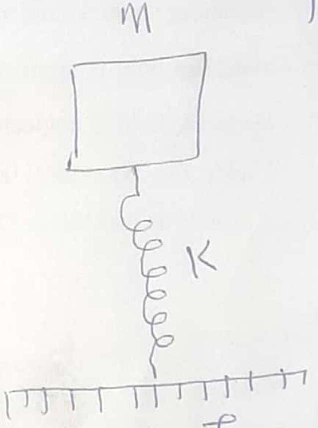
Example 1 A tray of mass 9kg is supported by a spring of force constant K as shown below. The tray is pressed slightly downward and then released. It seems to execute SHM of period 1.0s. When a block of mass M is placed on the tray, the period increase to 2.0s. Calculate the mass of the block.

Solution Since $w = \sqrt{\frac{K}{m}}$, and $w = \frac{2\pi}{T}$

then



$$\frac{4\pi^2}{T^2} = \frac{K}{m}$$

$$m = \frac{KT^2}{4\pi^2}$$

When the tray is empty $m = 9kg$ and $T = 1s$.

then

$$9 = \frac{K(1)^2}{4\pi^2}$$

and $T = 2s$. therefore

Since $m = 9 + M$

$$9 + M = K \times (2)^2 / 4\pi^2$$

$$\Rightarrow \quad \frac{9+M}{9} = 4.$$

therefore $M = 27kg$.

Example 2 A spring of force constant $1600Nm^{-1}$ is mounted on a horizontal table as shown below. A mass $m = 4.0kg$ attached to the free end of the spring is pulled horizontally.

Scanned with CamScanner

towards the right through a distance of 4.0 cm and then set free. Calculate
(i) frequency (ii) maximum acceleration and
(iii) maximum speed of the mass.

Solution $w = \sqrt{\frac{k}{m}} = \sqrt{\frac{1600}{4}} = 20 \, rad s^{-1}$

therefore $v = \frac{20}{2\pi} = 3.18 \, Hz$.

Maximum acceleration $= aw^2 = 0.04 \times 400 = 16 ms^{-2}$

and $V_{max} = aw = 0.04 \times 20 = 0.8 ms^{-1}$.

Moment of a Force

The moment of a force F about a point A is equal to $\pm Fd$ where

F is the magnitude of the force

d is the perpendicular distance from A to the line of action of the force.

The units of moment are newton metres (Nm)

Note: If the force is the same F, with the same line of action, but applied at a different point Q, the moment about A is unchanged.

Note that $+$ sign for rotation anticlockwise
$-$ sign for rotation clockwise.

Example 1 Find the moment of force $F = 6i - 3j$ applied at $(4,5)$ about the point $(1,2)$.

Solution $(x, y) = (4, 5)$

$(x_1, y_1) = (1, 2)$.

$F = (6, -3)$.

$$\text{moment} = -6(5-2) + (-3)(4-1) \quad (\text{clockwise})$$

$$= -18 - 9 = -27\,Nm$$

Example 2 Find the moment of a force of 6 newtons, applied in the direction of $i + 2j$ at point $(4, 2)$ about $(1, 1)$.

Solution F is parallel to $i + 2j$

So that $i + 2j$ is $\sqrt{1^2 + 2^2} = \sqrt{5}$.

unit vector in the direction of the force $\frac{1}{\sqrt{5}}(i + 2j)$

F ~~has magnitude~~ $6 = \frac{6}{\sqrt{5}}(i + 2j) = F$

Hence, $F_1 = \frac{6}{\sqrt{5}}$ and $F_2 = \frac{12}{\sqrt{5}}$.

