MTS 104

1.0 Linear Motion:

Motion is one of the key branches of physics. It finds applications in numerous fields such as engineering, medicine, geology, sport science and so on.

1.1 Variables (Quantities in Motion)

(i) Distance: is a change in position relative to a reference point. It is a scalar quantity measured in metre (m) and as such, it can only be positive.

(ii) Displacement: is a change in position relative to a reference point in a particular direction. It is a vector quantity and also measured in metre (m) and represented by d, s, and x. Displacement, being a vector can be positive or negative.

(iii) Average Speed: is the rate of change of distance. It is a scalar quantity, measured in (m) per second (s), m/s. Since speed is likely to change over the course of motion.

$$\text{Average Speed} = \frac{\text{total distance covered}}{\text{Total time taken}}$$

$$= \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1}$$

(iv) Instantaneous speed: is the speed recorded at a given point in time.

$$v = \frac{d}{t}$$

(V) Average velocity: is the rate of change of displacement and is also measured in m/s.

$$A_v = \frac{\text{Total displacement}}{\text{Total time taken}}$$

$$= \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$$

Instantaneous velocity: is a specified position or a particular point in time.

$$V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

(VI) Average Acceleration: is the rate of change of velocity and is measured in (m/s^2). It is also a vector quantity.

$$A_a = \frac{\text{change in velocity}}{\text{time taken}}$$

$$A_a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Acceleration occurs due to change in the

(i) magnitude of the velocity only,

(ii) direction of the velocity only, or

(iii) magnitude and direction of the velocity.

(VII) Instantaneous acceleration: is the acceleration at a specified position or particular point in time.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v}{t}.$$

(viii) Free Acceleration: This is experienced by an object undergoing vertical motion in the vicinity of the earth and this has value $g = 9.8\,ms^{-2}$

When an acceleration is negative, it is called deceleration.

1.2 Equation of Motion:

$$v = u + at \quad \text{———} \; (1)$$

$$s = ut + \frac{1}{2}at^2 \quad \text{———} \; (2)$$

$$v^2 = u^2 + 2as \quad \text{———} \; (3)$$

$$s = \frac{1}{2}(u+v)t \quad \text{———} \; (4)$$

$$s = vt - \frac{1}{2}at^2 \quad \text{———} \; (5)$$

Example 1 Find v when $u = 2ms^{-1}$, $a = 3ms^{-2}$ and $t = 4s$.

Soln Using $v = u + at$

$v = 2 + 3 \times 4$

$= 2 + 12$

$= 14ms^{-1}$

Example 2 Find s when $v = 0.2ms^{-1}$, $u = 3.8ms^{-1}$ and $t = 10s$.

Soln

$$s = \frac{(u+v)}{2}t$$

$$s = \frac{3.8 + 0.2}{2} \times 10$$

$s = 20m$.

Example 3 Find s, when $v = 15ms^{-1}$, $a = 6ms^{-2}$ and t = 5s.

Soln

$$s = vt - \frac{1}{2}at^2$$

$$s = 15 \times 5 - \frac{1}{2} \times 6 \times 5^2$$

$$= 75 - 75$$

s = 0m. this is possible, since we dealing with vector quantities.

Example 4 Find a when $v = 3ms^{-1}$, $u = 13ms^{-1}$ s = 8m

Soln

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s} = \frac{3^2 - 13^2}{2 \times 8} = \frac{9 - 169}{16} = -10m/s^2$$

Example 5 A plane flies from london Heathrow Airport at to Dubai Airport a distance of 5500km at an average of 1200 km/h. The return trip was made at an average speed of 1050 km/h. Find the average speed for the whole journey.

Soln

$s = 5500km$

$u_1 = 1200km/h$

$u_2 = 1050 km/h$

$$t_1 = \frac{\text{distance}}{\text{average speed}} = \frac{5500}{1200} = \frac{55}{12}$$

$t_1 = 4.58$ hours

$$t_2 = \frac{\text{distance}}{\text{average speed}}$$

$$= \frac{5500}{1050} = \frac{110}{21}$$

$t_2 = 5.24$ hours

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$= \frac{5500 + 5500}{4.58 + 5.24} = 1120 km/h$$

Example 6 the safe take-off velocity of a port london passenger plane is set at 210km/h. find the minimum acceleration that the airplane needs to move on a 2.2 km run away

Sol

$u = 0ms^{-1}$ $\quad v = \frac{210 \times 1000}{3600}$

$v = 210 \ km/h^{-1}$ $\quad = 58.3 ms^{-1}$

$s = 2.2km = 2200m$

$a = ?$

Using $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{58.3^2 - 0^2}{2 \times 2200} = 0.77ms^{-2}$$

Example 7 A taxi driver moving at a velocity of 10m/s realised that he had 35sec to get to his destination which is 300m away. He therefore accelerated at $3m/s^2$ for the rest of the journey. And he succeed in getting to this destination on time

Sol $u = 10ms^{-1}$ $\quad a = 3ms^{-2}$, $s = 300m$ $\quad t = ?$

Using $s = ut + \frac{1}{2}at^2$ by substitution:

$300 = 10 \times t + \frac{1}{2}(3)t^2$

$300 = 10t + 1.5t^2 \times 2$

$3t^2 + 20t - 1600 = 0$

$$3t^2 + 20t - 1600 = 0$$

$$(t - 20)(3t + 80) = 0$$

$$t - 20 = 0$$

$$t = 20$$

$$3t + 80 = 0$$

$$3t = -80$$

$$t = \frac{-80}{3}$$

Example A particle moves such that its position x metres at time t seconds is given by the expression $x = 3t^3 - 13t^2 - 2t$.

(i) Determine the position of the particle when time $t = 0, 1, 2, 3, 4,$ and 5.

Soln At $= 0$, then

$$x = 3t^3 - 13t^2 - 2t = 3(0)^3 - 13(0)^2 - 2(0)$$

$$= 0m.$$

At $t = 1$, $x = 3(1)^3 - 13(1)^2 - 2(1) = 3 - 13 - 2$

$$x = -12m$$

At. $t = 2$, $x = 3t^3 - 13t^2 - 2t$

$$= 3(2)^3 - 13(2)^2 - 2(2)$$

$$= 24 - 52 - 4.$$

At $t = 3$, $x = 3(3)^2 - 13(3)^2 - 2(3)$

$$= \cancel{3(27)}\ 81 - 117 - 6$$

$$x = -42$$

At $t = 4$ $x = 3t^3 - 13t^2 - 2t$

$$= 3(4)^2 - 13(4)^2 - 2(4)$$

$$= 192 - 208 - 8$$

$$x = -20m.$$

At $t = 5$, the position $x = 3t^3 - 13t^2 - 2t$

$$x = 3(5)^3 - 13(5)^2 - 2(5)$$

$$x = 375 - 325 - 10$$

$$x = 40m.$$

1.3 Free Fall Motion

Example 1 A metal coin is thrown straight upwards with an initial velocity of 20m/s. Calculate the distance covered from the point of projection and velocity after 2s. take $g = 10ms^{-2}$

Soln

$u = 20ms^{-1}$

$g = 10ms^{-2}$

$t = 2s$

$s = ?$

$v = ?$

$$s = ut + \frac{1}{2}gt^2$$

$$s = 20 \times 2 - \frac{1}{2}(10)(2)^2$$

$$= 40 - 20 = 20m.$$

Example 2 An apple fruit falls freely from a tree at a height of 2.4m. How long does it take to reach the ground?

Soln

$u = 0ms^{-1}$ $\quad g = 9.8ms^{-2}$

$h = 2.4m$ $\quad t = ?$

Using $s = ut + \frac{1}{2}gt^2$

$u = 0.$

$$s = 0 \times t + \frac{1}{2}gt^2$$

$$s = \frac{1}{2}gt^2$$

$$t^2 = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 2.4}{9.8}}$$

$$t = \frac{2\sqrt{6}}{7}$$

$$t = 0.70s$$

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Example 3 A 500g parcel is dropped from a height of 60m, from a plane which is moving upwards with a velocity of 5.0m/s. Determine:

(i) the initial velocity of the parcel.

(ii) the time taken for the parcel to reach the ground? take $g = 10m/s$

Soln

$h = 60m \quad g = 10m/s^{-2}$

$u = ? \quad t = ?$

Using $s = ut + \frac{1}{2}gt^2$

$60 = -5t + \frac{1}{2}10t^2$

$60 = -5t + 5t^2$

$5t^2 - 5t - 60 = 0$

$t^2 - t - 12 = 0$

$(t - 4)(t + 3) = 0$

$t - 4 = 0$

$t = 4$

$t + 3 = 0$

$t = -3$.

Since time cannot be negative, it follows that only solution is

$t = 4.0s$

1.4 Free fall motion

Example 1 An object is projected straight upwards with an initial velocity of u. If T is the time taken to return to the point of projection (time of flight), H the greatest height and g is the acceleration due to gravity, ~~show~~ that

(i) $T = \frac{2u}{g}$ (ii) $H = \frac{u^2}{2g}$

Soln

(i) $u = U ms^{-1}$

$t = T_s$

$s = H$

Using $v = u + gt$

$T = 2t$

$t = \frac{1}{2}T$ —— (2)

$0 = U - g\left(\frac{1}{2}T\right)$

$U = \frac{1}{2}gT$

$T = \frac{2U}{g}$

(ii) Using $v^2 = u^2 + 2gh$

$0 = U^2 - 2gH$

$2gH = U^2$

$H = \frac{U^2}{2g}$

Example 2 An object is thrown vertically upwards with an initial velocity of 3 m/s from a ladder which is 5m above ground level. Taking g to be $10 m/s^2$

(i) Express the height h of the object above the ground as a function of time t.

(ii) Use the expression in (i) to find the time the object hits the ground and velocity and velocity with which it strikes the ground.

Soln $h = 5m$, $g = 10ms^{-2}$, $u = 3ms^{-1}$

Using $s = ut + \frac{1}{2}gt^2$ ——— (1)

$h = 5 + s$ ——— (2)

$s = 3t - \frac{1}{2}10t^2$

$= 3t - 5t^2$

Therefore from (2)

$h = 5 + 3t - 5t^2$

Since the object hits the ground when the height is zero, thus

$0 = 5 + 3t - 5t^2$

$5t^2 - 3t - 5 = 0$

Using

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(-5)}}{2(5)} = \frac{3 \pm \sqrt{109}}{10}$$

$$t = \frac{3 + \sqrt{109}}{10} = 1.34 \quad \text{or} \quad t = \frac{3 - \sqrt{105}}{10} = -0.744$$

Since time cannot be negative, it implies that the time taken to reach the ground is $t = 1.34s$.

Using $v^2 = u^2 + 2gs$

$= (-3)^2 + 2(10)(5)$

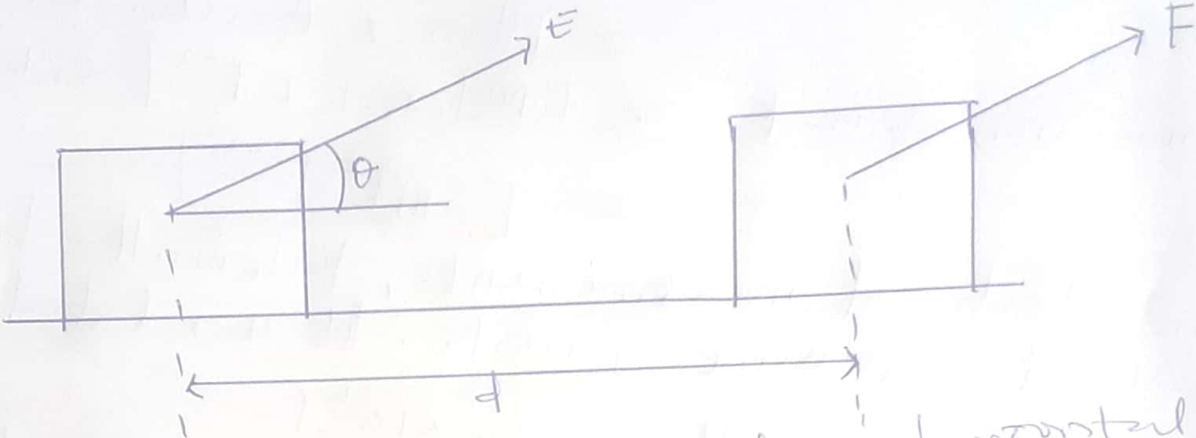
$= 9 + 100$

$v = \sqrt{109}$

$v = 10.4ms^{-1}$.

