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ARTICLE

Hybrid short-term freeway speed prediction methods based on periodic analysis

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Abstract: Short-term traffic speed forecasting is an important issue for developing Intelligent Transportation Systems applications. So far, a number of short-term speed prediction approaches have been developed. Recently, some multivariate approaches have been proposed to consider the spatial and temporal correlation of traffic data. However, as traffic data often demonstrates periodic patterns, the existing methodologies often fail to take into account spatial and temporal information as well as the periodic features of traffic data simultaneously in the multi-step prediction. This paper comprehensively evaluated the multi-step prediction performance of space time (ST) model, vector autoregression (VAR), and autoregressive integrated moving average (ARIMA) models using the 5 minute freeway speed data collected from five loop detectors located on an eastbound segment of Interstate 394 freeway, in Minnesota. To further consider the cyclical characteristics of freeway speed data, hybrid prediction approaches were proposed to decompose speed into two different components: a periodic trend and a residual part. A trigonometric regression function is introduced to capture the periodic component and the residual part is modeled by the ST, VAR, and ARIMA models. The prediction results suggest that for multi-step freeway speed prediction, as the time step increases, the ST model demonstrates advantages over the VAR and ARIMA models. Comparisons among the ST, VAR, ARIMA, and hybrid models demonstrated that modeling the periodicity and the residual part separately can better interpret the underlining structure of the speed data. The proposed hybrid prediction approach can accommodate the periodic trends and provide more accurate prediction results when the forecasting horizon is greater than 30 min.

Key words: spatial and temporal correlation, multi-step prediction, freeway speed, periodic analysis, trigonometric regression.

Résumé : Il est essentiel de prédire la vitesse de la circulation routière à court terme pour pouvoir concevoir des systèmes de transport intelligents. Jusqu'à présent, un certain nombre de méthodes de prédiction de la vitesse du trafic routier à court terme ont été élaborées. Récemment, des approches à variables multiples ont été proposées, qui tiennent compte de la corrélation spatiale et temporelle entre les données relatives à la circulation routière. Cependant, étant donné que ces données présentent souvent un caractère périodique, les méthodologies existantes ne parviennent souvent pas à tenir compte en même temps des informations spatiales et temporelles et de la périodicité des données relatives au trafic routier dans le cadre de prédictions multiniveaux. Dans le présent article, on a évalué de manière approfondie l'efficacité de la prédiction multiniveaux des modèles espace-temps (ET) et vectoriel autorégressif (VAR) et du modèle mixte intégré autorégressif et de moyennes mobiles (ARIMA), et ce à l'aide des données de vitesses recueillies pendant 5 minutes au moyen de cinq détecteurs à boucle sur la portion allant en direction de l'est de l'autoroute I394, dans le Minnesota. Pour davantage tenir compte des caractéristiques cycliques des données de vitesse collectées sur cette autoroute, on a proposé des méthodes prédictives hybrides permettant de décomposer la vitesse en deux composantes différentes : une composante périodique et une composante résiduelle. On décrit une fonction régressive trigonométrique, qui permet d'intégrer la première composante alors que la seconde est modélisée à l'aide des modèles ET, VAR et ARIMA. Les résultats montrent, dans le cas de la prédiction multiniveaux de la vitesse du trafic autoroutier, que lorsque l'intervalle de temps augmente, le modèle ET est plus performant que les modèles VAR et ARIMA. Des comparaisons entre les modèles ET, VAR, ARIMA et hybrides montrent que la modélisation effectuée séparément de la périodicité et de la composante résiduelle permet de mieux interpréter la structure sous-jacente des données de vitesse. La méthode prédictive hybride proposée peut tenir compte de la périodicité et fournit des prédictions plus précises lorsque l'on souhaite effectuer des prévisions sur des durées supérieures à 30 minutes. [Traduit par la Rédaction]

Mots-clés : corrélation spatiale et temporelle, prédiction multiniveaux, vitesse du trafic autoroutier, analyse périodique, régression trigonométrique.

