Pre-class Assessment 2 $_{ m MA~220}$

1. (a) Use differentiation to determine an extremum of the function

$$f(x) = \begin{cases} 10x^2 + 5x + 1 & : x \in (-10, 10) \\ 0 & : \text{ otherwise} \end{cases}$$

(b) Is the extrema you found in the previous part a maximum or a minimum of f?

(c) What is the value of f at this point?

(d) What are the other extrema of f? **Hint:** Calculus will not help here.

2. Compute the first derivatives with respect to x of the following functions:

(a)
$$g(x) = e^x$$

(b)
$$h(x) = e^{-x^2}$$

3. Evaluate the following definite integrals:

(a)
$$\int_a^b \frac{1}{b-a} dx$$
 where a and b are constants

(b)
$$\int_0^2 4x - 2x^2 dx$$

(c)
$$\int_0^\infty ce^{-cx} dx$$
 where c is a positive constant

(d)
$$\int_0^\infty x e^{-x} \, dx$$

4. What is the power series expansion (aka, Taylor series) of e^x ? What is its radius of convergence?

5. Expand $(a+b)^k$, where $k \in \mathbb{N}$, using the Binomial Theorem. Your answer should be in summation notation, *i.e.* in the form

$$(a+b)^k = \sum_{n=?}^{?}$$
?

- 6. How does $e^x e^y$ simplify?
- 7. Prove De Morgan's Laws. You may use a Venn Diagram in your proof.

Reminder: \cup denotes union, \cap denotes intersection, and ' denotes complement.

(a)
$$(A \cup B)' = A' \cap B'$$

(b)
$$(A \cap B)' = A' \cup B'$$