2.0 WORK ENERGY AND POWER.

Definition 2.1 The work done by a force is the product of the magnitude of force component in the direction of displacement and the displacement of this object as shown below.



A force F on a block moves it by a horizontal distance d. The direction of force makes an angle θ with the horizontal direction

If force F is acting at an angle θ with respect to the displacement d of the object, its component along d will be $F\cos\theta$. Then work done by the force F is given by

$$W = F\cos\theta\, d \qquad (1)$$

In vector form, the work done is given by

$$W = F \cdot d \qquad (2)$$

Remark 2.2 (i) If $d=0$, $W=0$, that is no work is done by a force, whatever its magnitude, if there is no displacement of the object.

(ii) If both force and displacement are vectors, work is a scalar.

The unit of work is Joule or Nm

One Joules is defined as the workdone by a force of one newton when it produces a displacement of one metre. Joules is the SI unit of work

Example 2.3 Find the dimensional formula of work

Solution
$$W = \text{Force} \times \text{Distance}$$
$$= \text{Mass} \times \text{Acceleration} \times \text{distance}$$

Dimension of work $= [M] \times [LT^{-2}] \times [L]$
$$= [ML^2T^{-2}]$$

In electrical measurements, Kilowatt-hour (KWh) is used as unit of work. It is related to Joule as
$$1 KWh = 3.6 \times 10^6 J.$$

Example 2.4 A force of 6N is applied on an object at an angle of 60° with the horizontal. Calculate the workdone in moving the object by 2m in the horizontal direction.

Solution
$$W = Fd\cos\theta$$
$$= 6 \times 2 \cos 60°$$
$$= 6 \times 2 \times \frac{1}{2}$$
$$= \underline{6J}$$

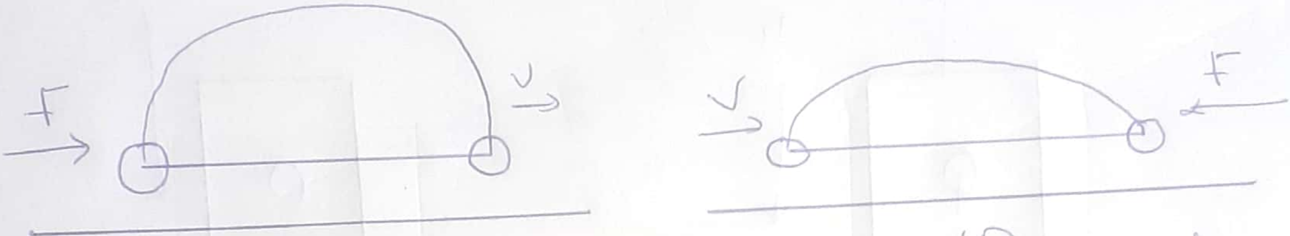
Example 2.5 A person lifts 5kg potatoes from the ground floor to a height of 4m to bring it to first floor. Calculate the work done.

Solution
Since work is done against gravity then
$$\text{Force} = mg$$
$$= 5kg \times 9.8ms^{-2}$$
$$= 49N.$$

Work done = 49×4 (Nm)

$= 196\,J$

there are situations where work becomes a positive or a negative quantity as described below.



(a) (b)

The figure (a) shows a car moving in $+x$ direction and a force F is applied in the same direction. The force and the displacement both are in the same direction i.e $\theta = 0°$. therefore

$$W = Fd \cos 0°$$
$$= Fd \text{ (this work is +ve)}$$

The figure (b) shows the same car moving in the $+x$ direction, but the force is applied in the opposite direction to stop ~~the~~ car. $\theta = 180°$.
therefore,

$$W = Fd \cos 180°$$
$$= -Fd. \text{ (this work is +ve)}$$

2.6 Work done by the force of Gravity

From the figures below, (a) shows that a mass m being lifted to a height h. and the work done against the force mg (downwards) and the displacement is upward ($\theta = 180°$). therefore

$$W = Fd \cos 180°$$
$$= -mgh.$$

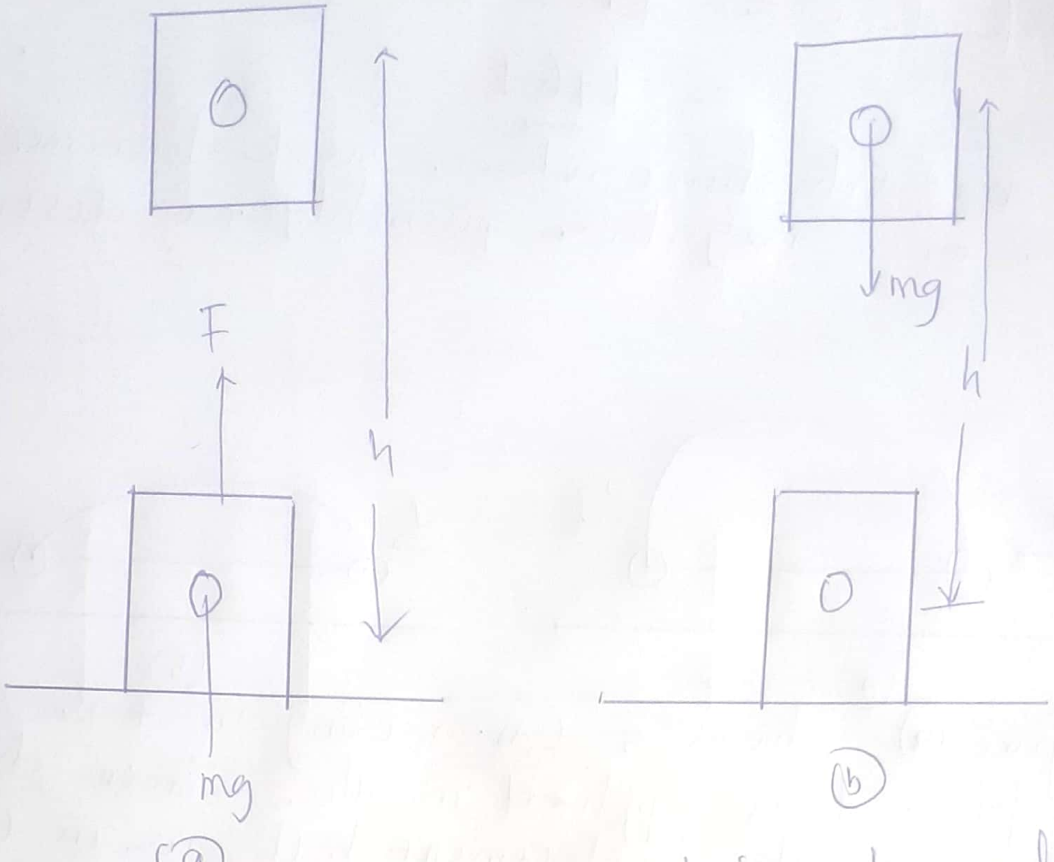


Figure (b) shows the mass is being lowered. the force mg and the displacement d are in the direction ($\theta = 0°$). Therefore, the work done

$$W = Fd\cos 0°$$
$$= +mgh$$

Remark 2.6 When the object is lifted up, the work done by the gravitional force is negative but the work done by the person lifting the object is positive.

2.7 Work done by a variable force

Consider a case in which the magnitude of force $F(x)$ changes with the position x of the object. then work is calculated over a large number of small intervals of displacements Δx so that

$$\Delta W = F(x)\,\Delta x.$$

Therefore, the total work done by the force between x_i and x_f is the sum of all the such areas i.e

$$W = \sum \Delta W$$

$$= \sum F(x)\,\Delta x$$

$$= \sum_{\lim \Delta x \to 0} F(x)\,\delta x$$

2.6 Work done by a spring

The work done of a variable force exerted by a spring can be expressed as

$$W = \text{Force} \times \text{displacement}$$

$$= \frac{F}{2} \cdot x$$

taking $F = K|x|$ ~~given~~ by (Hooke's Law).

Then

$$W = \frac{1}{2} Kx \times x$$

$$= \frac{1}{2} K x^2$$

Example 2.7 A mass ~~of~~ 2kg is attached to a light spring of force constant $K = 100\,Nm^{-1}$. Calculate the work done by an external force in stretching the spring by 10 cm

Solution:

$$W = \frac{1}{2} K x^2$$

$$= \frac{1}{2} \times 100 \times \left(\frac{10}{100}\right)^2$$

$$= \frac{1}{2} \times 100 \times 0.1^2 = 50 \times 0.01$$

$$= \frac{1}{2} \times 100 \qquad = 0.5J.$$

Remark 2.8 the work done by the externally force is positive but the workdone by the spring is negative and its magnitude is $(\frac{1}{2})kx^2$. Therefore the work done by the spring in example 2.7 is $-0.5J$

3.0 POWER

Definition 3.1 Power is the rate at which work is done.

If ΔW work is done in time Δt, the average power is defined as

$$P = \frac{\text{Work done}}{\text{time taken}} \quad \text{———} \ (1)$$

$$P = \frac{\Delta W}{\Delta t} \quad \text{———} \ (2)$$

If the rate of doing work is not constant, this rate may vary. In such cases, we define instantaneous power P:

$$P = \lim_{\Delta t \to 0} \left(\frac{\Delta W}{\Delta t}\right) = \frac{dW}{dt} \quad \text{———} \ (3)$$

the S.I unit of power is Joules/second = Watt denoted as W. Note that

$1KW = 10^3 W$ and $1MW = 10^6 W$.

Example 3.1.1 Determine the dimensions of power

Solution:

$$P = \frac{\text{work}}{\text{time}} = \text{Force} \times \frac{\text{Distance}}{\text{time}}$$

$$= [\text{Mass}] \times [\text{Acceleration}] \times \frac{[\text{Distance}]}{[\text{Time}]}$$

$$= [M] \times \left[\frac{L}{T^2}\right] \times \left[\frac{L}{T}\right]$$

$$= ML^2T^{-3}$$

In electricity, the power of a machine in terms of horse power denoted as hp can be expressed as

$$1hp = 746 W.$$

Also, in electrical measurement

$$\text{kilowatt-hour (kWh)} = 1 kW \times 1 \text{ hour}$$

$$= 10^3 W \times 3600 s$$

$$= \frac{10^3 J}{1s} \times 3600 s$$

$$= 36,00,000 J$$

$$= 3.6 \times 10^6 J.$$

or $1 kWh = 3.6 MJ$. (mega Joules).

3.2 WORK AND KINECTIC ENERGY

Consider an object of mass m moving along a straight line when a constant force of magnitude F acts on it along the direction of motion. This force produces a uniform acceleration a such that $F = ma$. Let v_1 be the speed of the object at time t_1. The speed becomes v_2 at another instant of time t_2. During this interval of time $t = (t_2 - t_1)$, the object covers a distance s.

Hence

$$v_2^2 = v_1^2 + 2as$$

$$a = \frac{v_2^2 - v_1^2}{2s}$$

Since $F = ma$

$$= m \times \frac{v_2^2 - v_1^2}{2s}$$

and $W = Fs$

$$= m \times \frac{v_2^2 - v_1^2}{2s} s$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$= K_2 - K_1$$

where $K_2 = \frac{1}{2} mv^2$ and $K_1 = \frac{1}{2} m v_1^2$ respectively denotes final and initial kinetic energies.

