# Impact Simplified

Watimagbo

## Tutorial Questions

Titiano

### Set theory

5) $n(\varepsilon) = 38$

$n(H) = 25$

$n(F) = 27$

$n(A) = 28$

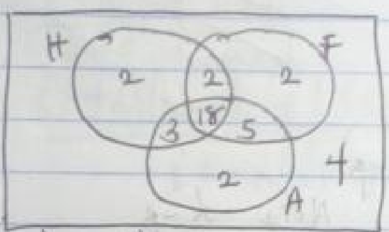
$n(H \cap F) = 20$

$n(F \cap A) = 23$

$n(H \cap A) = 21$

$n(H \cap F \cap A) = 18$

5a)




$n((H \cap A) \cap F') = 21 - 18$
$= 3$

$n((F \cap A) \cap H') = 23 - 18$
$= 5$

$n((H \cap F) \cap A') = 20 - 18$
$= 2$

$n(A \cap (H' \cap F')) = 28 - 3 - 18 - 5$
$= 2$

$n(F \cap (A' \cap H')) = 27 - 2 - 18 - 5$
$= 2$

$n(H \cap (F' \cap A')) = 25 - 2 - 18 - 3$
$= 2$

$n(H' \cap F' \cap A') = 38 - 2 - 2 - 2 - 3 - 18 - 5 - 2$
$= 4$

5b) Only one subjects $= 2 + 2 + 2 =$ <u>6 students</u>

5c) At least two subjects $= 2 + 3 + 5 + 18 =$ <u>28 students</u>

5d) None of the three subjects $=$ <u>4 students</u>

# Impact Simplified

watimaybo

## Polynomial and polynomial equation

7a) $x^2 + px + q = 0$

$\alpha + \beta$ and $\alpha - \beta$

$\alpha + \beta = \frac{-b}{a}$

$= -p/_1 = -p$

$\alpha\beta = \frac{c}{a}$

$= q/_1 = q$

$\therefore x^2 - \text{Sum}\,x + \text{product} = 0$

Sum of roots $= (\alpha + \beta) + (\alpha - \beta)$

Note $\alpha - \beta$

$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$(\alpha - \beta) = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$\therefore \alpha - \beta = \sqrt{p^2 - 4q}$

Sum of roots $= -p + \sqrt{p^2 - 4q}$

product of roots $= (\alpha + \beta)(\alpha - \beta)$

$= -p\sqrt{p^2 - 4q}$

$\therefore x^2 - \text{Sum}\,x + \text{product} = 0$

$x^2 - [p + \sqrt{p^2 - 4q}]\,x + [-p\sqrt{p^2 - 4q}] = 0$

$x^2 - [p + \sqrt{p^2 - 4q}]\,x - p\sqrt{p^2 - 4q} = 0$

OR

$x^2 - px - \sqrt{p^2 - 4q}\,x - p\sqrt{p^2 - 4q} = 0$

$x^2 - px - \sqrt{p^2 - 4q}\,[x + p] = 0$

Impact Simplified

Watimaybo

# Impact Simplified

Vatimago

7b) $x^2 - x - 1 = 0$

$\frac{1+\alpha}{2-\alpha}$ and $\frac{1+\beta}{2-\beta}$

$\alpha+\beta = \frac{-b}{a}$

$= \frac{--1}{1} = 1$

$\alpha\beta = \frac{c}{a}$

$= \frac{-1}{1} = -1$

$\text{Sum} = \frac{1+\alpha}{2-\alpha} + \frac{1+\beta}{2-\beta}$

$= \frac{(1+\alpha)(2-\beta) + (1+\beta)(2-\alpha)}{(2-\alpha)(2-\beta)}$

$= \frac{2-\beta+2\alpha-\alpha\beta+2-\alpha+2\beta-\alpha\beta}{4-2\beta-2\alpha+\alpha\beta}$

$= \frac{4+\alpha+\beta-2\alpha\beta}{4-2(\alpha+\beta)+\alpha\beta}$

$= \frac{4+1+2}{4-2-1}$

$= 7$

$\text{Product} = \frac{1+\alpha}{2-\alpha} \times \frac{1+\beta}{2-\beta}$

$= \frac{1+\alpha+\beta+\alpha\beta}{4-2\alpha-2\beta+\alpha\beta}$

$= \frac{1+1-1}{\cancel{4} \, 4-2(\alpha+\beta)-1}$

$= \frac{1}{3-2} = 1$

$\therefore x^2 - \text{Sum}\,x + \text{product} = 0$

$x^2 - 7x + 1 = 0$

Impact Simplified

Vatimago

# Impact Simplified

Watimagbo

## Matrix

6a) $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$

i) $A^2 = A \times A$

$$= \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 2-6 \\ 4-12 & 8+9 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$$

ii) $A^3 = A^2 \times A$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 9-16 & 18+12 \\ -8+68 & -16-51 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}$$

Hence $f(A) = 2A^3 + 3A^2 - 4$

$$= 2\begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix} + 3\begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix} + \begin{bmatrix} 27 & -12 \\ -24 & 51 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 48 \\ 96 & -83 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 48 \\ 96 & -87 \end{bmatrix}$$

# Impact Simplified

Watimagbo

6b) $2x+3y+z=11$
$x+y+z=6$
$5x-y+10z=34$

i) Inverse method

$$\begin{bmatrix}2&3&1\\1&1&1\\5&-1&10\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}11\\6\\34\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2&3&1\\1&1&1\\5&-1&10\end{bmatrix}^{-1}\begin{bmatrix}11\\6\\34\end{bmatrix}$$

