

# Followers Are Not Enough: Beyond Structural Communities in Online Social Networks

## Abstract

Community detection in online social networks is typically based on the analysis of the explicit connections between users, such as “friends” on Facebook and “followers” on Twitter. But online users often have hundreds or even thousands of such connections, and many of these connections do not correspond to real friendships or more generally to accounts that users interact with. We claim that community detection in online social networks should be question-oriented and rely on additional information beyond the simple structure of the network. The concept of ‘community’ is very general, and different questions such as “who do we interact with?” and “with whom do we share similar interests?” can lead to the discovery of different social groups. In this paper we focus on three types of communities beyond structural communities: activity-based, topic-based, and interaction-based. We analyze a Twitter dataset using three different weightings of the structural network meant to highlight these three community types, and then infer the communities associated with these weightings. We show that the communities obtained in the three weighted cases are highly different from each other, and from the communities obtained by considering only the unweighted structural network. Our results confirm that asking a precise question is an unavoidable first step in community detection in online social networks, and that different questions can lead to different insights into the network under study.

## Introduction

Networks play a central role in online social networks like Twitter, Facebook, and Google+. These services allow a user to interact with others based on the online social network they curate, also known as contact filtering (Cazabet, Leguistin, and Amblard 2012). For example, ‘friends’ on Facebook represent reciprocal links for sharing information, while ‘followers’ on Twitter allow a single user to broadcast information in a one-to-many fashion. Central to all of these interactions is the fact that the *structure* of the social network dictates the ways that information can be broadcast or diffuse through the service.

Because of the importance of structural networks, a large amount of work has focused on using structural networks to detect communities in online social media. Here, we use the term community with the standard definition: a collection of nodes (users) within the network who are more highly connected to each other than to users outside of the community (Newman and Girvan 2004). For instance, in (Java et al. 2009), the authors use a friendship network to determine communities within Twitter, and note that conversations tend to occur within these communities. The approach of focusing on structural networks makes sense for ‘real-world’ sociological experiments, where obtaining additional information about users interactions may be expensive and time-consuming. However, with the prevalence of large, rich data sets for online social networks, additional information beyond the structure alone may be incorporated, and more realistically reflects how users interact with each other on social media services (Grabowicz et al. 2012).

A large body of work exists on methods for automatic detection of communities within networks (Newman 2004; Newman and Girvan 2004; Rosvall and Bergstrom 2008; Blondel et al. 2008; Lancichinetti et al. 2011). See (Fortunato 2010) for a recent review. All these methods *begin* with a given network, and then attempt to uncover structure present in the network, i.e., they are agnostic to how the network was constructed. As opposed to this agnostic analysis, we propose and illustrate the importance of a deeper analysis. We believe that in order to understand the communities present in a data set, it is important to begin with a clear picture of the community type under consideration, and then to tailor the construction of the network and the use of detection algorithms towards that goal.

This is especially true for social network analysis. In online social networks, ‘community’ could refer to several possible structures. The simplest definition of community, as we have seen, might stem from the network of explicit connections between users on a service (friends, followers, etc.). On small time scales, these connections are more or less static, and we might instead determine communities based on who is talking to whom. On a more abstract level, a user might consider themselves part of a community of people discussing similar topics. We might also define communities as collections of people who exhibit similar behaviors on a service, as in a communities of teenagers vs. elderly users.

We can characterize these types of communities based on the types of questions we might ask about them:

- **Structure-based:** Who are you friends with? Who do you follow?
- **Activity-based:** Who do you act like?
- **Topic-based:** What do you talk about?
- **Interaction-based:** Who do you talk to? Who do you talk about?

We propose looking at when and how communities motivated by different questions overlap, and whether different approaches to asking the question, “What community are you in?” leads to different insights about a social network. For example, a user on Twitter might connect mostly with computational social scientists, talk mostly about machine learning, interact solely with close friends (who may or may not be computational social scientists), and utilize the service only on nights and weekends. Each of these different ‘profiles’ of the user highlight different views of the user’s social network, and represent different types of communities. We divide our approaches into four categories based on the questions outlined above: structure-based, activity-based, topic-based, and interaction-based. The structure-based approach, as outlined above, is most common, and for our data relies on reported follower relationships.

The activity-based approach is motivated by the question of which individuals act in a concerted manner, e.g., which users use a service with similar frequency and at similar times of day. The main tools for answering this question stem from information theory. We consider each user on an online social network as an information processing unit, but ignore the content of their messages. This viewpoint has been successfully applied to gain insight into local behavior in online social networks (Darmon et al. 2013b). The information processing framework applies equally well to spatially extended systems. In particular, our current activity-based approach was originally motivated by a methodology used to detect functional communities within populations of neurons (Shalizi, Camperi, and Klinkner 2007). This approach has been extended to social systems, detecting communities on Twitter based on undirected information flow (Darmon et al. 2013a). Others have successfully applied a similar viewpoint using transfer entropy, a measure of directed information flow, to perform link detection on Twitter (Ver Steeg and Galstyan 2012). However, transfer entropy has not been used for community detection, and we extend this previous work to incorporate transfer entropy-based communities.

