MTS 104

1.0 Linear Motion:

Motion is one of the key branches of physics. It finds applications in numerous fields such as engineering, medicine, geology, sport science and so on.

1.1 Variables (Quantities in Motion)

(i) Distance: is a change in position relative to a reference point. It is a scalar quantity measured in metre (m) and as such, it can only be positive.

(ii) Displacement: is a change in position relative to a reference point in a particular direction. It is a vector quantity and also measured in metre (m) and represented by d, s, and x. Displacement, being a vector can be positive or negative.

(iii) Average Speed: is the rate of change of distance. It is a scalar quantity, measured in (m) per second (s), m/s. Since speed is likely to change over the course of motion.

$$\text{Average Speed} = \frac{\text{total distance covered}}{\text{Total time taken}}$$

$$= \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1}.$$

(iv) Instantaneous speed: is the speed recorded at a given point in time.

$$v = \frac{d}{t}$$

(V) Average velocity: is the rate of change of displacement and is also measured in m/s.

$$A_v = \frac{\text{Total displacement}}{\text{Total time taken}}$$

$$= \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$$

Instantaneous velocity: is a specified position or a particular point in time.

$$V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

(VI) Average Acceleration: is the rate of change of velocity and is measured in (m/s^2). It is also a vector quantity.

$$A_a = \frac{\text{change in velocity}}{\text{time taken}}$$

$$A_a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Acceleration occurs due to change in the

i) magnitude of the velocity only,
(ii) direction of the velocity only, or
(iii) magnitude and direction of the velocity.

(VII) Instantaneous acceleration: is the acceleration at a specified position or particular point in time.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v}{t}$$

(viii) Free Acceleration: this is experienced by an object undergoing vertical motion in the vicinity of the earth and this has value $g = 9.8\,ms^{-2}$

When an acceleration is negative, it is called deceleration.

1.2 <u>Equation of Motion</u>:

$$v = u + at \quad \text{———} \quad (1)$$

$$s = ut + \frac{1}{2}at^2 \quad \text{———} \quad (2)$$

$$v^2 = u^2 + 2as \quad \text{———} \quad (3)$$

$$s = \frac{1}{2}(u+v)t \quad \text{———} \quad (4)$$

$$s = vt - \frac{1}{2}at^2 \quad \text{———} \quad (5)$$

<u>Example 1</u> Find v when $u = 2ms^{-1}$, $a = 3ms^{-2}$ and $t = 4s$.

<u>Soln</u> Using $v = u + at$

$$v = 2 + 3 \times 4$$
$$= 2 + 12$$
$$= 14ms^{-1}$$

<u>Example 2</u> Find s when $v = 0.2ms^{-1}$, $u = 3.8ms^{-1}$ and $t = 10s$.

<u>Soln</u>

$$s = \frac{(u+v)}{2}t$$

$$s = \frac{3.8 + 0.2}{2} \times 10$$

$$s = 20m.$$

Example 3 Find s, when $v = 15ms^{-1}$, $a = 6ms^{-2}$

and of $t = 5s$.

Soln

$$s = vt - \frac{1}{2}at^2$$

$$s = 15 \times 5 - \frac{1}{2} \times 6 \times 5^2$$

$$= 75 - 75$$

$s = 0m$. this is possible, since we dealing with vector quantities.

Example 4 Find a when $v = 3ms^{-1}$, $u = 13ms^{-1}$ $s = 8m$

Soln

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s} = \frac{3^2 - 13^2}{2 \times 8} = \frac{9 - 169}{16} = -10m/s^2$$

Example 5 A plane flies from london Heathrow Airport to Dubai Airport a distance of 5500km at an average of 1200 km/h. The return trip was made at an average speed of 1050 km/h. Find the average speed for the whole journey.

Soln

$s = 5500km$

$u_1 = 1200km/h$

$u_2 = 1050 km/h$

$$t_1 = \frac{\text{distance}}{\text{average speed}} = \frac{5500}{1200} = \frac{55}{12}$$

$t_1 = 4.58$ hours

$$t_2 = \frac{\text{distance}}{\text{average speed}}$$

$$= \frac{5500}{1050} = \frac{110}{21}$$

$t_2 = 5.24$ hours

Average speed = $\frac{\text{total distance}}{\text{Total time}}$

$= \frac{5500 + 5500}{4.58 + 5.24} = 1120 km/h$

Example 6 the safe take-off velocity of a part london passenger plane is set at 210km/h. find the minimum acceleration that the airplane needs to move on a 2.2 km run away

Sol

$u = 0ms^{-1}$ $\qquad v = \frac{210 \times 1000}{3600}$

$v = 210 km h^{-1}$

$s = 2.2km = 2200m$ $\qquad = 58.3 ms^{-1}$

$a = ?$

Using $v^2 = u^2 + 2as$

$a = \frac{v^2 - u^2}{2s} = \frac{58.3^2 - 0^2}{2 \times 2200} = 0.77 ms^{-2}$

Example 7 A taxi driver moving at a velocity of 10m/s realised that he had 35sec to get to his destination which is 300m away. He therefore accelerated at $3m/s^2$ for the rest of the journey. to this destination on And he succeed in getting to this destination on time

$t = ?$

Sol $u = 10ms^{-1}$ $a = 3ms^{-2}$, $s = 300m$

Using $s = ut + \frac{1}{2}at^2$ by substitution:

$300 = 10 \times t + \frac{1}{2}(3)t^2$

$800 = 10t + 1.5t^2 \times 2$

$3t^2 + 20t - 1600 = 0$

Scanned with CamScanner

$3t^2 + 20t - 1600 = 0$

$(t - 20)(3t + 80) = 0$

$t - 20 = 0$

$t = 20$

$3t + 80 = 0$

$3t = -80$

$t = -\frac{80}{3}$

Example A particle moves such that its position x metres at time t seconds is given by the expression $x = 3t^3 - 13t^2 - 2t$.

