Diagnostika pomocí zvuku: Měřič objemu tekutiny ve xylofonické nádobě

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I. Problem definition

THE aim of this semestral project is to develop and implement a piece of software for the estimation of liquid volume in a wine glass using the microphone data. The solution should be implemented in Matlab [1], work and show the estimations in real-time and have a function of re-calibration for each glass and liquid type.

II. Approaches overview

As the glass is rubbed or hit, it starts to produce sound by oscillating with its resonant frequency, defined by the shape of the glass, the amount of liquid inside of it, density of the liquid etc. These oscillations produce a sound close to a pure tone, which can be easily identified by performing a Discrete Fourier Transform (DFT) of the recorded sound.

As it is mentioned in [2] and [3], resonant glass frequency is described by complex formulas with many parameters with some simplifications usually considered. Still, both the works show that the resonant frequency can be estimated by not complicated relations after some of their parameters are known. For example, in [3], it is shown, that the glass liquid level can be well approximated by a function depending on the resonant frequencies of empty and filled glass and the glass height.

III. Implementation details

A. Assumptions

As the resonant frequency of the glass depends on many parameters and the sensors setup is limited to one laptop microphone, some assumptions about the problem and measurement process had to be made.

Firstly, all the assumptions for glass model from [3] (e.g. even wall thickness throughout the whole height) are considered here too. The model is quite precise for most glasses, and this simplifies the calculations significantly.

Secondly, it is assumed that during the measurement phase, the glass is located close to the microphone, such that the background noise has much lower amplitude than the sound of the glass itself.

B. Application Functionality

The application was developed using Matlab [1] of version 2020b (this is important here, as the GUI tools do not work well in some newer versions) and can be found here [1]. It uses the App Designer tool to provide a convenient Graphical User Interface (GUI). In the GUI, users can see the real-time signal from the microphone in time and frequency domains as well as liquid volume estimation and the interface for calibration.

Calibration can be done either from a file with measured correspondences of resonant frequency to liquid level or in real-time by putting known amounts of liquid into the glass, rubbing it and recording the sound. In both ways, many measurements may be used and in general case, the more measurements there are, the more precise the future estimation will be. Important to note here is the fact that there must always be a calibration point with no liquid in the glass and there must always be at least 2 calibration points.

Considering the fact that the only used metric of the signal in frequency domain is the dominant frequency, the original sound signal is multiplied by a square window before the Fast Fourier Transform (FFT) algorithm application.

For sound sampling, a window of length 1.25 s. is used with sampling frequency of 8000 Hz. This is motivated by the fact that increasing the window size brings complications to user as the sound should last for the whole window length to be properly recorded and classified. On the other hand, making the window length smaller would decrease the resolution in frequency domain, which is very important for low distinguishing between sounds of the glass with low liquid volumes. For the sampling rate, it is clearly better to set value as small as possible for performance reasons and to be able to update the GUI as fast as possible. The sampling rate of 8000 Hz is enough for proper signal detection and as soon as the only needed metric is the dominant frequency and considering the fact that glass resonant frequencies are unlikely to be larger than 4000 Hz. This allows to update GUI plots and perform FFT with 75 ms. period without any performance issues.

C. Graphical User Interface

The GUI itself is shown in Figure 4. On the right side, plots of sound signal in frequency and time domains are displayed in real time. If the calibration was already performed, estimation of current liquid volume is shown on two panels (one for each method described further) below the plots.

On the left side of the interface calibration tools are located. Here, known glass parameters may be specified and calibration may be performed either from a pre-recorded file or by recording samples in real time inside of the application. In either way, if method 2 is used, some glass parameters may be fixed while others will be deduced from measurements.

After calibration is performed successfully, such calibration results are plotted on the left panel:

[1] https://github.com/ddatsko/volume_by_sound_measurement_tool

- original calibration points,
- correspondence of frequency to volume predicted by method 1,
- correspondence of frequency to volume predicted by method 2.