Our topic-based and interaction-based approaches, in contrast to the activity-based approach, rely on the *content* of a user’s interaction and ignore its temporal component. The content contains a great deal of information about the communication between users. For example, a popular approach to analyzing social media data is to use Latent Dirichlet Allocation (LDA) to infer topics based on the prevalence of words within a status (Zhao et al. 2011; Michelson and Macskassy 2010). The LDA model can then be used to infer distributions over latent topics, and the

similarity of two users with respect to topics may be defined in terms of the distance between their associated topic distributions. Because our focus is not on topic identification, we apply a simpler approach using hashtags as a proxy for topics (Becker, Naaman, and Gravano 2011; Tsur and Rappoport 2012). We can then define the similarity of two users in terms of their hashtags, and use this similarity to build a topic-based network.

The interaction-based approach relies on meta-data about who a user converses with on the social media service. On Twitter, we can use mentions (indicating a directed communication) and retweets (indicating endorsement of another user) to identify conversation. Moreover, we can define a directed influence between two users by considering the attention paid to that user compared to all other users. This allows us to generate a network based on conversations and user interactions.

The activity-based, topic-based, and interaction-based networks allow us to build a more complete picture of the *latent* social network present in online social media, as opposed to the *explicit* social network indicated by structural links. In this paper, we explore the relation between these various possible networks and their corresponding communities. We begin by describing the methodologies used to generate the various types of networks, and infer their community structure. We then explore how the communities of users differ depending on the type of network used. **TK: Put a blurb here about the qualitative study.** Finally, we explore how communication patterns differ across and within the different community types.

## Related Work

Most previous work on communities in online social networks have focused on detecting communities based on a single type of network. For example, an early paper considered the communities, and associated statistics, inferred from a friendship network in Twitter (Java et al. 2009). More recent work has considered the dynamics of communities based on structural links in Facebook (Nguyen et al. 2011).

Information theoretic, activity-based approaches have been applied previously to the analysis of networks arising in online social media (Ver Steeg and Galstyan 2012; Darmon et al. 2013a), but to the best of our knowledge this is the first use of transfer entropy for community detection.

For interaction-based networks, (Conover et al. 2011) considered both mention and retweet networks in isolation for a collection of users chosen for their political orientation. In (Deitrick and Hu 2013), the authors construct a dynamic network based on simple time-windowed counts of mentions and retweets, and use the evolution of this network to aid in community detection.

There are two broad approaches to topic-based communities in the literature. (Rossi and Magnani 2012) used a set of users collected based on their use of a single hashtag, and tracked the formation of follower and friendship links within that set of users. In (Lim and Datta 2012a), the authors chose a set of topics to explore, and then seeded a network from a celebrity chosen to exemplify a particular topic. Both approaches thus begin with a particular topic in mind, and

perform the data collection accordingly. Other approaches use probabilistic models for the topics that treat community membership as a latent variable (Yin et al. 2012).

A notable exception to the analysis of isolated types of networks is (Lim and Datta 2012b), which considered both structure-based and interaction-based communities on Twitter. However, this study focuses on data collected based on particular topics (country music, tennis, and basketball), and not on a generic subpopulation of Twitter users. Moreover, it did not explore the differences in community structure resulting from the different network weightings, and focuses on aggregate statistics (community size, network statistics, etc.). Another notable exception is (Kao et al. 2013), where the authors use a tensor representation of user data to incorporate retweet and hashtag information into a study of the social media coverage of the Occupy Movement. The tensor can then be decomposed into factors in a generalization of the singular value decomposition of a matrix, and these factors can be used to determine ‘salient’ users. However, this approach focused on data for a particular topic (the Occupy Movement) and did not collect users based on a structural network.

## Methodology

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### Community Detection

As discussed in the introduction, we adopt the usual definition of *community*: a collection of nodes (users) within a network who are more densely connected to each other than with the rest of the network. Structural community detection is a well studied problem and several different methods and algorithms have been proposed; for a complete review of this subject we refer the reader to (Fortunato 2010). In this paper however we focus on a class of networks and communities that is far less studied, in particular we study networks which are both *weighted* and *directed* and communities within those weighted directed networks that can (but need not) *overlap*. When selecting a detection algorithm we propose that all three (weight, direction and overlap) are important for the following reasons: Twitter is a directed network and we wish to maintain this structure. But we are interested in not just structure but function, to capture function we use edge weightings which must be used in the community detection process. Finally, since people can belong to multiple and possibly overlapping social (e.g., college friends, co-workers, family, etc.) and topical (e.g., a user can be interested in both cycling and politics and use the network to discuss the two topics with the two different communities) communities, we are interested in finding *overlapping* modules, rather than partitions of the weighted directed network.

