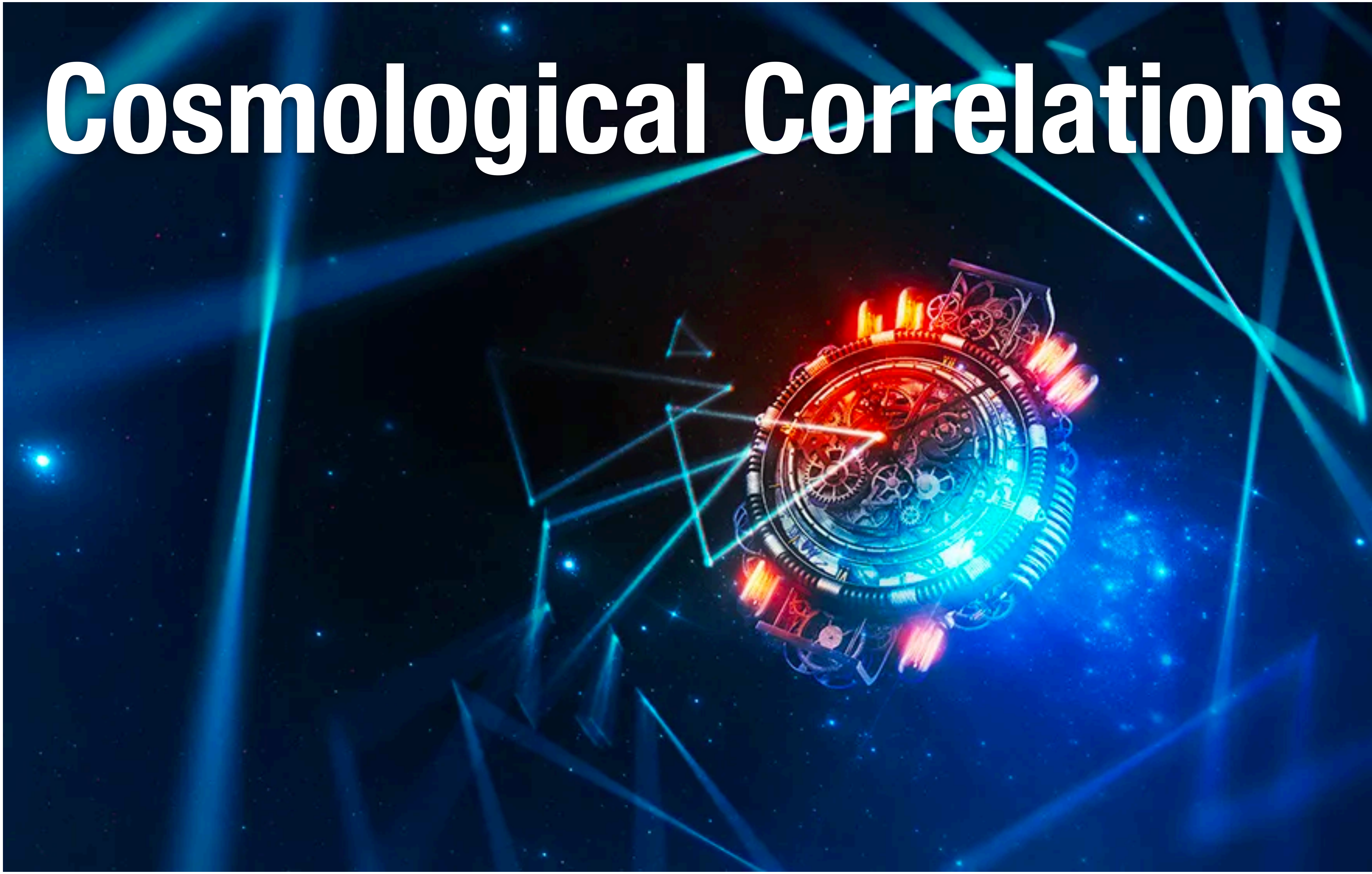
Cosmological Correlations



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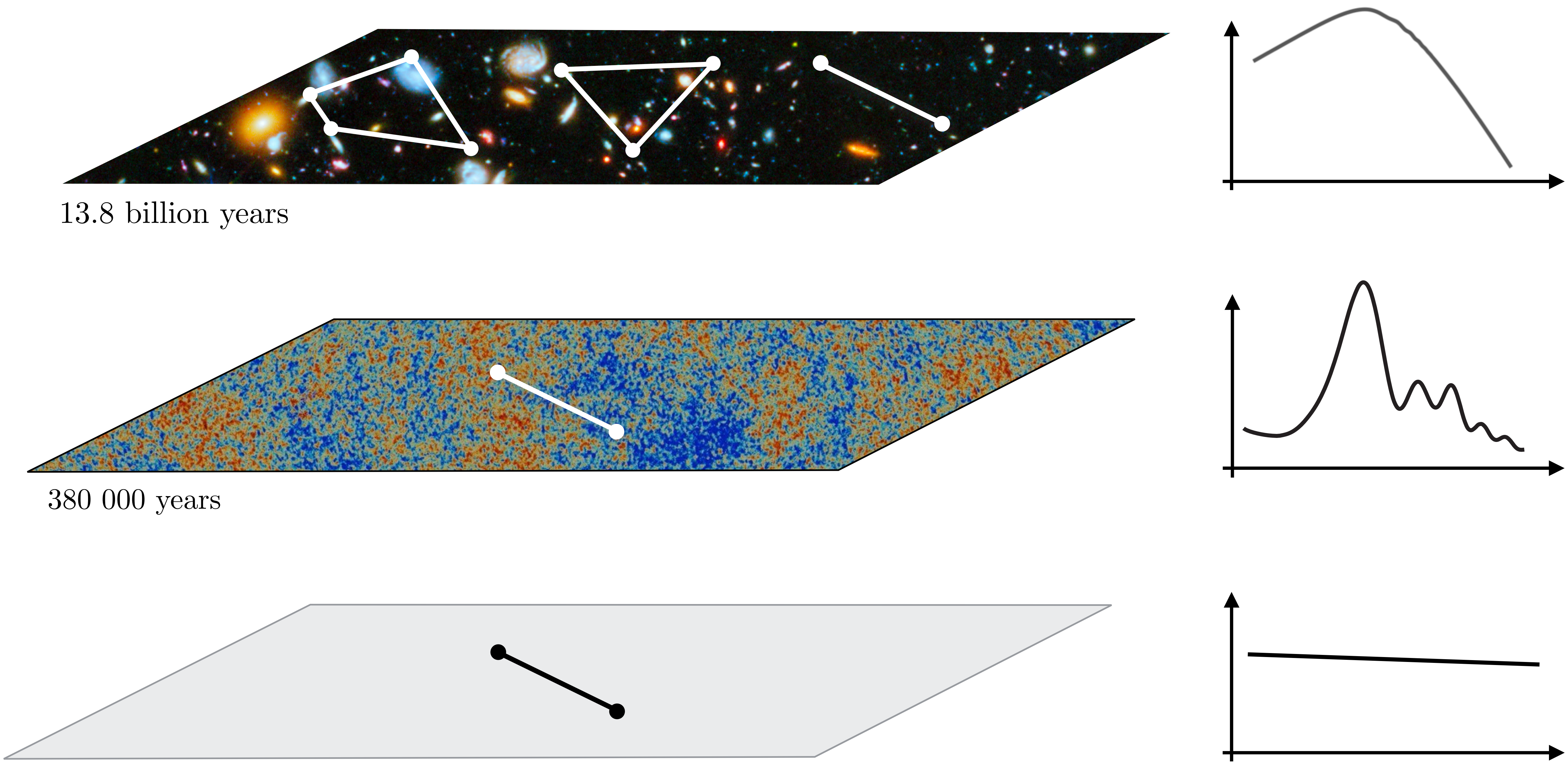
Kavli Asian Winter School
Jan 2023

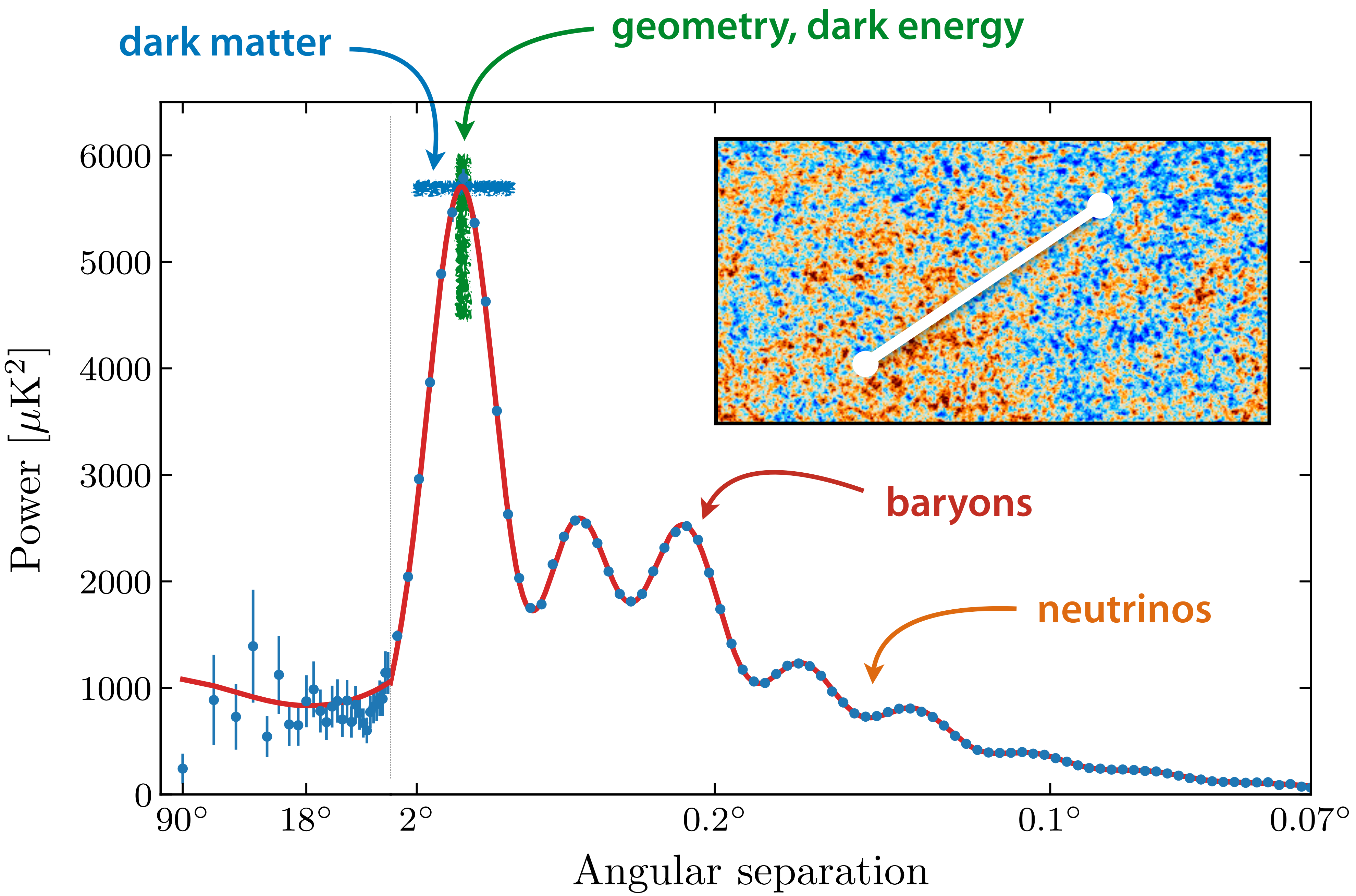
Motivation

By measuring cosmological correlations, we learn both about the **evolution** of the universe and its **initial conditions**:



Motivation

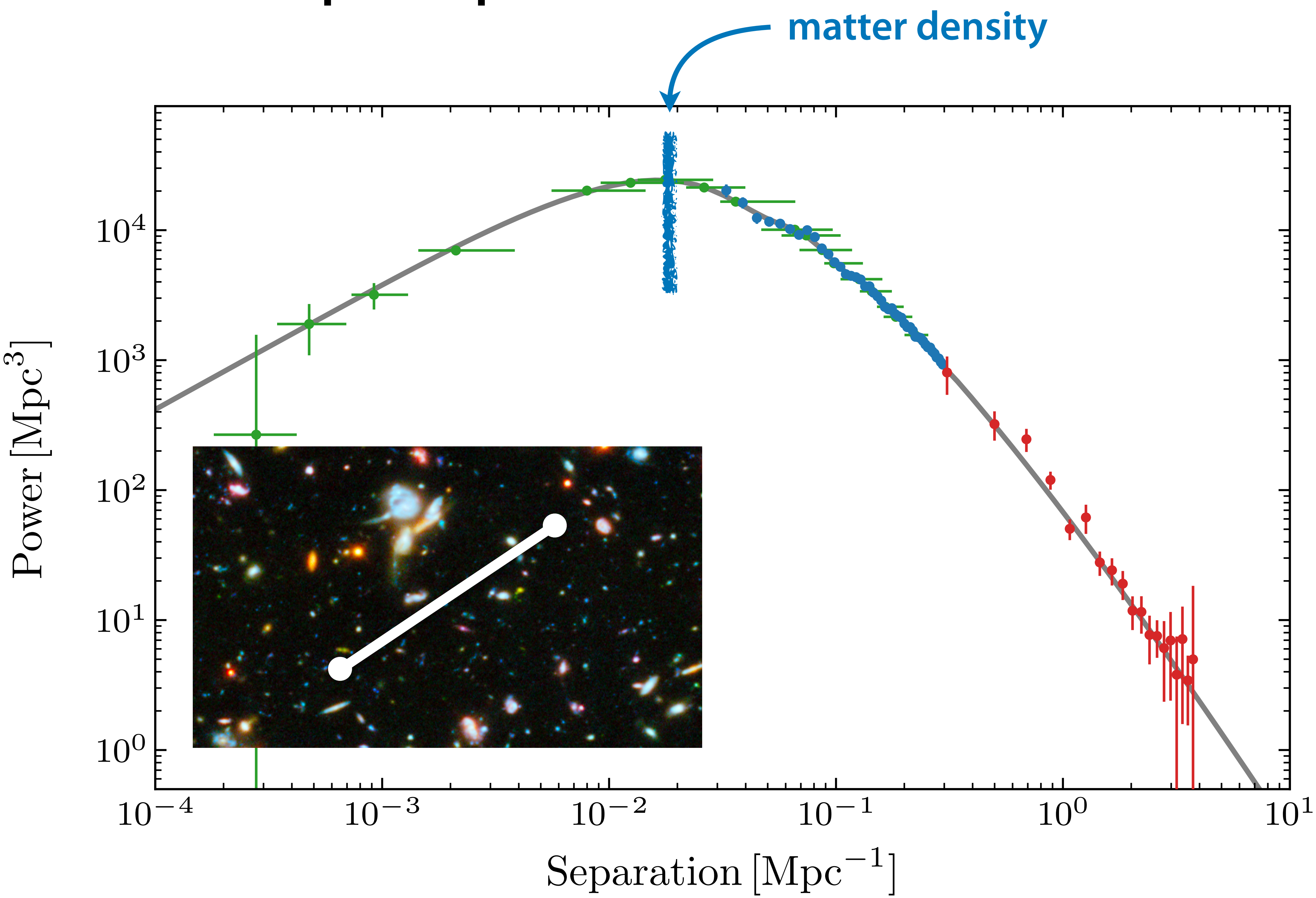
The correlations in the CMB temperature anisotropies have revealed a great deal about the **geometry** and **composition** of the universe:



Motivation

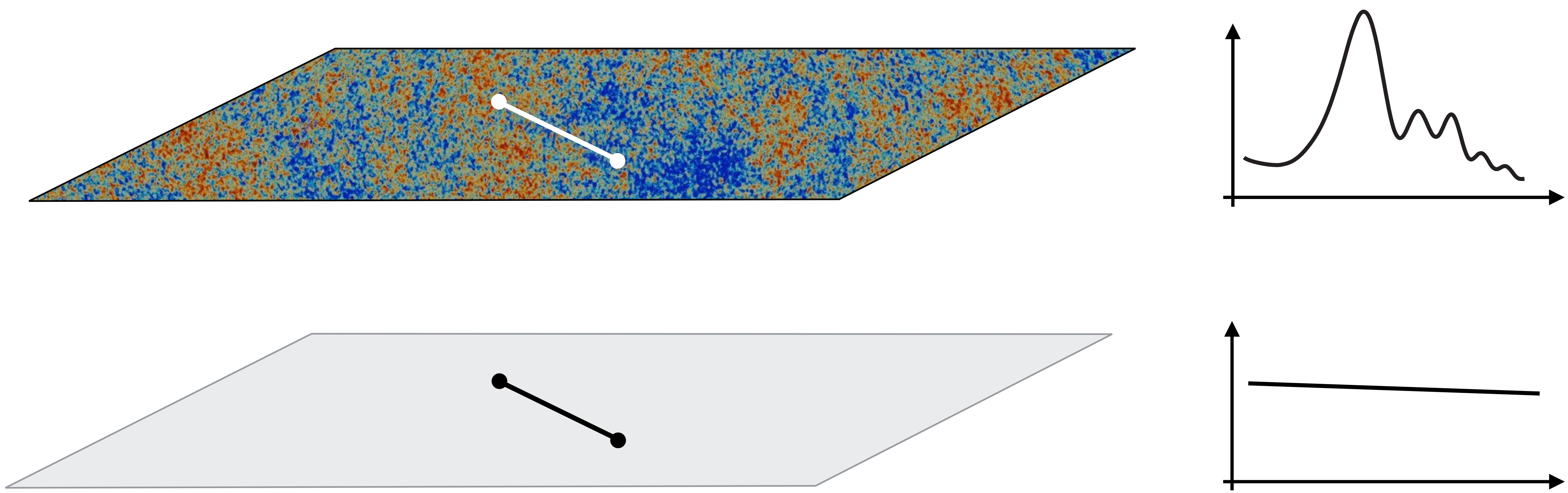
Correlations are also observed in the large-scale structure of the universe.