D. Glass model

For precise volume measurements after a calibration with only a couple of known values, we need a generalized glass model with a small number of unknown parameters. Here, results of [3] are used that show that relation of resonant frequency and liquid level can be well approximated with an equation

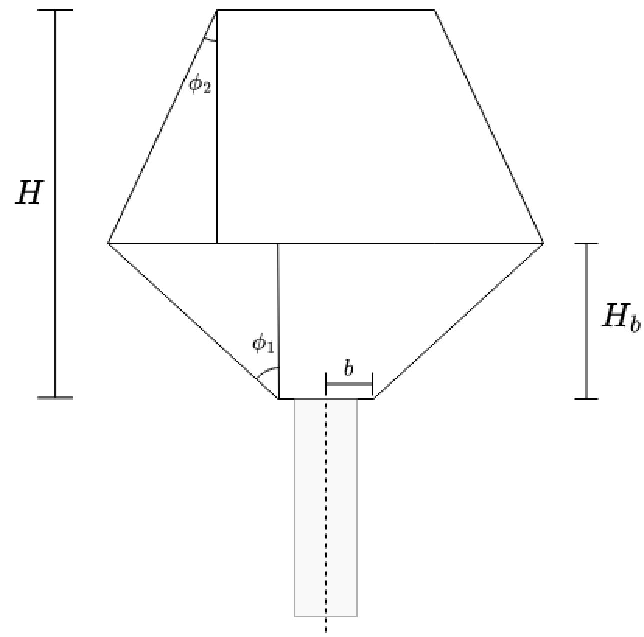
$$\left(\frac{\nu_0}{\nu_h}\right)^2 \approx 1 + \alpha \left(\frac{h}{H}\right)^4, \tag{1}$$

where ν_0 is the resonant frequency of an empty glass, h is the liquid level, ν_h is the frequency with water level h, H is the height of the glass and α is a constant describing glass parameters like glass radius, liquid and glass density and others combined together. While α is dimensionless, ν_0, ν_d, h, H can be expressed in any units (e.g., Hz and ml.) as long as the units are the same in the enumerator and denominator of each fraction.

However, when trying to guess the liquid volume instead of level, more complications arise. Simple assumption that the glass is cylindrical may work well if the number of calibration points is small, but many wine glasses do not have this shape. To somehow approximate and parametrize the shape of a wine glass, the model in Figure 1 is considered. In this simplified model it is considered that the side view of the glass has a shape of 2 symmetrical trapezoids. H is the height of the glass in cm, H_b is the height of the bottom trapezoid in cm, b is the radius of the bottom trapezoid at the base in cm, ϕ_1 and ϕ_2 are angles between the base and the side of the bottom and top trapezoids correspondingly in radians. From these parameters, all of other geometrical properties may be deduced.

Then, the volume can be easily calculated as the volume of 1 or 2 frustrums with dimensions dependent on a specific h.

Fig. 1. Glass shape model



E. Calibration

During calibration and prediction phases, 2 models may be used depending on the user's preference.

The first method utilizes Equation 1, considering a cylindrical glass with H and h being measured in ml directly so that H is the volume of the full glass. This way, only one parameter α has to be found. So, in theory, only one frequency measurement except of the one with empty glass should be enough for calibration. If there are m measurements in form of the volume to frequency correspondences (h_i, ν_i), the function

$$f_1(\alpha) = \sum_{i=1}^{m} \left(h_i - H \sqrt[4]{\frac{(\frac{\nu_0}{\nu_h})^2}{1+\alpha}} \right)^2 \tag{2}$$

is minimized. It is basically the sum of squared differences between the real volume and one predicted with α

For the second method, let us introduce the function $vol(h) = vol(h, b, H, H_b, \phi_1, \phi_2)$, accepting the liquid level and glass parameters and calculating the liquid volume in that glass. Then having the same measurements, the function

$$f_2(\alpha, b, H_b, \phi_1, \phi_2) = \sum_{i=1}^{m} \left[v_i - vol\left(H \sqrt[4]{\frac{(\frac{\nu_0}{\nu_h})^2}{1+\alpha}} \right) \right]^2, \tag{3}$$

where v_i is the known liquid volume, is minimized with respect to all the unknown parameters. The obvious disadvantage of this method is a large number of measurements required. To fit all the 5 parameters well, at least 5 measurements are usually needed. In the developed application this is partially avoided by allowing the user to fix some of parameters, as their physical representation is known. For example, one can simply physically measure b and H_b, which leads to fitting only 3 parameters, which requires less points and is more robust with a small number of them.

For minimization of both functions, a non-linear solver implemented in $fminsearch$ MATLAB function is used.

F. Prediction

After the calibration is done, the volume can be predicted by taking the fitted constants and guessing the volume h^* by

$$h^* = H \sqrt[4]{\frac{(\frac{\nu_0}{\nu_h})^2}{1+\alpha}} \tag{4}$$

or

$$h^* = vol\left(H \sqrt[4]{\frac{(\frac{\nu_0}{\nu_h})^2}{1+\alpha}} \right) \tag{5}$$

depending on the chosen method.

