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1.
$A - B = A \cap B^c$
let x be Universal set
$\Leftrightarrow x \in (A-B)$
$\Leftrightarrow x \in A$ and $x \notin B$
$\Leftrightarrow x \in A$ and $x \in B^c$
$\Leftrightarrow x \in A$ and B^c
$\therefore A - B = A \cap B^c$ Proved

2. $(A-B) - C = (A-C) - (B-C)$
let x be Universal set
$\Leftrightarrow x \in (A-B) - C$
$\Leftrightarrow x \in (A-B)$ and $x \notin C$
$\Leftrightarrow [x \in A$ and $x \notin B]$ and $x \notin C$
By distribution law
$\Leftrightarrow [x \in A$ and $x \notin C]$ and $[x \notin B$ and $x \notin C]$
$\Leftrightarrow [x \in A \cap C']$ and $[x \in B' \cap C']$
$\Leftrightarrow x \in [A \cap C']$ and $x \in [B \cup C]'$ \de morgans law
$x \in (A \cap C') \cap (B \cup C)'$
$x \in (A - C) - (B \cup C)$
$\therefore (A-B) - C \neq (A-C) - (B-C)$ disProved

$\therefore (A' \cap B') = (A \cup B)'$
$\therefore A - B = A \cap B'$

3. $(A \cap B) \cup C = A \cap (B \cup C)$

let x be Universal set

$\Leftrightarrow x \in (A \cap B) \cup C$

$\Leftrightarrow x \in (A \cap B)$ or $x \in C$

$\Leftrightarrow x \in A$ and $x \in B]$ or $x \in C$

By Associative law

$\Leftrightarrow x \in A$ and $[x \in B$ or $x \in C]$

$\Leftrightarrow x \in A \cap (B \cup C)$

$\therefore (A \cap B) \cup C = A \cap (B \cup C)$ Proved

Associative law

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

16.

$A \Delta B = (A-B) \cup (B-A) \quad \forall A, B \in U$

for Commutative:

$A \Delta B = B \Delta A$

$\boxed{\begin{array}{l} \text{let } D = B-A \\ \text{let } C = A-B \end{array}}$

$\underbrace{(A-B)}_{C} \cup \underbrace{(B-A)}_{D} = \underbrace{(B-A)}_{D} \cup \underbrace{(A-B)}_{C}$

$C \cup D = D \cup C$

By Commutative law $A \cap B = B \cap A$
$A \cup B = B \cup A$

$C \cup D = C \cup D$

Proved

for Associative:

$A \Delta (B \Delta C) = (A \Delta B) \Delta C$

$A \Delta (B \Delta C)$:

$\Leftrightarrow [A \cap (B \Delta C)'] \cup [(B \Delta C) \cap A']$

$\Leftrightarrow [A \cap [(B \cap C') \cup (C \cap B')]'] \cup [(B \cap C') \cup (C \cap B')] \cap A'$

$\Leftrightarrow [A \cap (B \cap C')' \cap (C \cap B')' \cup (B \cap C') \cap A') \cup ((C \cap B') \cap A')$

$\Leftrightarrow A \cap ((B' \cup C) \cap (C' \cup B)) \cup ((B \cap C') \cap A') \cup ((C \cap B') \cap A')$

$\Leftrightarrow A \cap [(B' \cap C') \cup (C \cap B)] \cup ((B \cap C') \cap A') \cup ((C \cap B') \cap A')$

$\Leftrightarrow (A \cap B' \cap C') \cup (A \cap C \cap B) \cup (B \cap C' \cap A') \cup (C \cap B' \cap A')$

$(A \Delta B) \Delta C$:

$\Leftrightarrow (C \cap A' \cap B') \cup (C \cap B \cap A) \cup (A \cap B' \cap C') \cup (B \cap A' \cap C')$

By Commutative

$(A \cap B' \cap C') \cup (A \cap C \cap B) \cup (B \cap C' A') \cup (C \cap B' A')$

$A \Delta (B \Delta C) = (A \Delta B) \Delta C$

Proved

for Closure:

$a * b = c \quad , c \in N \quad \text{if } a, b \in N$

The Operation is closure under Otimes.

$U = [A, B, C]$

(ii) $A \Delta B = (A - B) \cup (B - A)$

let E be the Identity

$A \Delta E = A$

$A \Delta E = (A - E) \cup (E - A)$

for $(A - E)$ exist when $E = \emptyset$ or null

for $(E - A)$ is Undefined, such that E Contains no element of A

$\therefore E = \emptyset$

The Identity is Null / Void

(iii) find the Inverse of any element $f \in U$

$f \Delta f^{-1} = e$

where e = Identity

$\forall f \in U, f^{-1} = f$

f is the Inverse of itself.

