UNIVERSITY OF AGRICULTURE, ABEOKUTA
BSc. MATHEMATICS EXAMINATION
FIRST SEMESTER - 100 LEVEL

Subject: MTS 101 - ALGEBRA
Insruction: Answer Any Four Questions
Time: 2½ Hours

Question 1

a. If A,B,C are any three sets, prove that

(i) $A - (B \cap C) = (A-B) \cup (A - C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b. To investigate the popularity of three brands of toilet soaps X,Y,Z manufacturers, ABC Soaps Industry, asked 150 housewives to fill questi[onnaire] and, the following information was obtained:
60 housewives had used X, 85 had used Y and 72 had used Z. 25 had u[sed X and] Y, 35 had used Y and Z, 17 had used X and Z. Find

(i) How many housewives had used all the three brands? 60

(ii) How many housewives had used just two of the brands? 47

Question 2

a. If $2x^2 - 5x - 7$ is divided by $ax + b$, the quotient is $x - 3$, and the remainder is c. Find the values of a, b and c.

b. Solve the equation

(i) $\log_{10}(X-8) = 1 - \log_{10}(X+1)$

(ii) $3^{x+2} = 5^{x-1}$

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c. The first term of an A.P. is 3, the common difference is 4, and the sum of all terms is 820. Find the number of terms and the last term.

Question 3

a. If α, β are the roots of the equation $ax^2 + bx + c = 0$. Find the equation whose roots are $\frac{1}{\alpha-4\beta}$, $\frac{1}{\beta-4\alpha}$

b. Find and sketch the solution set of the inequality $\frac{3x-5}{x-1} > \frac{x}{2}$

c. Simplify

(i) $\frac{3\sqrt{5} - \sqrt{3}}{2\sqrt{5} + 3\sqrt{3}}$

(ii) $(x^{1/3} - y^{1/3})(x^{1/3} + y^{1/3})(x^{2/3} + x^{1/3}y^{1/3} + y^{2/3})(x^{2/3} - x^{1/3}y^{1/3} + y^{2/3})$

161 87 204
5 +2 +4 =

FEDERAL UNIVERSITY OF AGRICULTURE, ABEOKUTA

DEPARTMENT OF MATHEMATICS MTS101 – ALGEBRA (2012/2013) CAT

1. List the members of the set $\{x \in \Re : x^2 + 1 = 0\}$
   A. $\emptyset$ B. $\{1\}$ C. $\{-1, 1\}$ D. $\{i, -i\}$
2. List the members of $\{x : x^2 = 16 \text{ and } 3x = 9\}$
   A. $\{3,4,-4\}$ B. $\emptyset$ C. $\{-4,4\}$ D. $\{3\}$
3. Of a group of 200 persons, 100 are interested in Music, 70 are interested in Photography and 40 like skiing. Also, 40 are interested in both Music and photography, 30 in both Mus and skiing, 20 in both photography and skiing, while 20 are interested in photography bu not in Music or skiing. How many persons are interested in all the three?
   A. 9 B. 10 C. 11 D. 12
4. $2(3\sqrt{5}-4)^{-2}$ equals
   A. $\frac{122}{841} - \frac{48}{841}\sqrt{5}$ B. $\frac{1}{841}(122+48\sqrt{5})$ C. $\frac{48}{841} - \frac{122}{841}\sqrt{5}$ D. $\frac{81}{841}(48+122\sqrt{5})$
5. Solve the equation $\sqrt{3x+1} - 1 = \sqrt{x+4}$
   A. 0 B. 2 C. 3 D. 5
6. Find x, if $2^{3x+1} - 5^{x+1} = 0$
   A. 1.92 B. 1.93 C. 1.94 D. 1.95
7. Simplify $\frac{4^{-x}.16^{2x-2}}{4^{2x-3}.128}$
   A. 2^{2x+9} B. 2^{2x-9} C. 2^{3x+9} D. 2^{3x-9}