This poses an interesting problem because the majority of community detection algorithms developed so far are built to find partitions of a network and very few are aimed at finding overlapping communities (Baumes et al. 2005; Palla et al. 2005; Zhang, Wang, and Zhang 2007; Gregory 2007;

Nepusz et al. 2008; Lancichinetti, Fortunato, and Kertész 2009a; Evans and Lambiotte 2009; Kovács et al. 2010). Among these methods, even fewer deal with directed or weighted networks. (Palla et al. 2005) clique percolation method can for example take into account these two features, but not both at the same time. (Lancichinetti et al. 2011) recently proposed OSLOM (Order Statistics Local Optimization Method) which is the first method that is able to deal with all of these features at the same time; their method “is based on the local optimization of a fitness function expressing the statistical significance of clusters with respect to random fluctuations” (**taken from their paper, to rephrase**). For this paper we use OSLOM as it allowed us to detect *overlapping* communities present in our *weighted and directed* network.

### The Initial Dataset and Network Construction

The dataset for this study consisted of the tweets of 15,000 Twitter users over a 9 week period (from April 25th to June 25th 2011). The users are embedded in a network collected by performing an intelligent breadth-first expansion from a random seed user. In particular, once the seed user was chosen, the network was expanded to include his/her followers, but only included users considered to be ‘active’ (i.e., users who tweeted at least once per day over the past one hundred days). Network collection continued in this fashion by considering the active followers of the active followers of the seed, and so on until 15,000 users were added to the network.

Since our goal is to explore the functional communities of this network, we filter this network down to the subset of users which actively interact with each other (e.g., retweets and mentions). We do this through a metric we call (incoming/outgoing) information events. We define an outgoing information event for a given user  $u$  as either a mention made by  $u$  of another user in the network, or if another user in the network retweets a tweet made by  $u$ . The logic for this definition is as follows, if  $u$  mentions a user  $v$  this can be thought of as  $u$  directly sending information to  $v$ , and if  $u$  is retweeted by  $v$  then  $v$  received information from  $u$  and rebroadcast it to their followers. In either case there was information outgoing from  $u$  which affected the network in the same way. Analogously, we define the incoming information event for  $u$  as either being mentioned by a different user in the network, or by retweeting another user in the network. With (incoming/outgoing) information events defined we filtered the network by eliminating all users with less than 9 outgoing *and* incoming information events, i.e., less than one information event per type per week on average. We then further restricted our analysis to the strong giant connected component of the network built from the (incoming/outgoing) information filtered set of users. In this study the link is directed from the user to the follower because this is the direction in which the information flows. **[[Josh: Is this the case for all the analysis or just the transfer entropy?]]** The final network consists of 6,917 nodes and 1,481,131 edges.

## Activity-Based Communities and Transfer Entropy

For the activity-based communities, we consider only the timing of each users tweets and ignore any additional content. From this starting point, we can view the behavior of a user  $u$  on Twitter as a point process, where at any instant  $t$  the user has either emitted a tweet ( $X_t(u) = 1$ ) or remained silent ( $X_t(u) = 0$ ). This is the view of a user's dynamics taken in (Ver Steeg and Galstyan 2012) and (Darmon et al. 2013b). Thus, we reduce all of the information generated by a user on Twitter to a time series  $\{X_t(u)\}$  where  $t$  ranges over the time interval for which we have data (9 weeks in this case). Because status updates are only collected in discrete, 1-second time intervals, it is natural to consider the only the discrete times  $t = 1, 2, \dots$ , relative to a reference time. We can then compute the flow of information between two users  $u$  and  $v$  by computing the transfer entropy between their time series  $X_t(u)$  and  $X_t(v)$ .

Let  $\{X_t\}$  and  $\{Y_t\}$  be two strong-sense stationary stochastic processes. We use the notation  $X_{t-k}^t$  to denote the values of the stochastic process from time  $t-k$  to time  $t$ ,  $X_{t-k}^t = (X_{t-k}, X_{t-k-1}, \dots, X_{t-1}, X_t)$ . The lag- $k$  transfer entropy of  $Y$  on  $X$  is defined as

$$\text{TE}_{Y \rightarrow X}^{(k)} = H[X_t | X_{t-k}^{t-1}] - H[X_t | X_{t-k}^{t-1}, Y_{t-k}^{t-1}], \quad (1)$$

where

$$H[X_t | X_{t-k}^{t-1}] = -E[\log_2 p(X_t | X_{t-k}^{t-1})] \quad (2)$$

and

$$H[X_t | X_{t-k}^{t-1}, Y_{t-k}^{t-1}] = -E[\log_2 p(X_t | X_{t-k}^{t-1}, Y_{t-k}^{t-1})] \quad (3)$$

