1) $\lim_{x \to 0} (\sin x + \cos e^x)$ in 4 d.p.

$\Rightarrow \sin 0 + \cos e^0$

$= 0 + \cos 1 = \cos 1$

$= 0.9998476952$

$= 0.9998$ (to 4 d.p)

2) $y = \dfrac{x+1}{x-1}$

$\dfrac{dy}{dx} = \dfrac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$

$\dfrac{dy}{dx} = \dfrac{(x-1) - x - 1}{(x-1)^2}$

$\dfrac{dy}{dx} = \dfrac{-2}{(x-1)^2} = -2(x-1)^{-2}$

$\dfrac{d^2y}{dx^2} = 4(1)(x-1)^{-3}$

$$\boxed{\dfrac{d^2y}{dx^2} = \dfrac{4}{(x-1)^3}}$$

$(x-1)\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx}$

$(x-1) \times \dfrac{4}{(x-1)^3} + 2\left(\dfrac{-2}{(x-1)^2}\right)$

$\Rightarrow \dfrac{4}{(x-1)^2} - \dfrac{4}{(x-1)^2} = 0$

3) $y^2(x+3) = 0$?

$y^2(1) + 2y\dfrac{dy}{dx} = 0$

$-2y\dfrac{dy}{dx} = y^2$

$\dfrac{dy}{dx} = \dfrac{y^2}{-2y} = -\dfrac{y}{2}$

At $x = 2$,

$y^2(2+3) = 0$

$5y^2 = 0$

$\therefore y = 0$

$\Rightarrow dy/dx = -y/2 = -0/2 = 0$

4) $y = \dfrac{8}{x^2 - 4}$

For vertical asymptotes,

$\Rightarrow x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

$\therefore x = 2$ or -2 are asymptotes

5) $\lim_{x \to 2} \dfrac{x-2}{x^2-4}$

$\lim_{x \to 2} \dfrac{x-2}{(x-2)(x+2)}$

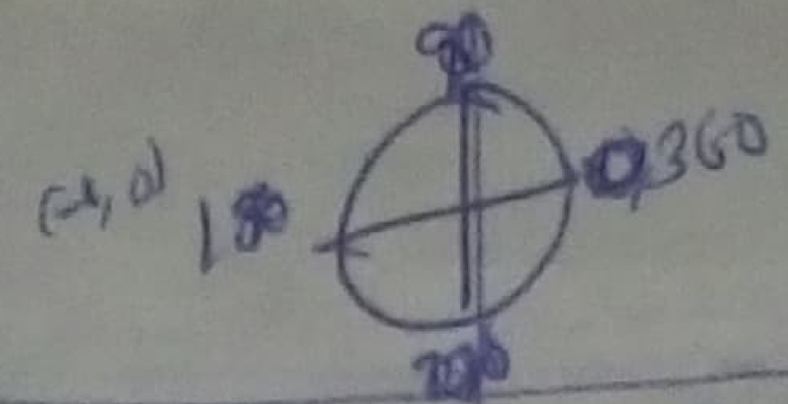
$\lim_{x \to 2} \dfrac{1}{x+2} = \dfrac{1}{2+2}$