The observed **matter power spectrum** is



Motivation

Under relatively mild assumptions, the observed correlations can be traced back to primordial correlations on the **reheating surface**:

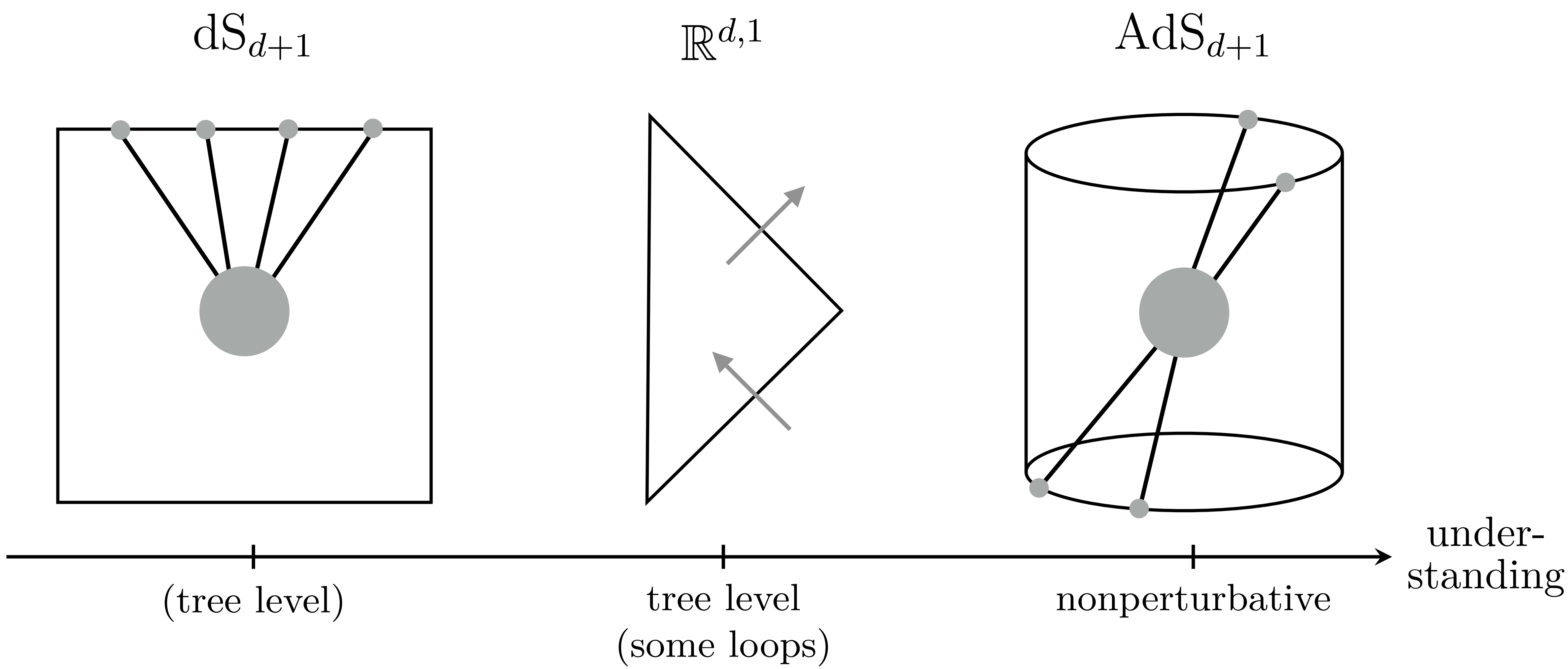


We have learned two interesting facts about these initial conditions:

1) The fluctuations were created **before the hot Big Bang**.
2) During a phase of time-translation invariance (= **inflation**).

Motivation

The study of cosmological correlators is also of **conceptual interest**:



Our understanding of quantum field theory in de Sitter space (cosmology) is still rather underdeveloped. **Opportunity for you to make progress!**

OUTLINE:

Cosmological Correlations	Wavefunction Approach	Cosmological Bootstrap
(Lecture 1)	(Lectures 2+3)	(Lectures 3+4)

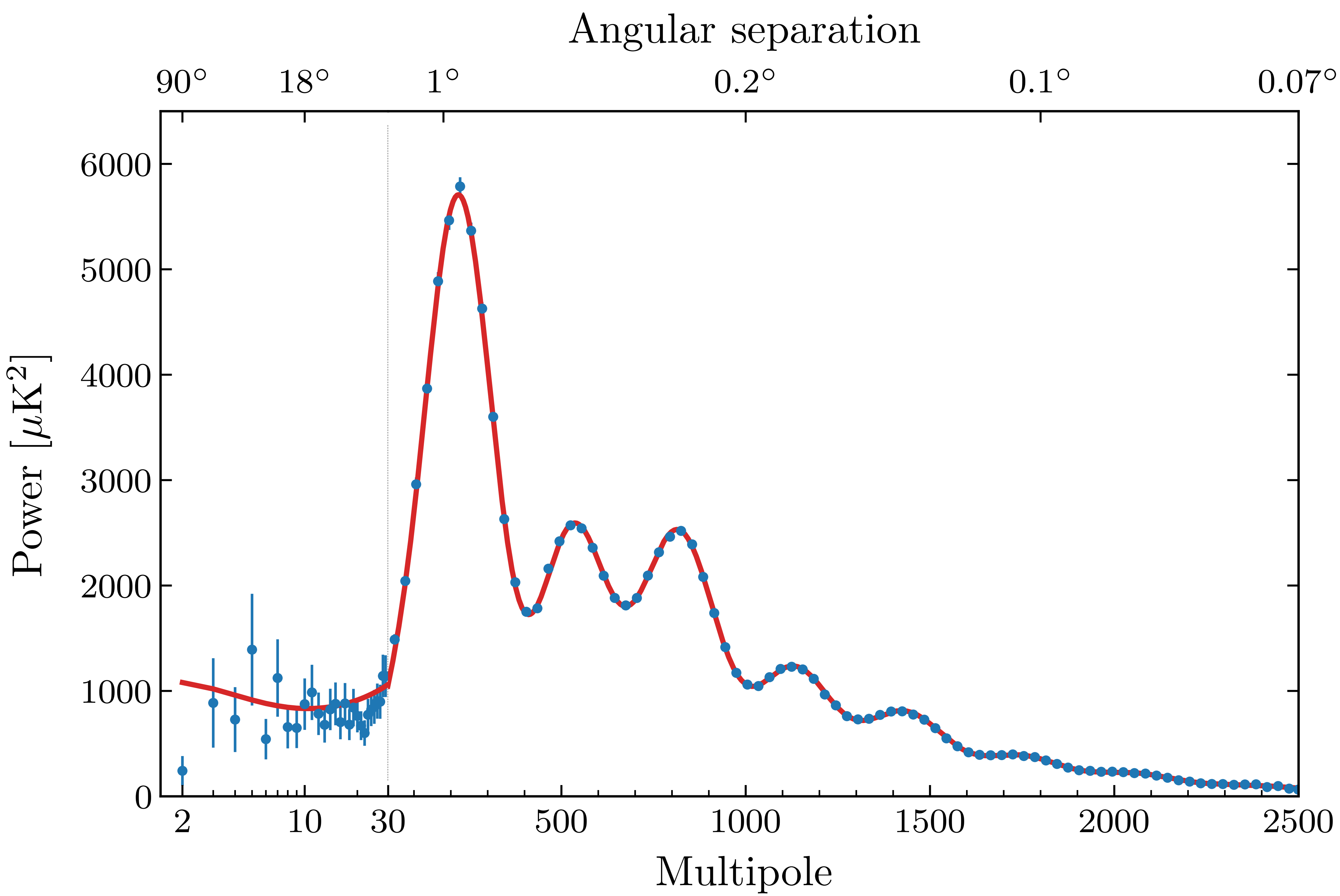
LECTURE NOTES:

https://github.com/ddbaumann/cosmo-correlators

written together with Austin Joyce.

Cosmological Correlations

CMB Power Spectrum



Large scales

Small scales

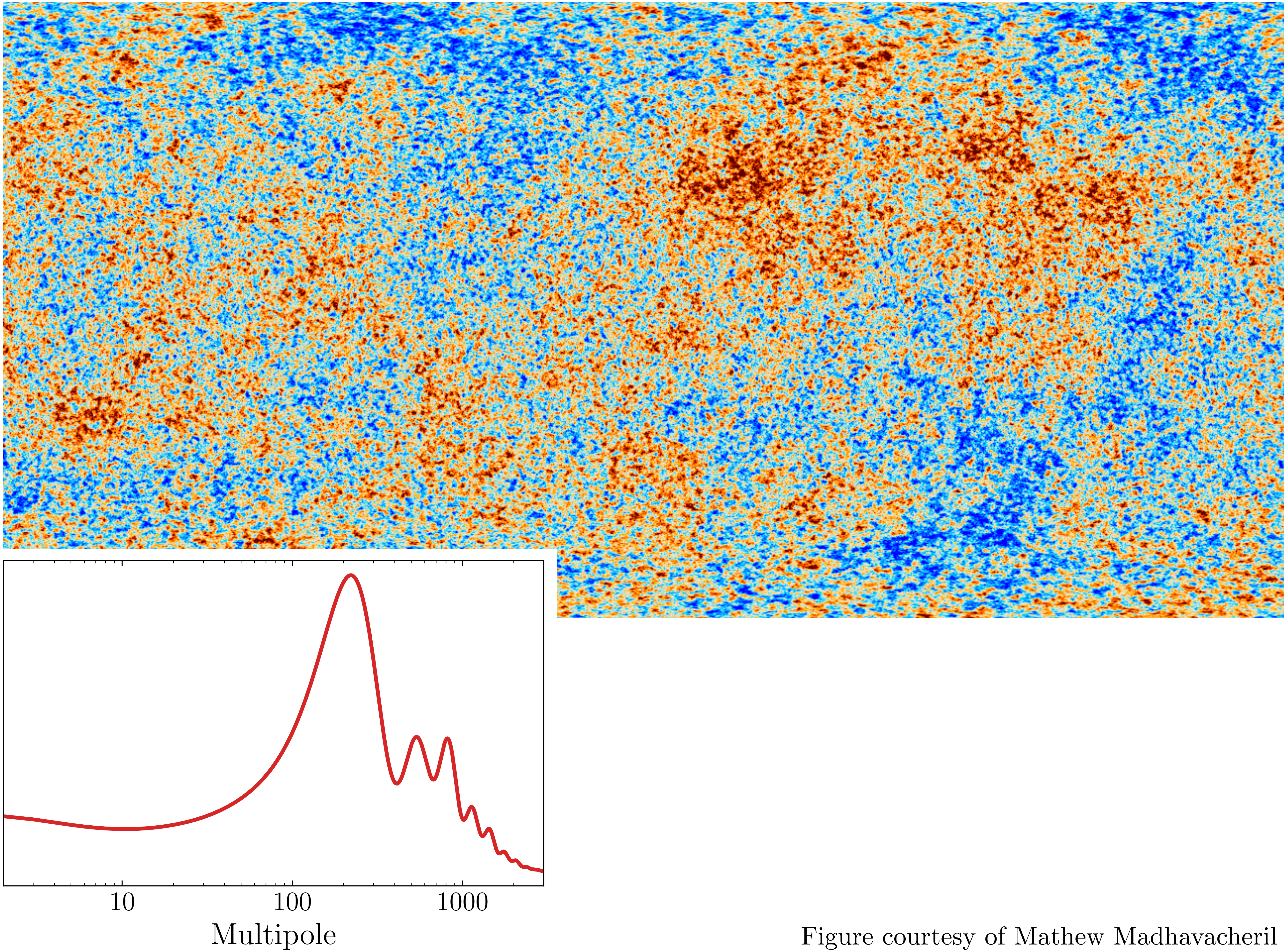
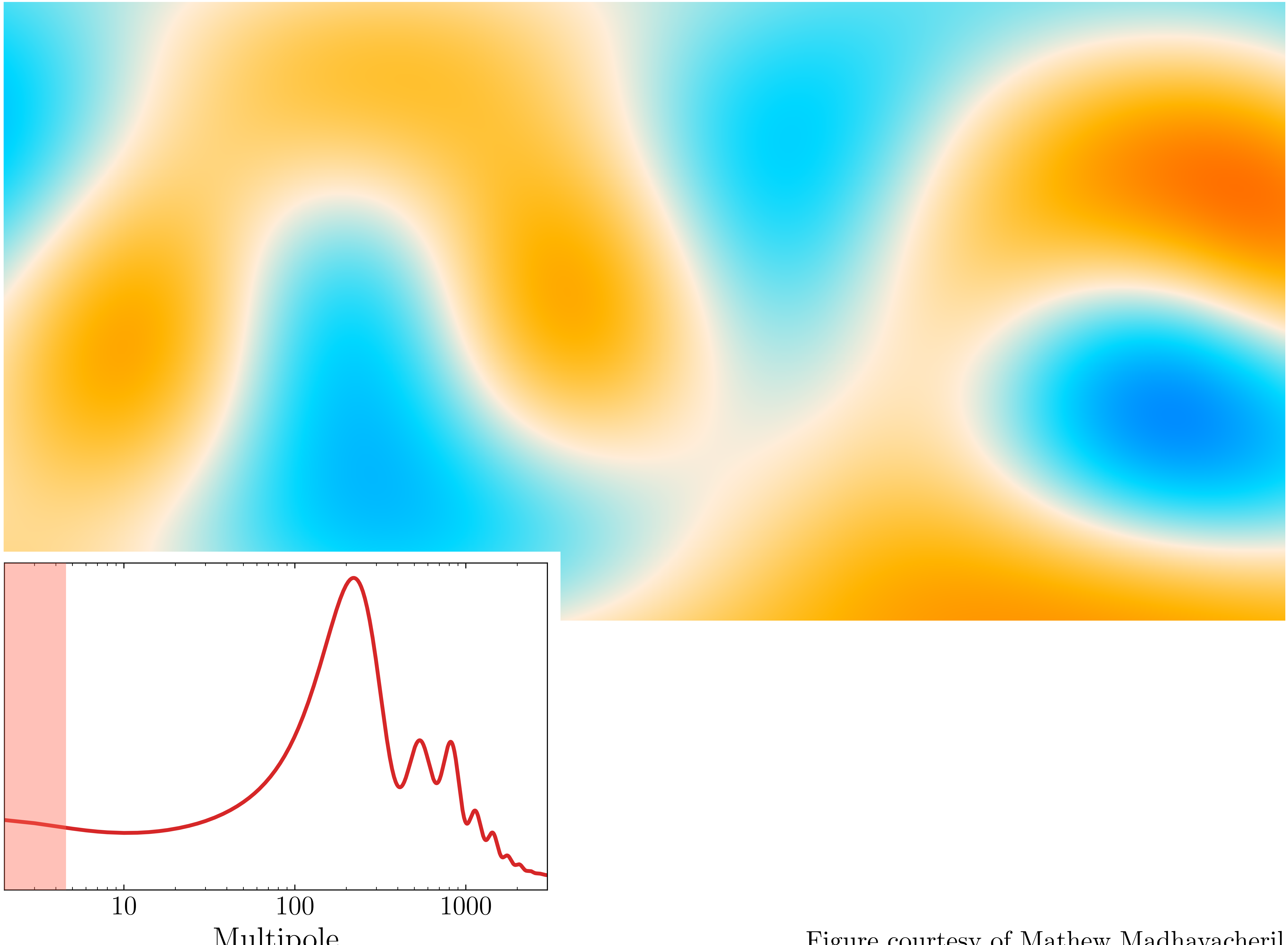


