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Math E-156

Week 5 Proof of the Week

$M(t) = E[e^{tX}]$

$e^{tX} = 1 + tX + \frac{(tX)^2}{2!} + \frac{(tX)^3}{3!} + \cdots$

$M(t) = E\left[1 + tX + \frac{(tX)^2}{2!} + \frac{(tX)^3}{3!} + \cdots\right] = 1 + tE[X] + \frac{t^2}{2!}E[X^2] + \frac{t^3}{3!}E[X^3] + \cdots$

$M'(t) = 0 + E[X] + tE[X^2] + \frac{t^2}{2!}E[X^3] + \frac{t^3}{3!}E[X^4] + \cdots$

$M'(0) = E[X]$

Eliminating all terms afterwards which are multiplied by t

$M''(t) = 0 + 0 + E[X^2] + tE[X^3] + \frac{t^2}{2!}E[X^4] + \cdots$

$M''(0) = E[X^2]$

Proceeding along the same logic,

$M^{(n)}(0) = E[X^n]$

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Given independent random variables X_1, X_2 with moment generating functions $M_1(t)$ and $M_2(t)$,

$M_{X_1+X_2}(t) = E\left[e^{t[X_1+X_2]}\right] = E[e^{tX_1}e^{tX_2}] = E[e^{tX_1}]E[e^{tX_2}] = M_{X_1}(t)M_{X_2}(t)$

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