For $P(x, y) = (4, 2)$ and $A = (x_1, y_1) = (1, 1)$

Then $$\text{moment} = -\frac{6}{\sqrt{5}}(2-1) + \frac{12}{\sqrt{5}}(4-1)$$

$$= \frac{30}{\sqrt{5}} = 6\sqrt{5}\,Nm$$

~~Resultants~~ Resultants of forces on bodies of non-zero size

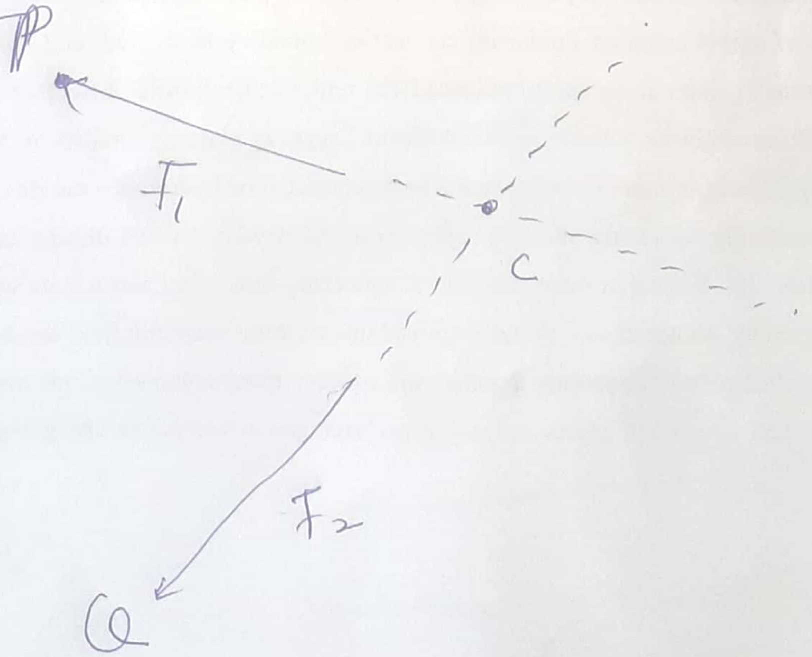
If different forces act at different points then the total moment about any point P is the algebraic sum of each moment about P.

Suppose we have two forces F_1 and F_2. The resultant force H is the force which has the same effect as F_1 and F_2 combined. Clearly the vector R is the vector sum of F_1 and F_2 so:

$$R = F_1 + F_2$$

Case 1:- F_1 and F_2 are not parallel

F_1 is applied at P and F_2 at Q as shown below:



The moment of F_1 is the same if it is applied at any point on the line of action.

likewise, the moment F_2 is the same if it is applied at any point on the line of action. therefore it is the same as if it is applied at C, where the lines of actions cross.

Hence, provided the line of action of R, passes through C, we get the correct total moment.

Example) Calculate the resultant and its line of action for the two forces.

$\vec{F_1} = 2\hat{i} + \hat{j}$ N applied at point $(1,0)$ and

$\vec{F_2} = -3\hat{i} + 3\hat{j}$ N applied at point $(0,0)$

Sol

$R = F_1 + F_2 = (2,1) + (-3,3) = (-1,4)$

Since vector $F_1 = (2,1)$, the gradient of its line of action is $\frac{1}{2}$.

the equation of the line of action is

$y = \frac{1}{2}x + c_1$ since it passes through $(1,0)$

$0 = \frac{1}{2} + c_1 \Rightarrow c_1 = -\frac{1}{2}$

$\Rightarrow y = \frac{1}{2}x - \frac{1}{2}$ ——— (1)

vector $F_2 = (-3,3)$, so the gradient of its line of action is $\frac{3}{-3} = -1$. the equation

The equation of the line of action is

$y = (-1)x + c_2$ since it passes through $(0,0)$

$0 = 0 + c_2$ ~~since~~

$\Rightarrow c_2 = 0$

$y = -x$ ——— (2)

Solving (1) and (2) simultaneously

$-x = \frac{1}{2}x - \frac{1}{2}$

$-2x = x - 1$

$x = \frac{1}{3}, \quad y = -\frac{1}{3}$

Vector $R = (-1, 4)$, so the gradient of its line of action is $\frac{4}{-1} = -4$.

~~The equation of~~

The equation of its line of action is

$y = -4x + c_3$ ~~so that~~

$-\frac{1}{3} = -4\frac{1}{3} + c_3 \Rightarrow c_3 = 1$

which gives

$y = -4x + 1$

Case 2 If the forces F_1 and F_2 are parallel then lines of action do not cross. The vector resultant $R = F_1 + F_2$ and we can find the line of action by taking moments about a point

Example 2 $F_1 = 2i + j$ N is applied at $A(2,0)$ $F_2 = 6i + 3j$ N is applied at $O(0,0)$. So that F_2 is parallel to F_1. Find the resultant R and the point B. whose its line of action crosses the X-axis. Hence, find the equation of the line of action.