Figure courtesy of Mathew Madhavacheril



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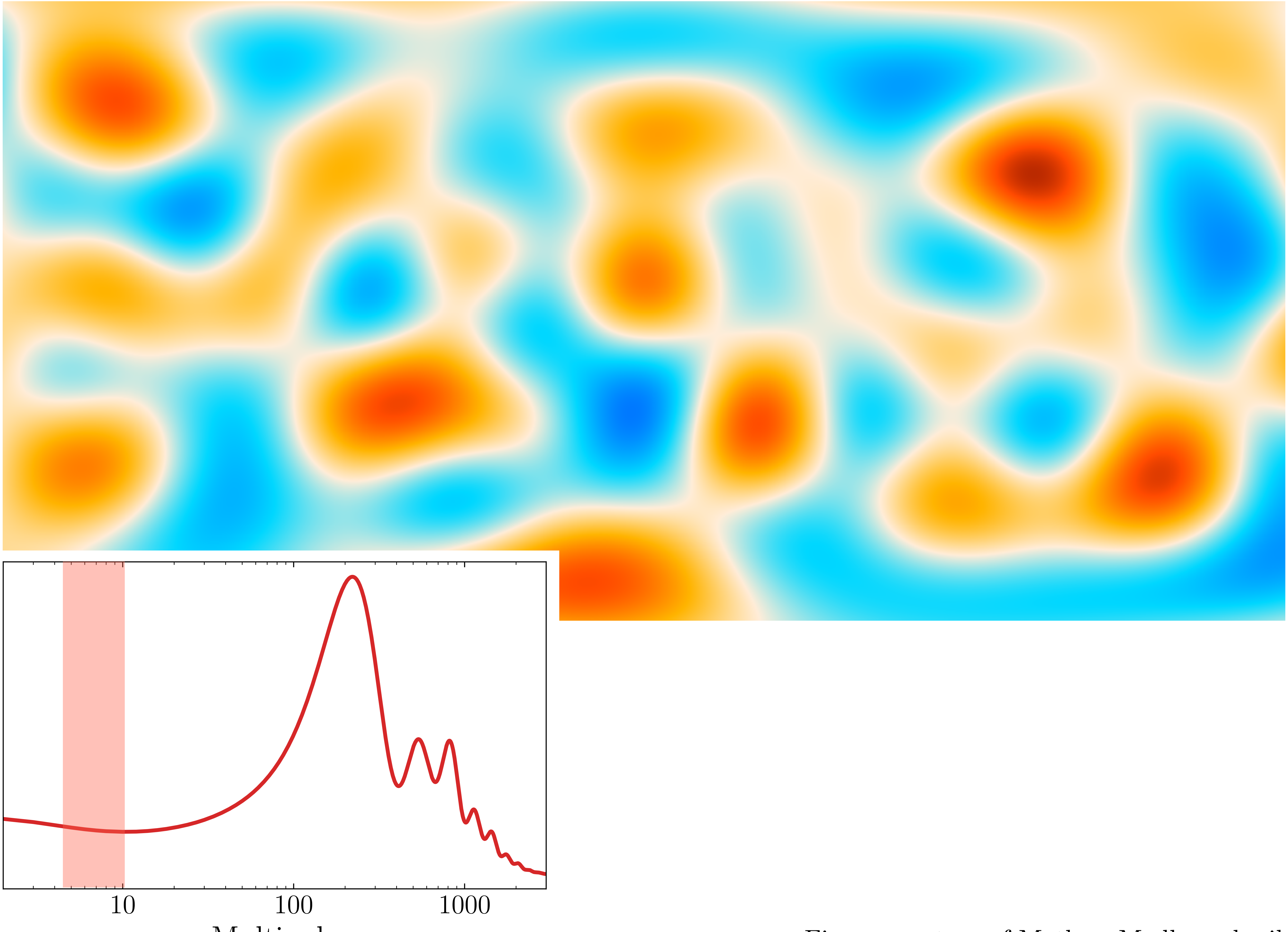

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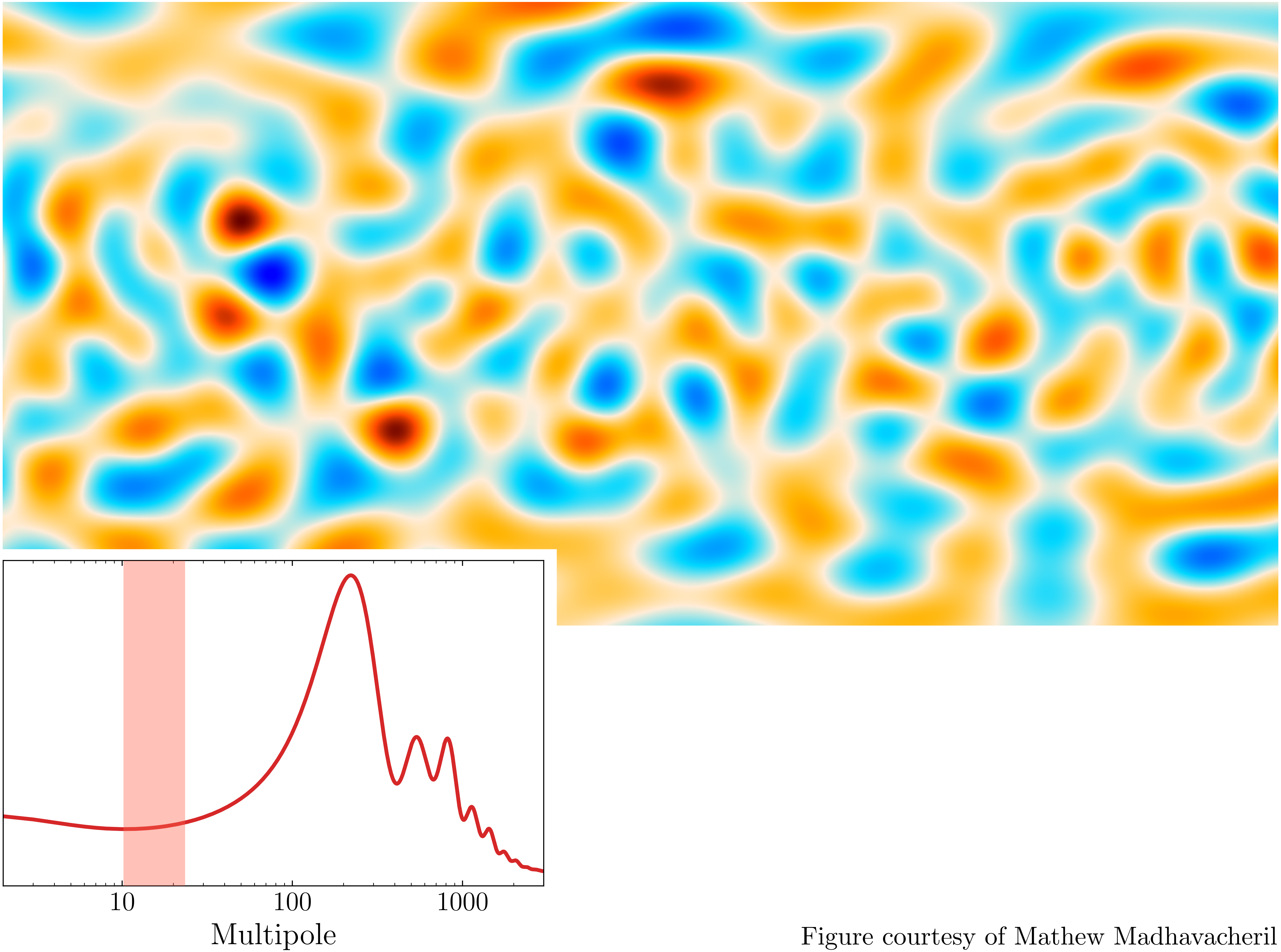

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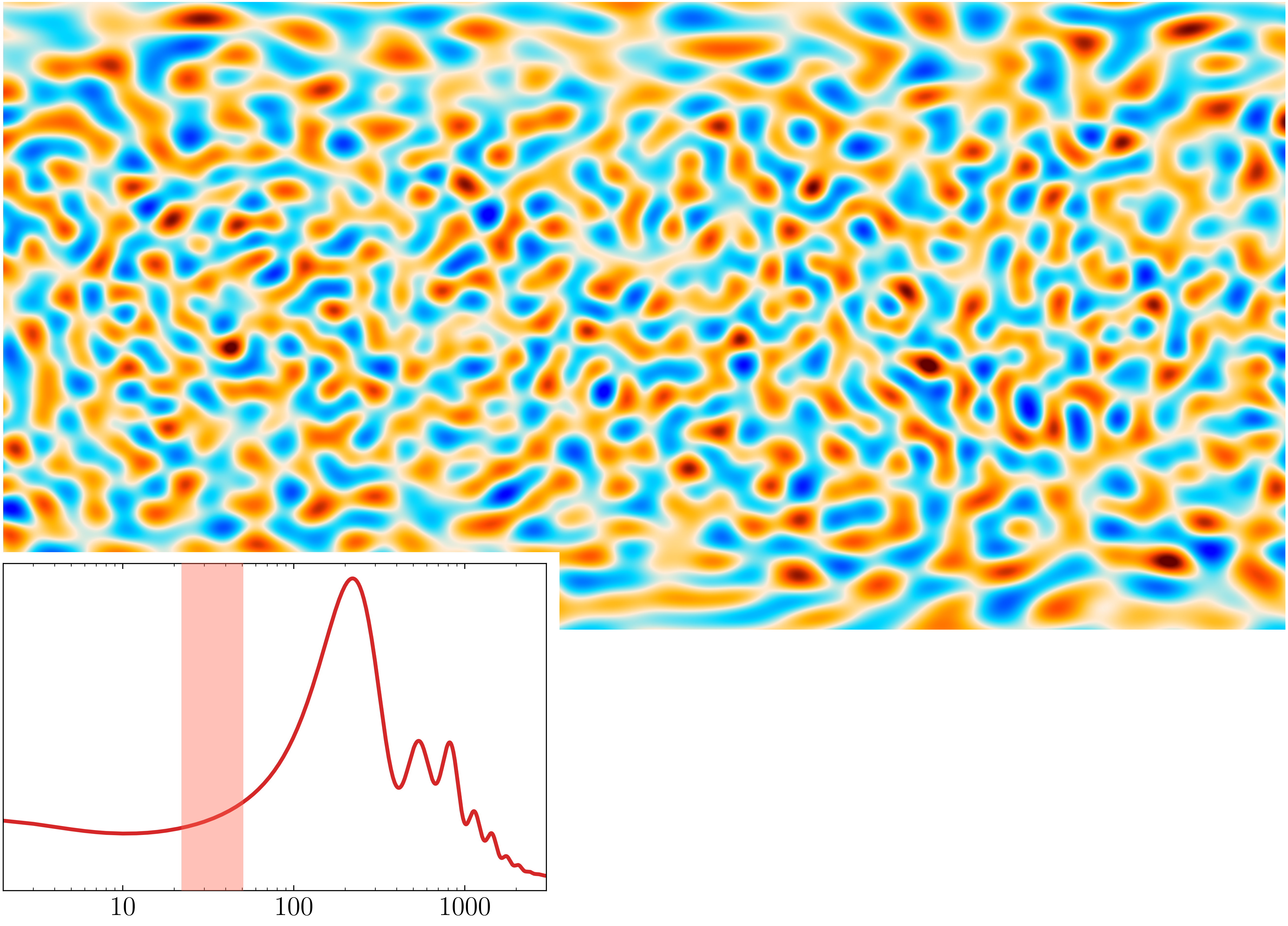

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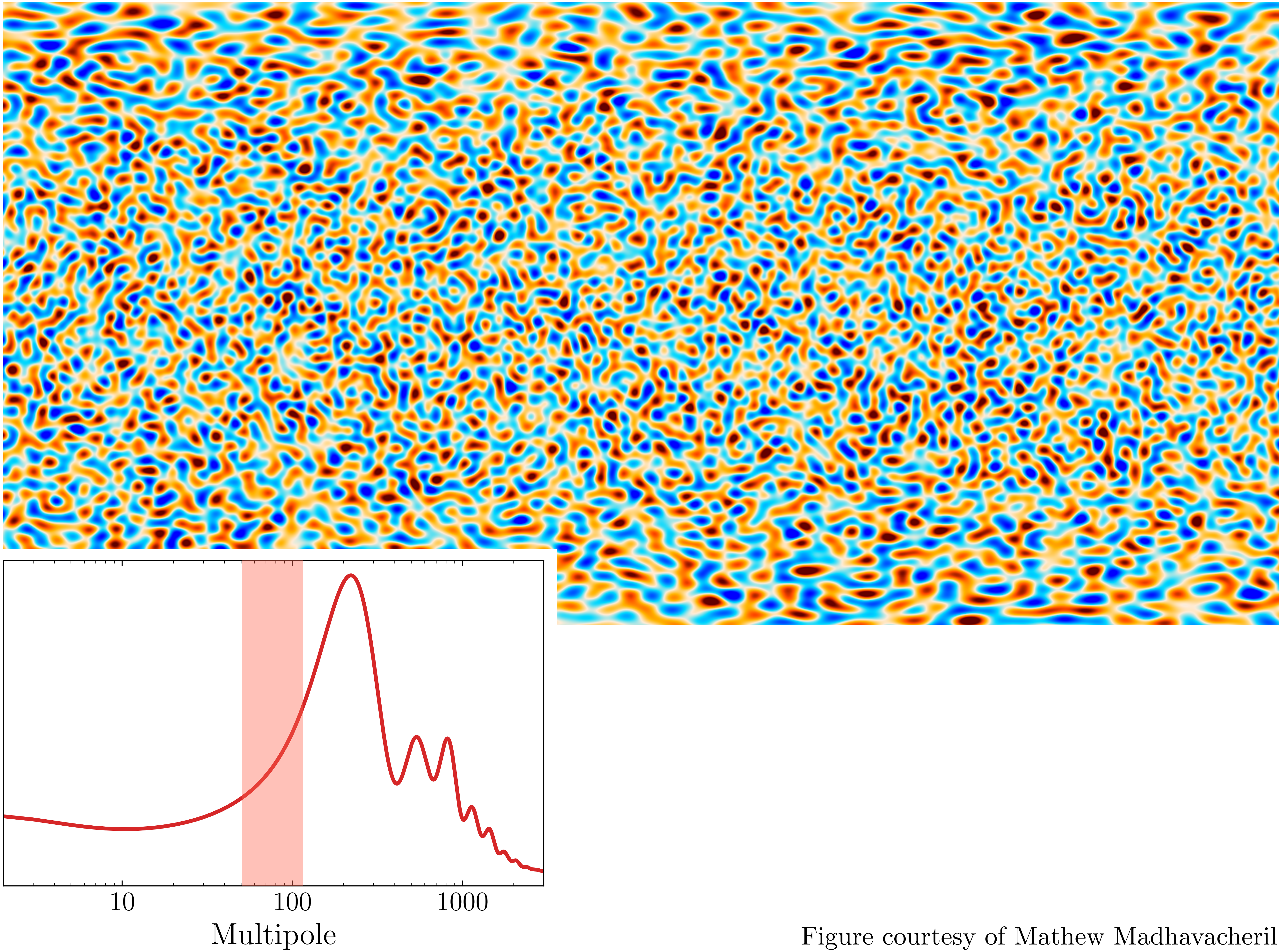

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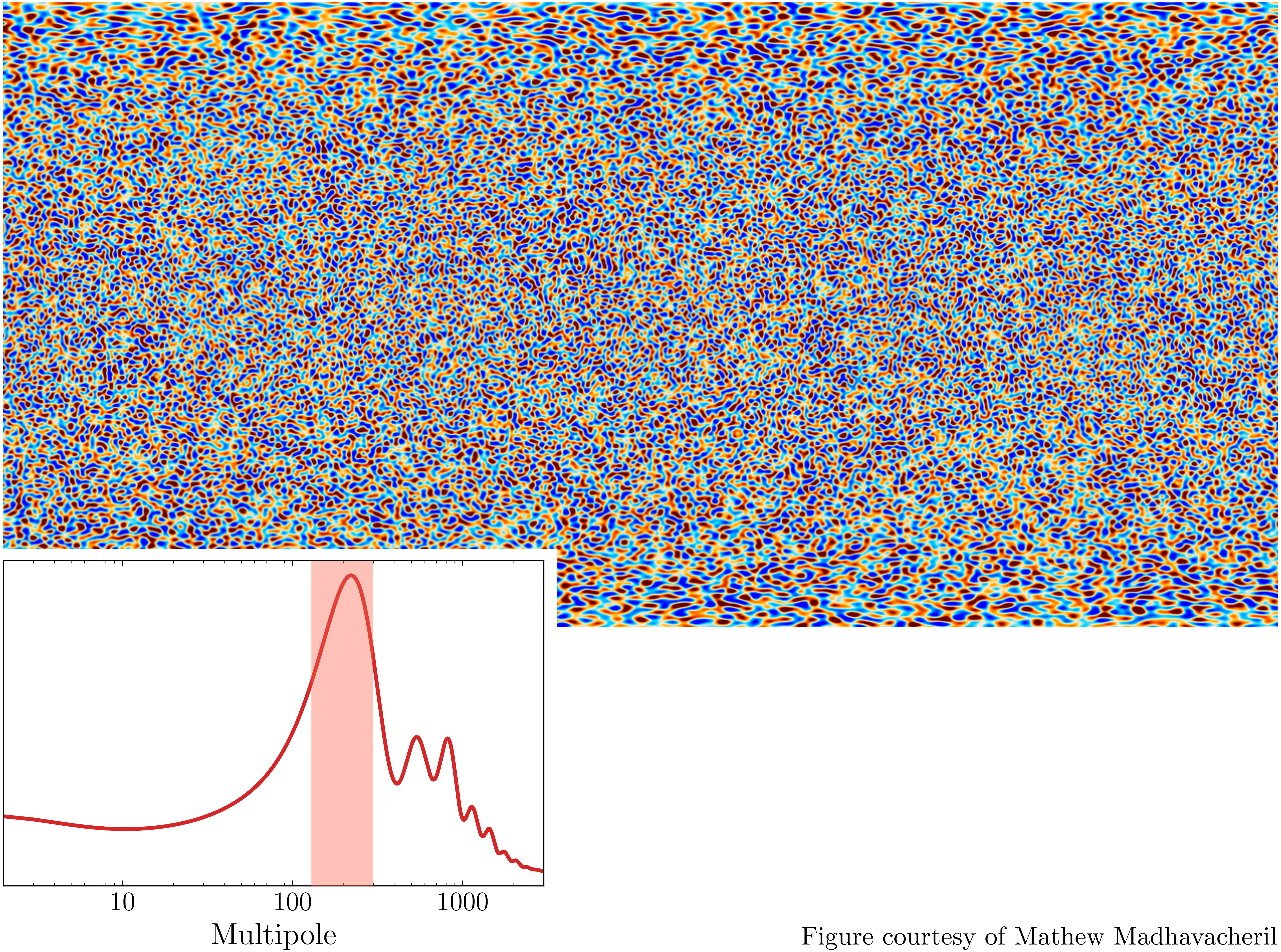