are the usual conditional entropies over the conditional (predictive) distributions  $p(x_t | x_{t-k}^{t-1})$  and  $p(x_t | x_{t-k}^{t-1}, y_{t-k}^{t-1})$ . This formulation was originally developed in (Schreiber 2000), where transfer entropy was proposed as an information theoretic measure of *directed* information flow. Formally, recalling that  $H[X_t | X_{t-k}^{t-1}]$  is the uncertainty in  $X_t$  given its values at the previous  $k$  time points, and that  $H[X_t | X_{t-k}^{t-1}, Y_{t-k}^{t-1}]$  is the uncertainty in  $X_t$  given the joint process  $\{(X_t, Y_t)\}$  at the previous  $k$  time points, transfer entropy measures the reduction in uncertainty of  $X_t$  by including information about  $Y_{t-k}^{t-1}$ , controlling for the information in  $X_{t-k}^{t-1}$ . By the ‘conditioning reduces entropy’ result (Cover and Thomas 2012)

$$H[X | Y, Z] \leq H[X | Y], \quad (4)$$

we can see that transfer entropy is always non-negative, and is zero precisely when  $H[X_t | X_{t-k}^{t-1}] = H[X_t | X_{t-k}^{t-1}, Y_{t-k}^{t-1}]$ , in which case knowing the past  $k$  lags of  $Y_t$  does not reduce the uncertainty in  $X_t$ . If the transfer entropy is positive, then  $\{Y_t\}$  is considered causal for  $\{X_t\}$  in the Granger sense (Granger 1963).

In estimating the transfer entropy from finite data, we will assume that the process  $(X_t, Y_t)$  is jointly stationary, which gives us that

$$p(x_t | x_{t-k}^{t-1}) = p(x_{k+1} | x_1^k) \quad (5)$$

and

$$p(x_t | x_{t-k}^{t-1}, y_{t-k}^{t-1}) = p(x_{k+1} | x_1^k, y_1^k) \quad (6)$$

for all  $t$ . That is, the predictive distribution only depends on the past, not on when the past is observed<sup>1</sup>. Given this assumption, we compute estimators for  $p(x_{k+1} | x_1^k)$  and  $p(x_{k+1} | x_1^k, y_1^k)$  by ‘counting’: for each possible past  $(x_1^k, y_1^k)$ , we count the number of times a future of type  $x_{k+1}$  occurs, and normalize. Call these estimators  $\hat{p}(x_{k+1} | x_1^k)$  and  $\hat{p}(x_{k+1} | x_1^k, y_1^k)$ . Then the plug-in estimator for the transfer entropy is

$$\widehat{\text{TE}}_{Y \rightarrow X}^{(k)} = \hat{H}[X_t | X_{t-k}^{t-1}] - \hat{H}[X_t | X_{t-k}^{t-1}, Y_{t-k}^{t-1}] \quad (7)$$

where we use the plug-in estimators  $\hat{H}[X_t | X_{t-k}^{t-1}]$  and  $\hat{H}[X_t | X_{t-k}^{t-1}, Y_{t-k}^{t-1}]$  for the entropies. It is well known that the plug-in estimator for entropy is biased (Paninski 2003). To account for this bias, we use the Miller-Madow adjustment to the plug-in estimator (Miller 1955).

For the communities based on transfer entropy, we weight each edge from a user  $u$  to a follower  $f$  by the estimated transfer entropy of the user  $u$  on  $f$ ,

$$w_{u \rightarrow f}^{\text{TE}(k)} = \widehat{\text{TE}}_{X(u) \rightarrow X(f)}^{(k)} \quad (8)$$

Operationally, we expect users to interact with Twitter on a human time scale, and thus the natural one-second time resolution is too fine. We coarsen each time series by considering non-overlapping time intervals ten minutes in length. For each time interval, we record a 1 if the user has tweeted during that time interval, and a 0 if they have not. Thus, the new coarsened time series now captures whether or not the user has been active on Twitter over any given ten minute time interval in our data set. We then compute the transfer entropy on these coarsened time series taking  $k$  to range from 1 to 6, which corresponds to a lag of ten minutes to an hour. The choice of lag must balance a trade-off between additional information and sparsity of samples: as we increase the lag, we account for longer range dependencies, but we also decrease the number of samples available to infer a higher dimensional predictive distribution. Ultimately, due to similarities in the underlying communities (see Figure 2), we chose a lag-4 transfer entropy. All references to activity-based weights, unless otherwise noted, refer to this choice of lag.

