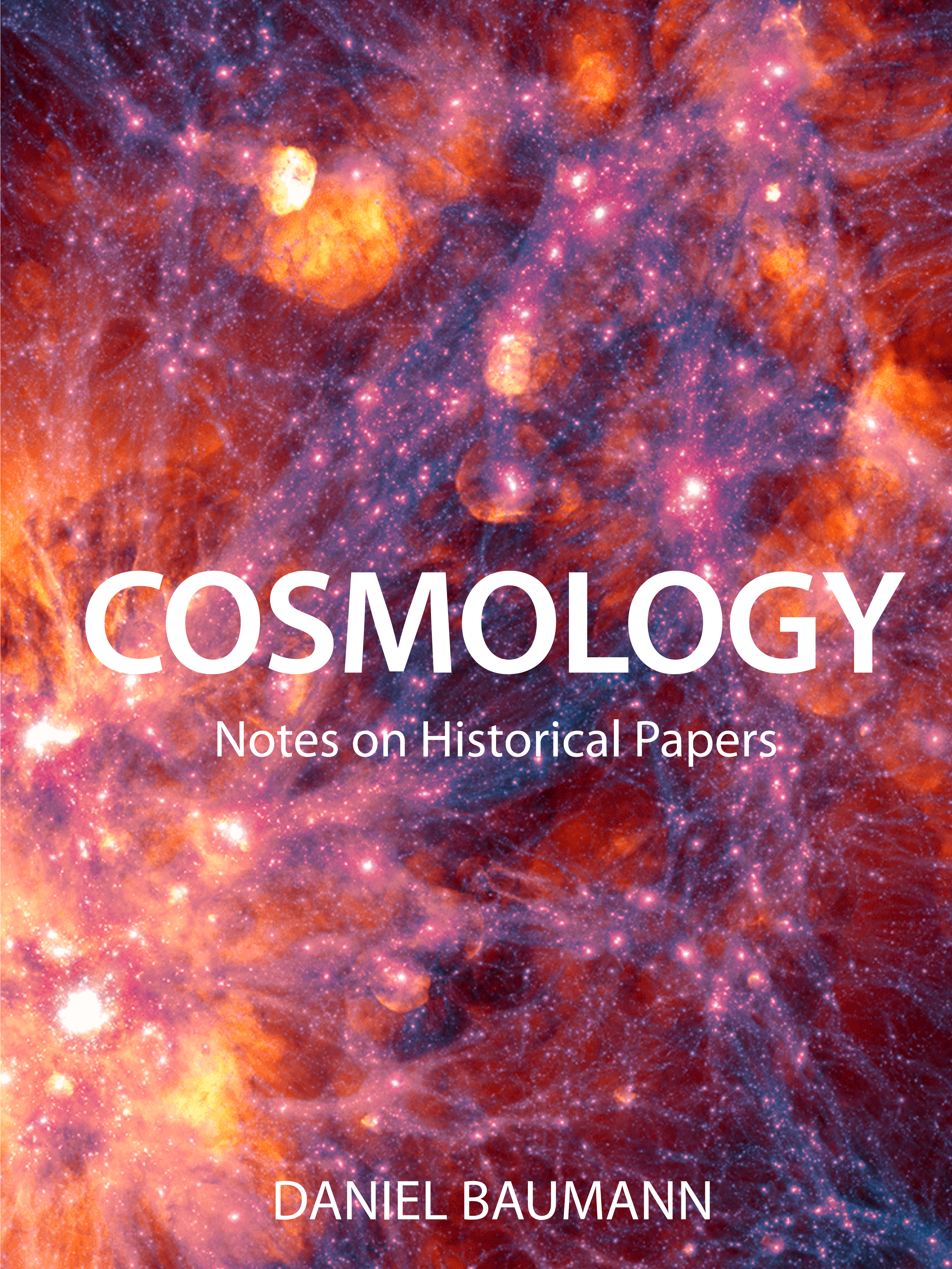

COSMOLOGY

Notes on Historical Papers

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Contents

Preface

The following are notes on papers of historical importance to the field of cosmology. It is *not* a rigorous treatment of the history of cosmology, which I am completely unqualified to write. Instead, I have just collected some key papers and put them in a chronological order. The text is arranged according to the chapters of my book *Cosmology*.

Given that I am not an expert on the topic, I very much welcome feedback and corrections. Please let me know if I am missing key references and/or if I am mischaracterizing anything.

I am very grateful to Malcolm Longair for his generous advice and correspondence on the history of cosmology. I have also been privileged to learn about the history of inflation from some of its main participants, especially Alan Guth, Andrei Linde and Paul Steinhardt. Finally, I have received helpful comments and corrections on a draft version of these notes from Daniel Green, Phillip Helbig, Soren Larsen, Eugene Lim, Andrei Linde, Alessandro Melchiorri, Antonio Padilla, Markus Pössel, Anze Slosar, Jaco de Swart and Roberto Trotta. Any remaining mistakes and inaccuracies are of course my fault.

Daniel Baumann
Amsterdam and Taipei, 2022

Further Reading

There are a number excellent books on the history of cosmology. I have found the following references particularly useful when preparing these notes:

- H. Kragh, *Cosmology and Controversy: The Historical Development of Two Theories of the Universe.* Princeton University Press, 1999.
- M. Longair, *The Cosmic Century: A History of Astrophysics and Cosmology.* Cambridge University Press, 2006.
- P.J.E. Peebles, *Cosmology's Century: An Inside History of our Modern Understanding of the Universe.* Princeton University Press, 2020.
- P.J.E. Peebles, L. Page, and B. Partridge, *Finding the Big Bang.* Cambridge University Press, 2009.

- M. Bartusiak, *Archives of the Universe: 100 Discoveries That Transformed Our Understanding of the Cosmos.* Knopf Doubleday Publishing Group, 2006.
- B. Jones, *Precision Cosmology.* Cambridge University Press, 2017.
- H. Nussbaumer and L. Bieri, *Discovering the Expanding Universe.* Cambridge University Press, 2009.
- H. Kragh and M. Longair, *The Oxford Handbook of the History of Modern Cosmology.* Oxford University Press, 2019.
- H. Kragh, *Conceptions of Cosmos: From Myths to the Accelerating Universe: A History of Cosmology.* Oxford University Press, 2013.
- E. Harrison, *Cosmology.* Cambridge University Press, 2000.
- A. Lightman and R. Brawer, *Origins: The Lives and Worlds of Modern Cosmologists.* Harvard University Press, 1990.
- J. Bernstein and G. Feinberg, *Cosmological Constants: Papers in Modern Cosmology.* Columbia University Press, 1986.

Popular books that cover the history of cosmology are:

- S. Weinberg, *The First Three Minutes.* Basic Books, 1993.
- D. Overbye, *Lonely Hearts of the Cosmos.* Back Bay Books,1999.
- S. Singh, *Big Bang: The Origin of the Universe.* Harper Perennial, 2005.
- M. Bartusiak, *The Day We Found the Universe.* Vintage Books, 2010.
- J. Barrow, *The Book of Universe.* W. W. Norton & Company, 2012.
- P. Halpern, *Flashes of Creation.* Basic Books, 2021.
- A. Guth, *The Inflationary Universe.* Basic Books, 1998.
- D. Hooper, *At the Edge of Time.* Princeton University Press, 2019.
- W. Kinney, *An Infinity of Worlds.* MIT Press, 2022.
- G. Schilling, *The Elephant in the Universe.* Harvard University Press, 2022.

2 The Expanding Universe

The discovery of the expanding universe has an interesting history. In the following, I list some important milestones:[1]

• **1915**: On November 25, Einstein presents the field equations of general relativity (GR) at the Prussian Academy of Sciences in Berlin. These equations became the foundation of modern cosmology.

Vesto Slipher of the Lowell Observatory publishes the spectra of 15 spiral nebulae [4]. The measure redshifts imply large radial velocities, exceeding the typical velocities of stars by a factor of 25.

• **1917**: Einstein applies GR to a homogeneous universe [5]. According to Einstein, "the most important fact that we draw from experience as to the distribution of matter is that the relative velocities of the stars are very small." This leads him to conclude that "there is a system of reference relatively to which matter may be looked upon as being permanently at rest." He further assumes that the universe if finite and finds the metric (and the energy-momentum tensor) for a static spherically symmetric space. He also notices, however, that this metric doesn't solve his field equations unless a cosmological constant is introduced to balance the gravitational attraction of the matter. This solution became known as the "Einstein static universe."