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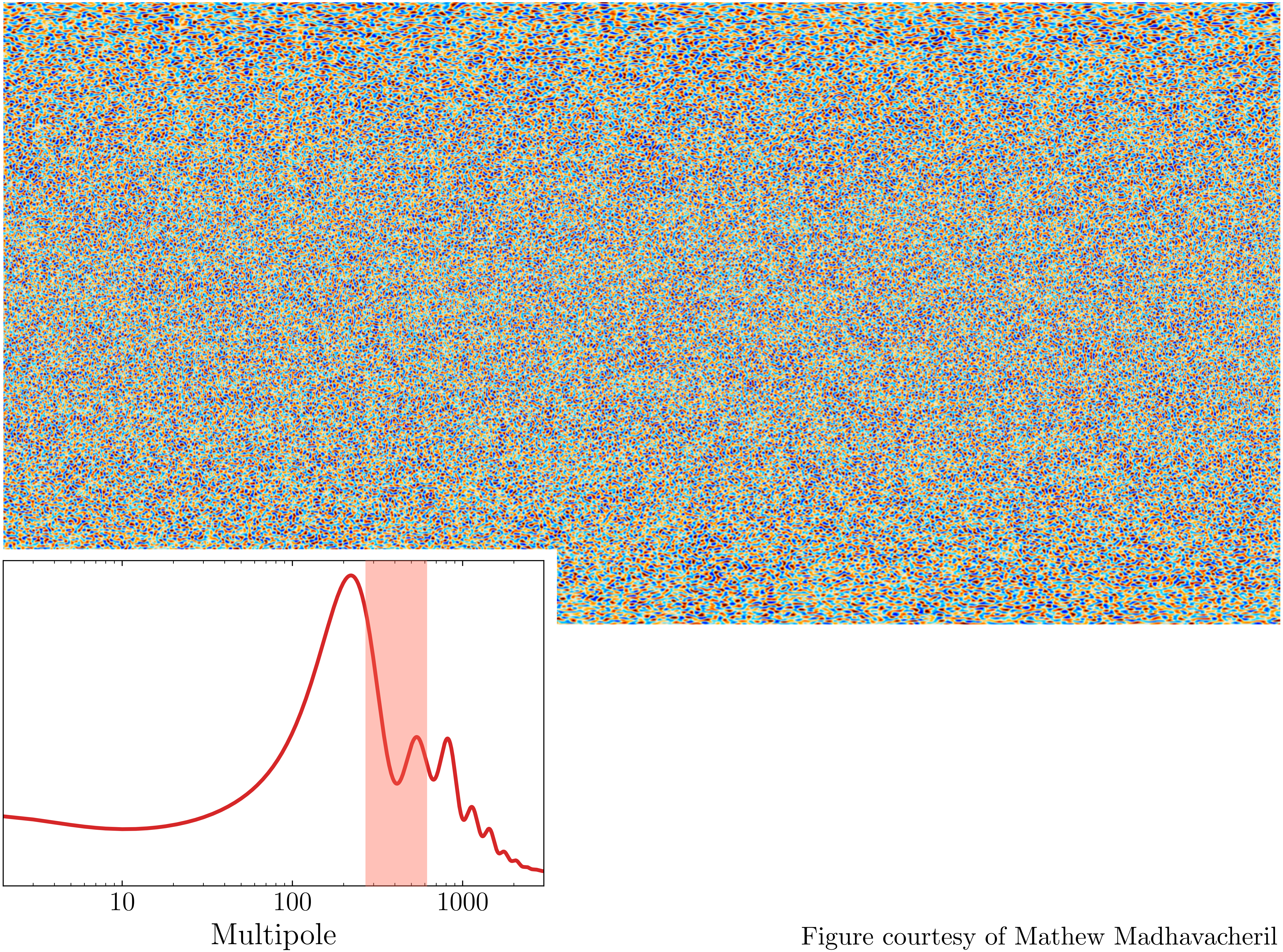

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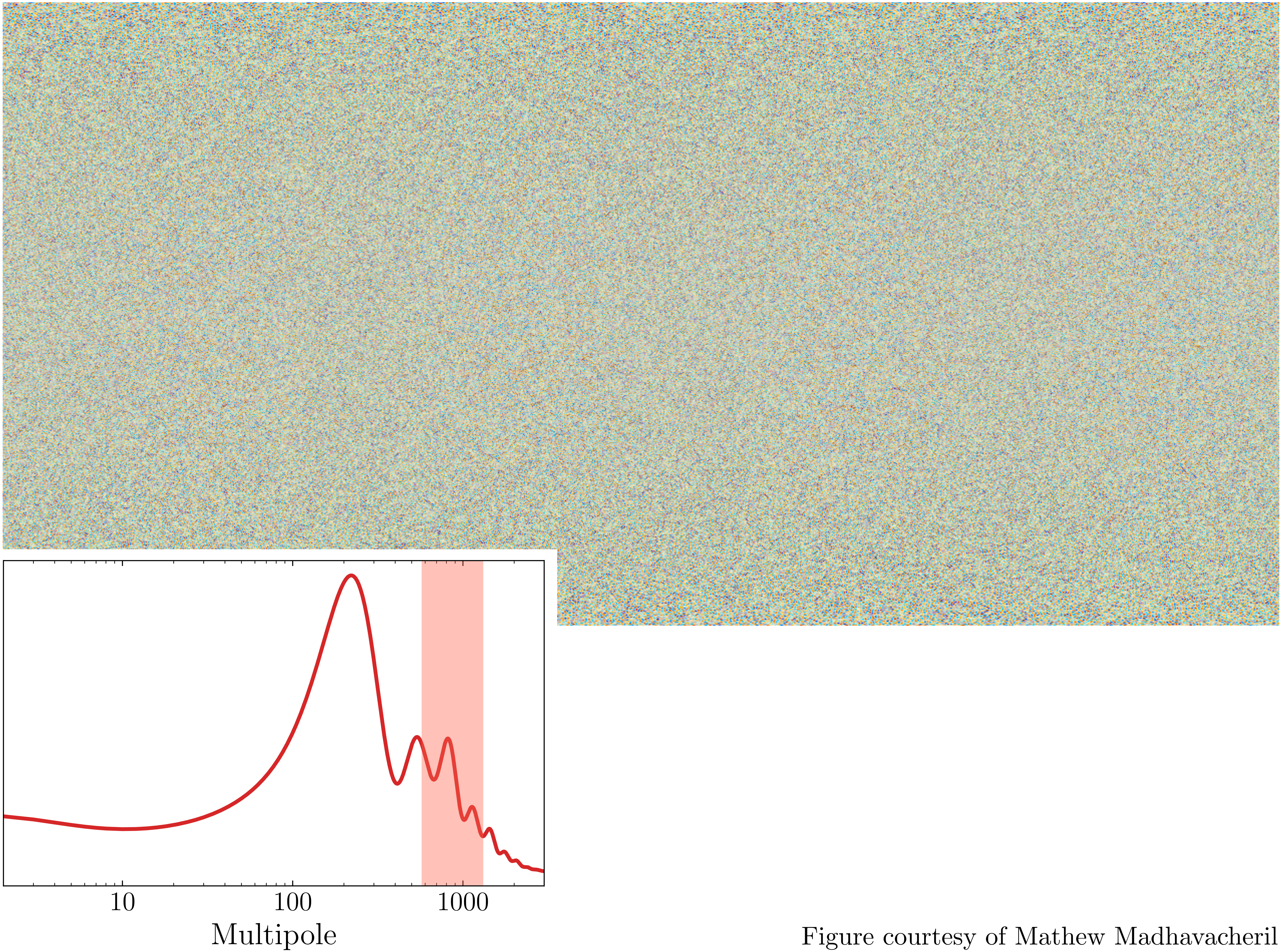

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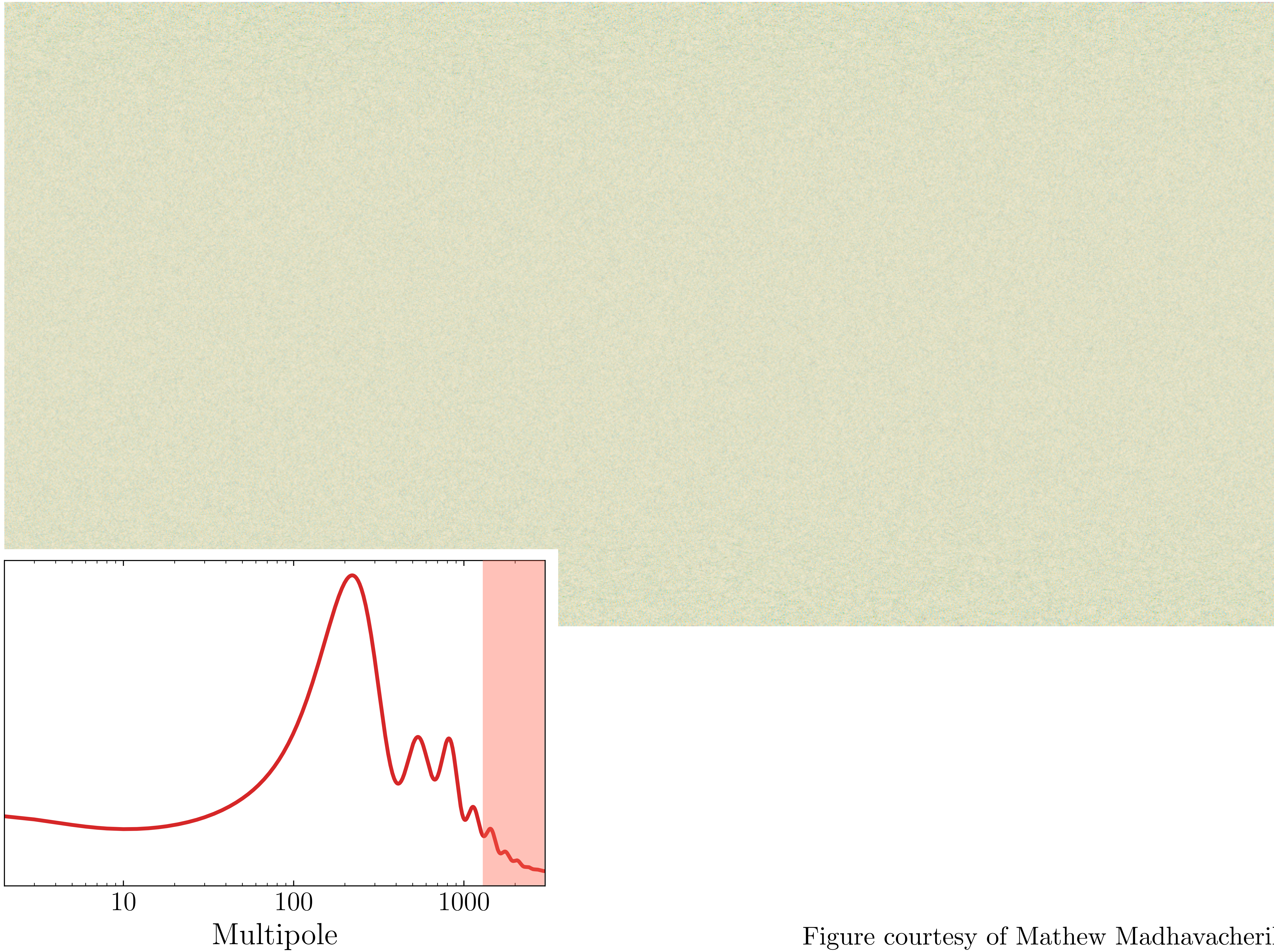

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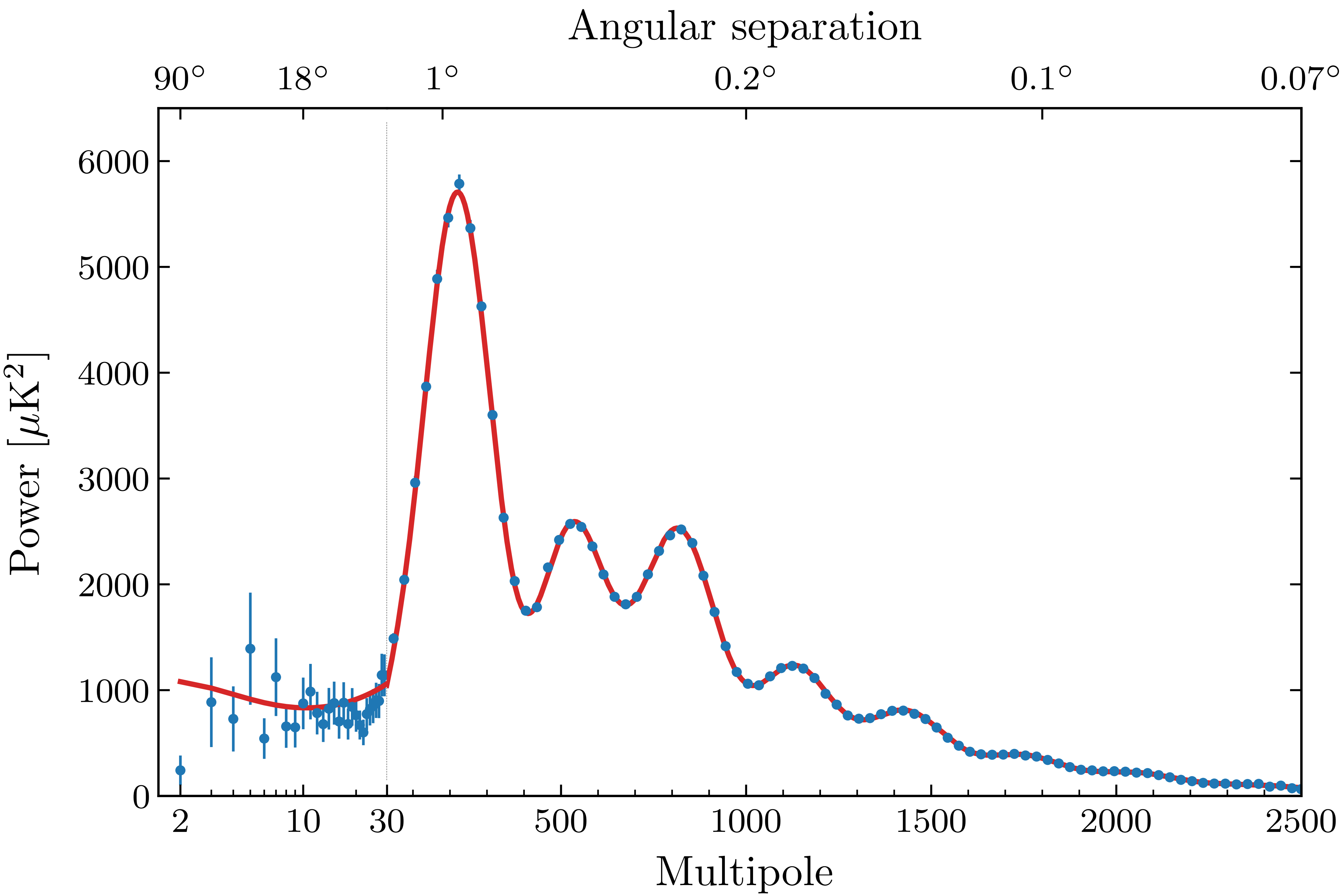
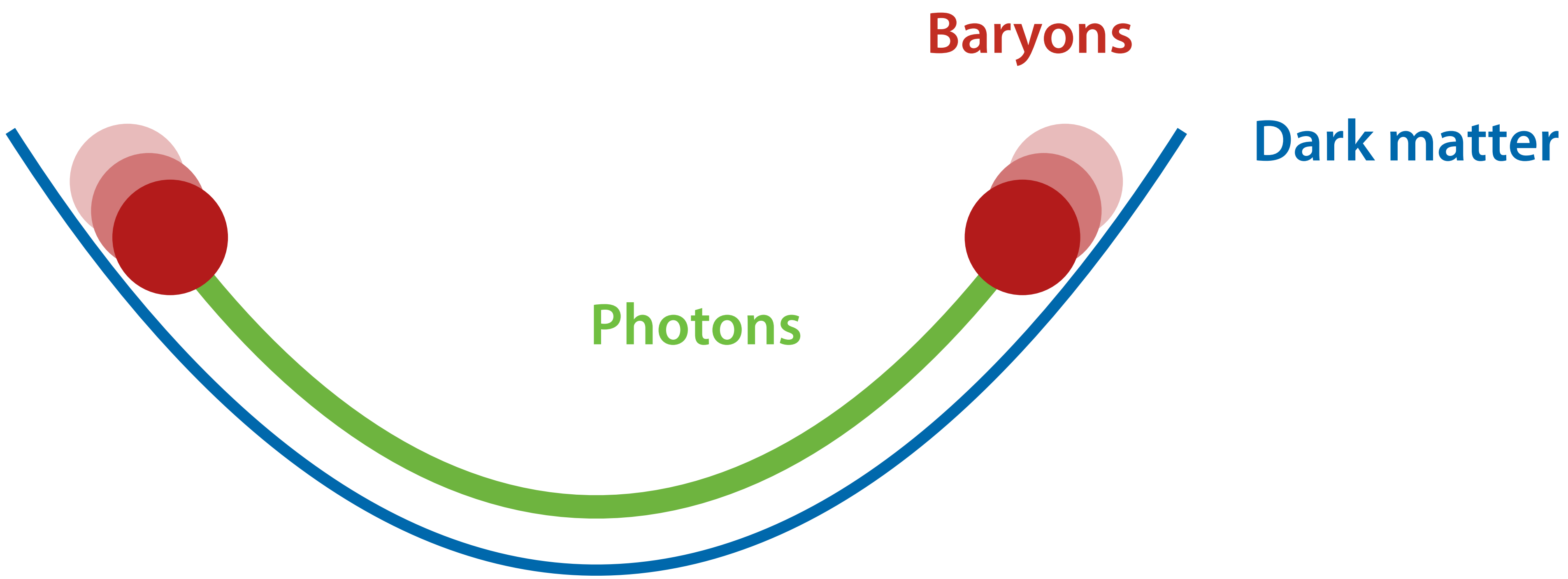
CMB Power Spectrum



