Discrete Math Question Set 4

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1. Curves in Space and Their Tangents:

   Suppose you are given a curve in space defined by the equation $x^2 + y^2 + z^2 = 1$. Determine the equation of the tangent line to this curve at the point $(1, 0, 0)$.

2. Arc Length in Space:

   Consider the curve defined by the equation $x = t^3 - 3t, y = t^2 - 1, andz = t + 1$ for t in the interval [0,3]. Determine the length of this curve.

3. The Chain Rule:

   Suppose you have a function $f(x)$ defined as $f(x) = (x^2 + 1)^3$. Determine the derivative of f(x) with respect to x.

4. Directional Derivatives and Gradient Vectors:

   Suppose you are given a function $f(x, y)$ defined as $f(x, y) = x^2 + y^2$. Determine the directional derivative of $f(x, y)$ in the direction of the vector $v = (1, 1)$.

5. Tangent Planes and Differentials:

   Suppose you are given a function $f(x, y)$ defined as $f(x, y) = x^2 + y^2$. Determine the equation of the tangent plane to the surface $z = f(x, y)$ at the point $(1, 1)$.

6. Extreme Values and Saddle Points:

   Suppose you are given a function $f(x, y)$ defined as $f(x, y) = x^2 - y^2$. Determine the local extrema and saddle points of $f(x, y)$.

7. Double and Iterated Integrals over Rectangle:

   Consider the function $f(x, y)$ defined as $f(x, y) = x^2 + y^2$. Determine the double integral of $f(x, y)$ over the rectangle defined by $0 < x < 1, 0 < y < 2$.

8. Double Integrals over General Regions:

   Consider the function $f(x, y)$ defined as $f(x, y) = x^2 + y^2$. Determine the double integral of $f(x, y)$ over the region defined by the circle $x^2 + y^2 = 1$.

9. Area by Double Integration:

   Consider the region in the xy-plane bounded by the curves $y = x^2 and y = 1$. Determine the area of this region using double integration.

10. Double Integrals in Polar Form:

    Consider the function $f(x, y)$ defined as $f(x, y) = x^2 + y^2$. Use polar coordinates to evaluate the double integral of $f(x, y)$ over the region defined by the circle $x^2 + y^2 = 1$.

11. Triple Integrals in Rectangular Coordinates:

    Consider the function $f(x, y, z)$ defined as $f(x, y, z) = x^2 + y^2 + z^2$. Determine the triple integral of $f(x, y, z)$ over the rectangular solid defined by $0 < x < 1, 0 < y < 2$, and $0 < z < 3$.