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8. Solve the simultaneous equations for $x + y$

$$2^x.4^{-y} = 2$$
$$3^{-x}.9^{2y} = 3$$

   A. 1 B. 2 C. 3 D. 4
9. Solve completely for a, $9\log_a 5 = \log_5 a$
   A. 5^3 or 5^{-3} B. 3^5 or 3^{-5} C. 3^5 or 5^3 D. 3^5 or 5^{-3}
10. Simplify $\log_a b + \log_a bx + \log_a bx^2$
   A. $\log_a b - \log_a x$ B. $\log_a b + \log_a x$ C. $3(\log_a b - \log_a x)$ D. $3(\log_a b + \log_a$
11. Find the remainder when $4x^3 - 7x^2 + 2x + 5$ is divided by x-3
   A. 56 B. -56 C. -160 D. 20
12. Find the factors of $x^3 + 3x^2 - 4x - 12$
   A. (x-2)(x+2)(x-3) B. (x+2)(x-2)(x+3) C. $(x-2)^2(x-3$ D. $(x^2+4)(x-3)$
13. Given that (x+3y) is a factor of $+6x^2y + 11xy^2 + 6y^3$, factorise the expression comp
   A. (x+3y)(x-y)(x+2y) B. (x-3y)(x+y)(x-2y) C. (x+3y) (x+y) (x+2y) D. (x+3y) (x+y) (x
14. Find the value of k if (x+1) is a factor of $x^3 + 5x^2 + kx + 3$, then find the other factors of the expression
   A. k=7, (x+1)(x+3) B. k=0, (x+1)(x+3) C. k= -7, (x+1)(x+3) D. k=1, (x-1)(x-
15. Find the values of a and b if (x+1) and (x+2) are both factors of $x^3 + ax^2 + bx + 8$
   A. $(-7,-14)$ B. $(7,-14)$ C. $(-7,14)$ D. $(7,14)$

16. Find the value of λ for which the equation $x^2 - x + 1 = \lambda(x^2 + x + 1)$, where $\lambda \neq 1$, has equal roots.

A. $(\frac{1}{3}, -3)$ B. $(\frac{1}{3}, 3)$ C. $(-\frac{1}{3}, 3)$ D. $(-\frac{1}{3}, -3)$

17. The roots of $(x-1)(x-2) = p$ are equal. Find the value of p.

A. $1/4$ B. $1/2$ C. $-1/2$ D. $-1/4$

18. Find the range of values of λ for which the equation $x^2 - x + 1 = \lambda(x^2 + x + 1)$, where $\lambda \neq 1$, has real and unequal roots.

$3 < \lambda < -\frac{1}{3}$ B. $-\frac{1}{3} < \lambda < -3$ C. $\frac{1}{3} < \lambda < 3$ D. $3 \leq \lambda < 1/3$

19. If α and β are the roots of the equation $px^2 + qx + r$ find in terms of p, q and r, the values of $\alpha^2 + \beta^2$.

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A. $\frac{q^2+2pr}{p^2}$ B. $\frac{q^2-2pr}{p^2}$ C. $\frac{-q^2+2pr}{p^2}$ D. $\frac{-q^2-2pr}{p^2}$

20. The roots of $3x^2 + 8x + 5 = 0$ are α and β. Find the quadratic equations with α^2 and β^2.

A. $x^2 - 34x + 25 = 0$ B. $9x^2 + 34x + 25 = 0$ C. $9x^2 - 34x + 25 = 0$ D. $x^2 - 34x - 25 = 0$

	A	B	C	D
1				

University of Agriculture, Abeokuta,
Department of Mathematics, 2011/2012 MTS 101 CAT
INSTRUCTION: Answer ALL Time:40 minutes

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1. Which of the following is not true for sets A, B and C.
(a) $A \cap (B \cap C) = (A \cap B) \cap C$ (b) $A \cup (B \cup C) = (A \cup B) \cup C$
(c) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$ (d) none of the above.

2. Solve $\frac{1}{2-x} < 3$ and give the solution set as an interval or union of intervals. (a) $(2, \infty)$ (b) $(-\infty, \frac{5}{3})$ (c) $(\frac{5}{3}, 2)$ (d) $(-\infty, \frac{5}{3}) \cup (2, \infty)$

3. Evaluate $log_y 3^y$. (a) y^3 (b) $\frac{1}{3}$ (c) y (d) 3.

4. Solve the equation $3^x = 7.83$ (a) $x = 0.8938$ (b) $x = 0.4771$
(c) $x = 3$ (d) $x = 1.87$

5. Let $A = \{1, 2, 3, 6, 8\}$, $B = \{2, 5, 6, 7, 9\}$ and $C = \{4, 5, 6, 8\}$ be sets, then find $B \triangle (A \cap C)$. (a) $\{2, 5, 7, 9\}$ (b) $\{6, 8\}$ (c) ϕ.
(d) $\{2, 5, 7, 8, 9\}$.