What created the features in the power spectrum?

Photon-Baryon Fluid

At early times, photons and baryons (mostly protons and electrons) are strongly coupled and act as a single fluid:



The **photon pressure** prevents the collapse of density fluctuations. This allows for **sound waves** (like for density fluctuations in air).

Cosmic Sound Waves

Decomposing the sound waves into Fourier modes, we have

$$\boxed{\frac{\delta\rho_\gamma}{\rho_\gamma} = A_{\mathbf{k}} \cos[c_s k \tau_*] + B_{\mathbf{k}} \sin[c_s k \tau_*]} \;\leftarrow\; 4\,\frac{\delta T}{T}$$

Time at recombination

Sound speed $c_s \approx 1/\sqrt{3}$

The wave amplitudes are random variables, with

$$\langle A_{\mathbf{k}} \rangle = \langle B_{\mathbf{k}} \rangle = 0$$

This is why we don't see a wavelike pattern in the CMB map.

To see the sound waves, we need to look at the statistics of the map.

CMB Power Spectrum

The two-point function in Fourier space is

$$\langle \delta T(\mathbf{k}) \delta T(\mathbf{k}') \rangle = \Big[P_A(k) \cos^2(c_s k \tau_*) + 2 C_{AB}(k) \sin(c_s k \tau_*) \cos(c_s k \tau_*) \\ + P_B(k) \sin^2(c_s k \tau_*) \Big] (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

If the sound waves were created by a random source, we would have

$$C_{AB}(k) = 0 \qquad\qquad P_A(k) = P_B(k) \equiv P(k)$$

and hence the two-point function becomes

$$\langle \delta T(\mathbf{k}) \delta T(\mathbf{k}') \rangle = P(k)\, (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

Where are the oscillations?

Coherent Phases

To explain the oscillations in the CMB power spectrum, we have to assume that the sound waves do **not** have random phases:

$$\delta T(\mathbf{k}) = A_{\mathbf{k}} \cos[c_s k \tau_*]$$

$$\downarrow$$

$$\langle \delta T(\mathbf{k}) \delta T(\mathbf{k}') \rangle' = P_A(k) \cos^2[c_s k \tau_*]$$

Oscillations in the CMB power spectrum

$$\boxed{\text{What created the coherent phases in the primordial sound waves?}}$$

Before the Big Bang

Phase coherence is created if the fluctuations were produced **before the hot Big Bang**. Let's see why.

Going back in time, all fluctuations have wavelengths > Hubble scale.

The solution on these large scales is

$$\frac{\delta\rho_\gamma}{\rho_\gamma} = C_{\mathbf{k}} + D_{\mathbf{k}}\, a^{-3}$$

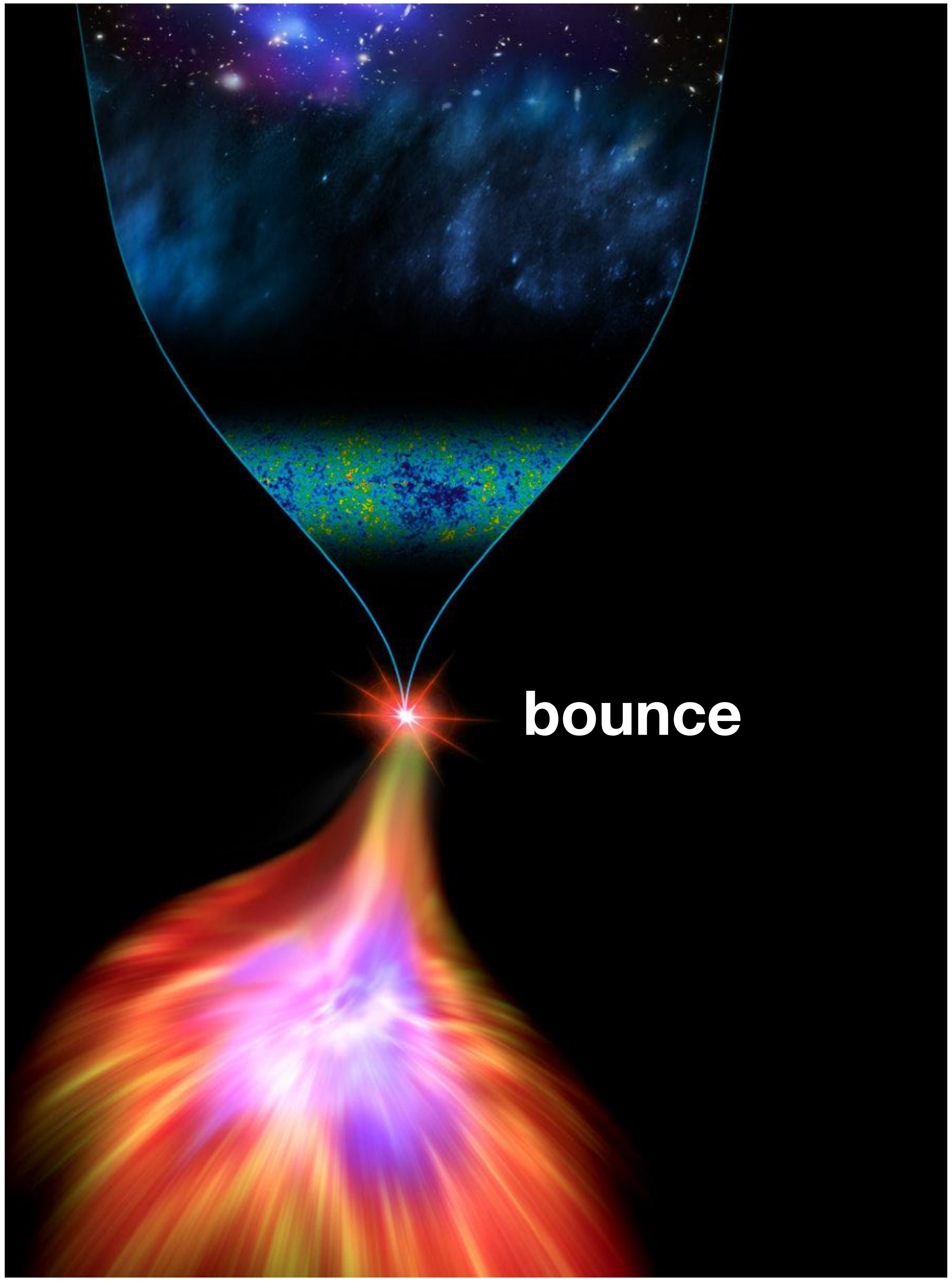
If the fluctuations were created long before the hot Big Bang, then only the constant growing mode survived.

This explains the phase coherence of the primordial sound waves

$$\frac{\delta\rho_\gamma}{\rho_\gamma} = C_{\mathbf{k}} \;\; \to \;\; A_{\mathbf{k}} \cos\left[c_s k\tau\right]$$

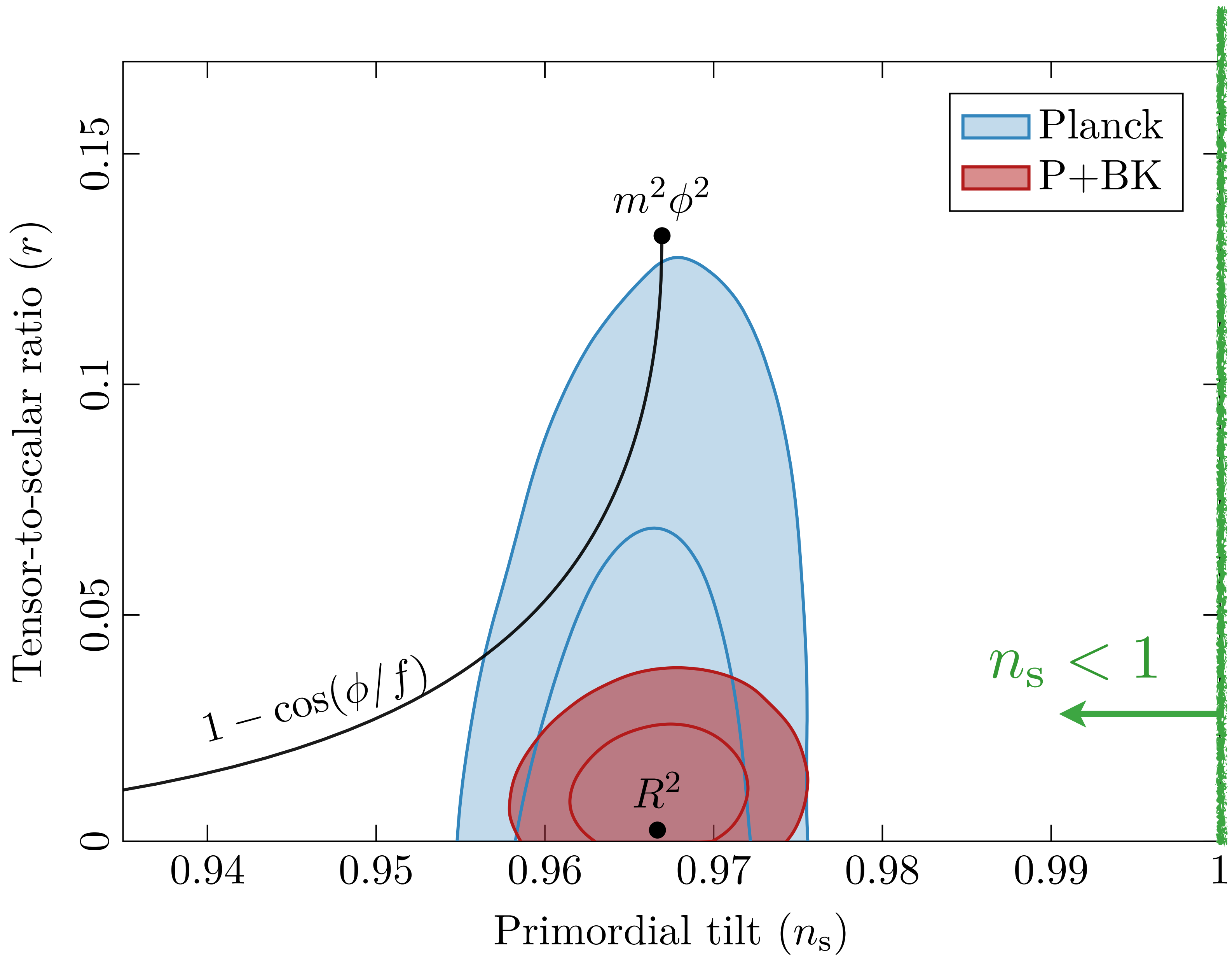
Before the Big Bang

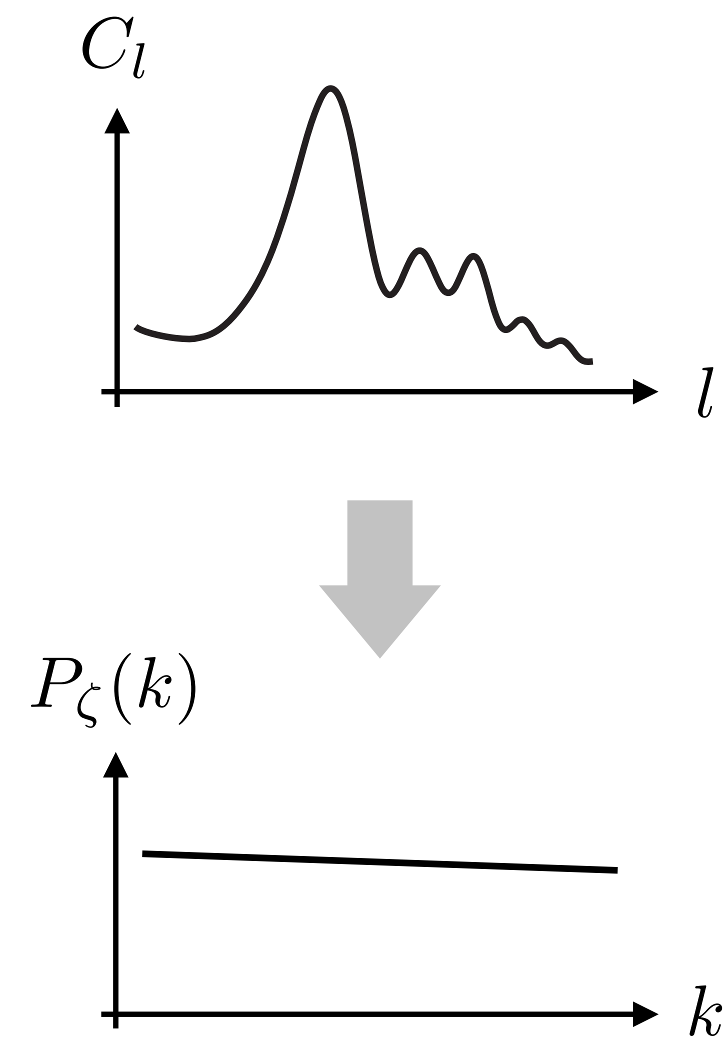
This suggests two options: **rapid expansion** or **slow contraction**?



Scale-Invariance

The primordial fluctuations are approximately **scale-invariant**:





This suggests that the fluctuations were created in a phase of approximate **time-translation invariance** (= inflation).

