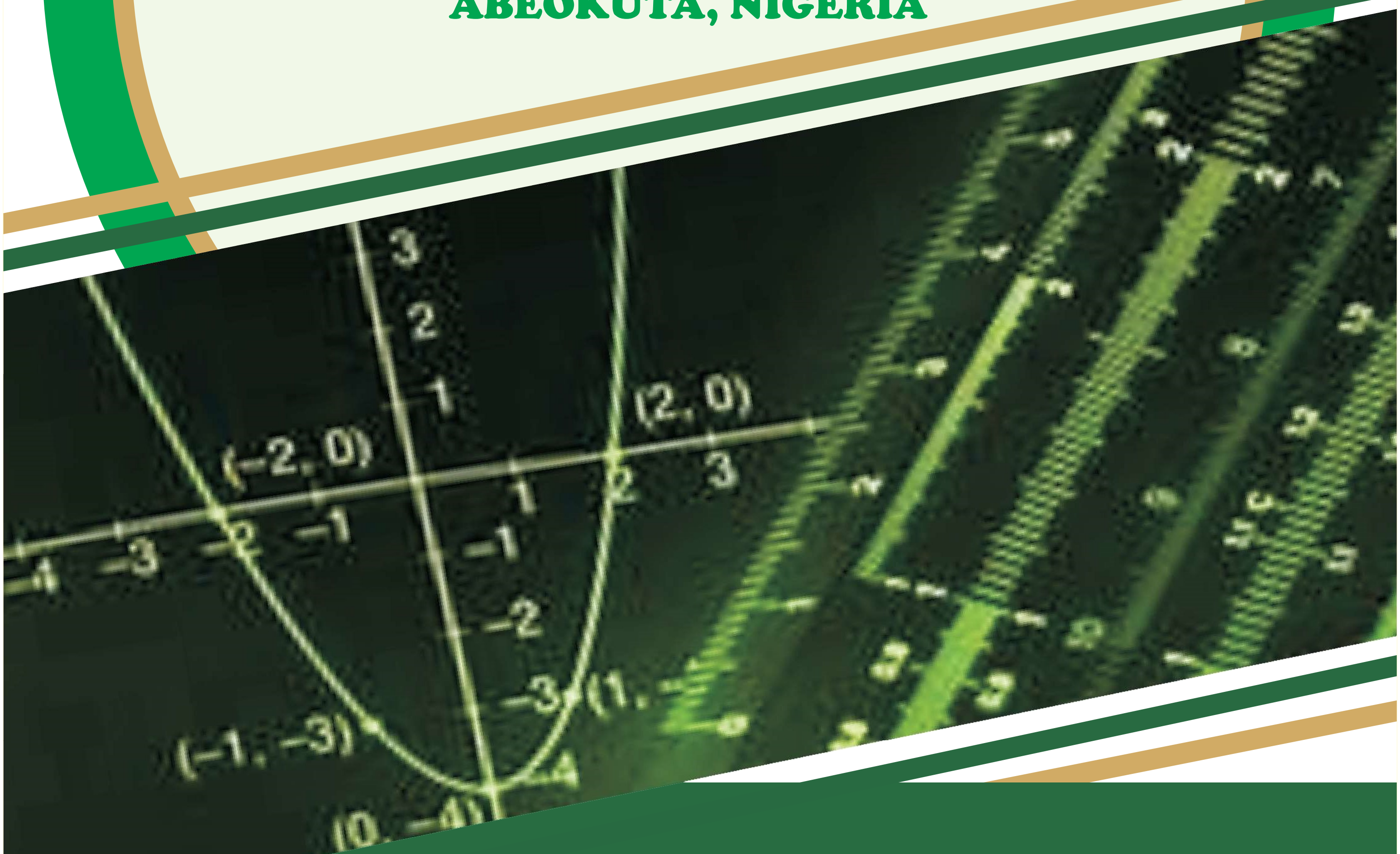
DEPARTMENT OF MATHEMATICS
COLLEGE OF PHYSICAL SCIENCES
FEDERAL UNIVERSITY OF AGRICULTURE
ABEOKUTA, NIGERIA



TUTORIAL WORKBOOK MTS 101 & MTS 102

FEDERAL UNIVERSITY OF AGRICULTURE ABEOKUTA

College of Physical Sciences

Department of Mathematics

Tutorial Workbook

for

MTS 101 & 102

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Preface

This workbook is a complement of Lecture Notes of Department of Mathematics, University of Agriculture, Abeokuta.