1. Introduction

Driven by the increasing need for developing intelligent transportation systems (ITS) applications, short-term traffic forecasting has been an important issue that attracted a high level of interest among researchers over the past 30 years. It is especially a critical input for the advanced traveler information systems (ATIS) to assist travelers in making more informed decisions about departure time, mode, and route choices. A good prediction algorithm usually requires advanced technologies and computational advances that are able to deal with and model massive amounts of data. In the literature, a large amount of algorithms have been proposed to address traffic prediction problems. Vlahogianni et al. (2004) summarized existing short-term traffic predictions algorithms up to 2003. And recently, Vlahogianni et al. (2014) updated the literature from

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2004 to 2013. Van Lint and Van Hinsbergen (2012) reviewed existing applications of neural network and artificial intelligence in short-term traffic forecasting. Existing traffic prediction algorithms range from statistical prediction methods (Cetin and Comert 2006; Shang et al. 2005; Chandra and Al-Deek 2008; Williams and Hoel 2003), neural networks (Ye et al. 2012; Chan et al. 2012; van Lint et al. 2005; Park and Rilett 1999; Rilett and Park 2001), support vector regression (Lam and Toan 2008), state space approach (Dong et al. 2015), and hybrid approaches (Hamad et al. 2009; Xie et al. 2007).

Based on whether or not data from only one location is used, traffic prediction methods can be divided into two categories: univariate and multivariate methods. Univariate methods study and forecast traffic parameters at each detector location individually, while multivariate methods take advantage of traffic information in adjacent locations to forecast traffic parameters. Existing studies mainly focused on the first type. However, traffic can be influenced by its upstream and downstream conditions; a multivariate approach potentially improves model performance by considering traffic information in its adjacent locations. Previously, Yin et al. (2002) developed a fuzzy-neural model to predict traffic flow by utilizing upstream flows in the current time interval. Kamarianakis and Prastacos (2003, 2005) compared and discussed different multivariate and univariate traffic prediction approaches. Van Lint (2006) proposed a state-space neural network model that utilizes upstream and downstream traffic as model input to predict travel time. Sun and Zhang (2007) modeled traffic flows among adjacent road links in a transportation network as a Bayesian network. Min and Wynter (2011) considered spatial–temporal correlation of traffic and predicted traffic speed and volume from 5 min to 60 min into the future. Sun and Xu (2011) developed Bayesian network approaches in traffic flow prediction by considering spatial correlation of traffic from adjacent road links. Pan et al. (2013) extended the stochastic cell transmission model to consider the spatial–temporal correlation of traffic flow in traffic state prediction. Li et al. (2013) developed a traffic flow imputation method and reduced imputing errors by using temporal–spatial dependence. Dong et al. (2014*a*) proposed a hybrid support vector machine that combines both statistical and heuristic models to consider the spatial-temporal patterns in traffic flow. Dong et al. (2014*b*) developed state space models for flow rate prediction under non-congested and congested traffic conditions based on the spatial–temporal patterns of traffic.

Traffic data often shows periodic patterns. Over a 24 h period during weekdays, there is usually one or two peak periods for each day, where vehicles travel in relatively lower speed. By considering periodic features in data, we gain better insights into the data and can improve prediction accuracy. Despite its importance, there are limited studies that consider periodic features when predicting traffic. Dendrinos (1994) considered traffic as a combination of periodic components and nonperiodic dynamics. Stathopoulos and Karlaftis (2001) studied common cyclical components of traffic flow between two successive loop detectors by using spectral analysis. Zhang et al. (2013) proposed a hybrid approach that uses a trigonometric regression function to model the cyclical patterns of the data and indicate that multi-step ahead prediction results can be improved by considering periodic features of traffic. Tchrakian et al. (2012) developed a real-time short-term traffic prediction algorithm based on spectral analysis.