Shortly thereafter, de Sitter finds a second solution to the Einstein equation corresponding to an empty universe dominated by the cosmological constant [6, 7]. De Sitter shows that the light emitted from particles at rest in the spacetime is redshifted,[2] as suggested by Slipher's observations of extra-galactic nebulae [4]. In his own words, "the frequency of light-vibrations diminishes with increasing distance from the origin of co-ordinates. The lines in the spectra of very distant stars or nebulae must therefore be systematically displaced towards the red, giving rise to a spurious radial velocity." Using the redshifts of three nebulae from Slipher's sample, de Sitter estimates the radius of (spacetime) curvature of his solution to be $R_c = 1.5\,\mathrm{Mpc}$.

• **1922**: Friedmann finds a solution of the Einstein equations for an expanding universe with matter and positive spatial curvature [11]. In this paper, he derives

[1] More detailed descriptions can by found in [1–3].
[2] De Sitter, in fact, found a quadratic relation between velocity and distance. The correct linear relationship was first derived by Weyl [8], and later rediscovered by Lemaître [9] and Robertson [10].

the famous equation which now bears his name. He also realizes that the expansion would imply a finite age for the universe. Unfortunately, Friedmann's work was dismissed by Einstein, so it did not receive much attention at the time. (Einstein later apologized for this.) Friedmann died three years later, before his work got the recognition it deserved.

Wirtz shows that their radial velocities of 29 galaxies increase with decreasing apparent brightness of the galaxies [12].

• **1924**: Friedmann presents a new solution of the field equations with constant negative curvature [13].

Lundmark [14] plots the radial velocities of the spiral nebulae versus their distances (as inferred from their apparent magnitudes). The plot suggests "that there may be a relation between these two quantities, although not a very definite one." In [15], Lundmark tries to fit a quadratic function to the data. A similar analysis was carried out independently by Stromberg [16].

• **1925**: Lemaître shows that de Sitter's solution corresponds an exponentially expanding universe [9] (which was not manifest in the coordinates used by de Sitter). He also derives the linear distance–redshift relation for the de Sitter universe.

• **1927**: Independently from Friedmann, Lemaître applies the Einstein equations to an expanding universe with matter and curvature [17]. Lemaître's remarkable paper also includes a derivation of Hubble's law and a measurement of Hubble's constant (two years before Hubble).[3] Unfortunately, Lemaître published his work in a rather obscure Belgian journal, so it did not receive much attention either.

• **1928**: Unaware of Lemaître's 1925 and 1927 papers, Robertson shows again that de Sitter's solution describes an exponentially expanding universe and derives the linear distance–redshift relation [10]. Combining distance measurements by Hubble with Slipher's redshifts, Robertson verifies the linear distance–redshift relation and provides an estimate of Hubble's constant.

• **1929**: Hubble's paper on the expansion of the universe is arguable one of the most important papers in the history of science [18]. Using the period–luminosity relationship of Cepheid variable stars [19], Hubble and Humason determine the distances to extra-galactic nebulae (galaxies) and finds a correlation with their redshifts (as measured by Slipher). This correlation is a key feature of the expansion of the universe, although Hubble himself resisted that interpretation for a long time and instead preferred to think of it in terms of the de Sitter effect. While Hubble's result established the correlation between distances and recession speeds, the measurements came with large systematic errors and the inferred value of the Hubble constant was off by a factor of 7.

[3] In the English translation of his article (published in 1931), Lemaître omitted the paragraph including his estimate of the Hubble constant, noting in his letter to the editor that "I did not find advisable to reprint the provisional discussion of radial velocities which is clearly of no actual interest."

• **1930**: Eddington points out that the Einstein static universe is unstable to small fluctuations in the matter density [20].

Lemaître sends his 1927 paper to Eddington who realizes its significance and begins to promote it more widely (with the help of de Sitter). The importance of Lemaître's work (and also that of Friedmann) is finally appreciated and Hubble's redshift–distance measurements are interpreted as the expansion of space.

• **1931**: Adding 46 new radial velocity measurements by Humason [21] (extending to larger distances), Hubble and Humason provide a much more convincing measurement of the linear distance–redshift relation [22].

Tolman [23] and Eddington [24] point out that the second law of thermodynamics (the increase in entropy) implies a finite age for the universe. They hesitate, however, to formulate the Big Bang hypothesis.

Lemaître publishes a short letter [25] describing his vision for the beginning of the universe from a "primeval atom" on "a day without yesterday." Here is a short passage from the letter:

> If the world had begun with a simple quantum, the notions of space and time would altogether fail to have any meaning at the beginning: they would only begin to have a sensible meaning when the original quantum had been divided into a sufficient number of quanta. If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time. [...] We could conceive the beginning of the universe in the form of a unique atom, the atomic weight of which is the total mass of the universe. This highly unstable atom would divide in smaller and smaller atoms by a kind of super-radioactive process.

Shortly after, Lemaître provides a more detailed account of his Big Bang hypothesis [26]. This paper includes an explicit solution of the Friedmann equations for a universe with matter and a cosmological constant. The initial singularity of the solution is identified with the beginning of the universe. (The name Big Bang had not been introduced yet.). The paper also emphasizes that the addition of the cosmological constant leads to a phase of stagnated expansion at intermediate times, which alleviates the age problem of the matter-only universe.

Einstein dispenses with the cosmological constant in his field equations [27].

• **1932**: Einstein and de Sitter write an influential two-page paper describing the evolution of a flat matter-dominated universe [28].[4] For a long time, this Einstein–de Sitter universe was the standard cosmological model. It is still a good approximation to the long matter-dominated period in the history of our universe.

• **1933**: Zwicky argues for the presence of dark matter in the Coma cluster [29, 30]. It took over 40 years until the rest of the community also got convinced of the

[4] Apparently, neither Einstein nor de Sitter were very enthusiastic about the paper. In a conversation with Eddington, Einstein remarked "I did not think the paper very important myself, but de Sitter was keen on it." On the other hand, de Sitter told Eddington that "I do not myself consider the result of much importance, but Einstein seemed to think it was."

existence of dark matter (mostly through measurements of galactic rotation curves by Vera Rubin and others [31–33]).

Robertson publishes an influential review on relativistic cosmology [34].

• **1934**: Lemaître identifies the cosmological constant with the energy density of empty space [35] (see also [36]).[5]

• **1935**: Robertson [39] and Walker [40] derive the FRW metric on the basis of homogeneity and isotropy alone. A few years earlier, Milne had shown that Hubble's law is a consequence of the cosmological principle [41].

• **1946**: Gamow initiates the study of Big Bang nucleosynthesis (BBN) [42] (more on this in Chapter 3).

• **1947**: Lemaître and Einstein exchange letters discussing the need for introducing a cosmological constant to solve the age problem of the Einstein–de Sitter universe. While Lemaître finds the cosmological constant a necessity, Einstein objects to it on aesthetic grounds ("I am unable to believe that such an ugly thing should be realized in nature").

• **1948**: Alpher and Herman provide important refinements of Gamow's theory of BBN [43]. The paper contains the first prediction of a left-over radiation from the hot Big Bang, and estimates its present temperature. Unfortunately, the prediction of the CMB is presented as a side remark and largely ignored.

Hoyle [44] and Bondi and Gold [45] introduce the "Steady State Theory" as an alternative to the hot Big Bang. The fascinating history of the Big Bang vs. Steady State controversy is reviewed in [1, 46].

• **1949**: Fred Hoyle coins the term "Big Bang" in a BBC radio programme.

• **1955**: Ryle and Scheuer show that there are more radio galaxies at large distances than there are nearby [47], thus indicating that the universe had evolved over time, in conflict with the expectation from the steady-state theory of Hoyle, Bondi and Gold. The robustness of these observations is a topic of active debate.

• **1963**: Maarten Schmidt discovers the first "quasi-stellar object" (*quasar*) [48]. More and more of the extremely luminous quasars are soon detected at large redshifts. The distant quasars are another sign that the universe was evolving and not in a steady state.

• **1965**: Penzias and Wilson make one of the biggest discoveries in the history of science. You wouldn't know it from their one-page paper with the modest title "A Measurement of Excess Antenna Temperature at 4080 Mc/s" [49]. The interpretation of the signal as relic radiation from the Big Bang is given in a companion paper by Dicke, Peebles, Roll and Wilkinson [50]. The discovery of the CMB put the nail in the coffin of the steady-state cosmology.

[5] That empty space is filled with quantum zero-point energy was first suggested by Walther Nernst in 1916 (one year before Einstein introduced the cosmological constant) [37]. This early history of dark energy is reviewed by Helge Kragh in [38].

• **1970**: For a long time, it was unclear whether the singularity of the FRW universe was just an artefact of assuming homogeneity and isotropy. Then, Hawking and Penrose prove their famous singularity theorem [51] (see also [52]): a universe filled with "ordinary matter" must have started with an initial singularity. Their theorem assumes the strong energy condition, and therefore doesn't apply to inflation.

