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Sensor Models and Multisensor Integration

Abstract

We maintain that the key to intelligent fusion of disparate sensory information is to provide an effective model of sensor capabilities. A sensor model is an abstraction of the actual sensing process. It describes the information a sensor is able to provide, how this information is limited by the environment, how it can be enhanced by information obtained from other sensors, and how it may be improved by active use of the physical sensing device. The importance of having a model of sensor performance is that capabilities can be estimated a priori *and, thus, sensor strategies developed in line with information requirements.*

We describe a technique for modeling sensors and the information they provide. This model treats each sensor as an individual decision maker, acting as a member of a team with common goals. Each sensor is considered as a source of uncertain geometric information, able to communicate to, and coordinate its activities with, other members of the sensing team. We treat three components of this sensor model: the observation model, which describes a sensor's measurement characteristics; the dependency model, which describes a sensor's dependence on information from other sources; and the state model, which describes how a sensor's observations are affected by its location and internal state. We show how this mechanism can be used to manipulate, communicate, and integrate uncertain sensor observations. We show that these sensor models can deal effectively with cooperative, competitive, and complementary interactions between different disparate information sources.

1. Introduction

Recently there has been an increasing interest in the development of robot systems capable of using many different sources of sensory information (Henderson 1987). This interest arises from a realization that there are *fundamental* limitations on any attempt at building descriptions of the environment based on a single source of information: A single source of sensory information can only provide partial information about an environment, information that is insufficient to constrain possible interpretations and that is limited in resolving ambiguity. Diverse information from many different sources can be used to overcome the limitations inherent in the use of single sensors through coordinated constraint of partial interpretations and by cooperative resolution of ambiguity.

The sensors of a multisensor system are characteristically diverse and necessarily logically distinct. The information obtained by the different sensors of a multisensor system is always uncertain, usually partial, occasionally spurious or incorrect, and often geographically or geometrically incomparable with other sensor views. The motivating goal behind all sensor integration techniques is to actively *utilize* this diversity of information in overcoming the limitations of any one sensor system. We will maintain that the only way to understand and utilize the disparity between different sensor views is to explicitly model the sensor and the information it provides in a manner that can describe both individual sensor abilities and the interactions that must take place between sensors. A sensor model is an abstraction of the physical sensing process whose purpose is to describe the ability of a sensor to extract descriptions of the environment in terms of the information available to the sensor itself. Sensor models should provide a quantitative ability to analyze sensor performance, allowing capabilities to be estimated *a priori* and decision procedures developed in line with information requirements.

We will develop a probabilistic model of individual sensor performance, embedding these descriptions in a team-theoretic framework to describe interactions between different sensors. The information obtained by each sensor is represented in terms of the geometric elements it is able to extract. This geometry is described probabilistically, enabling different sensors to communicate to each other in a common dimensionless language. This model of individual sensor performance is then embedded in a team structure which describes when information is communicated, and how this information can be used to improve the performance of the overall sensing system. Within this framework, we develop three important models: the observation model, describing the observations and decisions made by individual sensors; the dependency model, describing the communication of information between different sensors; and the state model, describing how a sensor's observations are affected by its location and internal state. By describing the resulting structure as a multi-Bayesian team, we develop some simple fusion procedures that demonstrate how these sensor models can be used to improve communication and cooperation between diverse sensing systems, how individual sensors can be controlled to acquire useful information, and how the description of competitive and complementary interactions between sensors can be used to improve the reliability and usefulness of multisensor robot systems. In conclusion, we argue for the development of other multisensor models and show how the team model described might fit into these.

2. Sensor Models

The realization that a multisensor system must be modeled to be understood is not a new idea. However, this need has often been subsumed into an implicit recognition of the limitations of a sensor with respect to some prescribed task. This approach to modeling leads to impossibly complex integration procedures and an inability to understand or utilize the potential power available from a multisensor system.

The most comprehensive rationalization of sensor abilities in the context of a multisensor robot system is probably Henderson's "logical" sensor descriptions (Henderson and Hansen 1985). These logical sensor models provide a description of sensor capabilities in terms of a set of rules specifying input-output characteristics of a device or algorithm. Logical models are defined in a modular manner, allowing different functions to be specified in terms of "characteristic building blocks," which can be used to construct a complete system. One advantage of this approach is that it concentrates on the information provided by different sensors and algorithms rather than on the physical sensing process itself. This lends itself to ideas of modularity and extensibility.

Flynn (1985) has also proposed a rule-based sensor model for integrating sonar and infrared sensor information as applied to mobile robot map building. Three rules were proposed for deciding when data from one or another sensor should be considered valid. Although this is a rather limited model, the way in which the rules were derived from direct observation of sensor data has a certain appeal. Allen (1987) has also used an implicit rule-based model of sensor performance in describing the ability of a tactile finger to explore geometries occluded during visual inspection.

Our principal objection to these heuristic, rule-based approaches to sensor modeling is their inability to *quantify* sensor performance. A quantitative model of sensor information is essential in providing a means of analyzing performance, in estimating system abilities, and in developing sensing strategies in line with task requirements. The importance of providing quantitative models of sensor performance is well established in conventional sensing domains. Powerful techniques exist for developing probabilistic noise models and for studying modes of sensor failure; Gelb (1974) describes a number of these models and the way they are used. The utility of existing probabilistic models of sensor information has been realized by a number of researchers in robotics. Most notably, Faugeras and Ayache (Faugeras and Ayache 1986; Ayache and Faugeras 1987) have employed a probabilistic model to fuse noisy line segment descriptions extracted from stereo visual images. Porril et al. (1987) and Bolle and Cooper (1986) have employed similar statistical models to estimate object locations from visual images. Durrant-Whyte (1987a) has developed probabilistic models of both visual and tactile information.

However, even in more mature sensing research areas, only single sensor models are well understood. The integration of information from many diverse, geographically disparate sensor systems is still an open research issue; see for example the discussion in Athans (1987). In robotics, multisensor integration is made more difficult because of often indivisible relationships between different sensing modalities (in manipulation or in hand-eye coordination, for example).

Terzopoulos (1986) has considered this problem, in describing a cooperative integration of sparse depth information with local surface constraints obtained from visual intensity arrays. A further exploration of the dynamics involved in cooperative multisensor interactions is introduced in Hager (1987) and Hager and Durrant-Whyte (1988). This study describes a number of mechanisms for communicating and coordinating information flow between sensors, and develops a simulated scenario, involving active sensors, to explore these ideas.

In general, a sensor model has two important components: an individual model of performance, and a group model of available information. The individual model describes to the sensor itself the observations that an independent sensor can obtain. The group model describes to a single sensor the observations made by the other sensors in the system. The purpose of these models is to provide each sensor with an ability to make decisions, based on its own observations and in the light of information provided by other sensors.

2.1. Characterizing Sensors

In modeling a sensor's ability to make observations of the environment, we should be concerned with a number of important sensing characteristics.

Device Complexity

When a sensor is composed of many physical devices, each making contributions to the observation extracted by the sensor, the exact distribution of uncertainty and character of the sensor is complex and difficult to describe as an exact model. For example, a camera system comprises a charge coupled device (CCD) array, lenses, digitizers, etc; their cumulative effect on the sensor character is difficult to predict. This problem is encountered in other sensor-based research areas as well as robotics.

Observation Error

The feature observations reconstructed from sensor data have more uncertainty associated with them than just sensor noise (Bajcsy, Krotkov, and Mintz 1986). Error may occur due to inaccuracies in the placement of devices, incorrect interpretation of measurements, or device failure, for example. Uncertainty also arises when a feature observation is incomplete or partial, when information is unavailable in one or more degrees of freedom, or all two-dimensional visual cues, for example.

Observation Disparity

Robot sensors are characterized by the diversity of observations that can be obtained: edges, normals, locations, or texture for example. If sensor information from many disparate sources is to be combined, we must have an ability to transform one type of uncertain geometric feature to another (edges to surface normals for example). This ability allows observations from different cues to be compared or used to complement each other in deriving a consensus description of the environment.

Multiple Viewpoints

When we have two or more sensors which are geographically separated, we must be able to transform their observations in to a common coordinate system so that they can be compared. In vision this is called the viewpoint problem — how to combine observations taken from a number of different viewpoints into a consensus description. The central issue here is to describe the ability of the sensor model to transform and manipulate uncertain descriptions of the environment. This is an important consideration in active sensor descriptions.