In light of the reviewed literature, although some existing time series approaches can accommodate the spatial and temporal correlation of traffic data, these methods often fail to consider the cyclical patterns in forecasting multi-step ahead freeway speed. Thus, this study focuses on multi-step ahead traffic speed prediction by considering both spatial–temporal correlations of traffic through a space time model, and introduced a trigonometric regression function to capture periodic patterns in data. Unlike the neural networks and fuzzy logic methods (van Lint et al. 2002; Zhang and Ye 2008) which use a "black box" approach to predict traffic conditions and often lack a good interpretation of the model, the proposed space time model incorporates geographically dispersed travel speed as predictors to obtain short-term prediction and can yield theoretically interpretable prediction models. By considering periodic features in data, the long-term pattern of data can be better captured, and is beneficial in terms of multi-step ahead prediction. To test the performance of the proposed method, this study comprehensively evaluates performance of space time (ST) model, vector autoregression (VAR), and autoregressive integrated moving average (ARIMA) under different scenarios: multi-step ahead prediction (1, 3, 6, 12 steps ahead predictions), consider periodic patterns and not consider periodic patterns. The experiment is performed by using every 5 min speed data collected on an eastbound segment of Interstate 394 (I-394) freeway, Minnesota. The contribution of this paper includes: introduces an advanced space time model in freeway traffic speed prediction by considering spatial–temporal correlations of its upstream and downstream traffic; incorporates a trigonometric regression function to model the periodic features of the data; and comprehensively evaluates and compares three models' performance under different scenarios.

2. Data description and preliminary data analysis

The speed data used in the study were collected on an eastbound segment of I-394 freeway, between Trunk Highway 100 and Peen Ave in Twin Cities Metro area, Minnesota. This freeway segment experiences significant traffic congestions during morning and afternoon peak hours. For the selected segment, 5 adjacent stations are located to collect traffic speed data. The distance between each two adjacent stations is approximately 0.8 km. The total length of the I-394 stretch of interest is approximately 2.7 km.

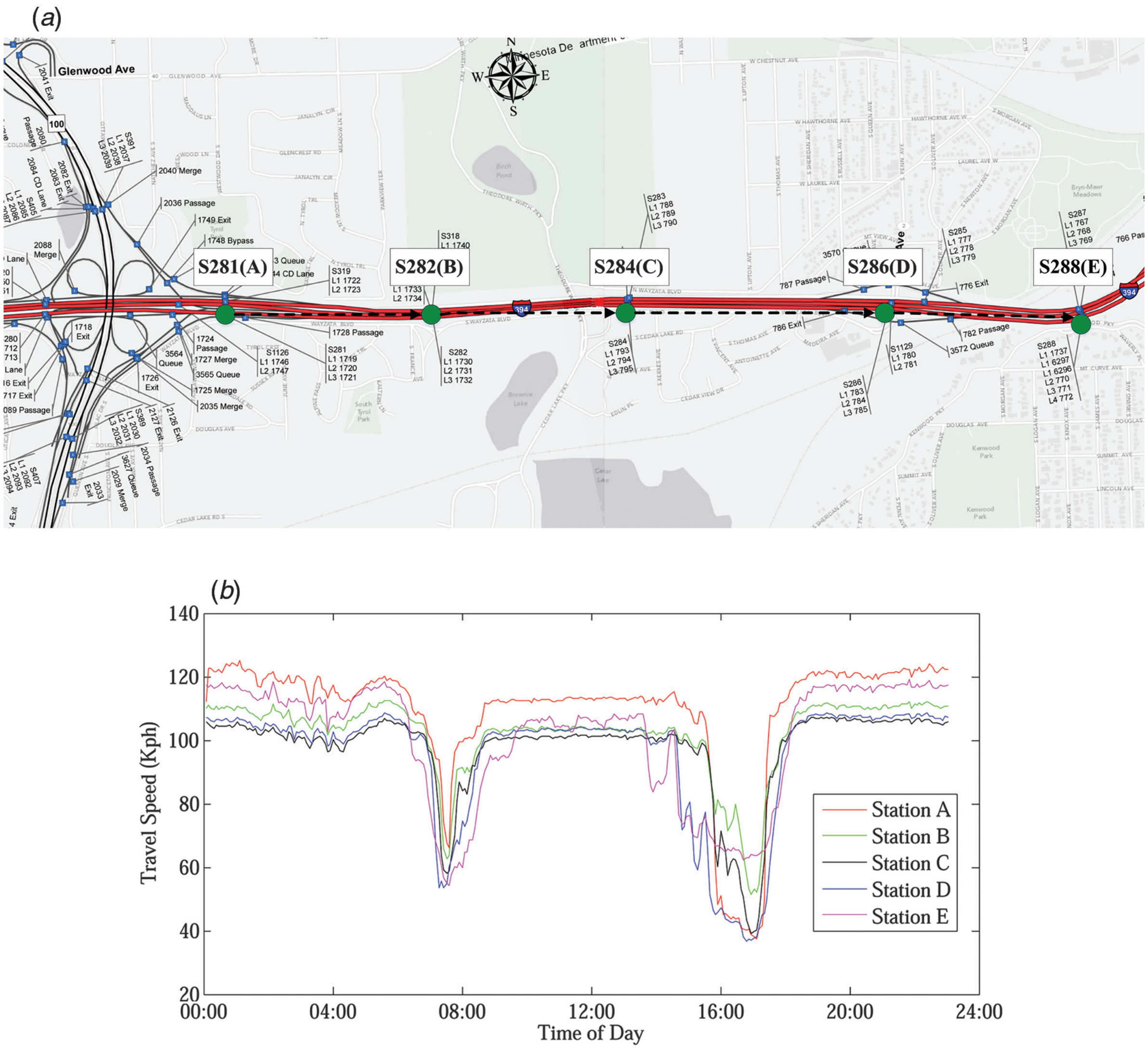
The travel speed data are publicly available using the data tool developed by Minnesota Department of Transportation. The traffic data were initially collected once every 30 s, 24 h a day from loop detectors. We calculated the arithmetic mean of travel speeds and aggregated the travel speeds into 5 min intervals in this study. The missing data for the five stations are all less than 5%, and historical averaged based data imputation method has been implemented to ensure the selected speed data are appropriate for model validation and evaluation in this study. It is known that the traffic speed pattern on weekdays are quite different from that on weekends; traffic during the night and weekend is normally smooth without congestions. Thus, this study considers and analyzes the speed data observed from 6:00 AM to 8:00 PM on weekdays with different traffic conditions (i.e., congested and uncongested traffic conditions). Speed data on weekdays with a time period of 7 months, from October 2012 to May 2013, were collected. Figure 1*a* shows the location of the selected I-394 segment.

