MTS 104

INTRODUCTION TO MECHANICS

Course synopsis

The notion of speed, displacement,speed,velocity and acceleration of particle. Newton's laws of motion and application to simple problems. Work,power and Energy.Application of the principle of conservation of energy to motion particles and these involving elastic strings and springs.Collision of smooth spheres.Simple problem on projectiles. Simple pendulum and simple harmonic motion. Resultant of any number of forces acting on a particle.Reduction of coplanar forces, parallel forces, couples.Laws of friction.Applications of principles of moments.Moment of inertial of simple bodies

OBJECTIVE QUESTIONS

1. A particle is projected with an initial velocity of 42m/s at an angle 60° to the horizontal. After 2 seconds, find the vertical velocity.
   A. 16.72m/s B. 12 .57m/s
   C. 17.67m/s D. 15.27m/s
2. The path or trajectory of a particle is
   A. hyperbola B. Parabola C. ellipse D. circus
3. A man fired a bullet from the top of a tree which is 64m tall. The bullet is fired with a velocity of 70m/s at an angle of 35° to the horizontal line from the eye view of the man. If in sitting position the man 68m tall Determine maximum taught reached from the ground.
   A. 145.05m. B 146.05m C. 147.05m D. 148.05m
4. Forces $2i-j+k$, $3i+4j+4k$, $-5i+3j+3k$ act thought points with position vector $3i+2k$, $4i-5j+\lambda k$, $-4i-2j+4k$ respectively. Find the value of λ, if the system of forces is in equilibrium.
   A. -1 B. -2 C. -3 D. -4
5. A force $F = 7i+2j-3k$ acts through the point p with positions vector $4i+2j+6k$.A line L passes through point A with position vector is $2j+2k$. Find the moment of F about L.
   A. $\frac{49\sqrt{17}}{17}$ B. $\frac{17}{49\sqrt{17}}$ C. 49 D. 17
6. A simple pendulum is formed by a bob of mass 2kg at the end of a cord 600mm long. How many complete oscillations will it

makes per second.

A. O.61Hz B. 0.62Hz C. 0.63Hz D. 0.64Hz

7. A uniform ladder rests at an angle of 60° with the horizontal against a smooth vertical wall and on rough ground. The ladder weight 60N and its length is 800cm. Find the reaction at the wall.

A. 3.17N B. 17.3 N C. 62.4N D. 4.62N

8. A uniform rod AB of weight 30N is jointed at a hinge on a vertical wall. It is held in a horizontal position by a sitting attached to the end and to a point. C of the wall vertical above A, where angle ACB=30°. Find the tension in the string.

A. 17.30N B. 17.31N C. 17.32N D. 17.33N

9. A parcel of mass 10kg rests on a lorry. When the Lorries accelerate at 1.5m/s², the parcel is just about to slide backwards. What is the coefficient of friction between the parcel and the lorry? (g= 2.8/ s²)

A. 15.3N B. 15.4N C. 15.5N D. 15.6N

Use the information below to answer question 11-12

The speed of a car increased from 18/s to 48/s in 6m.

11. What was the acceleration?

A. 10/53 sec B. 1/53sec C. 5/53 sec D. 53sec

12. How long did this take?

A. 165m/s² B. 105m/s² C. 65m/s² D. 125m/s²

13. A load of mass 80g is placed in a lift. Calculate the reaction between the floor of the lift and the load when the lift and the load when the lift moves upward at constant.

A.184N B.134N C. 708N D. 708N

14. A particle moves along a straight line so that after sec, it distance from fixed point O on the line is given by s= (t-1)² (t-2). Find its distance from O when the velocity is zero

A. 0 and 4/27 B. 0 and 4 C. 2/27 and 4/27

D. 4/27 and 5 E. 1 and -4/27

15. An inelastic particle of mass 20kg impinges on another particle of mass 2kg coming from the opposite direction with an equal velocity. If the common velocity after impact is 9m/s. Find the velocities before impact.

Five force p,2 ,5,Q and 8 Newton ... acting on body in direction 030°,045°,

090. 150° and 270 respectively are in equilibrium, calculate to 2 d.p

16. The value of P
A. 0.56N B. 0.78N C. 0.77N D. 0.95N

17. The value of Q
A. 2.50N B. 4.20N C. 2.40N D. 3.60N

18. An object of mass 15kg is pulled over a rough surface of 45N. The object accelerates at a rate of $2m/s^2$. The frictional force between the object and the force is
A. 15N B. 10N C. 5N D. 20N

Three point A,B and C have position vectors $i + j + k$, $i + 2k$, $3i + 2j + 3k$ respective to a fixed origin O.A particle P state form B at time $t = 0$, and moves along BC toward C with constant speed of 1 unit per second. Find the position of P after t seconds.

