# General Physics

## Phs 101 (A)/ Physics

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# Elasticity, Young's and Bulk Moduli

**The Mass Density** ($\rho$) of a body is its mass per unit volume:

$$\rho = \frac{\text{Mass of body}}{\text{Volume of body}} = \frac{m}{V}$$

The SI unit for mass density is $kg/m^3$, although $g/cm^3$ is also used: 1000 $kg/m^3 = 1\ g/cm^3$. The density of water is close to 1000 $kg/m^3$.

**The Specific Gravity** (sp gr) of a substance is the ratio of the density of the substance to the density of some standard substance. The standard is usually water (at 4 °C) for liquids and solids, while for gases, it is usually air.

$$\text{sp gr} = \frac{\rho}{\rho_{\text{standard}}}$$

Since sp gr is a dimensionless ratio, it has the same value for all systems

## Elasticity:

**Elasticity** is the property by which a body returns to its original size and shape when the forces that deformed it are removed.

- Ability of a body to stretch and not break..
- A force which produces a change in configuration of the object on applying it, is called a deforming force.
- A change in shape of an object due to the application of a force is known as a deformation.
- Objects or physical media under the action of external forces experiences deformation.

- **Perfectly Elastic Bodies**

Those bodies which regain its original configuration. immediately and completely after the removal of: deforming force

- **Perfectly Plastic Bodies**

Those bodies which does not regain its original configuration at all on the removal of deforming force

**Forces of Deformation**

- Stress
- Strain

**Stress** It is a quantity that describes the magnitude of forces that cause deformation.

## Types of Stress

- **Normal Stress:** If deforming force is applied normal to the area

(i) If there is an increase in length, then stress is called tensile stress.

(ii) If there is a decrease in length, then stress is called compression stress.

(iii) When an object is compressed from all sides it is called bulk (or volume) stress.

- **Tangential Stress:** If deforming force is applied tangentially to the object surface (shear stress).

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## Strain

- It is defined as a fractional change in configuration.
- It is the quantity that describes the deformation of an object due to stress.
- It is a dimensionless quantity.
- Strain under a tensile stress is called tensile strain, strain under bulk stress is called bulk strain, and that caused by shear stress is called shear strain.

## Elastic Moduli

- A force applied to an object can change its shape. In general, the response of a material to a given type of deforming force is characterized by an elastic modulus, which is defined as

- Elastic modulus $= \dfrac{stress(tensile)}{strain(tensile)}$

(Tensile) Stress – This is a force per unit area and it is measured in $Nm^{-2}$
(Tensile) Strain – This is a fractional change in a dimension or volume.

We shall discuss three elastic moduli; Young's modulus for solids, the shear modulus for solids and the bulk modulus for solids and fluids.

# Young's Modulus

- This is the **measure of the resistance** of a solid to a change in its length when a force is applied perpendicular to a face.

- Let us consider a rod with an unstressed length lo and cross - sectional area A.

- Its length changes by Δl when it is ***subject to equal and opposite forces F*** **along its axis and perpendicular to the end faces**.

- These forces tend to stretch the rod.

Tensile stress on the rod is defined as

$$\text{Tensile stress} = \frac{F}{A}$$

$$\text{Tensile strain} = \frac{\Delta l}{l_o}$$

Young's modulus for the material of the rod is defined as the ratio

$$\text{Young's Modulus} = \frac{(Tensile\ \ )stress}{(Tensile\ \ )strain}$$

$$\gamma = \frac{F / A}{\Delta l / l_o}$$

# Shear Modulus

- The shear modulus of a solid indicates **its resistance** to a shearing force, which is a ***force applied tangentially*** to a surface.

  - Shear stress = $\frac{Tangential\ Force}{Area} = \frac{F_1}{A}$ = $\sigma_s$
  - Shear strain = $\frac{\Delta x}{h}$ = $\varepsilon_s = \frac{\text{Displacement}}{\text{Height}}$

where h is the separation between the top and the bottom surfaces.

- The shear modulus S is defined as $S = \frac{shear\ stress}{shear\ strain}$

$$S = \frac{F_1/A}{\Delta x/h}$$

# Bulk Modulus

- The **bulk modulus** **of a** ***solid or a fluid*** indicates its ***resistance to a change in volume***.

- Let us consider a **cube of some material**, **solid or fluid**.

- All the faces experience the ***same force F normal to each face***.