Figure 1*b* shows the historical median travel speeds (Monday–Friday, from October 2012 to May 2013) at the five stations. It can be observed that there are two peak hours (one at approximately 8 AM and the other from 3:30 PM to 7:00 PM) for all five stations.

Previous studies have indicated that spatial and temporal correlation exists among traffic data observed at adjacent stations (Chandra and Al-Deek 2008, 2009; Yang et al. 2014). The traffic speed values observed at downstream stations are often influenced by speed values at upstream stations. Due to congestion, the downstream traffic conditions may also have an influence on upstream traffic. To describe the temporal and spatial correlation of speed data, cross-correlation function (CCF) was used.

In this study, station C is considered as the target location. Figure 2 provides the sample cross correlation results between station C and other neighboring stations. As shown in Fig. 2, the cross correlation functions of travel speeds demonstrate a steady decline when absolute value of lag increases. In other words, if the lag equals to 0, cross correlation values between the center station

Fig. 1. (*a*) Location of I-394 segment, from station S281 to station S288; (*b*) historical median travel speeds at the five stations.



C and the other stations reach the maximum. When the absolute values of lag increase to 20, the cross correlation values decrease to approximately 0.3. Another feature worth noting is that as the distance between two stations increases, a decreasing tendency of cross-correlation value between the two stations can be observed. In Fig. 2, the maximum cross-correlation values between station C and its adjacent stations A, B, D, and E are 0.8488, 0.9289, 0.8740, and 0.7087, respectively. This is reasonable because the stations further apart would have less impact on each other. Considering the factor of distance between two stations, travel speed data from four nearest neighboring stations were collected to predict speeds at station C.

