Monty Hall Solution

14 May 2021

$$X = \text{Location of Prize}$$
$$Y = \text{Contestant's First Choice}$$
$$Z = \text{Monty Hall's Choice}$$

$$X \sim \text{Uniform}(\{1, 2, 3\})$$
$$Y \sim \text{Uniform}(\{1, 2, 3\})$$
$$Z \mid X, Y \nsim \text{Uniform}(\{1, 2, 3\})$$

```r
Pxyz <- array(dim = c(3, 3, 3))

Pxyz[1, , ] <- matrix(c(0, 0, 0, 1/2, 0, 1, 1/2, 1, 0), nrow = 3)

Pxyz[2, , ] <- matrix(c(0, 1/2, 1, 0, 0, 0, 1, 1/2, 0), nrow = 3)

Pxyz[3, , ] <- matrix(c(0, 1, 1/2, 1, 0, 1/2, 0, 0, 0), nrow = 3)

Pxyz <- 1/3*1/3*Pxyz
```

$$P(X \mid Y, Z) = P(X, Y, Z)/P(Y, Z)$$

```r
Pyz <- matrix(0, nrow = 3, ncol = 3)

for (x in 1:3){
  for (y in 1:3){
    for (z in 1:3){
      Pyz[y, z] <- Pyz[y, z] + Pxyz[x, y, z]
    }
  }
}

PxGyz <- array(dim = c(3, 3, 3))

for (x in 1:3){
  for (y in 1:3){
    for (z in 1:3){
      PxGyz[x, y, z] <- Pxyz[x, y, z] / Pyz[y, z]
    }
  }
}
```

$$X = \text{Location of Prize}$$
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PxGyz

```
## , , 1
##
##      [,1]      [,2]      [,3]
## [1,]  NaN 0.0000000 0.0000000
## [2,]  NaN 0.3333333 0.6666667
## [3,]  NaN 0.6666667 0.3333333
##
## , , 2
##
##           [,1] [,2]      [,3]
## [1,] 0.3333333  NaN 0.6666667
## [2,] 0.0000000  NaN 0.0000000
## [3,] 0.6666667  NaN 0.3333333
##
## , , 3
##
##           [,1]      [,2] [,3]
## [1,] 0.3333333 0.6666667  NaN
## [2,] 0.6666667 0.3333333  NaN
## [3,] 0.0000000 0.0000000  NaN
```