CSC 642/841

Computer Performance Evaluation

Darrel Daquigan

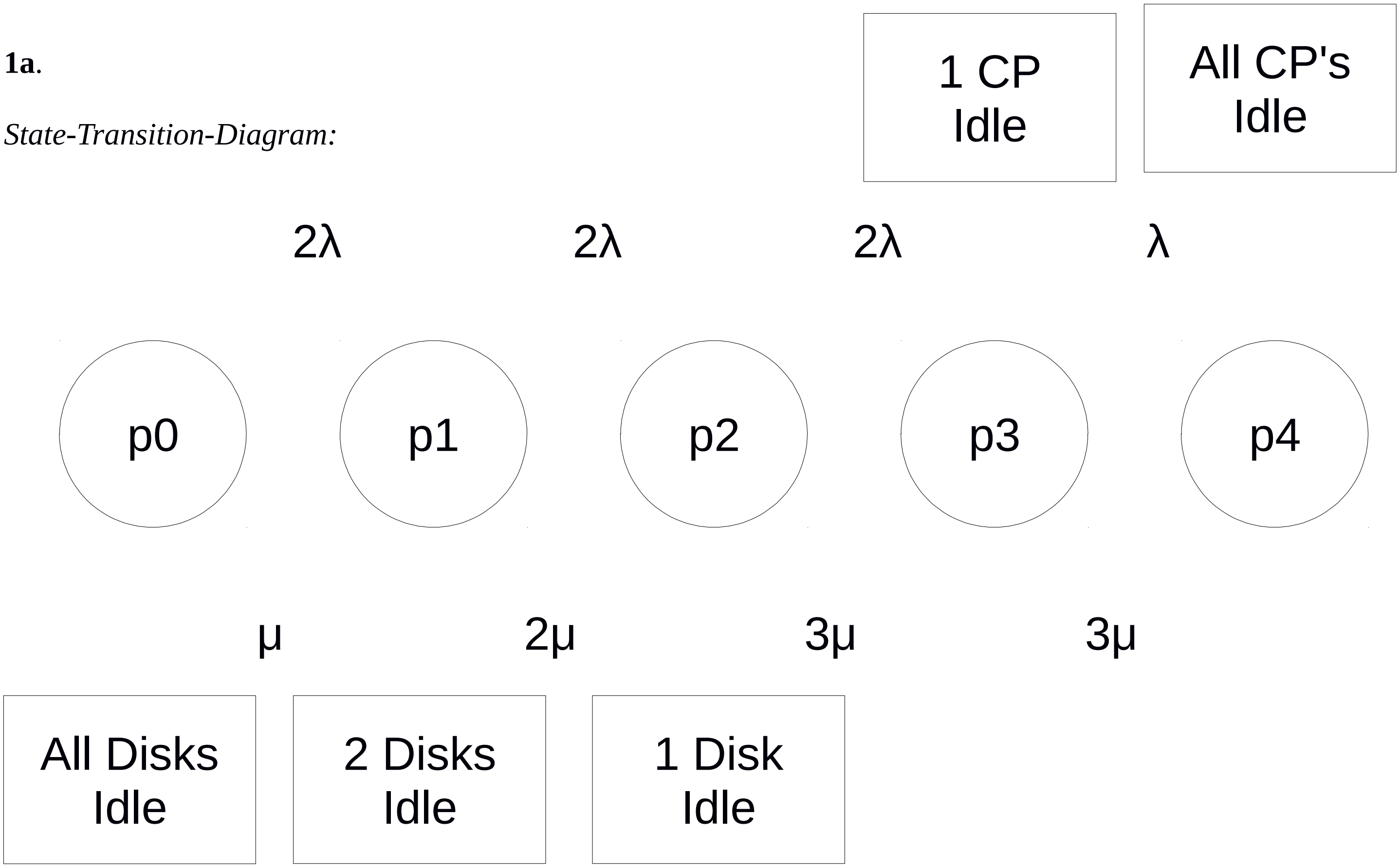
Homework #7

Closed Queuing Models

December 7, 2016

1a.

State-Transition-Diagram:



Given:

2 CP's with speed S_p
3 Disks with speed S_d
4 Programs needing 10 CPU minutes each
$S_d = 2S_p$

Derived:

$\lambda = 1/S_p$ $\quad\mu = 1/S_d = 1/2S_p$
$\rho = \lambda/\mu = 2$

Balanced Equations:

$2\lambda p_0 = \mu p_1 \quad\rightarrow\quad p_1 = 2\rho p_0 = 4p_0$
$2\lambda p_1 = 2\mu p_2 \quad\rightarrow\quad p_2 = \rho p_1 = 2p_1 = 8p_0$
$2\lambda p_2 = 3\mu p_3 \quad\rightarrow\quad p_3 = (2/3)\rho p_2 = (4/3)p_2 = (32/3)p_0$
$\lambda p_3 = 3\mu p_4 \quad\rightarrow\quad p_4 = (1/3)\rho p_3 = (2/3)p_3 = (64/9)p_0$