2.1. Stationary analysis

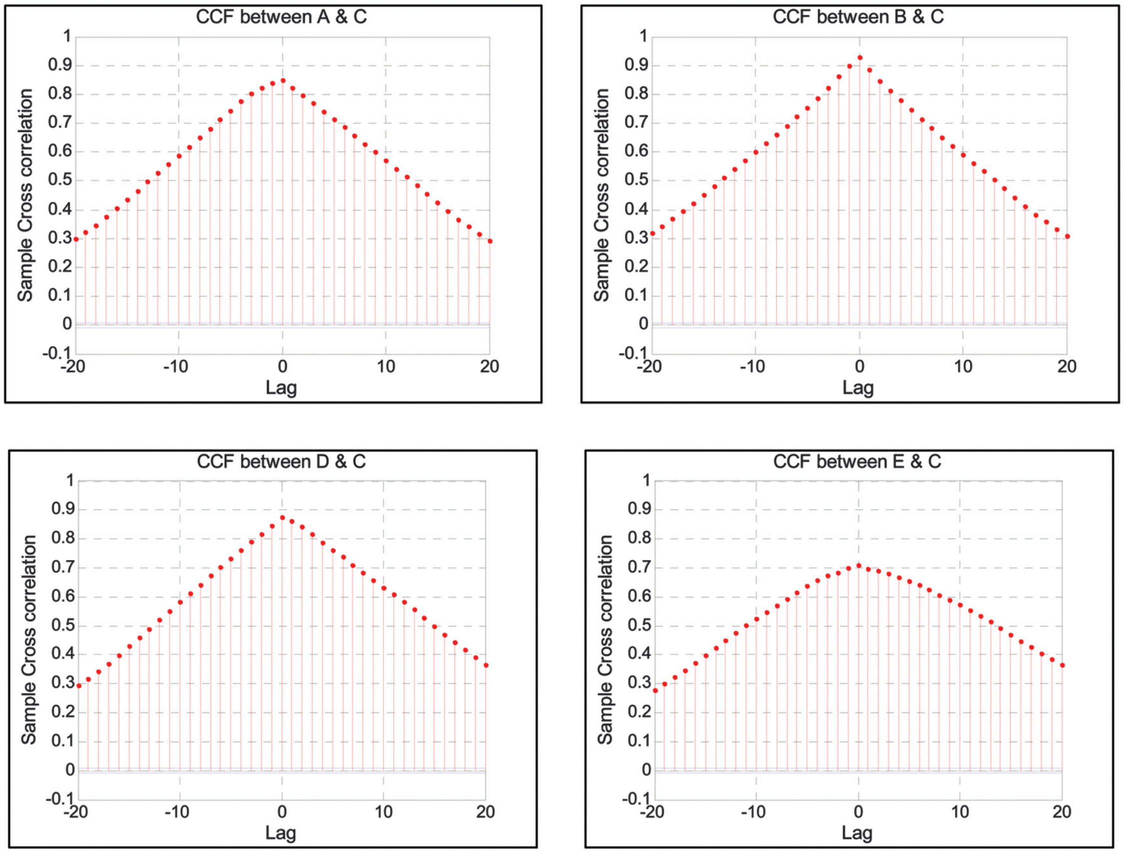
Some commonly used statistical forecasting methods, including the VAR model in the paper, are all based on an assumption that time series data are stationary or can be stationarized by some transformations. The autocorrelation function (ACF) is plotted for the travel speed time series.

Figure 3 shows the results of autocorrelation function of speed on five stations from lag 0 to 288. The results in Fig. 3 suggest that the original speed time series are non-stationary at all stations, since the autocorrelation values decay slowly. To obtain stationary speed time series, first difference is applied. Two statistical tests, Augmented Dickey Fuller test and Kwiatkowski Phillips Schmidt Shin test are conducted to test the stationary hypothesis and the results of statistical tests confirm the time series of travel speed after first difference are stationary.

2.2. Cyclical analysis

In addition to the temporal and spatial correlation analyzed above, another important characteristic of speed data are the cyclical pattern. Figure 4 displays the speed at station C during five consecutive typical weekdays. From Fig. 4, we can see that freeway speed shows a cyclical pattern over different weekdays. It is evident that speed drops significantly in the morning and afternoon peak hours. During off-peak hours, the traffic is in the free flow traffic condition and speed values fluctuate randomly. The travel speed during weekdays is periodic and recurs every 24 h. This cyclical pattern can also be found for other traffic variables (i.e., travel time, volume and vehicle occupancy) in previous studies (Zou et al. 2014*b*; Zhang et al. 2013; Xia et al. 2011). Note that the cyclical speed patterns are also observed at the other four stations. To take this periodic pattern into consideration, a hybrid prediction approach is proposed in the next section.

Fig. 2. Cross-correlation functions of travel speed between station C and other neighboring stations.