• **1981**: Guth shows that a period of exponential expansion ("inflation") solves the horizon, flatness and monopole problems of the standard Big Bang cosmology [53].

• **1987**: Weinberg provides an anthropic explanation for the smallness of the cosmological constant [54], which, at this point, had not been observed yet.

• **1992**: The COBE satellite measures the blackbody spectrum of the CMB [55] and provides the first detection of its anisotropies [56].

• **1998**: Two teams of astronomers—the Supernova Cosmology Project [57] and the High-Z Supernova Search Team [58]—make a striking discovery. By measuring the redshifts and apparent brightnesses of type Ia supernovae, they show that the rate of expansion is accelerating, rather than decelerating.

• **1999**: Turner introduces the term "dark energy" for the mysterious energy density driving the accelerated expansion of the universe [59].

• **2000**: BOOMERanG (a ballon-borne CMB experiment) measures enhanced CMB fluctuations on degree angular scales [60], thereby confirming that the geometry of the universe is close to spatially flat.

• **2003**: The WMAP satellite measures the power spectrum of CMB anisotropies to unprecedented accuracy [61]. The measurement initiates the era of precision cosmology and establishes the ΛCDM model.

• **2013**: The Planck satellite provides improved measurements of the CMB spectrum, yielding precise measurements of the parameters of the ΛCDM concordance cosmology [62].

• **2018**: The International Astronomical Union (IAU) votes to rename Hubble's law the *Hubble-Lemaître law*.

3 The Hot Big Bang

I have also enjoyed reading some of the pioneering papers establishing the hot Big Bang cosmology. The following is a brief sketch of key milestones in the subject:

• **1921**: Meghnad Saha develops a theory for the thermodynamic equilibrium of chemical reactions [1, 2], mostly to apply it to the ionization of elements in stellar atmospheres. The work culminates in the famous *Saha equation*.

• **1925**: Cecilia Payne-Gaposchkin publishes a seminal PhD thesis on the chemical composition of stars [3]. She discovers that stars are made mostly of hydrogen and helium, contrary to the conventional wisdom at the time which held that stars have approximately the same elemental composition as the Earth.

• **1926**: Eddington emphasizes in his classic book *The Internal Constitution of the Stars* [4] that the formation of helium from hydrogen inside of stars requires very high temperatures (to overcome the electric repulsion of the hydrogen nuclei). The book includes the following prophetic statement:

> The helium which we handle must have been put together at some time and some place. We do not argue with the critic who urges us that the stars are not hot enough for this process; we tell him to go and find *a hotter place*.

• **1929**: Atkinson and Houtermans show how quantum mechanical tunneling helps nuclei to overcome the energy barrier due to their electric repulsion and hence increases the efficiency of nuclear fusion inside stars [5].

• **1930**: In a letter to Lise Meitner, Pauli proposes the existence of neutrinos to explain how beta decay conserves energy and momentum [6].

• **1932**: Chadwick discovers the *neutron* [7].

Anderson discovers the *positron* (which he called “easily deflectable positives”) [8], and hence establishes the existence of antimatter (as predicted by Dirac [9]).

• **1934**: Tolman publishes his influential book *Relativity, Thermodynamics and Cosmology* [10]. He is also one of the first to apply thermodynamics to the expanding universe.

• **1936**: Anderson and Neddermeyer discover the *muon* in high-altitude observations of cosmic rays [11, 12] (see also [13]). The existence of the muon was so surprising that Rabi famously asked “Who ordered that?”.

• **1938**: Bethe explains why stars shine (through the fusion of protons with carbon and nitrogen in the CN cycle) [14]. He points out, however, that “no elements heavier than helium can be built up to any appreciable extent," so that "we must assume that the heavier elements were built up *before* the stars reached their present state of temperature and density."

Weizsäcker provides a sketch how the chemical elements might have been produced in the early universe [15]. He did not, however, incorporate the idea into the relativistic cosmologies of Friedmann and Lemaître.

• **1942**: Chandrasekhar and Henrich [16] apply equilibrium thermodynamics to estimate the relative abundances of the chemical elements produced in the hot beginnings of the universe. They fail to account for any appreciable amounts of the heavy elements.

• **1946**: Gamow, for the first time, includes the expansion of the universe and the associate non-equilibrium physics to describe the fusion of the nuclei in the early universe [17]. Although the paper still got many details wrong, it was foundational in developing the modern theory of Big Bang nucleosynthesis (BBN).

• **1947**: Powell discovers the *pion* [18] (predicted in 1935 by Hideki Yukawa [19]). The observational methods that led to this discovery were developed by Bibha Chowdhuri and Debendra Mohan Bose.

• **1948**: Gamow, together with Alpher and Herman, wrote no less than 11 papers studying BBN this year. The convoluted back-and-forth in these papers is reviewed by Peebles in [20] (see also [21, 22]). The papers at the beginning of the year—including the (in)famous[1] Alpher, Bethe, and Gamow paper [23]—were incomplete (and incorrect) in a number of way. In particular, the universe was taken to be matter-dominated during BBN. This was corrected in an important paper by Alpher and Herman [24]. The latter paper also includes the first prediction of the CMB and an estimate of its present temperature (5 K).[2]

• **1950**: The work of Alpher, Herman and Gamow, assumed that the universe was initially made up of mostly neutrons. These neutrons would decay into protons and electrons, and the heavier elements would then be built up by neutron capture.

[1] Although Bethe didn't contribute to this work, Gamow added his name to the paper because he liked the sound of the author list. Herman was not included because "he stubbornly refused to change his name to Delter."

[2] It is interesting to ask why this dramatic prediction of the Big Bang theory wasn't picked up by the astronomy community and didn't lead to a search for the CMB in the 1950s. This question is discussed in the books by Weinberg [25] and Kragh [21]. At a practical level, most astronomers simply weren't familiar with the work of Gamow, Alpher and Herman, since it wasn't published in one of their main journals like the *Astrophysical Journal*. Even if they had come across their work, the Big Bang theory was still new and according to Weinberg "it was extraordinary difficult for physicists to take seriously *any* theory of the early universe. ... [The] first three minutes are so remote from us in time, the conditions of temperature and density are so unfamiliar, that we feel uncomfortable in applying our ordinary theories of statistical mechanics and nuclear physics." Even the authors themselves didn't try to convince radio astronomers to look for the microwave background radiation. Weinberg concludes that "this is often the way it is in physics—our mistake is not that we take our theories too seriously, but that we don't take them seriously enough. It is always hard to realize that these numbers and equations we play with at our desks have something to do with the real world."

Chushiro Hayashi was the first to realize that at high temperatures neutron decay wasn't the only relevant process. He showed that reactions like $n + e^+ \leftrightarrow p + \bar{\nu}$ and $n + \nu \leftrightarrow p + e^-$ would create a significant proton-to-neutron ratio [26]. The paper predicts the correct neutron freeze-out and includes an essentially correct estimate of the final helium abundance.

• **1953**: The seminal paper by Alpher, Follin and Herman [27] establishes the modern theory of primordial nucleosynthesis. It was realized that BBN produces hardly any elements above helium (because of the absence of stable nuclei with mass numbers $A = 5$ and 8). At the time, this was viewed as a problem for the Big Bang theory and motivated the Steady State model of Hoyle, Bondi and Gold [28, 29]. The paper also states that the universe at the time consisted of many causally disconnected parts because the horizon size was much smaller that the radius of the universe. This fact later became the "horizon problem."

• **1954**: Fred Hoyle predicts the existence of an excited state in the carbon-12 nucleus, arguing that such a state is necessary for the production of carbon in stars [30]. This excited state was observed three years later by Willie Fowler's research group at Caltech [31].

• **1956**: Cowan and Reines discover the *neutrino* [32].

• **1957**: Margaret and Geoffrey Burbidge, Fowler and Hoyle (B^2FH) [33] show that the heavy elements (up to iron) can be synthesized inside of stars. This resolves the problem of the heavy elements that was the original motivation for BBN. Stellar nucleosynthesis, however, still failed to account for the cosmic helium abundance, which observationally was known to comprise 25% (by mass) of the baryonic matter in the universe.

• **1961**: Osterbrock and Rogerson provide an estimate of the primordial helium abundance using measurements of HII regions, the Sun and planetary nebulae [34].

• **1964**: Hoyle and Tayler show that the correct helium abundance can be produced in the hot Big Bang [35].

• **1965**: Penzias and Wilson serendipitously discover the cosmic background radiation [36]. Dicke, Peebles, Roll and Wilkinson explain the cosmological origin of the signal [37].

Zel'dovich shows that a universe with baryon-symmetric initial conditions cannot produce the observed baryon-to-photon ratio [38]. Specifically, he derives that the freeze-out of particle–antiparticle annihilations leads to $\eta \sim 10^{-18}$, almost 10 orders of magnitude smaller than the required value. This motivates the search for an alternative mechanism of baryogenesis.