Kinetic Energy is a scalar quantity.

3.2.1 <u>Work - Energy theorem</u>

The theorem states that the work done by the resultant of all forces acting on a body is equal to the change in kinetic energy of the body.

Example 3.2.2 A body of mass 10kg is initially moving with a speed of 4.0 ms⁻¹. A force of 30N is now applied to the change on the body for 2 seconds.

(i) What is the final speed of the body after 2 second?
(ii) How much work has been done during this period?
(iii) What is the initial kinetic energy?
(iv) What is the final kinetic energy?
(v) What is the distance covered during this period?
(vi) Show that the work done is equal to the change in kinetic energy?

Solution:

(i) $F = ma$

$a = F/m = 30/10 = 3ms^{-2}$

$v_2 = v_1 + at$

$= 4 + (3 \times 2) = 10ms^{-1}$

(ii) the distance covered in 2 seconds

$s = ut + \frac{1}{2}at^2$

$= (4 \times 2) + \frac{1}{2}(3 \times 4)$

$= 8 + 6 = 14m.$

$W = F \times S = 30 \times 14 = 420J.$

(iii) the initial kinetic energy

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(10 \times 16) = 80J$

(IV) The final kinetic energy

$$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(10 \times 100) = 500J.$$

(V) the distance covered as calculated above $= 14m$.

(VI) the change in kinetic energy is

$$K_2 - K_1 = (500 - 80) = 420J.$$

3.3 POTENTIAL ENERGY.

Definition 3.3.1 the energy possessed by an object due to their position in space is known as potential energy. Examples is the Gravitational potential energy possessed by a body in Gravitational field. The potential energy is described as

$$W = mgh.$$

Example 3.3.1 A truck is loaded with sugar bags. the total mass of the load and the truck together is 100,000kg. The truck moves on a winding path up a mountain to a height of 700m in 1hr. What average power must the engine produce to lift the material?

solution

$$W = mgh$$
$$= (100,000 kg) \times (9.8ms^{-2} \times 700m)$$
$$= 68.6 \times 10^7 J.$$

$$\text{Time taken} = 1hr = 60 \times 60s$$
$$= 3600s$$

$$\text{Average Power} = W/t = \frac{68.6 \times 10^7 J}{3600 s}$$

$$= 1.91 \times 10^5 W$$

Since $746 W = 1 hp$, then

$$P = \frac{1.91 \times 10^5}{746} = 2.56 \times 10^2 = 265 hp.$$

Example 3.3.2 Hydroelectric power generation uses falling water as a source of energy to turn turbine blades and generates electrical power. In a power station, $1000 \times 10^3 Kg$ water falls through a height of 51 m in one second.

(i) Calculate the work done by the falling water

(ii) How much power can be generated under ideal condition?

Solution:

$$P.E = mgh$$

$$= (1000 \times 10^3 kg) \times (9.8 m s^{-2}) \times (51 m)$$

$$= 9.8 \times 51 \times 10^6 J$$

$$= 500 \times 10^6 J.$$

Since

$$W = F \times S = mg \times h$$

$$= 1000 \times 10^3 \times 9.8 \times 51 J$$

$$= 500 \times 10^6$$

$$= 500 MJ$$

(ii) $P = W/t = \frac{500\,MJ}{1s}$

$= 500\,MW$

3.4 Potential Energy of springs

Recall that, an external force is required to compress or stretch a given spring, then

$$W = \frac{1}{2}Kx^2$$

This work is stored in the spring as elastic potential energy.

3.5 Conservation of Energy

The law of conservation of Energy states that the total energy of an isolated system always remains constant.

3.6 Elastic and Inelastic Collisions

When two bodies interact, it is termed as collision. there are no external forces acting on the system.

the collisions are of two kinds

(i) perfectly Elastic Collision:- If the forces of interaction between two bodies are conservative, the total kinetic energy is conserved. The total kinetic energy before collision is same as that after the collision.

(ii) Perfectly Inelastic Collision:- When two bodies stick together after the collision and move as as one single unit, it is termed as perfectly Inelastic Collision.

By applying the laws of conservation of momentum and kinetic energy, we have

$$m_A V_{Ai} + m_A V_{Bi} = m_A V_{Af} + m_B V_{Bf} \quad \text{--- (1)}$$

$$\frac{1}{2} m_A V_{Ai}^2 + \frac{1}{2} m_{Ai} V_{Bi}^2 = \frac{1}{2} m_A V_{Af}^2 + \frac{1}{2} m_B V_{Bf}^2 \quad \text{--- (2)}$$

Also

$$(V_{Bf} - V_{Af}) = -(V_{Bi} - V_{Ai}) \quad \text{--- (3)}$$

$$V_{Af} = \frac{2m_B V_{Bi}}{m_A + m_B} + \frac{V_{Ai}(m_A - m_B)}{m_A + m_B} \quad \text{--- (4)}$$

$$V_{Bf} = \frac{-2m_A V_{Ai}}{m_A + m_B} + \frac{(m_B - m_A)V_{Bi}}{(m_A + m_B)} \quad \text{--- (5)}$$

Remark (1) Suppose that the two balls colliding with each other are identical i.e $m_A = m_B = m$. then equation (4) & (5) result to

$$V_{Af} = V_{Bi} \quad \text{--- (6)}$$

$$V_{Bf} = V_{Ai} \quad \text{--- (7)}$$

(11) Suppose the collision of two particles of the unequal masses

then

$m_B >> m_A$ and $v_{Bi} = 0$

eqn from (4) and (5) reduced to

$v_{Af} = - v_{Ai}$

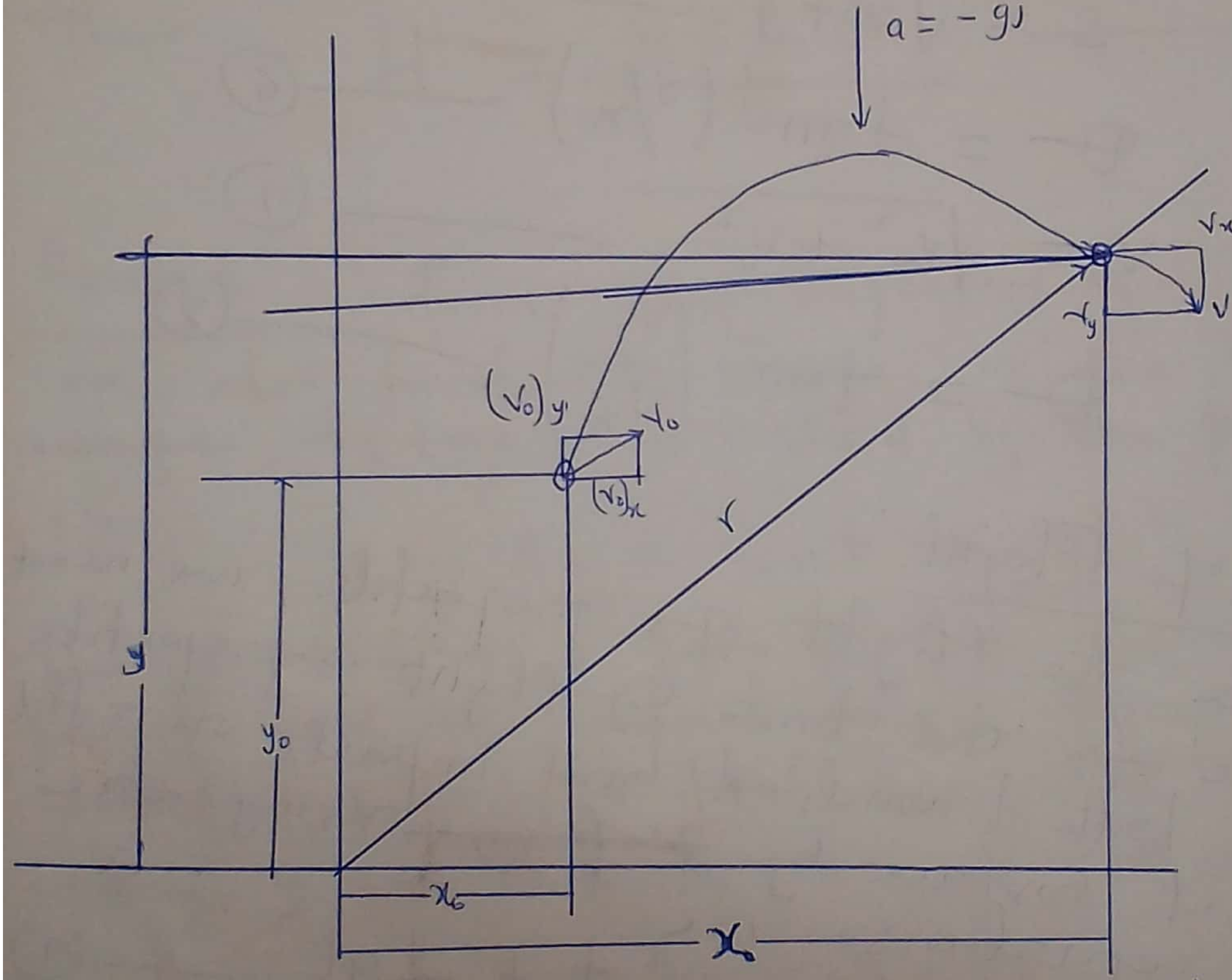
$v_{Bf} = 0$

PROJECTILE MOTION

Definition 3.1.1 Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration

KINEMATIC EQUATIONS:

Consider the diagram below



We have $a_x = 0$, the velocity in the horizontal direction remains constant $(V_x = V_{0x})$ and the position in the x direction can be determined by:

$$x = x_0 + V_{0x}(t) \quad \text{—— (1)}$$

why is a_x equal to zero (assuming movement through the air)?

Since the positive y-axis is directed upward, $a_y = -g$. Application of the constant acceleration equation yields

$$V_y = V_{0y} - g(t) \quad \text{——— (2)}$$

$$y = y_0 + V_{0y}(t) - \frac{1}{2}g(t)^2 \quad \text{——— (3)}$$

$$V_y^2 = V_{0y}^2 - 2g(y - y_0) \quad \text{——— (4)}$$

$$S = \sqrt{x^2 + y^2} \quad \text{——— (5)}$$

$$\theta = \tan^{-1}(y/x) \quad \text{——— (6)}$$

$$V = \sqrt{V_x^2 + V_y^2} \quad \text{——— (7)}$$

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) \quad \text{——— (8)}$$

Time of Flight

While the flight of a projectile, we need ~~to~~ solve for the time of flight of projectile that is both launched and impacts on a flat horizontal surface by ~~performing~~ using the kinetic equations. i.e

$$y - y_0 = V_{0y}t + \frac{1}{2}gt^2 \quad \text{——— (9)}$$

Solving for t, we have

$$T = \frac{2V_0 \sin\theta}{g} \quad \text{——— (10)}$$

Note that $V_x = V_0 \cos\theta$ $\quad x = (V_0 \cos\theta)t$

$V_y = V_0 \sin\theta$ $\quad y = (V_0 \sin\theta)t$

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Maximum height: the maximum height of trajectory of a projectile can be found from the kinematic equations. i.e

$$v^2 = v_0^2 + 2a\Delta x$$

so that if $v_y = 0$, then

$$0 = v_0^2 + 2a\Delta x$$

then

$$H = \frac{v^2 \sin^2\theta}{2g} \quad \text{———} \quad (11)$$

Range: From the trajectory equation, we can also find the range, or the horizontal ~~range~~ distance travelled by the projectile.

i.e

$$x - x_0 = v_{0x}t + \frac{1}{2}gt^2$$

the position y is zero for both the launch point and the impact point since we are considering only a flat horizontal surface. setting $y = 0$, then $x = 0$, so that