(i) Determine the position of the particle when time $t = 0, 1, 2, 3, 4$, and 5.

Soln At $= 0$, then

$x = 3t^3 - 13t^2 - 2t = 3(0)^3 - 13(0)^2 - 2(0)$

$= 0m$.

At $t = 1$, $x = 3(1)^3 - 13(1)^2 - 2(1) = 3 - 13 - 2$

$x = -12m$

At $t = 2$, $x = 3t^3 - 13t^2 - 2t$

$= 3(2)^3 - 13(2)^2 - 2(2)$

$= 24 - 52 - 4$.

At $t = 3$, $x = 3(3)^2 - 13(3)^2 - 2(3)$

$= \cancel{3(27)}\ 81 - 117 - 6$

$x = -42$

At $t = 4$ $x = 3t^3 - 13t^2 - 2t$

$= 3(4)^2 - 13(4)^2 - 2(4)$

$= 192 - 208 - 8$

$x = -20m$.

At $t = 5$, the position $x = 3t^3 - 13t^2 - 2t$

$$x = 3(5)^3 - 13(5)^2 - 2(5)$$

$$x = 375 - 325 - 10$$

$$x = 40m.$$

1.3 Free Fall Motion

Example 1 A metal coin is thrown straight upwards with an initial velocity of 20m/s. Calculate the distance covered from the point of projection and velocity after 2s. take $g = 10ms^{-2}$

Soln

$u = 20ms^{-1}$

$g = 10ms^{-2}$

$t = 2s$

$s = ?$

$v = ?$

$$s = ut + \frac{1}{2}gt^2$$

$$s = 20 \times 2 - \frac{1}{2}(10)(2)^2$$

$$= 40 - 20 = 20m.$$

Example 2 An apple fruit falls freely from a tree at a height of 2.4m. How long does it take to reach the ground?

Soln

$u = 0ms^{-1}$ $\quad g = 9.8ms^{-2}$

$h = 2.4m$ $\quad t = ?$

Using $s = ut + \frac{1}{2}gt^2$

$u = 0.$

$$s = 0 \times t + \frac{1}{2}gt^2$$

$$s = \frac{1}{2}gt^2$$

$$t^2 = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 2.4}{9.8}}$$

$$t = \frac{2\sqrt{6}}{7}$$

$$t = 0.70s$$

Example 3 A 500g parcel is dropped from a height of 60m, from a plane which is moving upwards with a velocity of 5.0m/s.
Determine:
(i) the initial velocity of the parcel.
(ii) the time taken for the parcel to reach the ground? take $g = 10m/s$

Soln

$h = 60m \quad g = 10m/s^{-2}$

$u = ? \quad t = ?$

Using $s = ut + \frac{1}{2}gt^2$

$60 = -5t + \frac{1}{2}10t^2$

$60 = -5t + 5t^2$

$5t^2 - 5t - 60 = 0$

$t^2 - t - 12 = 0$

$(t-4)(t+3) = 0$

$t - 4 = 0$

$t = 4$

$t + 3 = 0$

$t = -3$

Since time cannot be negative, it follows that only solution is

$t = 4.0s$

1.4 Free fall motion

Example 1 An object is projected straight upwards with an initial velocity of U. If T is the time taken to return to the point of projection (time of flight, H the greatest height and g is the acceleration due to gravity, show that

(i) $T = \frac{2V}{g}$ (ii) $H = \frac{U^2}{2g}$

Soln

(i) $u = U\,ms^{-1}$

$t = T_s$

$s = H$

Using $v = u + gt$

$T = 2t$

$t = \frac{1}{2}T$ ——— (2)

$0 = U - g\left(\frac{1}{2}T\right)$

$U = \frac{1}{2}gT$

$T = \frac{2U}{g}$

(ii) Using $v^2 = u^2 + 2gh$

$0 = U^2 - 2gH$

$2gH = U^2$

$H = \frac{U^2}{2g}$

Example 2 An object is thrown vertically upwards with an initial velocity of 3 m/s from a ladder which is 5m above ground level. Taking g to be $10\,m/s^2$

(i) Express the height h of the object above the ground as a function of time t.

(ii) Use the expression in (i) to find the time the object hits the ground and velocity and velocity with which it strikes the ground.

Soln $h = 5m$, $g = 10ms^{-2}$ $u = 3ms^{-1}$

Using $s = ut + \frac{1}{2}gt^2$ ——— (1)

$h = 5 + s$ ——— (2)

$s = 3t - \frac{1}{2}10t^2$

$= 3t - 5t^2$

Therefore from (2)

$h = 5 + 3t - 5t^2$

Since the object hits the ground when the height is zero, thus

$0 = 5 + 3t - 5t^2$

$5t^2 - 3t - 5 = 0$

Using $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \dfrac{3 \pm \sqrt{(-3)^2 - 4(5)(-5)}}{2(5)} = \dfrac{3 \pm \sqrt{109}}{10}$

$t = \dfrac{3 + \sqrt{109}}{10} = 1.34$ or $t = \dfrac{3 - \sqrt{105}}{10} = -0.744$

Since time cannot be negative, it implies that the time taken to reach the ground is $t = 1.34s$.

Using $v^2 = u^2 + 2gs$

$= (-3)^2 + 2(10)(5)$

$= 9 + 100$

$v = \sqrt{109}$

$v = 10.4ms^{-1}$.

Scanned with CamScanner