• **1966**: Peebles revisits the predictions of BBN (especially helium and deuterium) in light of the measurements of the CMB temperature and the mean mass density of the universe [39].

Gerstein and Zel'dovich derive the decoupling of neutrinos and predict their relic abundance [40]. The paper also provides a cosmological upper bound on the electron and muon neutrinos of 400 eV (tau neutrinos had not been discovered yet). A similar bound is derived a few years later by Cowsik and McClelland [41].

• **1967**: Wagoner, Fowler and Hoyle write the first detailed BBN code [42] (later developed further by Wagoner [43]). The code is used to predict the correct deuterium and lithium abundances. In essence, this code is still used to perform modern computations in BBN.

Sakharov presents his three conditions for successful baryogenesis [44].

• **1968**: Peebles develops his theory of recombination [45] (see also Zel'dovich, Kurt and Sunyaev [46]). These papers show that recombination did not proceed in Saha equilibrium. They also introduce the effective three-level atom and show that two-photon decays are important in the recombination dynamics.

• **1977**: Peccei and Quinn introduce a spontaneously broken $U(1)$ symmetry as a solution to the strong CP problem [47, 48]. Weinberg [49] and Wilczek [50] point out that this implies the existence of a Nambu–Goldstone boson, called the "axion." This axion becomes a popular dark matter candidate.

Steigman, Schramm and Gunn use the BBN prediction for primordial helium to derive an upper limit to the number of light neutrino species [51].

Lee and Weinberg use an estimate of the mass density of the universe to place a lower limit to the mass of a hypothetical heavy neutrino species [52, 53].

• **1979**: Yoshimura [54] and Weinberg [55] propose the first models of baryogenesis.

• **1981**: Dimopoulos and Georgi introduce the Minimal Supersymmetric Standard Model (MSSM) [56]. The lightest superpartners (neutralinos) become a popular dark matter candidate [57, 58].

• **1990**: The FIRAS detector on the COBE satellite measures the blackbody spectrum of the CMB [59]. This solidified the cosmological origin of the microwave background, making it a key observation of the Big Bang cosmology.

• **1999**: Seager, Sasselov and Scott develop the recombination code RecFast [60]. By modelling the non-equilibrium behaviour of the higher excited states, they predict the ionization history of the universe at the accuracy required for the era of CMB precision observations.

• **2003**: WMAP measures the CMB anisotropy spectrum to unprecedented accuracy and establishes the ΛCDM concordance cosmology [61].

• **2010**: Several advances were made between 2006 and 2010 that increased both the numerical efficiency and the precision of recombination codes, culminating in the new state-of-the art codes HyRec [62] and CosmoRec [63].

4 Cosmological Inflation

The following is a very incomplete survey of papers that have been important in the development of inflationary cosmology:[1]

• **1966**: Gliner shows that the energy-momentum tensor of a vacuum-dominated universe with a positive energy density leads to a de Sitter universe [2], but doesn't consider the origin of this energy density (see also related work by Zel'dovich [3]).

• **1968**: Misner formulates the "horizon problem" of the Big Bang cosmology [4].

• **1969**: Dicke describes the "flatness problem" in a lecture [5].

• **1974**: Linde shows that a symmetry breaking potential can act like an effective time-dependent cosmological constant [6] (see also the independent work of Dreitlein [7]). Years later, this will become an important element of slow-roll inflation.

• **1975**: Gliner and Dymnikova study the transition from a vacuum-dominated universe to a radiation-dominated universe, showing that it produces a large increase in the scale factor [8]. Their motivation for introducing an initial de Sitter phase was to remove the initial singularity of the Big Bang theory.

• **1979**: Dicke and Peebles point out [9] that the universe must have been flat to better than one part in 10^{15} around $t = 1\,\text{sec}$.

Preskill estimates [10] that the density of magnetic monopoles in Grand Unified Theories (GUTs) would be larger than the critical density by a factor of 10^{12}, reducing the age of the universe to just 30 000 years.

Various groups start to investigate the dynamical effects of phase transitions in the early universe [11]. Kolb and Wolfram show that can be dominated by a positive energy density during a first-order phase transition and that this would lead to an exponential expansion of the universe [12]. They also speculate that the phase transition might lead to density perturbations, but don't study this further.

• **1980**: Starobinsky shows that higher-derivative corrections to Einstein gravity can lead to a de Sitter solution [13], but doesn't relate it to the problems of the hot Big Bang. Instead Starobinsky's motivation was to find a way to avoid the Big Bang singularity.

Guth and Tye study the production of magnetic monopoles in GUTs and argue

[1] The early history of inflation is reviewed compresensively in [1]. My summary of "inflation before inflation" closely follows the treatment there.

that their abundance can be suppressed if the phase transition occurs after a long period of supercooling [14].

Kazanas explores the effects of a first-order phase transition on the expansion of the universe [15]. He suggests that a long period of exponential expansion could explain the large-scale isotropy of the universe.

• **1981**: Guth realizes that a universe dominated by a false vacuum expands exponentially and that this would solve the horizon, flatness and monopole problems of the standard Big Bang cosmology [16]. The paper ends with the admission that models of false vacuum inflation don't end smoothly, but instead produce large inhomogeneities after the tunneling to the true vacuum. This is the "graceful exit problem" of old inflation.

Independently, Sato studies first-order phase transitions in GUTs and shows that this would lead to an exponential increase in the particle horizon [17]. He also realizes the graceful exit problem in these scenarios [18].

• **1982**: Linde [19], and independently Albrecht and Steinhardt [20], solve the graceful exit problem by introducing the first models of slow-roll inflation (also called "new inflation" in contrast to Guth's old inflation).

There was concern that inflation would be too efficient, leading to an empty universe without any fluctuations in its matter density. It was therefore with some relief that is was realized that quantum zero-point fluctuations provide an inevitable source of perturbations (see Chapter 8). These primordial fluctuations were discussed at the famous "Nuffield meeting" in Cambridge and subsequently computed by several groups [21–25]. It was shown that the inflationary fluctuations have precisely the nearly scale-invariant form suggested by observations.

In a talk at the Nuffield meeting, Steinhardt points out that if inflation starts at the top of a flat potential, there may be regions of space where it doesn't stop and instead globally becomes eternal. The concept of "eternal inflation" is also mentioned in Steinhardt's contribution to the proceedings of the Nuffield meeting [26]. In the same proceedings, Linde explains that eternal inflation could lead to a "multiverse" with different regions of space having different physical properties.[2]

Albrecht, Steinhardt, Turner and Wilczek provide a first account of the "reheating" of the universe after inflation [28].

• **1983**: Vilenkin shows that eternal inflation is a generic feature of all new inflationary models [29].

Linde develops a variation of slow-roll inflation called "chaotic inflation" [30]. While both old and new inflation assumed thermal equilibrium of the pre-inflationary universe, chaotic inflation relaxes this constraint on the initial conditions.

• **1986**: Linde shows that models of chaotic inflation are typically also eternal [31]. The term "eternal inflation" is introduced in this paper.

[2] A brief history of eternal inflation and the inflationary multiverse can be found in [27].

• **1989**: Goldwirth and Piran study the problem of the inflationary initial conditions with numerical simulations in 1+1 dimensions [32, 33].

• **1990**: Freese, Frieman and Olinto develop “natural inflation” [34], a model in which the role of the inflaton is played by a pseudo-Nambu–Goldstone boson whose shift symmetry makes the flatness of the potential technically natural.

• **1997**: Kofman, Linde and Starobinsky provide a comprehensive treatment of perturbative reheating [35].

Armendariz-Picon, Damour and Mukhanov propose “k-inflation” [36] where the inflationary dynamics is driven not by a flat potential, but by a nontrivial kinetic term.

• **2000**: The BOOMERanG satellite measures the position of the first peak in the CMB anisotropy spectrum, confirming that the universe is spatially flat as predicted by inflation [37].

• **2001**: Khoury, Ovrut, Steinhardt, and Turok propose the “Ekpyrotic Universe” as an alternative to inflationary cosmology [38]. In this scenario, the primordial perturbations are created in a phase of slow contraction before the universe “bounces” and transitions to the expanding hot Big Bang cosmology.

• **2002**: Steinhardt and Turok introduce the “Cyclic Universe” as version of the ekpyrotic scenario with periodically repeating bounces separating phases of contraction and expansion [39].

• **2003**: WMAP detects a small deviation from scale-invariance in the primordial density fluctuation [40], as predicted by all inflationary models.

