1 Background

The purpose of this assignment is to compare the leapfrog and predictor-corrector schemes in modeling a low amplitude wave using the Benjamin-Bona-Mahony (BBM) partial differential equation and to describe the model's accuracy and computation time with respect to time and position step size. The BBM equation is shown in equation 1.

$$u_t + ux + uu_x - u_{xxt} = 0 \tag{1}$$

The time derivative can be isolated as shown in Equation 2.

$$u_t = -(1 - \frac{d^2}{dx^2})^{-1}\frac{d}{dx}(u + u^2) = B(u + u^2) \tag{2}$$

Equation 2 can be iterated by separately describing matrices for $\frac{d}{dx}(u+u^2)$ and $(1-\frac{d^2}{dx^2})^{-1}$.

The center differencing scheme is used to take the derivative of $u + u^2$. The necessary matrix populates $\frac{1}{2dx}$ on the upper diagonal and $\frac{-1}{2dx}$ on the lower diagonal.

The result of the matrix operating on $u + u^2$ is shown in Equation 3.

$$\frac{d(u + u^2)}{dx} = \frac{(u + u^2)_{i+1} - (u + u^2)_{i-1}}{2dx} \tag{3}$$

The matrix describing $1 - \frac{d^2}{dx^2}$ populates the upper and lower diagonals with $\frac{-1}{dx^2}$, and the main diagonal with $1 + \frac{2}{dx^2}$.

The iterating loop for the predictor corrector methods is shown below,

```
def iteratePredCor(nts, yuMtx, dt, invSvm, fod):
   for t in range(nts):
       prediction = yuMtx + eulerStep(dt, invSvm, fod, yuMtx)
       yuMtx = .5 * (prediction + yuMtx + eulerStep(dt, invSvm, fod, prediction))
return yuMtx
```

where eulerStep is defined as

```
def eulerStep(dt, invSvm, fod, yuMtx):
    return -dt * (invSvm * (fod * (yuMtx + sqrVals(yuMtx))))
```

The iterating loop for the leapfrog method is shown below.

```
def iterateLeapFrog(nts, yuMtx, dt, invSvm, fod):
    yuLast = yuMtx.copy()
    yuMtx = iteratePredCor(1, yuMtx, dt, invSvm, fod)
    nts-=1
    for t in range(nts):
       store = yuMtx.copy()
       yuMtx = yuLast + 2 * eulerStep(dt, invSvm, fod, yuMtx)
       yuLast = store
return yuMtx
```

The leapfrog method must store a copy of the last step, making copying necessary.

The BBM equation can be integrated into equation 4

$$u(x,t) = \frac{3}{2} asech^2 (\frac{1}{2}\sqrt{\frac{a}{a+1}}(x-(1+a)t)) \tag{4}$$

where a is the amplitude of the wave. This will be used to compare the results of the iterations.