

## **Contents**

Preface			page xiii		
	xvi				
	Ackr	nowledg	ments	xviii	
1	Introduction			1	
	1.1	Scales	s of the Universe	1	
	1.2	.2 The Invisible Universe			
	1.3	4			
	1.4	7			
	1.5	8			
	Further Reading			9	
	Prob	Problems			
			Part I The Homogeneous Univers	se	
2	The	Expand	ling Universe	15	
	2.1	Geometry		16	
		2.1.1	Spacetime and Relativity	16	
		2.1.2	Symmetric Three-Spaces	17	
		2.1.3	Robertson–Walker Metric	19	
	2.2	Kinen		21	
			Geodesics	21	
			Redshift	26	
		2.2.3	Distances*	29	
	2.3 Dynamics			35	
			Perfect Fluids	35	
		2.3.2		39	
		2.3.3	5,	41	
		2.3.4	Spacetime Curvature	44	
		2.3.5	Friedmann Equations	47	
		2.3.6	Exact Solutions	49	
	2.4 Our Universe			57	
	2.5 Summary			62	
	Further Reading			63	
	Problems			64	

vii



viii Contents

3	The	Hot Big	g Bang	70
	3.1	Thermal Equilibrium		72
		3.1.1	Some Statistical Mechanics	72
		3.1.2	The Primordial Plasma	74
		3.1.3	Entropy and Expansion History	82
		3.1.4	Cosmic Neutrino Background	85
		3.1.5	Cosmic Microwave Background	89
	3.2	Beyon	nd Equilibrium	97
		3.2.1	The Boltzmann Equation	97
		3.2.2	Dark Matter Freeze-Out	99
		3.2.3	Baryogenesis: A Sketch*	103
		3.2.4	Big Bang Nucleosynthesis	107
		3.2.5	Recombination Revisited*	117
	3.3	Sumn	mary	124
	Furt	her Re	ading	126
	Prob	olems		127
4	Cosr	nologi	cal Inflation	132
	4.1	Probl	133	
		4.1.1	The Horizon Problem	133
		4.1.2	The Flatness Problem	137
		4.1.3	Superhorizon Correlations	138
	4.2	Before	e the Hot Big Bang	139
		4.2.1	A Shrinking Hubble Sphere	139
		4.2.2	Horizon Problem Revisited	140
		4.2.3	Flatness Problem Revisited	142
		4.2.4	Superhorizon Correlations	144
		4.2.5	Duration of Inflation	144
	4.3 The Physics of Inflation		Physics of Inflation	146
		4.3.1	Scalar Field Dynamics	147
		4.3.2	Slow-Roll Inflation	150
		4.3.3	Creating the Hot Universe	153
	4.4	Open	Problems*	154
		4.4.1	Ultraviolet Sensitivity	154
		4.4.2	Initial Conditions	156
		4.4.3	Eternal Inflation	158
	4.5	Sumn	mary	160
	Further Reading			161
	Prob	olems	161	



ix Contents

## Part II The Inhomogeneous Universe

5	Stru	cture F	Formation	167
	5.1	Newto	onian Perturbation Theory	168
		5.1.1	Fluid Dynamics	168
		5.1.2	Adding Gravity	170
		5.1.3	Adding Expansion	171
	5.2	Grow	th of Matter Perturbations	174
		5.2.1	Jeans Instability	174
		5.2.2	Linear Growth Function	175
		5.2.3	Transfer Function	178
	5.3	Statis	stical Properties	180
		5.3.1	Correlation Functions	180
		5.3.2	Gaussian Random Fields	182
		5.3.3	Harrison–Zel'dovich Spectrum	183
		5.3.4	Matter Power Spectrum	184
	5.4	Nonli	near Clustering*	185
		5.4.1	Spherical Collapse	186
		5.4.2	Virialization and Halos	189
		5.4.3	A Bound on Lambda	190
		5.4.4	Press–Schechter Theory	191
	5.5 Summary			198
	Furt	her Re	200	
	Prol	blems		200
6	Rela	itivistic	: Perturbation Theory	204
	6.1	Linea	206	
		6.1.1	Metric Perturbations	207
		6.1.2	Matter Perturbations	212
		6.1.3	Conservation Equations	217
		6.1.4	Einstein Equations	222
	6.2	Initia	d Conditions	226
		6.2.1	Superhorizon Limit	227
		6.2.2	Adiabatic Perturbations	229
		6.2.3	Isocurvature Perturbations*	230
		6.2.4	Curvature Perturbations	231
		6.2.5	Primordial Power Spectrum	233
	6.3	Grow	th of Matter Perturbations	234
		6.3.1	Evolution of the Potential	234
		6.3.2	Clustering of Dark Matter	237
		6.3.3	Matter Power Spectrum	240
	6.4	Evolu	ntion of Photons and Baryons	241
		6.4.1	Radiation Fluctuations	242
		6.4.2	Photon-Baryon Fluid	244