Kachru, Kallosh, Linde and Trivedi (KKLT) introduce a compactification of string theory with de Sitter vacua [41]. Together with Maldacena and McAllister, they further argue that a moving D3-brane can lead to slow-roll inflation in this setting [42] (see also [43]). Silverstein and Tong show that inflation can even occur if the brane moves relativistically, exploiting the fact that nonlinearities in the Dirac-Born-Infeld action for the brane become important in this limit [44, 45].

Susskind relates the “landscape” of string theory vacua to the anthropic principle and Weinberg’s solution to the cosmological constant problem [46] (following earlier work by Bousso and Polchinski [47]).

Borde, Guth and Vilenkin prove that inflationary spacetimes are *not* past eternal and therefore don’t evade the problem of the initial singularity [48].

• **2008**: Silverstein, Westphal and McAllister introduce “axion monodromy inflation” [49, 50], a promising candidate for large-field inflation in string theory.

Cheung, Creminelli, Fitzpatrick, Kaplan and Senatore develop an effective theory to describe inflationary models with large self-interactions [51] (see also [52]). The inflationary fluctuations are identified with the Goldstone boson of spontaneously broken time translations, whose interactions are constrained by the nonlinearly realized symmetry.

• **2015**: East et al. [53] and Clough et al. [54] revisit the problem of inflationary initial conditions with numerical GR simulations in 3+1 dimensions. (See also the earlier work by Laguna, Kurki-Suonio and Matzner [55].)

5 Structure Formation

A wonderful summary of the history of cosmological structure formation can be found in the books by Peebles [1] and Longair [2]. Moreover, the history of dark matter and its role in structure formation is reviewed in the article by Bertone and Hopper [3]. A few milestone and important papers are listed below.

• **pre-1900**: It wasn't until the beginning of the last century that the scale of the universe and of the structures within it was understood.

Between 1771 and 1784, Charles Messier catalogs over 100 fuzzy 'nebulae' in the sky (to avoid confusing them with comets, his main objects of interest). The systematic cataloguing of these nebulae is carried forward by William Herschel, who, in 1786, and with assistance from his sister Caroline, publishes a catalogue of over 1000 nebulae. In 1864, John Herschel (son of William) publishes the *General Catalogue of Nebulae* containing over 5000 objects, and, in 1888, Dreyer releases the *New General Catalogue of Nebulae* (NGC) with over 15 000 objects. NGC and Messier numbers are still used to refer to nearby galaxies.

• **1902**: Jeans studies the growth of matter perturbations in a static space, finding the exponential instability that is now named after him [4].

• **1912**: Henrietta Leavitt discovers the period–luminosity relationship for Cepheid variable stars in the Magellanic clouds [5].

• **1920**: On 26 April, the National Academy of Sciences in Washington organizes the famous *Great Debate* between Harlow Shapley and Heber Curtis. While Shapley believes that the nebulae are objects inside our own Milky Way galaxy, Curtis argues that they are distant "island universes" (galaxies). Their arguments are summarized in [6], but the data at the time wasn't good enough to resolve the issue conclusively.

• **1925**: Hubble observes Cepheids in nearby nebulae and uses them to show that these nebulae are objects outside of our own galaxy [7].

• **1929**: Hubble discovers the expansion of the universe [8] (see Chapter 2).

• **1933**: Lemaître finds that a spherically-symmetric perturbation in an expanding, matter-dominated universe grows as $\delta\rho/\rho \propto t^{2/3}$ [9]. Lifshitz later establishes the same result without assuming spherical symmetry [10] (see also [11, 12]). The growth of the large-scale structure of the universe therefore requires initial perturbations of a sufficiently large amplitude, which leads Lemaître and Lifshitz to conclude erroneously that galaxies were *not* formed by gravitational collapse.

Zwicky provides the first evidence for the existence of dark matter in the Coma cluster [13, 14]. He also proposes to measure the masses of galaxies and clusters through gravitational lensing.

• **1946**: Lifshitz studies Einstein's equations in linear perturbation theory [10]. He derives the solutions to the linearized equation for both matter perturbations and radiation. This remarkable paper initiated cosmological perturbation theory in the fully relativistic context (see Chapter 6).

• **1948**: Gamow highlights the significance of matter–radiation equality for structure formation [15], which was later made rigorous by Mészáros [16].

• **1952**: Neyman and Scott initiate a statistical treatment of the spatial distribution of galaxies [17, 18]. Their work also introduces the two-point correlation function, $\xi(r)$, to describe the large-scale clustering of galaxies.

• **1957**: Bonnor derives the perturbation equations from the fluid equations in Newtonian gravity [19].

• **1964**: Novikov argues that the formation of galaxies requires the initial density perturbations to have $\delta\rho/\rho \sim 10^{-4}$ [20].

• **1965**: Peebles proposes that structure forms from the "bottom-up" through the hierarchical clustering of small objects into larger ones [21].

Gunn and Peterson predict that the presence of neutral hydrogen in the intergalactic medium (IGM) would lead to a characteristic feature in the spectra of high-redshift quasars—the "Gunn–Peterson trough" [22].

• **1969**: Totsuji and Kihara show that the galaxy correlation function can be approximated by a power law over a large range of scales, $\xi(r) = (r/r_0)^{-\gamma}$ [23].

• **1970**: Harrison [24] and Peebles and Yu [25] propose a scale-invariant spectrum as the most natural initial conditions. This was later picked up by Zel'dovich [26] and is now called the "Harrison–Zel'dovich spectrum."

Rubin and Ford provide new evidence for the existence of dark matter by studying galactic rotation curves [27] (see also [28, 29]). Over the course of the next decade, astronomers finally get convinced that dark matter exists around galaxies and clusters of galaxies [30].

• **1971**: Peebles publishes his book *Physical Cosmology* [31].

• **1972**: Gunn and Gott introduce the spherical collapse model for nonlinear structure formation [32].

Cowsik and McClelland present a cosmological upper bound on the mass of neutrinos [33] (see also [34]), but don't discuss the potential role of neutrinos as dark matter.

• **1974**: Press and Schechter develop a theory of the statistics of dark matter halos [35]. Although the Press–Schechter theory makes a number of unjustified assumptions, it initiates a large body of work on structure formation in the nonlinear regime.

Ostriker et al. [36] and Einasto et al. [37] use a variety of datasets to argue for the existence of dark matter over a wide range of scales. These papers established the cosmological significance of dark matter.

• **1976**: Marx and Szalay suggest that 10 eV neutrinos could be the dark matter [38].

• **1977**: Lee and Weinberg show that very heavy neutrinos ($m_\nu > 2\,\mathrm{GeV}$) are still consistent with cosmological constraints and propose that these neutrinos could be the dark matter [39]. A number of similar papers appeared almost simultaneously [40–43].

• **1980**: Studies of tritium beta decay claim a measurement of the mass of the electron anti-neutrino of about 30 eV [44]. Although this result did not survive further scrutiny, at the time, it provided significant motivation for exploring the consequences of neutrinos as hot dark matter (HDM).

• **1981**: Guth introduces the concept of cosmological inflation [45]. A key prediction of the theory—the spatial flatness of the universe—seems to be in tension with the observed matter density, $\Omega_m \approx 0.3$.

• **1982**: A number of groups compute the spectrum of density perturbations created by quantum fluctuations during inflation [46–50].

The CfA redshift survey releases the first extensive 3D survey of galaxies [51].

Dick Bond introduces the term “cold dark matter” (CDM) to describe massive particles that decoupled from the thermal plasma after they had become non-relativistic. Peebles discussed structure formation in a CDM cosmology [52].

• **1983**: Numerical simulations by White, Frenk and Davis rule out the HDM model [53].

• **1984**: Davis, Efstathiou, Frenk and White (the “gang of four”) perform the first numerical simulations of structure formation in the (Λ)CDM cosmology [54].

Blumenthal, Faber, Primack and Rees [55] compare the predictions of the CDM model to data from the CfA survey.

Turner, Steigman and Krauss [56] argue that a cosmological constant can make the low matter density of the universe consistent with the spatial flatness predicted by inflation. See also the paper by Peebles of the same year [57].

Kaiser introduces the concept of “galaxy biasing” to explain the difference in the correlation strengths of galaxies and clusters of galaxies [58]. He shows that clusters are naturally highly biased tracers of the underlying matter distribution since they form only at the highest density peaks of the mass distribution.

• **1986**: Bardeen, Bond, Kaiser and Szalay (BBKS) provide further mathematical details of Kaiser’s idea of galaxy biasing [59].

• **1987**: Weinberg derives an anthropic bound on the value of the cosmological constant [60].

• **1989**: Using data from the second CfA survey, Geller and Huchra discover the *Great Wall*, one of the largest known superstructures in the observable universe [61].

• **1990**: Efstathiou, Sutherland and Maddox argue that the low matter density inferred from large-scale structure observations implies the need for a cosmological constant [62].