Under Cayley table, f_2 elements Inverts itself

$f_2 = \{0, 1\}$

$\star$	0	1
0	0	1
1	1	0

Show that

1 $\log_{y^n} x = \frac{1}{n} \log_y x$

$\log_{y^n} x$

$\Rightarrow \frac{\log_a x}{\log_a y^n}$ → Change of Base

$\Rightarrow \frac{\log_a x}{n \log_a y} \Rightarrow \frac{1}{n} \cdot \frac{\log_a x}{\log_a y}$

$\Rightarrow \frac{1}{n} \log_y x$

4 $\log_{y^n} x^n = \log_y x$

$\log_{y^n} x^n$

$\Rightarrow \frac{\log_b x^n}{\log_b y^n}$ → Change of Base.

$\Rightarrow \frac{\cancel{n} \log_b x}{\cancel{n} \log_b y} \Rightarrow \frac{\log_b x}{\log_b y}$

$\Rightarrow \log_y x$

Hence show that:

$$81^{\frac{1}{\log_{6561} 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 6}} + \sqrt{7}^{\frac{2}{\log_5 125}} - 125^{\log_{125} 25} = \sqrt{27} + \sqrt[3]{49} - 22$$

$$81^{\frac{1}{\log_{9^4} 9}} + 3^{\frac{3}{\log_{6^{1/2}} 6}} + \sqrt{7}^{\frac{2}{\log_5 5^3}} - 125^{\log_{5^3} 5^2}$$

$$81^{\frac{1}{\frac{1}{4}\log_9 9}} + 3^{\frac{3}{2\log_6 6}} + \sqrt{7}^{\frac{2}{3\log_5 5}} - 125^{\frac{2}{3}\log_5 5}$$

from $\log_a a = 1$

$$81^4 + 3^{\frac{3}{2}} + \sqrt{7}^{2/3} - 125^{2/3}$$

$$81^4 + (3^3)^{1/2} + (\cancel{\sqrt{7}}^{2})^{1/3} - (5^{\cancel{3}})^{2/\cancel{3}}$$

$$81^4 + \sqrt{27} + \sqrt[3]{7} - 25 \neq \sqrt{27} + \sqrt[3]{49} - 22$$

2b.

Solve the equation

$\log_2 x + \log_x 2 = 2.5$

let m represent $\log_x 2$

$\frac{1}{m} + m = \frac{25}{10}$

find the l.c.m

$\frac{m^2+1}{m} = \frac{25}{10}$

Cross multiply

$25m = 10(m^2+1)$

$25m = 10m^2 + 10$

like term

$10m^2 - 25m + 10 = 0$

divide through by 5.

$2m^2 - 5m + 2 = 0$

$2m^2 - 4m - 1m + 2 = 0$

$2m(m-2) - 1(m-2) = 0$

$(2m-1)(m-2)$

$2m - 1 = 0$

$2m = 1$ OR $m - 2 = 0$

$m = \frac{1}{2}$. $m = 2$

$\therefore \log_x 2 = m$. when $m = \frac{1}{2}$

$x^m = 2$.

$x^{1/2} = 2$

$x = \sqrt{2}$

$\because 2^2 = x$. when $m = 2$ $[\log_2 x = m]$

$x = 4$ $2^m = x$

$X = \sqrt{2}$ or $X = 4$

2 ii

$\log_x 5 \times \log_x 2 = 10$

$\log_x 5+2 = 10$

$\log_x 7 = 10$

$x^{10} = 7$

~~Multiply~~ through ln

$\ln 10x = \ln 7$

divide through by 10

$\ln x = \frac{\ln 7}{10}$

Add $e^{\wedge}$ to both side

$\boxed{\therefore e^{\ln} = \text{Cancel out}}$

$e^{\ln x} = e^{\frac{\ln 7}{10}}$

$x = e^{\frac{\ln 7}{10}}$

$x = 1.214814044 04$

(3a) $x^4 + 12x^3 + 46x^2 + px + q$.

$\Rightarrow (x^2 + cx + d)^2$

$\Rightarrow (x^2 + cx + d)(x^2 + cx + d)$

$\Rightarrow x^4 + cx^3 + dx^2 + cx^3 + 2c^2x^2 + cdx + dx^2 + cdx + d^2$

$\Rightarrow x^4 + 2cx^3 + 2dx^2 + 2c^2x^2 + 2cdx + d^2$

By Comparison.

x^3:

$12 = 2c$

$c = 12/2$

$c = 6$ //

x^2:

$46 = 2d + c^2$

$46 = 2d + 36$

$10 = 2d$

$d = 5$ //

q:

$q = d^2$

$q = 5^2$

$q = 25$ //

$p = 2cd$.