3. Methodology

The objective of this study focuses on multi-step ahead traffic speed prediction by considering spatial–temporal correlations and periodic pattern of traffic through time series methods. As pointed out by Karlaftis and Vlahogianni (2011), the comparisons between statistical and neural network models (or other machine-learning models) are 'unfair', particularly because complex neural networks (essentially highly nonlinear models) are compared to simple linear regression or linear time series models. In addition, although machine-learning models may provide higher prediction accuracy than the statistical methods, these black-box methods have the limited inherent explanatory power. Thus, in this section, we describe three statistical methods (i.e., ST, VAR, and ARIMA models) to predict the 5 min freeway traffic speed.

Both ST and VAR models can make use of geographically dispersed speed observations as predictors to obtain short-term prediction. Previously, the ST method was first proposed in the prediction of wind speed (Gneiting et al. 2006; Hering and Genton 2010). Considering wind speed and freeway traffic speed share some common characteristics (i.e., temporal and spatial correlation, periodic pattern, etc.), this approach is used to predict short-term freeway speed in this study. The VAR and ARIMA models have been extensively used in short-term traffic prediction (Williams 2001; Chandra and Al-Deek 2009). In addition, a hybrid prediction approach is proposed to include the trigonometric regression function into the analysis. In this study, station C is selected as the target location because this loop detector experiences significant speed drop during peak hours, and a greater number of model structures can be examined because of the availability of speed data from the upstream and downstream loop detectors. Since there exists strong correlation between the target station and neighboring stations, the speed data observed from neighboring loop detectors for preceding time periods can be used to predict the current speed at the target loop detector. In the following sections, we denote the 5 min average speed at stations A, B, C, D, and E at time (5 min period) t by A_t, B_t, C_t, D_t, and E_t. The considered time period is 6:00 AM–8:00 PM (Monday to Friday, October 2012 to May 2013). The proposed prediction models are used to provide multiple time-step ahead speed prediction at station C, and the desired look-ahead prediction interval ranges from 5 min to 1 h; C_{t+p} is the predicted speed at station C, where p represents the time step (for example, C_{t+1} denotes the 5 min ahead prediction, or one-step ahead prediction; and C_{t+12} denotes the 60 min ahead prediction, or 12-step ahead prediction).

3.1. The space time model

The proposed ST model is a probabilistic modeling approach that can provide the point prediction of future speed and its corresponding prediction intervals. In probabilistic speed prediction, it is necessary to determine the appropriate distributions for speed data. Previously, normal, log-normal, skew-*t* and other forms of distribution have been proposed to fit speed data (Zou et al. 2014*a*). Considering the normal distribution is the most widely used model for describing speed, it is assumed that the speed at time $t + p$ at station C, C_{t+p}, follows a normal distribution, that is, $C_{t+p} \sim N(\mu_{t+p}, \sigma^2_{t+p})$ with location parameter μ_{t+p} and scale parameter σ_{t+p}. Note that the ST model yields the probabilistic prediction of 5-minute average speed. The point prediction of C_{t+p} is the mean, μ_{t+p}, of the normal distribution, and the α quantile (used to determine the prediction interval) is given by

$$z_\alpha = \mu_{t+p} + \sigma_{t+p} \times \Phi^{-1}[\alpha] \tag{1}$$

Fig. 3. Autocorrelation functions of travel speeds at five study stations.