To provide a homogeneous method for describing these diverse characteristics, we will develop a model of sensor observations in terms of uncertain geometry; see Durrant-Whyte (1988) for a complete description of this subject. All geometric objects (features, locations and relations) can be described by a parameter vector $\mathbf{p}$ and a vector function

$$\mathbf{g}(\mathbf{x}, \mathbf{p}) = \mathbf{0}, \quad \mathbf{x} \subseteq \Re^n, \quad \mathbf{p} \in \Re^m. \tag{1}$$

This function can be interpreted as a model of the physical geometric object that maps a (compact) *region* $x \subseteq \Re^n$ in Euclidean n-space to a *point* $\mathbf{p} \in \Re^m$ in the parameter space. Each function $\mathbf{g}$ describes a particular type of geometric object: all straight lines, or the family of quadratic surfaces, for example. Each value of $\mathbf{p}$ specifies a particular instance of a geometric object molded by $\mathbf{g}$. For example, *all* plane surfaces can be represented by the equation

$$g(\mathbf{x}, \mathbf{p}) = p_x x + p_y y + p_z z + 1 = 0, \qquad (2)$$

with $\mathbf{x} = [x, y, z]^T$ and $\mathbf{p} = [p_x, p_y, p_z]^T$. A specific plane can be represented as a point $\mathbf{p} \in \Re^3$ in the parameter space. A sensor can be considered as observing values of $\mathbf{p}$. The uncertain *event* that the sensor "sees" a specific instance of this geometric object can be described by taking the parameter vector $\mathbf{p}$ to be a random variable. The likelihood that a particular instance of a geometric feature is observed can now be described by a probability density function (p.d.f.) $f_g(\mathbf{p})$. For a specific type of feature represented by Eq. (1), this p.d.f. describes the probability or likelihood of observing a particular instance of the associated geometric object. It should be noted that this is *not* just a noise model.

The advantage of describing sensor information in terms of an uncertain parameterized function is that geometric description itself can easily be transformed between different coordinate systems and different object representations, providing a simple but effective means of communicating information between different sensors. We will consider every sensor as a "geometry extractor," able to extract uncertain geometric information about the environment and communicate this information to the system in terms of a p.d.f. $f(\mathbf{p})$ on the observed geometric parameter vector $\mathbf{p}$. The communication of information then reduced to the transformation of stochastic geometric functions of the form of Eq. (1).

This geometric description of sensor information provides two essential components: first, a natural way to embed noise models into the structure, and second a mechanism to communicate information between different sensors in the common language of geometry.

2.2. Sensors as Members of a Team

We will develop a description of a multisensor system as a *team* of decision makers. Each sensor of this team will be considered as an individual decision maker taking observations, making local decisions, and implementing its own actions. Together, the sensors must coordinate their activities, guide each other to view areas of interest, and ultimately come to some team-consensus view of the environment.

Team decision theory was originally developed to provide a quantitative mechanism for describing economic organizations. The basis for the analysis of cooperation among structures with differing opinions or interest was formulated by Nash (1950) in the well-known bargaining problem. In their book, Marshak and Radnor (1972) describe the extension of the bargaining problem to multiperson organizations. The primary emphasis of this work is to provide a means of coordinating a distributed decision making process. Each decision maker is only allowed access to partial state information, from which local decisions must be made, reflecting some global optimality criteria. This team structure has also been developed as a means of describing problems in multiperson control. In a series of papers, Ho (Ho and Chu 1972; Ho 1980) has investigated the distributed linear quadratic Gaussian control problem, as applied to a group of sequential decision makers.

The characteristics of a team structure capture many of the desirable properties of a multisensor system: Team members can make local decisions based on a team goal, providing a means of delegating observation tasks and actions. Team members can help each other by exchanging information unattainable to individual members, thus providing the team with a more complete description of events. Team members can disagree with each other as to what is being observed; resolving differences of opinion can supply a mechanism for validating each other's operation. The most important observation about a team is that the coordinated activity of the members is more effective than the sum of their individual actions.

The idea of a team is essentially simple. It provides a mechanism for describing the observations made by a number of individual decision makers, how this information is communicated to other members of a team, and how team decisions and actions can be arrived at. In general, the group decision problem can be very complex. Our development of multisensor teams eliminates much of this complexity by reducing the more general group decision problem to one of team estimation. Further simplifications arise from providing a common, geometric, communication language, and restricting the basis of local decision making to likelihood comparisons rather than, more generally, utility.

We propose to describe a multisensor system as a team in the following sense: The sensors are considered as members of the team, each observing the environment and making local decisions based on the information available to them. The observations $\mathbf{z}$ made by a sensor are described by an information structure η. Each sensor can make a decision δ, based on these observations, resulting in an action $\mathbf{a}$, usually an estimate of a feature description, describing the opinion of the sensor. The opinions of each sensor are integrated to provide a team decision and action.

Our interest in this team structure centers on the model of individual decision makers as described by each team member's information structure.

2.3. Information Structure

The ith team member is described by an *information structure* η_i. This structure is a model of sensor capabilities. Formally an information structure is defined by the relation between observations, state and decisions.

DEFINITION The *information structure* of the ith sensor or team member ($i = 1, \ldots, n$) is a function η_i which describes the observations $\mathbf{z}_i$ made by a sensor in terms of its physical state $\mathbf{x}_i$, available prior information about the state of the environment $\mathbf{p}_i \in \mathscr{P}_i$, and the other sensors' or team members' actions $\mathbf{a}_j \in \mathscr{A}_j$, $j = 1, \ldots, n$. Thus $\mathbf{z}_i = \eta_i(\mathbf{x}_i, \mathbf{p}_i, \mathbf{a}_1, \ldots, \mathbf{a}_{i-1}, \mathbf{a}_{i+1}, \ldots, \mathbf{a}_n)$.

Collectively the n-tuple $\boldsymbol{\eta} = (\eta_1, \ldots, \eta_n)$ is called the information structure of the team. The action $\mathbf{a}_i$ of the ith team member is related to its information $\mathbf{z}_i$ by a decision function $\delta \in \mathscr{D}_i$ as $\mathbf{a}_i = \delta_i(\mathbf{z}_i)$. Collectively the n-tuple $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_n)$ is called the team decision function.

We will be concerned with the model of sensor performance provided by the information structure. We consider three types of sensor model: an *observation* model, a *dependence* model, and *state* model. The observation model η_i^p describes the character of measurements given the state of the sensor and all other sensor actions. The dependence model η_i^δ describes the effect of other sensor actions (observations) on the sensor measurements. The state model η_i^x describes the observation dependence on the internal state and location of the sensor. We will embed all these models in a common information structure η_i.

Within this structure, we will describe a multisensor system as a team of observers, taking observations of geometric features, and making decisions which are estimates of geometric descriptions $\delta_i(\mathbf{z}_i) \mapsto \mathbf{p}_i \in \mathscr{P}$. With this interpretation, the formal definition of the information structure can be simplified and considered as a function transforming uncertain geometric feature estimates from one sensor into prior information or predictive hypotheses for use by another sensor.

$$\mathbf{z}_i = \eta_i(\mathbf{x}_i, \mathbf{p}_i, \delta_1(\mathbf{z}_1), \ldots, \delta_{i-1}(\mathbf{z}_{i-1}), \delta_{i+1}(\mathbf{z}_{i+1}), \ldots, \delta_n(\mathbf{z}_n)), \tag{3}$$

where the observations $\mathbf{z}_i$ are random vectors and η_i is a stochastic function transforming the decisions $\delta_j(\cdot) \in \mathscr{P}_j$, the prior information $\mathbf{p}_i \in \mathscr{P}_i$ and the sensor state $\mathbf{x}_i$ into elements of $\mathscr{P}$.