A conscientious and careful use of this workbook along with the Lecture Notes will go a long way to help students who are offering MTS 101 and 102 to have a good background for University Mathematics.

The workbook is equally useful to candidates at the Polytechnics, Colleges of Education and those preparing for GCE Advance Level.

The Head of Department

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STUDENT DATA

MATRIC NO:______________________________

DEPT:______________________________

COLLEGE:______________________________

MTS 101 TUTORIAL QUESTIONS

1 Elementary Theory of Sets

1. Let A, B and C be sets, prove from first principle that

   (a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

   (b) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

   (c) $(A - B) \cap (A \cap B) \cap (B - A) = \{\}$

   (d) For (a) - (c), prove using Venn diagrams.

2. If A, B, C are finite sets whose elements are from the same universal set U and $n(A)$ denotes the number of elements in the set A.

   (a) Show by means of Venn diagram that

   $$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

   (b) Using the fact that $(A \cup B \cup C) = (A \cup B) \cup C = A \cup (B \cup C)$ deduce an expression for $n(A \cup B \cup C)$

   (c) If $n(A \cup B) = n(A \cap B)$, what can be said about the set A and B? How did you reach your conclusion.

3. Let A, B, C be subsets of a universal U. If A^c denotes the complement of A and $*$ an operation on the subsets of U is defined as $A * B = A \cap B^c$; Prove that

   (a) $A * (B \cup C) = (A * B) \cap (A * C)$ Express $A * (B * C)$ and $(A * B) * C$; each in as simple a form as possible. Show by an example that

   $$(A * B) * C \neq A * (B * C)$$

   (b) Prove that $A \cap (B * C) = (A \cap B) * (A \cap C)$.

4. Prove that if A and B are sets.

$$A \cup B = [A - (A \cap B)] \cup [B - (A \cap B)] \cup (A \cap B).$$

5. In preparing a table for a class of 38 students the following facts were taken into consideration: 25 students take History, 27 take French, 28 take Agriculural Science, 20 take both History and French, 23 take both French and Agricultural Science, while 21 take both History and Agricultural Science. If 18 take all the three subjects

   (a) Express these in a Venn diagram showing the number of students who offer the three subjects.

   (b) How many students take only one subject?

   (c) How many students take at least two subjects?

   (d) How many students offered none of the three subjects?

6. In a cultural gathering of 400 people, there are 270 men and 200 musicians. Of the latter, 80 are singers. 60 of the women are not musicians and 220 of the men are not singers. How many of the women are musicians but not singers, if there are 150 singers altogether and 40 men are both musicians and singers?

2 Real Numbers

Mathematical Induction

Use the method of Mathematical induction to establish the following

1. $1 + 3 + 5 + ... + (2n - 1) = 2^n$

2. $2^n > n$

3. 7^{2n+1} is divisible by 8

4. $3^{4n+2} + 2.4^{3n+1}$ is exactly divisible by 17

5. $(1 \times 2 \times 3) + (2 \times 3 \times 4) + ... + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

6. $1^3 + 2^3 + 3^3 + ... + n^3 = \frac{1}{4}n^2(n+1)^2$

7. $n^4 + 4n^2 + 11$ is a multiple of 16 for all odd positive integer n.

Indices and Logarithms

1. (a) Simplify $\frac{(x^3yz^{-2})^{-3}(x^{-1}y^2z)}{(yz)^{1/2}}$

   (b) If $x = (p+q)^{\frac{1}{3}} + (p-q)^{\frac{1}{3}}$ and $r = (p^2 - q^2)^{\frac{1}{3}}$. Prove that $x^3 - 3rx = 2p$.

   (c) Simplify $6^{\frac{1}{2}n} \times 12^{n+1} \times 27^{-\frac{1}{2}n}32^{\frac{1}{2}n}$.

   (d) Simplify $\frac{1}{x}\left[\frac{1}{2}x^{1/2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}(1+x)^{1/2}\right]$.

   (e) Simplify $\frac{3(2^{n+1})-4(2^{n-1})}{2^{n+1}-2^n}$.

   (f) Express x in terms of y if $8^{2x} = 64 \times 4^{3y}$.