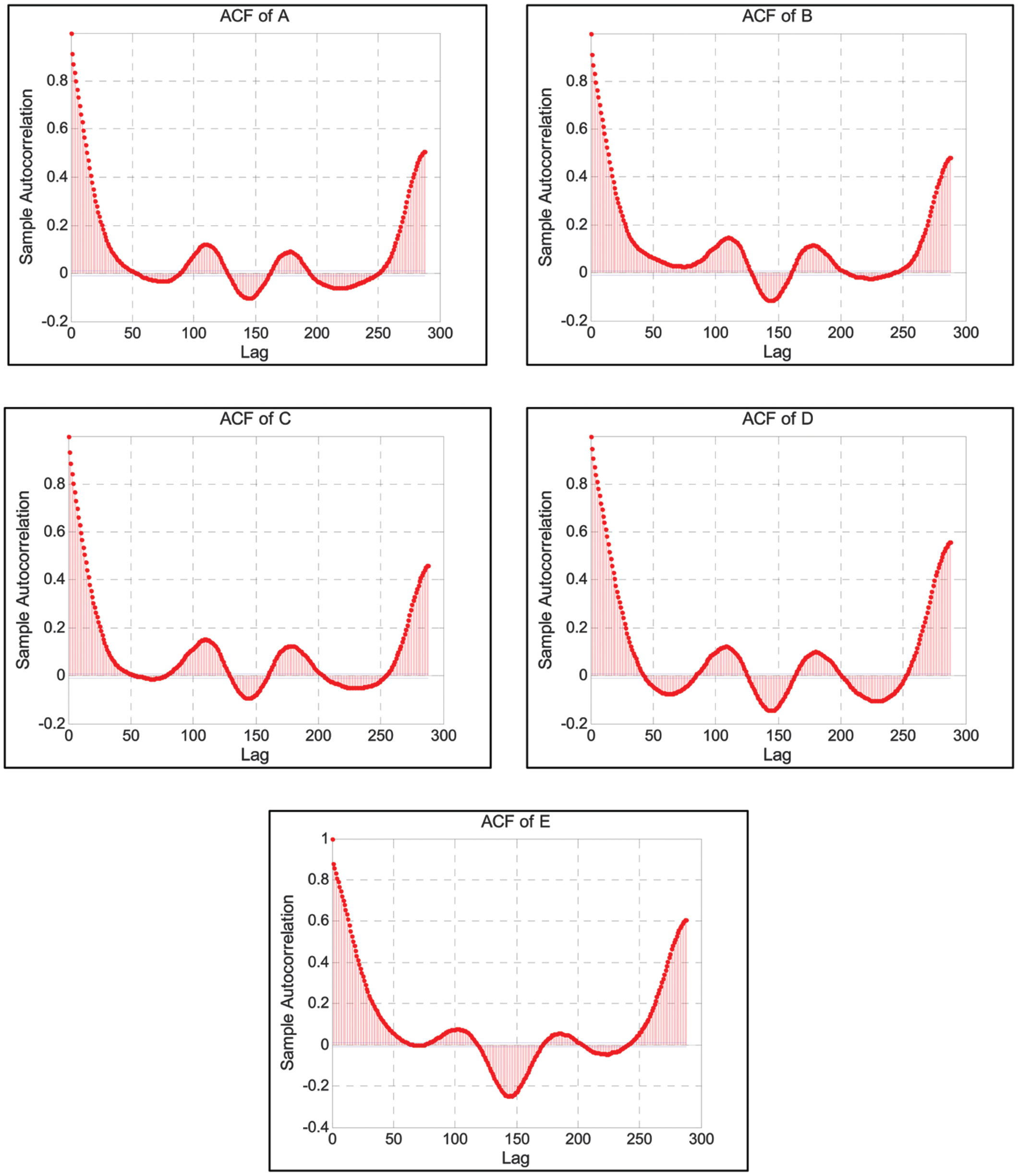
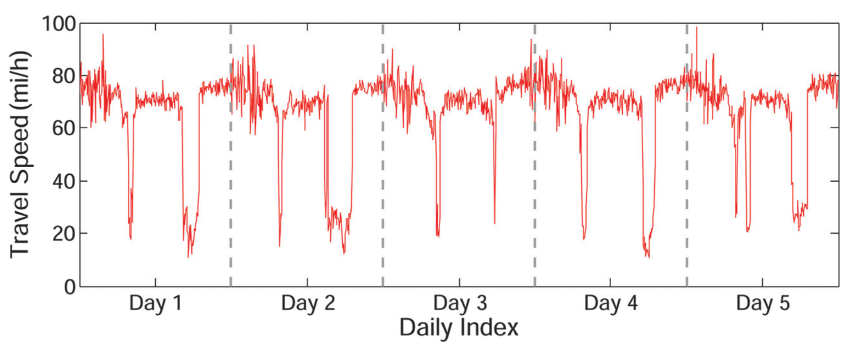


Fig. 4. Cyclical pattern of speed at station C during typical weekdays.



where $p = 1, 2, \ldots, 12$, μ_{t+p} and σ_{t+p} are location parameter and scale parameter of $N(\mu_{t+p}, \sigma^2_{t+p})$, and Φ represents the cumulative density function (cdf) of a standard normal distribution.