## Interaction-Based Communities and Mention / Retweet Weighting

Retweets and mentions are two useful features of Twitter networks which can be used to track information flow through the network. With mentions users are sending directed information to other users and with retweets users are rebroadcasting information from a user they follow to all of their followers. This type of information flow defines a community in a much different way than transfer entropy

<sup>1</sup>We really only need *conditional* stationarity (Caires and Ferreira 2003), but stationarity implies conditional stationarity.

did. Instead of defining communities by the loss of uncertainty in one users tweeting history based on anothers, we define an interaction-based communities by weighting the user-follower network with a measure proportional to the number of mentions and/or retweets between users. For the interaction-based communities we consider three weighting schema: proportional retweets

$$w_{u \rightarrow f}^R = pR = \frac{\# \text{ retweets of } u \text{ by } f}{\# \text{ total retweets made by } f} \quad (9)$$

proportional mentions

$$w_{u \rightarrow f}^M = pM = \frac{\# \text{ mentions of } f \text{ by } u}{\# \text{ total mentions of } f} \quad (10)$$

and mention-retweet as the arithmetic mean of these two measures:

$$w_{u \rightarrow f}^{MR} = \frac{(pM + pR)}{2}. \quad (11)$$

### Topic-based Communities and Hashtag Weighting

The final community we consider is a topic-based or topical community, i.e., a community defined by the content (topics) they discuss. So in a topical community, users are defined to be a member of a community if they tweet *about* similar topics as other members of the community. In order to detect the topical communities, we weight the edges of the user-follower network through a measure based on the number of common hashtags between pairs of users. Hashtags are a good proxy for a tweets content as hashtags are explicitly meant to be keywords indicating the topic of the tweet. Moreover they are widely used and straightforward to detect.

To this end, we characterize each user  $u$  by a vector  $\vec{h}(u)$  of length equal to the number of unique hashtags in the dataset, and whose elements are defined as

$$h_i(u) = n_i(u) * \log \frac{N}{n_i} \quad (12)$$

where  $n_i(u)$  is the frequency of hashtag  $i$  occurring in user  $u$ 's tweets,  $N$  is the total number of users, and  $n_i$  is the number of users that have used the hashtag  $i$  in their tweets. This adapted term frequency-inverse document frequency (tf-idf) measure (Salton and McGill 1983) captures the importance of a hashtag in the users's tweets through the first factor, but at the same time smooths it through the second factor by giving less importance to hashtags that are too widely used (as  $\frac{N}{n_i}$  approaches one, its logarithm approaches zero).

For the topical communities we weight each directed edge from a user  $u$  to a follower  $f$  with the cosine similarity of their respective vectors  $\vec{h}(u)$  and  $\vec{h}(f)$ :

$$w_{u \rightarrow f}^{HT} = \frac{\vec{h}(u) \cdot \vec{h}(f)}{\|\vec{h}(u)\| \|\vec{h}(f)\|}. \quad (13)$$

This weight captures the similarity between users in terms of topic discussed in their tweets.

## Results and Discussion

### Comparing Aggregate Statistics of Community Structure

We begin by examining the overall statistics for the communities inferred by OSLOM using the weightings defined in the previous sections. The number of communities by community type is given in Table 1. We see that the topic- and interaction-based networks admit the most communities. The activity-based network admits the least number of communities. One advantage of the OSLOM over many other community detection algorithms is that it explicitly accounts for singleton 'communities': those nodes who do not belong to *any* extant communities. This is especially important when a network is collected via a breadth-first search, as in our network, where we begin from a seed node and then branch out. Such a search, once terminated, will result in a collection of nodes on the periphery of the network that may not belong to any community in the core.

The number of singletons by community type is also shown in Table 1. We see that the topic- and interaction-based communities have the most singletons, with the activity-based community dominating this measure. This result for the activity-based community is an artifact of a property of the retweet/mention weighting: 717 of the users were removed in the process **TK: Fix after clarification from Elisa**.

Table 1: Number of non-singleton communities and singletons by community type: S(tructural), A(ctivity -based), T(opic-based), and I(nteraction-based).

Community Type	# of Communities	# of Singletons
S	201	308
A, Lag 1	101	951
A, Lag 2	99	600
A, Lag 3	106	611
A, Lag 4	105	668
A, Lag 5	107	632
A, Lag 6	106	642
T	289	1064
I	252	2436 (1719)

Next we consider the distribution of community sizes across the community types. The complementary cumulative distribution of community sizes is given in Figure 1. Note that both axes are plotted on log-scale, and the horizontal axis begins with non-singleton communities. Thus, for a fixed community size  $c$ , Figure 1 shows the proportion of communities of size greater than  $c$  for each community type. We see that the community distributions have longer tails for the non-structural networks, and that the interaction-based network has the longest tail. The largest communities for the structural, activity-based, topic-based, and interaction-based networks have 198, 358, 338, and 811 members, respectively. Most importantly, we see that the distributions are heterogeneous across the network types, highlighting that the different networks give rise to different large-scale community structure dependent on the particular weighting of

the structural network.

