

Surreal Trajectories and the Quantum Potential

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Overview

Quantum Controversy! Background on Surreal Trajectories

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Surreal Walking Droplets *in Silico*

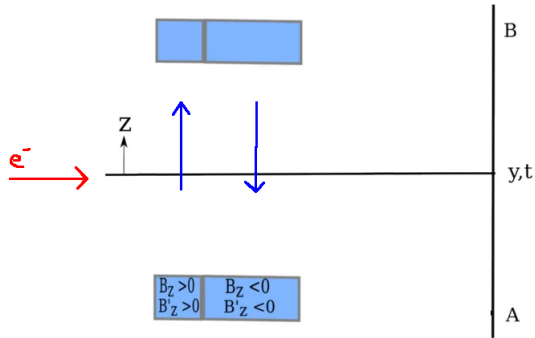
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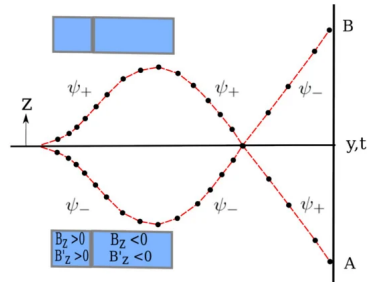
Looking Ahead: the Hydrodynamic Quantum Potential

ESSW Thought Experiment



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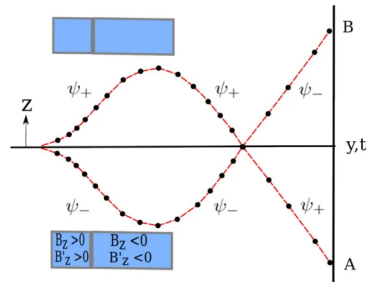
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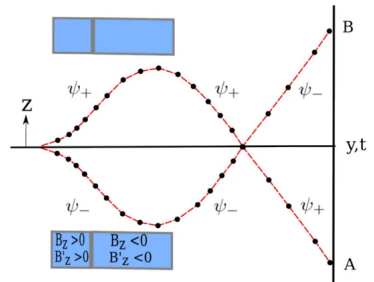
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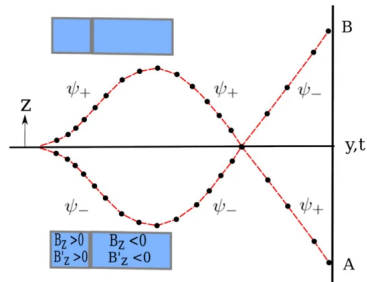
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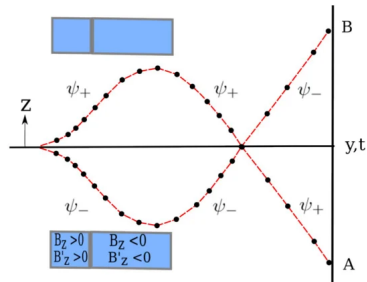


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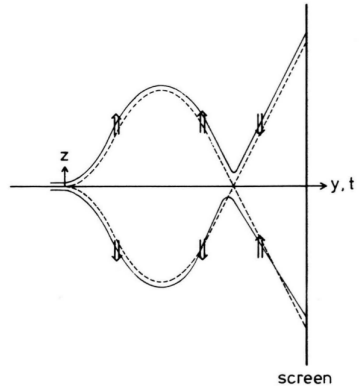
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4. Solve by decomposing

$$|\psi(z, t)\rangle = \psi_+(z, t) \otimes |\uparrow\rangle + \psi_-(z, t) \otimes |\downarrow\rangle.$$