x Contents

		6.4.3	Cosmic Sound Waves	245
	6.5	Gravi	249	
	6.6	Sumn	· ·	252
	Furt	Further Reading		
	Prob	olems		255
7	Cosr	nic Mic	rowave Background	259
	7.1	Anisotropies in the First Light		261
		7.1.1	Angular Power Spectrum	262
		7.1.2	A Road Map	264
	7.2	Photo	ons in a Clumpy Universe	265
		7.2.1	Gravitational Redshift	265
		7.2.2	Line-of-Sight Solution	267
		7.2.3	Fluctuations at Last-Scattering	268
	7.3	Aniso	tropies from Inhomogeneities	271
		7.3.1	Spatial-to-Angular Projection	271
		7.3.2	Large Scales: Sachs–Wolfe Effect	274
		7.3.3	Small Scales: Sound Waves	275
	7.4	Primo	ordial Sound Waves	276
		7.4.1	Photon–Baryon Dynamics	278
		7.4.2	High-Frequency Solution	279
		7.4.3	Semi-Analytic Solution*	282
		7.4.4	Small-Scale Damping	290
		7.4.5	Summary of Results	294
	7.5	Understanding the Power Spectrum		295
		7.5.1	Peak Locations	295
		7.5.2	Peak Heights	296
		7.5.3	LCDM Cosmology	299
		7.5.4	Beyond LCDM	305
	7.6	A Glimpse at CMB Polarization*		308
		7.6.1	Polarization from Scattering	308
		7.6.2	Statistics of CMB Polarization	310
		7.6.3	Visualizing E- and B-modes	315
			E-modes from Scalars	317
		7.6.5	B-modes from Tensors	324
	7.7	Sumn	328	
	Furt	331		
	Prob	olems		331
8	Qua	336		
	8.1	Inflat	ionary Perturbations	337
		8.1.1	Equation of Motion	338
		8.1.2	From Micro to Macro	341
	8.2	Quan	tum Fluctuations	343
		8.2.1	Quantum Harmonic Oscillators	343



χi

Contents 8.2.2 Inflationary Vacuum Fluctuations 347 8.3 Primordial Power Spectra 351 Curvature Perturbations 352 8.3.2 Gravitational Waves 354 8.3.3 Slow-Roll Predictions 359 8.4 Observational Constraints\* 361 8.4.1 Current Results 361 8.4.2 Future Tests 365 8.5 Summary 372 Further Reading 373 Problems 374 9 Outlook 378 **Appendices Appendix A Elements of General Relativity** 379 A.1 Spacetime and Relativity 379 A.2 Gravity Is Geometry 388 A.3 Motion in Curved Spacetime 394 A.4 The Einstein Equation 403 A.5 Summary 414 Further Reading 415 **Appendix B Details of the CMB Analysis** 416 **Boltzmann Equation** 416 B.2 Free Streaming and Projection 422 B.3 Evolution Before Decoupling 424 B.4 CMB Polarization 431 **Useful Quantities and Relations** Appendix C 435 C.1 Units and Conversions 435 Constants and Parameters 437 C.3Important Relations 439 **Appendix D Special Functions** 442 D.1 Fourier Transforms 442 D.2 Spherical Harmonics 442 D.3 Legendre Polynomials 443 D.4 Spherical Bessel Functions 444 D.5 Bessel and Hankel Functions 445 D.6 Gamma and Zeta Functions 446 References 447 Index458