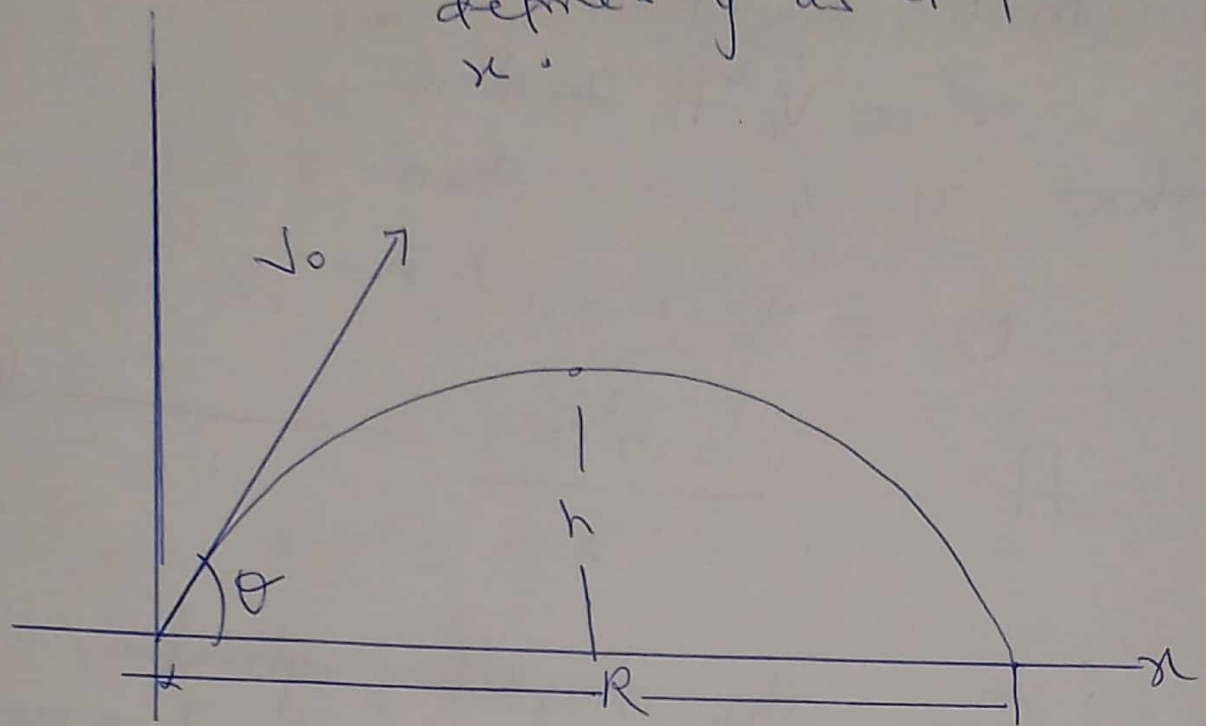
$$x - x_0 = v_{0x}t$$

Using the identity $2\sin\theta\cos\theta = \sin 2\theta$

such that $x = R$, then

$$R = \frac{v_0 \sin 2\theta}{2g} \quad \text{———} \quad (12)$$

Example 3.2 Given V_0 and θ from the diagram below: Find the equation that defines y as a function of x.



Solution: Using $V_x = V_0 \cos\theta$ and $V_y = V_0 \sin\theta$

we have $x = (v_0 \cos\theta)t$. or $t = \dfrac{x}{v_0 \cos\theta}$

$$y = (v_0 \sin\theta)t - \frac{1}{2}g(t)^2$$

Substituting for t

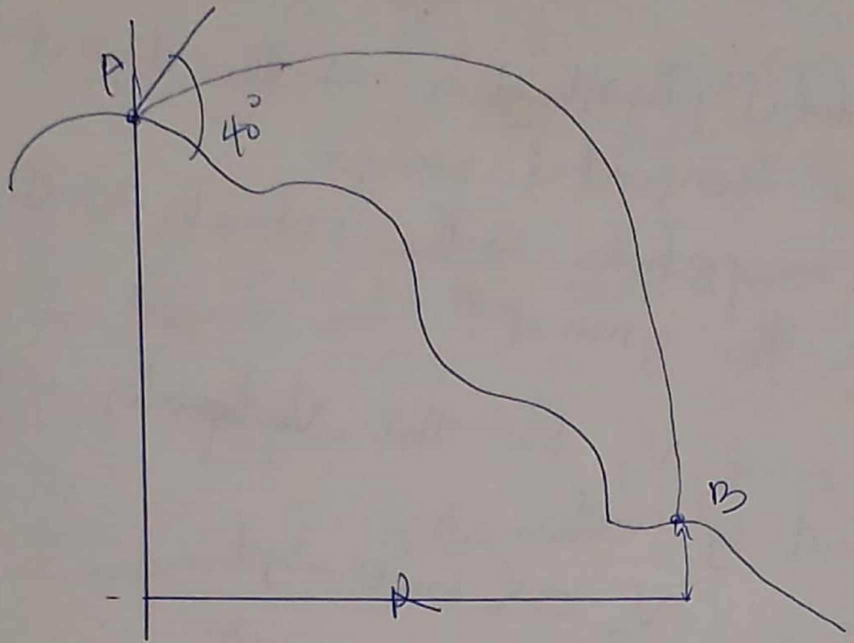
$$y = (v_0 \sin\theta)\left(\frac{x}{v_0\cos\theta}\right) - \frac{g}{2}\left(\frac{x}{v_0\cos\theta}\right)^2$$

By simplification,

$$y = x\tan\theta - \left(\frac{gx^2}{2v_0^2}\right)(1+\tan^2\theta) \qquad (13)$$

The above equation (13) is called the "path equation" which describes the path of a particle in projectile motion.

Example 3.3 The below diagram shows a snowmobile going 15m/s at point A. Find the horizontal distance (R) it travels and the time in the air.



Solution from the equation for the horizontal motion

$$X_B = X_A + V_{Ax} t_{AB}, \quad V_{Ax} = V_{ox} \cos\theta \text{ m/s}$$
$$= 15 \cos 40° \quad \text{——— (1)}$$

~~horizontal motion~~.

Vertical motion equation:

$$y_B = y_A + V_{Ay} t_{AB} - \frac{1}{2} g t_{AB}^2 \quad \text{(3)} \qquad V_{Ay} = 15 \sin 40° \text{ m/s} \quad \text{(2)}$$

Note that

$$X_B = R, \quad x_A = 0, \quad y_B = \left(-3/4\right) R. \text{—— (4)}$$

and $y_A = 0$. Solving (3) & (4), we have

$R = 42.8$m and $t_{AB} = 3.72$s

Example 3.4 An object is launched at a velocity of 20m/s in a direction making an angle of 25° upward with the horizontal.

(i) What is the maximum height reached by the object?.

(ii) What is the total flight time of the object?

(iii) What is the horizontal range

(iv) What is the magnitude of the velocity of the object just before it hits the ground?

Soln Let V_x and V_y be the velocity with respect to x and y. then

$$V_x = V_0 \cos\theta, \quad V_y = V_0 \sin\theta - gt.$$

$$V_0 = 20m/s, \quad \theta = 25°, \quad g = 9.8ms^{-2}.$$

so that $V_y = V_0 \sin\theta - gt = 0.$

$$t = \frac{V_0 \sin\theta}{g} = \frac{20 \sin 25°}{9.8} = 0.86 \text{ secs}$$

To find the maximum height.

$$S = V\sin\theta t - \frac{1}{2}gt^2.$$

$$= 20\sin 25°(0.86) - \frac{1}{2} 9.8 (0.86)^2 = 3.64m$$

(ii) At $t = t_1$ and $t = t_2$, $y = 0$. Hence

$$V_0 \sin\theta t - \frac{1}{2}gt^2 = 0$$

$$t = t_1 = 0 \quad \text{and} \quad t = t_2 = \frac{2V_0 \sin\theta}{g}$$

$$\text{Time of flight} = t_2 - t_1 = \frac{2(20)\sin\theta}{g}$$

$$= \frac{2 \times 20 \sin 25°}{9.8} = 1.72 \text{ secs}$$

(iii) The horizontal Range is

$$R = \frac{2V_0 \cos\theta V_0 \sin\theta}{g} = \frac{V_0^2 \sin 2\theta}{g}$$

$$= \frac{20^2 \times \sin 2(25)}{9.8} = \underline{\underline{31.26m}}$$

(iv) The object hits the ground at $t = t_2 = \frac{2V_0 \sin\theta}{g}$.

so that $V_x = V_0 \cos\theta$, $V_y = V_0 \sin\theta - gt$.

The component of the velocity at $t = \frac{2V_0 \sin\theta}{g}$

are

$$V_x = V_0 \cos\theta = 20\cos 25^\circ$$

$$V_y = V_0 \sin 25^\circ - g\left(\frac{2 \cdot V_0 \sin 25^\circ}{g}\right) = -V_0 \sin 25^\circ$$

The magnitude V of the velocity is given by

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(20\cos 25^\circ)^2 + (-V_0 \sin 25^\circ)^2}$$

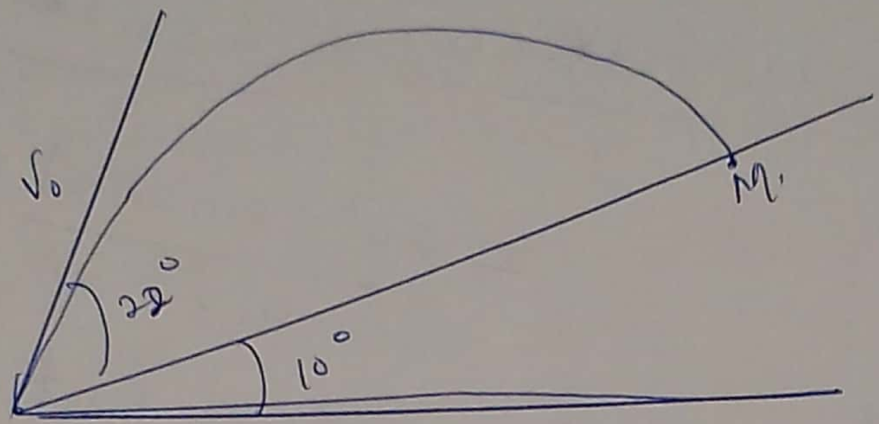
$$= V_0 = 20m/s$$

__Example 3-5__ A projectile is lanched from the point O at angle of 22° with an initial velocity of 15m/s up an inclined plane that makes an angle of 10° with the horizontal. The projectile hits the incline plane at point M.

(i) Find the time it takes for the projectile to hit the incline plane

(ii) Find the distance OM.

Solution



the x and y component of the displacement are given by

$$x = V_0 \cos\theta \, t \qquad y = V_0 \sin\theta \, t - \frac{1}{2} g t^2 \quad (1)$$

with $\theta = 22^\circ + 10^\circ = 32^\circ$, $V_0 = 15$ m/s.

the relationship between the coordinate x and y on the incline is given by.

$$\tan 10^\circ = \frac{y}{x} \quad (2)$$

Substitute x and y by the expression into (2)

$$\tan 10^\circ = \frac{\left(V_0 \sin\theta - \frac{1}{2} g t^2\right)}{V \cos\theta \, t}$$

By simplification to obtain t.

$$\frac{1}{2} g t + V_0 \cos\theta \tan 10^\circ - V_0 \sin\theta = 0$$

Solve for t.

$$t = \frac{V_0 \sin\theta - V_0 \cos\theta \tan 10^\circ}{(0.2)\, g} = \frac{15 \sin 32^\circ - 15 \cos 32^\circ \tan 10^\circ}{0.5 \times 9.8}$$

$$= 1.16 \text{ s.}$$

(ii)

$$OM = \sqrt{(v_0 \cos \theta t)^2 + (v_0 \sin \theta t - \frac{1}{2} g t^2)^2}$$

~~at~~ $t = 1.16$

$$OM = \sqrt{15 \cos 32 (1.16)^2 + (15 \sin 32 (1.16) - \frac{1}{2} 9.8 (1.16)^2)^2}$$

$$= \underline{\underline{15m}}$$

__Example 3.6__ Two balls A and B of masses 100grams and 300grams respectively are pushed horizontally from a table of height 3 metres. Ball has a pushed so that its initial velocity is 10m/s and ball B is pushed so that its initial velocity is 15m/s. Find (i) the time it takes each ball to hit the ground.