We will model the observations $\mathbf{z}_i$ by a p.d.f. $f_i(\mathbf{z}_i | \eta_i(\cdot))$. This density function serves as the natural realization of the information structure; it describes the observations as a probabilistic function of the information provided to the sensor. The description of observations is now dimensionless, so we can manipulate this function using standard probabilistic techniques and communicate information between different sensory sources by transformations of uncertain geometric feature descriptions. The decisions $\delta_i(\cdot)$ can be related through f_i to any common decision philosophies—maximum likelihood, for example. This will enable us to develop decision procedures capable of using many disparate sources of information. Another important advantage of incorporating the information structure into a probability distribution function is that the variables (prior information, state, and other sensor decisions) can be separated from each other by expanding f_i as a series of conditional probability distributions. This in turn allows us to decouple the three types of sensor model (η^p, η^δ, and η^x) from each other. Let

$$\bar{\delta}_i = (\delta_1, \ldots, \delta_{i-1}, \delta_{i+1}, \ldots, \delta_n)$$

be the information communicated to the ith sensor by all other sensors. Assuming that the observation's dependence on state, prior information, and other sensor decisions is separable, we can write

$$\begin{aligned} f(\mathbf{z}_i|\eta_i(\mathbf{x}_i, \mathbf{p}_i, \bar{\delta}_i)) &= f_{\eta i}(\mathbf{z}_i|\mathbf{x}_i, \mathbf{p}_i, \bar{\delta}_i) \\ &= f_x(\mathbf{z}_i|\mathbf{x}_i) f_p(\mathbf{z}_i|\mathbf{p}_i) f_\delta(\mathbf{z}_i|\bar{\delta}_i) \\ &= f_x(\eta_i^x) f_p(\eta_i^p) f_\delta(\eta_i^\delta). \end{aligned} \tag{4}$$

The state model $f_x(\eta_i^x)$ now describes the dependence of a sensor's observations on its location and internal state *given* any prior information and all other sensor opinions. The observation model $f_p(\eta_i^p)$ describes the dependence of sensor measurements on the state of the environment *given* all other sensor decisions. The dependence model $f_\delta(\eta_i^\delta)$ describes the prior information supplied by the other sensors in the system. The product of these three models describes the observations made by the sensor.

This ability to decouple sensor models enables the effect of different types of information to be analyzed independently. This in turn provides a powerful framework in which to develop descriptions of sensor performance. In the remainder of this article, we will develop these three different types of sensor models.

2.4. Observation Models

An observation model describes the measurements made by a sensor, given all prior information, other sensors' information, and the state of the sensor itself. It is essentially a model of sensor noise and error. Observation models have been studied extensively in related sensor research areas but are still relatively underused in robotics. We will briefly discuss some more popular models and introduce our own favorite.

Consider a sensor taking observations of a geometric feature, an instance of a given family described by $\mathbf{g}(\mathbf{x}, \mathbf{p}) = \mathbf{0}$ and parameterized by the vector $\mathbf{p}$. The observation model of the ith sensor is represented as a conditional probability distribution function $f_i(\mathbf{z}_i|\mathbf{p}_i)$, describing the likelihood of feature observation given all prior information about $\mathbf{p}_i$. The exact form of $f(\cdot|\mathbf{p})$ will depend on many physical factors. It is unlikely that we can obtain an exact description of the probabilistic character of observations in all but the simplest cases. It may in fact be undesirable to use an exact model even if it were available because of its likely computational complexity and its inability to model non-noise errors such as software failures or algorithmic misclassifications. It is usual to assume a Gaussian or uniform distribution model for the conditional observations (Faugeras and Ayache 1986). These models allow the development of computationally simple decision procedures. However, they are not able to represent poor information and can fail with catastrophic results even when the observations deviate only a small amount from the assumed model. In the absence of an exact observation description, it is possible to build robust approximations to the true character of sensor measurements. There are two different ways to approach this approximation problem: We can model the observation noise as a *class* of possible distributions, or as some nominal distribution *together* with an additional unknown likelihood of errors and mistakes. McKendall and Mintz (1987) have conducted experiments on a stereo camera system and have proposed modeling visual noise as a class of probability distributions. Applying a minimax philosophy to this class results in a soft quantizer as a decision procedure. In the long term these models show great promise, but they currently have two major problems: the relative computational complexity of the decision procedures, and their inability to describe non-noise errors. An alternative to this approach is to use an approximation that describes the observations by some nominal distribution together with an unknown likelihood of errors or mistakes. Then we shall require that our decision procedures be robust to the possibility of error, but otherwise assume a nominal noise model. These distributions are termed *gross error models* (Huber 1981) and have the general form

$$\mathcal{P}_\epsilon(F_0) = \{F | F = (1 - \epsilon)F_0 + \epsilon H,\ H \in \mathcal{M}\}. \quad (5)$$

These models are described by a set of distributions F which is composed of some nominal distribution F_0 together with a small fraction ϵ of a second probability measure H. This second measure is often assumed unknown and acts to contaminate the nominal distribution with unexpected observations. This model results in decision procedures which cluster observations and trim outliers from consideration in the integration process. If the nominal model has finite moments, then this clustering process converges to a Gaussian observation model. For this reason, we consider a particular case of the gross error model called the contaminated Gaussian distribution, which has the general form

$$\begin{aligned} f(\mathbf{z}|\mathbf{p}) = & \frac{1-\epsilon}{(2\pi)m/2|\Lambda_1|^{1/2}} \\ & \times \exp\left[-\frac{1}{2}(\mathbf{z}-\mathbf{p})^T\Lambda_1^{-1}(\mathbf{z}-\mathbf{p})\right] \\ & + \frac{\epsilon}{(2\pi)m/2|\Lambda_2|^{1/2}} \\ & \times \exp\left[-\frac{1}{2}(\mathbf{z}-\mathbf{p})^T\Lambda_2^{-1}(\mathbf{z}-\mathbf{p})\right], \end{aligned} \quad (6)$$

with $0.01 < \epsilon < 0.05$ and $|\Lambda_1| \ll |\Lambda_2|$. The spirit of this model is that the sensor behaves as $N(\mathbf{p}, \Lambda_1)$ most of the time but submits occasional spurious measurements from $N(\mathbf{p}, \Lambda_2)$. With the type of sensors that we are considering (vision, tactile, range, etc.), the contaminated model is intended to represent the fact that we would normally expect quite accurate observations within a specific range, but must be robust to problems like miscalibration, spurious matching, and software failures. We assume that we have knowledge of the value of Λ_1 from the character of the sensor, but we do not explicitly assume anything other than bounds on the values of Λ_2 and ϵ. This has the property of forcing any reasonable integration policy developed to be robust to a wide variety of sensor observations and malfunctions. This model is thought to be a sufficiently conservative estimate of sensor behavior, in that by choosing Λ_1, Λ_2, and ϵ sufficiently large, we can encompass all the possible uncertainty characteristics we may expect to encounter. The intention of this model is to approximate a sensor's true characteristics without having to analyze possible sensor responses. This forces us to develop decision procedures which are robust to the exact specification of observation model, which provide efficient results in the light of our poor knowledge of sensor characteristics, and that will be robust to spurious or gross contaminations.

The observation model developed here describes the distribution of a single feature measurement. We must also be concerned with multiple-sample characteristics, the statistical correlation between observations, the likely density of observations, and the effect of algorithms that aggregate observations into higher-level environment descriptions. We maintain that these issues should be considered in terms of decision procedures rather than in the sensor model *per se.* The ϵ-contamination model has been used to develop decision rules which are sympathetic to the *actual* character of sensor observations and which are robust with respect to spurious information and model specification (Durrant-Whyte 1987b).

2.5. Dependency Models

To understand and utilize diverse sensor capabilities, it is important that we be able to model the interactions and exchange of information between different sensors. Unlike the development of observation models, there has been essentially no research in dynamic sensor models in robotics or any other sensor-based research areas. We will develop a model of dependence between sensory systems by considering the dependence model $f_\delta(\mathbf{z}_i | \cdot)$ in terms of a set of conditional probabilities or functions describing the information provided by other sensors and cues.