2. (a) Prove from the definition of logarithm that if m, n, x are positive integers, then

$$logmn^x = \frac{logn^x}{1 + logn^m}$$

   (b) Find without using tables, calculator, etc, the value of

   (i) $\frac{log27 + log8 - log125}{log5 - log6}$ (ii) $\frac{log27 + log5 - log1000}{log(0.6)}$

(c) Show that $loga + logax + logax^2 = 3log(loga + logx)$

(d) If $2logy^x + 2logx^y = 5$, Show that $logy^x$ is either $1/2$ or 2. Hence find all pairs of values of x and y which satisfy simultaneously the equation above and the equation $xy = 27$.

3. (a) Prove from the definition of logarithm that if m, n, x are positive integers, then

$$logmn^x = \frac{logn^x}{1 + logn^m}$$

(b) Which one of the two numbers $\sqrt{3} + 2\sqrt{2}$ and $\sqrt{5} + \sqrt{5}$ is smaller?

(c) If $loga^b = logb^c = logc^a$, show that $a = b = c$.

(d) Show that $loga + logax + logax^2 = 3log(loga + logx)$.

(e) If $2logy^x + 2logx^y = 5$, show that $logy^x$ is either $1/2$ or 2. Hence find all pairs of values of x and y which satisfy simultaneously the equation above and the equation $xy = 27$.

4. (a) Simplify $\frac{(x^2yz^{-3})^{-2}(x^{-3}y^2z)}{(yz)^{7/3}}$.

(b) If $x = (p+q)^{1/3} + (p-q)^{1/3}$ and $r = (p^2 - q^2)^{1/2}$. Prove that

$$x^3 - 3rx = 2p.$$

5. (a) If $(log_4x)^2 = log_2xlog_kx$, find the value of k.

(b) If p and q are both real numbers and $q > 4$, show that $\frac{(x^2+px+p)}{(x^2+qx+q)}$ can not be between $\frac{p}{q}$ and $\frac{(p-4)}{(q-4)}$ when x is real.

(c) If $y = \frac{(x^2+2x+\lambda)}{(2x-3)}$ and x is real, find the greatest value of λ for which y can take all real values.

Polynomials and Polynomial Equations

1. Solve for x and y in the following simultaneous equations

   (a) $\frac{x}{y} + \frac{y}{x} = \frac{17}{4}, \quad x^2 - 4xy + y^2 = 1$

   (b) $xy - \frac{x}{y} = 5, \quad xy - \frac{x}{y} = \frac{1}{5}$

   (c) $2^x + 3^y = 44, \quad 2^{x+2} + 3^{y+2} = 221$

   (d) $x + 2y = 3, \quad x^2 - xy + 5y^2 + 2y = 7$

2. Solve the equations $xy = 1, yz = 9, xz = 16$ and deduce the : olution of the equations

$$(y+z)(z+x) = 1$$

$$(z+x)(x+y) = 9$$

$$(x+y)(y+z) = 16$$

3. (a) Prove that $(a+b+c)(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} > 9$ unless $a = b = c$, where $a, b.c$ are real constants.

   (b) Prove that $x^4 + y^4 > 2x^2y^2$ unless $x = y$

   (c) From $(p-q)^2 + (q-r)^2 + (r-p)^2 > 0$, prove that $p^2 + q^2 + r^2 > pq + qr + rp$. Hence deduce that for any real p, q, r, $(p+q+r)^2 \geq 3(pq + qr + rp)$.

4. Solve the inequality

   (a) $\frac{2x^3 - 3x + 5}{x^2 + 2x + 6} < \frac{1}{2}$

   (b) $\frac{x-2}{x-4} > \frac{x-3}{x+4}$

   (c) $\frac{2x^2 - 3x - 5}{x^2 + 2x + 6} < \frac{1}{2}$.

   (d) $-3 \leq \frac{(x-1)(x-6)}{(x-3)} \leq -3$

   (e) Find y if $\left| \frac{y-3}{y+1} \right| < 2$.

   (f) Prove that $p^4 + p^3q + pq^3 + q^4 \geq 0$ where p, q are real constants.

5. (a) If x and y are unequal real numbers, prove that $x^2 + y^2 > 2xy$

   (b) If x and y are real numbers, show that $x^2 + y^2 \geq \frac{1}{2}(x+y)^2$.