(ii) What is the difference in the distance between the points of impact of the two balls on the ground?.

__Sol__ the two balls are subject to the same gravitational acceleration and therefore will hit the ground at the same time t :.

By equation $-3 = -\frac{1}{2} g t^2$

$$t = \sqrt{\frac{3(2)}{9.8}} = 0.785.$$

(i) Horizontal distance X_A ~~of~~ ball A

$X_A = 10m/s \cdot 0.785 = 7.8m$

$X_B = 15 m/s \times 0.78s = 11.7m$

Difference in distance X_A & X_B

$|X_B - X_A| = |11.7 - 7.8| = \underline{\underline{3.9m}}$

4.0 Simple Harmonic Motion

Definition 4.1.1 Simple Harmonic Motion or SHM is defined as a motion in which the restoring force is directly proportional to the displacement of the body from its mean position. The direction of this restoring force is always towards the mean position. The acceleration of a particle executing simple harmonic motion is given by $a(t) = \omega^2 x(t)$ where ω is the angular velocity of the particle.

All the Simple Harmonic Motions are oscillatory and also periodic, but not all oscillatory motions are SHM. The study of Simple Harmonic Motion is very useful and forms an important tool in understanding the characteristic of sound waves, light waves and alternating current.

4.1.2 Types of Simple Harmonic Motion

- Linear SHM
- Angular SHM.

4.1.3 Linear Simple Harmonic Motion

When a particle moves to and fro about a fixed point (called equilibrium position) along with a straight line then its motion is called linear Simple Harmonic Motion. Example Spring-mass system.

$$\vec{F} \propto -\vec{x}$$

$$\vec{a} \propto -\vec{x}$$

$\vec{x}$ = displacement of particle from equilibrium

$\vec{F}$ = Restoring force

$\vec{a}$ = acceleration

4.1.4 Angular Simple Harmonic Motion

When a system oscillates angular long with respect to a fixed axis, then the motion is called angular simple harmonic motion.

$$T \propto \theta$$

or

$$a \propto \theta$$

where

T = Torque

α = angular acceleration

θ = angular displacement.

4.1.5 Time period and frequency of SHM.

the minimum time after which the particle keeps on repeating its motion is known as the period or the shortest time taken to complete one oscillation

$$T = \frac{2\pi}{w}$$

where frequency $= \frac{1}{T}$ so that $w = 2\pi f = \frac{2\pi}{T}$.

4.1.6 Phase in SHM.

The phase of a vibrating particle at any instant is the state of the vibrating (oscillating) particle regarding its displacement and direction of vibration at that particular instant.

the expression is given as

$$x = A \sin(wt + \phi)$$

where $(wt + \phi)$ is the phase of the particle, the phase at time $t = 0$ is known as the initial phase

4.1.7 Phase Difference

The ~~difference~~ difference of ~~total~~ phase angles of two particles executing simple harmonic motion with respect to the mean position is known as the phase difference.

Two vibrating particles are said to be in the same phase, the phase difference between them is an even multiple of π.

$\nabla \phi = n\pi$ where $n = 0, 1, 2, 3 \ldots$

Two vibrating particles are said to be in the opposite phases if the phase difference between them is an odd multiple of π

$\nabla \phi = (2n+1)\pi$ where $n = 0, 1, 2, 3 \ldots$

Recall that $\vec{F} = -K\vec{x}$ where $-K$ is a +ve constant.

$\Rightarrow \vec{F} = m\vec{a}$, where $\vec{a}$ is the acceleration

~~Hence~~,

$\Rightarrow m\vec{a} = -K\vec{x}$

$\Rightarrow \vec{a} = -\left(\frac{K}{m}\right)\vec{x}$

put $\frac{K}{m} = w^2$

$\Rightarrow w = \sqrt{\frac{K}{m}}$

$\Rightarrow \vec{a} = -\left(\frac{K}{m}\right)\vec{m} = -w^2\vec{x}$

Since $\vec{a} = \frac{d^2x}{dt^2}$

therefore $\frac{d^2x}{dt} = -w^2\vec{x}$.

So $\frac{d^2x}{dt^2} + w^2 x = 0$, which is the differential equation for linear simple harmonic motion.

Also

Recall that $T \propto -\theta$

$$T = -K\theta$$

$$T = Ia$$

where $a = -K\theta$

$$I \frac{d^2\theta}{dt^2} = -K\theta$$

$$\frac{d^2\theta}{dt^2} = \left(\frac{K}{I}\right)\theta = -w^2\theta$$

$$\frac{d^2\theta}{dt^2} = -w^2\theta = 0$$ is the differential equation of an angular simple Harmonic Motion.

Conditions for Simple Harmonic Motion

Since $\vec{F} \propto -\vec{x}$

$$\vec{a} \propto -\vec{x}$$

Then $\vec{a} = -w^2 x$

$$\vec{a} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

$$\vec{a} = v\frac{dv}{dx} = -w^2 x.$$

$$\int_0^v v\,dv = \int_0^x -w^2 x\,dx$$

$$\frac{v^2}{2} = \frac{-w^2x^2}{2} + c \quad \text{———} \ (1)$$

At the point $v=0$, $[x=A)$ equation (1) becomes

$$\frac{v^2}{2} = \frac{-w^2A^2}{2} + c$$

$v=0$,

$$0 = \frac{-w^2A^2}{2} + c$$

$$c = \frac{w^2A^2}{2}$$

Substituting c into equation (1)

$$\frac{v^2}{2} = -\frac{w^2x^2}{2} + \frac{w^2A^2}{2}$$

$$\Rightarrow \quad v^2 = -w^2x^2 + w^2A^2$$

$$\Rightarrow \quad v^2 = w^2(A^2 - x^2)$$

$$v = \sqrt{w^2(A^2 - x^2)}$$

$$v = w\sqrt{A^2 - x^2} \quad \text{———} \ (2)$$

where v is the velocity of the particle executing simple harmonic motion from the definition of instantaneous velocity

$$v = \frac{dx}{dt} = w\sqrt{A^2 - x^2}$$

$$\Rightarrow \int \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t w\,dt$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{A}\right) = wt + \phi$$

so that

$$x = A\sin(wt + \phi) \quad \text{———— (3)}$$

Equation (3) is the equation of position of a particle as a function of time.

4.2 Energy ~~in~~ Simple Harmonic Motion (SHM)

the system that executes SHM is called the harmonic oscillator.

Consider a particle of mass m, executing linear simple harmonic motion of angular frequency w and amplitude (A). the displacement $(\vec{x})$, velocity $(\vec{v})$ and acceleration $(\vec{a})$ at any time t are given by:

$$x = A\sin(wt + \phi) \quad [\text{by differentiation}]$$

$$v = Aw\cos(wt+\phi) = w\sqrt{A^2 - x^2}$$

$$a = -w^2 A\sin(wt + \phi) = -w^2 x^2$$

The restoring force $(\vec{F})$ acting on the particle are given by

$$F = -Kx, \quad \text{where } K = mw^2.$$

4.2.1 Kinetic Energy of a Particle in SHM.

Given that

$$K = \frac{1}{2}mv^2, \quad \text{Since } v^2 = A^2 w^2 \cos^2(wt+\phi)$$

$$K = \frac{1}{2} m w^2 A^2 \cos^2 (wt + \phi)$$
$$= \frac{1}{2} m w^2 (A^2 - x^2)$$

therefore $K = \frac{1}{2} m w^2 A^2 \cos^2 (wt + \phi) = \frac{1}{2} m w^2 (A^2 - x^2)$

4.2.2 Potential Energy of SHM.
the total work done by the restoring force in displacing the particle from $x = 0$ to $x = x$,
When the particle has been displaced from x to $x + dx$, the work done by restoring force

$$dw = F dx = -Kx\, dx$$

$$w = \int dw = \int_0^x -Kx\, dx = -\frac{Kx^2}{2}$$

$$= -\frac{m w^2 x^2}{2} \quad \left[K = m w^2 \right]$$

$$w = -\frac{m w^2 A^2}{2} \sin^2 (wt + \phi) \text{ . is the}$$

potential Energy

$$\text{ie } \frac{m w^2 x^2}{2} = \frac{m w^2 A^2}{2} \sin^2 (wt + \phi)$$

<u>4.2.3</u> Total Mechanical Energy of particle Execution SHM.

$$E = K.E + P.E$$

$$= \frac{1}{2} m w^2 (A^2 - x^2) + \frac{1}{2} m w^2 x^2$$

$$7 \quad E = \frac{1}{2} m w^2 A^2$$

Example 1 Consider a particle undergoing simple harmonic motion. the velocity of the particle at position x_1 is v_1 and velocity of the particle at position x_2 is V_2. Show that the ratio of time period and amplitude is

$$\frac{T}{A} = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - V_2^2 x_1^2}}$$

Solution

Using the equation

$$v = w\sqrt{A^2 - x^2} \Rightarrow v^2 = w^2(A^2 - x^2)$$

then

$$v_1^2 = w^2(A^2 - x_1^2) \quad (1)$$

and

$$v_2^2 = w^2(A^2 - x_2^2) \quad (2)$$

Subtracting (2) from (1), then

$$v_1^2 - v_2^2 = w^2(A^2 - x_1^2) - w^2(A^2 - x_2^2)$$

$$= w^2(x_2^2 - x_1^2)$$

$$w = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \Rightarrow T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}} \quad (3)$$

Dividing (1) and (2), then

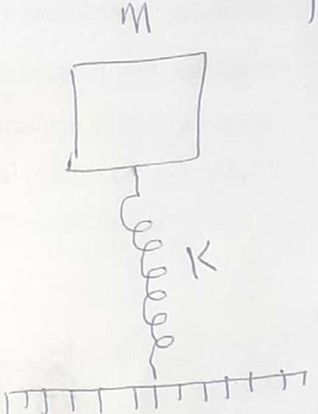
$$\frac{v_1^2}{v_2^2} = \frac{w^2(A^2 - x_1^2)}{w^2(A^2 - x_2^2)} \Rightarrow A = \sqrt{\frac{v_1^2 x_2^2 - V_2^2 x_1^2}{v_1^2 - V_2^2}} \quad (4)$$

Dividing (3) & (4), then

x

Example 1 A tray of mass 9kg is supported by a spring of force constant K as shown below. The tray is pressed slightly downward and then released. It seems to execute SHM of period 1.0s. When a block of mass M is placed on the tray, the period increase to 2.0s. Calculate the mass of the block.

Solution Since $w = \sqrt{\frac{K}{m}}$, and $w = \frac{2\pi}{T}$



then

$$\frac{4\pi^2}{T^2} = \frac{K}{m}$$

$$m = \frac{KT^2}{4\pi^2}$$

When the tray is empty $m = 9kg$ and $T = 1s$.

then

$$9 = \frac{K(1)^2}{4\pi^2}$$

and $T = 2s$. therefore

Since $m = 9 + M$

$$9 + M = K \times (2)^2 / 4\pi^2$$

$$\Rightarrow \frac{9+M}{9} = 4$$

therefore $M = 27kg$.

Example 2 A spring of force constant $1600Nm^{-1}$ is mounted on a horizontal table as shown below. A mass $m = 4.0kg$ attached to the free end of the spring is pulled horizontally

towards the right through a distance of 4.0 cm and then set free. Calculate
(i) frequency (ii) maximum acceleration and
(iii) maximum speed of the mass.