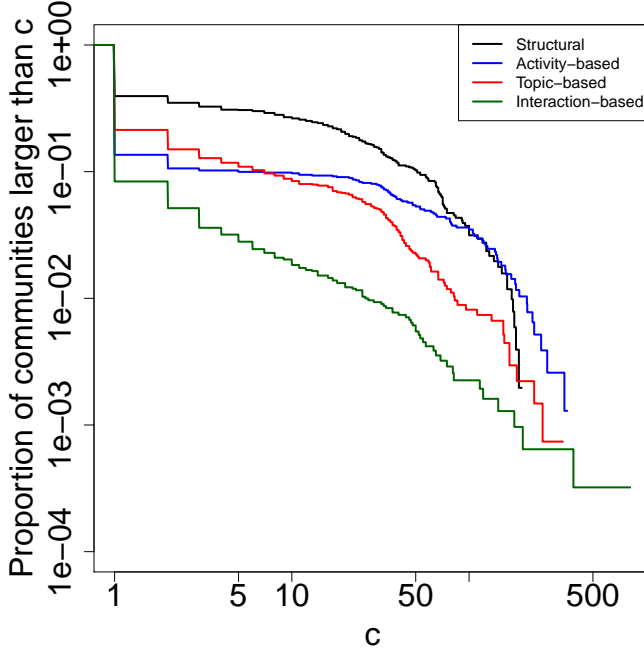


Figure 1: The proportion of communities greater than  $c$  in size, across the different community types. Note the logarithmic scale on the horizontal and vertical axes.

### Comparing Community Structure with Normalized Mutual Information

In the previous section, we saw that the large scale statistics of the communities were highly dependent on the type of community under consideration. However, macroscale network statistics do not account for differences in community structure that result from operations such as splitting or merging of communities. Moreover, this view does not account for which users belong to which communities, and in particular which users belong to the same communities across community types. To answer this question, we invoke methods for cluster comparisons: given two different clusterings of nodes into communities, how similar are the two clusters? The standard approach to answering this question is to define a metric on the space of clusterings. Because OSLOM detects *coverings* rather than *partitions* of users, standard cluster comparison metrics like variation of information (Meilă 2003) are not appropriate. Instead, we use a generalization of variation of information first introduced in (Lancichinetti, Fortunato, and Kertész 2009b), the normalized mutual information. The normalized mutual information stems from treating clustering as a community identification problem: given that we know a node’s community membership(s) in the first clustering, how much information do we have about its community membership(s) in the second clustering, and vice versa? Consider the two coverings  $\mathcal{C}_1$  and  $\mathcal{C}_2$ . We think of the community memberships of a randomly chosen node in  $\mathcal{C}_1$  as a binary random

vector  $\mathbf{X} \in \{0, 1\}^{|\mathcal{C}_1|}$  where the  $i^{\text{th}}$  entry of the vector is 1 if the node belongs to community  $i$  and 0 otherwise. Similarly,  $\mathbf{Y} \in \{0, 1\}^{|\mathcal{C}_2|}$  is a binary random vector indicating the community memberships of the node in  $\mathcal{C}_2$ . Then the normalized mutual information is defined as

$$\text{NMI}(\mathcal{C}_1, \mathcal{C}_2) = 1 - \frac{1}{2} \left( \frac{H[\mathbf{X}|\mathbf{Y}]}{H[\mathbf{X}]} + \frac{H[\mathbf{Y}|\mathbf{X}]}{H[\mathbf{Y}]} \right) \quad (14)$$

where  $H[\cdot]$  denotes a marginal entropy and  $H[\cdot|\cdot]$  denotes a conditional entropy. The normalized mutual information varies from 0 to 1, attaining the value of 1 only when  $\mathcal{C}_1$  and  $\mathcal{C}_2$  are identical coverings up to a permutation of their labels. See the appendix of (Lancichinetti, Fortunato, and Kertész 2009b) for more details.

We considered the normalized mutual information between the communities inferred from the structural network and the networks weighted with lag 1 through 6 transfer entropies, hashtag similarity, and mention, retweet, and mention-retweet activity. The resulting  $\text{NMI}(\mathcal{C}_i, \mathcal{C}_j)$  are shown in Figure 2. We see that similarity between the coverings is dictated by the generic community type (structural, activity-based, etc.). That is, the transfer entropy coverings are more similar to each other than to any of the other coverings, with a similar result for the mention, retweet, and mention-retweet coverings. Interestingly, the coverings resulting from the different weightings are all more similar to each other than to the structural covering from the unweighted network. Also note that the covering based on the hashtag similarities are different from all of the other weight-based coverings.

Thus, we see that although the activity-based, interaction-based, and topic-based communities relied on the structural network, their community structure differs *the most* from the community structure of the follower network. This agrees with the results from the previous section, and reinforces that the follower network is a necessary but not sufficient part of detecting communities in online social networks.