   (c) Find the zeros of $x^4 - 4x^3 + 4x^2 - 9$ and hence factorize the polynomial.

   (d) Solve the following equation

$$\frac{2}{3x^2 - x - 2} + \frac{5}{3x^2 - x + 1} = \frac{2}{3x^2 - x - 3}$$

   (e) Prove that if the difference between the roots of the equation $ax^2 + bx + c = 0$ is 1, then $a^2 = b^2 - 4ac$.

6. If the roots of the equation $3x^2 - 5x + 1 = 0$ are α and β, find the values of

   (a) $\alpha\beta^2 + \alpha^2\beta$

   (b) $\alpha^2 - \alpha\beta + \beta^2$

   (c) $\alpha^3 + \beta^3$

   (d) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

7. (a) If α and β are the roots of the equation $x^2 + px + q = 0$, form the equation whose roots are $\alpha + \beta$ and $\alpha - \beta$.

   (b) Given that the roots of the equation $x^2 - x - 1 = 0$ are α and β, find, in its simplest form, the quadratic equation with numerical coefficient whose roots are $\frac{1+\alpha}{2-\alpha}$ and $\frac{1+\beta}{2-\beta}$.

8. Solve the equations:

   (a) $\sqrt{x} - \frac{6}{\sqrt{x}} = 1$

   (b) $\sqrt{x+1} + \sqrt{5x+1} = 2\sqrt{x+6}$

Remainder and Factor Theorem

1. (a) Find the remainder when $3x^5 + 11x^4 + 90x^2 - 19x + 53$ is divided by $x + 5$.

(b) Find the connection between a and b such that $2x^4 - 7x^3 + ax + b$ is divisible by $x + 5$.

(c) If $x^5 - 5ax + 4b$ is divisible by $(x-c)^2$, show that $a^5 = b^4$.

2. (a) If $((x+1)(x+2) = a + b(2x+1) + c(3x^2+3x+1) + 4dx^3$ for $x = 1, 2, 3, 4$, find a, b, c and d.

(b) Find m and n if the remainder when $8x^3 + mx^2 - 6x + n$ is divided by $x - 1$ and $2x - 3$ are 2 and 8 respectively.

(c) Find a and b if $x^4 + ax^3 - 2x^2 + bx - 8$ is divisible by $x^2 - 4$.

(d) When the expression $x^3 + ax^2 + bx + c$ is divided by $x^2 - 4$, the remainder is $18 - x$, and when it is divided by $x + 3$, the remainder is 21. Find the remainder when the expression is divided is divided by $x + 1$.

3. (a) Given that $2x^3 + bx^2 + cx + 18$ has the factors $(x-2)$ and $(x+3)$, find b and c and complete the factorization.

(b) Find the remainder when $x^4 - 3x^3 + 4x^2 - 6x + 7$ is divided by $x - 1$.

(c) Find the values of a, b and c such that $x^3 + 1$ is a factor of $x^6 + ax^4 + bx^3 + cx^2 + 3x + 2$.

4. Find the relation which holds betweenthe numbers p and q if $x^3 + 4x^2 + x + p$ is exactly divisble by $x + q$. Determine for what value of p this relationship is satisfied when $q = -1$ and hence find the factors of $x^3 + 4x^2 + x + p$ for this value of p.

5. Find m and n if the remainders when $8x^3 + mx^2 - 6x + n$ is divided by $x - 1$ and $x - 3$ are 2 and 8 respectively.

6. (a) Show that $x + y + z$ is a factor of $x^3 + y^3 + z^3 - 3xyz$. Find the other factors.

(b) Find the connection between a and b when $2x^4 - 7x^3 + ax + b$ is divided by $x + 5$.

(c) If $x^5 - 5ax + 4b$ is divisible by $(x-c)^2$, Show that $a^5 = b^4$.

(d) Find a and b if $x^4 + ax^3 - 2x^2 + bx - 8$ is divisible by $x^2 - 4$.

3 Complex Number

1. Simplify

   (a) $(5+2i)+(3-7i)$

   (b) $(2+5i)(4-3i)$

   (c) $\frac{4-3i}{-3+4i}$

   (d) $\frac{i+i^2+i^3+i^4+i^5}{1-i}$

   (e) $|4-3i||3+4i|$

   (f) $|\frac{1}{1+3i}-\frac{1}{1-3i}|$

2. (a) Let z_1 and z_2 be two complex numbers. Prove that $|z_1z_2|=|z_1||z_2|$.

