Discrete Math Question Set 3

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1. Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

   (a) x is a multiple of y

   R is reflexive because $(x, x) \in$ R and x is always a multiple of itself.

   R is not symmetric because for $(4, 2) \in$ R, but $(2, 4) \notin$ R.

   R is antisymmetric because $(x, y) \in$ R implies x | y (x divides y). This implies that $|x| >= |y|$. A number can only be a multiple of anotehr number if the multiple is bigger than that other number. Therefore, if $(x, y) \in R$, then $(y, x) \notin R$ because y will be less than x.

   R is transitive because if $(x, y) \in R$, it implies $x = ky$ and $(y, z) \in R$, it implies $y = jz$ with k and j being integers. $x = k(jz)$ implies $x = (kj)z$ which makes z a multiple of x.

   (b) R is the relation on $\mathbb{Z}$ where x R y $x + y$ is odd

   R is not reflexive because x+x = 2x. Any number added to itself will become an even number.

   R is symmetric because addition is commutative. x+y will equal the same as y+x.

   R is not transitive because let x=2,y=3,z=4. $(2, 3) \in R$ because 2+3 = 5, and $(3, 4) \in R$ beause 3+4 = 7, but $(2, 4) \notin R$ because 2+4 = 6.

2. In the previous question, determine whether the relation is a partial ordering or equivalence relation or none of these. Explain why.

   Part a is a partial ordering relation because it is reflexinve, antisymmetric and transitive. Part b is neither beause it is not reflexive, and not transitive.

3.

The input/output table below defines a function $f(x, y, z)$. Create and fill in a new column for $\overline{f(x, y, z)}$. Then find a DNF expression equivalent to $\overline{f(x, y, z)}$.

x	y	z	$f(x, y, z)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$f(x, y, z) = \overline{x}\overline{y}z + \overline{x}y\overline{z} + x\overline{y}\overline{z} + xy\overline{z}$

$\overline{f(x, y, z)} = xy\overline{z} + x\overline{y}z + \overline{x}yz + \overline{x}\overline{y}z$

x	y	z	$\overline{f(x, y, z)}$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

4. Describe a general method for taking a Boolean function defined by an input/output table and finding an equivalent CNF expression.

First, look for all rows where the function evaluates to 0. The form for a CNF function is $\overline{row1} \cdot \overline{row2} \cdot ...$

If the variable is 1, complement it, else do nothing. Multiply every row with each other.

The CNF function for the function in question 3 is $f(x, y, z) = (xyz)(x\overline{y}z)(\overline{x}yz)\overline{(xyz)}$. We use rows 1,3,5,8. We take all the values of x,y, and z, complementing when we see a 1 and multiply all rows together.

5. For each expression below, give an equivalent expression that uses only the NAND operation. Then give an equivalent expression that uses only the NOR operation. (using boolean algebraic notations)

   (a) $\overline{x} + y$

      $\overline{x} + y = (x \uparrow \overline{y}) = x \uparrow (y \uparrow y)$

      $\overline{x} + y = \overline{x\overline{y}} = ((x \downarrow x) \downarrow y) \downarrow ((x \downarrow x) \downarrow y)$

   (b) $\overline{(xy)}$

      $\overline{(xy)} = x \uparrow y$ This is what NAND is.

      $\overline{(xy)} = ((x \downarrow x) \downarrow (y \downarrow y)) \downarrow ((x \downarrow x) \downarrow (y \downarrow y))$

6a. Keeping the order of the elements fixed as 1, 2, 3, 4, 5, determine the (0, 1) relation matrix for each of the following equivalence relations.

$R_1 = (1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5)$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$R_2 = (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 4), (4, 5), (5, 4), (5, 5).$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{2}$$

6b. Do the results of part (a) lead to any generalization?

Both of these graphs have 1's on the diagnols, which prove the fact that equivalence relations are reflexive.

Both of these graphs also supposed to be symmetric, since that is a defining property of equivalence relations. If we raise both matrix to any power, we still get that matrix.

7. Let $|A| = 5$

   (a) How many directed graphs can one construct on A?

      The number of directed graphs we can make is given by the equation

$$2^{n(n-1)}$$

      If we plug in n=5, we get that the number of graphs we can create with 5 vertices is 2^20.

   (b) How many of the graphs in part (a) are actually undirected?

      We can use the equation $2^{\frac{n(n-1)}{2}}$ which gives us 2^{10} undirected graphs.