6. The element(s) of the set $Y = \{x | x^2 + 5x - 14 = 0\}$ is/are (a) -2
(b) -7 (c) $-2, -7$ (d) $-7, 2$

7. Evaluate $81^{\frac{1}{2}} \times 9^{\frac{-1}{2}}$. (a) $\frac{1}{3}$ (b) 3 (c) 1 (d) 9.

8. Simplify $\frac{8.2^{n+1} - 2^{n+2}}{2^{n+3} - 2^n}$. (a) $\frac{12}{17}$ (b) $\frac{4}{3}$ (c) $\frac{6}{7}$ (d) $\frac{3}{4}$.

9. The power set of the set $B = \{1, 2, 3\}$ is (a) $\{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$
(b) $\{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$
(c) $\{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \phi, \{1, 2, 3\}\}$ (d) None of the above

10. Simplify $(x^4 y^2 z^{-3})^{\frac{1}{2}} (x^5 y^2 z)^{\frac{1}{2}} \div x z^{\frac{7}{2}}$. (a) $x^9 y^4 z^7$ (b) $x^{\frac{-7}{2}} y^2 z^{\frac{9}{2}}$
(c) $x^{\frac{7}{2}} y z^{\frac{9}{2}}$ (d) $x^{\frac{7}{2}} y^2 z^{-\frac{9}{2}}$.

11. Express $\frac{2}{9}$ as a repeating decimal and use bar to indicate the repeating digit. (a) $0.\overline{32}$ (b) $0.\overline{15}$ (c) $0.\overline{22}$ (d) $0.\overline{2}$

12. Express $0.\overline{12}$ as a quotient of integers in lowest terms. (a) $\frac{4}{33}$ (b) $\frac{2}{13}$
(c) $\frac{3}{14}$ (d) $\frac{1}{11}$

13. Obtain the rational number represented by 0.14285. (a) $\frac{7}{11}$ (b) $\frac{1}{7}$
(c) $\frac{5}{7}$ (d) $\frac{6}{7}$

14. Solve for x in the equation $\sqrt{x+5} = 5 - \sqrt{x}$. (a) 5 (b) 3 (c) 4
(d) 2.

15. Which of the following is not a subset of $A = \{a, e, i, o, u\}$. (a) $\{a, i, o\}$ (b) $\{e, u\}$ (c) $\{a, p, o\}$ (d) $\{a, e, i, o, u\}$.

16. The set $(A \cup B) \cap (A' \cup C) \cap (A \cup B') \cap (A' \cup C')$ is (a) undefined (b) empty (c) Universal set (d) none of the above.

17. The simplification of set $(A \cup B)' \cup (A' \cap B')'$ gives (a) $A \cup B$ (b) $A \cap B$ (c) Universal set (d) ϕ.

18. In a certain gathering of 200 students, 60 percent of them like Economics while 77 percent of them like History. How many of them like both History and Economics. (a) 58 (b) 37 (c) 5 (d) 74.

19. Let $A = \{1, 2, 3, 6, 8\}$, $B = \{2, 5, 6, 7, 9\}$ be sets, then find $A \triangle B$. (a) $\{1, 3, 5\}$ (b) $\{1, 3, 5, 7\}$ (c) $\{1, 3, 5, 7, 8\}$ (d) $\{1, 3, 5, 7, 8, 9\}$

20. Solve the system of inequalities $3 \leq 2x + 1 \leq 5$. (a) $[1, \infty)$ (b) $(-\infty, 2]$ (c) $[1, 2]$ (d) $(1, 2)$

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S/N	A	B	C	D	S/N	A	B	C	D

DEPT OF MATHS FUNAAB MTS 101 CAT TUE 26/06/18 ANSWER ALL 1 HOUR

NAME MATRIC NO DEPT COLLEGE

1. (a) Let A and B be nonempty subsets of the universal set X. Show that:

(i) $[(A \cup A') \cap (B \cup B')] \cup (A \cap B) = X$ (ii) $(A \cup B) \cap (B \cup A') = B$ (iii) $(A \cup B)' = A' \cap B'$

(b) i. Express $(\sqrt{5})^{-2} + (\sqrt{5})^{-1} + (\sqrt{5})^{0} + (\sqrt{5})^{1} + (\sqrt{5})^{2}$ in the form $p + q\sqrt{5}$ where p and q are rational numbers. State the values of p and q.

ii. Given that $\log_8(x+2) + \log_8 y = z - \frac{1}{3}$ and $\log_2(x-2) - \log_2 y = 2z+1$. Show that $x^2 = 4 + 32^z$. Given that $z = 1$, find all the possible values of x and y.

