Proof 4-3
Larson Hogstrom - Math E156 - 2014

Prove that the Poisson distribution with parameter λ has mean and variance both equal to λ. Solution:

$$E[X] = \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$E[X] = \sum_{x=1}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$E[X] = \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-1)!}$$

$$E[X] = \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda\lambda^{x-1}}{(x-1)!}$$

$$E[X] = e^{-\lambda}\lambda \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

let k = x-1

$$E[X] = e^{-\lambda}\lambda \sum_{k=1}^{\infty} \frac{\lambda^k}{k!}$$

$$E[X] = e^{-\lambda}\lambda e^{\lambda} = \lambda$$

* Taylor Series expansion of exponential function

Prove that if X_1 and X_2 are independent Poisson random variables with parameters λ_1 and λ_2 respectively, then $X_1 + X_2$ is Poisson with parameter $\lambda_1 + \lambda_2$.

$$P(X_1 + X_2 = m) = \sum_{j=0}^{m} P(X_1 = j)P(X_2 = m - j)$$

$$= \sum_{j=0}^{m} e^{-\lambda_1} \frac{\lambda_1^j}{j!} e^{-\lambda_2} \frac{\lambda_2^{m-j}}{(m-j)!}$$

$$= \sum_{j=0}^{m} e^{-(\lambda_1+\lambda_2)} \frac{\lambda_1^j}{j!} \frac{\lambda_2^{m-j}}{(m-j)!}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{m!} \sum_{j=0}^{m} \frac{m!}{j!(m-j)!} \lambda_1^j \lambda_2^{m-j}$$

*Binomial to obtain $(\lambda_1 + \lambda_2)^m$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{m!} (\lambda_1 + \lambda_2)^m$$