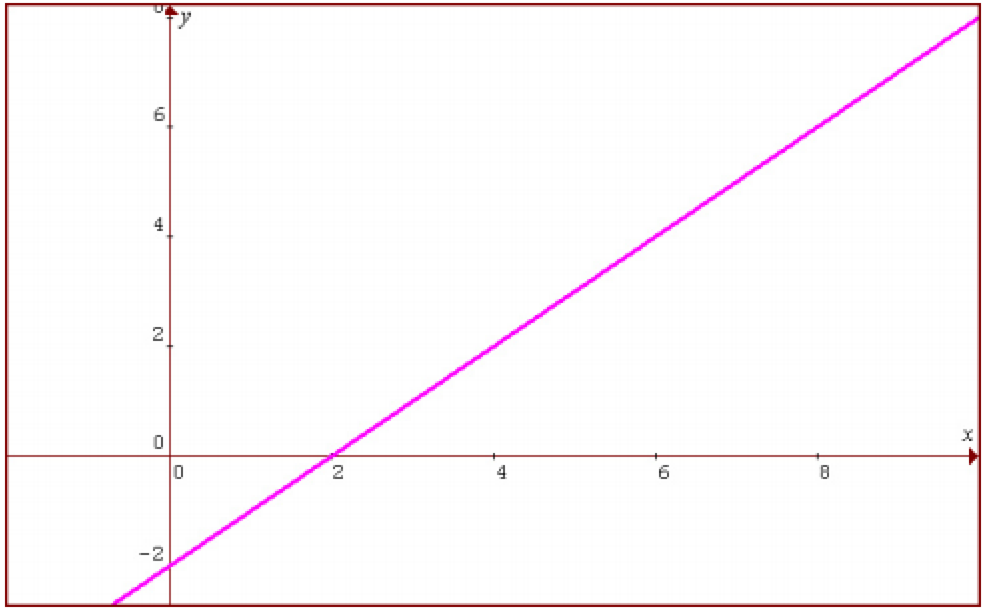
$$r(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ polynomials.}$$

3. Trigonometric functions and inverse trigonometric functions.

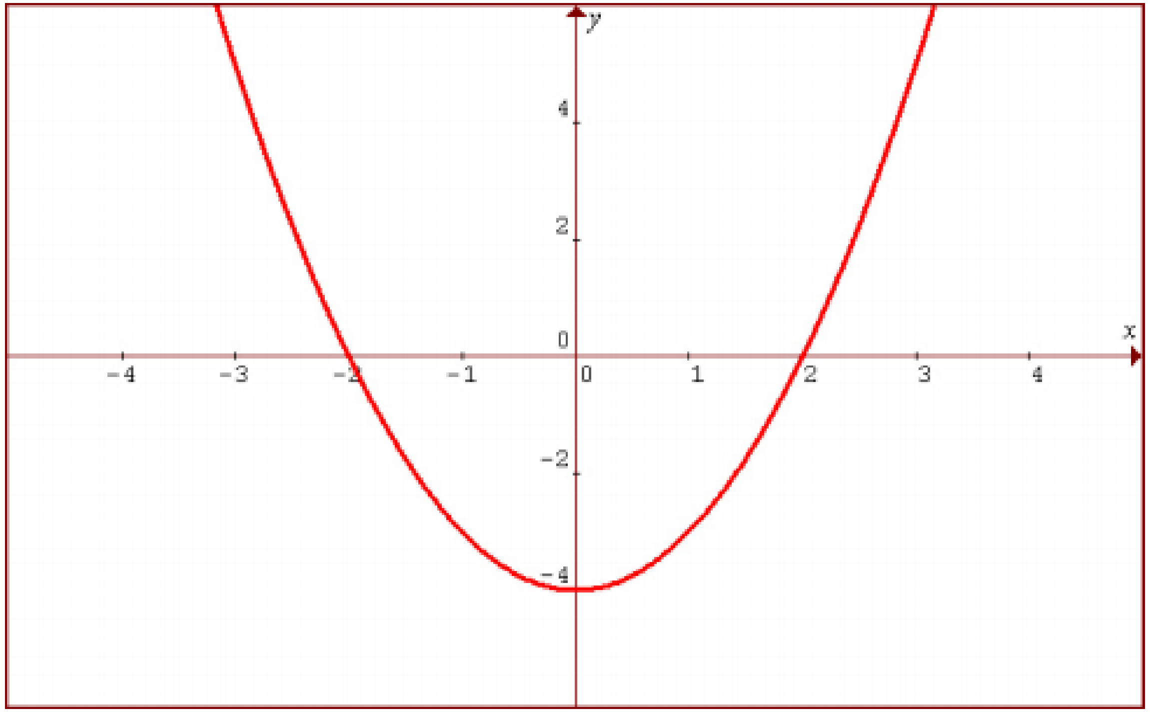
4. Exponential and logarithmic functions.