Consider the dependence of the ith sensor or team members' observations on other sensor information as described by the probabilistic dependence model

$$f_\delta(\mathbf{z}_i|\bar{\delta}_i) = f_\delta(\mathbf{z}_i|\delta_1(\mathbf{z}_1), \ldots, \delta_{i-1}(\mathbf{z}_{i-1}), \ldots, \delta_{i+1}(\mathbf{z}_{i+1}), \delta_n(\mathbf{z}_n)). \tag{7}$$

The decisions made by other sensors, communicated to the ith sensor can be described probabilistically by the distribution function $f_\delta(\bar{\delta}_i)$. With this definition, we can interpret the dependence model as a posterior probability of observations, and the information communicated to the sensor as a prior probability. This interpretation makes statistical sense as the joint feature density is found by multiplying the conditional observation model by the prior information, so by following Bayes' rule, we have $f_\delta(\mathbf{z}, \bar{\delta}_i) = f_\delta(\mathbf{z}_i|\bar{\delta}_i) f_\delta(\bar{\delta}_i)$. This interpretation is also intuitively appealing because it is the observations made by other sensors, communicated to the ith sensor, that provide the initial prior information.

Physically, the information provided by other sensors, as described by the distribution $f_\delta(\bar{\delta}_i)$, is geometric: The decisions $\delta_j(\cdot) \in \mathcal{P}_j$ communicated to the ith sensor are parameter vectors of the geometric feature observed by the jth sensor. The distribution function $f_\delta(\cdot)$ describes the uncertainty in these communicated decisions. This then leads us to interpret the dependency model $f_\delta(\mathbf{z}_i|\cdot)$ as a geometric function, describing the transformation of geometric features observed by the jth sensor into geometric features observed by the ith sensor. This transformation is stochastic and must make use of the tools developed in the field of uncertain geometry (Durrant-Whyte 1988; Faugeras and Ayache 1986). Our interest now centers on the communication of uncertain geometric information between sensor systems.

The internal structure of the distribution $f_\delta(\cdot)$ must, in general, describe the interdependence of all sensor information contributing to the ith sensor's information structure $f_\delta(\mathbf{z}_i|\cdot)$. These dependencies should represent the sequence in which information is passed between sensors and describe the way in which observations combine in different ways to provide a complete environment description. Each observation $\mathbf{z}_i$ made by a sensor is a measurement of a specific type of feature $\mathbf{p}_i \in \mathcal{P}_i$; the decision $\delta_i \in \mathcal{P}_i$ is an estimate of this feature based on the observation, such as the parameter vector of an edge or surface equation. The information structure is an interpretation of this estimate in terms of a distribution function defined on the parameter vector. The dependency model $f_\delta(\mathbf{z}_i| \ldots, \delta_j, \ldots)$ is therefore a *transformation* of feature descriptions obtained by the jth sensor into feature descriptions required by the ith sensor. This transformation represents a change in stochastic feature descriptions.

The interpretation of the dependency model as a prior probability distribution allow us to expand $f_\delta(\cdot)$ as a series of conditional distributions describing the effect of each individual sensor on the ith sensor's prior information. For example, if the numeric order of decision making is also the natural precedence, then

$$f_\delta(\bar{\delta}_i) = f^i_\delta(\delta_1, \ldots, \delta_n) = f^i_\delta(\delta_1|\delta_2, \ldots, \delta_n) f^i_\delta(\delta_2|\delta_3, \ldots, \delta_n) \ldots f^i_\delta(\delta_n).$$

Each term $f^i_\delta(\delta_j|\delta_k)$ describes the information contributed by the jth sensor to the ith sensor's prior information, given that the information provided by the kth sensor is already known. The transformation affected by $\eta^i_j(\delta_j)$ takes the jth sensor observation and interprets it in terms of observations made by the ith sensor: $\eta^i_j(\cdot) \in \mathcal{P}_i$. The term $\eta^\delta_j(\delta_k)$ is the dependence model describing the kth sensor's contribution to the ith sensor's prior information. This in turn can be described by the probability distribution $f^j_\delta(\delta_k)$.

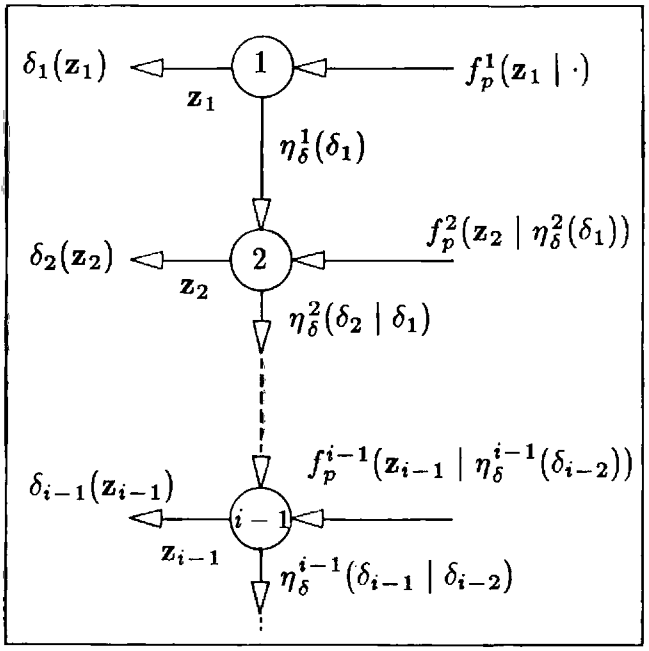
This decomposition by conditionals can, in general, be written in any appropriate order. If there is a natural precedence order in which sensors take observations, then an expansion in that order is appropriate. For example, if the decision δ_j only depends on information provided by the $(j-1)$th sensor, then the ith dependence model can be represented by

$$f^i_\delta(\delta_1, \ldots, \delta_{i-1}) = f^i_\delta(\delta_{i-1}|\delta_{i-2}) \ldots f^i_\delta(\delta_2|\delta_1) f^i_\delta(\delta_1), \tag{8}$$

describing a Markovian chain of decision makers, as shown in Fig. 1.

The use of conditional probability distributions in this way induces a *network* structure of relations between different sensors and cues. This network is a constraint exposing description of sensor capabilities: Each arc in the network describes a dependence be-

Fig. 1. The Markovian team decision network.



tween sensor observations, represented by a constraining transformation and implemented by the propagation of prior information between sensors. Each decision-making node or sensor uses this prior information to guide or constrain the acquisition of new observations. There are two distinct processes going on during this interaction: the local decision made by each sensor, and the transformation of this information between decision-making nodes. Each sensor takes observations described by the observation model $f_p(\mathbf{z}_i|\mathbf{p})$ and is provided with prior information from other sensors through the dependence model $f_\delta(\mathbf{z}_i|\cdot)$. The sensor makes a decision $\delta_i(\cdot)$ based on the product of observed and prior information. This is passed on to the next sensor and interpreted in terms of its dependence model $f_\delta(\mathbf{z}_{i+1}|\delta_i(\cdot), \ldots)$, which transforms the information provided by δ_i into the feature description understood by δ_{i+1}. This transformation and combination of information from disparate sources demonstrates the clear advantage of describing information in terms of a dimensionless probability distribution function.

These dependency models and opinion networks provide a powerful means of describing the dynamic exchange of information between different sensor systems. These models will be developed further in the context of the decision problem in the next section.

2.6. State Models

Many robot sensors are able to change their observation characteristics by either relocating themselves or altering their internal state. For example, a camera with controllable zoom and focus (Krotkov 1986), a mobile ranging device with variable viewing angle, or a tactile probe taking observations by touching objects in the environment (Cameron, Daniel, and Durrant-Whyte 1988). To be able to use these *active* sensor capabilities in an intelligent manner, we must be able to model the dependency of observations on sensor state or location. A model of this dependency will allow us to develop sensor control strategies and provide a mechanism with which to actively observe and localize objects in the environment.