   (b) Show that $z\bar{z}$ is real and equal $|z|^2$.

   (c) Find the moduli and arguments of $3+4i, 4-2i, -1+\sqrt{3}$ and write each in polar form. Furthermore, plot each of the numbers on Argand diagram.

3. (a) If $x=1+i$ is a root of the equation $x^4+ax^3+bx^2+8=0$, where $a,b\in\Re$. find the other roots and determine the values of a and b.

   (b) Solve the equation $x^3-2x-4=0$.

   (c) Solve the equation $\left[\frac{z+i}{z-i}\right]^2=i$. Find the locus represented by $|\frac{z-3i}{z-3}|=2$.

4 Rational Functions and Partial Fractions

1. Sketch the following functions:

   (i) $r(x)=\frac{3x-9}{x^2-x-2}$ (ii) $r(x)=\frac{x}{x-1}$ (iii) $r(x)=\frac{1}{x^2-1}$

   (iv) $r(x)=\frac{x^2+2x-3}{x^2-4}$ (v) $r(x)=\frac{x^2-4x+1}{x^2-4x+4}$ (vi) $r(x)=\frac{1}{x}$

2. Find the ranges of values of k for which the equation $r(x)\frac{3x-1}{x(x+1)}=k$ has real roots. Hence write down the maximum and minimum values of the function $\frac{3x-1}{x(x+1)}$ and

find the corresponding values of x. Sketch the graph of the function $r(x)$, showing clearly the behaviour of $r(x)$ as $x \to 0, x \to -1$.

3. (a) If $f(x) = 3^x$, prove that $f(m).f(n) = f(m+n)$; $\frac{f(m)}{f(n)} = f(m-n)$; $f(x+2) - f(x+1) = 6f(x)$; $\frac{f(x+3)}{f(x-1)} = f(4)$.

   (b) Let $f(x) = \frac{(x-2)(x-6)}{(x-1)(x-3)}$; $x \neq 1, x \neq 3$. Show that for all real values of x, the function $f(x)$ can assume all real values.

   (c) For what values of k does the equation $x^2 + 9 = (4+k)x$ has real roots?

4. Resolve the following rational expressions into partial fractions:

   (i) $\frac{x^2+1}{x^2-1}$ (ii) $\frac{1}{x^4-1}$ (iii) $\frac{1}{(x-1)^4}$

   (iv) $\frac{2x^4-4x^3-42}{(x-2)(x^2+3)}$ (v) $\frac{x^4+3x-1}{(x+2)(x^2-2x+1)}$ (vi) $\frac{7x+2}{125x^3-8}$

   (vi) $\frac{1}{x^2(x^2+x+1)}$ (vii) $\frac{2x+3}{x^3-x^2+1}$. (viii) $\frac{3x^4+9x^3+16x^2+9x+13}{(x-1)^2(x^2+2x+2)^2}$

5. Let $g(x) = \frac{5-x}{(1+x^2)(1-x)}$. Express $g(x)$ in partial fractions. Hence or otherwise determine the values of a, b, c, d when $g(x)$ is expanded as a series in ascending powers of x in the form $g(x) = a + bx + cx^2 + dx^3 + \ldots$

5 Binomial Expansion

1. Expand the following functions in ascending powers of x

   (i) $\frac{3}{(1-x)((1+2x)}$ (ii) $\frac{x-1}{x^2+2x+1}$

   (iii) $\frac{5}{1-x-6x^2}$ (iv) $\frac{x+3}{(x-2)^2}$

2. (a) If the first three terms of the expansion $(1+px)^n$ in ascending powers of x are $1+20x+160x^2$, find the values of n and p.

   (b) Write down the first four terms of the binomial expansion of $(2+\frac{1}{2}x)^7$, simplifying all its coefficient.

   Use your result above to evaluate 1.998^7, correct to four decimal places.

   (c) Find the

   (i) general term,

   (ii) coefficients of x^4, x^5, x^{10},

   (iii) term independent of x,

   in the binomial expansion of $(2x^2 - \frac{1}{2x})^{12}$.