Definition . If f is a function, then the graph of f is the set of all points (x, y) in a given plane for which $(x, y) \in f$.

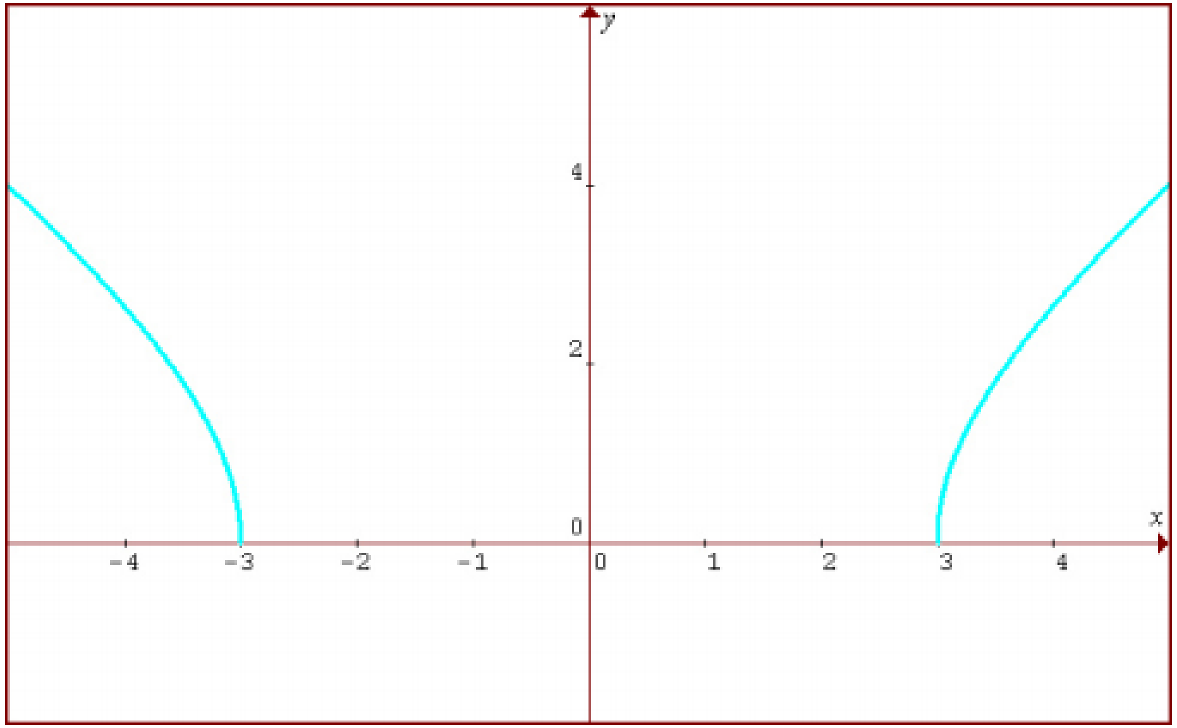
Example . The graph of $f(x) = x - 2$ is given by



Example 3. The graph of $g(x) = x^2 - 4$ is given by



Example . The graph of $h(x) = \sqrt{x^2 - 9}$ is given by



Def. Let $f : D \to \mathbb{R}$ and let c be an accumulation point of D. A number L is the **limit of** f **at** c if to each $\epsilon > 0$ there corresponds a $\delta > 0$ such that

$$|f(x) - L| < \epsilon$$

whenever

$$x \in D \quad \text{and} \quad 0 < |x - c| < \delta.$$

Equivalently:

Def. Let $f : D \to \mathbb{R}$ and let c be an accumulation point of D. A number L is the **limit of** f **at** c if to each neighborhood V of L there corresponds a deleted neighborhood U of c such that $f(U \cap D) \subset V$.

Definition . (Informal Definition of Limit) Let $f(x)$ be defined on an open interval containing a, except possibly at a itself. If $f(x)$ gets arbitrarily close to L for all x sufficiently close to a, we say that f approaches the limit L as x approaches a, and we write

$$\lim_{x \to a} f(x) = L.$$

Example . How does the function $f(x) = \dfrac{2x^2 + x - 3}{x - 1}$ behave near $x = 1$?

x	$f(x) = \dfrac{2x^2 + x - 3}{x - 1}$
0	0.3
0.25	3.5
0.5	4
0.75	4.8
0.9	4.98
0.99	4.998
0.999	4.9998
0.9999	4.99998
0.99999	4.999998

x less than 1

x	$f(x) = \dfrac{2x^2 + x - 3}{x - 1}$
2	7
1.75	6.5
1.5	6.0
1.25	5.5
1.1	5.2
1.01	5.02
1.001	5.002
1.0001	5.0002
1.00001	5.00002

x greater than 1

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Examples:

1. $\lim\limits_{x \to 3} (5x - 3) = 12.$

2. $\lim\limits_{x \to 2} \dfrac{2x^2 + 4x - 16}{x - 2} = 12.$

THEOREM : (Arithmetic)

Let $f,\ g : D \to \mathbb{R}$ and let c be an accumulation point of D. If

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M,$$

1. $\lim_{x \to c} [f(x) + g(x)] = L + M$,

2. $\lim_{x \to c} [f(x) - g(x)] = L - M$,

3. $\lim_{x \to c} [f(x)g(x)] = LM$,

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4 $\lim_{x \to c} [k\, f(x)] = kL, \;\; k$ constant,

5 $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}$ provided $M \neq 0$.

