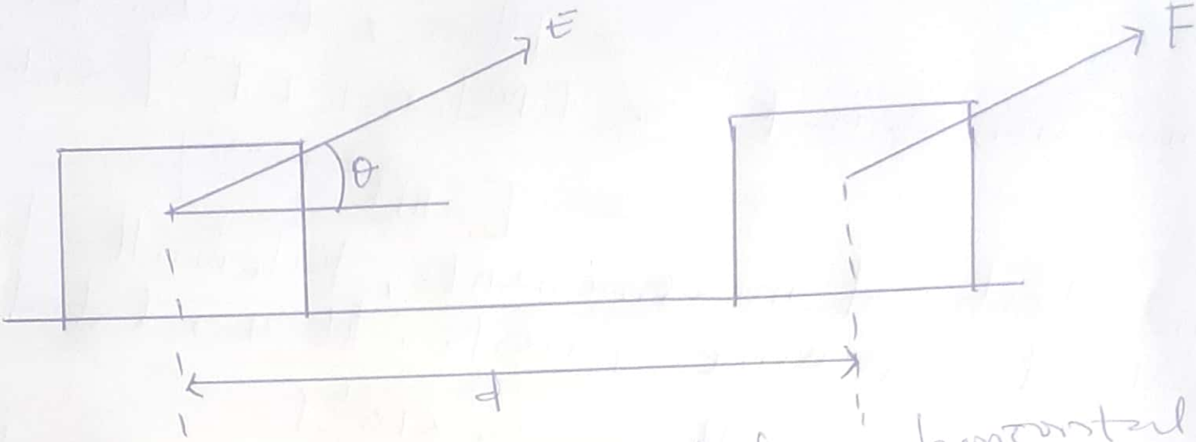
2.0 WORK ENERGY AND POWER.

Definition 2.1 The work done by a force is the product of the magnitude of force component in the direction of displacement and the displacement of this object as shown below.



A force F on a block moves it by a horizontal distance d. The direction of force makes an angle θ with the horizontal direction.

If force F is acting at an angle θ with respect to the displacement d of the object, its component along d will be F cos θ. Then work done by the force F is given by ——— (1)

$$W = F \cos\theta \, d.$$

In vector form, the work done is given by ——— (2)

$$W = F \cdot d.$$

Remark 2.2 (i) If $d = 0$, $W = 0$, that is no work is done by a force, whatever its magnitude, if there is no displacement of the object.

(ii) If both force and displacement are vectors, work is a scalar.

The unit of work is Joule. or Nm

One Joules is defined as the workdone by a force of one newton when it produces a displacement of one metre. Joules is the SI unit of work

Example 2.3 Find the dimensional formula of work

solution

$$W = \text{Force} \times \text{Distance}$$

$$= \text{Mass} \times \text{Acceleration} \times \text{distance}$$

Dimension of work $= [M] \times [LT^{-2}] \times [L]$

$$= [ML^2T^{-2}]$$

In electrical measurements, Kilowatt-hour (KWh) is used as unit of work. It is related to Joule as

$$1KWh = 3.6 \times 10^6 J.$$

Example 2.4 A force of 6N is applied on an object at an angle of 60° with the horizontal. Calculate the workdone in moving the object by 2m in the horizontal direction.

Solution

$$W = Fd\cos\theta$$

$$= 6 \times 2 \cos 60^\circ$$

$$= 6 \times 2 \times \frac{1}{2}$$

$$= 6J$$

Example 2.5 A person lifts 5kg potatoes from the ground floor to a height of 4m to bring it to first floor. Calculate the work done.

solution Since work is done against gravity

then Force $= mg$

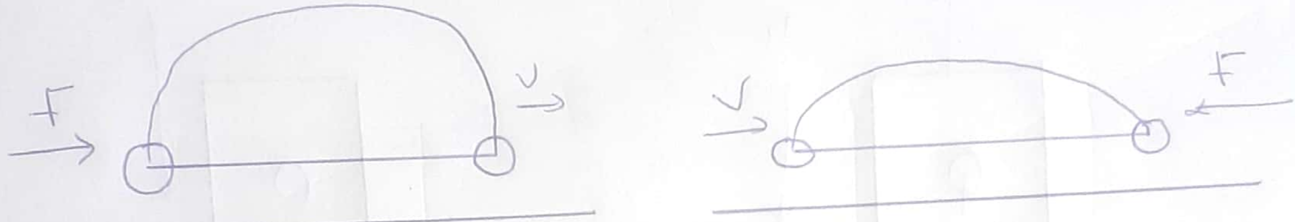
$$= 5kg \times 9.8ms^{-2}$$

$$= 49N.$$

Work done = 49 × 4 (Nm)

= 196 J

there are situations where work becomes a positive or a negative quantity as described below.