1.(a)(i) $[(A \cup A') \cap (B \cup B')] \cup (A \cap B)$

$= (X \cap X) \cup (A \cap B)$ 1/2

$= X \cup (A \cap B) = X.$ 1/2

ii) $(A \cup B) \cap (B \cup A') = (B \cup A) \cap (B \cup A')$ 1/2

$= B \cup (A \cap A') = B \cup \phi$

$= B.$ 1/2 AMBASSADOR

iii) Let $x \in (A \cup B)'$ 1/2

$\Longleftrightarrow x \notin A \cup B$ 1/2

$\Longleftrightarrow x \notin A$ or $x \notin B$ 1/2

$\Longleftrightarrow x \in A'$ and $x \in B'$ 1/2

$\Longleftrightarrow x \in A' \cap B'$ 1/2

$\therefore (A \cup B)' = A' \cap B'$ 1/2

b)(i) $(\sqrt{5})^{-2} + (\sqrt{5})^{-1} + (\sqrt{5})^{0} + (\sqrt{5})^{1} + (\sqrt{5})^{2}$

$= \frac{1}{5} + \frac{1}{\sqrt{5}} + 1 + \sqrt{5} + 5$ 1/2

$= 6 + \frac{1}{5} + \frac{1}{\sqrt{5}} + \sqrt{5}$

$= \frac{31}{5} + \frac{6}{\sqrt{5}} = \frac{31}{5} + \frac{6\sqrt{5}}{5}$ 1/2

$= \frac{31}{5} + \frac{6}{5}\sqrt{5} = p + q\sqrt{5}$ 1/2

$\therefore p = \frac{31}{5}, \quad q = \frac{6}{5}$ 1/2

(ii) $\log_8(x+2) + \log_8 y = z - \frac{1}{3}$

$\Rightarrow \log_8 (x+2)y = z - \frac{1}{3}$ 1/2

$\Rightarrow 8^{z - \frac{1}{3}} = y(x+2)$ 1/2

$\Rightarrow 2^{(3z-1)} = y(x+2)$ 1/2

$\log_2(x-2) - \log_2 y = 2z+1$

$\Rightarrow \log_2\left(\frac{x-2}{y}\right) = 2z+1$ 1/2

$\Rightarrow 2^{(2z+1)} = \frac{x-2}{y}$ 1/2 (2)

Eqn (1) × Eqn (2) gives 1/2

$2^{(3z-1+2z+1)} = (x+2)(x-2) = x^2 - 4$

$\Rightarrow x^2 = 4 + 2^{5z} = 4 + (2^5)^z$ 1/2

$\therefore x^2 = 4 + 32^z$ 1/2

When $z = 1$, $x^2 = 36$ $\therefore x = \pm 6$ 1/2

When $x = 6$, from (1), $8y = 4$

When $x = -6$, from (1), $-4y = 4$

13 m

1 HOUR

(iii) $(A \cup B)' = A' \cap B'$

are rational

(a) If $(x-1)$ is a factor of the polynomial $f(x) = 3x^4 + px^3 + 12x^2 + qx + 4$ and $f(x)$ leaves a remainder 18 when divided by $(x+2)$, find the values of p and q and hence find the remainder (in the simplified form) when $f(x)$ is divided by $(kx+2)$ where k is an integer.

(b) If $f(x) = 6x^3 - 7x^2 - 7x + 6$ is a given polynomial, factorize $f(x)$ completely and hence state all the values of x for which $f(x) = 0$.