Example-'

Find (a) $\lim_{x\to 1}\left[2x^2(x+\sqrt{x})+3x^{\frac{1}{3}}-\frac{14}{x}\right]$ (b) $\lim_{x\to\infty}\frac{8x^2+16x+3}{2x^3-x+3}$

Solution:

(a) $\lim_{x\to 1}\left[2x^2(x+\sqrt{x})+3x^{\frac{1}{3}}-\frac{14}{x}\right]$

$$= (2\lim_{x\to 1} x^2).(\lim_{x\to 1} x + \lim_{x\to 1}\sqrt{x}) + 3\lim_{x\to 1} x^{1/3} - 14\lim_{x\to 1}\frac{1}{x}$$

$$= 2\,(1^2)\,(1+\sqrt{1}) + 3\,(1^{1/3}) - 14\left(\frac{1}{1}\right) = -7.$$

(b) $\lim_{x\to\infty}\frac{8x^2+16x+3}{2x^3-x+3}$

$$= \lim_{x\to\infty}\frac{\frac{8x^2+16x+3}{x^3}}{\frac{2x^3-x+3}{x^3}}$$

$$= \lim_{x\to\infty}\frac{\frac{8}{x}+\frac{16}{x^2}+\frac{3}{x^3}}{2-\frac{1}{x^2}+\frac{3}{x^3}}$$

$$= \frac{0}{2}\left[where,\ \lim_{x\to\infty}\frac{1}{x}=0\right]$$

$$= 0.$$

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Example

Evaluate : $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$.

Solution : $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

$$= \lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)} = \lim_{x \to 2} \left(x^2 + 2x + 4\right) \qquad [\because x \neq 2]$$

$$= 2^2 + 2 \times 2 + 4 = 12$$

Example

Evaluate : $\lim_{x \to 2} \frac{\sqrt{3-x} - 1}{2 - x}$.

Solution : Rationalizing the numerator, we have

$$\frac{\sqrt{3-x}-1}{2-x} = \frac{\sqrt{3-x}-1}{2-x} \times \frac{\sqrt{3-x}+1}{\sqrt{3-x}+1} = \frac{3-x-1}{(2-x)\left(\sqrt{3-x}+1\right)}$$

$$= \frac{2-x}{(2-x)\left(\sqrt{3-x}+1\right)}$$

$$\therefore \quad \lim_{x \to 2} \frac{\sqrt{3-x}-1}{2-x} = \lim_{x \to 2} \frac{2-x}{(2-x)\left(\sqrt{3-x}+1\right)}$$

$$= \lim_{x \to 2} \frac{1}{\left(\sqrt{3-x}+1\right)} = \frac{1}{\left(\sqrt{3-2}+1\right)} = \frac{1}{1+1} = \frac{1}{2}$$

Example Evaluate : $\lim\limits_{x\to 3} \dfrac{\sqrt{12-x}-x}{\sqrt{6+x}-3}$.

Solution : Rationalizing the numerator as well as the denominator, we get

$$\lim_{x\to 3} \frac{\sqrt{12-x}-x}{\sqrt{6+x}-3} = \lim_{x\to 3} \frac{\left(\sqrt{12-x}-x\right)\left(\sqrt{12-x}+x\right)\cdot\left(\sqrt{6+x}+3\right)}{\sqrt{6+x}-3\left(\sqrt{6+x}+3\right)\left(\sqrt{12-x}+x\right)}$$

$$= \lim_{x\to 3} \frac{\left(12-x-x^2\right)}{6+x-9}\cdot \lim_{x\to 3} \frac{\sqrt{6+x}+3}{\sqrt{12-x}+x}$$

$$= \lim_{x\to 3} \frac{-(x+4)(x-3)}{(x-3)}\cdot \lim_{x\to 3} \frac{\sqrt{6+x}+3}{\sqrt{12-x}+x} \quad [\because x \neq 3]$$

$$= -(3+4)\cdot\frac{6}{6} = -7$$

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Exercises:
Find the indicated limit .