$p_0 + p_1 + p_2 + p_3 + p_4 = 1$
$p_0\,(1 + 4 + 8 + (32/3) + (64/9)) = 1$

$p_0 = 9/277$
$p_1 = 4p_0 = 36/277$
$p_2 = 8p_0 = 72/277$
$p_3 = (32/3)p_0 = 96/277$
$p_4 = (64/9)p_0 = 64/277$

$U_p = p_0 + p_1 + p_2 + (1/2)p_3$

$= 9/277 + 36/277 + 72/277 + 48/277$

$= 165 / 277$

$\underline{\mathbf{U_p \approx 0.5957}}$

1b.

$2RU_p = 4T_p$

$R = 2T_p/U_p$

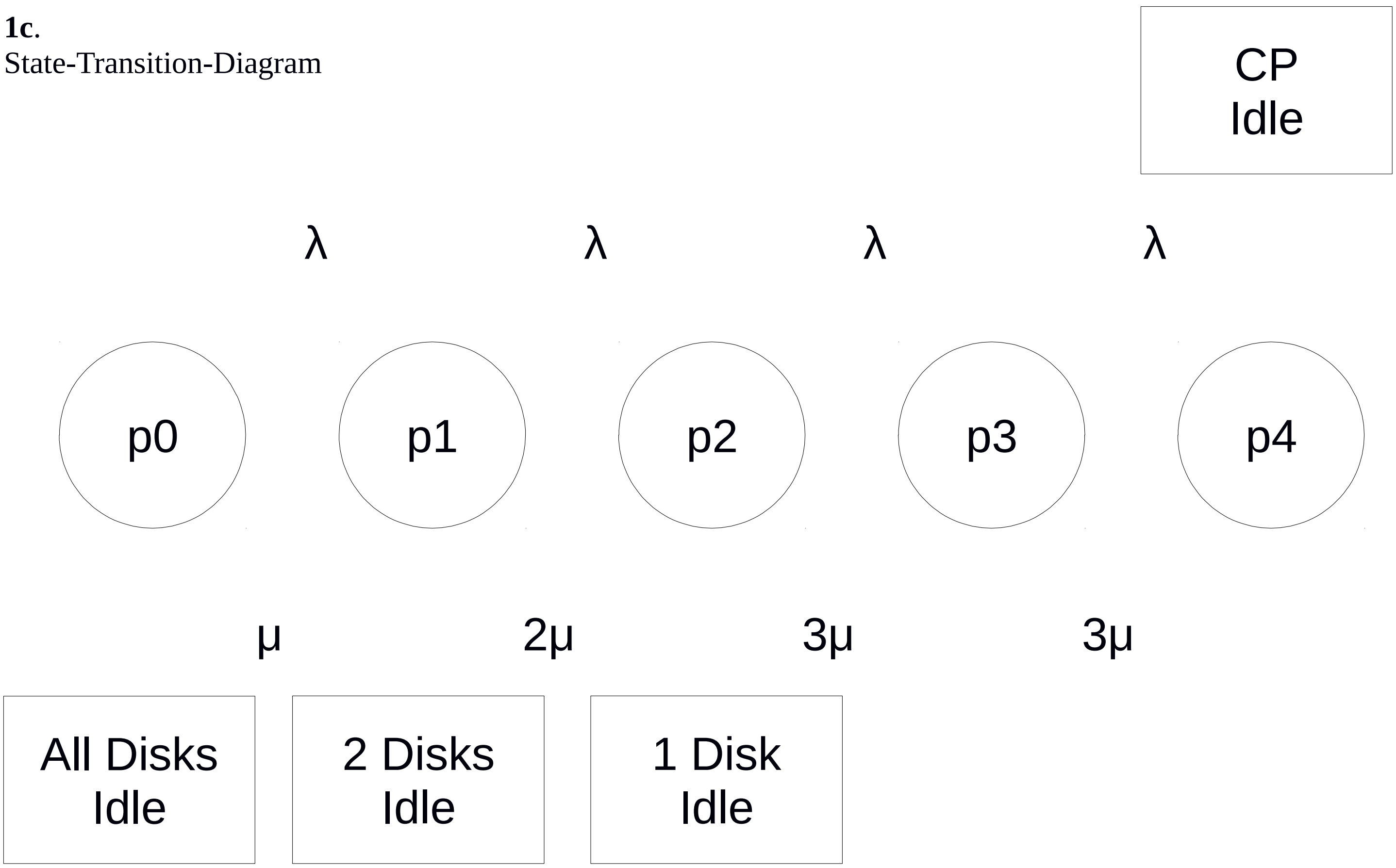
$= (2*10)/(165/277)$

$= 5540/165$

$\underline{\mathbf{R \approx 33.5758 \text{ CPU minutes}}}$

1c.

