Check up problem 1

Impact Simplified

1) $X = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{1,3,5,7,9\}$

$B = \{1,2,4,6,8\}$

$C = \{1,2,4,3,9\}$

$P \Delta Q = (P-Q) \cup (Q-P)$

Evaluate $(A^c \Delta (B^c \Delta C))^c$

$B^c = \{3,5,7,9,10\}$

$A^c = \{2,4,6,8,10\}$

let $B^c \Delta C = P$

$P = (B^c - C) \cup (C - B^c)$

$P = \{5,7,10\} \cup \{1,2,4\}$

$P = \{1,2,4,5,7,10\}$

Watimagbo

$A^c \Delta P = (A^c - P) \cup (P - A^c)$

$= \{6,8\} \cup \{1,5,7\}$

$= \{1,5,6,7,8\}$

$[(A^c \Delta (B^c \Delta C)]^c = \{2,3,4,9,10\}$

T. Fiaro

T. Fiaro

2) $n^3+(n+1)^3+(n+2)^3$ is divisible by 9

let $n=1$ $\quad 1^3+(1+1)^3+(1+2)^3$

$1+8+27$

$=36$

$=9\times 4$

True (It's divisible by 9)

let $n=k$ $\quad k^3+(k+1)^3+(k+2)^3$

$=k^3+k^3+3k^2+3k+1+k^3+3k^2(2)+3k(2)^2+8$

$=3k^3+9k^2+15k+9$

$=9\left[\frac{k^3}{3}+k^2+\frac{15k}{9}+1\right]$

True

let $n=k+1$

$(k+1)^3+(k+2)^3+(k+3)^3$

$=k^3+3k^2+3k+1+k^3+3k^2(2)+3k(2)^2+2^3+k^3+3k^2(3)+3k(3)^2+3^3$

$=3k^3+18k^2+42k+36$

$=9\left[\frac{k^3}{3}+2k^2+\frac{14}{3}k+4\right]$

True

Therefore since $n=1, k, k+1$ is true

hence $n^3+(n+1)^3+(n+2)^3$ is divisible by 9

proved

11) $\frac{n^3}{3} + \frac{n^7}{7} + \frac{11n}{21}$

Impact Simplified

let $n = 1$ $\quad \frac{1}{3} + \frac{1}{7} + \frac{11}{21} = 1$

let $n = k$ $\quad \frac{k^3}{3} + \frac{k^7}{7} + \frac{11k}{21} = p$

let $n = k+1$ $\quad \frac{(k+1)^3}{3} + \frac{(k+1)^7}{7} + \frac{11(k+1)}{21}$

$= \frac{1}{3}\left[(k+1)^3\right] + \frac{1}{7}(k+1)^7 + \frac{11}{21}(k+1)$

Watimagbo

$= \frac{1}{3}(k^3+3k^2+3k+1) + \frac{1}{7}\left[k^7+7k^6+21k^5+35k^4+35k^3+21k^2+7k+1\right]$

$+ \frac{11}{21}(k+1)$

$= \frac{1}{3}k^3 + k^2 + k + \frac{1}{3} + \frac{1}{7}k^7 + k^6 + 3k^5 + 5k^4 + 5k^3 + 3k^2 + k + \frac{1}{7} + \frac{11}{21}k + \frac{11}{21}$

Remove all whole numbers because they are Perfect integers

$= \frac{1}{3}k^3 + \frac{1}{3} + \frac{1}{7}k^7 + \frac{1}{7} + \frac{11}{21}k + \frac{11}{21}$

$= \left[\frac{k^3}{3} + \frac{k^7}{7} + \frac{11}{21}k\right] + \left[\frac{1}{3} + \frac{1}{7} + \frac{11}{21}\right]$