(a) (b)

The figure (a) shows a car moving in +x direction and a force F is applied in the same direction. The force and the displacement both are in the same direction i.e $\theta = 0^\circ$. therefore

$$W = Fd \cos 0^\circ$$

$$= Fd \text{ (this work is +ve)}$$

The figure (b) shows the same car moving in the +x direction, but the force is applied in the opposite direction to stop ~~the~~ car. $\theta = 180^\circ$. therefore,

$$W = Fd \cos 180^\circ$$

$$= -Fd \text{ (this work is +ve)}$$

2.6 Work done by the force of Gravity

From the figures below, (a) shows that a mass m being lifted to a height h. and the work done against the force mg (downwards) and the displacement is upward ($\theta = 180^\circ$). therefore

$$W = Fd \cos 180^\circ$$

$$= -mgh.$$

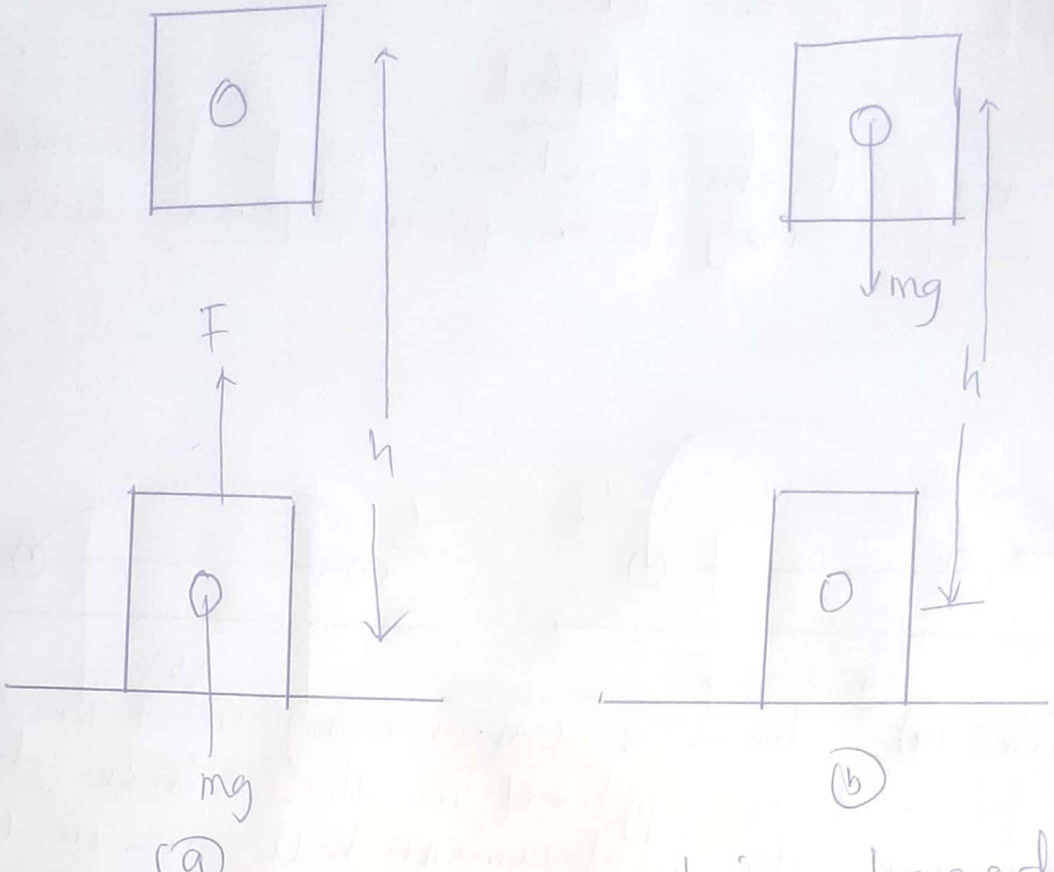


Figure (b) shows the mass is being lowered. the force mg and the displacement d are in the direction ($\theta = 0^\circ$). Therefore, the work done

$$W = Fd\cos 0^\circ$$
$$= +mgh$$

Remark 2.6 When the object is lifted up. the work done by the gravitational force is negative but the work done by the person lifting the object is positive.