Soln $R = F_1 + F_2 = 8i + 4j$ so that

$|R| = \sqrt{80}$ N $= \sqrt{8^2 + 4^2}$

Let the line of action of R cross X-axis at $B(d, 0)$. Consider moments about O.

~~At~~ $A(2,0)$, F_1 consists of for $2i$ which has moment $2 \times 0 = 0$ Nm about O. Together with $1j$ which has moment $1 \times 2 = 2$ Nm about O. The total moment of F_1 about O is thus 2Nm. The line of action of F_2 passes through O so the moment about O is zero.

At $B(d, 0)$, R consist of force $8i$ which has moment $8 \times 0 = 0$ Nm about O, together with $4j$ which has moment $4 \times d = 4d$ Nm about O. The total moment of R about O is thus 4d Nm. The total moment of R must equal the total moment of F_1 and F_2 so

$$4d = 2 + 0 \Rightarrow d = 0.5$$

Thus the line of action of R passes through $(0.5, 0)$

Since $R = 8i + 4j$ its line of action must have gradient $\frac{4}{8} = \frac{1}{2}$. the equation of the line of action is thus

$$y = \frac{1}{2}x + c$$

Since it goes through $(0.5, 0)$, $0 = 0.5 \times 0.5 + c$

$$\Rightarrow c = -0.25$$

The equation of the line of action is thus

$y = 0.5x - 0.25$.

Couples

Given $F_2 = -F_1$, then, the line of action of F_2 is in the opposite direction to that of F_1. Then we have a turning effect even though the resultant is zero.

$$R = F_1 + F_2 = 0$$

The situation, where there is a turning effect but with resultant equal to zero is called a Couple.

Remark the magnitude of the Couple depends on F and d only, but is independent of the distance.

Equilibrium of Coplanar forces

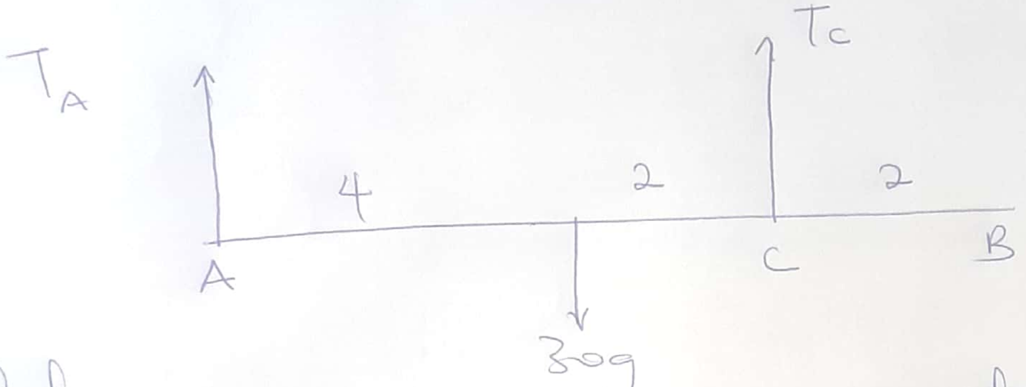
Coplanar means that the lines of actions of all the forces are in a single plane. or parallel to the plane.

A body will be in equilibrium under the action of coplanar forces if

(i) the resultant is zero

(ii) they donot reduce to a Couple

Example 1 A uniform plank, AB, is 8m long and has mass 30kg. It is supported in equilibrium in a horizontal position by two vertical inextensible ropes, as shown below. Find the tension in each rope.



Solution

Resolving Into Components

$T_A + T_C - 30g = 0$ ——— (1)