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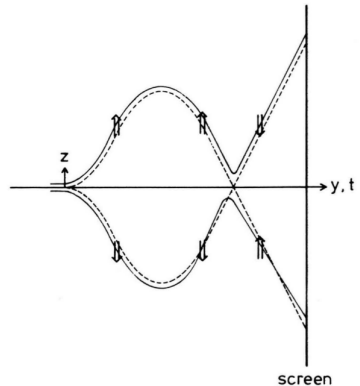
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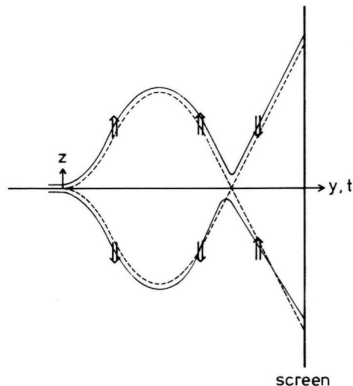
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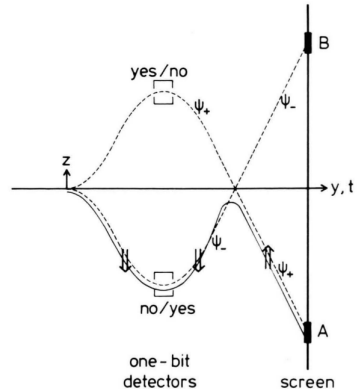
But, this contradicts the Bohmian picture!

- ▶ Symmetry arguments show that the Bohmian velocity field is odd in z .
- ▶ (ESSW, '92) calculated* the field directly, and found the right-hand picture.



Possible Experimental Evidence?

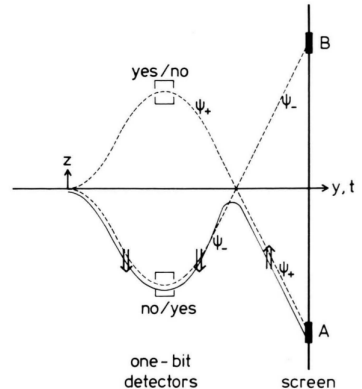
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Possible Experimental Evidence?

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They calculate that the *top* sensor would be triggered, but the Bohmian trajectory goes along the *bottom*.



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Moreover, it shows that Bohmian mechanics is “at variance with common sense” (Scully, '98).

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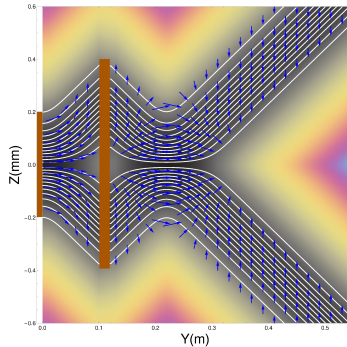
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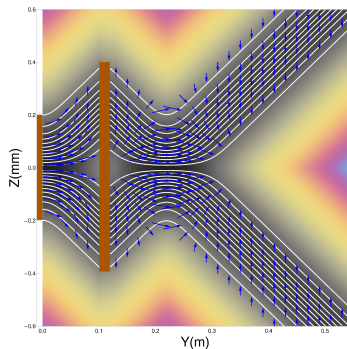
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1. Their predicted wave packets are correct, but wave packet trajectories are distinct from particle trajectories.
2. They calculated trajectories using a spin-0 version of Bohmian mechanics.
3. They neglected to show probability currents in the Schrödinger picture (shown below, calculated in (Hiley and Reeth, '18)).
4. Bohmian mechanics is entirely equivalent to Schrödinger mechanics, and they cannot be mathematically or experimentally distinguished.



Takeaways

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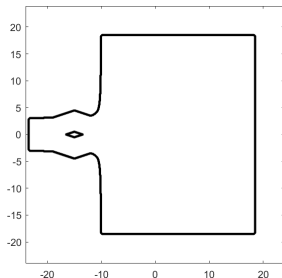
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- ▶ Surreal trajectories are not unique to Bohmian mechanics.
- ▶ Nothing is unique to Bohmian mechanics.

Can we replicate this?

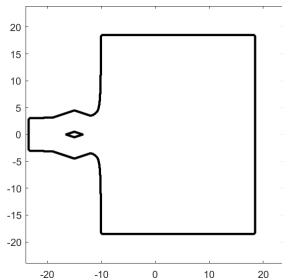
To try to replicate these dynamics *in silico*, we used a modified version of the code from (Faria, 2017).