$= \dfrac{1}{4}$

6) Gottfried Leibniz & Sir Isaac Newton

7) $f(x) = \dfrac{x-1}{x+1}$

$\lim_{k \to 0} \dfrac{f(x+k) - f(x)}{k}$



Roof of shop

$\Rightarrow \dfrac{\dfrac{(x+k)-1}{(x+k)+1} - \dfrac{x-1}{x+1}}{k}$

$\left(\dfrac{x+k-1}{x+k+1} - \dfrac{x-1}{x+1}\right) \times \dfrac{1}{k}$

$\dfrac{(x+k-1)(x+1) - (x-1)(x+k+1)}{(x+k+1)(x+1)} \times \dfrac{1}{k}$

$\dfrac{x^2+kx-x+x+k-1-x^2-kx-x+x+k+1}{k(x+k+1)(x+1)}$

$\dfrac{2k}{k(x+k+1)(x+1)}$

$= \dfrac{2}{(x+k+1)(x+1)}$

$\sin(\pi+\alpha) = \sin\pi\cos\alpha + \cos\pi\sin\alpha$

$= 0(\cos\alpha) + (-1)\sin\alpha$

$= -\sin\alpha$

$\sinh y = \cosh x$, $\dfrac{dy}{dx}$ at $(1,0)$

$\cosh y \dfrac{dy}{dx} = \sinh x$

$\dfrac{dy}{dx} = \dfrac{\sinh x}{\cosh y}$

At $(1,0)$,

$dy/dx = \dfrac{\sinh(1)}{\cosh(0)} = \dfrac{\sinh(1)}{1}$

$= 1.175$

10) $f(x) = \dfrac{3x^2-2x+1}{2x-1}$

As $x \to 1$,

$f(1) = \dfrac{3(1)^2-2(1)+1}{2(1)-1} = \dfrac{3-2+1}{2-1}$

$= \dfrac{2}{1} = 2$

11) $\lim_{x\to\infty^+} \sqrt{x}\sin x$

$\Rightarrow \sqrt{\infty}\sin\infty$

(DNE)

12) $y = \tan^{-1}(x^2+1)$

$\dfrac{dy}{dx}$ Let $u = x^2+1$

$u' = 2x$

$\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx}$

$\Rightarrow \dfrac{d}{du}(\tan^{-1} u)\cdot(2x)$

$= 2x \times \dfrac{1}{1+u^2}$

$= \dfrac{2x}{1+(x^2+1)^2}$

$\Rightarrow \dfrac{2x}{(x^2+1)^2+1}$

At $x=-1$,

$y' = \dfrac{2(-1)}{((-1)^2+1)^2+1}$

$y' = \dfrac{-2}{5}$

13) $Ax^2+By^2=9$

$\Rightarrow 2Ax + 2By\,dy/dx = 0$

$\dfrac{dy}{dx} = \dfrac{-2Ax}{2By}$

$\dfrac{dy}{dx} = -\dfrac{Ax}{By}$

At $dy/dx = -4/4$, $x=1$, $y=2$

$$-\frac{1}{4} = \frac{A(1)}{B(2)}$$

$$\frac{A}{2B} = -\frac{1}{4}$$

$$\Rightarrow \frac{A}{B} = -\frac{1}{4} \times 2 = -\frac{1}{2}$$

$$x^3 + y^3 = 27xy$$

From algebraic identities,

$$x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

$$\Rightarrow 27xy = (x+y)^3 - 3xy(x+y)$$

5 $x = t^3 - 4t^2 + 4t$

$$v = \frac{dx}{dt} = 3t^2 - 8t + 4$$

At rest, $v = 0$

$$\Rightarrow 3t^2 - 8t + 4 = 0$$

$$\Rightarrow t = 2 \text{ or } 2/3$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$= 6t - 8$$

At $t = 2$,

$a = 6(2) - 8 = 4 m/s^2$ OR

At $t = 2/3$,

$$a = 6\left(\frac{2}{3}\right) - 8 = -4 m/s^2$$

16) $\sin 120°$

$$\sin 120 = \sin(180 - 60)$$

$$= \sin 60 = \sqrt{3}/2$$

17) $\sin\theta = 5/13 \quad 0° \leq \theta \leq 90°$

But $\sin^2\theta + \cos^2\theta = 1$

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

$$\cos\theta = \sqrt{1 - (5/13)^2}$$

$$= \sqrt{1 - 25/169} = \sqrt{144/169}$$

$$= \frac{12}{13}$$

18) $f(x) = 2x^3$

$$\frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow \frac{2(x+h)^3 - 2x^3}{h}$$

$$= \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$$

$$\Rightarrow \frac{\cancel{2x^3} + 6x^2h + 6xh^2 + h^3 - \cancel{2x^3}}{h}$$

$$= \frac{h(6x^2 + 6xh + h^2)}{h}$$

$$= 6x^2 + 6xh + h^2$$

19) $y = \sqrt{4x^3 + 2x}$

$$y = (4x^3 + 2x)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(12x^2 + 2)(4x^3 + 2x)^{-1/2}$$