(a) $f(1) = 0$

$\Rightarrow 3 + p + 12 + q + 4 = 0$

$\therefore p + q = -19$ $\frac{1}{2}$ (1)

$f(-2) = 18$

$\Rightarrow 48 - 8p + 48 - 2q + 4 = 18$

$\therefore 4p + q = 41$ $\frac{1}{2}$ (2)

Eqn (2) − Eqn (1) gives

$3p = 60 \therefore p = 20$ $\frac{1}{2}$

and frm (1), $q = -39$ $\frac{1}{2}$

$\therefore f(x) = 3x^4 + 20x^3 + 12x^2 - 39x + 4$ $\frac{1}{2}$

Now,

$f\left(\frac{-2}{K}\right) = \frac{48}{K^4} - \frac{160}{K^3} + \frac{48}{K^2} - \frac{78}{K} + 4$ $\frac{1}{2}$

$= \frac{4K^4 - 78K^3 + 48K^2 - 160K + 48}{K^4}$ $\frac{1}{2}$

which is the remainder when $f(x)$ is divided by $(Kx+2)$.

(b) $f(x) = 6x^3 - 7x^2 - 7x + 6$

$f(-1) = -6 - 7 + 7 + 6 = 0$

$\therefore x+1$ is a factor. $\frac{1}{2}$

By long division:

$$
\begin{array}{r|l}
 & 6x^2 - 13x + 6 \\
\hline
x+1 & 6x^3 - 7x^2 - 7x + 6 \\
 & 6x^3 + 6x^2 \\
\hline
 & -13x^2 - 7x \\
 & -13x^2 - 13x \\
\hline
 & 6x + 6 \\
 & 6x + 6 \\
\hline
\end{array}
$$

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$\frac{1}{2}$

$\therefore f(x) = (x+1)(6x^2 - 13x + 6)$ $\frac{1}{2}$

$= (x+1)(2x-3)(3x-2)$

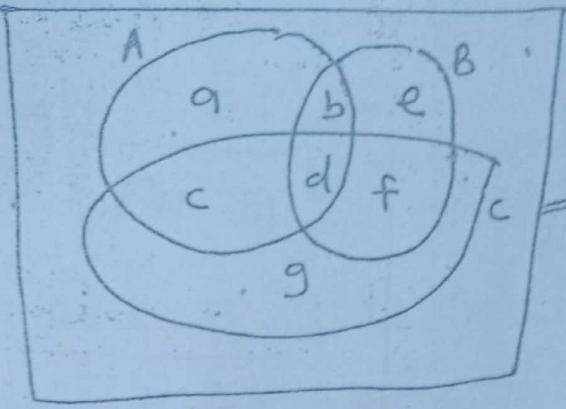
When $f(x) = 0$, then

$(x+1)(2x-3)(3x-2) = 0$

$\therefore x = -1$ or $3/2$

1. (a)

X:



1

$|A \cup B \cup C| = a+b+c+d+e+f+g$

$= (a+b+c+d) + (b+d+e+f) + (c+d+f+g) - (b+d) - (d+f) - (c+d) + d$ 1

$= |A| + |B| + |C| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$. 1

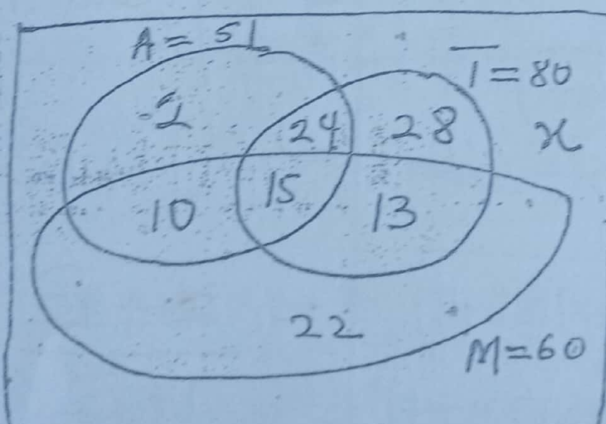
(b) Let A — Algebra

T — Trigonometry

M — Matrices

$\mathcal{E} = 150$

(i)