1. $\lim_{x \to -4} (5x + 2)$

2. $\lim_{x \to 3} (2x^2 - 4x + 5)$

3. $\lim_{y \to -1} (y^3 - 2y^2 + 3y - 4)$

4. $\lim_{x \to 2} \frac{3x + 4}{8x - 1}$

5. $\lim_{x \to -1} \frac{2x + 1}{x^2 - 3x + 4}$

6. $\lim_{x \to 2} \sqrt{\frac{x^2 + 3x + 4}{x^3 + 1}}$

7. $\lim_{x \to -3} \sqrt[3]{\frac{5 + 2x}{5 - x}}$

8. $\lim_{z \to -5} \frac{z^2 - 25}{z + 5}$

9. $\lim_{x \to 1/3} \frac{3x - 1}{9x^2 - 1}$

10. $\lim_{x \to -1} \frac{\sqrt{x + 5} - 2}{x + 1}$

1. Evaluate each of the following limits :

(a) $\lim_{x\to 2}[2(x+3)+7]$ (b) $\lim_{x\to 0}\left(x^2+3x+7\right)$ (c) $\lim_{x\to 1}\left[(x+3)^2-16\right]$

(d) $\lim_{x\to -1}\left[(x+1)^2+2\right]$ (e) $\lim_{x\to 0}\left[(2x+1)^3-5\right]$ (f) $\lim_{x\to 1}(3x+1)(x+1)$

2. Find the limits of each of the following functions :

(a) $\lim_{x\to 5}\frac{x-5}{x+2}$ (b) $\lim_{x\to 1}\frac{x+2}{x+1}$ (c) $\lim_{x\to -1}\frac{3x+5}{x-10}$

(d) $\lim_{x\to 0}\frac{px+q}{ax+b}$ (e) $\lim_{x\to 3}\frac{x^2-9}{x-3}$ (f) $\lim_{x\to -5}\frac{x^2-25}{x+5}$

(g) $\lim_{x\to 2}\frac{x^2-x-2}{x^2-3x+2}$ (h) $\lim_{x\to \frac{1}{3}}\frac{9x^2-1}{3x-1}$

3. Evaluate each of the following limits:

(a) $\lim_{x\to 1}\frac{x^3-1}{x-1}$ (b) $\lim_{x\to 0}\frac{x^3+7x}{x^2+2x}$ (c) $\lim_{x\to 1}\frac{x^4-1}{x-1}$

(d) $\lim_{x\to 1}\left[\frac{1}{x-1}-\frac{2}{x^2-1}\right]$

4. Evaluate each of the following limits :

(a) $\lim\limits_{x\to 0} \dfrac{\sqrt{4+x}-\sqrt{4-x}}{x}$ (b) $\lim\limits_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$ (c) $\lim\limits_{x\to 3} \dfrac{\sqrt{3+x}-\sqrt{6}}{x-3}$

(d) $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{1+x}-1}$ (e) $\lim\limits_{x\to 2} \dfrac{\sqrt{3x-2}-x}{2-\sqrt{6-x}}$

5. (a) Find $\lim\limits_{x\to 0} \dfrac{2}{x}$, if it exists. (b) Find $\lim\limits_{x\to 2} \dfrac{1}{x-2}$, if it exists.

6. Find the values of the limits given below :

(a) $\lim\limits_{x\to 0} \dfrac{x}{5-|x|}$ (b) $\lim\limits_{x\to 2} \dfrac{1}{|x+2|}$ (c) $\lim\limits_{x\to 2} \dfrac{1}{|x-2|}$

(d) Show that $\lim\limits_{x\to 5} \dfrac{|x-5|}{x-5}$ does not exist.

Continuity

Definition (Definition of a function Continuous at a Number) The function f is said to be continuous at the number a if and only if the following three conditions are satisfied:

(i) $f(a)$ exists;

(ii) $\lim_{x\to a} f(x)$ exists;

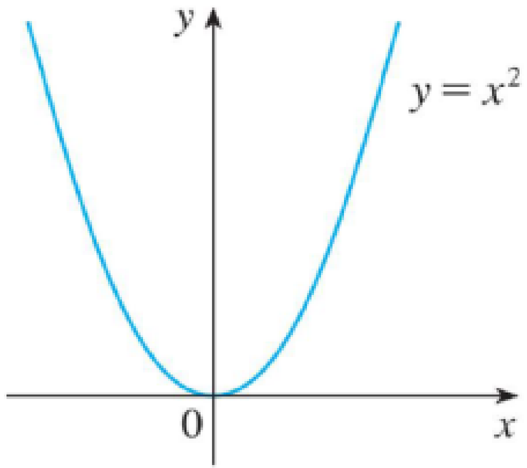
(iii) $\lim_{x\to a} f(x) = f(a)$.

If one or more of these three conditions fails to hold at a, the function f is said to be discontinuous at a.