$$\frac{dy}{dx} = \frac{6x^2 + 1}{(4x^3 + 2x)^{1/2}}$$

20)
$y = \sqrt{\sqrt{x}} + \frac{1}{\sqrt{x}}$

$y = (x^{1/2})^{1/2} + \frac{1}{x^{1/2}}$

$y = x^{1/4} + x^{-1/2}$

$y = \frac{1}{4} x^{-3/4} - \frac{1}{2}(x^{-3/2})$

$y = \frac{1}{4(\sqrt[4]{x})^3} - \frac{1}{2(\sqrt{x})^3}$

21)
$y = 2x^3 - x^2 + 3x + 1$

$\frac{dy}{dx} = 6x^2 - 2x + 3$

At $x = 1$,

$dy/dx = 6(0)^2 - 2(1) + 3$

$= 0 - 2 + 3 = 1$

At $x = 1$, $y = 2(1)^3 - (1)^2 + 3(1) + 1$

$= 2 - 1 + 3 + 1 = 5$

Equ. of tangent

$y - y_1 = m(x - x_1)$

$y - 5 = 1(x - 1)$

$y - 5 = x - 1$

$\therefore x - y + 4 = 0$

22)
$y = 2x^2 + 5x - 3$

$dy/dx = 4x + 5$

At $x = 2$,

$dy/dx = 4(2) + 5 = 8 + 5$

$= 13$

23)
$y = 2\phi x + \phi^2$?

24)
2.3 rad

$= 2.3 \times \frac{180}{\pi}°$

$= 131.78°$

25)
$y = \frac{x^4 - 3x^3 - 4x^2 + 5}{x^2}$

$y = x^2 - 3x - 4 + 5/x^2$

$\frac{dy}{dx} = 2x - 3 + 5(-2)x^{-3}$

$\frac{dy}{dx} = 2x - 3 - 10/x^3$

$\frac{dy}{dx} = \frac{2x^4 - 3x^3 - 10}{x^3}$

28)
$y = \sin x$

$dy/dx = \cos x$

$d^2y/dx^2 = -\sin x$

$d^3y/dx^2 = -\cos x$

$d^4y/dy^4 = \sin x = y$

27)
$f(x) = x^2 - 1$

$f(\sin x - \cos x) = (\sin x - \cos x)^2 - 1$

$= \sin^2 x - 2\sin x \cos x + \cos^2 x - 1$

$= (\sin^2 x + \cos^2 x) - 1 - 2\sin x \cos x$

$= 1 - 1 - 2\sin x \cos x$

$= -2\sin x \cos x = -2\sin 2x$

28)
$f(x) = \frac{x^2 + x - 6}{x^2 + kx - 3}$

~~If not continuous at x = 2,~~

The function $f(x)$ is not continuous when $x^2+kx-3=0$

At $x=3$,

$3^2+3(k)-3=0$

$9+3k-3=0$

$3k+6=0 \Rightarrow k=-2$

29) $-\cos(90+\beta)$

$= -[\cos 90 \cos\beta - \sin 90 \sin\beta]$

$= -(0 \times \cos\beta - 1 \times \sin\beta)$

$= -(-\sin\beta) = \sin\beta$

30) $f(x) = \dfrac{\sqrt{(x-3)(x+2)}}{x-1}$

Domain of $f(x)$:

$(x-3)(x+2) > 0$ and $x-1 \neq 0$

$\therefore -2 < x < 3$ and $x \neq 1$

$\Rightarrow$ Domain

$\{x: -2 < x < 3, x \in R\} / 1$

31) $x^2 + xy + y^2 = 3$

$\Rightarrow 2x + x\dfrac{dy}{dx} + y + 2y\dfrac{dy}{dx} = 0$

$(x+2y)\dfrac{dy}{dx} = -(2x+y)$

$\dfrac{dy}{dx} = \dfrac{-(2x+y)}{x+2y}$

At 1, 1,

$dy/dx = \dfrac{-(2+1)}{1+2} = -3/3$

$dy/dx = -1$

Let m' be the gradient of the normal

$\therefore m' = -1/m$

$= -1/-1 = 1$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x-1)$

$y - 1 = x - 1$

$\therefore x = y$