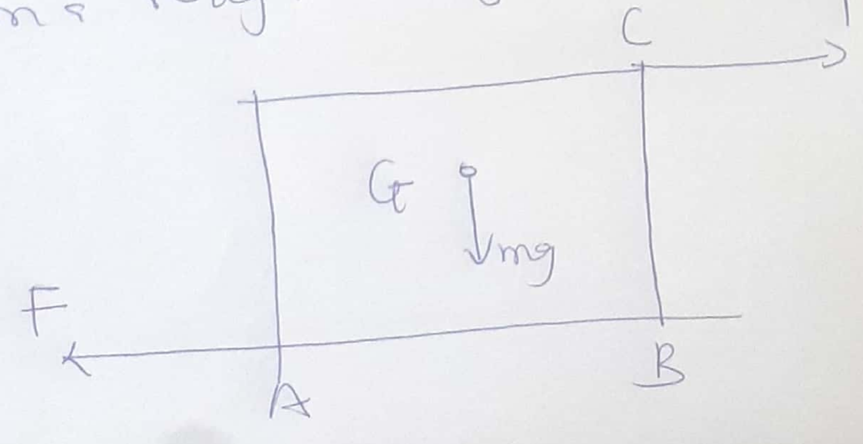
Moments about A ——— (2)

$6T_C - 30g \times 4 = 0$

From (2) $\Rightarrow T_C = \frac{30 \times 9.81 \times 4}{6} = 196.2N$

From (1) $T_A = 30g - T_C = 98.1N$

Example 2 A Cube of side 2m and mass m kg rests on a rough horizontal surface as shown below



Solution

For the cube to tilt, the total moment about B must be greater than 0.

The total moment about B is $mg \times 1 - T \times 2$

$$T = \frac{1}{2} mg \; H$$

Also, by resolving vertically

$$R - mg = 0$$

resolving horizontally $T - F = 0$

$$R = mg \quad \text{and} \quad F = T = 0.5mg$$

The maximum friction available is $F = \mu R = \mu mg$

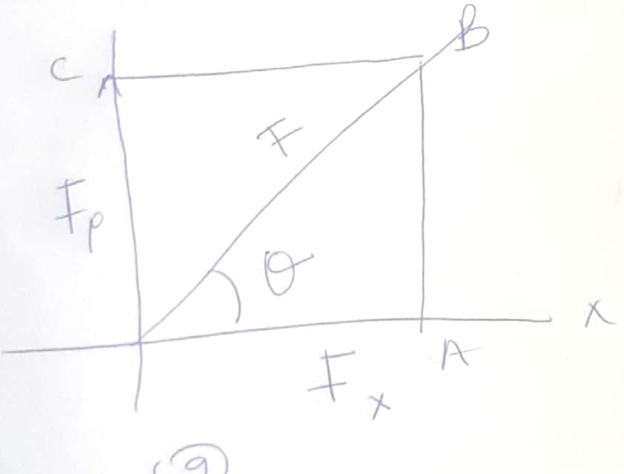
Thus, if μ is greater than 0.5, the cube will tilt first. If it is less than 0.5, it will slide first.

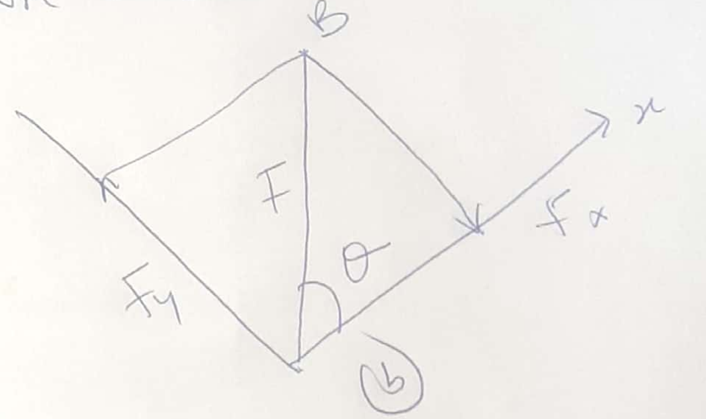
1.0 Resolution of Forces

A given force F can be resolved into two forces which together produce the same effects as that of force F. These forces are called the components of the force F. This process is known as resolution of a force into components.

1.1 Resolution of a force into rectangular components

Consider a force F acting on a particle O inclined at an angle θ as shown below.