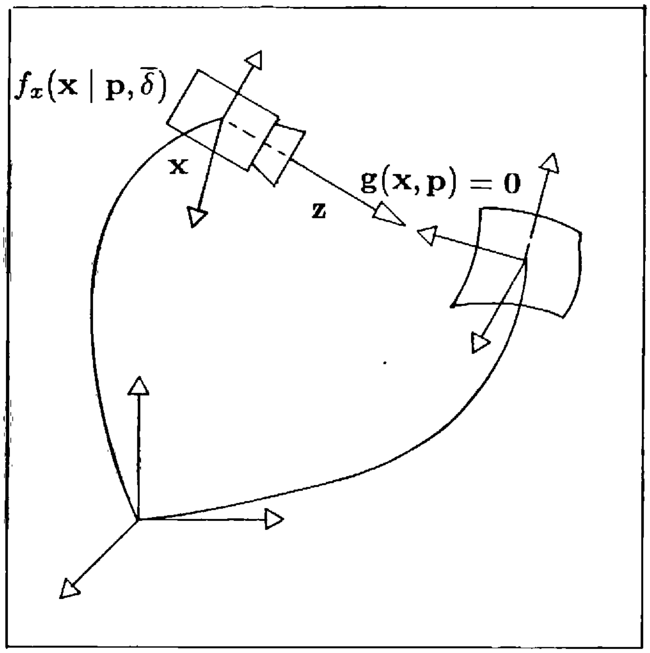
We will describe the dependence of sensor observations on location and state by a state model $f_x(\mathbf{z}_i|\mathbf{x}_i)$, which describes the posterior likelihood of feature observation in terms of a state vector $\mathbf{x}_i$. This model acts as a "view-modifier" on the other sources of information supplied to the sensor, a function that transforms the prior information $f_\delta(\mathbf{z}_i|\cdot)$ and observation model $f_p(\mathbf{z}_i|\cdot)$ to the current viewpoint of the sensor. This transformation is just a product of distribution functions:

$$f_i(\mathbf{z}_i|\mathbf{x}_i, \mathbf{p}_i, \bar{\delta}_i) = f_x(\mathbf{z}_i|\mathbf{x}_i)[f_p(\mathbf{z}_i|\mathbf{p}_i)f_\delta(\mathbf{z}_i|\bar{\delta}_i)]. \qquad (9)$$

There are two related parts to this description of a state model: the dependency of observation uncertainty on sensor state, and the transformation of prior world information to the current sensor location to determine if a feature is in view. Both of these considerations involve a process of transforming feature descriptions between coordinate systems.

Consider a mobile sensor located in space by the vector $\mathbf{x} = [x, y, z, \phi, \theta, \psi]^T$, observing a given feature $\mathbf{g}(\mathbf{x}, \mathbf{p}) = \mathbf{0}$ parameterized by the vector $\mathbf{p}$ as shown in Fig. 2. Suppose that the observations made by this sensor when in a fixed location can be described by a

Fig. 2. A mobile sensor observing a single geometric feature.



Gaussian distribution $\mathbf{z}_i \sim N(\hat{\mathbf{p}}, \Lambda_p)$. This is our observation model. When a Gaussian feature is transformed between coordinate systems as $\mathbf{p}_j = {}^j\mathbf{h}_i(\mathbf{p}_i)$, the mean vector follows the usual laws of geometry (Durrant-Whyte 1988), $\hat{\mathbf{p}}_j = {}^j\mathbf{h}_i(\hat{\mathbf{p}}_i)$, and with ${}^j\mathbf{J}_i = \partial^j\mathbf{h}_i/\partial\mathbf{p}_i$ the variance matrix is transformed by

$$^j\Lambda_p = {}^j\mathbf{J}_i(\mathbf{x})^i\Lambda_p{}^j\mathbf{J}_i^T(\mathbf{x}).$$

This transform can be interpreted as a change in sensor location resulting in a change in feature observation. In this case, the observation model will be state dependent:

$$\mathbf{z}_i \sim N({}^j\mathbf{h}_i(\hat{\mathbf{p}}_i), \mathbf{J}\Lambda_p\mathbf{J}^T) \sim N(\hat{\mathbf{p}}_j, \Lambda_p(\mathbf{x})),$$

describing how the uncertainty and perspective in feature observation are affected by the location of the sensor. Now suppose that we have some prior information about a feature to be observed, either from a prior world model or from observations made by other sensors, described again as $\mathbf{g}(\mathbf{x}, \mathbf{p}) = \mathbf{0}$, with $\mathbf{p} \sim N(\hat{\mathbf{p}}, \Lambda_p)$. Neglecting the possibility of occlusion, this information can be transformed into the coordinate system of the sensor through the relation $\mathbf{p}' = \mathbf{h}(\mathbf{p})$. This transformation is described by the state model $f_x(\cdot \mid \mathbf{x})$ and implemented by equations relating the mean and variance in different coordinate systems. This allows us to describe the prior information available to a sensor in terms of the features it may actually view.

The development of sensor strategies follows naturally from this description of observation dependence on sensor state. The sensor observation model described in terms of the mean and variance of sensor observations is now a function of the state and location of the sensor. If, for example, information about some particular feature is required, then this dependence of feature description on state can be used to determine the value of $\mathbf{x}$ which would give the measurements made by the sensor some appropriate characteristics. For example, knowing the function $\Lambda_\mathbf{p}(\mathbf{x})$, we can proceed to find the sensor state $\mathbf{x}$ that minimizes (in some sense) the uncertainty in taking observations of $\mathbf{p}$.

A second important problem in modeling state dependencies is describing the effect on a sensor's observations of *other* sensor states. This most obviously occurs when two or more sensors obtain information from the same physical device. A harder problem arises when observations made by two different physical devices are dependent on each other's state, such as the problem of hand-eye coordination. A simple example of this is discussed in Hager and Durrant-Whyte 1988).

3. Integrating Sensor Information

We are primarily interested in teams of observers, sensors making observations of the state of the environment. In this case individual team members can be considered as Bayesian estimators. The team decision is to come to a consensus view of the observed state of nature. The static team of estimators is often called a multi-Bayesian system (Weerahandi and Zidek 1981; 1983). Each individual sensor or team member communicates information in the common language of uncertain geometry. This allows different sensors to exchange information and utilize diverse opinions from other, different, sources. Each sensor must also communicate a preference order on its decisions to provide a means of resolving any differences of opinion among sensors. In a team of Bayesians, this preference order is described by a likelihood function. The description of preferences in terms of a probability distribution provides a dimensionless means of comparing estimates of disparate geometric features in a common framework. The communication of observations and preferences forms the basis for coordinating the acquisition of information and provides a mechanism for the generation and verification of team hypotheses.

The description of a multisensor system as a team of decision makers contains a number of important elements:

How individual sensors make local decisions based on their own observations
How the capabilities and information provided by each sensor are described to other members of the team
How information obtained by one sensor is described and communicated to other sensors
How local decisions made by individual sensors can be integrated to provide a team decision and action.

We will develop a description of a sensor as a Bayesian observer of geometric states in the environment. The capabilities of each sensor are represented by the probabilistic information structure, describing the observation of geometric features with respect to the state of the sensor and the availability of prior information. The communication of information will be described by the transformation and interpretation of uncertain geometric features. Decisions will be made by testing of dimensionless likelihoods on different geometric observations and consequent integration using Bayesian decision procedures.

3.1. The Team Structure

The observations $\mathbf{z}_i$ made by different sensors can be described, in terms of uncertain geometry, as a probability distribution function. Each sensor must make a decision δ_i, based on these observations to estimate some geometric feature in the environment $\delta_i(\mathbf{z}_i) \in \mathcal{P}_i$.

The decisions made by each sensor must be evaluated with respect to some measure of preference. This preference ordering, or utility function $u_i(\cdot, \delta_i) \in \Re$, allows different observations to be compared in a common framework. Consider for example two sensors taking observations $\mathbf{z}_1$ and $\mathbf{z}_2$ of two physically disparate geometric features $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively. Suppose that the observed features are related in some way so that the estimate $\delta_1(\mathbf{z}_1)$ is constrained by the estimate $\delta_2(\mathbf{z}_2)$. In general, these two estimates cannot be compared directly and so the constraints between them cannot be made explicit. However, we can compare the contribution or utility they each provide to some joint consensus. Thus by first choosing some consensus estimate, we can compare diverse opinions by evaluating $u_i(\cdot, \delta_i(\mathbf{z}_i))$. The local decision is to maximize u_i, the group decision is to incorporate geometric constraint between sensors in this maximization.

A sensor or team member will be considered *rational* if for each observation $\mathbf{z}_i$ of some prior feature $\mathbf{p}_i \in \mathcal{P}_i$ it makes the estimate $\delta_i(\mathbf{z}_i) \in \mathcal{P}_i$, which maximizes its individual utility $u_i(\mathbf{p}_i, \delta_i(\mathbf{z})_i) \in \Re$. In this sense, utility is just a metric for constructing a complete lattice of decisions, thus allowing any two decisions to be compared in a common framework.