2.7 Work done by a variable force

Consider a case in which the magnitude of force $F(x)$ changes with the position x of the object. then work is calculated over a large number of small intervals of displacements Δx so that

$$\Delta W = F(x)\,\Delta x.$$

therefore, the total work done by the force between x_i and x_f is the sum of all the such areas i.e

$$W = \sum \Delta W$$

$$= \sum F(x)\, \Delta x$$

$$= \sum_{\lim \Delta x \to 0} F(x)\, \delta x$$

2.6 Work done by a spring

The work done of a variable force exerted by a spring can be expressed as

$$W = \text{force} \times \text{displacement}$$

$$= \frac{F}{2} \cdot x$$

taking $F = K|x|$. ~~given~~ by (Hooke's Law).

$$W = \frac{1}{2} Kx \times x$$

Then

$$= \frac{1}{2} K x^2$$

Example 2.7 A mass of 2kg is attached to a light spring of force constant $K = 100\,Nm^{-1}$. Calculate the work done by an external force in stretching the spring by 10 cm

Solution:

$$W = \frac{1}{2} Kx^2$$

$$= \frac{1}{2} \times 100 \times \left(\frac{10}{100}\right)^2$$

$$= \frac{1}{2} \times 100 \times 0.1^2 = 50 \times 0.01$$

$$= 0.5\,J.$$

Remark 2.8 the work done by the externally force is positive but the work done by the spring is negative and its magnitude is $(\frac{1}{2})kx^2$: Therefore the work done by the spring in example 2.7 is $-0.5J$

3.0 POWER

Definition 3.1 Power is the rate at which work is done.

If ΔW work is done in time Δt, the average power is defined as

$$P = \frac{\text{Work done}}{\text{time taken}} \quad \text{—— (1)}$$

$$P = \frac{\Delta W}{\Delta t} \quad \text{—— (2)}$$

If the rate of doing work is not constant, this rate may vary. In such cases, we define instantaneous power P:

$$P = \lim_{\Delta t \to 0} \left(\frac{\Delta W}{\Delta t}\right) = \frac{dW}{dt} \quad \text{—— (3)}$$

the S.I unit of power is Joules/second = Watt denoted as W. Note that

$1KW = 10^3 W$ and $1MW = 10^6 W$.

Example 3.1.1 Determine the dimensions of power

Solution: $P = \frac{\text{Work}}{\text{time}} = \text{Force} \times \frac{\text{Distance}}{\text{time}}$

$$= [\text{Mass}] \times [\text{Acceleration}] \times \frac{[\text{Distance}]}{[\text{Time}]}.$$

$$= [M] \times \left[\frac{L}{T^2}\right] \times \left[\frac{L}{T}\right]$$

$$= ML^2T^{-3}$$

In electricity, the power of a machine in terms of horse power denoted as hp can be expressed as

$$1hp = 746\,W.$$

Also, in electrical measurement

$$\text{kilowatt-hour } (kWh) = 1 kW \times 1 \text{ hour}$$

$$= 10^3 W \times 3600 s$$

$$= \frac{10^3 J}{1 s} \times 3600 s$$

$$= 36,00,000 J$$

$$= 3.6 \times 10^6 J.$$

or $1 kWh = 3.6 MJ$. (mega Joules).

3.2 WORK AND KINECTIC ENERGY

Consider an object of mass m moving along a straight line when a constant force of magnitude F acts on it along the direction of motion. this force produces a uniform acceleration a such that F = ma. Let v_1 be the speed of the object at time t_1. the speed becomes v_2 at another instant of time t_2. During this interval of time $t = (t_2 - t_1)$, the object covers a distance s.

Hence

$$v_2^2 = v_1^2 + 2as$$

$$a = \frac{v_2^2 - v_1^2}{2s}$$

Since $F = ma$

$$= m \times \frac{v_2^2 - v_1^2}{2s}$$

and $W = Fs$

$$= m \times \frac{v_2^2 - v_1^2}{2s} s$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$= K_2 - K_1$$

where $K_2 = \frac{1}{2} mv^2$ and $K_1 = \frac{1}{2} m v_1^2$ respectively denotes final and initial kinetic energies.

Kinetic Energy is a scalar quantity.

3.2.1 <u>Work - Energy theorem</u>

The theorem states that the work done by the resultant of all forces acting on a body is equal to the change in kinetic energy of the body.

Example 3.2.2 A body of mass 10kg is initially moving with a speed of 4.0 ms⁻¹. A force of 30N is now applied to the change on the body for 2 seconds.

(i) What is the final speed of the body after 2 second?

(ii) How much work has been done during this period?

(iii) What is the initial kinetic energy?

(iv) What is the final kinetic energy?

(v) What is the distance covered during this period?

(vi) Show that the work done is equal to the change in kinetic energy?

