MATHEMATICS E-156, FALL 2014

Mathematical Foundations of Statistical Software

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Proof of the Week #: 3

Theorem:

1. Proof of the week

   Prove that the sum of n independent Bernoulli random variables, each with parameter p, is a binomial random variable $Y \sim \text{Binom}(n, p)$, and that

   $$E[Y] = np, \text{ Var } Y = np(1-p).$$

   (a) A Bernoulli random variable X has the value 1 with probability p, 0 with probability $1-p$. Calculate its expectation and variance.

   (b) (The easy way) A binomial random variable Y is the sum of n independent Bernoulli random variables: $Y = X_1 + X_2 + \cdots + X_n$. Calculate its expectation and variance from this property alone.

   (c) If Y has the value r, then r of the X_i have the value 1, $n-r$ have the value 0. Calculate the probability of this event, which can happen in many ways, and so determine the mass (density) function $P(Y = r)$.

Proof:

1. (a)

   $$E[X] = (1-p) * 0 + p * 1 = p$$

   $$E[X^2] = (1-p) * 0 + p * 1 = p$$

   $$Var[X] = E[X^2] - (E[X])^2 = p - p^2 = p(1-p)$$

   (b)

   $$Y = X_1 + X_2 + X_3 + ... + X_n$$

   $$E[Y] = \sum E[X_i] = np$$

   $$Var[Y] = \sum Var[X_i] = np(1-p)$$

   (c)

   $$P(Y = r) = \binom{n}{r} p^r (1-p)^{n-p}$$