Itiano

Compare with when $n = 1$ and $n = k$

$= p + 1 \in N$ which signifies an integer

Proved

3) $r(x) = \dfrac{x^3-1}{x^3+2x^2+2x+1}$ Impact Simplified

Find the value(s) of x given that $r\left(\dfrac{x-1}{x+1}\right) = r\left(\dfrac{-3}{8}\right)$

$$\left[\left(\frac{x-1}{x+1}\right)^3-1\right] \div \left[\left(\frac{x-1}{x+1}\right)^3+2\left(\frac{x-1}{x+1}\right)^2+2\left(\frac{x-1}{x+1}\right)+1\right] = \left[\left(\frac{-3}{8}\right)^3-1\right] \div \left[\left(\frac{-3}{8}\right)^3+2\left(\frac{-3}{8}\right)^2+2\left(\frac{-3}{8}\right)+1\right]$$

$$\left[\frac{(x-1)^3}{(x+1)^3}-1\right] \div \left[\frac{(x-1)^3}{(x+1)^3}+\frac{2(x-1)^2}{(x+1)^2}+\frac{2(x-1)}{(x+1)}+1\right] = \frac{-11}{5}$$

$$\left[\frac{(x-1)^3-(x+1)^3}{(x+1)^3}\right] \div \left[\frac{(x-1)^3+2(x-1)^2(x+1)+2(x-1)(x+1)^2+(x+1)^3}{(x+1)^3}\right] = \frac{-11}{5}$$

$$\left[\frac{(x^3-3x^2+3x-1)-(x^3+3x^2+3x+1)}{(x+1)^3}\right] \div \left[\frac{x^3-3x^2+3x-1+(2x+2)(x^2-2x+1)+(2x-2)(x^2+2x+1)+x^3+3x^2+3x+1}{(x+1)^3}\right] = \frac{-11}{5}$$

$$\frac{-6x^2-2}{2x^3+6x+2x^3-4x^2+2x+2x^2-4x+2+2x^3+4x^2+2x-2x^2-4x-2} = \frac{-11}{5}$$

$$\frac{-6x^2-2}{4x^3+2x^3+2x} = \frac{-11}{5}$$

$$\frac{-6x^2-2}{6x^3+2x} = \frac{-11}{5}$$

$$\frac{-(6x^2+2)}{x(6x^2+2)} = \frac{-11}{5}$$

$$\frac{-1}{x} = \frac{-11}{5}$$

$$x = 5/11$$

OR we can also make use of polynomial approach

Wotimago

Irtiano

$$r(x) = \frac{x^3-1}{x^3+2x^2+2x+1}$$

factorizing both numerator and denominator

x^3-1

$x=1$ is a factor

$x-1=0$

$$\begin{array}{r|l} & x^2+x+1 \\ \hline x-1 & x^3-1 \\ - & x^3-x^2 \\ \hline & x^2-1 \\ & -x^2-x \\ \hline & x-1 \\ & -x-1 \\ \hline & 0 \end{array}$$

$x^3-1 = (x-1)(x^2+x+1)$

x^3+2x^2+2x+1

$x=-1$ is a factor

$x+1=0$

$$\begin{array}{r|l} & x^2+x+1 \\ \hline x+1 & x^3+2x^2+2x+1 \\ - & x^3+x^2 \\ \hline & x^2+2x+1 \\ & -x^2+x \\ \hline & x+1 \\ & -x+1 \\ \hline & 0 \end{array}$$

$x^3+2x^2+2x+1 = (x+1)(x^2+x+1)$

$$\therefore r(x) = \frac{x^3-1}{x^3+2x^2+2x+1}$$

$$r(x) = \frac{(x-1)(x^2+x+1)}{(x+1)(x^2+x+1)}$$

$$r(x) = \frac{x-1}{(x+1)}$$

$$\therefore r\left(\frac{x-1}{x+1}\right) = r\left(\frac{-3}{8}\right)$$

$$\frac{x-1}{x+1} = \frac{-3}{8}$$

$8x-8 = -3x-3$

$11x = 5$

$x = 5/11$