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MATRIC Number: 20203398.

i $A - B = A \cap B^c$

Proposition 1

$A - B \subseteq A \cap B^c$

$x \in A - B$

$x \in A$ and $x \notin B$

$x \in A \cap B^c$

$\therefore A - B \subseteq A \cap B^c$

Proposition 2

$A \cap B^c \subseteq A - B$

$x \in A \cap B^c$

$x \in A$ and $x \notin B$

$x \in A - B$

$\therefore A \cap B^c \subseteq A - B$

Hence $A - B = A \cap B^c$

ii $(A-B) - C = (A-C) - (B-C)$

Proposition 1

$(A-B) - C \subseteq (A-C) - (B-C)$

$x \in (A-B) - C$

$x \in (A-B)$ and $x \notin C$

$x \in A$ and $x \notin B$ and $x \notin C$

$x \in A$ and $x \notin C$ and $x \notin B$ and $x \notin C$

$x \in (A-C)$ and $x \notin B-C$

$x \in (A-C) - (B-C)$

$\therefore (A-B) - C \subseteq (A-C) - (B-C)$

Proposition 2

$(A-C) - (B-C) \subseteq (A-B) - C$

$x \in (A-C) - (B-C)$

$x \in (A-C)$ and $x \notin B-C$

$x \in A$ and $x \notin C$ and $x \notin B$ and $x \notin C$

$x \in A$ and $x \notin B$ and $x \notin C$

$x \in (A-B)$ and $x \notin C$

$x \in (A-B) - C$

$\therefore (A-C)-(B-C) \subseteq (A-B)-C$

Hence $(A-B)-C = (A-C)-(B-C)$

iii $(A \cap B) \cup C = A \cap (B \cup C)$

~~let~~ Proposition 1

$(A \cap B) \cup C \subseteq A \cap (B \cup C)$

$x \in (A \cap B) \cup C$

$x \in (A \cap B)$ or $x \in C$

$x \in A$ and $x \in B$ or $x \in C$

$x \in A$ and $x \in (B \cup C)$

$x \in A \cap (B \cup C)$

$\therefore (A \cap B) \cup C = A \cap (B \cup C)$

Proposition 2

$A \cap (B \cup C) \subseteq (A \cap B) \cup C$

$x \in A \cap (B \cup C)$

$x \in A$ and $x \in (B \cup C)$

$x \in A$ and $x \in B$ or $x \in C$

$x \in A \cap B$ or $x \in C$

$x \in (A \cap B) \cup C$

$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup C$

Hence $(A \cap B) \cup C = A \cap (B \cup C)$

1b $A \Delta B = (A-B) \cup (B-A)$

i Δ is commutative

$A \Delta B = B \Delta A$

$A \Delta B = (A-B) \cup (B-A)$

$B \Delta A = (B-A) \cup (A-B)$

Hence Δ is commutative.

ii $A \Delta B = A \times B = AB$, $AB \in U$

Hence Δ is closed.

iii Δ is associative.

$(A \Delta B) \Delta C = A \Delta (B \Delta C)$

$(A \Delta B) \Delta C = [(A-B) \cup (B-A) - C] \cup [C - (A-B) \cup (B-A)]$

$A \Delta (B \Delta C) = [A - (B-C) \cup (C-B)] \cup [(B-C) \cup (C-B) - A]$

iii Find E for Δ

Properties of identity shows that

$A \Delta E = A$

$(A - E) \cup (E - A) = A$

For this expression to be true E must be $\{\emptyset\}$

$A - E = A - \emptyset = A$

$E - A = \emptyset - A = \emptyset$

$A \cup \emptyset = A$

$\therefore E = \{\emptyset\}$

ii Inverse F for Δ

let $F = A'$

$A \Delta A' = E$

$(A - A') \cup (A' - A) = \emptyset$

$\emptyset \cup \emptyset = \emptyset$

$\therefore F = \{\emptyset\}$

2. $\log_{y^n} x = \frac{1}{n} \log_y x$

$\log_{y^n} x.$

Using change of base

$$\frac{\log_a x}{\log_a y^n} = \frac{\log_a x}{n \log_a y} = \frac{1}{n}\left(\frac{\log_a x}{\log_a y}\right)$$

$$= \frac{1}{n}\left(\log_y x\right)$$

Proved.

