MTS 105

Topics to be covered:

(i) Indices and Logarithms

(ii) Surds

(iii) Remainder Theorem

(iv) Partial Fractions

Recommended Textbooks

(i) Additional Mathematics by Godman and Talbert

(ii) Pure Mathematics for Advanced Level by B.D. Bunday and M. Mulholland.

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Indices

$$x^n$$

x ÷ variable

n ÷ power, exponent or index.

Laws of indices.

1. $x^m \times x^n = x^{m+n}$

(ii) $x^m \div x^n = x^{m-n}$

(iii) $x^0 = 1$; $x \neq 0$.

(iv) $x^{1/n} = \sqrt[n]{x}$

(v) $(x^m)^n = x^{mn}$

(vi) $x^n \cdot y^n = (xy)^n$.

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$\frac{6^2}{3^2} = \frac{6\times6}{3\times3} = 2\times4.$

1. Evaluate (a) $x^{\frac{1}{4}} \times x^{\frac{3}{4}}$ (b) $x^2 \div x^{\frac{3}{2}}$

(c) $\frac{x^2y^3}{x^4y}$ (d) $\frac{6^2}{3^2} = \left(\frac{6}{3}\right)^2 = 2^2 = 4.$

(a) $x^{\frac{1}{4}} \times x^{\frac{3}{4}} = x^{\frac{1}{4}+\frac{3}{4}} = x^{\frac{4}{4}} = x^1 = x.$

(b) $x^2 \div x^{\frac{3}{2}} = x^{2-\frac{3}{2}} = x^{\frac{4-3}{2}} = x^{\frac{1}{2}} = \sqrt{x}.$

(c) $\frac{x^2y^3}{x^4y} = x^2y^3 \div x^4y = x^2 \div x^4 \times y^3 \div y$

$= x^{2-4} \cdot y^{3-1} = x^{-2}y^2 = y^2/x^2 = \left(\frac{y}{x}\right)^2.$

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Logarithms

The Logarithm of a positive number N to the base a is defined as the power of a which is equal to N. Thus, if

$$a^x = N$$

$$x = \log_a N.$$

evaluate $x = \log_3 9 \Longrightarrow 3^x = 9 = 3^2$

$\Longrightarrow x = 2.$

$$x = \log_4 2$$

$$4^x = 2$$

$$(2^2)^x = 2^1$$

$$2^{2x} = 2^1$$

$$\Longrightarrow 2x = 1$$

$$x = \frac{1}{2}.$$

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Rules of logarithms

(i) $\log_a (b \times c) = \log_a b + \log_a c$.

(ii) $\log_a \left(\frac{b}{c}\right) = \log_a b - \log_a c$

(iii) $\log_a b^p = p \log_a b$.

(iv) $\log_a N$

$= \frac{\log_b N}{\log_b a}$.

Note:

(i) $\log_a 1 = 0$.

(ii) $\log_b b = 1$

(iii) $\log_a (\sqrt[n]{x}) = \frac{1}{n} \log_a x$.

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Example:

Find the value of x given that

$$5^x = 2(3^x).$$

$$\log 5^x = \log[2(3^x)]$$

$$x\log 5 = \log 2 + \log 3^x$$

$$x\log 5 = \log 2 + x\log 3$$

$$x\log 5 - x\log 3 = \log 2.$$

$$x(\log 5 - \log 3) = \log 2.$$

$$x \log\left(\frac{5}{3}\right) = \log 2$$

$$x = \frac{\log 2}{\log \frac{5}{3}}.$$

$$= \frac{0.30103}{0.2219} = 1.36.$$

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Exercise : Solve for x,

$$4^x + 6^x = 9^x.$$

Remark.

$$\log_e x = \ln x$$

where $e = 2.718281 - - - - -$.

e is called the Euler constant.

$\ln$ is the Niperian log.

$$x(\log 5 - \log 3) = \log 2.$$

$$x \log\left(\frac{5}{3}\right) = \log 2$$

$$x = \frac{\log 2}{\log \frac{5}{3}}.$$

$$= \frac{0.30103}{0.2219} = 1.36.$$

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \ldots$

General rules of Surds.

(a) Multiplication of Surds.