Solution:

(i) $F = ma$

$a = F/m = 30/10 = 3ms^{-2}$

So $v_2 = v_1 + at$

$= 4 + (3 \times 2) = 10ms^{-1}$

(ii) the distance covered in 2 seconds

$s = ut + \frac{1}{2}at^2$

$= (4 \times 2) + \frac{1}{2}(3 \times 4)$

$= 8 + 6 = 14m.$

$W = F \times S = 30 \times 14 = 420J.$

(iii) the initial kinetic energy

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(10 \times 16) = 80J$

(IV) The final kinetic Energy

$$K_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}(10 \times 100) = 500J.$$

(V) the distance covered as calculated above $= 14m$.

(VI) the change in kinetic energy is

$$K_2 - K_1 = (500 - 80) = 420J.$$

3.3 POTENTIAL ENERGY.

Definition 3.3.1 the energy possessed by an object due to their position in space is knows as potential Energy. Examples is the Gravitational potential energy possessed by a body in Gravitational field. The potential energy is described as

$$W = mgh.$$

Example 3.3.1 A truck is loaded with sugar bags. the total mass of the load and the truck together is 100,000kg. the truck moves on a winding path up a mountain to a height of 700m in 1hr. What average power must the engine produce to lift the material?

Solution

$$W = mgh$$
$$= (100,000\,kg) \times (9.8ms^{-2} \times 700m)$$
$$= 68.6 \times 10^7 J.$$

Time taken $= 1hr = 60 \times 60s$
$= 3600s$

$$\text{Average Power} = W/t = \frac{68.6 \times 10^7 J}{3600 s}$$

$$= 1.91 \times 10^5 W$$

Since $746 W = 1 hp$, then

$$P = \frac{1.91 \times 10^5}{746} = 2.56 \times 10^2 = 265 hp.$$

Example 3.3.2 Hydroelectric power generation uses falling water as a source of energy to turn turbine blades and generates electrical power. In a power station, $1000 \times 10^3 Kg$ water falls through a height of 51 m in one second.

(i) Calculate the work done by the falling water

(ii) How much power can be generated under ideal condition?

Solution:

$$P.E = mgh$$
$$= (1000 \times 10^3 kg) \times (9.8 ms^{-2}) \times (51m)$$
$$= 9.8 \times 51 \times 10^6 J$$
$$= 500 \times 10^6 J.$$

Since

$$W = F \times S = mg \times h$$
$$= 1000 \times 10^3 \times 9.8 \times 51 J$$
$$= 500 \times 10^6$$
$$= 500 MJ$$

(ii) $P = W/t = \frac{500\,mJ}{1\,s}$

$= 500\,mW$

3.4 Potential Energy of Springs

Recall that, an external force is required to compress or stretch a given spring then

$$W = \frac{1}{2}Kx^2$$

This work is stored in the spring as elastic potential energy.

3.5 Conservation of Energy

The law of conservation of Energy states that the total energy of an isolated system always remains constant.

3.6 Elastic and Inelastic Collisions

When two bodies interact, it is termed as collision. There are no external forces acting on the system.

The collisions are of two kinds

(i) Perfectly Elastic Collision:- If the forces of interaction between two bodies are conservative, the total kinetic energy is conserved. The total kinetic energy before collision is same as that after the collision.

(ii) Perfectly Inelastic Collision:- When two bodies stick together after the collision and move as as one single unit, it is termed as perfectly Inelastic Collision.

By applying the laws of Conservation of momentum and kinetic energy, we have

$$m_A v_{Ai} + m_A v_{Bi} = m_A v_{Af} + m_B v_{Bf} \quad \text{—(1)}$$

$$\frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_{Ai} v_{Bi}^2 = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 \quad \text{—(2)}$$

Also

$$(v_{Bf} - v_{Af}) = -(v_{Bi} - v_{Ai}) \quad \text{—(3)}$$

$$v_{Af} = \frac{2m_B v_{Bi}}{m_A + m_B} + \frac{v_{Ai}(m_A - m_B)}{m_A + m_B} \quad \text{—(4)}$$

$$v_{Bf} = \frac{-2m_A v_{Ai}}{m_A + m_B} + \frac{(m_B - m_A) v_{Bi}}{(m_A + m_B)} \quad \text{—(5)}$$

Remark (1) Suppose that the two balls colliding with each other are identical i.e $m_A = m_B = m$. then equation (4) & (5) result to

$$v_{Af} = v_{Bi} \quad \text{—(6)}$$

$$v_{Bf} = v_{Ai} \quad \text{—(7)}$$

(II) Suppose the collision of two particles are of unequal masses then

$m_B >> m_A$ and $v_{Bi} = 0$

eqnation (4) and (5) reduced to

$$v_{Af} = -v_{Ai}$$

$$v_{Bf} = 0$$