- The ***pressure*** on the **cube is the normal force per unit area.**

$$P = \frac{F}{A}$$

- When the **pressure on a body is increased**, its volume decreases. The **change in pressure ΔP** is called the ***volume stress*** and the fractional change in volume $\Delta V / V$ is called the volume strain i. e

$$\text{Volume stress} = \Delta P \qquad \text{Volume strain} = -\frac{\Delta V}{V_0}$$

- The bulk modulus B of the material is defined as

$$\text{Bulk Modulus} = \frac{\text{Volume Stress}}{\text{Volume Strain}} \qquad B = -\frac{\Delta P}{\Delta V / V}$$

The minus sign is used so as to cancel the negative numerical value of $\Delta V$ and thereby make $B$ a positive number. The bulk modulus has the units of pressure.

$$K = \frac{1}{B} = compressibility$$

## Terms Associated with Elasticity

- **Limit of Elasticity:** This is the maximum value of deforming force for which elasticity is present in the body
- **Breaking Stress:** This is the minimum value of stress required to break a wire.
- **Safety factor =** $\dfrac{Breaking - stress}{Working - stress}$
- **Elastic Relaxation Time:** This is the time delay in restoring the original configuration after removal of deforming force.

- **Poisson's Ratio:**

➢ When a deforming force is applied at the free end of a suspended wire of length l and radius R, then its length increases by dl but its radius decreases by dR.

➢ It occurs when a single force produces two types of strains.

➢ The two strains produced are; longitudinal and lateral strain

$$Poisson's-Ratio(\sigma)=\frac{Lateral-strain}{Longitudinal-strain}$$

➢ The theoretical value of Poisson's ratio lies between – 1 and 0.5 while its practical value lies between 0 and 0.5.

## Example:

Find the density and specific gravity of gasoline if 51 g occupies 75 $cm^3$.

(Density of Gasoline $6.8 \times 10^2\ kg/m^3$ )

## Solution:

**NOTE that** 1.0 $cm^3 = 1.0 \times 10^{-6}\ m^3$..

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{0.051\ kg}{75 \times 10^{-6}\ m^3} = 6.8 \times 10^2\ kg/m^3$$

$$\text{sp gr} = \frac{\text{Density of gasoline}}{\text{Density of water}} = \frac{6.8 \times 10^2\ kg/m^3}{1000\ kg/m^3} = 0.68$$

# Example

A solid cylindrical steel column is 4.0 m long and 9.0 cm in diameter. What will be its decrease in length when carrying a load of 80 000 kg? $Y=1.9 \times 10^{11}$ Pa.

## Solution:

Cross-sectional area of column $= \pi r^2 = \pi(0.045\text{ m})^2 = 6.36 \times 10^{-3}\text{ m}^2$

Then, from $Y=(F/A)/(\Delta L/L^0)$, →Young's Modulus

$$\Delta L = \frac{FL_0}{AY} = \frac{[(8.00 \times 10^4)(9.81)\text{ N}](4.0\text{ m})}{(6.36 \times 10^{-3}\text{ m}^2)(1.9 \times 10^{11}\text{ Pa})} = 2.6 \times 10^{-3}\text{ m} = 2.6\text{ mm}$$

# Example

The bulk modulus of water is 2.1 GPa. Compute the volume contraction of 100 mL of water when subjected to a pressure of 1.5 MPa.

**Solution:**

$$1 \text{ GPa} = 1 \times 10^9 \text{ Pa}.$$

**Using** $B = \Delta P/(\Delta V/V_0)$,

$$\Delta V = -\frac{V_0 \Delta P}{B} = -\frac{(100 \text{ mL})(1.5 \times 10^6 \text{ Pa})}{2.1 \times 10^9 \text{ Pa}} = -0.071 \text{ mL}$$

# Example

A box-shaped piece of gelatin dessert has a top area of 15 $cm^2$ and a height of 3.0 cm. When a shearing force of 0.50 N is applied to the upper surface, the upper surface displaces 4.0 mm relative to the bottom surface. What are the shearing stress, the shearing strain, and the shear modulus for the gelatin?

**Solution:**

$$\sigma_s = \frac{\text{Tangential force}}{\text{Area of face}} = \frac{0.50\ \text{N}}{15 \times 10^{-4}\ \text{m}^2} = 0.33\ \text{kPa}$$

$$\varepsilon_s = \frac{\text{Displacement}}{\text{Height}} = \frac{0.40\ \text{cm}}{3.0\ \text{cm}} = 0.13$$

$$S = \frac{0.33\ \text{kPa}}{0.13} = 2.5\ \text{kPa}$$