Numerical Solution of Landau-Lifshitz-Gilbert-Slonczewski equation

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Abstract

will be added later

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1. Introduction

The basic equation that governs the motion of the nanomagnet is described by Landau-Lifshitz-Gilbert-Slonczewski(LLGS) equation. It is given by [eq. (8) of Ref. [1]]

$$\frac{d\hat{m}}{dt} = -\frac{\gamma\mu_0}{1+\alpha^2}\left(\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{\gamma\mu_0\alpha}{1+\alpha^2}\left(\hat{m}\times\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{1}{(1+\alpha^2)qN_s}\left(\hat{m}\times\hat{m}\times\overrightarrow{I}_s\right) + \frac{\alpha}{(1+\alpha^2)qN_s}\left(\hat{m}\times\overrightarrow{I}_s\right) \tag{1}$$

Here $\gamma = 1.76\times 10^{11}$ rad/(S.T) is the Gyromagnetic ratio, $\mu_0 = 4\pi\times 10^{-7}$ T.m/A is the free space permeability and α is the damping constant. $\hat{m}$ is the unit magnetization vector whose dynamics we would describe by solving the LLGS equation. $\overrightarrow{H}_{eff}$ is the effective magnetic field which consists of uniaxial field($\overrightarrow{H}_{uni}$), demagnetization field($\overrightarrow{H}_{demag}$), and applied field($\overrightarrow{H}_{app}$). These fields gives the deterministic motion, but apart from that thermal noise can also influence the motion. This can be included by adding the thermal field($\overrightarrow{H}_{Therm}$) which is generated due to the thermal noise. So the effective field is given by

$$\overrightarrow{H}_{eff} = \overrightarrow{H}_{uni} + \overrightarrow{H}_{demag} + \overrightarrow{H}_{app} + \overrightarrow{H}_{Therm} \tag{2}$$

$\overrightarrow{H}_{therm}$ is given by the following formula

$$\overrightarrow{H}_{therm} = \sqrt{\frac{\alpha}{1+\alpha^2}\frac{2K_BT_K}{\gamma M_s V\delta_t}}\ \overrightarrow{G}_{0,1} \tag{3}$$

Here, $K_B = 1.38\times 10^{-23}$ J/K is the Boltzmann constant, T_K is the temperature, M_s is the saturation magnetization, V is the volume of the nanomagnet and δ_t is the simulation step. $\overrightarrow{G}_{0,1}$ is the Gaussian distribution with zero mean and unit standard deviation.

In Eq.(1), q is the electronic charge, $N_s = \frac{M_sV}{\mu_B}$ is the number of free spins that are present in the free layer of the MTJ(μ_B is the Bohr magneton). $\overrightarrow{I}_s$ in Eq.(1) denotes the spin current vector that enters into the free layer. μ_B is written as $\mu_B = \frac{\gamma\hbar}{2}$, so total number of free spins N_s can be written as

$$N_s = \frac{M_sV}{\mu_B} = \frac{2M_sV}{\gamma\hbar}$$

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Substituting this into Eq.(1), we get,

$$\frac{d\hat{m}}{dt} = -\frac{\gamma\mu_0}{1+\alpha^2}\left(\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{\gamma\mu_0\alpha}{1+\alpha^2}\left(\hat{m}\times\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{\gamma\hbar}{(1+\alpha^2)q2M_sV}\left(\hat{m}\times\hat{m}\times\overrightarrow{I}_s\right) + \frac{\alpha\gamma\hbar}{(1+\alpha^2)q2M_sV}\left(\hat{m}\times\overrightarrow{I}_s\right) \quad (4)$$

This Eq. can further be simplified by considering $\overrightarrow{I}_s = |I_s|\hat{m}_p$, where $|I_s|$ denotes the magnitude of the spin current and $\hat{m}_p$ is the spin polarization direction.

$$\frac{dm}{dt} = -\frac{\gamma\mu_0}{1+\alpha^2}\left(\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{\gamma\mu_0\alpha}{1+\alpha^2}\left(\hat{m}\times\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{\gamma\hbar|I_s|}{(1+\alpha^2)2qM_sV}(\hat{m}\times\hat{m}\times\hat{m}_p) + \frac{\alpha\gamma\hbar|I_s|}{(1+\alpha^2)2qM_sV}(\hat{m}\times\hat{m}_p) \quad (5)$$

The volume of the nanomagnet can be written as $V = A\times t_{FL}$, where A is the cross-sectional area and the t_{FL} is the thickness of the free layer MTJ. So, denoting $\frac{|I_s|}{A} = J_{MTJ}$, spin current density, we can modify Eq. (5), such that,

$$\frac{dm}{dt} = -\frac{\gamma\mu_0}{1+\alpha^2}\left(\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{\gamma\mu_0\alpha}{1+\alpha^2}\left(\hat{m}\times\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{\gamma\hbar J_{MTJ}}{(1+\alpha^2)2qM_st_{FL}}(\hat{m}\times\hat{m}\times\hat{m}_p) + \frac{\alpha\gamma\hbar J_{MTJ}}{(1+\alpha^2)2qM_st_{FL}}(\hat{m}\times\hat{m}_p) \quad (6)$$

Assume, $\beta = \frac{\gamma\hbar J_{MTJ}}{2qM_st_{FL}}$ and substitute this in the above equation, we get,

$$\frac{dm}{dt} = -\frac{\gamma\mu_0}{1+\alpha^2}\left(\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{\gamma\mu_0\alpha}{1+\alpha^2}\left(\hat{m}\times\hat{m}\times\overrightarrow{H}_{eff}\right) - \frac{\beta}{1+\alpha^2}(\hat{m}\times\hat{m}\times\hat{m}_p) + \frac{\alpha\beta}{1+\alpha^2}(\hat{m}\times\hat{m}_p) \quad (7)$$

Eq. (7) denotes the final LLGS equation that we need to solve for the magnetization dynamics.

2. In-plane anisotropy

For in-plane anisotropy suppose the easy axis is along the z-axis and the demagnetization field is along the y-axis, perpendicular to plane. So the associated fields are:

Uniaxial field: H_{uni}=$[0, 0, H_km_z]$, here $H_k = \frac{2K_u}{\mu_0M_s}$

Demagnetization field: H_{demag}=$[0, -H_dm_y, 0]$, where $H_d = M_s$(for S.I. unit)

Critical current: I_{sc}=$\frac{2q\alpha}{\hbar}\,[\mu_0(H_k + 0.5H_d)M_sV]$

3. Perpendicular anisotropy

For perpendicular anisotropy, both, uniaxial and the demagnetization field are perpendicular to plane but they oriented opposite to each other.

Uniaxial field: H_{uni}=$[0, 0, H_km_z]$

Demagnetization field: H_{demag}=$[0, 0, -H_dm_z]$

Critical current: I_{sc}=$\frac{2q\alpha}{\hbar}\,[\mu_0(H_k - H_d)M_sV]$

4. References

[1] A. Sengupta, K. Roy, Encoding neural and synaptic functionalities in electron spin: A pathway to efficient neuromorphic computing, Applied Physics Reviews 4 (4) (2017) 041105.