State-Transition-Diagram



Given:

1 CP with speed S_p

3 Disks with speed S_d

4 Programs needing 10 CPU minutes each

$S_d = 2S_p$

Derived:

$\rho = 2$

Balanced Equations:

$\lambda p_0 = \mu p_1 \quad \rightarrow \quad p_1 = \rho p_0 = 2p_0$

$\lambda p_1 = 2\mu p_2 \quad \rightarrow \quad p_2 = (1/2)\rho p_1 = p_1 = 2p_0$

$\lambda p_2 = 3\mu p_3 \quad \rightarrow \quad p_3 = (1/3)\rho\, p_2 = (2/3)p_2 = (4/3)p_0$

$\lambda p_3 = 3\mu p_4 \quad \rightarrow \quad p_4 = (1/3)\rho p_3 = (2/3)p_3 = (8/9)p_0$

$p_0\,(1 + 2 + 2 + (4/3) + (8/9)) = 1$

$p_0 = 9/65$

$p_1 = 2p_0 = 18/65$

$p_2 = 2p_0 = 18/65$

$p_3 = (4/3)p_0 = 12/65$

$p_4 = (8/9)\,p_0 = 8/65$

$U_p = 1 - p_4$

$= 57/65$

$U_p \approx 0.8308$

$RU_p = 4T_p$

$R = 4T_p/U_p$

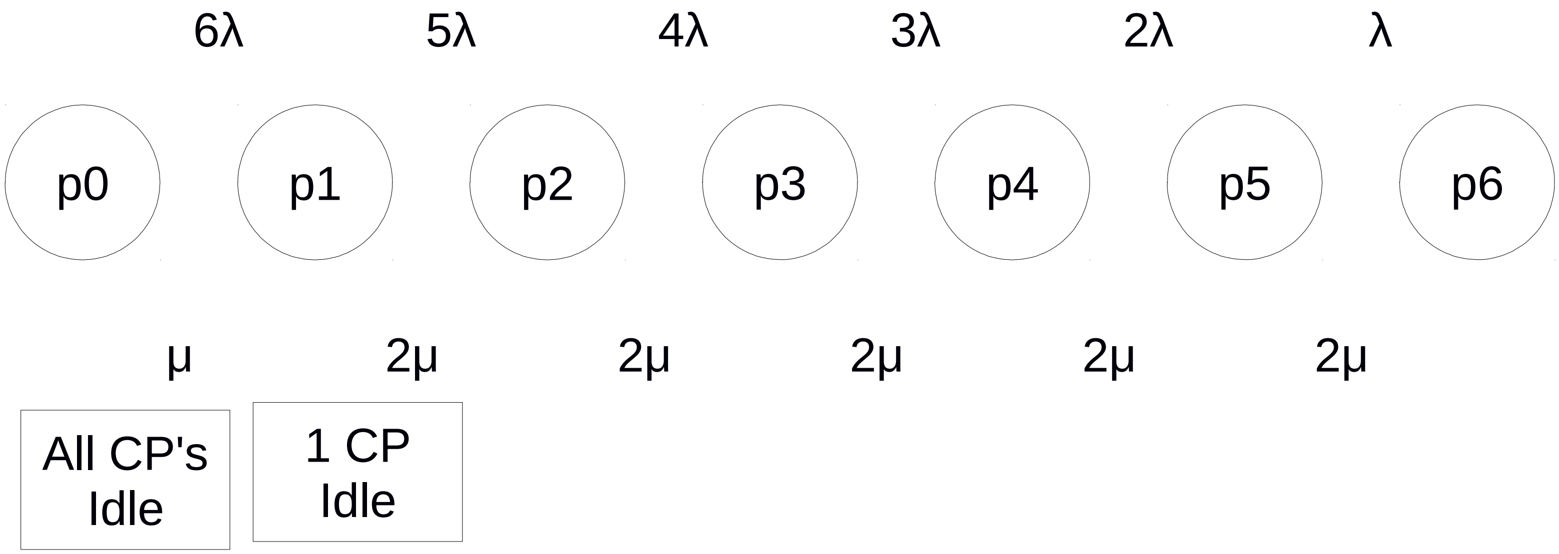
$= (4*10)/(57/65)$

$= 2600/57$

R ≈ 45.6140 CPU minutes

2a.