$p = 2 \times 6 \times 5$

$p = 60$ //

$\therefore$ p and q are 60 and 25 respectively

$p = 60$

$q = 25$.

Scanned with CamScanner

3ii

$$x^2+(p+q)x+p^2-pq+q^2$$

$$\left[\frac{2x+p+q}{2}\right]^2=\frac{4pq-4p^2+q^2+p^2+q^2+2pq}{4}$$

$$\left[\frac{2x+p+q}{2}\right]^2=\frac{6pq-3p^2-3q^2}{4}$$

$$\left[\frac{2x+p+q}{2}\right]^2=-\frac{3(p-q)^2}{4}$$

$$\Longrightarrow\left[\frac{2x+p+q}{2}\right]^2+3\left[\frac{p-2}{4}\right]^2=0$$

3b
$$\frac{n+3}{(n-1)n(n+1)} = \frac{A}{(n-1)} + \frac{B}{n} + \frac{C}{n+1}$$

$$\frac{n+3}{\cancel{(n-1)n(n+3)}} = \frac{A(n)(n+1) + B(n-1)(n+1) + C(n-1)n}{(n-1)n(n+3)}$$

When $n = 1$

$1+3 = A(1)(1+1) + B(0) + C(0)$

$4 = 2A$

$A = 2$

When $n = 0$

$0+3 = A(0) + -B + C(0)$

$3 = -B$

$B = -3$

When $n = -1$

$-1+3 = A(0) + B(0) + C(-1-1)-1$

$2 = 2C$

$C = 1$

$\therefore A = 2 \quad B = -3 \quad C = 1$

Hence show that

$$\frac{5}{1\times2\times3} + \frac{6}{2\times3\times4} + \frac{7}{3\times4\times5} + \dots + \frac{n+3}{(n-1)n(n+1)} = \frac{3}{2} - \frac{n+2}{n(n+1)}$$

We know that $\dfrac{n+3}{(n-1)n(n+1)} = \dfrac{2}{(n-1)} - \dfrac{3}{n} + \dfrac{1}{n+1}$ $\Rightarrow$ Using Telescoping Series

when $n = 2$ $\left[\dfrac{2}{2-1} - \dfrac{3}{2} + \dfrac{1}{2+1}\right]$

when $n = 3$ $\left[\dfrac{2}{2} - \dfrac{3}{3} + \dfrac{1}{4}\right]$

when $n = 4$ $\left[\dfrac{2}{3} - \dfrac{3}{4} + \dfrac{1}{5}\right]$

when $n = 5$ $\left[\dfrac{2}{4} - \dfrac{3}{5} + \dfrac{1}{6}\right]$

when $n = n-3$ $\left[\dfrac{2}{n-3-1} - \dfrac{3}{n-3} + \dfrac{1}{n-3+1}\right] \Rightarrow \left[\dfrac{2}{n-4} - \dfrac{3}{n-3} + \dfrac{1}{n-2}\right]$

when $n = n-2$ $\left[\dfrac{2}{n-2-1} - \dfrac{3}{n-2} + \dfrac{1}{n-2+1}\right] \Rightarrow \left[\dfrac{2}{n-3} - \dfrac{3}{n-2} + \dfrac{1}{n-1}\right]$

when $n = n-1$ $\left[\dfrac{2}{n-1-1} - \dfrac{3}{n-1} + \dfrac{1}{n-1+1}\right] \Rightarrow \left[\dfrac{2}{n-2} - \dfrac{3}{n-1} + \dfrac{1}{n}\right]$

when $n = n$ $\left[\dfrac{2}{n-1} - \dfrac{3}{n} + \dfrac{1}{n+1}\right]$

$$\sum \frac{n+3}{(n-1)n(n+1)} = \frac{2}{1} - \frac{3}{2} + \frac{2}{2} + \frac{1}{n} - \frac{3}{n} + \frac{1}{n+1}$$

$$= \frac{4-3+2}{2} \qquad \frac{(n+1) - 3(n+1) + n}{n(n+1)}$$

$$= \frac{3}{2} + \frac{n+1-3n-3+n}{n(n+1)}$$

$$= \frac{3}{2} - \frac{2n+2}{n(n+1)}$$