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- ▶ $\text{memory} = 0.905$
- ▶ $r_{\text{drop}} = 3.9 \times 10^{-4}$
- ▶ $w_{\text{channel}} = 6$
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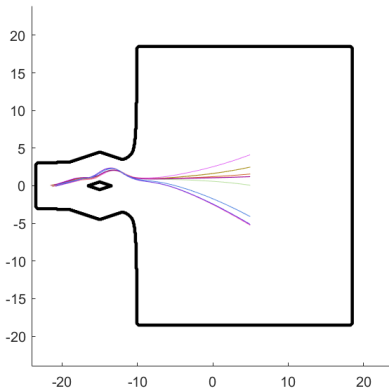
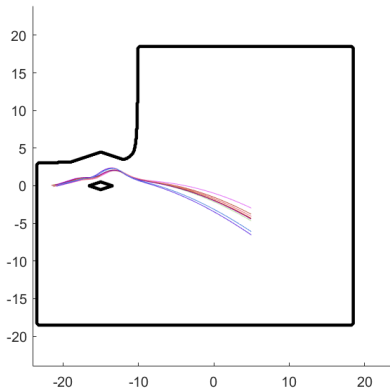
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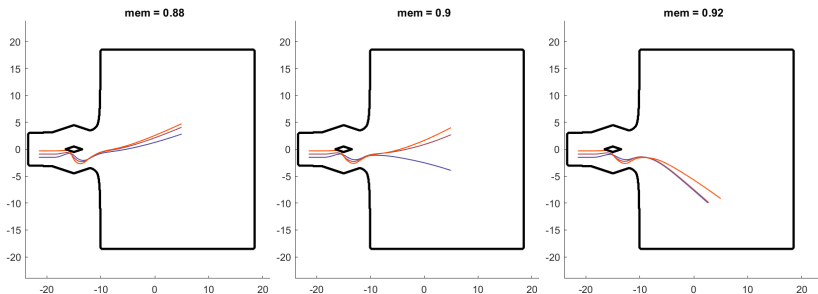


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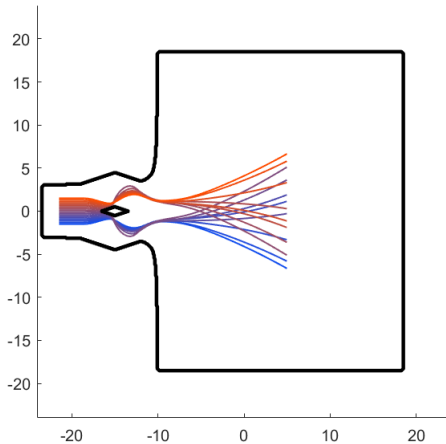


Dependence on Memory



Note (not displayed): at high memory, we see surreal trajectories without the bottom reflector, because of the finite domain.

Dependence on Impact Parameter



The Quantum Potential

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The Bohmian picture evolves according to a set of Hamilton–Jacobi equations:

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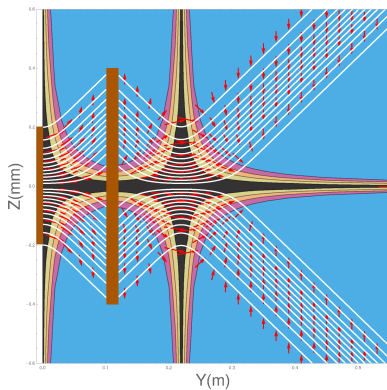
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This is simply a statistical Newtonian system with velocity field $\mathbf{J} = \nabla S$ and an extra, nonlocal potential

$$V_q = \rho^{-1/2} \nabla \rho^{1/2}.$$

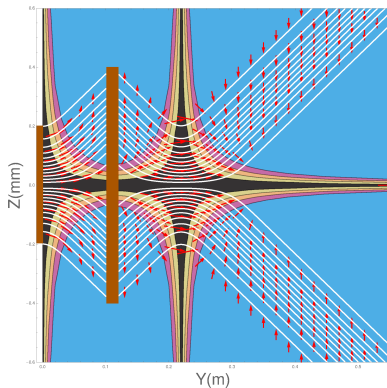
V_q Examples:

(Hiley and Reeth, '18)

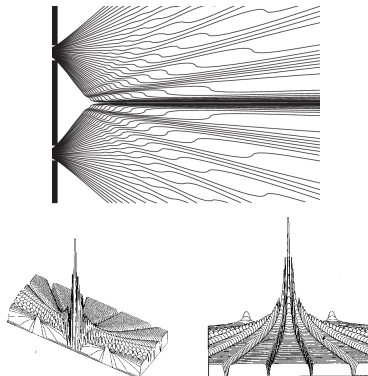


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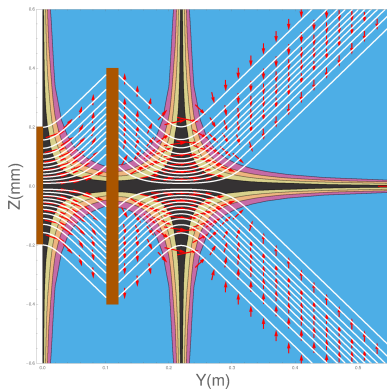


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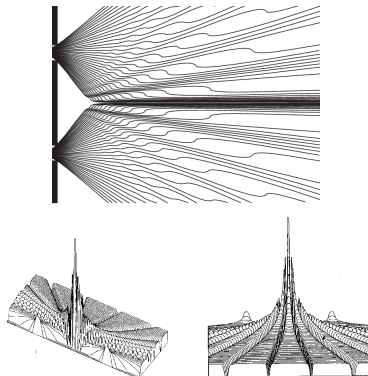


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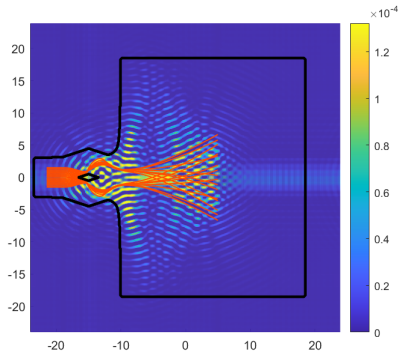
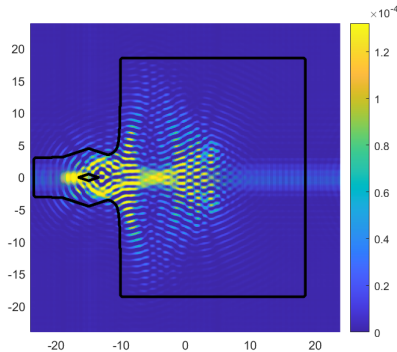


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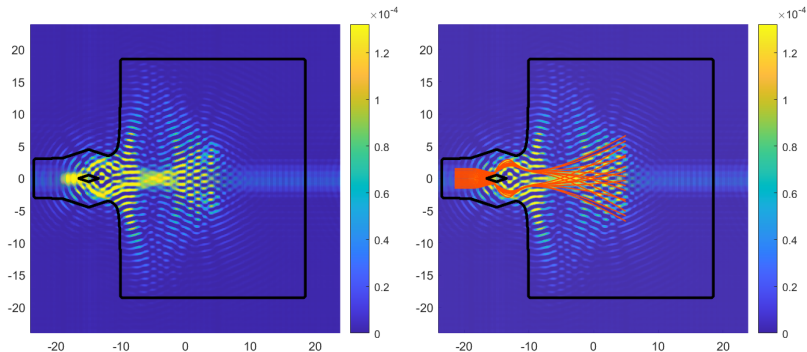


It is important to note that these are all *1-dimensional* quantum potentials, with a $y \sim t$ axis cutting across them.

$$V_q \sim \langle \text{wave height} \rangle$$



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Note similar features:

- ▶ Vertical beams following the reflectors
- ▶ Horizontal beam to the right
- ▶ Large peaks at points of re-convergence
- ▶ “Diffraction-like” front at the right

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- ▶ 1-D replica of walking droplet
- ▶ Mapping out particle trajectories themselves
- ▶ Heatmaps of stored momentum in field
- ▶ Splices—rather than averages—of wave heights