Solution $w = \sqrt{\frac{k}{m}} = \sqrt{\frac{1600}{4}} = 20 \, rad s^{-1}$

therefore $v = \frac{20}{2\pi} = 3.18 \, Hz.$

Maximum acceleration $= aw^2 = 0.04 \times 400 = 16 \, ms^{-2}$

and $V_{max} = aw = 0.04 \times 20 = 0.8 \, ms^{-1}.$

Moment of a Force

The moment of a force F about a point A is equal to $\pm Fd$ where

F is the magnitude of the force

d is the perpendicular distance from A to the line of action of the force.

The units of moment are newton metres (Nm)

Note: If the force is the same F, with the same line of action, but applied at a different point O, the moment about A is unchanged.

Note that + sign for rotation anticlockwise
− sign for rotation clockwise.

Example 1 Find the moment of force $F = 6i - 3j$ applied at (4, 5) about the point (1, 2).

Solution $(x, y) = (4, 5)$

$(x, \gamma) = (1, 2)$.

$F = (6, -3)$.

$$\text{moment} = -6(5-2) + (-3)(4-1)$$

$$= -18 - 9 = -27 Nm \text{ (clockwise)}$$

Example 2 Find the moment of a force of 6 newtons, applied in the direction of $i+2j$ at point $(4, 2)$ about $(1, 1)$.

Solution F is parallel to $i+2j$

so that $i + 2j$ is $\sqrt{1^2 + 2^2} = \sqrt{5}$.

unit vector in the direction of the force $\frac{1}{\sqrt{5}}(i+2j)$

F ~~has magnitude~~ $6 = \frac{6}{\sqrt{5}}(i+2j) = F$

Hence, $F_1 = \frac{6}{\sqrt{5}}$ and $F_2 = \frac{12}{\sqrt{5}}$.

For $P(x, y) = (4, 2)$ and $A = (x, \gamma) = (1, 1)$

Then $$\text{moment} = -\frac{6}{\sqrt{5}}(2-1) + \frac{12}{\sqrt{5}}(4-1)$$

$$= \frac{30}{\sqrt{5}} = 6\sqrt{5} \text{ Nm}$$

Resultants of forces on bodies of non-zero size

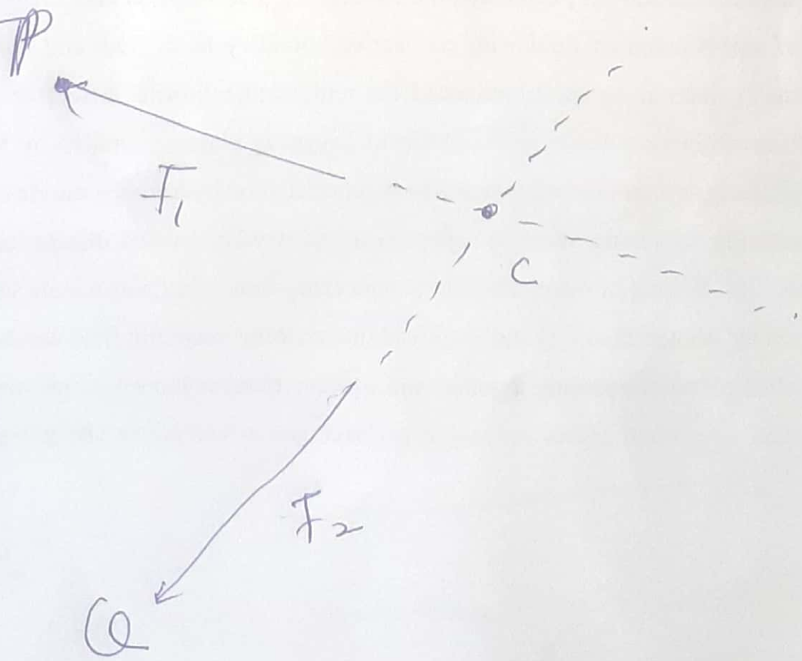
If different forces act at different points. Then the total moment about any point P is the algebraic sum of each moment about P.

Suppose we have two forces F_1 and F_2. The resultant force H is the force which has the same effect as F_1 and F_2 combined. Clearly the vector R is the vector sum of F_1 and F_2 so:

$$R = F_1 + F_2$$

Case 1: F_1 and F_2 are not parallel

F_1 is applied at P and F_2 at Q as shown below:



The moment of F_1 is the same if it is applied at any point on the line of action.

likewise, the moment F_2 is the same if it is applied at any point on the line of action. therefore it is the same as if it is applied at C, where the lines of actions cross.

Hence, provided the line of action of R, passes through C, we get the correct total moment.

Example) Calculate the resultant and its line of action for the two forces.

$\vec{F_1} = 2i + j$ N applied at point $(1,0)$ and

$\vec{F_2} = -3i + 3j$ N applied at point $(0,0)$

Sol $R = F_1 + F_2 = (2,1) + (-3,3) = (-1,4)$

Since vector $F_1 = (2,1)$, the gradient of its line of action is $\frac{1}{2}$.

the equation of the line of action is

$y = \frac{1}{2}x + c_1$ since it passes through $(1,0)$

$0 = \frac{1}{2} + c_1 \Rightarrow c_1 = -\frac{1}{2}$

$\Rightarrow y = \frac{1}{2}x - \frac{1}{2}$ ——— (1)

vector $F_2 = (-3,3)$, so the gradient of its line of action is $\frac{3}{-3} = -1$. the equation

The equation of the line of action is

$y = (-1)x + c_2$ since it passes through $(0,0)$

$0 = 0 + c_2$

$\Rightarrow c_2 = 0$

$y = -x$ ——— (2)

Solving (1) and (2) simultaneously

$-x = \frac{1}{2}x - \frac{1}{2}$

$-2x = x - 1$

$x = \frac{1}{3}, \quad y = -\frac{1}{3}$

Vector $R = (-1, 4)$, so the gradient of its line of action is $\frac{4}{-1} = -4$.

~~The equation of~~

The equation of its line of action is

$y = -4x + c_3$ so that

$-\frac{1}{3} = -4\frac{1}{3} + c_3 \Rightarrow c_3 = 1$

which gives

$y = -4x + 1$

Case 2 If the forces F_1 and F_2 are parallel then lines of action do not cross. The vector resultant $R = F_1 + F_2$ and we can find the line of action by taking moments about a point

Example 2 $F_1 = 2i + j$ N is applied at $A(2,0)$ $F_2 = 6i + 3j$ N is applied at $O(0,0)$. So that F_2 is parallel to F_1. Find the resultant R and the point B. whose its line of action crosses the X-axis. Hence, find the equation of the line of action.

Soln

$$R = F_1 + F_2 = 8i + 4j \text{ so that}$$

$$|R| = \sqrt{80}\ N = \sqrt{8^2 + 4^2}$$

Let the line of action of R cross X-axis at $B(d, 0)$. Consider moments about O.

~~At~~ $A(2,0)$, F_1 consists of for $2i$ which has moment $2 \times 0 = 0$ Nm about O. Together with $1j$ which has moment $1 \times 2 = 2$ Nm about O. the total moment of F_1 about O is thus 2Nm

The line of action of F_2 passes through O so the moment about O is zero.

At $B(d, 0)$, R consist of force $8i$ which has moment $8 \times 0 = 0$ Nm about O, together with $4j$ which has moment $4 \times d = 4d$ Nm about O. the total moment of R about O is thus $4d$ Nm

The total moment of R must equal the total moment of F_1 and F_2 so

$$4d = 2 + 0 \Rightarrow d = 0.5$$

Thus the line of action of R passes through $(0.5, 0)$

Since $R = 8i + 4j$ its line of action must have gradient $\frac{4}{8} = \frac{1}{2}$. the equation of the line of action is thus

$$y = \frac{1}{2}x + c$$

Since it goes through $(0.5, 0)$, $0 = 0.5 \times 0.5 + c$
$\Rightarrow c = -0.25$
the equation of the line of action is thus
$y = 0.5x - 0.25$.

Couples

Given $F_2 = -F_1$, then, the line of action of F_2 is in the opposite direction to that of F_1. then we have a turning effect even though the resultant is zero.

$$R = F_1 + F_2 = 0$$

the situation, where there is a turning effect but with resultant equal to zero is called a Couple.

<u>Remark</u> the magnitude of the Couple depends on F and d only, but is independent of the distance.

Equilibrium of Coplanar forces

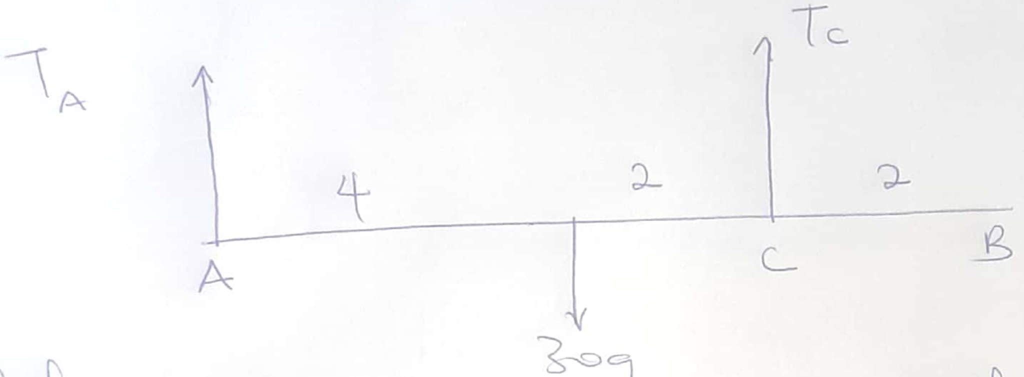
Coplanar means that the lines of actions of all the forces are in a single plane. or parallel to the plane.

A body will be in equilibrium under the action of coplanar forces if

(i) the resultant is zero

(ii) they donot reduce to a Couple

Example 1 A uniform plank, AB, is 8m long and has mass 30kg. It is supported in equilibrium in a horizontal position by two vertical inextensible ropes, as shown below. Find the tension in each rope.



Solution

Resolving into Components

$$T_A + T_C - 30g = 0 \quad \text{——— (1)}$$

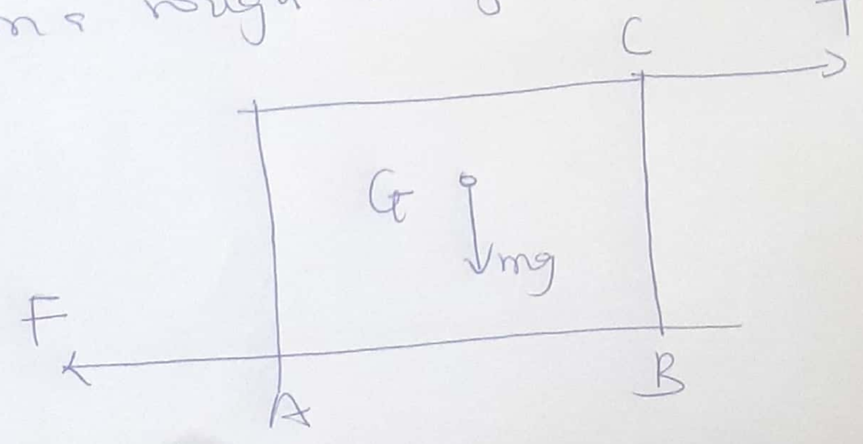
Moments about A ——— (2)

$$6T_C - 30g \times 4 = 0$$

From (2) $\Rightarrow T_C = \dfrac{30 \times 9.81 \times 4}{6} = 196.2N$

From (1) $T_A = 30g - T_C = 98.1N$

Example 2 A Cube of side 2m and mass m kg rests on a rough horizontal surface as shown below



Solution

For the cube to tilt, the total moment about B must be greater than 0.