ii $\log_{y^n} x^n = \log_y x$

$\log_{y^n} x^n$

Using change of base

$$\frac{\log_a x^n}{\log_a y^n} = \frac{n \log_a x}{n \log_a y}$$

$$= \frac{\log_a x}{\log_a y}$$

$$= \log_y x$$

$$(81)^{\frac{1}{\log_{6561} 9}} + (3)^{\frac{3}{\log_{\sqrt{6}} 6}} + (\sqrt{7})^{\frac{2}{\log_5 125}} - (125)^{\log_{125} 25} = \sqrt{27} + \sqrt[3]{49} - 22$$

$$(81)^{\frac{1}{\log_{9^4} 9}} + (3)^{\frac{3}{\log_{6^{1/2}} 6}} + (\sqrt{7})^{\frac{2}{\log_{5^3} 5}} - (125)^{\log_{5^3} 5^2} = \sqrt{27} + \sqrt[3]{49} - 22$$

$$(81)^{\frac{1}{\frac{1}{4}\log_9 9}} + (3)^{\frac{3}{2\log_6 6}} + (\sqrt{7})^{\frac{2}{3\log_5 5}} - (125)^{\frac{2}{3}\log_5 5} = \sqrt{27} + \sqrt[3]{49} - 22$$

$$(81)^4 + (3)^{3/2} + (\sqrt{7})^{\frac{2}{3}} - \cancel{125^{\frac{2}{3}}} - 125^{2/3} = 3\sqrt{3} + 7^{1/3} - 22$$

$$(81)^4 + 3\sqrt{3} + \sqrt[3]{7} - 125^{2/3} = 3\sqrt{3} + 7^{1/3} - 22$$

$$(81)^4 + 3\sqrt{3} + 7^{1/3} - 25 \neq 3\sqrt{3} + 7^{1/3} - 22$$

b. $\log_2 x + \log_x 2 = 2.5$

$\log_2 x + \frac{\log_2 2}{\log_2 x} = 2.5$

$\log_2 x + \frac{1}{\log_2 x} = 2.5$

let $\log_2 x$ be p

$p + \frac{1}{p} = 2.5$

$\frac{p^2+1}{p} = 2.5p$

$p^2 + 1 = 2.5p$

$p^2 - 2.5p + 1 = 0$

multiply through by 10

$10p^2 - 25p + 10 = 0$

$p = 1/2$ or $p = 2$

recall that $p = \log_2 x$

when $p = 2$

$\log_2 x = 2$

$x = 2^2 = 4$

when $x = 1/2$

$\log_2 x = 1/2$

$x = 2^{1/2}$

$x = \sqrt{2}$

$x = 4$ or $\sqrt{2}$

3a $x^4 + 12x^3 + 46x^2 + px + q$

$(x^2 + ax + b)^2$

$(x^2 + ax + b)(x^2 + ax + b)$

$x^4 + ax^3 + bx^2 + ax^3 + a^2x^2 + abx + bx^2 + abx + b^2$

$x^4 + 2ax^3 + (a^2 + 2b)x^2 + 2abx + b^2$

Comparing

$2a = 12, a = 6$

$(a^2 + 2b)x^2 = 46x^2$

$(6)^2 + 2b = 46$

$36 + 2b = 46$

$46 - 36 = 2b$

$b = \frac{10}{2} = 5$

$2abx = px$

$2 \times 6 \times 5 = p$

$p = 60$

$q = b^2$

$q = 5^2 = 25$

$\therefore x^4 + 12x^3 + 46x^2 + 60x + 25$

9.

$$\frac{n+3}{(n-1)n(n+1)} = \frac{A}{n-1} + \frac{B}{n} + \frac{C}{n+1}$$

$$n+3 = A[n(n+1)] + B(n-1)(n+1) + C[(n-1)n]$$

Put $n=0$

$$0+3 = B(0-1)(0+1)$$

$$B = -3$$

Put $n=-1$

$$-1+3 = C[(-1-1)-1]$$

$$-1+3 = 2C$$

$$C = 1$$

Put $n=1$

$$1+3 = A[1(1+1)]$$

$$4 = 2A$$

$$A = 2$$

$$\frac{n+3}{(n-1)n(n+1)} = \frac{2}{n-1} - \frac{3}{n} + \frac{1}{n+1}$$