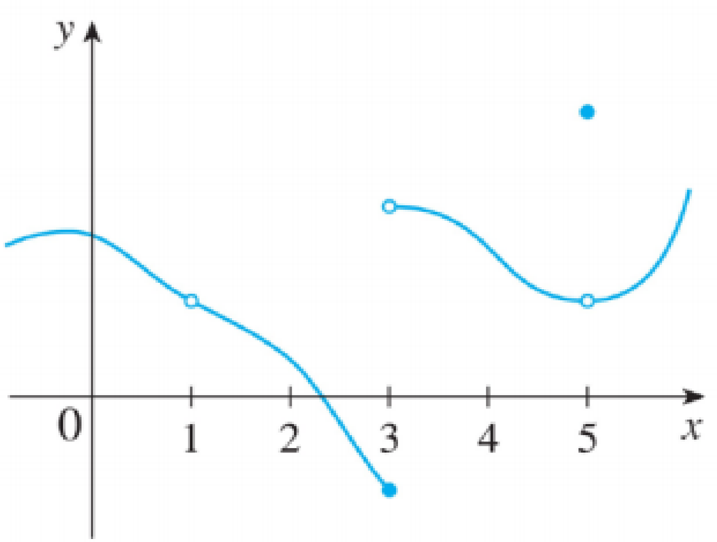
- A function is continuous if there are no holes, breaks or jumps in its graph.

- It is often said that a function is continuous if you can draw it's graph |"*without lifting your pencil* " from the paper.

Continuous Function



Discontinuous Function

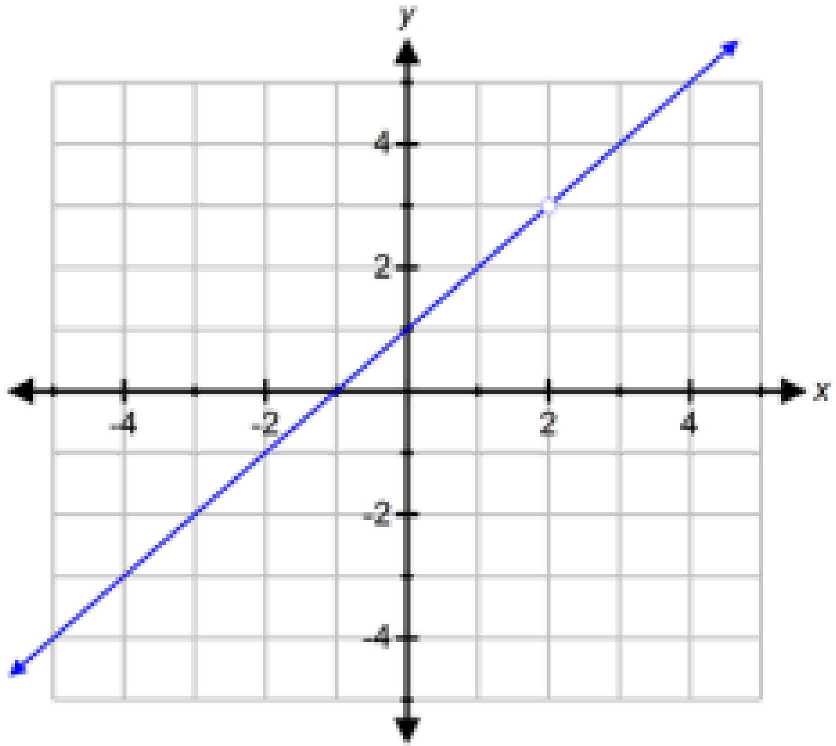


Removable discontinuity: $f(x)$ is defined every where in an interval containing a except at $x = a$ and limit exists at $x = a$ OR $f(x)$ is defined also at $x = a$ and limit is NOT equal to function value at $x = a$. Then we say that $f(x)$ has removable discontinuity at $x = a$. These functions can be extended as continuous functions by defining the value of f to be the limit value at $x = a$.

Example . $f(x) = \begin{cases} \frac{\sin x}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$. *Here limit as $x \to 0$ is 1. But $f(0)$ is defined to be 0.*

Removable Discontinuity

$$f(x) = \frac{x^2 - x - 2}{x - 2}$$



Example :

Show that $f(x) = \dfrac{x^2 - 4}{x - 2}$ is not continuous $x = 2$ but continuous at $x = 3$.

Solution:

The conditions to be satisfied by a function before we can say that it is continuous at a particular point say $x = a$ are: $f(a)$, $\lim\limits_{x \to a-} f(x)$ and $\lim\limits_{x \to a+} f(x)$ should have definite and finite values and these are all equal.

Let us examine whether these conditions are satisfied by $f(x) = \dfrac{x^2 - 4}{x - 2}$ for $x = 2$.

Here $x = 2$, therefore we have

(i) $f(2) = \dfrac{2^2 - 4}{2 - 2} = \dfrac{0}{0}$, which is undefined.

Again by the method of finding the left hand and right hand side limits, we have

(ii) $\lim\limits_{x \to 2^-} \dfrac{x^2 - 4}{x - 2} = \lim\limits_{x \to 2^-} \dfrac{(x + 2)(x - 2)}{(x - 2)}$

$= \lim\limits_{x \to 2^-} (x + 2)$

$= \lim\limits_{h \to 0} (2 - h + 2) = 4$

$\therefore$ L.H.S. limit = 4.

