Linear Regression

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RLLAB

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Overview

	Univariate	Multivariate	General
Model	$h_w(x_j) = wx_j + b$ $x_j, w, b \in \mathbb{R}$	$h_{\mathbf{w}}(\mathbf{x}_j) = \mathbf{w}^T\mathbf{x}_j + b$ $\mathbf{x}_j, \mathbf{w} \in \mathbb{R}^n$ $b \in \mathbb{R}$	$h_{\mathbf{w}} = \mathbf{w}^T\boldsymbol{\phi}(\mathbf{x}_j) + b$ $\mathbf{x}_j, \mathbf{w}, \boldsymbol{\phi}(\mathbf{x}_j) \in \mathbb{R}^n$ $\phi_i(\mathbf{x}_j), b \in \mathbb{R}$ $\phi_i(\mathbf{x}_j) : \mathbb{R}^n \longrightarrow \mathbb{R}$
MSE Loss		$\sum_{j=1}^{N}(y_j - h_{\mathbf{w}})^2$	
	$\sum_{j=1}^{N}\left(y_j - (wx_j + b)\right)^2$	$\sum_{j=1}^{N}\left(y_j - (\mathbf{w}^T\mathbf{x}_j + b)\right)^2$	$\sum_{j=1}^{N}\left(y_j - (\mathbf{w}^T\boldsymbol{\phi}(\mathbf{x}_j) + b)\right)^2$
Gradients	$\frac{\partial Loss}{\partial w} = \sum_{j=1}^{N} -2\left(y_j - (wx_j + b)\right)x_j \in \mathbb{R}$	$\frac{\partial Loss}{\partial \mathbf{w}} = \sum_{j=1}^{N} -2\left(y_j - (\mathbf{w}^T\mathbf{x}_j + b)\right)\mathbf{x}_j \in \mathbb{R}^n$	$\frac{\partial Loss}{\partial \mathbf{w}} = \sum_{j=1}^{N} -2\left(y_j - (\mathbf{w}^T\boldsymbol{\phi}(\mathbf{x}_j) + b)\right)\boldsymbol{\phi}(\mathbf{x}_j) \in \mathbb{R}^n$
	$\frac{\partial Loss}{\partial b} = \sum_{j=1}^{N} -2\left(y_j - (wx_j + b)\right) \in \mathbb{R}$	$\frac{\partial Loss}{\partial b} = \sum_{j=1}^{N} -2\left(y_j - (\mathbf{w}^T\mathbf{x}_j + b)\right) \in \mathbb{R}$	$\frac{\partial Loss}{\partial b} = \sum_{j=1}^{N} -2\left(y_j - (\mathbf{w}^T\boldsymbol{\phi}(\mathbf{x}_j) + b)\right) \in \mathbb{R}$
Training Set	$\{\mathbf{x}_j, y_j\}_{j=1}^{N}$, where: $\mathbf{x}_j = [x_{j,1} \quad x_{j,2} \quad \cdots \quad x_{j,i} \quad \cdots \quad x_{j,n}]^T \in \mathbb{R}^n, \quad y_j \in \mathbb{R}$		
Parameter Update	$\mathbf{w}_{new} = \mathbf{w}_{old} - \alpha\frac{\partial Loss}{\partial \mathbf{w}}$, $b_{new} = b_{old} - \alpha\frac{\partial Loss}{\partial b}$, $\alpha \in \mathbb{R}$: step size / learning rate (≈0.01)		

Let’s move on to the code

‘Linear Regression.ipynb’