3. Use the binomial expansion to find the values of

   (i) $(16.32)^{\frac{1}{4}}$ (ii) $\sqrt{9.09}$ (iii) $\frac{1}{(10.04)^2}$

   (iv) $\frac{1}{\sqrt{17}}$ (v) $(1.03)^{\frac{1}{3}}$ (vi) $(1.05)^5$

4. Show that if x is so small that x^3 and higher powers of x can be neglected

$$\sqrt{\frac{1+x}{1-x}} = 1 + x + 1/2x^2.$$

   By putting $x = 1/7$, show that $\sqrt{3}$ is approximately $\frac{196}{113}$.

5. Express $\frac{1}{(x+2)^2(2x+1)}$ in partial fractions, and hence expand the expression as a series in ascending powers of x, giving the first four terms and the coefficient of x^n. Show that for values of x so small that x^4 may be neglected, the given expression can be represented by $\frac{1}{(3x+2)^2} + kx^3$ for some number k independent of x, find k.

6. (a) Using the binomial theorem, expand $(1+2x)^5$, simplifying all the terms. Use your expansion to calculate the value of 1.02^5 to six significant figures.

   (b) If the first three terms of the expansion of $(1+px)^n$ in ascending powers of x are $1+20x+160x^2$, find the values of n and p.

7. (a) Write down the first four terms of the binomial expansion of $(1-y)^8$ in ascending powers of y.

   (b) By putting $y=\frac{1}{2}x(1-x)$ in your expansion, find the values of p and q if $\left[1-\frac{1}{2}(1-x)\right]^8 = 1-4x+px^2+qx^3+\ldots$

   (c) By substituting $x=0.01$ in the expansion, calculate correct to four decimal places, the values of 0.99505^8.

8. (a) Use binomial theroem to expand $\sqrt{\frac{2+x}{3-x}}$ in ascending powers of x as far as the term in x^3.

   (b) Write down the first four terms of the binomial expansion $(2+\frac{1}{2}x)^7$, simplifying all its coefficients. Use your result to evaluate 1.998^7, correct to four decimal places.

   (c) Find the; (i) general term (ii) coefficient of x^4, x^5 and x^10 (iii) term independent of x, in the binomial expansion of $(2x^2-\frac{1}{2x})^{12}$.

6 Sequences and Series

1. (a) Evaluate the first five terms of the sequence whose n^{th} term U_n is $2^n + n^2$

   (b) Find the formula for U_n for the sequence 1, -4, 9, -16, 25, ...

   (c) If $U_1 = -1, U_2 = -5$, and $U_n = a + bn$. Find a, b and U_5.

   (d) If $U_n = log_{10}n$, show that

$$\sum_{n=1}^{10} U_n = \sum_{n=1}^{10} log_{10}n = log_{10}3628800$$

2. (a) The number of terms in an arithmetic progression is 40 and the last is -54. Given that the sum. Given that the sum of the 15 terms addede to the sum of the first 30 terms is zero, calculate

   (i) The first term and common difference,

   (ii) the sum of the progression.

   (b) The fourth term of a geometric progression is 6 and the seventh term is -48. Calculate

   (i) the common ratio,

   (ii) the first term,

   (iii) the sum of the first eleven terms.

   (c) The first term of a geometric progression is a and the common ration is r. Given that $a = 2r$ and the sum to infinity is 4, calculate the third term.

3. A sequence of numbers $U_1, U_2, U_3, ..., U_n$ satisfies the relation

$$U_{n+1} + n^2 = nU_n + 2$$

for all integers $n \geq 1$. If $U_1 = 2$, find:

   (a) the values of U_2, U_3 and U_4,

   (b) an expression for U_n in terms of n,

   (c) the sum of the first n terms of the sequence,

(d) the value of $(U_nU_{n+1} + U_{n+2})$, when $n = 20$.

4. (a) The sum of three numbers in arithmetic progression is 18 and the sum of their square is 206. Find the numbers.

   (b) The third term of an arithmetic progression is 18, the seventh term is 30. Find the sum of the first 33 terms.

   (c) If a and r are both positive, prove that the series

$$log a + log ar + log ar^2 + ... + log ar^{n-1}$$

   is an arithmetic series and find the sum of the terms.

   (d) Find the sum of first 17 terms of the series

$$log 2 + log 6 + log 18 + ...$$

5. (a) If S is the sum of n terms of a geometric progression. P is the product of the numbers, and R is the sum of the reciprocals of the terms. Show that $(\frac{S}{R})^n = P^2$.