• **1992**: COBE measures the amplitude of CMB temperature fluctuations [63]. The size of the fluctuations indicates that dark matter is required to explain the growth of structure.

• **1997**: The Two-degree-Field Galaxy Redshift Survey (2dF) begins operation.

• **2000**: The Sloan Digital Sky Survey (SDSS) begins operation.

• **2001**: SDSS discovers the Gunn–Peterson trough in a $z = 6.28$ quasar. This provides evidence that the universe had undergone a transition from neutral to ionized gas around $z \sim 6$.

• **2003**: WMAP confirms the predictions of the ΛCDM concordance cosmology with increasing precision [64] .

• **2004**: Tegmark et al. [65] provide a joint-analysis of data from WMAP and SDSS.

• **2005**: The Virgo Consortium runs the “Millennium Simulation” based on the N-body code GADGET [66]. The simulation follows the evolution of over 10 billion “particles” in a volume with a side-length of over 2 billion light years. The simulation ran for over a month producing 25 terabytes of data.

• **2010**: Together with Nicolis, Senatore and Zaldarriaga, I show that dark matter fluctuations can be described by an “effective fluid” [67]. This is later expanded into a rigorous effective theory of large-scale structure (EFT-of-LSS) by Senatore and collaborators [68].

• **2019**: The EFT-of-LSS is applied for the first time to measure cosmological parameters from the BOSS galaxy survey [69, 70].

Jim Peebles receives the Nobel prize for his groundbreaking work in physical cosmology.

• **2021**: The Dark Energy Survey (DES) releases its first cosmological results from galaxy clustering and weak leasing data [71].

The Dark Energy Spectroscopic Instrument (DESI) begins operation.

6 Relativistic Perturbation Theory

The following are a few papers that have played an important role in the development of cosmological perturbation theory:

• **1946**: Lifshitz provides a seminal analysis of linearized general relativity in cosmological spacetimes [1]. It is remarkable how much of our modern understanding of cosmological perturbation theory (as described in my book) is in this paper. The paper introduces the scalar-vector-tensor decomposition of metric and matter fluctuations. It derives the linearized Einstein equations for $k = \pm 1$ (but not $k = 0$!) backgrounds in synchronous gauge. The issue of unphysical gauge modes is treated carefully. Solutions to the equations are presented for scalar, vector and tensor in FRW backgrounds with matter and radiation.

Ironically, the introduction of the paper ends with the following sentences:

> We shall see that in the expanding universe of the general relativity theory the perturbations of most types decrease with time, thus showing no tendency to spontaneous increase. There exist also such perturbations which increase with time, but so slowly that they cannot produce large condensations. Thus we can apparently conclude that gravitational instability is not the source of condensation of matter into separate nebulae.

• **1959**: Arnowitt, Deser and Misner introduce a Hamiltonian formulation of general relativity [2, 3]. In this so-called *ADM formalism*, the metric is separated into dynamical and constrained degrees of freedom, which allows for a convenient formulation of the initial value problem in GR. The ADM parameterization plays an important role in numerical relativity and also has found applications in cosmological perturbation theory [4].

• **1963**: Lifshitz and Khalatnikov extend and correct aspects of Lifshitz' 1946 paper [5]. They also study matter perturbations in a contracting universe, showing that scalar perturbations are unstable and hence grow rapidly.

• **1966**: Hawking develops a formalism to study perturbation in the curvature tensor directly (rather than in the metric), thereby avoiding gauge ambiguities [6].

• **1967**: Sachs and Wolfe extend Lifshitz' treatment to include perturbations in the radiation. They discuss the effect of cosmological perturbations on the CMB anisotropies. (Remember that the CMB was only discovered two years earlier.) The paper presents the linearized Einstein equations now also for $k = 0$ backgrounds, but still in synchronous gauge.

Harrison derives the linearized Einstein equations in conformal Newtonian gauge (longitudinal gauge) [7].

• **1969**: Nariai studies cosmological perturbations in comoving gauge [8].

• **1970**: Peebles and Yu develop the Boltzmann formalism for the evolution of photons and neutrinos in the early universe [9], which is the basis of all modern CMB codes.

• **1980**: Press and Vishniac show how to eliminate the two unphysical gauge modes in synchronous gauge and interpret ambiguous density perturbations on superhorizon scales [10].

Bardeen introduces a gauge-invariant formulation of cosmological perturbation theory clearing up a persistent confusion about fictitious gauge modes [11]. Classic reviews of the gauge-invariant approach are by Kodama and Sasaki [12] and Mukhanov, Feldman and Brandenberger [13].

• **1982**: The primordial perturbations from inflation are computed in a number of papers [14–18]. The paper by Bardeen, Steinhardt, and Turner includes the first definition of the conserved curvature perturbation, ζ [15] (see also [19, 20]).

• **1984**: Lyth introduces the curvature perturbation in comoving gauge, $\mathcal{R}$ [21].

• **1995**: Hu and Sugiyama provide a semi-analytic treatment of small-scale cosmological perturbations [22].

• **1996**: Ma and Bertschinger present the equations of linearized cosmological perturbations in both conformal Newtonian and synchronous gauges [23].

Seljak and Zaldarriaga release the Boltzmann code CMBFast [24]. Besides computing the CMB spectra, the code outputs linear transfer functions for all fluctuations.

• **1997**: Eisenstein and Hu give an analytic derivation of the BAO feature in the matter power spectrum [25].

• **1999**: Bucher, Moodley and Turok provide a comprehensive analysis of isocurvature perturbations [26].

• **2000**: Using only the local conservation of energy and momentum, Wands, Malik, Lyth and Liddle prove that curvature perturbations are conserved on large scales when non-adiabatic pressure perturbations are negligible [20]. Different versions of the proof have also appeared in [27–29].

• **2005**: The Baryon Oscillation Spectroscopic Survey (BOSS) of SDSS-III detects the BAO feature in the galaxy power spectrum [30].

7 Cosmic Microwave Background

Observations of the cosmic microwave background (CMB) have revolutionized cosmology (see Table 7.1 for a list of selected experiments). The existence of the CMB is a cornerstone of the modern Big Bang theory and observations of the CMB anisotropies have been instrumental in establishing the ΛCDM concordance cosmology. The fascinating history of the cosmic microwave background is chronicled in [1]. Here is a list of key events:

• **1934**: Tolman shows that blackbody radiation in an expanding universe cools but remains thermal [2].

• **1941**: McKellar measures a background radiation with an effective temperature of 2.3 K using stellar absorption lines [3]. The cosmic significance of this measurement, however, was not appreciated until 1966, one year after Penzias and Wilson's discovery of the CMB.

• **1946**: Gamow estimates the present temperature of the universe to be 50 K (assuming the universe to be 3 billion year old). He comments that this "is in reasonable agreement with the actual temperature of interstellar space", but does not mention a background radiation.

• **1948**: Alpher and Herman predict the CMB with a temperature of 5 K [4]. However, their prediction is not picked up by the astronomical community and does not lead to an experimental search for the CMB.

• **1964**: Doroshkevich and Novikov emphasize that the background radiation from the Big Bang should be observable by radio observations [5]. The same was realized by Dicke and his group in Princeton, who initiate an experimental search for the relic radiation.

• **1965**: Following a suggestion by Zel'dovich, Sakharov shows that density perturbations in a relativistic fluid develop pressure-supported acoustic waves on small scales [6]. Essentially the same physics is responsible for the oscillations in the primordial photon–baryon fluid, although Sakharov didn't make that connection. Instead, Sakharov was studying quantum fluctuations of the density in a cold universe (based on Zel'dovich's hypothesis that the initial temperature of matter in the universe was zero). Nevertheless, Sakharov understood that these acoustic oscillations lead to a preferred scale in the mass spectrum of the large-scale structure of the universe, as they do in the CMB anisotropy spectrum [7, 8].

Penzias and Wilson accidentally discover the cosmic background radiation [9]. Dicke, Peebles, Roll and Wilkinson explain its cosmological significance [10].

Table 7.1 List of selected CMB experiments (adapted from https://lambda.gsfc.nasa.gov/).