19. Relative to O.
A. $(t-1)i + 2tj + 3k$ B $(\frac{2}{3}t + 1)\, i + tj + 3k$
C. $(\frac{2}{3}t + 1)\, i + \frac{2}{3}j + (t/3 + 22)k$ D. $(t/3 - 1)\, i + \frac{2}{3}\, t\, j - 3k$

20. relative to A
A. $\frac{2}{3}t\, i + (\frac{2}{3}t - 1)j + (t/3 + 1k)$ B. $(\frac{2}{3}t + 1)i + tj + 3k$
C. $(\frac{2}{3}t + 1)i + \frac{2}{3}j + (t/3 + 22)k$ D. $(t/3 - 1)i + \frac{2}{3}tj - 3k$

21. If the angle PAB = 0, then Cos0 is
A. $\sqrt{\frac{3 - t\,3t}{2 - 2t + 6}}$ B. $\sqrt{\frac{3 + t}{6\,(3t^2 + 2t + 6)}}$
C. $\sqrt{\frac{0 - t}{6\,(3t^2 - 2t + 6)}}$ D. $\sqrt{\frac{6 + t}{3t^2 - 2t + 6}}$

An electric train takes 3 minutes to travel between two stations 2.97km apart. The train accelerates uniformly to a speed of 18m/s and then travels for at time at this speed before retarding uniformly to rest at the second station. If the acceleration and retardation are in the ratio 2:3.

22. Calculate the time for which the train was accelerating
A. 20sec B. 12sec. C.15sec D.189sec

23. Calculate the time for which the train was travelling at steady speed
A. 200sec B.120sec C.150sec D.180sec.

24. Fine theacceleration of the train
A. $12m/s^2$ B. $1.0m/s^2$ C. $1.2m/s^2$ D.$1.8m/s^2$

25. Fine the retardation of the train

A. $1.0m/s^2$ B. $1.2m/s^2$ C. $1.5m/s^2$ D. $1.8m/s^2$

A particle is projected with an initial speed of 40m/s at an angle 0 to the horizontal group. If it hits the group at a point 60m away from the point of project. Then

26. it's time of flight is
A. 8 SinΘsec. B. 4 SinΘsec C. 16SinsΘsec. D. 6SinΘsec.

27. Sin2Θ is
A. $\frac{5}{16}$ B. $\frac{3}{8}$ C. $\frac{5}{8}$ D. $\frac{3}{16}$

28. A stone is thrown vertically upwards with a speed of 20m/s. one second later; a second stone is thrown vertically upward with a speed of 30m/s. At what height above the ground do they collide?
A. $\frac{210}{16}$ m B. $\frac{250}{49}$ m C. $\frac{315}{16}$ m D. $\frac{305}{16}$ m

A car state from rest, accelerates at $1m/s^2$ for 10 seconds. it then continues at a steady speed for a further 20 seconds and decelerates to rest in 5 seconds.

29. The total distance travelled is
A. 215m B. 305m C.415m D.275m

30. The average speed is
A. 43/7m/s B. 61/7 m/s C. 83/7m/s D. 55/7 m/s

31. The total distance travelled is
A. 215sec B. 18.75sec C. 16.75sec D. 15.50sec

32. Equal force act on two bodies which are initially at rest for equal time if the ratio of the masses is 4:3, the ratio of the distances covered is
A. 3:4 B. 9:16 C. 4.5 D. 2.5

33. About the origin
A. $2i - j + k$ B. $i + 2j + k$ C. $i - j + k$ D. $i - 2j + k$

34. About a point Q with position vector $i - 2j + 3k$.
A. $3i + 11j + 7k$ B. $3i - 11j + 7k$
C. $5i - 11j - 7k$ D. $5i + 11j + 7K$

35. Fine the vector equation of the axis XY through Q.
A. $i + 2j - 3k + (3i + 11j - 7k)$
B. $i - 2j - 3k + (5i + 11j + 7k)$
C. $i + 2j - 3k + (3i - 11j + 7k)$

D. $i - 2j + 3k + (5i - 11j - 7k)$

THEORECTICAL QUESTIONS

1(a) State the Newton's laws of motion.

(b) Two particles string which passes over a fixed smooth pulley. Obtain the acceleration of the particles and the tension in the string.

(c) If the position vector r(t) of a particle at time t is given by

$\hat{r}(t) = (5\sin 2t - 3)i + 5\cos 2tj$

Show that

(i) The particle moves in a circle in the xy plane.

(ii) The acceleration of the particle is always directed towards the center of a circle.