Individual rationality alone imposes insufficient structure on the organization of team members: If each sensor is allowed to make its own decisions regardless of other sensor opinions, then a group consensus will never be reached. We must provide a structure which allows for sensor opinions to be constrained or biased toward a common team objective. To do this, we will define a *team* utility function defined on the state of the environment $\mathbf{p}$ and the team decision function $\boldsymbol{\delta}$:

$$U = U(\mathbf{p}, \delta_1, \delta_2, \ldots, \delta_n). \tag{10}$$

This is a real-valued function which for every team decision $\boldsymbol{\delta}$ assigns a team utility, admitting a preference (in some sense) on group decisions. The fundamental problem in team decision theory is based on finding this preference ordering on $\boldsymbol{\delta}$. This problem, the formulation of group rationality axioms, can be quite complex and is still an open research issue; see, for example, Harsanyi (1977) and Bachrach (1975).

We can considerably simplify the team decision problem by restricting the type of group preference orderings that can be considered "rational." Consider a multisensor team taking observations of a number of different features in the environment. Each sensor has a preference ordering on the decisions that it can make; if it estimates the location of a feature, then it would prefer to have this estimate verified rather than disproved or relocated by the group decision. Clearly it must allow its decisions to be changed by at least a small amount so that it can be brought into line with a consistent interpretation of all sensors' observations. However, it should not allow its opinion to be changed so much that the group opinion now bears no resemblance to the sensors' original observations. This dichotomy between consensus and disagreement is fundamental to the multisensor decision problem.

If the sensor system suggests an explanation for all the different sensor observations, then each sensor must interpret this decision in terms of an explanation for its own measurements. If the sensor cannot reconcile its views with those of the team, then it must disagree and not support the team decision. If, however, it can support the team interpretation of sensor observations, the sensor should reflect this in its individual preference ordering. In this situation, the sensor system will try to find decisions that explain as many different sensor observations as possible, suggesting consistent interpretations for the different opinions, but allowing sensors to disagree. If sensors were not allowed to disagree, then they may be required to support hypotheses that are actually incorrect. The role of the sensors and sensor system in this case is to dynamically find a consistent, consensus estimate of the state of the environment by alternately suggesting and trying to verify possible environment hypotheses. This process of dynamic interaction, agreement, and disagreement is called the *bargaining* problem (Nash 1950).

3.2. The Multi-Bayesian Team

We will consider a team structure composed of individual Bayesian observers, each making observations of uncertain geometric features in the environment, communicating prior information to each other through geometric transforms, and making decisions based on likelihood.

A multi-Bayesian team works by considering the likelihood function $f_i(\cdot|\mathbf{p})$ of each team member as the (normalized) utility of individual observers. Then the team utility is considered to be the joint posterior distribution function $F(\mathbf{p}|\mathbf{z}_1, \ldots, \mathbf{z}_n)$ after each sensor i has made the observation $\mathbf{z}_i$. The advantage of considering the team problem in this framework is that both individual and team utilities are normalized so that interperson comparison of decisions can be performed easily, supplying a simple and transparent interpretation to the group rationality problem.

Before developing the general multi-Bayesian system, it is helpful to study the simpler case of two scalar homogeneous observers. Consider two team members, each observing the same scalar variable $p \in \mathcal{P}$, with observation density $f_i(\cdot|p)$, $i = 1, 2$. Suppose each observer takes a single observation z_i, considered independent and derived from a Gaussian distribution with mean $\hat{p}$ and variance σ_i^2. The goal of this team is to come to some consensus estimate of state $\bar{p} \in \mathcal{P}$, based on the two observations z_1 and z_2. In the multi-Bayesian system, each team member's individual utility function is given by the posterior likelihood $f(p|z_i) \sim N(\hat{p}, \sigma_i^2)$. The team utility function is given by the joint posterior likelihood $F(p|z_1, z_2) = f_1(p|z_1)f_2(p|z_2)$. A team member will be considered *individually* rational if it chooses the estimate $\bar{p} \in \mathcal{P}$ which maximizes its local posterior density:

$$\bar{p} = \arg\max_{p \in \mathcal{P}} f_i(p|z_i), \qquad i = 1, 2. \tag{11}$$

The team itself will be considered group rational if together the team members choose the estimate $\bar{p} \in \mathcal{P}$ which maximizes the joint posterior density:

$$\begin{aligned} \bar{p} &= \arg\max_{p \in \mathcal{P}} F(p|z_1, z_2) \\ &= \arg\max_{p \in \mathcal{P}} f_1(p|z_1)f_2(p|z_2). \end{aligned} \tag{12}$$

Whether or not the individual team members will arrive at a consensus team estimate will depend on some measure of how much they disagree, $|z_1 - z_2|$. If z_1 and z_2 are "close enough," then the two Bayesians should agree to use the posterior density $F(p|z_1, z_2)$ as their joint utility function and provide a joint estimate that satisfies Eq. (12). As $|z_1 - z_2|$ increases, they should "agree to disagree" and use their individual posterior densities as utility functions, providing individual estimates satisfying Eq. (11).

In a multi-Bayesian system, the point of disagreement occurs when the likelihood of individual estimates exceeds that of the group estimate. The best way to visualize this is by considering the *set*

$$\mathbf{f}(p) \times [f_1(p|z_1), f_2(p|z_2)]^T \in \Re^2$$

describing the preference space of the two observers. When $|z_1 - z_2|$ is small, this space is convex, and a joint utility vector can be found which is larger than any individual utility vector. As $|z_1 - z_2|$ increases, this space becomes concave, so that individual utility vectors become larger than any joint consensus (Weerahandi and Zidek 1983).

To find the point at which this space is no longer convex and disagreement occurs, all we need ensure is that the second derivative of the function $F(p|\cdot)$ is positive. Differentiating, we obtain

$$\begin{aligned}\frac{\partial^2 F}{\partial p^2} &= \frac{1}{f_1}\frac{d^2 f_1}{dp^2} + \frac{1}{f_2}\frac{d^2 f_2}{dp^2} + \frac{2}{f_1 f_2}\frac{df_1}{dp}\frac{df_2}{dp} \\ &= (\sigma_1^{-2} + \sigma_2^{-2}) - [\sigma_1^{-2}(p - z_1) + \sigma_2^{-2}(p - z_2)]^2.\end{aligned}$$

For this to be positive and hence $\mathbf{f}(p)$ to be convex, we are required to find a consensus p which satisfies

$$[\sigma_1^{-2}(p - z_1) + \sigma_2^{-2}(p - z_2)]^2[\sigma_1^{-2} + \sigma_2^{-2}]^{-1} \leq 1. \quad (13)$$

Notice that Eq. (13) is no more than a normalized weighted sum, a scalar equivalent to the Kalman gain matrix. It follows, therefore, that if a p can be found which satisifes Eq. (13), then the consensus $\bar{p}$ which maximizes F will be given by

$$\bar{p} = (\sigma_1^{-2} z_1 + \sigma_2^{-2} z_2)/(\sigma_1^{-2} + \sigma_2^{-2}). \quad (14)$$

Substituting Eq. (14) into Eq. (13), we can write

$$(z_1 - z_2)(\sigma_1^2 + \sigma_2^2)^{-1}(z_1 - z_2) = D_{12}(z_1, z_2),$$

where $D_{12} \leq 1$ in Eq. (13). Figure 3 shows a plot of the set $\mathbf{f}(p)$ for two Bayesians. The disagreement measure $D_{12} = D_{12}(z_1, z_2)$ is called the Mahalanobis distance. Figure 3 shows that for $D_{12} \leq 1$, the set $\mathbf{f}(p)$ is convex and a consensus, given by Eq. (14), exists. As the difference $|z_1 - z_2|$ increases, D_{12} becomes larger and eventually the set $\mathbf{f}(p)$ becomes concave, indicating disagreement. The consensus value of p describes an envelope of extrema of $\mathbf{f}(p)$. This provides an important visualization of agreement or disagreement between a set of Bayesians.