Name	Full Name	Type	Duration
RELIKT	Relikt	Satellite	1983 – 1984
COBE	Cosmic Background Explorer	Satellite	1989 – 1992
MAXIMA	Millimeter Anisotropy eXperiment Imaging Array	Balloon	1998 – 1999
BOOMERanG	Balloon Observations Of Millimetric Extragalactic Radiation and Geophysics	Balloon	1997 – 2003
DASI	Degree Angular Scale Interferometer	Ground	2001 – 2003
WMAP	Wilkinson Microwave Anisotropy Probe	Satellite	2001 – 2010
CBI	Cosmic Background Imager	Ground	2002 – 2008
CAPMAP	Cosmic Anisotropy Polarization MAPper	Ground	2002 – 2008
KECKArray	Keck Array	Ground	2003 –
BICEP	Background Imaging of Cosmic Extragalactic Polarization	Ground	2006 –
SPT	South Pole Telescope	Ground	2007 –
ACT	Atacama Cosmology Telescope	Ground	2008 –
QUIET	QU Imaging ExperimenT	Ground	2008 – 2010
Planck	Planck	Satellite	2009 – 2013
ABS	Atacama B-mode Search	Ground	2011 – 2014
POLARBear	Polarization of Background Microwave Radiation	Ground	2012 –
EBEX	The E and B Experiment	Balloon	2012 – 2013
SPTpol	SPT: Polarization	Ground	2012 –
ACTPol	ACT: Polarization	Ground	2013 –
SPIDER	Spider	Balloon	2015 –
CLASS	Cosmology Large-Angular Scale Surveyor	Ground	2016 –
SPT-3G	SPT: Third Generation	Ground	2018 –
SO	Simons Observatory	Ground	Future
CMB-S4	CMB Stage-4	Ground	Future
LiteBIRD	(Lite) B-mode polarization and Inflation from cosmic background Radiation Detection	Satellite	Future

• **1967**: Sachs and Wolfe study the effect of density fluctuations on the CMB temperature anisotropies [11].

• **1968**: Rees predicts that the CMB should be polarized [12].

Rees and Sciama predict that CMB fluctuations are created when photons travel through time-dependent gravitational potentials [13].

• **1969**: Sunyaev and Zel'dovich show that, before recombination, photons and baryons are strongly coupled and can be described by a single fluid [14] (see also the earlier paper by Weymann [15]).

• **1970**: Peebles and Yu study the evolution of fluctuations in the primordial photon–baryon fluid [7]. Amongst many other things, this remarkable paper contains the first derivation of the CMB power spectrum. The paper also sets up the Boltzmann formalism used in modern CMB codes. Independently, Sunyaev and Zel'dovich [8] explore the same physics. They also derive the oscillatory feature in the CMB power spectra. Their abstract ends with the following sentences:

A detailed investigation of the spectrum of fluctuations may, in principle, lead to an understanding of the nature of initial density perturbations since a distinct periodic dependence of the spectral density of perturbations on wavelength (mass) is peculiar to adiabatic perturbations. Practical observations are quite difficult due to the smallness of the effects and the presence of fluctuations connected with discrete sources of radio emission.

• **1971**: Paul Henry reports the first measurement of the CMB dipole [16]. The work was part of his PhD thesis under the supervision of Dave Wilkinson. It was published as a single authored paper because at the time Princeton required such papers for a successful PhD thesis. Henry's measurement was later confirmed by Corey and Wilkinson [17], as well as Smooth, Gorenstein and Muller [18].

• **1972**: Sunyaev and Zel'dovich show that the inverse Compton scattering of microwave background photons by the hot gas in galaxy clusters produces a spectral distortion of the CMB fluctuations on small scales (the SZ effect) [19].

• **1983**: Kaiser computes the damping rate of the photon–baryon plasma [20] and Silk shows how this leads to a suppression of small-scale CMB fluctuations [21].

• **1984**: Birkinshaw, Gull, and Hardebeck report the first detection of the SZ effect [22].

• **1985**: Polnarev computes the CMB polarization spectrum for small angular separations [23]. See also the later treatments [24–26].

• **1987**: Bond and Efstathiou provide a very comprehensive analysis of the CMB anisotropies in a CDM cosmology [27].

• **1990**: The COBE satellite measures the blackbody spectrum of the CMB [28], thereby confirming the cosmological origin of the signal.

• **1992**: COBE announces the first detection of CMB anisotropies on large scales [29].

• **1995**: Bertschinger releases the Boltzmann code COSMICS [30]. The code is slow and takes hours to compute a single CMB spectrum.

Hu and Sugiyama provide a semi-analytic solution for the sound waves in the primordial plasma [31].

• **1996**: Zaldarriaga and Seljak [32], and independently Kamionkowski, Kosowsky and Stebbins [33], introduce the E/B decomposition of CMB polarization and provide an all-sky analysis of the corresponding power spectra.

Seljak and Zaldarriaga release the Boltzmann code CMBFast [34] which significantly speeds up the computation of the CMB spectra. The code includes E/B modes, CMB lensing, and applies to curved universes.

• **1999**: Lewis and Challinor release the Boltzmann code CAMB [35] as an alternative to CMBFast.

Seager, Sasselov and Scott create the recombination code RecFast [36].

• **2000**: BOOMERanG measures enhanced CMB fluctuations on degree angular scales [37], thereby confirming that the geometry of the universe is close to spatially flat. This was further corroborated by measurements of the TOCO [38] and MAXIMA [39] experiments.

• **2002**: The polarization of the CMB is detected by the Degree Angular Scale Interferometer (DASI) [40].

• **2003**: The first E-mode polarization spectrum is measured by the Cosmic Background Imager (CBI) [41].

WMAP detects a small scale dependence, $n_\mathrm{s} \neq 1$, in the primordial density fluctuations [42], as predicted by all inflationary models. Stephen Hawking calls it "the discovery of the century, if not of all time." WMAP also measures the large-scale TE correlation [43] proving that there was time before the hot Big Bang [44].

• **2004**: Bashinsky and Seljak derive the neutrino-induced phase shift of the CMB anisotropy spectrum [45].

• **2005**: Alpher is awarded the National Medal of Science for his foundational work in nucleosynthesis and the prediction of the CMB.

• **2006**: Smoot and Mather receive the Nobel prize for their work on COBE.

• **2007**: Smith, Zahn and Doré provide the first detection of CMB lensing using cross correlation with radio galaxy counts [46].

• **2011**: ACT [47] and SPT [48] measure the CMB lensing power spectrum.

Lesgourgues, Tram and others release the Boltzmann code CLASS [49], a significantly restructured alternative to CAMB and CMBFast.

• **2014**: The BICEP collaboration announces a detection of primordial B-modes [50]. Doubts are soon raised about the primordial origin of the signal [51].

• **2015**: Follin et al. [52] measure the phase shift expected from the cosmic neutrino background in the Planck data.

In a joint analysis [53], the Planck and BICEP collaborations show that the previously announced B-mode detection was not due to primordial gravitational waves, but came from dust in our own galaxy.

- **2018**: The Planck satellite releases its final data [54].

8 Quantum Initial Conditions

The history of inflation was sketched in Chapter 4. In the following, I will add a few more details on the key developments establishing the quantum generation of perturbations during inflation.

• **1975**: Hawking shows that black holes radiate due to the effect of quantum fluctuations near the black hole's event horizon [1]. This is the famous *Hawking radiation*.

• **1977**: Gibbons and Hawking show that the concept of Hawking radiation also applies to cosmological horizon [2]. This provides the conceptual basis for the computation of the vacuum fluctuations in inflation.

• **1978**: Bunch and Davies define a preferred vacuum state for quantum fluctuations in de Sitter space [3]. See also the earlier work by Chernikov and Tagirov [4].

• **1979**: Starobinsky derives the spectrum of gravitational waves generated in a de Sitter background [5].

• **1981**: Mukhanov and Chibisov [6] compute the spectrum of scalar fluctuations in the Starobinsky model. The red tilt of the spectrum, $n_s < 1$, is predicted.

• **1982**: The general theory of quantum fluctuations during inflation is developed at the "Nuffield workshop" in Cambridge. This results in a series of papers computing the power spectrum of density fluctuations [7–10].

• **1983**: Hartle and Hawking introduce the "no-boundary state" as an initial condition for the universe [11].

• **1990**: Salopek and Bond [12] estimate the amount of non-Gaussianity in slow-roll inflation (see also [13, 14]). The paper also contains a nonlinear definition of the comoving curvature perturbation.

• **1996**: Lyth derives his famous bound on the field variation in inflationary models with observable gravitational waves [15].

• **1997**: Spergel and Zaldarriaga [16] point out that the cross-correlation of CMB temperature fluctuations and polarization on large scales is an important test of inflation (or more general a period that preceded the hot Big Bang). A few years later, this feature is measured by the WMAP satellite [17] proving that the hot Big Bang was not the beginning of time.

• **2003**: Maldacena provides the first rigorous calculation of the inflationary non-Gaussianity using the in-in formalism [18].

WMAP measures the scale dependence in the primordial density fluctuations [19] expected from inflationary models.

• **2004**: Creminelli and Zaldarriaga prove a consistency relation for single-field inflation, relating the squeezed limit of the bispectrum to the scale dependence of the power spectrum [20]. This proves that a large signal in the squeezed limit can only come from additional particles during inflation.

Silverstein and Tong introduce Dirac-Born-Infeld (DBI) inflation [21], a model of inflation inspired by string theory that allows for large equilateral non-Gaussianity coming from higher-derivative interactions.