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

e.g. $\sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12} = \sqrt{36} = 6$

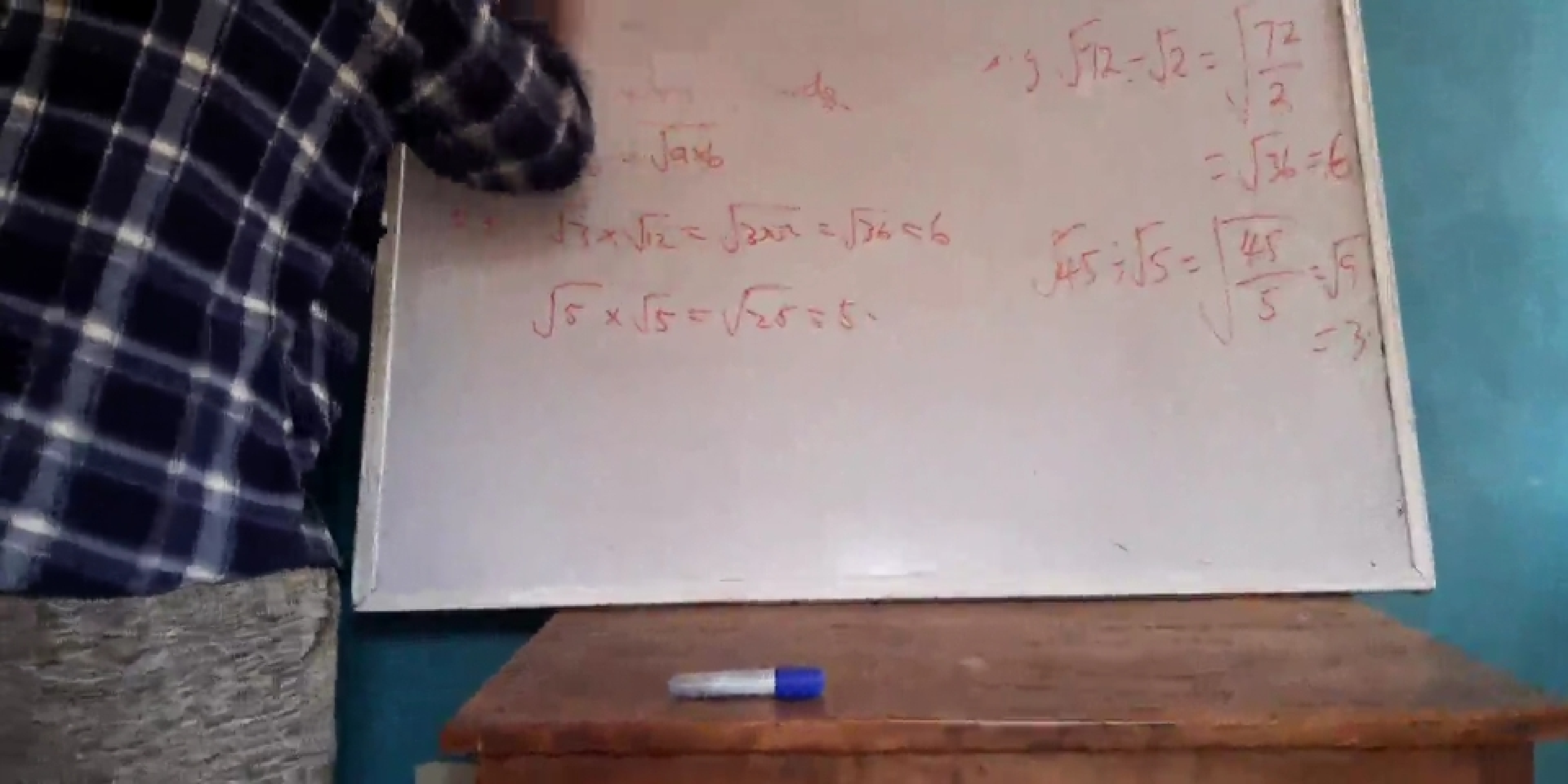
$\sqrt{5} \times \sqrt{5} = \sqrt{25} = 5.$

e.g $\sqrt{72} \div \sqrt{2} = \sqrt{\frac{72}{2}}$

$= \sqrt{36} = 6$

$\sqrt{45} \div \sqrt{5} = \sqrt{\frac{45}{5}} = \sqrt{9}$

$= 3$



$= \sqrt{9\times6}$
$\sqrt{3}\times\sqrt{12}=\sqrt{3\times12}=\sqrt{36}=6$
$\sqrt{5}\times\sqrt{5}=\sqrt{25}=5.$
$\sqrt{72}\div\sqrt{2}=\sqrt{\frac{72}{2}}$
$=\sqrt{36}=6$
$\sqrt{45}\div\sqrt{5}=\sqrt{\frac{45}{5}}=\sqrt{9}$
$=3$