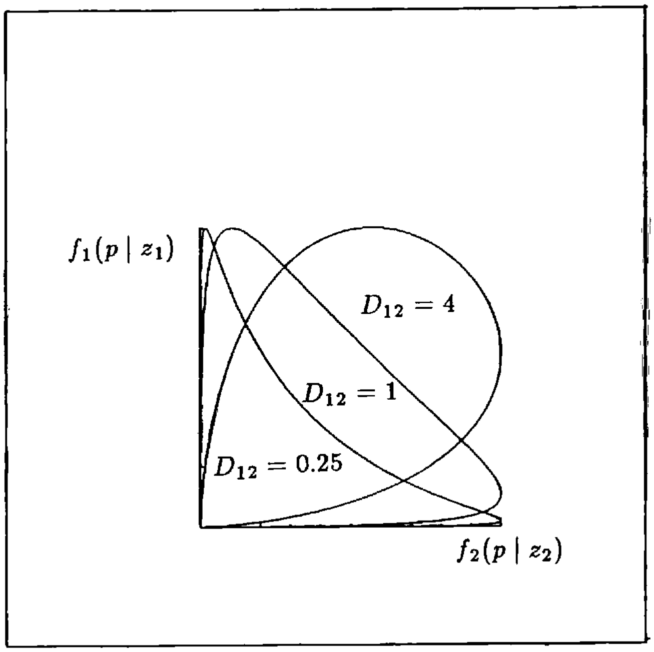


Fig. 3. The space of preferences for a two-Bayesian team.

We will now generalize this two-Bayesian system to a team of n Bayesians all taking different observations of geometric features in the environment. Consider n sensors each taking observations $\mathbf{z}_i \in \mathcal{P}_i$ of different geometric features $\mathbf{p}_i \in \mathcal{P}_i$, $\mathcal{P}_i \neq \mathcal{P}_j$. To compare geometrically disparate observations, we must hypothesize the existence of some common geometric object $\mathbf{p} \in \mathcal{P}$ to which these observations are related. For example, this object could be the location of a centroid or the description of a common surface. The observations made by a sensor can be related to this common object through the decision function $\delta_i(\mathbf{z}_i) \in \mathcal{P}$. Although $\delta_i^{-1}(\mathbf{p}) \in \mathcal{P}_i$ is usually well defined, $\delta_i(\mathbf{z}_i) \in \mathcal{P}$ is often only a partial mapping. For example, given the centroid of a known polyhedra, we can uniquely determine the location of all its edges and surfaces. Conversely, given an observed edge of this polyhedra, it is not possible to uniquely determine the object's centroid without recourse to other constraining information. However, although the decision $\delta_i(\mathbf{z}_i)$ contains only partial information about $\mathbf{p}$, we can consider the observation in terms of uncertain geometry and thus manipulate or transform it using the techniques described in the previous section.

Following our development of the two-Bayesian team, we will consider an individual sensor's preference ordering on consensus descriptions of a common geometric object $\mathbf{p}$ as the posterior density $f_i(\mathbf{p}|\delta_i(\mathbf{z}_i))$, and the joint preference ordering as the joint posterior density:

$$F(\mathbf{p}|\delta_1(\mathbf{z}_1), \ . \ . \ . \ , \delta_n(\mathbf{z}_n)) = \prod_{i=1}^{n} f_i(\mathbf{p}|\delta_i(\mathbf{z}_i)). \qquad (15)$$

We will call the set

$$\mathbf{f}(\mathbf{p}) = [f_1(\mathbf{p}|\delta_1(\mathbf{z}_1)), \ . \ . \ . \ , f_2(\mathbf{p}|\delta_1(\mathbf{z}_n))]^{\mathrm{T}} \subseteq \Re^n$$

the opinion space of the n-sensor team. This is an n-dimensional space with basis axis corresponding to individual sensor preferences f_i. The set $\mathbf{f}(\mathbf{p})$ describes an $(n-1)$-dimensional surface in this space, parameterized by $\mathbf{p}$ and enclosing a volume $v(\mathbf{z}_1, \ . \ . \ . \ , \mathbf{z}_n) \in \Re^n$. A consensus of *all* sensors requires that this volume be convex along each axis. The consensus value lies on the surface enclosing this volume. If $v \in \Re^n$ is concave along the jth axis, then the jth sensor observation will be unable to agree with other sensor opinions; its individual maximum likelihood estimate is preferred to the team decision. Thus if the volume v is concave in one or more directions f_i, then coalitions supporting different decisions will be formed. This occurrence is quite natural; individual sensor systems may indeed be viewing different features and so should not agree with a single consensus decision. Further, a partitioning of the f_i into coalitions should tell the sensor system that its suggested consensus is only partially correct and should be altered to bring the different coalitions into a single, convex opinion.

Following similar arguments to those of the two-Bayesian system, the observations $\mathbf{z}_i$ will only provide a consensus estimate for $\mathbf{p}$ when the subspaces of the opinion space are convex. To determine convexity, we need only ensure that the hessian of F is positive semidefinite. It can be shown (Durrant-Whyte 1987b) that a consensus value of $\mathbf{p}$ which satisfies this also satisfies

$$\left[\sum_{i=1}^{n} \Lambda_i^{-1}(\mathbf{p} - \delta_i(\mathbf{z}_i))\right]^{\mathrm{T}} \left[\sum_{i=1}^{n} \Lambda_i^{-1}\right]^{-1} \left[\sum_{i=1}^{n} \Lambda_i^{-1}(\mathbf{p} - \delta_i(\mathbf{z}_i))\right] \leq 1. \qquad (16)$$

If it exists, the consensus parameter estimate $\hat{\mathbf{p}}$ that maximizes Eq. (15) and minimizes the left side of Eq. (16) is given by the modified Kalman minimum-variance estimate

$$\hat{\mathbf{p}} = \left[\sum_{i=1}^{n} \Lambda_i^{-1}\right]^{-1} \left[\sum_{i=1}^{n} \Lambda_i^{-1}\delta_i(\mathbf{z}_i)\right]. \qquad (17)$$

To compare and integrate different sensor information, we must first allow each sensor to make its best geometric decision $\delta_i(\cdot)$, based on its own observations, and transform this decision geometrically into a description understood by other sensors. In this common framework, different decisions can then be compared using Eq. (16) and, if there are grounds for agreement, integrated using Eq. (17).

3.3. Integrating Sensor Information

The hypothesis $\{\mathbf{p}_i\}$ generated by each sensor from its observations can be interpreted in terms of partial hypothesis $\{\mathbf{p}\}$ of some underlying global geometric environment description $\mathbf{g}(\mathbf{x}, \mathbf{p}) = \mathbf{0}$, $\mathbf{p} \in \mathscr{P}$. The fact that individual sensor hypothesis $\mathbf{p}_i$ can only ever supply partial estimates of the global geometry $\mathbf{p}$ is a primary motivation for the use of many sources of sensory information. The partial hypothesis on global geometry provided by the sensors provides a means of comparing and combining different sensor views.

Consider a single sensor taking observations $\{\mathbf{z}_i\}$ of a particular type of feature in the environment $\mathbf{g}_i(\mathbf{x}, \mathbf{p}_i) = \mathbf{0}$. Each sensor observation is described through the observation model $f_i^p = f_i(\mathbf{z}_i|\mathbf{p}_i)$. The features described by $\{\mathbf{p}_i\}$ are considered to be related to some underlying global geometric description $\mathbf{g}(\mathbf{x}, \mathbf{p}) = \mathbf{0}$ through the transformation $\{\mathbf{p}_i\} = \mathbf{h}_i(\mathbf{p})$, $\mathbf{p}_i \in \mathscr{P}_i$, $\mathbf{p} \in \mathscr{P}$. This is a stochastic transformation of one type of geometry to another. The forward transformation $\mathbf{h}_i(\cdot)$ is usually well defined, and if we have some prior information about $\mathbf{p}$, $\mathbf{h}_i$ can be used to obtain prior feature information. However, in general no prior information will be available. In this case we must use the inverse transform to generate hypotheses about the underlying geometry: $\mathbf{p} = \mathbf{h}_i^{-1}(\mathbf{p}_i)$. The inverse transform is usually indeterminant, a single estimate $\mathbf{p}_i$ insufficient to generate a complete hypothesis $\mathbf{p}$.