To get the Inverse

Let $A=\begin{bmatrix}2&3&1\\1&1&1\\5&-1&10\end{bmatrix}$

Impact Simplified

$$A^{-1}=\frac{Adj A}{|A|}=\frac{(cofac A)^T}{|A|}$$

$$|A|=+2\begin{vmatrix}1&1\\-1&10\end{vmatrix}-3\begin{vmatrix}1&1\\5&10\end{vmatrix}+1\begin{vmatrix}1&1\\5&-1\end{vmatrix}$$

$=2(10--1)-3(10-5)+1(-1-5)$
$=2(11)-3(5)+1(-6)$
$=1$

Watimagbo

$Adj A=(cofac A)^T$

Cofac A

$+2=+\begin{vmatrix}1&1\\-1&10\end{vmatrix}=10--1=11$

$-3=-\begin{vmatrix}1&1\\5&10\end{vmatrix}=-(10-5)=-5$

$+1=+\begin{vmatrix}1&1\\5&-1\end{vmatrix}=+(-1-5)=-6$

$-1=-\begin{vmatrix}3&1\\-1&10\end{vmatrix}=-(30--1)=-31$

$+1=+\begin{vmatrix}2&1\\5&10\end{vmatrix}=+(20-5)=15$

$-1=-\begin{vmatrix}2&3\\5&-1\end{vmatrix}=-(-2-15)=17$

$+5=+\begin{vmatrix}3&1\\1&1\end{vmatrix}=+(3-1)=2$

$$-(-1) = -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = -(2-1) = -1$$

$$+(10) = +\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = +(2-3) = -1$$

$$\text{Cofac}A = \begin{bmatrix} 11 & -5 & -6 \\ -31 & 15 & 17 \\ 2 & -1 & -1 \end{bmatrix}$$

$$\text{Adj}A = (\text{Cofac}A)^T$$

$$= \begin{bmatrix} 11 & -31 & 2 \\ -5 & 15 & -1 \\ -6 & 17 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{|A|} = \begin{bmatrix} 11 & -31 & 2 \\ -5 & 15 & -1 \\ -6 & 17 & -1 \end{bmatrix}$$



Recall

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 1 \\ 5 & -1 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 11 \\ 6 \\ 34 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 & -31 & 2 \\ -5 & 15 & -1 \\ -6 & 17 & -1 \end{bmatrix} \begin{bmatrix} 11 \\ 6 \\ 34 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 121 - 186 + 68 \\ -55 + 90 - 34 \\ -66 + 102 - 34 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$\therefore x = 3, y = 1, z = 2$$

# Impact Simplified

Watimagbo

11) Crammers method

$$\frac{1}{\Delta_0} = \frac{x}{\Delta_x} = \frac{y}{\Delta y} \neq \frac{z}{\Delta_z}$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 1 \\ 5 & -1 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \\ 34 \end{bmatrix}$$

$$\Delta_0 = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 1 & 1 \\ 5 & -1 & 10 \end{vmatrix} = 1$$

$$\Delta_x = \begin{vmatrix} \overset{+}{11} & \overset{-}{3} & \overset{+}{1} \\ 6 & 1 & 1 \\ 34 & -1 & 10 \end{vmatrix}$$

$$= +11\begin{vmatrix} 1 & 1 \\ -1 & 10 \end{vmatrix} - 3\begin{vmatrix} 6 & 1 \\ 34 & 10 \end{vmatrix} + 1\begin{vmatrix} 6 & 1 \\ 34 & -1 \end{vmatrix}$$

$$= +11(10 - -1) - 3(60 - 34) + 1(-6 - 34)$$

$$= 11 \times 11 - 78 - 40$$

$$= 121 - 118$$

$$= 3$$

$$\therefore \frac{x}{\Delta x} = \frac{1}{\Delta 0}$$

$$x = \frac{3}{1} = 3$$

$$\Delta y = \begin{vmatrix} \overset{+}{2} & \overset{-}{11} & \overset{+}{1} \\ 1 & 6 & 1 \\ 5 & 34 & 10 \end{vmatrix}$$

$$= +2\begin{vmatrix} 6 & 1 \\ 34 & 10 \end{vmatrix} - 11\begin{vmatrix} 1 & 1 \\ 5 & 10 \end{vmatrix} + 1\begin{vmatrix} 1 & 6 \\ 5 & 34 \end{vmatrix}$$

$$= +2(60 - 34) - 11(10 - 5) + 1(34 - 30)$$

$$= 52 - 55 + 4$$

$$= 1$$

$\Delta y = 1$

$$\therefore \frac{y}{\Delta y} = \frac{1}{\Delta 0}$$

$$y = 1/1 = 1$$

# Impact Simplified

watimagbo

$$\Delta z = \begin{vmatrix} 2 & 3 & 11 \\ 1 & 1 & 6 \\ 5 & -1 & 34 \end{vmatrix}$$

$$= +2\begin{vmatrix} 1 & 6 \\ -1 & 34 \end{vmatrix} - 3\begin{vmatrix} 1 & 6 \\ 5 & 34 \end{vmatrix} + 11\begin{vmatrix} 1 & 1 \\ 5 & -1 \end{vmatrix}$$

$$= 2(34 - -6) - 3(34 - 30) + 11(-1 - 5)$$

$$= 2(40) - 3(4) + 11(-6)$$

$$= 80 - 12 - 66$$

$$= 2$$

$$\therefore \frac{z}{\Delta z} = \frac{1}{\Delta 0}$$

$$z = \frac{2}{1} = 2$$

$$\therefore x = 3, y = 1, z = 2$$