2(a) Define "simple harmonic motion" and give two area of application

(b) One end of light inelastic string of length L is fixed. To the other end is attached a particle of mass m which hangs freely under gravity. The particle is pulled sideways through some distance keeping the taut and released. Show that

(i) The string remains taut throughout the motion.

(ii) For small initial displacement the motion is approximately simple harmonic with period $2\pi\sqrt{L/g}$

C. The acceleration of a particle moving in a straight line is given at time t by a $(t) = 3t - 7t^2$ at $t = 0$, the particle is at the origin and line zero velocity. Find

(i) The fastest distance the particle is from its starting point before returning there.

2 when it returns to its starting point

3(a) state the law of conservation of linear momentum during impact

(b) A ball of mass 10kg with speed 60m/s impinges on a second ball of equal radius but with mass 30kg at a speed $20\sqrt{2}$m/s. If their velocity immediately before impact is inclined at 30° and 40° respectively to the line of center at the instant of impact, determine their velocity immediately after impact given that $e = 1/2$.

(c) A bird is flying with velocity of 14m/s at a constant of elevation of 60°. At the instant when it is 10m vertically above a boy, the latter throws a stone at it at an initial angle of $\alpha \geq \tan^{-1}(2+\sqrt{3})$.

4(a) (1) Define "moment of a force".

(ii) When is a system of forces said to form a couple?

(iii) When is a system of forces said to be in equilibrium?

(b) ABCD is a rectangle with AB = 6m and BC = 4m. Force of 8N, 8N, PN, PN act along AB, CD, CB and AD, respectively in the direction indicated by the order of the letters. Find the value of P When

i. The forces are in equilibrium,

ii. The forces form a couple with moment, 8Nm in the same, ABCD

(C) Consider like forces of PN and QN acting at points A and B and at right angles to the line AB. Let the length of AB be a meters. If the resultant of PN and QN act through a point C on AB, where AC = x meters, show that AC: BC = Q : P.

5(a) A forces (3t+2j)N, (-5i-4j)N and (2i+2j)N act at point with position vectors (i+3j)m, (4i-j)m and (-2i-j)m respectively show that these forces form a couple and find the moment of the couple.

(b) A uniform bean AB of mass 50kg and length 4m rest on supports at A and B. Objects of mass 20kg and 40kg are hung on the beam at C and D respectively, where AC =1.4m and AD = 3.2m. Find the reaction at the supports.

(c) A ladder AB of mass 40kg rests on smooth horizontal ground and leans against a vertical wall. The inclination of the ladder to the wall is 30°. The ladder is kept in position by a horizontal force PN applied to the bottom of the ladder. Find the value of P and the reactions at the wall and ground.

6.(a) A block of mass 10kg rests on a horizontal floor (coefficient of friction 0.4).(α) What force is required just to make the block move when

(i) Pulling horizontally, (ii) pulling at an angle of 60^0 to the horizontal?(β) If the block is pulled with a horizontal force of 50N, with what acceleration does it move? ($g = 9.8ms^{-2}$)(b) A mass of 20kg is placed on a plane inclined at an angle of 30° to the horizontal. What force parallel to the plane is required

(i) to hold the mass at rest, (ii) to just make it move up the plane?

(iii) What horizontal force will it move up the plane? Take n = 0.4, $g = 9.8ms^{-1}$

7. (A) (i) show that for a particle performing a rectilinear motion with constant acceleration a and velocity v(t) at time t, with v_0 as its initial velocity : $v^2 = v_0^2 + 2a(x - x_0)$

Where x is the displacement of the particle and x_0 is the initial displacement reference to an origin.

(ii) A point moving in a straight line is retarding uniformly and travels distances a and b in successive time intervals t and t'. Prove that the [illegible] $t+t'$ If the above distance respectively are 56cm

and 69cm in time intervals 2s and 3s, find the initial speed and the total distance described before coming to rest.

(b) Consider a particle with displacement r and mass m = 50kg where $r = 3t \cos ti + 7t \sin tj + 5t \tan tk$, angle in radian. Find the velocity, acceleration and the force acting on the body after 5s.

8. Given a particle of mass (m) projected with a velocity (v) from a reference point O in a direction an angle a with the horizontal axis; show that:

(a) The vertex of the trajectory parabola is

$$\left(\frac{u^2 \sin a \cos a}{g}, \frac{v^2 \sin^2 a}{2g}\right)$$

Where g is the acceleration due to gravity

(b) The range is $\frac{v^2 \sin 2\alpha}{g}$ and hence the

Best angle of projection for a long Jumper is 45^0 and that

(c) The time of flight T of the projectile is $\left(\frac{2v \sin \alpha}{g}\right)$

(d) If a bomb due to explode in 5s is thrown with velocity 30 ms/s at elevation 60^0. Find the height, range and velocity of the bomb at the instant of explosion. Take $g = 110\ ms^{-2}$

(8a) The aggregate force acting on body is given by body is displaced in the direction given by required to find;

i. The magnitude of the force,
ii. The distance covered,
iii. The work done

(b) A particle of mass (m) is constrained to execute a rectilinear simple harmonic motion under a force towards O, of magnitude $m\omega^2 r$. (x) being the particle displacement from O. when passing through O its velocity is (v) and when its velocity has become ½ v in the same direction, an impulse (I) is applied to the particle in the direction of the motion .assuming the same law of force, fine the time and total distance travelled from O the first position of instantaneous rest.