Consider now a multisensor system comprising n sensors $S_1, \ldots, S_n$, each taking a set of geometric observations $\{\mathbf{z}_i\}$ of geometric features $\{\mathbf{p}_i\}$ in the environment. From these observations, each sensor can make individual estimates $\delta_i(\mathbf{z}_i) \in \mathscr{P}_i$ of possible features of a particular type, resulting in a set of feature hypotheses $\{\mathbf{p}_i'\} = \{\delta_i(\mathbf{z}_{i,1}), \ldots, \delta_i(\mathbf{z}_{i,1})\} = \{\mathbf{p}_{i,1}', \ldots, \mathbf{p}_{i,1}'\}$. We will consider the posterior distribution $f(\mathbf{p}_i|\mathbf{z}_i)$ as the preference order induced by the observations $\mathbf{z}_i$ on possible hypothesis $\mathbf{p}_i$. With this identification, we can consider the estimates $\mathbf{p}_i'$ as modeled by a Gaussian $\mathbf{p}_{i,j}' \sim N(\mathbf{p}_{i,j}, \Lambda_{i,j})$. To transform these feature hypotheses on some underlying geometry, we need to apply the transformation $\mathbf{h}^{-1}(\cdot)$ to the set $\{\mathbf{p}_i'\}$. If the elements of this hypothesis set are considered Gaussian, we can approximate this transform by a transformation of mean and variance, so that for each $\delta_i(\mathbf{z}_i)$, we have

$$\hat{\mathbf{p}}' = \mathbf{h}_i^{-1}[\mathbf{p}_i'] = \mathbf{h}_i^{-1}[\delta(\mathbf{z}_i)], \qquad \Lambda_p^{-1} = \left(\frac{\partial \mathbf{h}_i^{-1}}{\partial \mathbf{p}_i}\right) \Lambda_i^{-1} \left(\frac{\partial \mathbf{h}_i^{-1}}{\partial \mathbf{p}_i}\right)^{\mathrm{T}}. \tag{18}$$

Thus by transforming each feature hypothesis, we can obtain from each sensor S_i a set of hypotheses $\{\mathbf{p}\}_i$ on the underlying environment geometry, each of which can be described by a mean and variance. The information matrix Λ_p^{-1} will be singular, no information in one or more degrees of freedom, because the transform $\mathbf{h}_i^{-1}(\cdot)$ is indeterminant. This provides for the feature hypotheses $\mathbf{p}_i'$ to generate only partial estimates of the underlying geometry.

We now have a means of generating partial hypotheses of the environment geometry, regardless of the existence of prior information, in a language common to all sensors. It remains now to compare, verify, and combine these partial estimates to provide a complete description of the underlying environment geometry. We will formulate this problem in terms of a multi-Bayesian team, verifying individual sensor hypotheses by comparing their contribution to some global environment description.

Consider the preference placed on environment descriptions $\mathbf{p}$ by each sensor's partial estimates $\mathbf{p}'$ as the posterior intensity $f_i(\mathbf{p}|\mathbf{p}') = f_i(\mathbf{p}|\mathbf{h}_i^{-1}[\delta_i(\mathbf{z}_i)])$, $i = 1, \ldots, n$. This preference ordering represents an *individual* sensor's preferred contribution to a team consensus. We will define the *team* preference ordering as the joint posterior density on all contributions:

$$F(\mathbf{p}|\mathbf{h}_1^{-1}[\delta_1(\mathbf{z}_1)], \ldots, \mathbf{h}_n^{-1}[\delta_n(\mathbf{z}_n)]) = \prod_{i=1}^{n} f_i(\mathbf{p}|\mathbf{h}_i^{-1}[\delta_i(\mathbf{z}_i)]). \tag{19}$$

We will seek consensus values of $\mathbf{p}$ which make the opinion space convex. The hypotheses generated from different sensor observations will be associated with many different values of $\mathbf{p}$. Rather than comparing all of these hypotheses in one opinion pool, it makes sense to compare different estimates pairwise, recursively clustering hypotheses into groups, one associated with each value of $\mathbf{p}$.

From Eq. (16), the pairwise comparison of two hypotheses $\mathbf{h}_i^{-1}[\delta_i(\mathbf{z}_i)]$ and $\mathbf{h}_j^{-1}[\delta_j(\mathbf{z}_j)]$ requires that we find a consensus hypothesis $\mathbf{p}$ which satisifes

$$\bar{\mathbf{p}} = [\Lambda_i^{-1} + \Lambda_j^{-1}]^{-1} \times [\Lambda_i^{-1}\mathbf{h}_i^{-1}[\delta_i(\mathbf{z}_i)] + \Lambda_j^{-1}\mathbf{h}_j^{-1}[\delta_j(\mathbf{z}_j)]]. \tag{20}$$

Thus for two sensors to agree on a hypothesis, their interpretations must satisfy

$$(\mathbf{h}_i^{-1}[\delta_i(\mathbf{z}_i)] - \mathbf{h}_j^{-1}[\delta_j(\mathbf{z}_j)])(\Lambda_i + \Lambda_j)^{-1} \times (\mathbf{h}_i^{-1}[\delta_i(\mathbf{z}_i)] - \mathbf{h}_j^{-1}[\delta_j(\mathbf{z}_j)])^{\mathrm{T}} \leq 1. \tag{21}$$

Note that the transformation of observations is often only partial, so the sensor may have invariant (or indifferent) opinion preference to certain degrees of freedom of the team hypothesis.

3.4. Coordinating Information Flow

Our development of multisensor system models has relied on the probabilistic description and transformation of uncertain geometric objects. We have outlined how these objects can be communicated to other sensors, compared, and integrated to provide a group estimate of geometric elements describing the environment. An important aspect of this development is the dynamic exchange of information between different sensors. The dynamic use of information may occur in a number of ways, all of which can be classified in terms of either competitive, complementary, or cooperative information intercation. The dependence model can be used to describe all three of these dynamic effects.

1. *Competitive information* interaction occurs whenever two (or more) sensors supply information in the same location and degrees of freedom. For a consensus decision to be made, the opinion space formed by $f_\delta^i(\mathbf{p}\,|\,\cdot)$ must be convex; otherwise differences of opinion must be resolved by resorting to further information. Note that sensors can describe quite different geometric features but, when transformed by f_δ^i to a common description, can still be competitive. Examples of competitive information interaction include feature clustering algorithms or any homogeneous observation integration problem.
2. *Complementary information* interaction occurs whenever two (or more) information sources supply different information about the same geometric feature, in different degrees of freedom. In this case, the joint information is composed of different nonoverlapping information sources. It follows that each dependence model f_δ^i must only supply partial information about the *i*th sensor's observations and, consequently, expresses no preference ordering on the information obtained. A simple example of this is the observation of different beacons of a triangulation system. Complementary information can often be considered in terms of one sensor "filling in the gaps" in another sensor's observations.
3. *Cooperative information* interaction occurs whenever one sensor relies on another for information, prior to observation. Of particular importance is the use of one sensor's information to guide the search for new observations; the guiding of a tactile sensor by initial visual inspection is an example of this.

These different mechanisms for information exchange provide a basis for the coordination of information flow between sensors. They provide a mechanism to resolve conflicts or disagreements and to allow the development of dynamic sensing strategies.

4. Summary and Conclusions

We have argued that the development of use of sensor models is essential in providing a basis for understanding and utilizing multisensor systems. We have described a method of modeling sensors which considers the sensors of a multisensor system as members of a team communicating, cooperating, and coordinating their actions toward the solution of a common goal. The sensors of this team communicate information to each other in terms of a probabilistic description of observed geometric features. The description of sensors and sensor information in geometric terms allows this communication of information to be performed using well-understood tools from geometry and probability theory.

The sensor models developed are based on the idea of a team information structure, describing a sensor's observations as a probabilistic function of state and decisions communicated from other information sources. We have described three models based on this structure: the observation model, the dependence model, and the state model. These models, when used in conjunction with our description of sensor information as uncertain geometry, provide a probabilistic description of sensor capabilities which allow the development of general methods for the integration, coordination, and control of multisensor robot systems.

We have presented a general outline of how these models can be used to provide sensing strategies in multisensor systems. These techniques are far short of full development, and much could be done to improve their utility in more general settings. In particular, we would encourage the development of better methods to manipulate and interpret geometric information. Further, it is clear that these techniques would benefit from being developed in conjunction with other types of sensor models, able to describe aspects of sensing which are not geometric.

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