Again, $\lim\limits_{x\to 2^+} \dfrac{x^2-4}{x-2} = \lim\limits_{x\to 2^+} \dfrac{(x+2)(x-2)}{(x-2)}$

$$= \lim_{x\to 2^+} (x+2)$$

$$= \lim_{h\to 0} (2+h+2) = 4$$

$\therefore$ R.H.S. limit = 4.

Here, $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) \neq f(2)$

$\therefore f(x) = \dfrac{x^2-4}{x-2}$ is not continuous at $x = 2$.

Now, for $x = 3$,

i) $f(3) = \dfrac{3^2-4}{3-2} = 5$ and

ii) $\lim_{x\to 3^-} \frac{x^2-4}{x-2} = \lim_{x\to 3^-} \frac{(x+2)(x-2)}{(x-2)}$

$$= \lim_{h\to 0} (3 - h + 2) = 5$$

$\therefore$ L.H.S. limit = 5.

Again, $\lim_{x\to 3^+} \frac{x^2-4}{x-2} = \lim_{x\to 3^+} \frac{(x+2)(x-2)}{(x-2)}$

$$= \lim_{h\to 0} (3 + h + 2) = 5$$

$\therefore$ R.H.S. limit = 5.

Here, $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) \neq f(3)$

$\therefore f(x) = \frac{x^2-4}{x-2}$ is continuous at $x = 3$.

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Example- :

Find the points of discontinuity of the function $\dfrac{x^2 - 3x - 4}{x^3 - 2x^2 - 5x + 6}$

Solution:

Let $f(x) = \dfrac{x^2 - 3x - 4}{x^3 - 2x^2 - 5x + 6}$

We know that if a function is undefined at $x = a$, then $x = a$ is a point of discontinuity of the function. Therefore, the points of discontinuity of $f(x)$ are the values of x at which $f(x)$ becomes undefined. The values of x for which $f(x)$ is undefined are the roots of the equation.

$x^3 - 2x^2 - 5x + 6 = 0$

or, $x^2(x - 1) - x(x - 1) - 6(x - 1) = 0$

or, $(x - 1)(x^2 - x - 6) = 0$

or, $(x - 1)(x^2 - 3x + 2x - 6) = 0$

or, $(x - 1)[x(x - 3) + 2(x - 3)] = 0$

or, $(x - 1)(x + 2)(x - 3) = 0$

$\therefore x = 1$ or $x = -2$ or $x = 3$

Hence, the points of discontinuity of $f(x)$ are : $x = 1$, $x = 3$ and $x = -2$.

Inverse Functions

What is an Inverse Function?

An inverse function is a function that will “undo” anything that the original function does. For example, we all have a way of tying our shoes, and how we tie our shoes could be called a function. So, what would be the inverse function of tying our shoes? The inverse function would be “untying” our shoes, because “untying” our shoes will “undo” the original function of tying our shoes.

Let’s look at an inverse function from a mathematical point of view. Consider the function $f(x) = 2x – 5$. If we take any value of x and plug it into f(x) what would happen to that value of x? First, the value of x would get multiplied by 2 and then we would subtract 5. The two mathematical operations that are taking place in the function f(x) are multiplication and subtraction. Now let’s consider the inverse function. What two mathematical operations would be needed to “undo” f(x)? Division and addition would be needed to “undo” the multiplication and subtraction. A little farther down the page we will find the inverse of $f(x) = 2x – 5$, and hopefully the inverse function will contain both division and addition (see example 5).

Notation

If f(x) represents a function, then the notation $f^{-1}(x)$, read "f inverse of x", will be used to denote the inverse of f(x). Similarly, the notation $g^{-1}(x)$, read "g inverse of x", will be used to denote the inverse of g(x).

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$. It is very important not to confuse function notation with negative exponents.

Does the Function have an Inverse?

Not all functions have an inverse, so it is important to determine whether or not a function has an inverse before we try and find the inverse. If a function does not have an inverse, then we need to realize the function does not have an inverse so we do not waste time trying to find something that does not exist.

So how do we know if a function has an inverse? To determine if a function has an inverse function, we need to talk about a special type of function called a **One-to-One Function**. A one-to-one function is a function where each input (x-value) has a unique output (y-value). To put it another way, every time we plug in a value of x we will get a unique value of y, the same y-value will never appear more than once. A one-to-one function is special because only one-to-one functions have an inverse function.