State-Transition-Diagram:



Given:

2 CP's with speed S = 2sec
6 workstations with think time Z = 8 sec

Derived:

$\lambda = 1/Z$ $\quad\quad$ $\mu = 1/S$
$\rho = \lambda/\mu = S/Z = 1/4$

Balanced Equations:

$6\lambda p_0 = \mu p_1$	→	$p_1 = 6\rho p_0 = (3/2)p_0$		→	$p_1 = (12288/8192)p_0$
$5\lambda p_1 = 2\mu p_2$	→	$p_2 = (5/2)\rho p_1 = (5/8)p_1 = (15/16)p_0$		→	$p_2 = (7680/8192)p_0$
$4\lambda p_2 = 2\mu p_3$	→	$p_3 = 2\rho p_2 = (1/2)p_2 = (15/32)p_0$		→	$p_3 = (3840/8192)p_0$
$3\lambda p_3 = 2\mu p_4$	→	$p_4 = (3/2)\rho p_3 = (3/8)p_3 = (45/256)p_0$		→	$p_4 = (1440/8192)p_0$
$2\lambda p_4 = 2\mu p_5$	→	$p_5 = \rho p_4 = (1/4)p_4 = (45/1024)p_0$		→	$p_5 = (360/8192)p_0$
$\lambda p_5 = 2\mu p_6$	→	$p_6 = (1/2)\rho p_5 = (1/8)p_5 = (45/8192)p_0$		→	$p_6 = (45/8192)p_0$

$p_0 + p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1$
$p_0\,(8192 + 12288 + 7680 + 3840 + 1440 + 360 + 45)/8192 = 1$
$p_0 = 8192/3845$

$p_0 = 8192/\ 33845$
$p_1 = 12288/33845$
$p_2 = 7680/33845$
$p_3 = 3840/33845$
$p_4 = 1440/33845$
$p_5 = 360/33845$
$p_6 = 45/33845$

$U_p = 1 - p_0 - (1/2)p_1$

$= 1 - (8192/33845) - (1/2)(12288/33845)$

$= (33845 - 8192 - 12288)/33845$

$= 13365/33845$

<u>**$U_p \approx 0.3949$**</u>

$(R + Z)mU_p = nS$

$R = (nS/mU_p) - Z$

$= ((6*2)/(2(13365/33845))) - 8$

$= (203070/13365) - 8$

$= 96150 / 13365$

<u>**R ≈ 7.1941 sec**</u>

2b:

Given:

1 CP with speed S = 1sec

6 workstations with think time Z = 8 sec

Derived:

$\lambda = 1/Z \qquad \mu = 1/S$

$\rho = \lambda/\mu = S/Z = 1/8$

Balanced Equations:

$6\lambda p_0 = \mu p_1$	→	$p_1 = 6\rho p_0 = (3/4)p_0$	→	$p_1 = (12288/16384)p_0$
$5\lambda p_1 = \mu p_2$	→	$p_2 = 5\rho p_1 = (5/8)p_1 = (15/32)p_0$	→	$P_2 = (7680/16384)p_0$
$4\lambda p_2 = \mu p_3$	→	$p_3 = 4\rho p_2 = (1/2)p_2 = (15/64)p_0$	→	$p_3 = (3840/16384)p_0$
$3\lambda p_3 = \mu p_4$	→	$p_4 = 3\rho p_3 = (3/8)p_3 = (45/512)p_0$	→	$p_4 = (1440/16384)p_0$
$2\lambda p_4 = \mu p_5$	→	$p_5 = 2\rho p_4 = (1/4)p_4 = (45/2048)p_0$	→	$p_5 = (360/16384)p_0$
$\lambda p_5 = \mu p_6$	→	$p_6 = \rho p_5 = (1/8)p_5 = (45/16384)p_0$	→	$p_6 = (45/16384)p_0$

$p_0 + p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1$

$p_0\,(16384 + 12288 + 7680 + 3840 + 1440 + 360 + 45)/16384 = 1$

$p_0 = 16384/30977$

$p_0 = 16384/30977$

$p_1 = 12288/30977$

$p_2 = 7680/30977$

$p_3 = 3840/30977$

$p_4 = 1440/30977$

$p_5 = 360/30977$

$p_6 = 45/30977$

$U_p = 1 - p_0$

$= 1 - 16384/30977$

$= 14593/30977$

$U_p \approx 0.4711$

$(R + Z)\, mU_p = nS$

$R = (nS/mU_p) - Z$

$= ((6*1)/(1* 14593/30977)) - 8$

$= (185862/14593) - 8$

$= 69118/14593$

$R \approx 4.7364$ sec

The single processor with twice the speed has a better response time than the two processors. This is because the number of users is below the critical number of users for either model. When there are an adequate amount of users, the parallelism of the two server model will play a bigger factor in the utilization and response time of the processors.

2c.

$n^* = m(Z + S)/S$

$= 1(8 + 1)/1$

$n^* = 9$ users

$R = (nS/mU_p) - Z$

$= ((30*1)/(1*14593/30977)) - 8$

$= (929310/14593) - 8$

$= 812566/14593$

$R \approx 55.6819$ sec