Discrete Mathematics Question Set 1

Abrar Habib

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1. Simplify the statements so that no negation is outside a quantifier or an expression involving logical connectives. (20 pt.)

a. $\neg\forall x\forall y(P(x,y) \vee Q(x,y))$

$$\neg\forall x\forall y(P(x,y) \vee Q(x,y))$$

We can change the universal quantifier to the existential quantifier because $\neg\forall = \exists$ and factor the $\neg$ into the $\wedge$ operation. The equation now becomes:

$$\exists x\forall y\neg(P(x,y) \vee Q(x,y))$$

Apply DeMorgans Law:

$$\exists x\forall y(\neg P(x,y) \wedge \neg Q(x,y))$$

We cannot simplify any more.

b. $\neg(\exists x\exists y\neg P(x,y) \wedge \forall x\forall y Q(x,y))$ Distribute the outer negation and using DeMorgans Law:

$$\neg\exists x\neg\exists y\neg\neg P(x,y) \vee \neg\forall x\neg\forall y\neg Q(x,y)$$

Simplify statements:

$$\forall x\forall y P(x,y) \vee \exists x\exists y\neg Q(x,y)$$

We cannot simplify any more.

2. Prove the validity of this argument:
Premises:

$$(p \wedge t) \rightarrow (r \vee s) \tag{1}$$
$$q \rightarrow (u \wedge t) \tag{2}$$
$$u \rightarrow p \tag{3}$$
$$\neg s \tag{4}$$

Conclusion:

$$q \rightarrow r$$

1)	$(p \wedge t) \rightarrow (r \vee s)$	Premise
2)	$(p \wedge t)$	Premise
3)	p	2, Specification
4)	t	2, Specification
5)	q	Premise
6)	$q \rightarrow (u \wedge t)$	Premise
7)	$u \wedge t$	5,6 Modus Ponens
8)	$u \rightarrow p$	Premise
9)	$u \rightarrow (p \wedge t)$	3, 4 Conjunction
10)	$(p \wedge t) \rightarrow (r \vee s)$	Premise
11)	$u \rightarrow (r \vee s)$	9, 10 Hypothetical Syllogism
12)	$\neg s$	Premise
13)	$u \rightarrow r$	11, 12 Disjunctive Syllogism
14)	$(u \wedge t) \rightarrow r$	7, 13 Conjunction
15)	$q \rightarrow (u \wedge t)$	Premise
16)	$q \rightarrow r$	14, 15 Hypothetical Syllogism

Other Method:

1)	q	Premise
2)	$q \rightarrow (u \wedge t)$	Premise
3)	$u \wedge t$	1,2 Modues Ponens
4)	u	3, Specification
5)	t	3, Specification
6)	$u \rightarrow p$	Premise
7)	p	4, 6 Modus Ponens
8)	$p \wedge t$	5, 7 Specification
9)	$(p \wedge t) \rightarrow (r \vee s)$	Premise
10)	$(r \vee s)$	8,9 Modus Ponens
11)	$\neg s$	Premise
12)	r	10, 11 Disjunctive Syllogism
13)	$q \rightarrow r$	1,12 Conditional Proof

Because q is implied as a premise, and we have derived r using rules of inference, we can use conditional proof as a method of saying $q \rightarrow r$.

3. Is this statement a tautology? (Explain using a truth table) (10 pts.)

$$(\neg q \wedge (p \to q)) \to \neg p$$

P	Q	not p	not q	p -> q	not q and (p implies q)	(not q and (p implies q)) implies not p
1	1	0	0	1	0	1
1	0	0	1	0	0	1
0	1	1	0	1	0	1
0	0	1	1	1	1	1

This statement is a tautology.

4. Prove that if n is an integer and $n^3 + 5$ is odd, then n is even using (20 pts.)

a) proof by contraposition

b) proof by contradiction

Proof 1 *Assume n is an odd integer.*
So $n = 2k + 1$ for some integer k.
Plugging in $n = 2k + 1$ into $n^3 + 5$ we get

$$\begin{aligned} (2k+1)^3 + 5 \\ 8k^3 + 12k^2 + 6k + 1 + 5 \\ 8k^3 + 12k^2 + 6k + 6 \end{aligned}$$

We can factor out a 2 from this equation to get

$$2(4k^3 + 6k^2 + 3k + 3)$$

Since k was an integer, $8k^3 + 12k^2 + 6k + 6$ is also an integer.
Because we assumed n was odd and $n^3 + 5$ can be represented as $2(j)$ where $j = 4k^3 + 6k^2 + 3k + 3$,
n has to be even.

Proof 2 *Suppose that if n is an integer and $n^3 + 5$ is even, then n has to be odd.*
If n is odd, it can be represented as $2k + 1$ for some integer k.
We can plug it into $n^3 + 5$

$$\begin{aligned} (2k+1)^3 + 5 \\ 8k^3 + 12k^2 + 6k + 1 + 5 \\ 8k^3 + 12k^2 + 6k + 6 \\ 2(4k^3 + 6k^2 + 3k + 3) \end{aligned}$$

Since we assumed that n was an integer, $2(4k^3 + 6k^2 + 3k + 3)$ must also be an integer.
Because $n^3 + 5$ can be represented as $2(j)$ where $j = (4k^3 + 6k^2 + 3k + 3)$, this directly contradicts with our claim that n has to be odd.
Therefore, n is even.

5. Determine whether each of these statements is true or false and explain the reason briefly. (30 pts.)

a) $x \in \{x\}$

This is true. The set $\{x\}$ contains one element which is x.

b) $\{x\} \subseteq \{x\}$

This is true. Every element in $\{x\}$ is part of the set $\{x\}$.

c) $\{x\} \in \{x\}$

This is false. The set $\{x\}$ does not have within itself another set, only an element x.

6 Let A, B, and C be sets. Using membership or Venn diagram show that

a) $(A \cup B) \subseteq (A \cup B \cup C)$
$A \cup B \cup C$ is a union of all three sets such that it contains all elements of all three sets.
Therefore, the elements of $A \cup B$ are part of the union of all three sets.
Therefore, $A \cup B \subseteq (A \cup B \cup C)$ is true.

b) $(A - B) - C \subseteq A - C$
$(A - B) - C$ is the set of all elements that are unique to only A.
The set $A - C$ is the set of all elements that are unique to A, but also includes elements from B as well.
Therefore, the elements in $(A - B) - C$ which only includes elements unique to A is a subset of $A - C$ which includes all elements unique to set A, plus elements of set B.