One-to-one Functions A function f is one-to-one if it never takes the same value twice or

$$\boxed{f(x_1) \neq f(x_2) \quad \text{whenever} \quad x_1 \neq x_2.}$$

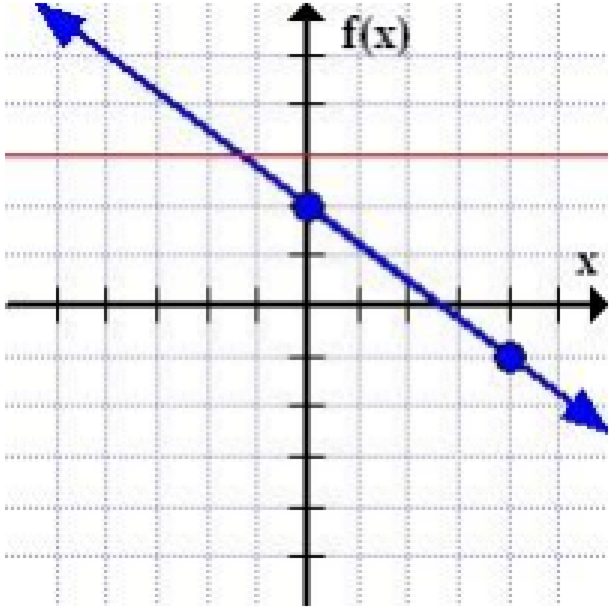
Example The function $f(x) = x$ is one to one, because if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.

On the other hand the function $g(x) = x^2$ is not a one-to-one function, because $g(-1) = g(1)$.

Horizontal Line Test – The HLT says that a function is a one-to-one function if there is no horizontal line that intersects the graph of the function at more than one point.

Example 3: Determine if the function $f(x) = -\frac{3}{4}x + 2$ is a one-to-one function.

To determine if f(x) is a one-to-one function, we need to look at the graph of f(x). Since f(x) is a linear equation the graph of f(x) is a line with a slope of –3/4 and a y-intercept of (0, 2).

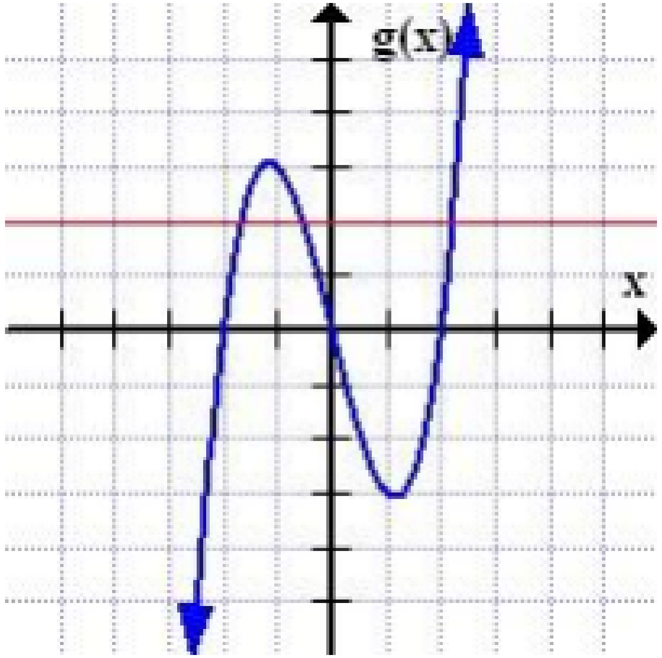


In looking at the graph, you can see that any horizontal line (shown in red) drawn on the graph will intersect the graph of f(x) only once.

Therefore, f(x) is a one-to-one function and f(x) must have an inverse.

Example 4: Determine if the function $g(x) = x^3 - 4x$ is a one-to-one function.

To determine if g(x) is a one-to-one function, we need to look at the graph of g(x).



In looking at the graph, you can see that the horizontal line (shown in red) drawn on the graph intersects the graph of g(x) more than once.

Therefore, g(x) is not a one-to-one function and g(x) does not have an inverse.

How to Find the Inverse Function

Now that we have discussed what an inverse function is, the notation used to represent inverse functions, one-to-one functions, and the Horizontal Line Test, we are ready to try and find an inverse function.

Here are the steps required to find the inverse function:

Step 1: Determine if the function has an inverse. Is the function a one-to-one function? If the function is a one-to-one function, go to step 2. If the function is not a one-to-one function, then say that the function does not have an inverse and stop.

Step 2: Change $f(x)$ to y.

Step 3: Switch x and y.

Step 4: Solve for y.

Step 5: Change y back to $f^{-1}(x)$.

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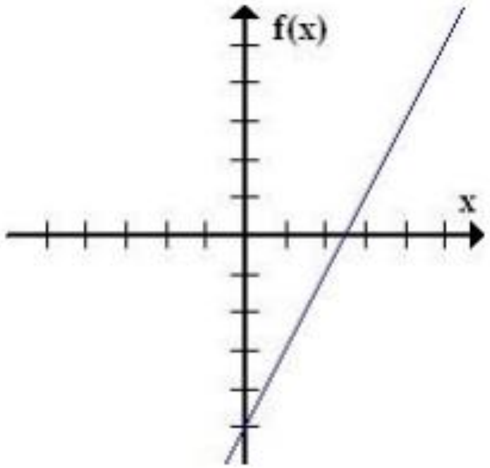
Example 5: If f(x) = 2x – 5, find the inverse.