The total moment about B is $mg \times 1 - T \times 2$

$$T = \frac{1}{2} mg \; H$$

Also, by resolving vertically

$$R - mg = 0$$

resolving horizontally $T - F = 0$

$$R = mg \quad \text{and} \quad F = T = 0.5mg$$

The maximum friction available is $F = \mu R = \mu mg$

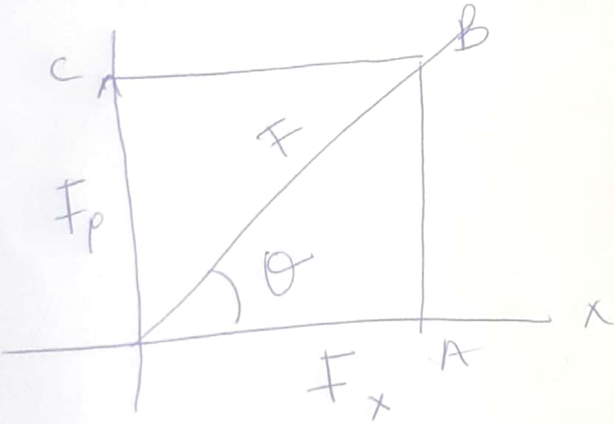
Thus, if μ is greater than 0.5, the cube will tilt first. If it is less than 0.5, it will slide first.

1.0 Resolution of Forces

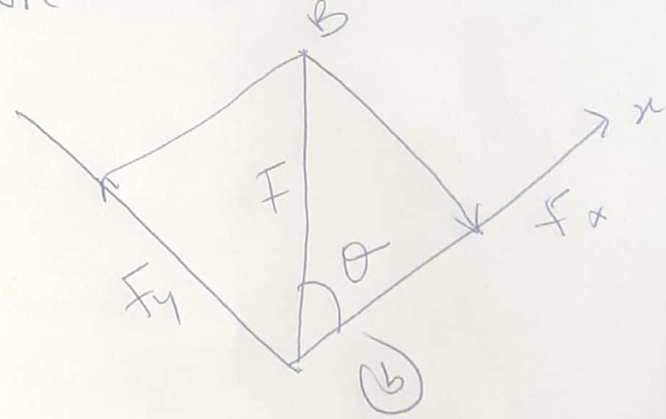
A given force F can be resolved into two forces which together produce the same effects that of force F. These forces are called the component of force F. This process is known as resolution of a force into components.

1.1 Resolution of a force into rectangular components

Consider a force F acting on a particle O inclined at an angle θ as shown below.



(a)

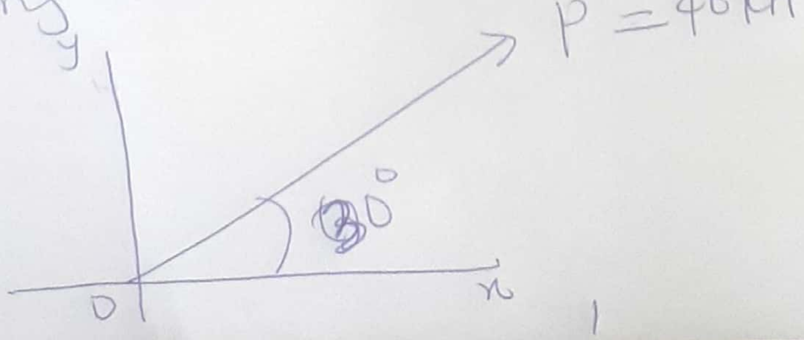


(b)

Then, the two rectangular components of the force F are

$$F_x = F\cos\theta, \quad F_y = F\sin\theta$$

Example 1.1.2 Determine the component of force $P = 40kN$ along x and y as shown below



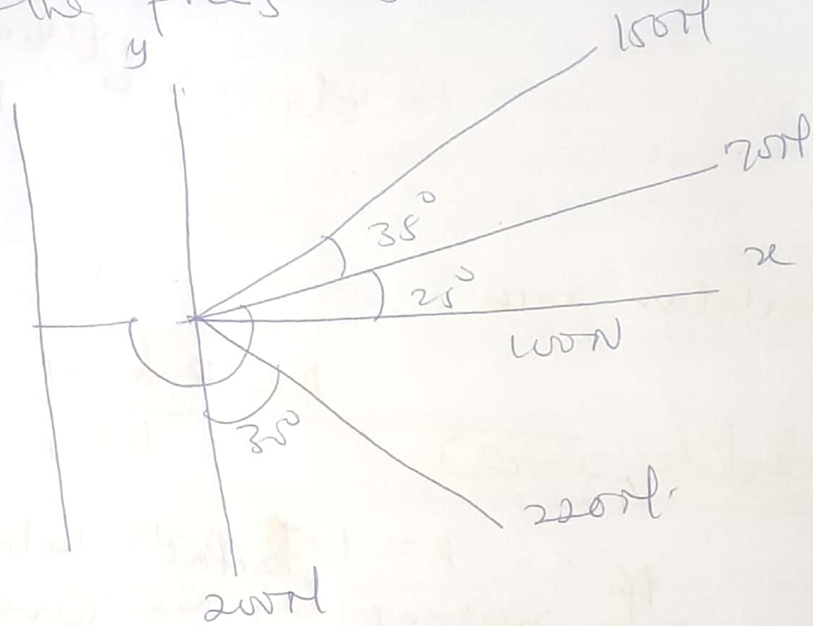
Solution

$$P_y = P\sin 30^\circ = 40\sin 30^\circ = 20KN$$

$$P_x = P\cos 30 = 40\cos 30^\circ = 34.64\ KN$$

Note that, the direction of P_x and P_y are determined based on the direction of P.

<u>Example 2</u> Determine the x and y components of each of the forces as shown below.



<u>Solution</u> $P_x = P\cos\theta$ $\qquad P_y = P\sin\theta$

100N	$100\cos 0^\circ = -100N$	$100\sin 0^\circ = 0$
70N	$70\cos 25^\circ = 63.44$	$70\sin 25^\circ = 29.58N$
150N	$150\cos 60^\circ = 75N$	$150\sin 60^\circ = 129.90N$
220N	$220\cos 60^\circ = 110N$	$220\sin 60^\circ = 190.53N$
200N	$200\cos 90^\circ = 0$	$200\sin 90^\circ = 200N$

1.2 Resolution of force into Inclined components

It is essential to know the components of a force which are not perpendicular to one another. Such components are known as inclined component or non-rectangular component.

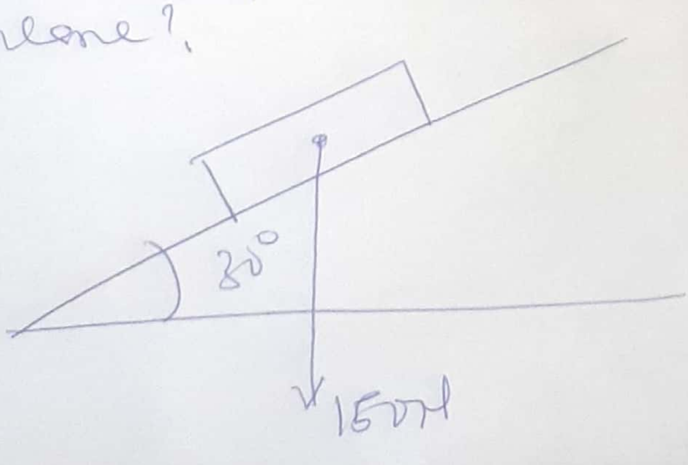
1.2.1 Triangular Law of forces

If two forces P and Q are acting on a particle A, then the two forces can be added or combined to form a single force F.

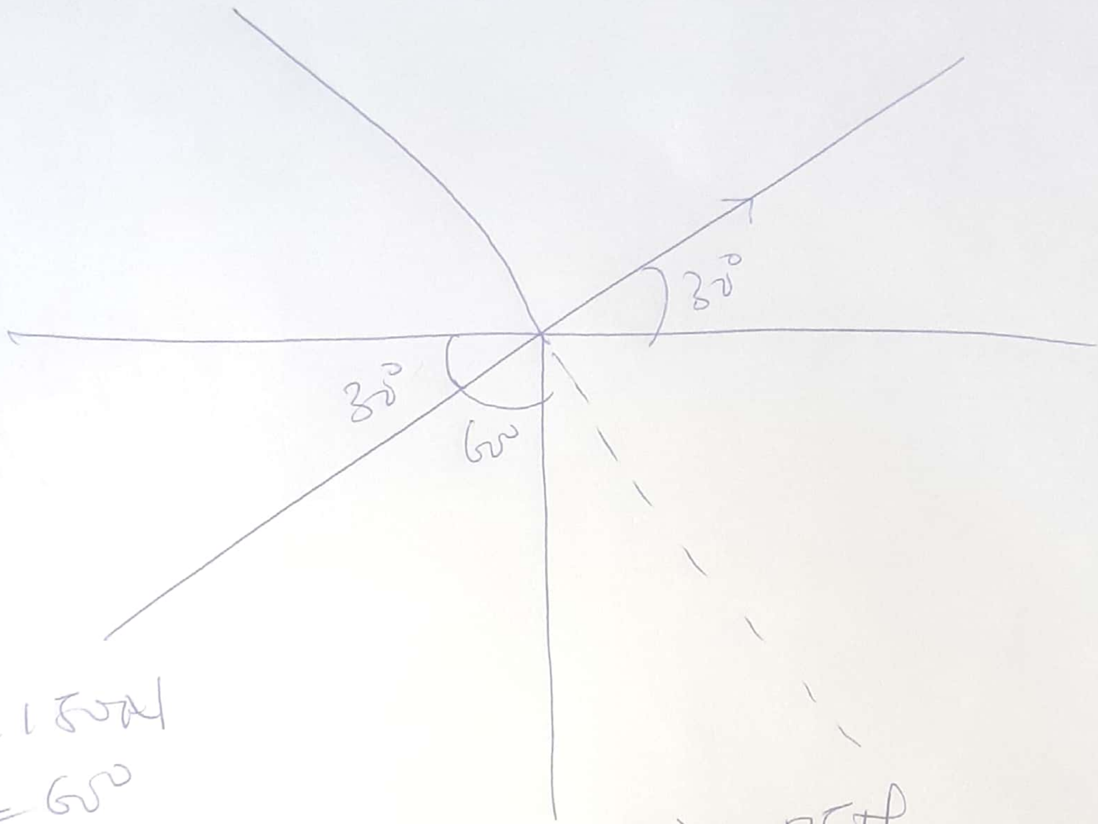
1.2.2 Law of Parallelogram of forces

This law states that two forces acting on a particle may be replaced by a single force obtained by drawing the diagonal of a parallelogram whose two adjacent sides are equal to the given two forces.

Example 1.2.3 A small block of weight 150N is placed on an inclined plane which makes an angle $\theta = 30^\circ$ with the horizontal. What is the component of this weight parallel to the inclined plane and perpendicular to the inclined plane?



Solution By resolution, we have the other diagram below -



$F = 150N$

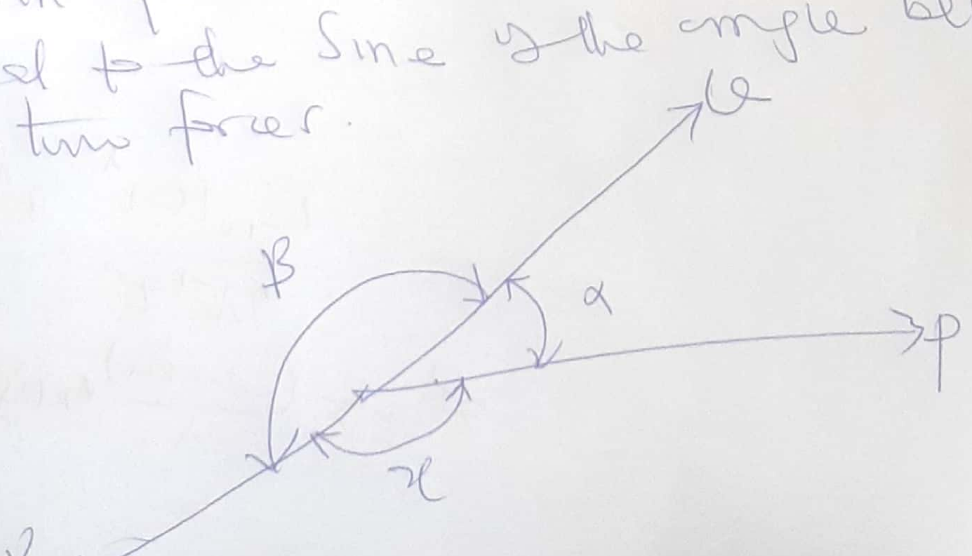
$\theta = 60^\circ$

$F_x = F\cos\theta = 150\cos 60^\circ = 75N$

$F_y = F\sin\theta = 150\sin 60^\circ = 129.90N$

1.2.4 Lami's theorem

It states that "If three forces acting at a point are in equilibrium each force will be proportional to the sine of the angle between the other two forces.