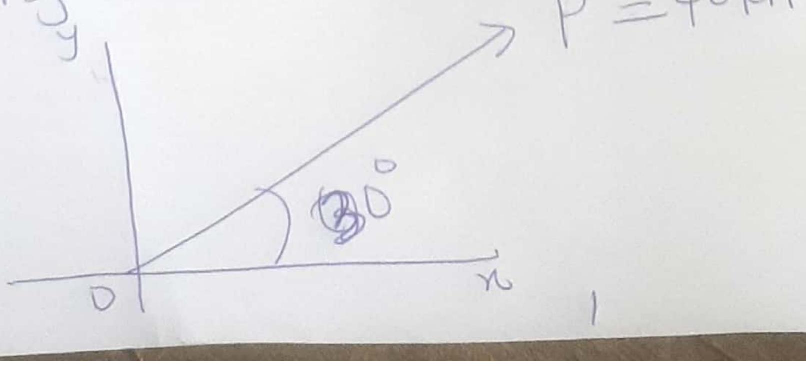




Then, the two rectangular components of the force F are

$$F_x = F\cos\theta \quad , \quad F_y = F\sin\theta$$

Example 1.1.2 Determine the component of force $P = 40kN$ along x and y as shown below



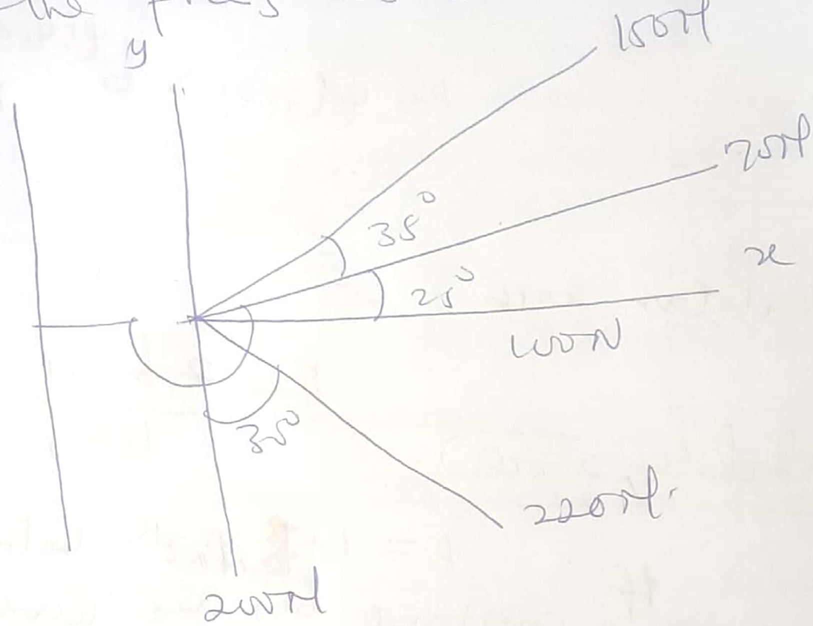
Solution

$$P_y = P \sin 30^\circ = 40 \sin 30^\circ = 20KN$$

$$P_x = P \cos 30 = 40 \cos 30^\circ = 34.64 \text{ KN}$$

Note that, the direction of P_x and P_y are determined based on the direction of P

Example 2 Determine the x and y components of each of the forces as shown below:



Solution $\quad P_x = P \cos \theta \qquad P_y = P \sin \theta$

100N	$100 \cos 0^\circ = -100N$	$100 \sin 0^\circ = 0$
70N	$70 \cos 25^\circ = 63.44$	$70 \sin 25^\circ = 29.58N$
150N	$150 \cos 60^\circ = 75N$	$150 \sin 60^\circ = 129.90N$
220N	$220 \cos 60^\circ = 110N$	$220 \sin 60^\circ = 190.53N$
200N	$200 \cos 90^\circ = 0$	$200 \sin 90^\circ = 200N$

1.2 Resolution of force into Inclined components

It is essential to know the components of a force which are not perpendicular to one another. Such components are known as inclined component or non-rectangular component.

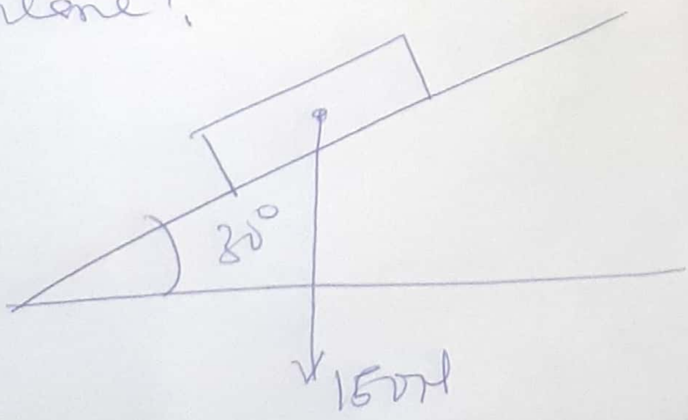
1.2.1 Triangular Law of forces

If two forces P and Q are acting on a particle A, then the two forces can be added or combined to form a single force F.