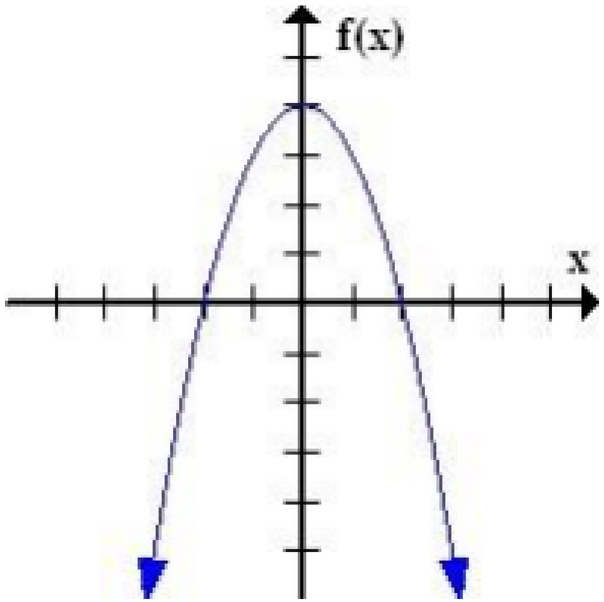


This function passes the Horizontal Line Test which means it is a one-to-one function that has an inverse.

$y = 2x - 5$	Change f(x) to y.
$x = 2y - 5$	Switch x and y.
$x + 5 = 2y$	Solve for y by adding 5 to each side and then dividing each side by 2.
$\dfrac{x+5}{2} = y$	
$f^{-1}(x) = \dfrac{x+5}{2} = \dfrac{1}{2}x + \dfrac{5}{2}$	Change y back to $f^{-1}(x)$.

Therefore, $f^{-1}(x) = \dfrac{x+5}{2}$ or $f^{-1}(x) = \dfrac{1}{2}x + \dfrac{5}{2}$.

Example 6: If $f(x) = -x^2 + 4$, find $f^{-1}(x)$.



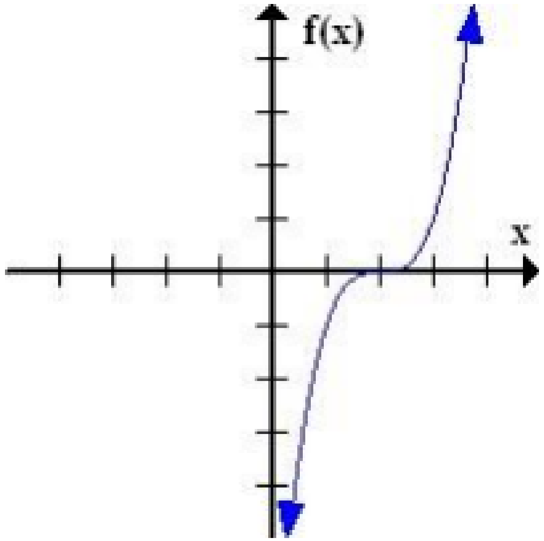
This function does not pass the Horizontal Line Test which means it is not a one-to-one function.

$f^{-1}(x)$ does not exist

f(x) is not a one-to-one, so $f(x) = -x^2 + 4$ does not have an inverse.

Therefore, $f^{-1}(x)$ does not exist.

Example 7: If $f(x) = (x-2)^3$, find the inverse.



This function passes the Horizontal Line Test which means it is a one-to-one function that has an inverse.

$y = (x-2)^3$ Change f(x) to y.

$x = (y-2)^3$ Switch x and y.

$\sqrt[3]{x} = \sqrt[3]{(y-2)^3}$

$\sqrt[3]{x} = y - 2$

$\sqrt[3]{x} + 2 = y$

Solve for y by taking the cube root of each side and then adding 2 to each side.

Change y back to $f^{-1}(x)$.

Therefore, $f^{-1}(x) = \sqrt[3]{x} + 2$.

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Practice Problems

Now it is your turn to try a few practice problems on your own. Work on each of the problems below and then click on the link at the end to check your answers.

Problem 1: If $f(x) = \frac{4x-3}{2x+1}$, find $f^{-1}(x)$.

Problem 2: If $f(x) = \frac{5}{6}x - \frac{3}{4}$, find $f^{-1}(x)$.

Problem 3: If $f(x) = -(x+2)^2 - 1$, find $f^{-1}(x)$.

Problem 4: If $f(x) = -3x + 11$, find $f^{-1}(x)$.

Problem 5: If $f(x) = \sqrt[5]{x+2} - 3$, find $f^{-1}(x)$.

Problem 6: If $f(x) = \frac{2x-5}{3}$, find $f^{-1}(x)$.