

Step1

$$y_{n(t)} W_t^T x_{n(t)} \leq 0 \text{ then } \left\lfloor \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} \right\rfloor \geq 0 \text{ and } \left\lceil \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} \right\rceil \geq 1$$

Step2

$$\begin{aligned} W_f^T W_{t+1} &= W_f^T \left(W_t + y_{n(t)} x_{n(t)} * \left\lfloor \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} + 1 \right\rfloor \right) \\ &\geq W_f^T W_t + y_{n(t)} W_f^T x_{n(t)} * \left\lfloor \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} \right\rfloor + y_{n(t)} W_f^T x_{n(t)} \quad \because [X + 1] = [X] + 1 \\ &\geq W_f^T W_t + y_{n(t)} W_f^T x_{n(t)} \quad \because y_{n(t)} W_f^T x_{n(t)} > 0 \text{ and } \left\lfloor \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} \right\rfloor \geq 0 \\ &\geq W_f^T W_t + \min_n y_n W_f^T x_n \end{aligned}$$

Step3

$$\begin{aligned} \|W_{t+1}\|^2 &\leq \left\| W_t + y_{n(t)} x_{n(t)} * \left\lfloor \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} + 1 \right\rfloor \right\|^2 \\ &= \|W_t\|^2 + 2 * y_{n(t)} W_t^T x_{n(t)} * \left\lfloor \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} + 1 \right\rfloor + \left\| y_{n(t)} x_{n(t)} * \left\lfloor \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} + 1 \right\rfloor \right\|^2 \\ &\leq \|W_t\|^2 + 2 * y_{n(t)} W_t^T x_{n(t)} * \left(\frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} + 1 \right) + \|y_{n(t)}\|^2 \|x_{n(t)}\|^2 \left\| \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} + 1 \right\|^2 \\ &\quad \because [X] \leq X \text{ and } [X]^2 \leq X^2 \text{ when } X \geq 0 \\ &= \|W_t\|^2 + 2 * y_{n(t)} W_t^T x_{n(t)} * \left(\frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|^2} + 1 \right) + \left\| \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|} + \|x_{n(t)}\| \right\|^2 \\ &= \|W_t\|^2 - 2 * \left(\frac{y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|} \right)^2 + 2 * y_{n(t)} W_t^T x_{n(t)} + \left\| \frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|} \right\|^2 \\ &\quad + 2 * \left(\frac{-y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|} \right) * \|x_{n(t)}\| + \|x_{n(t)}\|^2 \\ &= \|W_t\|^2 - 2 * \left\| \frac{y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|} \right\|^2 + 2 * y_{n(t)} W_t^T x_{n(t)} \\ &\quad + \left\| \frac{y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|} \right\|^2 + 2 * (-y_{n(t)} W_t^T x_{n(t)}) + \|x_{n(t)}\|^2 \\ &= \|W_t\|^2 - \left\| \frac{y_{n(t)} W_t^T x_{n(t)}}{\|x_{n(t)}\|} \right\|^2 + \|x_{n(t)}\|^2 \\ &\leq \|W_t\|^2 + \|x_{n(t)}\|^2 \leq \|W_t\|^2 + \max_n \|x_{n(t)}\|^2 \end{aligned}$$

Start from $W_0 = 0$, after T mistake corrections,

$$W_f^T W_T \geq T * \min_n y_n W_f^T x_n \cdots (1)$$

$$\|W_T\|^2 \leq T * \max_n \|x_{n(t)}\|^2 \rightarrow \|W_T\| \leq \sqrt{T} * \max_n \|x_{n(t)}\| \cdots (2)$$

Combining inequality (1) and (2), we can derive to inequality $1 \geq \frac{W_f^T W_T}{\|W_T\| \|W_f\|} \geq \sqrt{T} * \frac{\min_n y_n W_f^T x_n}{\|W_f\| * \max_n \|x_{n(t)}\|}$

When T grows to $\frac{1}{constant^2}$, $\frac{W_f^T W_T}{\|W_T\| \|W_f\|}$ will be 1 and make W_T equal to W_f .