• **2005**: Weinberg provides a systematic treatment of the "in-in formalism" [22] to compute higher-order Gaussian and non-Gaussian correlations in cosmology. While previous calculations were restricted to tree graphs, this paper also considers loops.

• **2006**: Chen, Huang, Kachru and Shiu [23] derive the bispectrum in $P(X)$ theories (i.e. inflationary models with an arbitrary kinetic term $X \equiv (\partial\phi)^2$). Creminelli [24] had previously calculated the non-Gaussianity coming from the leading higher-derivative interaction X^2.

• **2008**: Cheung et al. [25] introduce an influential effective theory of inflation. In this effective theory, the inflationary fluctuations are associated with the Goldstone boson of broken time translations during inflation.

Dalal et al. show that local non-Gaussianity leads to a scale-dependent bias that can be measured in the galaxy power spectrum [26].

• **2009**: Chen and Wang [27] show that inflationary models with additional massive particles close to the Hubble scale, $m \lesssim \frac{3}{2}H$, lead to a distinct non-analytic scaling in the squeezed limit of the bispectrum. They called such scenarios "quasi-single-field inflation." Daniel Green and I argue that extra fields with masses close to the Hubble scale arise naturally in inflationary models with spontaneously broken supersymmetry [28].

• **2015**: Arkani-Hamed and Maldacena [29] show that massive particles with $m > \frac{3}{2}H$ lead to characteristic oscillatory signatures in the squeezed limit of the bispectrum. These oscillations are the analog of a resonance in ordinary collider physics. Making this analogy precise, they initiate the field of "cosmological collider physics."

A A Cosmic Chronology

1917 Einstein applies general relativity to a homogeneous universe.
He introduces the cosmological constant to achieve a static universe.

De Sitter finds a solution for an empty universe with a cosmological constant.

1920 Shapley and Curtis debate the distances to the spiral nebulae.

1922 Friedmann derives new solutions to the Einstein field equations for an expanding universe.

1925 Hubble shows that the spiral nebulae are extragalactic objects.

1927 Lemaître applies the Einstein equations to an expanding universe with matter and curvature. He predicts the distance–redshift relation (the Hubble–Lemaître law).

1929 Hubble measures the linear distance–redshift relation and thus establishes the expansion of the universe.

1930 Eddington rediscovers and promotes Lemaître's 1927 paper.

Pauli proposes the existence of neutrinos.

1931 Lemaître introduces the "primevial atom" as the initial state of the universe.

Einstein drops the cosmological constant from his field equations.

1932 Chadwick discovers the neutron.

1933 Zwicky argues for the existence of dark matter in the Coma cluster.

Lemaître studies the gravitational collapse of matter perturbations in an expanding universe.

1934 Lemaître identifies the cosmological constant with vacuum energy.

1935 Robertson and Walker derive the FRW metric.

1938 Bethe explains the energy production inside stars via the CN cycle.

1946 Gamov initiates the study of Big Bang nucleosynthesis (BBN).

Lifshitz studies cosmological perturbations in general relativity.

1948 Gamov, Alpher, and Herman develop further details of BBN.
Alpher and Herman predict a 5K background radiation.

Bondi, Gold, and Hoyle propose the steady-state cosmology.

1949 Fred Hoyle coins the term "Big Bang."

1950 Hayashi derives the primordial neutron-to-proton ratio and predicts the helium abundance from BBN.

1953 Alpher, Follin and Herman present a more refined version of BBN, incorporating recent advances in nuclear physics.

1956 Cowan and Reines discover the neutrino.

1957 Burbidge, Burbidge, Fowler and Hoyle (B^2FH) show that the heavy elements are produced inside of stars.

1965 Penzias and Wilson discover the CMB.

1967 Sakharov presents the requirements for baryogenesis.

Wagner, Fowler, and Hoyle show that BBN predicts the correct deuterium and lithium abundances.

Sachs and Wolfe study the effects of cosmological perturbations on the CMB anisotropies.

1968 Peebles develops the non-equilibrium theory of recombination.

Rees predicts that the CMB should be polarized.

Misner formulates the horizon problem of the Big Bang theory.

1969 Dicke highlights the flatness problem.

1970 Rubin and Ford provide decisive evidence for dark matter through the measurement of galaxy rotation curves.

Hawking and Penrose prove the cosmological singularity theorem.

The Harrison–Zel'dovich spectrum is introduced as the natural initial condition for cosmological perturbations.

Peebles and Yu derive the CMB power spectrum.

1972 Gunn and Gott study the spherical collapse model of nonlinear structure formation.

Cowsik and McClelland present an upper bound on neutrino masses.

1974 Press and Schechter develop a theory for the statistics of dark matter halos.

1975 Hawking discovers that black holes radiate quantum mechanically.

1976 Marx and Szalay suggest neutrinos as hot dark matter (HDM).

1977 Peccei and Quinn introduce the axion to solve the strong CP problem. Axion-like particles become a popular dark matter candidate.

1979 Starobinsky derives the spectrum of gravitational waves generated in a de Sitter background.

1980 Starobinsky shows that high-energy corrections to Einstein gravity can lead to a de Sitter solution.

Bardeen introduces a gauge-invariant formulation of cosmological perturbation theory.

1981 Guth proposes inflation as a solution to the horizon and flatness problems.

Mukhanov and Chibisov suggest that quantum fluctuations during inflation could lead to primordial density fluctuations.

Dimopoulos and Georgi introduce the MSSM. The neutralino becomes a popular dark matter candidate.

1982 Linde, and independently Albrecht and Steinhardt, produce the first models of slow-roll inflation. Several groups compute the density fluctuations predicted by quantum fluctuations during inflation.

The first CfA galaxy redshift survey is completed.

Peebles introduces the cold dark matter (CDM) cosmology.

Steinhardt introduces the first example of eternal inflation.

1983 White, Frenk and Davis rule out the HDM cosmology.

Silk derives the damping of small-scale CMB fluctuations.

1984 Davis, Efstathiou, Frenk and White perform the first numerical simulations in ΛCDM cosmology.

Turner, Steigman and Krauss argue that a cosmological constant is needed to make the low matter density of the universe consistent with the spatial flatness predicted by inflation.

1984 Polnarev computes the CMB polarization spectrum on small scales.

1987 Weinberg provides an anthropic explanation for the small value of the cosmological constant.

1990 COBE measures the blackbody spectrum of the CMB.

Efstathiou, Sutherland and Maddox argue that large-scale structure observation imply the need for a cosmological constant.

Freese, Frieman and Olinto develop the theory of “natural inflation.”

Salopek and Bond estimate the amount of non-Gaussianity in slow-roll inflation.

1992 COBE announces the first detection of CMB anisotropies.

1996 Seljak and Zaldarriaga release the Boltzmann code CMBFast.

Zaldarriaga and Seljak, and independently Kamionkowski, Kosowsky and Stebbins, introduce the E/B decomposition of CMB polarization.

Lyth shows that inflationary models with observable gravitational waves involve super-Planckian field excursions.

1997 Kofman, Linde and Starobinsky provide a comprehensive analysis of perturbative reheating.

1998 The Supernova Cosmology Project and High-Z Supernova Search Team discover the accelerated expansion of the universe.

1999 Seager, Sasselov and Scott develop the recombination code RecFast.

Turner introduces the term “dark energy.”

2000 BOOMERanG measures the position of the first peak in the CMB spectrum, indicating that the universe is spatially flat.

2002 DASI discovers CMB polarization.

2003 The 2dF Survey shows that the matter density is 25% of the critical density, giving independent evidence for dark energy.

CBI measures the E-mode polarization spectrum.

WMAP measures the CMB spectrum to unprecedented accuracy. It discovers the deviation from scale-invariance predicted by inflation.

Maldacena provides the first rigorous calculation of the bispectrum of slow-roll inflation.

2004 A joint-analysis of WMAP and SDSS data provides precision constraints on the ΛCDM concordance cosmology.

Creminelli and Zaldarriaga prove the consistency relation for non-Gaussianity in single-field inflation.

2005 SDSS discovers the BAO feature in the galaxy power spectrum.

2006 Smoot and Mather receive the Nobel prize for their work on COBE.

2011 ACT and SPT measure the CMB lensing power spectrum.

2012 The ATLAS and CMS experiments at the Large Hadron Collider (LHC) report the detection of the Higgs boson.

2013 The Planck satellite provides precision measurements of the standard cosmological parameters.

2014 BICEP announces the detection of primordial B-modes; the signal is later shown to come from dust in our own galaxy.

2016 LIGO announces the detection of gravitational waves from black hole mergers.

2019 Peebles receives the Nobel prize for his groundbreaking work in physical cosmology.

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Chapter 8

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