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(ii) From the Venn diagram,

$x + 144 = 150$

$\therefore x = 150 - 144 = 3$

$\therefore$ 36 students prefer of the topics. 1

(iii) No of students prefer one of the topics $= 2 + 2 \ldots = 52$. 1

(iv) No of students pref just two of the topics $= 10 + 24 + 13 = 47$

(v) Probability $= \frac{10}{150} = \frac{1}{15} = 0.067$. 1

(c) Let $\log_b a = x$

$\Rightarrow b^x = a$ 1/2

$\Rightarrow \log_c b^x = \log_c a$

$\Rightarrow x \log_c b = \log_c a$ 1/2

$\therefore x = \frac{\log_c a}{\log_c b} = \log_b a$

$\log_{10} e \times \ln(x^2+1) - 2\log_{10} e \times \ldots = \log_{10} 5 = \frac{\ln 5}{\ln 10}$ 1

$\Rightarrow \frac{\ln e}{\ln 10} \times \ln(x^2+1) - 2 \ldots = \log_{10} 5 =$

$\Rightarrow \ln(x^2+1) - \ln x^2 = \ln 5$

$\Rightarrow \ln\left[\frac{x^2+1}{x^2}\right] = \ln 5$ 1

$\Rightarrow \frac{x^2+1}{x^2} = 5$ 1

$\Rightarrow 4x^2 - 1 = 0$

$\Rightarrow (2x-1)(2x+1) = 0$

$\therefore x = 1/2$. 1

(d) (i) $5^{2x} - 5^{x+1} + 4 = 0$

$\Rightarrow (5^x)^2 - 5 \times 5^x + 4 = 0$ 1

$\Rightarrow (5^x - 1)(5^x - 4) = 0$

$\therefore 5^x = 1$ or $5^x = 4$ 1

$\therefore 5^x = 5^0$ or $x \log 5 = \log 4$

$\therefore x = 0$ or $x = \frac{\log 4}{\log 5}$ 1

(ii) $\sqrt{3x+4} - \sqrt{x+2} = \sqrt{x-3}$

$\Rightarrow (3x+4) - 2\sqrt{(x+2)(3x+4)} + (x+2) = x-3$ 1

$\Rightarrow 3(x+3) = 2\sqrt{(x+2)(3x+4)}$

$\Rightarrow 9(x+3)^2 = 4(x+2)(3x+4)$ 1

$\Rightarrow 3x^2 - 14x - 49 = 0$

$\Rightarrow (x-7)(3x+7)$ ...

$\therefore x = 7$ or $-\frac{7}{3}$

(e) $\left[\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right]^2 = \frac{2-2\sqrt{6}}{2+2\sqrt{6}}$

$= \frac{5-2\sqrt{6}}{5+2\sqrt{6}}$

$= \frac{(5-2\sqrt{6})^2}{5^2-(2\sqrt{6})^2}$ 1

$= \frac{25-20\sqrt{6}+24}{25-24}$

$= 49 - 20\sqrt{6}$ 1

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(2) (a) (i)

$c(\ln(1+\sqrt{2}))$

$= \frac{1}{2}\left[e^{\ln(1+\sqrt{2})} + e^{-\ln ...}\right.$ 1

$= \frac{1}{2}\left[1+\sqrt{2} + \frac{1}{1+\sqrt{2}}\right.$

$= \frac{2+\sqrt{2}}{1+\sqrt{2}} = \frac{(2+\sqrt{2})(...}{1^2-(\sqrt{...}}$

$= \frac{-\sqrt{2}}{-1} = \sqrt{...}$

$2\sinh(x) + 6\cosh(x) = 9$

$\Rightarrow 2\times\frac{1}{2}(e^x - e^{-x}) + 6\times\frac{1}{2}(e^x + e^{-x}) = 9$

$\Rightarrow e^x - e^{-x} + 3(e^x + e^{-x}) = 9$

$\Rightarrow 4e^x + 2e^{-x} - 9 = 0$

$\Rightarrow 4e^{2x} - 9e^x + 2 = 0$

$\Rightarrow 4(e^x)^2 - 9e^x + 2 = 0$

$\Rightarrow (4e^x - 1)(e^x - 2) = 0$

$\Rightarrow e^x = 1/4$ or $e^x = 2$

$\therefore x = \ln 2$ or $x = \ln(1/4)$

$= -\ln 4.$

(b) (i) $f(x,y) = (x+y)^3 - x^3 - y^3$

$\Rightarrow f(y,x) = (y+x)^3 - y^3 - x^3$

$= (x+y)^3 - x^3 - y^3$

$= f(x,y)$

$\therefore f(x,y)$ is a symmetrical function.