© If a force f is applied on a body to cause a displacement x, under what condition will the work done be zero?

9.(a) let an object be acted upon by a set of coplanar forces.
i. State the possible action of the effective force on the object.
ii. State possible independent modes of movement of the object.
iii. Use (i) and (ii) to set conditions for the object equilibrium of the object.
iv. State any 3 theorems you know that deal with equilibrium of three forces acting on an object.

(b) If an object is acted upon by forces 2i-j+k, 3i+4j-4k and 5i-3j+3k through the point position vectors 3i+2k, 4i+5j-3k and -4i 2j+4k respectively, show that the is in equilibrium.

(c) The end A of a uniform rod AB, of length (2L) and mass(m) is smoothly hinged at a fixed point to a vertical wall. A light string has one end tied at B and the other is end tied to a fixed point C in the wall, the point C being vertically above A and AC = 1. Given that the rod and the string are of the same length, show that the magnitude of the tension in the string is (mg)

Calculate the magnitude and direction of the force exerted by the hinge on the rod.
Given further that he string is elastic of natural length (1), find, in terms of (m) and (g), the modulus of elasticity of the string. When a particles of mass (αm) is attached to the rod at B, the system rests in equilibrium with AB horizontal, find the value of α.

10 ai. State the laws of solid friction

ii. Define the angle of friction and show how it could be obtained with respect to an object placed on a rough inclined plane.

(b) A block of weight 50 N rests on a rough horizontal plane of inclination 60^0 The coefficient of friction being 1/3. Use angle of friction method to find the least horizontal force that will be required

i. Just to prevent it slipping down

ii To make it begin to slide up the plane

© A triangular vedge is fixed with one face horizontal. The vertical cross section ABC triangle with vertex A, and with BC horizontal and below the level of A. The angle ABC is 120^0 and AB = BC =2L. Two particles P and Q, of masses (4m) and (m) respectively, are joined by a light inextensible string of length (2L) pasSing over a smooth pulley at A lying in the vertical plane ABC. The coefficient of friction between each particle and the wedge is $\frac{1}{6}\sqrt{3}$ Initially P is at A on the face through AB and Q is at C. the system is released

from rest. When the particles are at the same horizontal level the string breaks. Show that at this instant the speed of each particle is

$\sqrt{1/_{m}gl}$

Find also the speed of P when it reaches B.

11(a)i. State Newton's laws of motion.

ii. State the law of conservation of linear momentum.

iii. State Newton's law of restitution and use the coefficient of restitution to distinguish between elastic, inelastic and perfect collisions.

(b) A mass A of 0.2kg is help at the top of a smooth slope which is 5m long and inclined at an angle 0 to the horizontal, where Sin Θ = 1/10. Another mass B of 0.4kg is at rest at the bottom of the slope. Simultaneously. B is projected directly up the slope with speed 3 m/s, while A is projected down the slope at 2 m/s, both motions being in the same plane. Find

i. how far up the slope B will travel before colliding with A

ii. Their speeds just before the impact;

iii. If B is brought momentarily to rest by the impact, find the velocity of A immediately afterwards

iv. Find the speed of B when it reaches the foot of the slope again Take (g = 10 ms^{-2})

© Three perfectly elastic uniform spheres A, B and C of equal radius but masses (2m), (m) and (3m) respectively lie at rest on a smooth horizontal table with their centers in a straight line, Sphere A is projected directly toward sphere B with speed V and after collision, sphere B moves directly towards and collides with sphere C. is twice that of sphere A.

Show that sphere C will experience only one collision.

(12) What is a projectile motion? Hence show from the equation of motion that

I equation of trajectory is

$y = \tan_{\theta} x - (g/2\, v_{0}^{2}\cos^{2}\theta)\, x^{2}$

ii the range of the projectile is

$$R=\frac{V^{2}\sin 2\theta}{g}$$

(b) A particle is projected with velocity 30m /s in a direction making an angle 15° with the horizontal. Find its position and speed after five seconds. (g = 9.8m/s^{2})

13. A ball of mass 5kg moving with speed 30m/s impacts on a second ball of