Surds

Simplify $\sqrt{243} - \sqrt{12} + 2\sqrt{75}$

$$\sqrt{243} - \sqrt{12} + 2\sqrt{75}$$

$$= \sqrt{81 \times 3} - \sqrt{3 \times 4} + 2\sqrt{25 \times 3}$$

$$= \sqrt{81} \times \sqrt{3} - \sqrt{3} \times \sqrt{4} + 2 \times \sqrt{25} \times \sqrt{3}$$

$$= 9\sqrt{3} - 2\sqrt{3} + 2 \times 5\sqrt{3}$$

$$= 7\sqrt{3} + 10\sqrt{3}$$

$$= 17\sqrt{3}.$$

$$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$$

e.g. $\sqrt{72} \div \sqrt{2} = \sqrt{\frac{72}{2}}$

$$= \sqrt{36} = 6$$

$$\sqrt{45} \div \sqrt{5} = \sqrt{\frac{45}{5}} = \sqrt{9}$$

$$= 3$$

Surds

$\sqrt{50} + \sqrt{8} + \sqrt{32}$

$= \sqrt{25 \times 2} + \sqrt{4 \times 2} + \sqrt{16 \times 2}$

$= \sqrt{25} \times \sqrt{2} + \sqrt{4} \times \sqrt{2} + \sqrt{16} \times \sqrt{2}$

$= 5\sqrt{2} + 2\sqrt{2} + 4\sqrt{2}$

$= 11\sqrt{2}$

Surds

$a^2-b^2=(a+)(a-b)$.

$$\frac{a}{\sqrt{b}}\times\frac{\sqrt{b}}{\sqrt{b}}=\frac{a\sqrt{b}}{b}$$

$\sqrt{m}+\sqrt{n}$ and $\sqrt{m}-\sqrt{n}$

$$(\sqrt{m}+\sqrt{n})(\sqrt{m}-\sqrt{n})$$

$$=(\sqrt{m})^2-(\sqrt{n})^2$$

$$=m-n$$

$$\frac{5}{\sqrt{3}}=\frac{5}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{5\times\sqrt{3}}{3}$$

$$=\frac{5\sqrt{3}}{3}$$

Surds $a^2-b^2=(a+b)(a-b)$.

$$\frac{4}{\sqrt{3}+\sqrt{7}} = \frac{4}{\sqrt{3}+\sqrt{7}} \times \frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}-\sqrt{7}}$$

$$= \frac{4(\sqrt{3}-\sqrt{7})}{(\sqrt{3})^2-(\sqrt{7})^2}$$

$$= \frac{4(\sqrt{3}-\sqrt{7})}{3-7}$$

$$= \frac{\cancel{4}(\sqrt{3}-\sqrt{7})}{-\cancel{4}}$$

$$= -(\sqrt{3}-\sqrt{7})$$

$$= \sqrt{7}-\sqrt{3}.$$

Simplify

$$\frac{1}{3-\sqrt{2}} + \frac{1}{3+\sqrt{2}}$$

$$= \frac{3+\sqrt{2}+3-\sqrt{2}}{(3-\sqrt{2})(3+\sqrt{2})}$$

$$= \frac{6}{3^2-(\sqrt{2})^2} = \frac{6}{9-2}$$

$$= \frac{6}{7}$$

Surds

Find the square roots of surds.