   (b) Prove that if P, Q and R are the p^{th}, q^{th} and r^{th} terms of an arithmetic progression, then

$$p(Q - R) + q(R - P) + r(P - Q) = 0.$$

   (c) The p^{th}, q^{th} and r^{th} terms of an arithmetic sequence are in geometric progression. Show that the common ration is $\frac{q-r}{p-q}$ or $\frac{p-q}{q-r}$.

6. (a) The sum to infinity of a geometric series is S. The sum to infinity of the square of the terms is $2S$. The sum to infinity of the cubes of the terms is $\frac{64}{13}S$. Find S and the first three terms of the original series.

   (b) Find the sum of the infinite series whose n^{th} term is given by $U_n = \frac{n}{(n+1)(n+2)(n+3)}$.

   (c) Solve the equation

$$1 + x + x^2 + x^3 + ... + x^{11} = x + 3 + \frac{x^{12}}{x - 1}$$

7 Matrices

1. (a) If $A = \begin{bmatrix} 2 & 5 & 3 \\ 0 & 2 & 1 \\ -1 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -5 & 0 \\ 01 & 2 & 6 \end{bmatrix}$

   Find (a) AB (b) BA (c) A^2 (d) $5B^2$.

   (b) If $A = \begin{bmatrix} 1 & 5 \\ -2 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$,

   (a) Find $A^2 + 3A + 4I$ where I is the unit matrix of order 2

   (b) Verify whether or not $BC = -CB$

   (c) Very if $B^2 = C^2 = I$.

2. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, *and* $C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

   Find (i) $A^2 + B^2 - C^2$ (ii)$(A - B)(A + B)$ (iii)$A(B + C)$

   (iv) $AB - BC$ (v)$AB + AC$

3. (a) Show that the determinant of the matrix

   (i) $\begin{bmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{bmatrix}$ is zero

   (ii) $\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 c^2 & \end{bmatrix}$ *is* $(a-b)(b-c)(c-a)$

   (iii) $\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$ *is* $(a+b+c)^3$

(b) Solve for x if the determinant of the following matrix is zero

(i) $\begin{bmatrix} x+1 & -5 & -6 \\ -1 & x & 2 \\ -3 & 2 & x+1 \end{bmatrix}$ (ii) $\begin{bmatrix} x+1 & x+2 & 3 \\ 2 & x+3 & x+1 \\ x+3 & 1 & x+2 \end{bmatrix}$

(iii) $\begin{bmatrix} x & 2 & 3 \\ 2 & x+3 & 6 \\ 3 & 4 & x+6 \end{bmatrix}$ (iv) $\begin{bmatrix} 1+sin^2x & cos^2x & 4sin2x \\ sin^2x & 1+cos^2x & 4sin2x \\ sin^2x & cos^2x & 1+4sin2x \end{bmatrix}$

4. (a) Show that $A(Adj.\ A) = |A|\ I$ given that

(a) $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 4 & 0 \end{bmatrix}$ (b) $A = \begin{bmatrix} 3 & 7 & 2 \\ 1 & 2 & 3 \\ 1 & 4 & 5 \end{bmatrix}$.

(c) $A = \begin{bmatrix} 8 & 4 & 2 \\ 2 & 8 & 4 \\ 1 & 2 & 8 \end{bmatrix}$ (d) $A = \begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix}$

(b) Find A^{-1} and $(A^T)^{-1}$ given that

(a) $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 4 & 0 \end{bmatrix}$ (b) $A = \begin{bmatrix} -2 & 2 & 4 \\ 1 & -3 & 2 \\ 1 & 5 & 2 \end{bmatrix}$

(c) Show that $(A^{-1})^T = (A^T)^{-1}$ in each case.

5. Show that $M^2 = 4I + 5I$ is satisfied by the matrix M, given by $M = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

Use the equation to find the inverse of the matrix M.

6. (a) Let $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$. Find (i) A^2 (ii) A^3. Hence compute $f(A)$ given that

$f(x) = 2x^3 + 3x^2 - 4$

(b) Solve the system of equations

$$2x + 3y + z = 11$$

$$x + y + z = 6$$

$$5x - y + 10z = 34$$

using (a) inverse method (b) Crammer's rule.