Inflation

During inflation, the expansion rate is nearly constant:

$$\boxed{H(t) \equiv \frac{1}{a}\frac{da}{dt} \approx \text{const}}$$

- → Exponential expansion: $a(t) \approx \exp(Ht)$
- → Quasi-de Sitter geometry

Inflation needs to end, so there has to be some time-dependence:

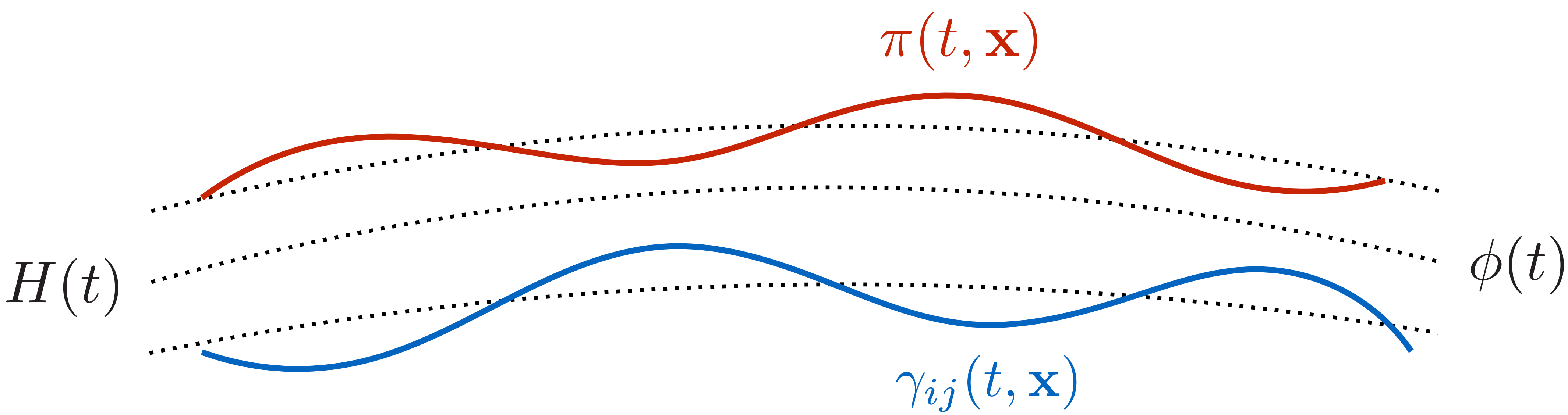
$$\boxed{\varepsilon \equiv -\frac{\dot{H}}{H^2} \ll 1}$$

In slow-roll inflation, this is generated by a field rolling down a flat potential. However, there are other ways to achieve inflation.

EFT of Inflation

Inflation is a **symmetry breaking phenomenon**:

Creminelli et al. [2006]
Cheung et al. [2008]



The low-energy EFT is parameterized by two massless fields:

- **Goldstone boson**
  of broken time translations

$$\delta\phi = \phi(t + \pi) - \bar{\phi}(t)$$

- **Graviton**

EFT of Inflation

Inflation is a **symmetry breaking phenomenon**:

Creminelli et al. [2006]
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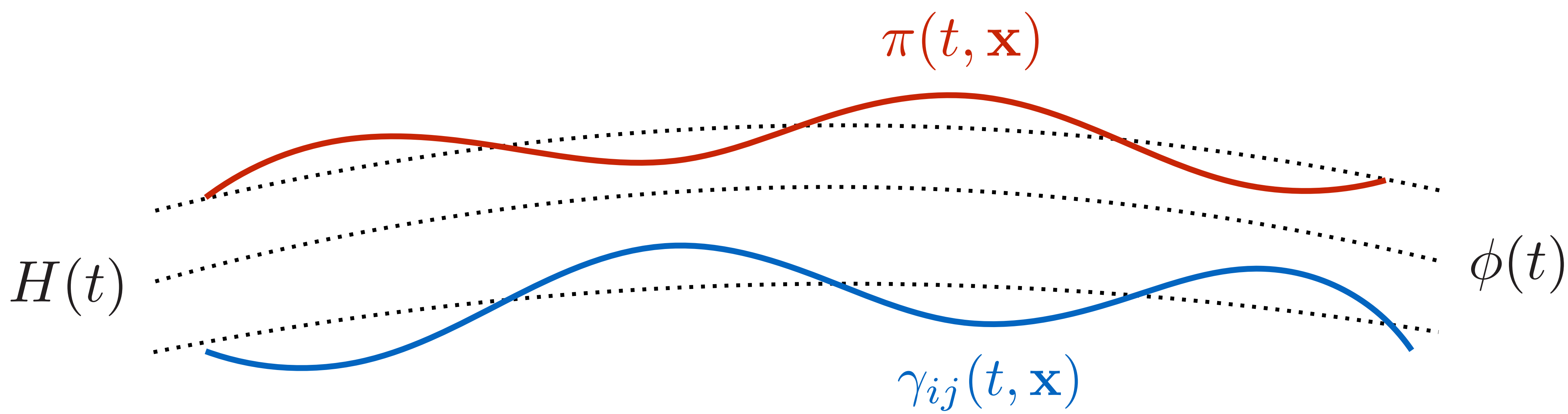


In comoving gauge, the Goldstone boson gets eaten by the metric:

$$g_{ij} = a^2 e^{2\zeta} [e^{\gamma}]_{ij} \qquad \zeta = -H\pi$$

Curvature perturbation

EFT of Inflation

The Goldstone Lagrangian is

Creminelli et al. [2006]
Cheung et al. [2008]

$$\mathcal{L}_\pi = M_{\rm pl}^2 \dot{H} (\partial\pi)^2 + \sum_{n=2}^{\infty} \frac{M_n^4}{n!} \left[-2\dot{\pi} + (\partial\pi)^2\right]^n + \cdots$$

Slow-roll inflation

$$\tfrac{1}{2}(\partial\phi)^2 - V(\phi)$$

Higher-derivative corrections

$$(\partial\phi)^{2n}$$

The fluctuations can have a small **sound speed** and large **interactions**:

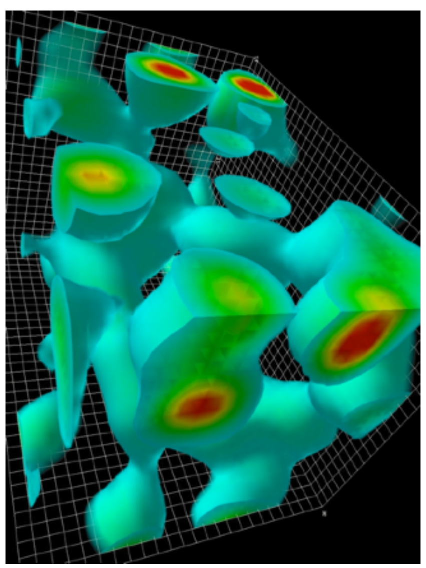
$$\mathcal{L}_\pi = \frac{M_{\rm pl}^2 |\dot{H}|}{c_s^2} \Big[\left(\dot{\pi}^2 - c_s^2 (\partial_i \pi)^2\right) - (1 - c_s^2)\dot{\pi}(\partial_\mu \pi)^2 + \cdots \Big]$$

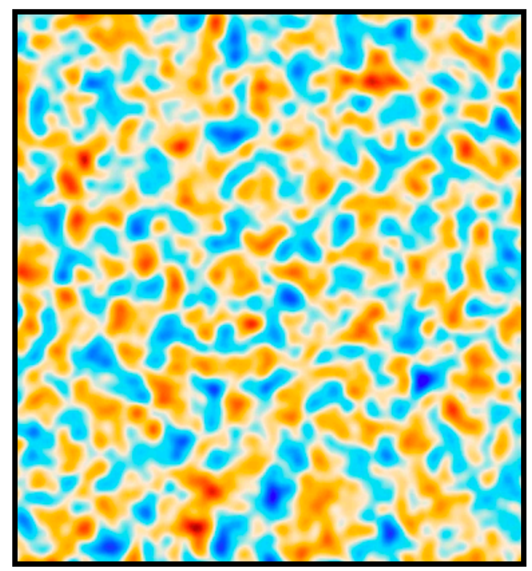
nonlinearly realized symmetry

Quantum Fluctuations

Quantum fluctuations of the Goldstone produced the density fluctuations in the early universe:

10^{-32} s 370 000 yrs 1 billion yrs





Let us derive this.

Quantum Fluctuations

We start from the quadratic Goldstone action

$$S = \int \mathrm{d}^4x\sqrt{-g}\, M_{\mathrm{Pl}}^2 \dot{H}\, g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi$$
$$= \int \mathrm{d}\tau \mathrm{d}^3x\, a^2(\tau)\, M_{\mathrm{Pl}}^2|\dot{H}|\Big((\pi')^2 - (\nabla\pi)^2\Big)$$

where $a(\tau) \approx -(H\tau)^{-1}$ (de Sitter).

It is convenient to introduce the canonically normalized field

$$u \equiv a(\tau)\sqrt{2M_{\mathrm{Pl}}|\dot{H}|}\,\pi$$

so that $$S = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x \left(\underbrace{(u')^2 - (\nabla u)^2}_{\text{Minkowski}} + \underbrace{\frac{a''}{a}}_{\text{Time-dependent mass}}u^2\right)$$

Classical Dynamics

The equation of motion then is

$$\boxed{u''_{\mathbf{k}} + \left(k^2 - \frac{2}{\tau^2}\right) u_{\mathbf{k}} = 0}$$

- At early times, $-k\tau \gg 1$, this becomes

$$u''_{\mathbf{k}} + k^2 u_{\mathbf{k}} \approx 0 \quad \longrightarrow \quad u_{\mathbf{k}}(\tau) = \frac{c_\pm}{\sqrt{2k}}\, e^{\pm ik\tau}$$

Harmonic oscillator

- At late times, $-k\tau \to 0$, this becomes

$$u''_{\mathbf{k}} - \frac{2}{\tau^2} u_{\mathbf{k}} \approx 0 \quad \longrightarrow \quad u_{\mathbf{k}}(\tau) = c_1 \tau^{-1} + c_2 \tau^2$$

Growing mode

Fixed by harmonic oscillator

Classical Dynamics

The equation of motion then is

$$\boxed{u''_{\mathbf{k}} + \left(k^2 - \frac{2}{\tau^2}\right) u_{\mathbf{k}} = 0}$$

- The curvature perturbation becomes a constant at late times:

$$\begin{aligned} \zeta = -H\pi &\propto a^{-1} u \\ &\propto \tau \times \tau^{-1} = \text{const} \end{aligned}$$

- The complete solution is

$$u_{\mathbf{k}}(\tau) = c_1 \frac{1}{\sqrt{2k}} \left(1 + \frac{i}{k\tau}\right) e^{ik\tau} + c_2 \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}$$

where the constants will be fixed by quantum initial conditions.

Canonical Quantization

Quantization starts from the action of the canonically normalized field:

$$S = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x \left((u')^2 - (\nabla u)^2 + \frac{a''}{a}u^2 \right)$$

- Define the conjugate momentum: $p = \delta\mathcal{L}/\delta u' = u'$
- Promote fields to operators: $u, p \to \hat{u}, \hat{p}$
- Impose canonical commutation relations: $[\hat{u}(\tau, \mathbf{x}), \hat{p}(\tau, \mathbf{x}')] = i\hbar\,\delta(\mathbf{x} - \mathbf{x}')$
- Define the mode expansion of the field operator:

$$\hat{u}(\tau, \mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \left(u_k^*(\tau)\,\hat{a}_{\mathbf{k}} + u_k(\tau)\,\hat{a}^\dagger_{-\mathbf{k}} \right) e^{i\mathbf{k}\cdot\mathbf{x}}$$

where $u_k(\tau)$ is a solution to the classical equation of motion and

$$[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta(\mathbf{k} - \mathbf{k}')$$

Annihilation Creation

Canonical Quantization

- Define the vacuum state: $\hat{a}_{\mathbf{k}}|0\rangle = 0$

- The expectation value of the Hamiltonian in this state is minimized for the Bunch-Davies mode function:

positive frequency

$$u_k(\tau) = \frac{1}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right)\boxed{e^{ik\tau}}$$

- The two-point function of the field operator (in that state) is

$$\begin{aligned}\langle 0|\hat{u}_{\mathbf{k}}(\tau)\hat{u}_{\mathbf{k}'}(\tau)|0\rangle &= \langle 0|\left(u_k^*(\tau)\,\hat{a}_{\mathbf{k}} + u_k(\tau)\,\hat{a}^\dagger_{-\mathbf{k}}\right)\left(u_{k'}^*(\tau)\,\hat{a}_{\mathbf{k}'} + u_{k'}(\tau)\,\hat{a}^\dagger_{-\mathbf{k}'}\right)|0\rangle \\ &= (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\,\boxed{|u_k(\tau)|^2}\end{aligned}$$

Power spectrum

Power Spectrum

Using the Bunch-Davies mode function, we obtain the following power spectrum for the primordial curvature perturbations:

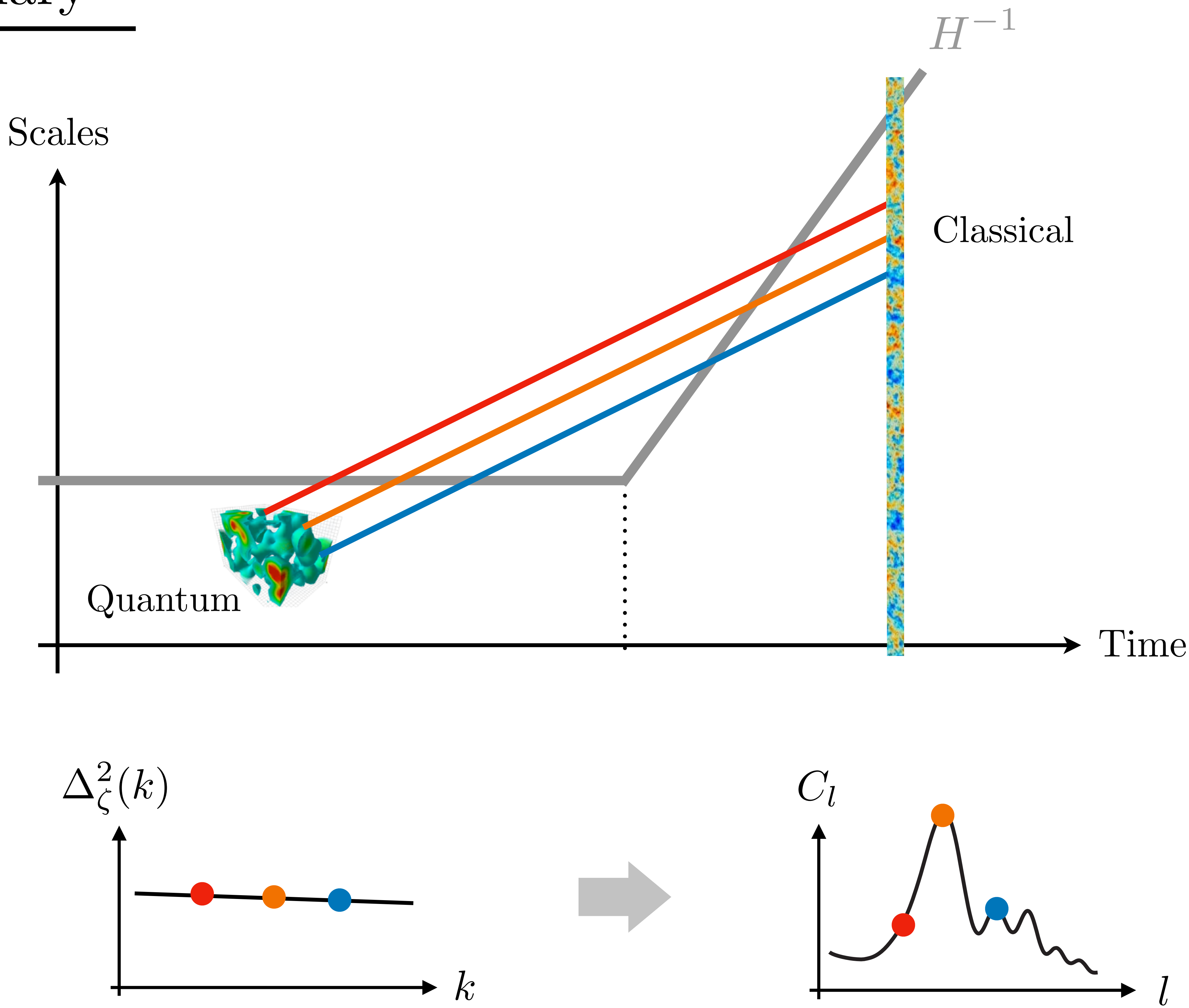
$$P_\zeta(k,\tau) \equiv \frac{H^2}{2M_{\mathrm{Pl}}^2|\dot{H}|}\frac{|u_k(\tau)|^2}{a^2(\tau)} = \frac{H^2}{2M_{\mathrm{Pl}}^2\varepsilon}\frac{1}{k^3}(1+k^2\tau^2) \xrightarrow{k\tau\to 0} \boxed{\frac{H^2}{2M_{\mathrm{Pl}}^2\varepsilon}\frac{1}{k^3}}$$

- The k^{-3} scaling is the characteristic of a scale-invariant spectrum.
- During inflation, both $H(t)$ and $\varepsilon(t)$ depend on time, which leads to a small scale-dependence

$$\boxed{\frac{k^3}{2\pi^2}P_\zeta(k) = A_{\mathrm{s}}\left(\frac{k}{k_0}\right)^{n_{\mathrm{s}}-1}} \equiv \Delta_\zeta^2(k)$$

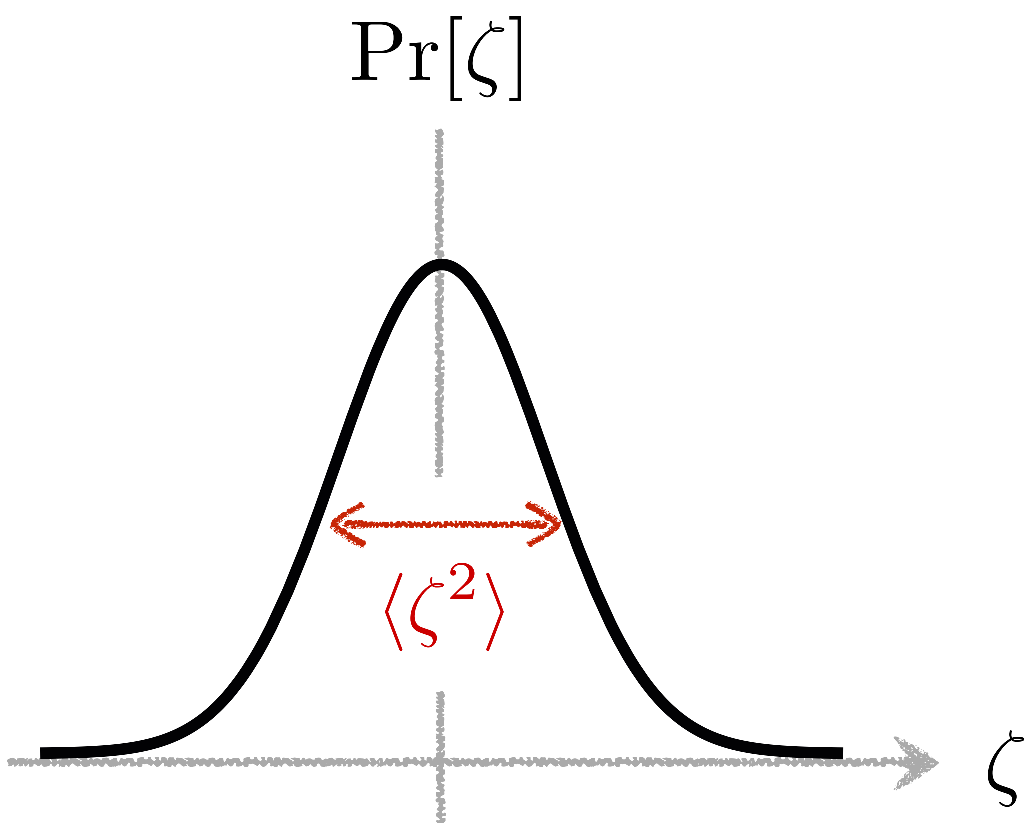
- Observations find $n_{\mathrm{s}} = 0.9667 \pm 0.0040$.