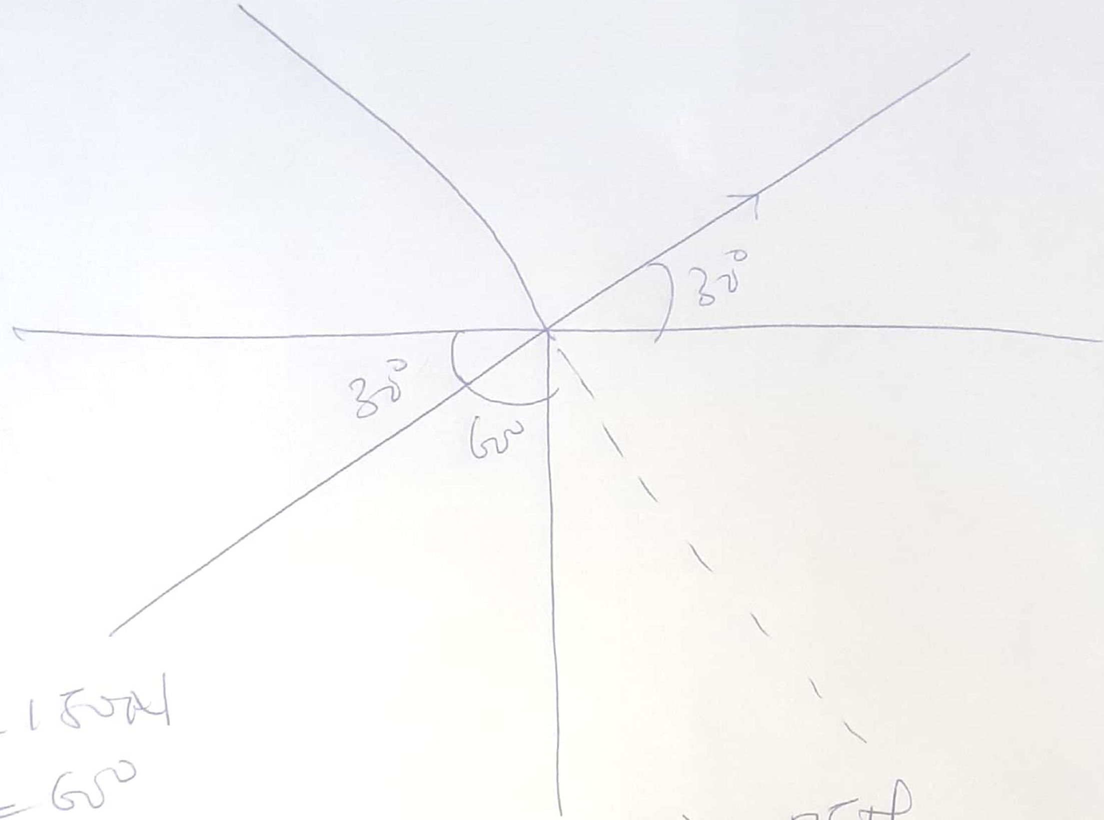
1.2.2 Law of Parallelogram of forces

This law states that two forces acting on a particle may be replaced by a single force obtained by drawing the diagonal of a parallelogram whose two adjacent sides are equal to the given two forces.

Example 1.2.3 A small block of weight 150N is placed on an inclined plane which makes an angle $\theta = 30^\circ$ with the horizontal. What is the component of this weight parallel to the inclined plane and perpendicular to the inclined plane?



Solution By resolution, we have the
the diagram below -



$F = 150N$

$\theta = 60^\circ$

$F_x = F\cos\theta = 150\cos 60^\circ = 75N$

$F_y = F\sin\theta = 150\sin 60^\circ = 129.9N$

1.2.4 Lami's theorem

It states that "If three forces acting at a point are in equilibrium each force will be proportional to the sine of the angle between the other two forces.