Given a surd $a+\sqrt{b}$.

$$\sqrt{a+\sqrt{b}} = \sqrt{x}+\sqrt{y}$$

Square both sides

$$a+\sqrt{b} = x+y+2\sqrt{xy}$$
$$= x+y+\sqrt{4xy}$$

$$x+y=a \quad \text{--- (1)}$$
$$4xy=b \quad \text{--- (2)}$$

From (1), $y=a-x$ and substitute into 2.

$$4x(a-x)=b$$
$$4ax-4x^2=b$$
$$4x^2-4ax+b=0$$

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HW

1X

Determine the square root of

$7+2\sqrt{10}$

$= \sqrt{p}+\sqrt{q}$

$= (\sqrt{p}+\sqrt{q})^2$

$p+q+2\sqrt{pq}$

$+\sqrt{4pq}$

From (1), $p = 7-q$

$q(7-q) = 10$

$7q - q^2 = 10$

$q^2 - 7q + 10 = 0$

$(q-5)(q-2) = 0$

$q = 5$ or 2

$p = 2$ or 5

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Determine the square root of

$7+2\sqrt{10}$.

$\sqrt{7+2\sqrt{10}} = \sqrt{p}+\sqrt{q}$

$7+2\sqrt{10} = (\sqrt{p}+\sqrt{q})^2$

$7+2\sqrt{10} = p+q+2\sqrt{pq}$

$7+2\sqrt{10} = p+q+\sqrt{4pq}$

$p+q = 7 \longrightarrow (1)$

$pq = 10 \longrightarrow (2)$

$p = 7-q$

$7q-q^2 = 10$

$q^2-7q+10 = 0$

$(q-5)(q-2) = 0$

$q = 5$ or 2

$p = 2$ or 5

Hence square root of

$\sqrt{7+2\sqrt{10}} = \sqrt{2}+\sqrt{5}$

$\therefore 7+2\sqrt{10} = p+q+2\sqrt{pq}$

$7+2\sqrt{10} = p+q+\sqrt{4pq}$

$p+q = 7 \longrightarrow (1)$

$pq = 10 \longrightarrow (2)$

From (1), $p = 7-q$

$q(7-q) = 10$

$7q - q^2 = 10$

$q^2 - 7q + 10 = 0$

$(q-5)(q-2) = 0$

$q = 5$ or 2

$p = 2$ or 5

$q = 2$ or 12.

$p = 12$ or 2.

Square roots of $14 - 4\sqrt{6}$

are $\sqrt{12} - \sqrt{2} = 2\sqrt{3} - \sqrt{2}$

and $\sqrt{2} - \sqrt{12} = \sqrt{2} - 2\sqrt{3}$.

$$14 - \sqrt{96} = p + q - \sqrt{4pq}$$

$$p + q = 14 \quad (1)$$

$$pq = 24 \quad (2)$$

$$96 = 4pq$$

$$pq = \frac{96}{4} = 24$$

$$p = 14 - q$$

$$q(14 - q) = 24$$

$$14q - q^2 = 24$$

$$q^2 - 14q + 24 = 0$$

$$q^2 - 12q - 2q + 24 = 0$$

$$q(q - 12) - 2(q - 12) = 0$$

$$(q - 2)(q - 12) = 0$$

[illegible]