1.0 Linear Momentum and Collisions

<u>Defn</u> 1.1 The linear momentum p of an object is equal to the product of the mass m and velocity v.

$$\vec{p} = m\vec{v} \qquad \text{——— (1)}$$

or in Component form: $\vec{p} = p_x\hat{x} + p_y\hat{y} = mv_x\hat{x} + mv_y\hat{y}$.

By Newton's second law, which can be written as $\vec{F} = \frac{\text{change in momentum}}{\text{time interval}} = \frac{\Delta\vec{p}}{\Delta t}$

then

$$v_f = v_i + at = v_i + a\Delta t$$

$$a\Delta t = v_f - v_i = \Delta v$$

$$a = \frac{\Delta v}{\Delta t}$$

So that

$$\Delta p = m\Delta v \Rightarrow \Delta v = \frac{\Delta p}{m}$$

Since $a = \frac{\Delta v}{\Delta t} = \frac{\Delta p/m}{\Delta t} = \frac{\Delta p}{m\Delta t}$

So that $\frac{\Delta P}{\Delta t} = ma = F$. ——— (2)

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We have

$$\vec{F}\Delta t = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i) \quad \text{--- (B)}$$

where $\vec{F}\Delta t$ is called the impulse of the force, with $\vec{F}$ as the average force during a collision.

Example 1.1.2 (a) Show that the kinetic energy of a particle of mass m is related to the magnitude of the momentum p of that particle by $K.E = \frac{p^2}{2m}$

Solution

recall that $K.E = \frac{1}{2}mv^2$

since $p = mv$, then

$$v^2 = \frac{2KE}{m}, \quad v^2 = \frac{p^2}{m^2} \text{ from } v = \frac{p}{m}$$

$$\frac{2KE}{m} = \frac{p^2}{m^2}$$

$$K.E = \frac{mp^2}{2m^2} = \frac{p^2}{2m}$$

(5) Suppose an object is moving so that K.E = 150J and the absolute value of its momentum is 30. Kg m/s. What is the mass of the object and at what velocity is it travelling?

Solution

Since $2K \cdot E = mv^2$, $p = mv$

$$m = \frac{2KE}{v^2}, \quad m = \frac{p}{v}$$

by equating $m \Rightarrow \frac{p}{v} = \frac{2KE}{v^2}$

$$v = \frac{2KE}{p} = \frac{2(150J)}{30.0 kg\,m/s}$$

$$v = \frac{300.\,kg\,m^2/s^2}{30.0\,kg\,m/s} = 10.0\,m/s$$

Since $m = \frac{p}{v} = \frac{30.\,kg\,m/s}{10.\,m/s} = 3.00\,kg$

Example 1.1.3 A tennis player receives a shot with the ball (0.0600kg) travelling horizontally at 50.0 m/s and returns the shot with the ball traveling horizontally at 40.0 m/s in the opposite direction

(a) What is the impulse delivered to the ball by the racquet? (b) What work does the racquet do on the ball?

Solution (a)

Assume the ball is initially in the $-x$ direction away from the net. then $v_i = -50.0$ m/s and $v_f = +40.0$ m/s. Using

$$I \Delta t = \Delta P = m(v_f - v_i)$$

$$= 0.0600 [40.0 \text{ m/s} - (-50.0 \text{ m/s})]$$

$$= 5.40 \text{ kg m/s}$$

(b) The work is just the change of kinetic energy as given by

$$W = \Delta KE = \frac{1}{2} m(v_f^2 - v_i^2)$$

$$= \frac{0.0600}{2} [40^2 - (+50)^2]$$

$$= -27.0 \text{ J}$$

where the negative sign means that the racquet is supplying work to the ball.

1.2 Conservation of Linear Momentum

Let $\vec{F}_1 \Delta t = m_1 \vec{v}_{1f} - m_1 \vec{v}_{1i}$ —— (1)

$\vec{F}_2 \Delta t = m_2 \vec{v}_{2f} - m_2 \vec{v}_{2i}$ —— (2)

By Newton's 3rd law

$$\vec{F}_1 = -\vec{F}_2$$

$$\vec{F}_1 \Delta t = -\vec{F}_2 \Delta t$$

$$m_1 \vec{v}_{1f} - m_1 \vec{v}_{1i} = -(m_2 \vec{v}_{2f} - m_2 \vec{v}_{2i})$$

$$m_1 \vec{v}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{V}_{2f} \quad \text{—— (3)}$$

Equation (3) is the Conservation of linear momentum.

Example 1.2.1 An 80.0 kg astronaut is working on the engines of her spaceship, which is drifting through space with a constant velocity. The astronaut, wishing to get a better view of the universe pushes against the ship and later find herself 30.0 m behind the ship and moving so slowly that she can be considered at rest with respect to the ship. Without a thruster, the only way to return to the ship is to throw a 0.500 kg wrench with a speed of 20.0 m/s in the opposite direction from the ship. How long will it take to get back to the ship once the wrench thrown?

Solution $U_{wf} = -20.0\,m/s$ $\quad V_{af} = ?$

$m_w = 0.500\,kg$ $\quad m_a = 80.1\,kg$

Wrench + astronaut initial mom = Wrench + astronaut final mo

$$P_{wi} + P_{ai} = P_{wf} + P_{af}$$

$P_{ai} = 0$, since $V_{ai} = 0$

$P_{wi} = 0$ since $U_{wi} = 0$

By momentum, then

$$0 = m_w V_{wf} + m_a V_{af}$$

$$V_{af} = -\frac{m_w}{m_a} V_{wf} = -\frac{0.500}{80.1} = (-20.0\,m/s)$$

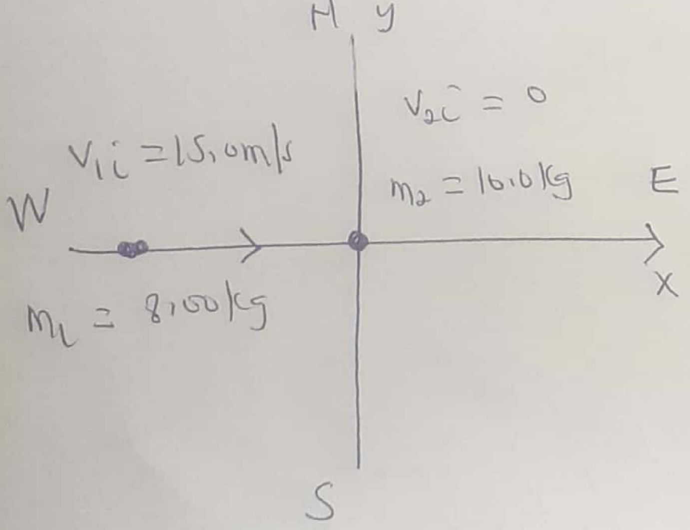
$$= 0.125\,m/s$$

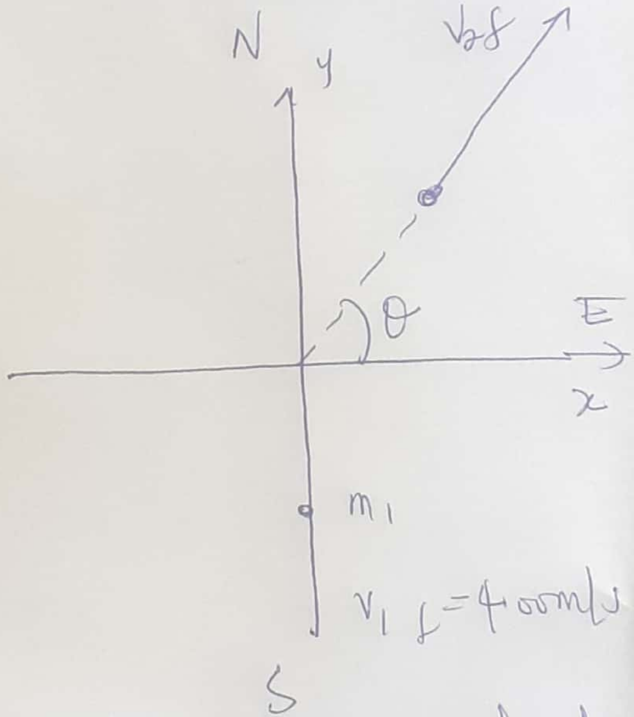
Since the velocity of the astronaut will be constant once the wrench is thrown $a = 0$, so $V_{af} = x_a/t$

$$t = \frac{x_a}{V_{af}} = \frac{30.0\,m}{0.125\,m/s} = 240\,s \times \frac{1\,min}{60\,s}$$

$$= 4.00\,m$$

Example 1.2.3 An 8.00 kg object moving east at 15.0 m/s on a frictionless horizontal surface collides with a 10.0 kg object that is initially at rest. After the collision, the 8.00 kg object moves south at 4.00 m/s (a) What is the velocity of the 10.0 kg object after the collision? (b) What percentage of the initial kinetic energy is lost in the collision?





By Conservation of linear momentum in the x-direction

$$m_1 V_{1ix} + m_2 V_{2ix} = m_1 V_{1fx} + m_2 V_{2fx}$$

$$m_1 V_{1ix} + 0 = 0 + m_2 V_{2ix}$$

$$V_{2fx} = \frac{m_1 V_{1ix}}{m_2}$$

$$= \left(\frac{8}{10}\right)(15.0) = 12.0 \text{ m/s}$$

Conservation of linear momentum in the y direction

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

$$0 + 0 = m_1 v_{1fy} + m_2 v_{2fy}$$

$$-m_2 v_{2fy} = m_1 v_{1fy}$$

$$v_{2fy} = \frac{-m_1}{m_2} v_{1fy}$$

$$= -\left(\frac{8}{10}\right)(-4.0 \, m/s)$$

$$= 3.20 \, m/s$$

the magnitude of the final velocity of object 2 is then found from the Pythagoras theorem

$$v_{2f} = \sqrt{v_{2fx}^2 + v_{2fy}^2}$$

$$= \sqrt{12^2 + 3.2^2} = 12.4 m/s$$

the angle θ can now easily be found with

$$\tan\theta = \frac{v_{2fy}}{v_{2fx}} = \tan^{-1}\left(\frac{3.20}{12.0}\right) = 14.9^0$$

$$\theta = \tan^{-1}\left(\frac{v_{2fy}}{v_{2fx}}\right)$$

Thus $v_{2f} = 12.4\,m/s$ at $14.9°$ N of E

(b) the percentage of kinetic energy lost is given by

$$\frac{KE_{loss}}{KE_i} = \frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i}$$

$$KE_i = \frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2}(8.0)(15.0)^2 = 900\,J$$

$$KE_f = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$= \frac{1}{2} \times 8 \cdot (-4)^2 + \frac{1}{2}(10)(12.4)^2$$

$$= 64.0\,J + 770\,J = 834\,J.$$

Hence, the percentage lost is

$$\frac{KE_{loss}}{KE_i} = 1 - \frac{834\,J}{900\,J} = 1 - 0.928$$

$$= 0.072$$

or 7.2% of the original kinetic energy is lost in the collision.