G. Calibration precision

To perform tests, two glasses were used with different geometrical properties. They both are shown on Figure 2 and will be referred to as glass A (left) and glass B (right) further in this section.

Fig. 2. Test glasses

For calibration and error estimation, a dataset was formed with correspondences shown in Table I, where h is the water volume in ml, ν_{hA} and ν_{hB} are the dominant frequencies of glass A and B correspondingly. For glass A it is visualized in Figure 3 additionally with two models calibrated on all the points with no parameters fixed for the method 2.

Looking at the plot, some challenges with the estimation can be clearly seen there. The most important one is the difference in frequencies between 0 and 200 ml water volumes is very low. Combined with the error in measurements, it leads to large errors when estimating liquid volume of an almost empty glass. Also some errors in measurements can be clearly seen from the plot. Taking a look at the fitted lines, it becomes more clear that if all the calibration points are measurements with little volume, the calibration may be not very precise for bigger volumes due to the model sensitivity.

It can be also seen that the approximation with method 1 is not very precise for this glass of a complex shape. Even with a big amount of points for fitting, the estimations for volumes over 200 ml have a significant error.

In Table III and Table II the precision of each method is estimated after calibration with different measurements. The first column there represents the volumes V from dataset in Table I that were used for the calibration. E_1, E_2 and E_3 represent the average prediction error calculated on all the dataset points after calibration with method 1, method 2 with no fixed variables and method 2 with $b = 1$, $H_b = 4$ values fixed correspondingly. Errors are calculated as

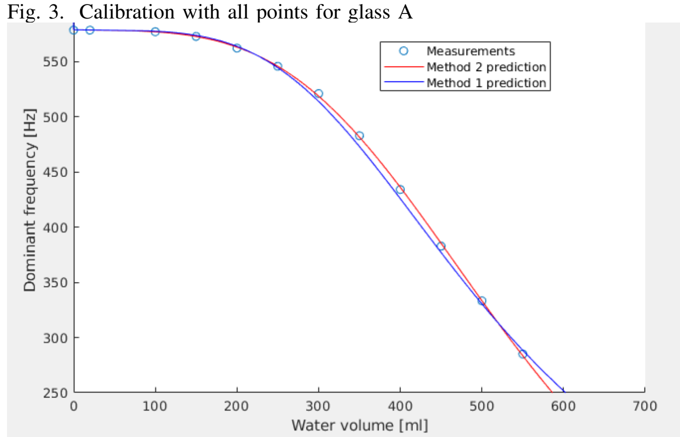
$$E_{1|2|3} = \frac{1}{m} \sum_{i=1}^{m} |h_i^* - h_i|, \tag{6}$$

where m is the size of dataset and h_i^* is the predicted volume, calculated by Equation 2 for E_1 and Equation 3 for E_2 and E_3. Some values for E_2 are skipped as minimizing the function

TABLE I
ERROR ESTIMATION DATASET

h [ml]	ν_{hB} [Hz]	ν_{hA} [Hz]
0	589.44	578.66
20	-	578.64
40	589.26	-
60	588.38	-
80	586.09	-
100	584.13	577.19
120	581.01	-
140	573.77	-
150	-	572.96
160	568.75	-
180	562.17	-
200	553.68	562.32
250	528.27	545.88
300	483.48	521.15
350	424.37	482.91
400	346.06	434.2
450	275.3	382.86
500	-	333.27
550	-	285.00

Fig. 3. Calibration with all points for glass A



over 5 parameters with less than 5 points often leads to different and unexpected results.

As can be seen from the results, if a small amount of points, but the volumes in measurements have significant difference, the first method works well, and in some cases even better than method 2 that ”overfits” there. A clear example of this is in the third column of Table III. There, an error in measurements of 20 ml calibration point combined with model sensitivity to calibration points with low water volume made a larger error for method 2 than for method 1 which is more robust.

On the other hand, if the number of measurements is sufficient, the 2-nd and 3-rd methods work better than the first one.

IV. Conclusion

As a result of this project, an application with GUI that estimates the liquid level in a glass was implemented. With 2 proposed models it can be seen that if the calibration is done properly, on the test glasses the average measurement error is less than 15 ml for both of models.