Solve the radical equation

$$2\sqrt{3x+4}+x=36$$

$$2\sqrt{3x+4}=36-x$$

Square both sides

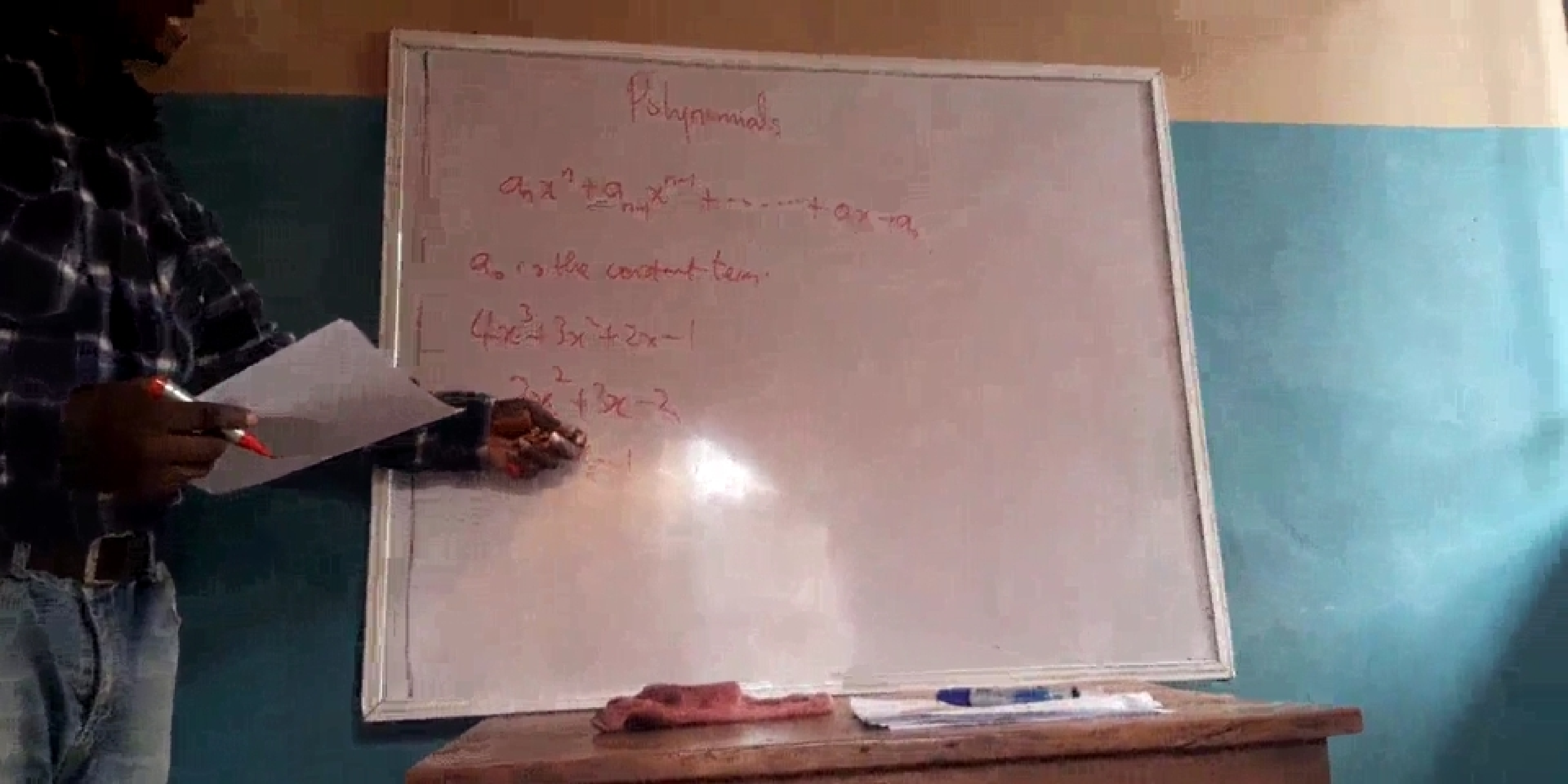
$$4(3x+4)=(36-x)^2$$

$$12x+16=1296-72x+x^2$$

$$x^2-84x+1280=0$$

$$(x-20)(x-64)=0$$

$$\Rightarrow x=20 \text{ or } x=64$$


Polynomials
$a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$
a_0 is the constant term.
$4x^3 + 3x^2 + 2x - 1$
$2x^2 + 3x - 2$

Polynomials

Addition and subtraction of Polynomials

Example 1. Consider two polynomials given by

$$P_1(x) = 2x^3 - 3x^2 + 5x - 7$$

$$P_2(x) = x^3 + x^2 - x + 1$$

Find (a) $P_1 + P_2$ (b) $P_1 - P_2$ (c) $2P_1 - 3P_2$.

$$P_1(x) + P_2(x) = 2x^3 - 3x^2 + 5x - 7 + x^3 + x^2 - x + 1$$

$$\quad {}^3 - 3x^2 + x^2 + 5x - x - 7 + 1$$

$$= 3x^3 - 2x^2 + 4x - 6.$$

$P_1 - P_2$

$$= 2x^3 - 3x^2 + 5x - 7 - (x^3 + x^2 - x + 1)$$

$$= 2x^3 - 3x^2 + 5x - 7 - x^3 - x^2 + x - 1$$

$$= x^3 - 4x^2 + 6x - 8.$$