Next,

$f(kx,ky) = (kx+ky)^3$

$-(kx)^3 - (ky)^3$

$= (k(x+y))^3 - k^3x^3 - k^3y^3$

$= k^3(x+y)^3 - k^3x^3 - k^3y^3$

$= k^3[(x+y)^3 - x^3 - y^3]$

$= k^3 f(x,y)$

$\therefore f(x,y)$ is a symmet[ric] function of order 3

(ii) Let

$f(x) = (x+y)^3 - x^3 - y^3$

$\Rightarrow f(-y) = (-y+y)^3 - (-y)^3 -$

$= 0 + y^3 - y^3 =$

$\therefore x+y$ is a factor [of] $f(x,y)$ by factor th[eorem]

Next,

$f(x,y) = (x+y)^3 - x^3 -$

$= x^3 + 3x^2y + 3xy^2 + y^3 - x$

$= 3x^2y + 3xy^2$

$= 3xy(x+y).$

(c) (i) $r(K) = \dfrac{K+4}{K^3+3K^2+2K}$

$= \dfrac{K+4}{K(K+1)(K+2)} \quad \frac{1}{1} = \dfrac{A}{K} + \dfrac{B}{(K+1)} + \dfrac{C}{K+2} \quad 1/2$

$= \dfrac{A(K+1)(K+2) + BK(K+2) + CK(K+1)}{K(K+1)(K+2)} \quad 1/2$

$\Longleftrightarrow K+4 \equiv A(K+1)(K+2) + BK(K+2) + CK(K+1) \quad 1/2$

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putting $K=0$:

$2A = 4 \quad \therefore A = 2 \quad 1$

putting $K=-1$,

$-B = 3 \quad \therefore B = -3 \quad 1$

putting $K=-2$,

$2C = 2 \quad \therefore C = 1 \quad 1$

$\therefore r(K) = \dfrac{2}{K} - \dfrac{3}{K+1} + \dfrac{1}{K+2} \quad 1/2$

(ii) $\dfrac{5}{1\times2\times3} + \dfrac{6}{2\times3\times4} + \dfrac{7}{3\times4\times5} + \ldots$

$= \sum_{K=1}^{n} \dfrac{K+4}{K(K+1)(K+2)}$

$= \sum_{K=1}^{n} \left[\dfrac{2}{K} - \dfrac{3}{K+1} + \dfrac{1}{K+2}\right] \quad 1/2$

$= \dfrac{2}{1} - \dfrac{3}{2} + \dfrac{1}{3}, \quad K=1$

$+ \dfrac{2}{2} - \dfrac{3}{3} + \dfrac{1}{4}, \quad K=2$

$+ \dfrac{2}{3} - \dfrac{3}{4} + \dfrac{1}{5}, \quad K=3$

$+ \dfrac{2}{4} - \dfrac{3}{5} + \dfrac{1}{6}, \quad K=4 \quad 1/2$

$\vdots$

$+ \dfrac{2}{n-3} - \dfrac{3}{n-2} + \dfrac{1}{n-1}, \quad K=r$

$+ \dfrac{2}{n-2} - \dfrac{3}{n-1} + \dfrac{1}{n}, \quad K=r$

$+ \dfrac{2}{n-1} - \dfrac{3}{n} + \dfrac{1}{n+1}, \quad K=$

$+ \dfrac{2}{} - \dfrac{3}{} + \dfrac{1}{}$

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$\left(3-\frac{3}{2}\right)+\frac{1}{n+2}-\frac{2}{n+1}$

$$= \frac{3}{2} - \frac{n+3}{(n+1)(n+2)} \quad 1$$

$$\therefore S_n = \frac{3}{2} - \frac{n+3}{(n+1)(n+2)} \quad 1/2$$

(iii) $S_\infty = \lim_{n\to\infty} S_n$

$$= \lim_{n\to\infty}\left[\frac{3}{2} - \frac{n+3}{(n+1)(n+2)}\right] \quad 1/2$$

$$= \frac{3}{2} - \lim_{n\to\infty}\frac{n+3}{(n+1)(n+2)}$$

$$= \frac{3}{2} - 0 \quad 1/2$$

$$= \frac{3}{2} = 1\frac{1}{2} \quad 1/2$$

50 marks

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