### Comparing Edges Across Different Community Types

We have defined communities using four different criteria: structure, activity, interaction, and topic. We next explore how edge weights, and thus different forms of information flow, differ between community types. For a fixed community type, edges for a particular community may be partitioned into three sets: those from a user in the community to another user in the community (internal-to-internal), those from a user in the community to a user outside of the community (internal-to-external), and those from a user outside the community to a user inside the community (external-to-internal). See Figure 3 for a schematic of this edge partitioning. For a meaningful community, we expect the distribution of weights within the community (internal-to-internal weights) to be different from the distribution of weights without the community (internal-to-external and external-to-internal).

As an example, Figure 4 shows the distributions of hashtag-based weights for the largest community in the mention-retweet network. We see that the distribution of

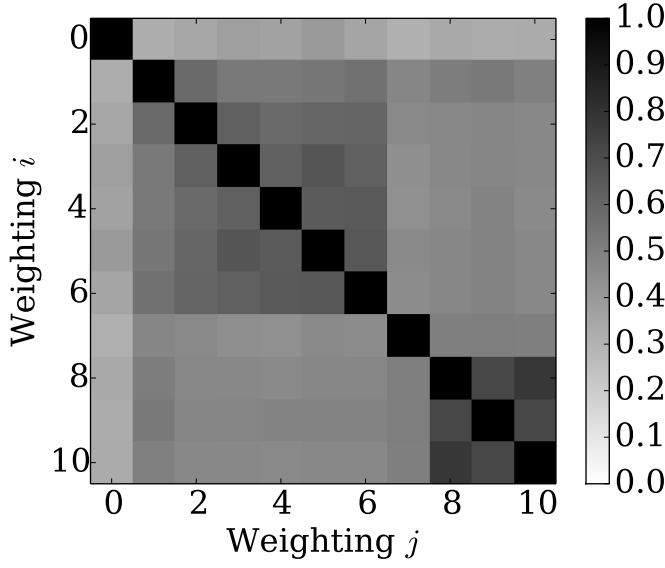


Figure 2: The normalized mutual information between the communities using the different weightings. Weighting 0 corresponds to the structural (binary weighting) network, weightings 1 through 6 correspond to the weighting using the transfer entropies with lag 1 through 6, weighting 7 corresponds to the hashtag similarity, and weightings 8, 9, and 10 correspond to the mention, retweet, and mention-retweet weightings. Values of normalized mutual information close to 1 indicate similarity in the community structure, while values close to 0 indicate dissimilarity. The normalized mutual information is computed with the singletons removed.

internal-to-internal hashtag weights has a longer tail than either the external-to-internal or internal-to-external hashtag weights, with edges within the community having higher weights than edges crossing the boundary of the community. Thus, while the community was defined in terms of interactions, we still see a shift in the distribution of topic-similarity.

This change in the tail of the distribution between edge types was typical of many of the community type / weight pairings. A useful summary statistic to quantify the change involves the median weights across the three types of edges, as demonstrated in Figure 4. In particular, by computing the ratio of the median weight for the internal-to-internal edges to the median weight for the internal/external-to-external/internal edges, we can quantify the ratio change in weight strength internal vs. external to a community. We computed this quantity for each of the top 100 largest communities defined by a particular community type (structure-based, activity-based, interaction-based, or topic-based), and report the median value across the 100 largest communities for each type in Table 2. This statistic represents the typical ratio shift for each community type / weight pairing. Values greater than 1 indicate that the edge weights tend to be higher within the community, and values less than 1 indicate that the edge weights tend to be higher for those edges

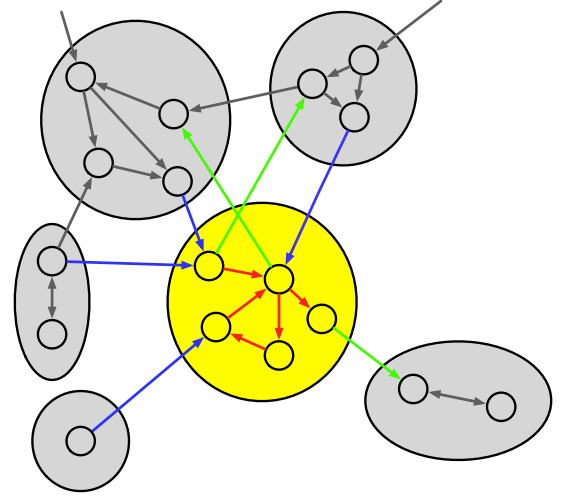


Figure 3: An example of the edges considered in determining the edge weight distribution for a given community (the focal community is in yellow). We focus on the internal-to-internal (red), internal-to-external (green), and external-to-internal (blue) edges. For a given focal community, all other edges (grey) are not considered.

crossing the community boundary.

We see that for all weight types except the transfer entropy, the weight on the edges internal to the communities tend to be higher than those entering or exiting the communities, ranging from a factor of 1.5 times larger for the activity-based/mention-retweet pairing to a factor of 28 times larger for the topic-based/hashtag similarity pairing. As stated above, we expect this ratio to be high for community / weight pairings that match (e.g. considering mention-retweet weighting for interaction-based communities), and we see that this is the case for all but the activity-based / transfer entropy pairing. Moreover, for both the mention-retweet and hashtag weightings, the ratio is largest when they match with the interaction-based and topic-based communities, respectively.

