

Discrete Math Question Set 5

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December 22, 2022

1. A chain letter starts with a person sending a letter out to 10 others. Each person is asked to send the letter out to 10 others, and each letter contains a list of the previous six people in the chain. Unless there are fewer than six names in the list, each person sends one dollar to the first person in this list, removes the name of this person from the list, moves up each of the other five names one position, and inserts his or her name at the end of this list. If no person breaks the chain and no one receives more than one letter, how much money will a person in the chain ultimately receive?

Each person introduces to 10 other people, so we get $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^6 = 1,000,000$. The first person on the list is on everyone else's list. so he gets one dollar from everyone else. \$1,000,000

2. What is the value of this expression? $- * 2/843$

Take the middle item and apply it to the items to its left. We get $- * 2(8/4)3 \rightarrow - * 223$. Then we take the multiplication operator and apply it to 2 and 2 to get -43 which we finally evaluate to get $4 - 3 = 1$.

3. Describe the trees produced by breadth-first search and depth-first search of the wheel graph W_n (that was explained in the class), starting at the vertex of degree n , where n is an integer with $n \geq 3$. Explain the reason for your answer.

Breadth-first search will produce one graph while depth-first search has the possibility to create more than one graph.

4. Draw the graphs of all nonisomorphic trees on six vertices.

Refer to attached image.

5. Let $F_1 = (V, E)$ be a forest of seven trees where $|E| = 40$. What is $|V|$? (A forest is a group of trees)

It is given that there are 40 edges. Each tree in the forest has $n-1$ vertices. We can relate this to the total number of edges to vertices through the equation $n_1 - 1 + n_2 - 1 + n_3 - 1 + n_4 - 1 + n_5 - 1 + n_6 - 1 + n_7 - 1 = 40$. where each n represents the number of vertices in each tree. This equation works because the number of edges to vertices are related such that $E = V - 1$. Solving the equation gives us that all the edges add up to 47.

6. Is this graph bipartite? Why? (Didn't want to draw the graph in LaTeX sorry)

The graph is not bipartite because it doesn't meet the requirements. Vertex E is in the middle and therefore makes it impossible to separate the graph into 2 individual groups. E is connected to every other vertex.

7. Does this graph has a Hamilton circuit? If yes, write the circuit. If not, explain why it doesn't have any.

A hamiltonian circuit is a path which goes through every vertex once. There is a hamiltonian vertex in the graph with the path being a,b,e,d,c.

8. Does this graph has an Euler circuit? If yes, write the circuit. If no, determine whether the graph has an Euler trail. Construct Euler trail if it exists.

An Euler circuit is a circuit which goes through every vertex once and goes back to where it started. There is no euler circuit in the graph because not all the vertices have an even degree. The graph does not have an euler trail because for this directed graph, the in degree of vertex A is not the same as its out degree.

9. Is this graph planar? If yes, draw it so that no edge cross.

The graph is not planar. Using euler's formula we can see that it is not able to be drawn planar.

10. What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state? [Hint: Use the pigeonhole principle.]

$n = 50$, $r = 100$. Use formule

$$n(r - 1) + 1$$

which evaluates to 4951.

11. I skipped number 11. It's extra credit.

12. The nine members of a coed intramural volleyball team are to be randomly selected from nine college men and ten college women. To be classified as coed the team must include at least one player of each gender. What is the probability the selected team includes more women than men?

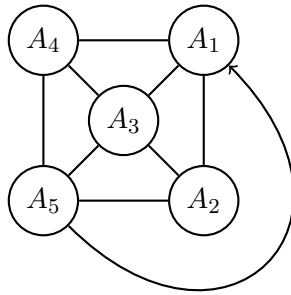
The number of men is 9 and the number of women is 10. The probability for men is $\frac{9}{19}$ and the probability for women is $\frac{10}{19}$. I subtracted $\frac{10}{19} - \frac{9}{19} = \frac{1}{19}$

13. The graph intersection of a collection of sets A_1, A_2, \dots, A_n is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph for of the following collection of sets.

$$A_1 \cap A_5 = \emptyset$$

$$A_2 \cap A_4 = \emptyset$$

I had to make a graph in which A_1 and A_5 aren't connected as well as A_2 and A_4 are not connected.



14. Describe the following graph.

$$\overline{K_n}$$

Since the graph K_n is the graph which has n vertices, $n-1$ edges and 1 edge between each vertex, $\overline{K_n}$ will be the graph with k vertices with no edges. To take the complement of a graph, you take the connections that the original graph has and remove them while adding connections that were not present in the original graph.

15. Describe the trees produced by the breadth-first search and depth-first search of the complete bipartite graph $K_{m,n}$, starting at a vertex of degree m , where m and n are positive integers. Justify your answers.

Breadth-first: The graph produced by this algorithm will be starting at the root, visiting any vertices that are adjacent to the root until there are no more to search. Then go visit the first vertex visited from root and repeat.

Deapth-first: The graph produced by this algorithm will be starting at the root, visit every vertex adjacent to it and repeat.

Refer to attached image for graph drawings.