There is always a problem with measurements with a low liquid volume in the glass that were not completely solved

TABLE II
ERRORS AFTER CALIBRATION FOR GLASS B

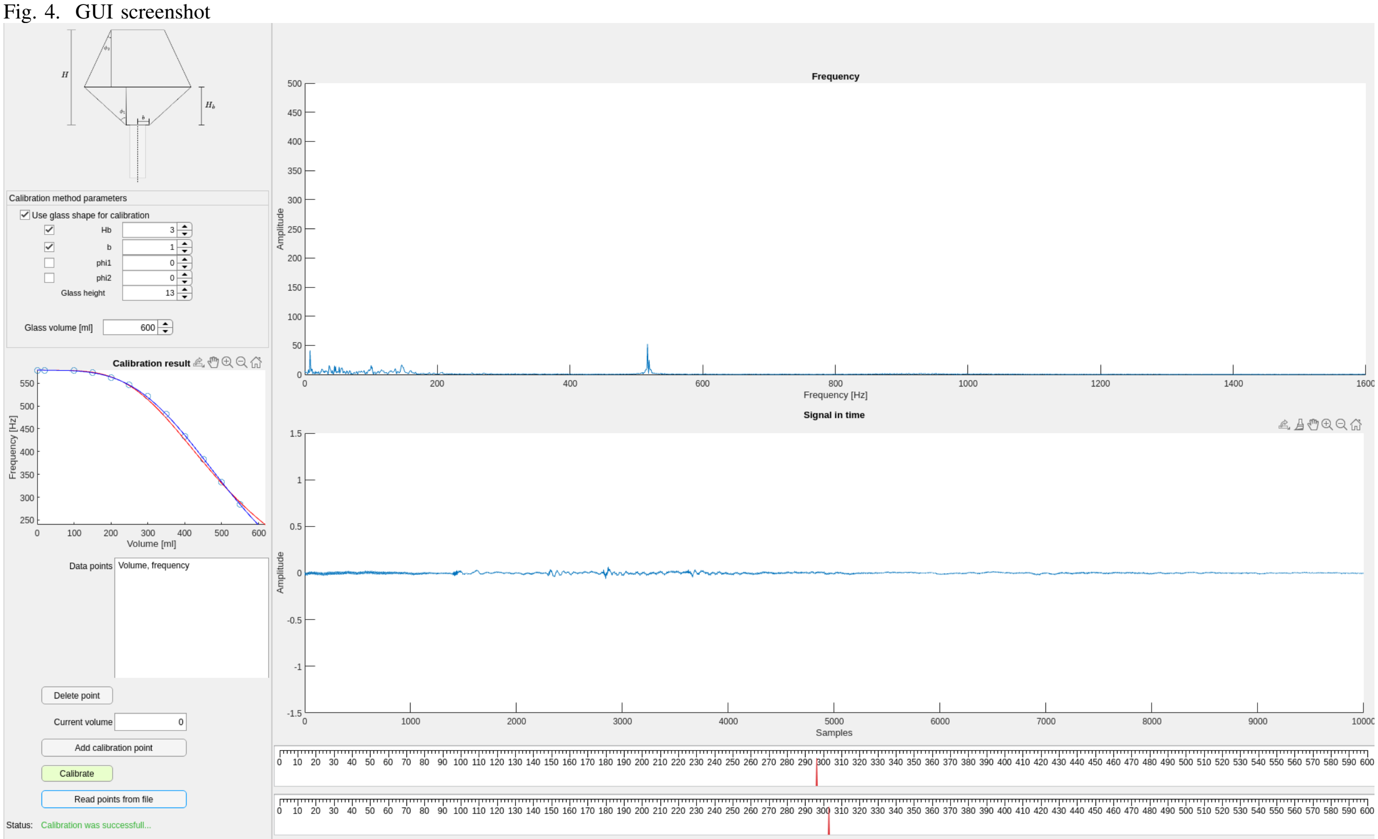
Calibration measurements	E_1	E_2	E_3
All	14.9	2.3	6.6
0, 200	14.7	-	-
0, 200, 300	20.3	-	11.9
0, 40, 60	45.4	-	57.6
0, 40, 200	14.7	-	18.4
0, 40, 200, 300	20.3	15.8	16.1
0, 40, 100, 200, 350	19.6	11.8	11.9

TABLE III
ERRORS AFTER CALIBRATION FOR GLASS A

Calibration measurements	E_1	E_2	E_3
All	7.5	1.5	1.5
0, 200	11.8	-	-
0, 20, 200, 300	9.6	20.3	13.8
0, 550	8.2	-	-
0, 200, 500	8.2	-	2.7

here. As was described above, the main reason for them is small differences in resonant frequencies and model sensitivity combined with errors in calibration and real-time measured data.

Fig. 4. GUI screenshot



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