Table 2: The median value across the 100 largest communities for the ratio of the median internal-to-internal weight to the median external-to-internal / internal-to-external weight for the different community / weight pairings. Note: For mention-retweet weights, zero weight edges were excluded from the computation of the median. We indicate such cases with an asterisk.

		Weight Type		
		TE	MR	HT
Community Type	S	0.96/0.94	1.7/2.1*	9.0/8.0
	A	1.0/0.96	1.5/2.4*	24/17
	I	0.83/0.86	3.2/4.4	10/8.5
	T	0.9/0.89	2.4/2.6*	28/26



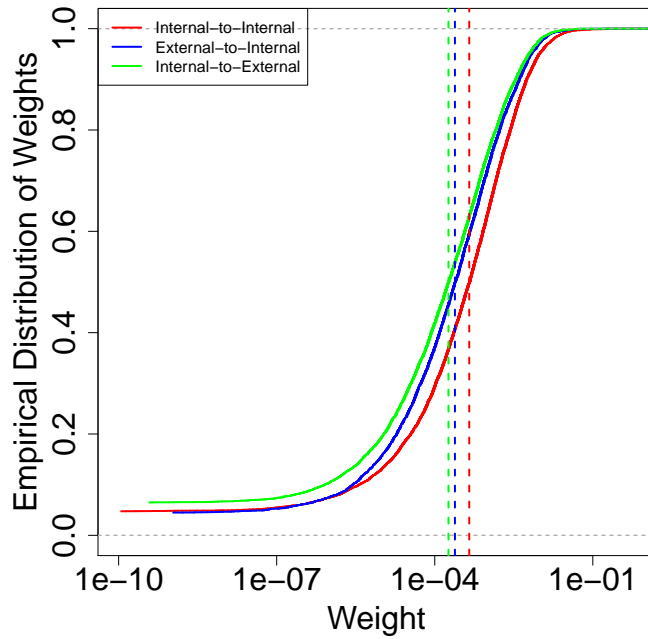


Figure 4: The proportion of edges with a weight at least as large as the weight on the horizontal axis, across the types of edges described in Figure 3. The community is defined by user interactions, and the edge weights are determined by topic similarity. The dashed horizontal lines indicate the median weight for each type of edge. Note the logarithmic scale on the horizontal axis.

For all four community types, the transfer entropy tended to be higher for edges crossing community boundaries than for those internal to community boundaries. Recall that the transfer entropy  $TE_{X(u) \rightarrow X(f)}$  quantifies the reduction in uncertainty about a follower  $f$ 's activity from knowing the activity of a user  $u$ . This result therefore implies that, in terms of prediction, it is more useful to know the time series of a user followed outside of the community compared to a user followed inside of the community. Thus, in an information theoretic sense, we see that novel information useful for prediction is more likely to flow *across* community boundaries than *within* community boundaries.

Note that the communities defined by the follower network do tend to have higher edge weights internal compared to across community boundaries. Thus, we do see that the structural communities capture some information about the functional behavior of users in terms of topics and interaction. However, the ratio is not as large as when we explicitly seek out communities based on a particular type of functional community. This again emphasizes the importance of properly formulating the goal of a community detection study in the context of online social networks.

## Qualitative Analysis of Community Memberships Across Types

As demonstrated in (Good, de Montjoye, and Clauset 2010) in the context of modularity maximization-based community detection, an exponential number of nearby partitions may exist that nearly maximize an objective function used to measure the goodness-of-fit of the graph partition used for community detection. Because of this and related issues, it is always wise to perform some sort of curated study of the communities returned by any community detection algorithm to verify their meaningfulness with respect to the scientific question at hand. In this section, we consider a collection of communities in such a study.

In the topic-based communities, we find a single community consisting of 83 users who tweet about environmental issues and frequently use hashtags such as #green, #eco and #sustainability. We also find a different community of 47 users who tweet about small businesses and entrepreneurship, using hashtags such as #smallbiz, #marketing and #entrepreneur. In both cases most members of the topic-based communities are not found in the same community in the other networks, indicating that while these people talk about the same things and can therefore be considered a community based on their content, they do not strongly interact with each other nor behave the same, and so belong to different social groups with respect to interactions and behavior.

Another interesting example is the one of a community whose topics tend focus on Denver and Colorado. These users do not belong to the same community in the interaction-based network, but most of them do belong to the same community in the activity-based network. This indicates that these users react to the same events and issues regarding Colorado and are therefore strongly connected in the topic-based and activity-based networks, but at the same time they do not directly interact with each other and are therefore more loosely connected in the interaction-based networks, where they belong to different communities.

## Conclusions

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