Summary



Gaussianity

The primordial fluctuations were highly **Gaussian** (as expected for the ground state of a harmonic oscillator):



$$F_{\rm NL} \equiv \frac{\langle\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^{3/2}} \lesssim 10^{-3}$$

The universe is more Gaussian than flat.

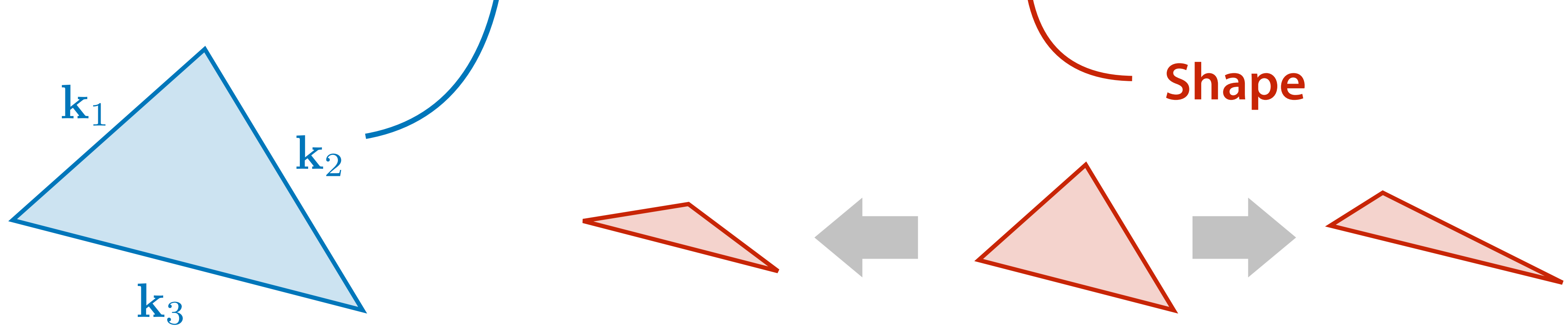
So far, we have only studied the free theory.

Interactions during inflation can lead to **non-Gaussianity**.

Primordial Non-Gaussianity

The main diagnostic of primordial non-Gaussianity is the **bispectrum**:

$$\boxed{\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \, \frac{(2\pi^2)^2}{(k_1 k_2 k_3)^2} \, B_\zeta(k_1, k_2, k_3)}$$

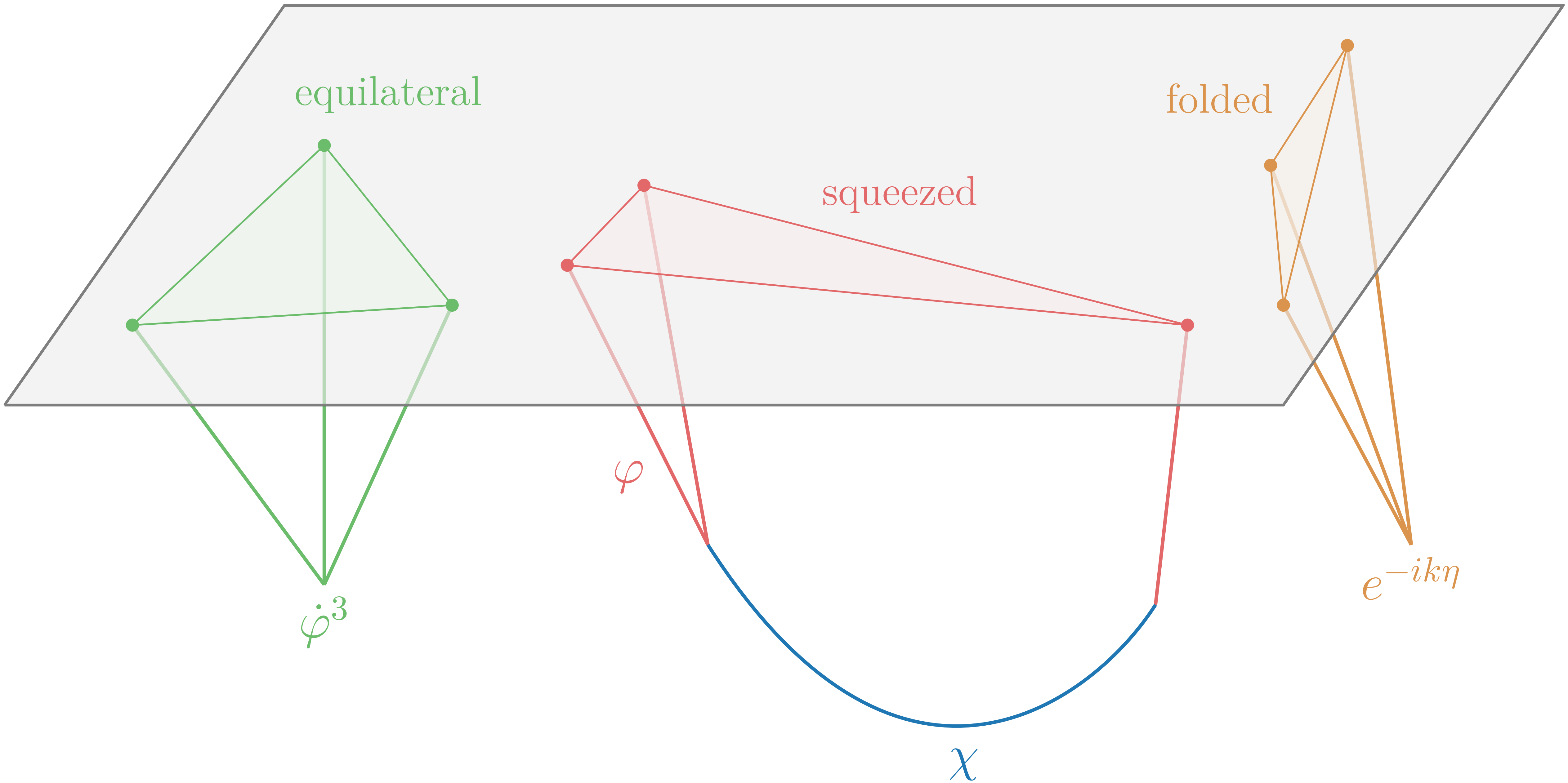


- The **amplitude** of the non-Gaussianity is defined as the size of the bispectrum in the equilateral configuration:

$$\boxed{F_{\rm NL}(k) \equiv \frac{5}{18} \frac{B_\zeta(k,k,k)}{\Delta_\zeta^3(k)}}$$

Shapes of Non-Gaussianity

- The **shape** of the non-Gaussianity contains a lot of information about the microphysics of inflation:



Equilateral Non-Gaussianity

The Goldstone mode during inflation can have large **self-interactions**:

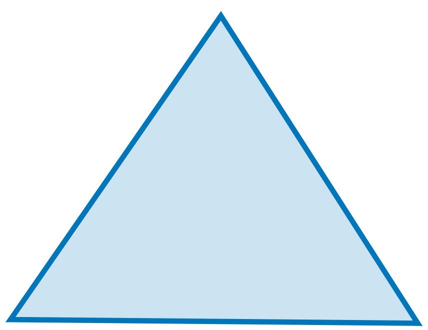
$$\mathcal{L}_\pi = \frac{M_{\rm pl}^2|\dot{H}|}{c_s^2}\Big[\left(\dot\pi^2 - c_s^2(\partial_i\pi)^2\right) - (1-c_s^2)\,\dot\pi(\partial_\mu\pi)^2 + \cdots\Big]$$

nonlinearly realized symmetry

$$F_{\rm NL} \propto c_s^{-2}$$

Correlations are largest when the fluctuations have comparable wavelengths:

equilateral NG:



$$B_{\dot\pi^3} \propto \frac{k_1k_2k_3}{(k_1+k_2+k_3)^3}$$

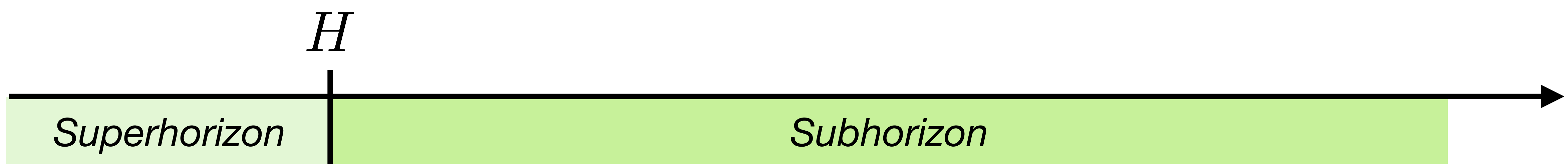
What would measuring (or constraining) equilateral NG teach us about the physics of inflation?

Equilateral Non-Gaussianity

Minimal models of inflation are characterized by three **energy scales**:

Equilateral Non-Gaussianity

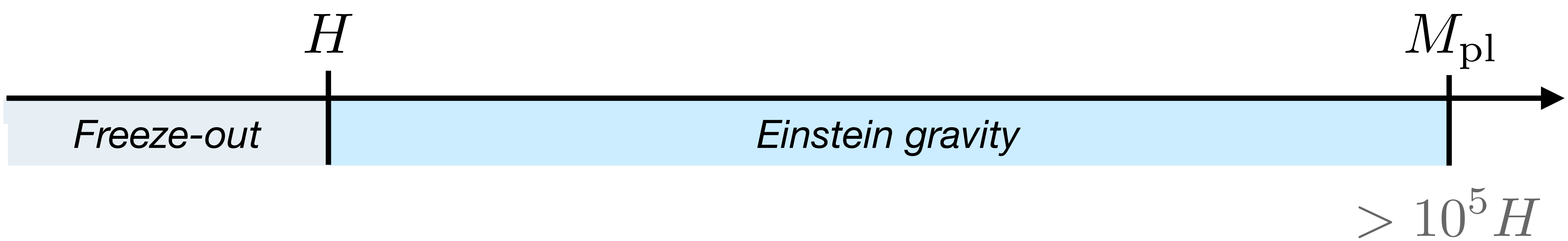
Minimal models of inflation are characterized by three **energy scales**:



- At the **Hubble scale**, modes exit the horizon and freeze out.
- This sets the **energy scale of the experiment**.

Equilateral Non-Gaussianity

Minimal models of inflation are characterized by three **energy scales**:



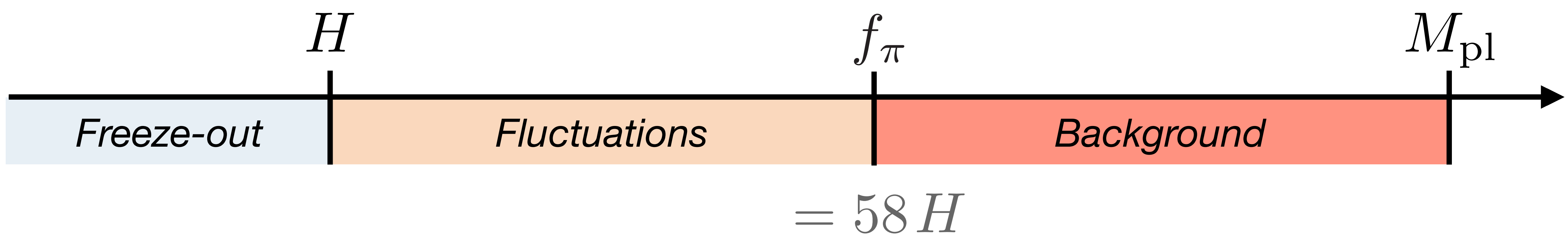
- At the **Planck scale**, gravitational interactions become strong.
- This sets the **amplitude of tensor fluctuations**:

$$\Delta_\gamma^2 = \frac{2}{\pi^2}\left(\frac{H}{M_{\rm pl}}\right)^2 < 0.1\,\Delta_\zeta^2 \approx 10^{-10}$$

No B-modes

Equilateral Non-Gaussianity

Minimal models of inflation are characterized by three **energy scales**:



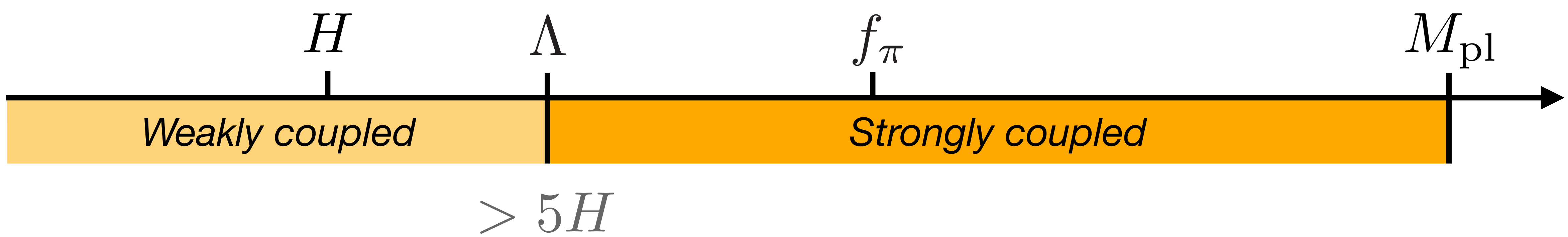
- At the **symmetry breaking scale**, time translations are broken.
- This sets the **amplitude of scalar fluctuations**:

$$\Delta_\zeta^2 = \frac{1}{4\pi^2}\left(\frac{H}{f_\pi}\right)^4 \approx 2.2 \times 10^{-9}$$

$$f_\pi^4 = M_{\rm pl}^2 |\dot{H}| c_s \to \dot{\phi}^2 \text{ (for SR)}$$

Equilateral Non-Gaussianity

Closer inspection may reveal additional scales:



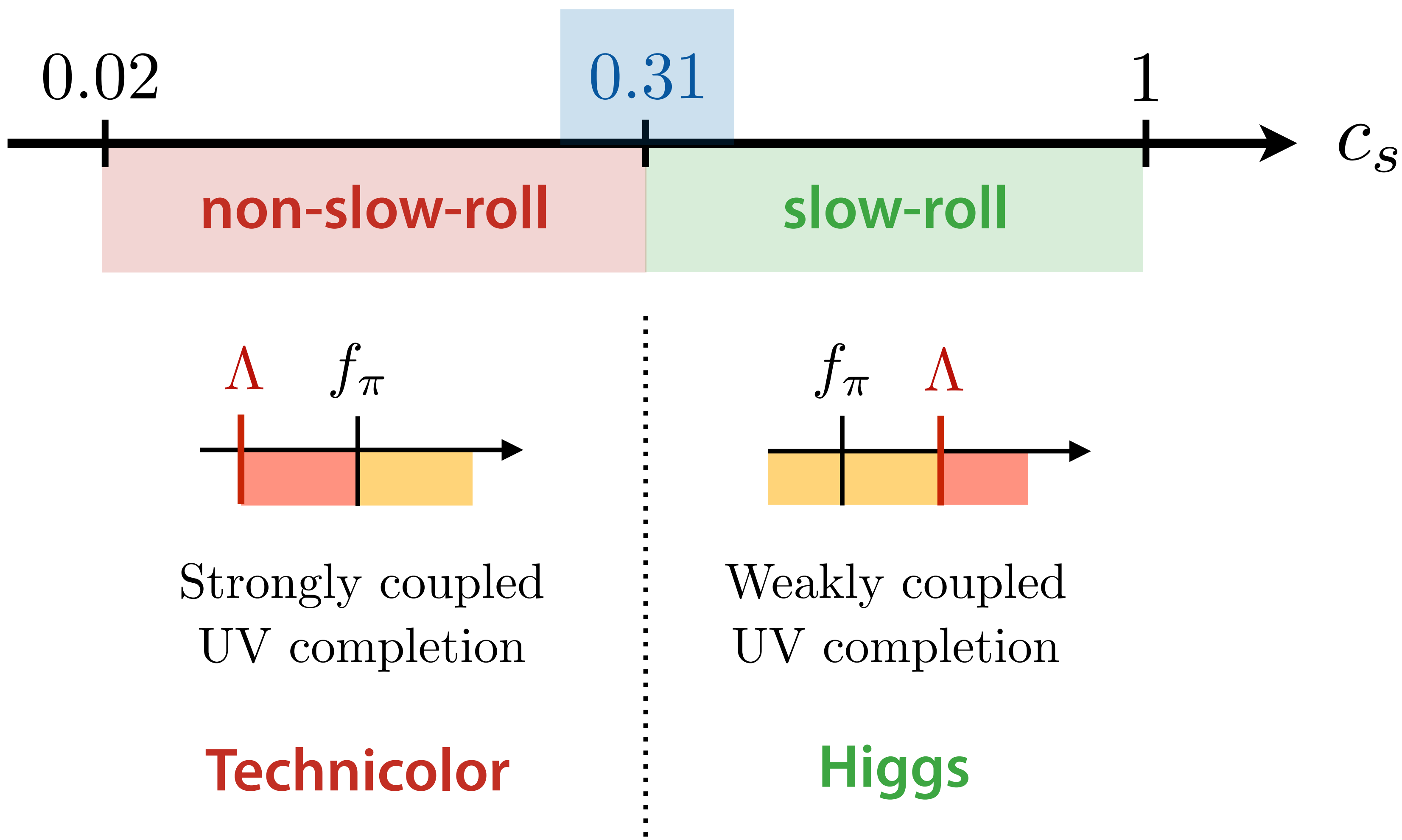
- At the **strong coupling scale**, interactions become strongly coupled.
- This sets the **amplitude of interactions**:

$$F_{\rm NL}^{\rm equil} \sim \frac{H^2}{\Lambda^2} < 0.01$$

No PNG

Equilateral Non-Gaussianity

Asking for the theory to be weakly coupled up to the symmetry breaking scale implies a critical value for the sound speed:



$$F_{\rm NL}^{\rm equil} \sim \Delta_\zeta$$

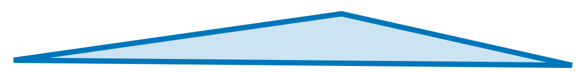
Hard to measure!

DB, Green, Lee and Porto [2015]
Grall and Mellville [2020]

Folded Non-Gaussianity

Excited initial states (negative frequency modes) lead to enhanced correlations when two wave vectors become collinear:

folded NG:



$$B \propto \frac{1}{k_1 + k_2 - k_3}$$

$$e^{-ik_3\tau}$$

- The inflationary expansion dilutes any pre-inflationary particles. In minimal scenarios, we therefore don't expect to see folded NG.

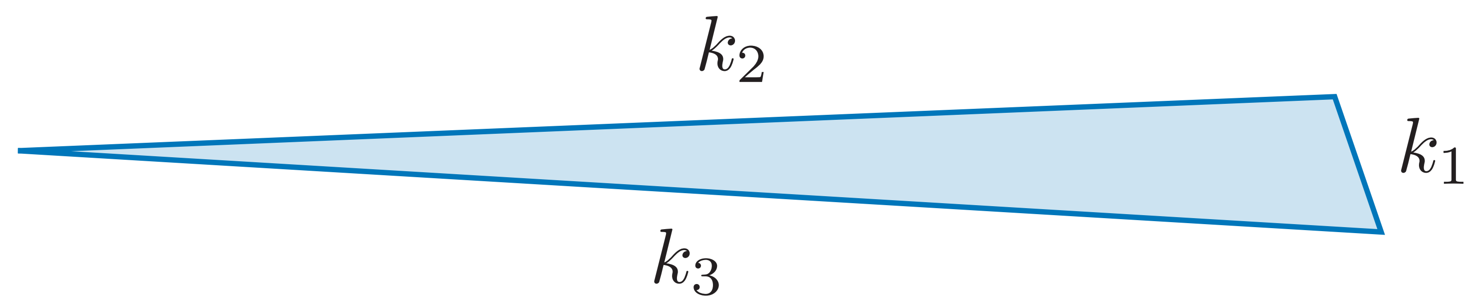
- Models with continuous particle production during inflation can still give a signal in the folded limit.

Flauger, Green and Porto [2013]
Flauger, Mirbabayi, Senatore and Silverstein [2017]
Green and Porto [2020]

Squeezed Non-Gaussianity

In single-field inflation, correlations must vanish in the **squeezed limit**:

$\lim_{k_1 \to 1}$



~ 0

Maldacena [2003]
Creminelli and Zaldarriaga [2004]

The signal in the squeezed limit therefore acts as a **particle detector**.

Chen and Wang [2009]
DB and Green [2011]
Noumi, Yamaguchi andYokoyama [2013]
Arkani-Hamed and Maldacena [2015]
Lee, DB and Pimentel [2016]
DB, Goon, Lee and Pimentel [2017]
Kumar and Sundrum [2018]
Jazayeri and Renaux-Petel [2022]
Pimentel and Wang [2022]

Cosmological Collider Physics

Massive particles are spontaneously
created in an expanding spacetime.

Arkani-Hamed and Maldacena [2015]

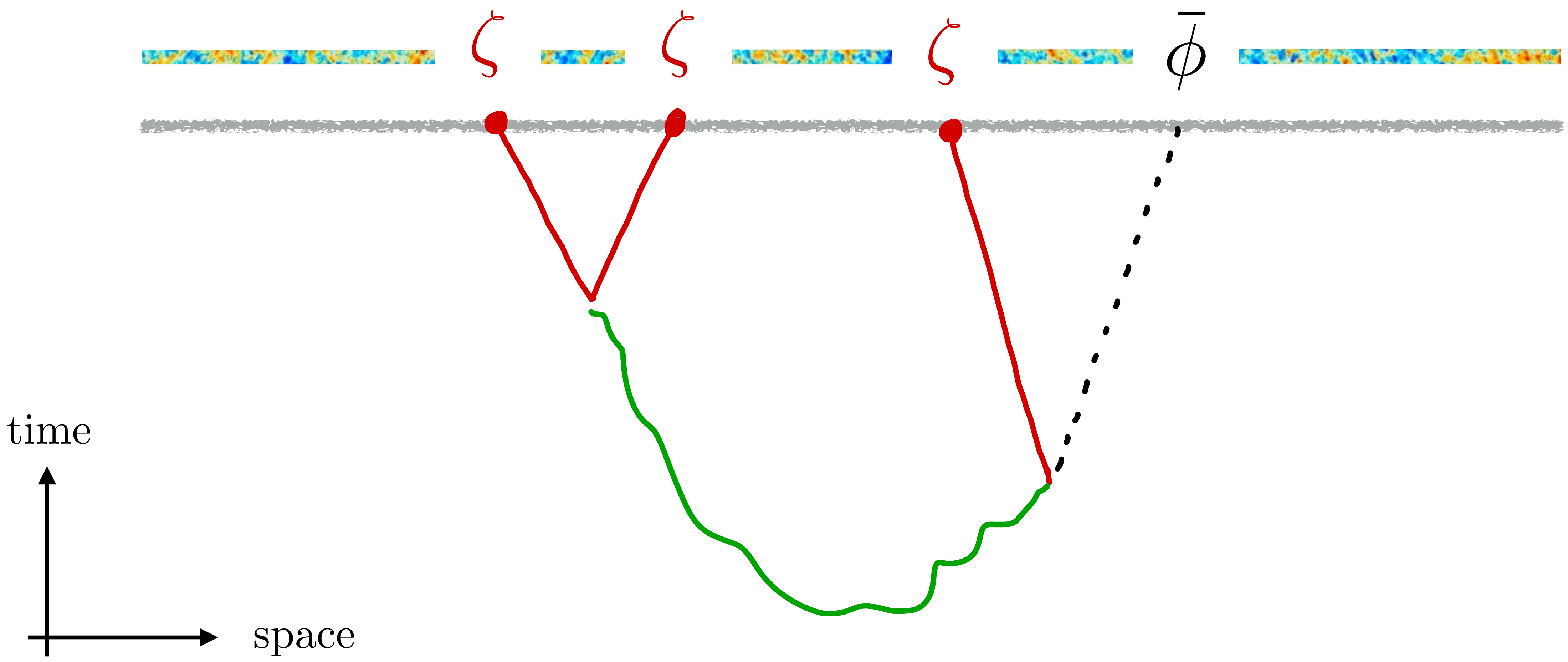
Cosmological Collider Physics

However, they cannot be directly observed at late times.

Arkani-Hamed and Maldacena [2015]

Cosmological Collider Physics

Instead, they decay into light fields.

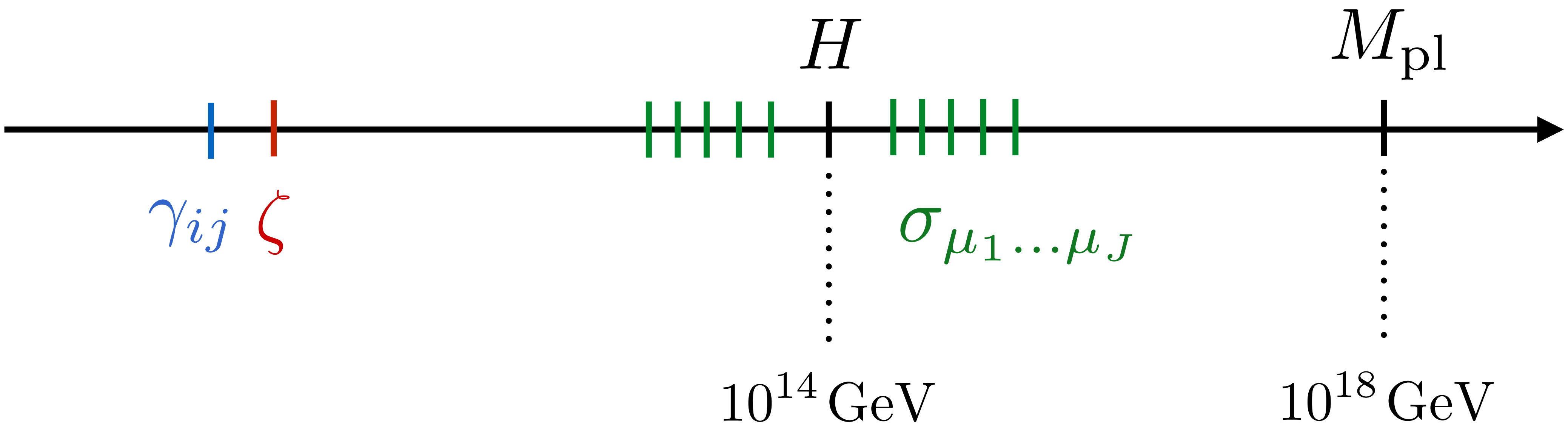


These correlated decays create distinct higher-order correlations in the inflationary perturbations.

Arkani-Hamed and Maldacena [2015]

Cosmological Collider Physics

This allows us to probe the particle spectrum at inflationary energies:

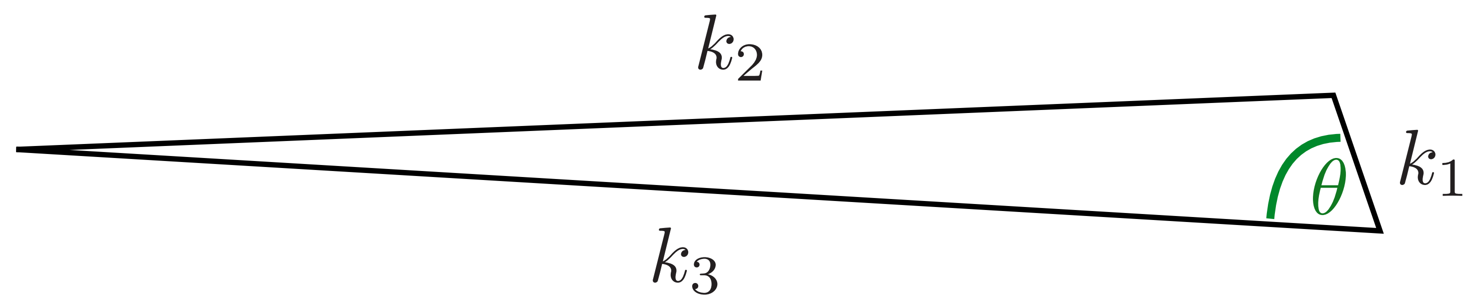


This can be many orders of magnitudes above the scales probed by terrestrial colliders.

Arkani-Hamed and Maldacena [2015]

Cosmological Collider Physics

The signal depends on the masses and spins of the new particles:



$$\lim_{k_1 \to 0} \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle' \;\propto\; \begin{cases} \left(\dfrac{k_1}{k_3}\right)^{\Delta} & m < \frac{3}{2}H \\[2ex] \left(\dfrac{k_1}{k_3}\right)^{3/2} \cos\left[\mu \ln \dfrac{k_1}{k_3}\right] & m > \frac{3}{2}H \end{cases}$$

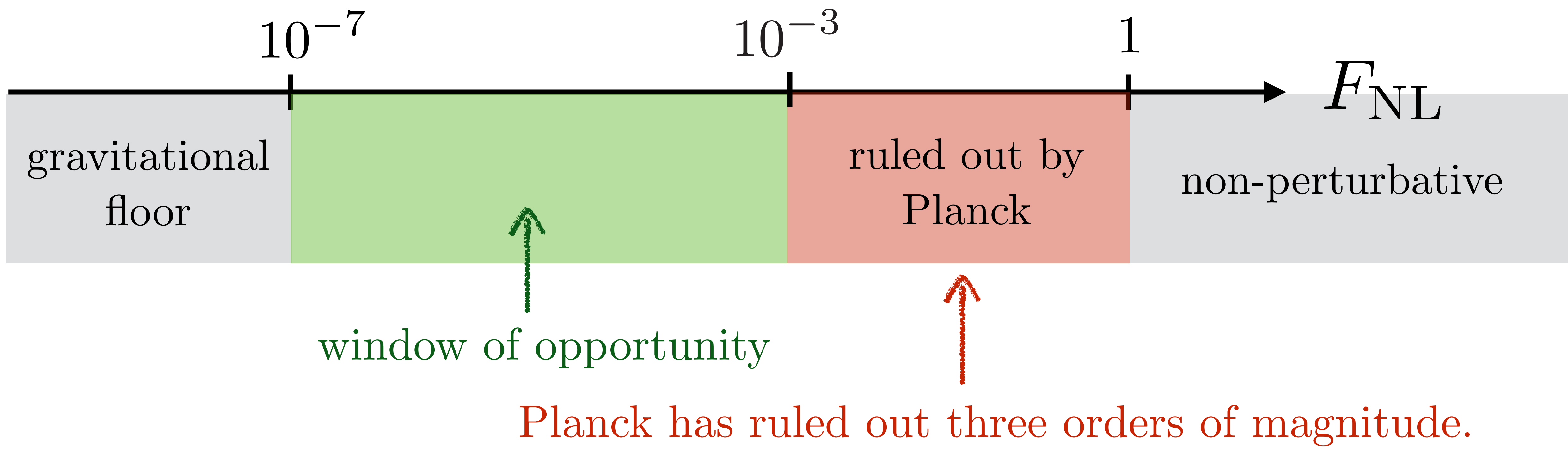
$$\propto \quad P_J(\cos\theta)$$

spin

Arkani-Hamed and Maldacena [2015]

Observational Constraints

The theoretically interesting regime of non-Gaussianity spans about seven orders of magnitude:



There is still room for new particles to leave their mark.

Any Questions?

So far, I have focussed mostly on the **practical motivations** for studying cosmological correlators.

In the rest of these lectures, I will introduce three different ways to compute primordial correlation functions:

- In-In Formalism
- Wavefunction Approach
- Cosmological Bootstrap

Notes

Lectures

I will leads us to more **conceptual questions** about the theory of quantum fields in cosmological spacetimes.