

## Problem J

# Gas Station

Time limit: 3 seconds

### Problem Description

Alex is planning rest area placements on a simplified model of Taiwan's freeway system. The system contains  $n$  interchanges, connected by  $n - 1$  bidirectional roads. The network is connected, and there is exactly one shortest route between any pair of interchanges. The  $i$ -th road connects interchanges  $u_i$  and  $v_i$ , and has a length of  $l_i$ .

Exactly  $k$  rest areas with gas stations can be built, each located at an interchange. A driver may start a trip from any interchange and travel to any other, always following the unique shortest path. They begin each trip with a full tank of gas and can refuel only at interchanges that have a rest area.

Alex is curious about the smallest possible fuel tank capacity  $d$  such that it's possible to place the  $k$  rest areas in a way that ensures no driver will ever run out of gas. On any trip, the driver must never have to travel more than  $d$  units along the path without passing through a rest area, including at the beginning or end of the journey. The goal is to figure out the minimum such  $d$ , assuming the rest areas are placed in the best possible way.

### Input Format

The first line contains two integer  $n, k$ .

Followed by  $n - 1$  lines, the  $i$ -th of which contains three integers  $u_i, v_i, l_i$ , representing the  $i$ -th road connects interchanges  $u_i$  and  $v_i$  with a length  $l_i$ .

### Output Format

Output one integer, the smallest possible fuel tank capacity  $d$ .

### Technical Specification

- $2 \leq n \leq 2 \times 10^5$
- $0 \leq k \leq n$
- $1 \leq u_i, v_i \leq n$
- $1 \leq l_i \leq 10^9$
- It is guaranteed that the input roads form a tree.

### Sample Input 1

```
5 1
1 2 3
1 5 2
2 3 3
2 4 1
```

### Sample Output 1

```
5
```

### Sample Input 2

```
5 2
1 2 3
1 5 2
2 3 3
2 4 1
```

### Sample Output 2

```
3
```