Then, μ_{t+p} is fitted by a linear combination of the present and past values of the speed series at all stations. For example, when $p = 1$ (i.e., 5 min ahead prediction),

$$\mu_{t+1} = \alpha_0 + \alpha_1 C_t + \alpha_2 C_{t-1} + \alpha_3 B_t + \alpha_4 D_t + \alpha_5 D_{t-1} + \alpha_6 A_t + \alpha_7 A_{t-1} + \alpha_8 E_{t-1} \tag{2}$$

where A_t, B_t, C_t, D_t, and E_t are the 5-minute average speed at stations A, B, C, D, and E at time t; $\alpha_0, \alpha_1, \ldots, \alpha_8$ are model coefficients.

Predictor variables for μ_{t+p} (for example, for μ_{t+1} in eq. (2) and eq. (14) below) are selected based on an analysis of the speed data from October to December 2012. Different combinations of predictor variables for μ_{t+p} are considered and we started from the simplest model and added predictor variables in a stepwise forward search until no further improvement in the Bayesian information criterion was obtained (see Gneiting et al. 2006 for details about predictor variable selection algorithm).

To model the predictive spread, σ_{t+p}, the ST model allows for conditional heteroscedasticity by modeling σ_{t+p} as a linear function of the volatility value v_t,

$$\sigma_{t+p} = b_0 + b_1 v_t \tag{3}$$

where coefficients b_0 and b_1 are constrained to be nonnegative, and the volatility value, is modeled as

$$v_t = \left(\frac{1}{10}\sum_{i=0}^{1}\left((A_{t-i} - A_{t-i-1})^2 + (B_{t-i} - B_{t-i-1})^2 + (C_{t-i} - C_{t-i-1})^2 + (D_{t-i} - D_{t-i-1})^2 + (E_{t-i} - E_{t-i-1})^2\right)\right)^{1/2} \tag{4}$$

where A_{t-i}, B_{t-i}, C_{t-i}, D_{t-i}, and E_{t-i} are the 5 min average speed at stations A, B, C, D, and E at time $t - i$.

The volatility value can reflect the magnitude of the recent variations in travel speed. A similar formulation of volatility value was first proposed in Gneiting et al. (2006).

3.2. Vector autoregressive models

Vector autoregressive model is a commonly adopted statistical approach for predicting systems of interrelated time series. The VAR model is capable of including the effect of the upstream and downstream stations in predicting the future speed at the target station. In this study, a 5-equation VAR(m) model is applied and its structure can be defined as

$$\mathbf{X}_{t+1} = \boldsymbol{\varphi}_0 + \boldsymbol{\varphi}_1 \mathbf{X}_t + \boldsymbol{\varphi}_2 \mathbf{X}_{t-1} + \cdots + \boldsymbol{\varphi}_m \mathbf{X}_{t-m+1} + \boldsymbol{u}_{t+1} \tag{5}$$

where $\mathbf{X}_{t+1} = (A_{t+1}, B_{t+1}, C_{t+1}, D_{t+1}, E_{t+1})^{\mathrm{T}}$, is the 5×1 vector of variables, $\boldsymbol{\varphi}_0$ is the 5×1 constant term, $\boldsymbol{\varphi}_1$ through $\boldsymbol{\varphi}_m$ are 5×5 coefficient matrices, and $\boldsymbol{u}_{t+1}$ is the corresponding 5×1 independently and identically distributed random vector with $E(\boldsymbol{u}_{t+1}) = 0$ and time invariant positive definite covariance matrix $E(\boldsymbol{u}_{t+1}\boldsymbol{u}^{\mathrm{T}}_{t+1}) = \Sigma_{\mathbf{u}}$.

The stability of the VAR(m) model can be ensured by evaluating the characteristic polynomial

$$\det(I_5 - \boldsymbol{\varphi}_1 z - \cdots - \boldsymbol{\varphi}_m z^m) \neq 0 \text{ for } |z| \leq 1 \tag{6}$$

where I_5 is a 5×5 identity matrix, and $\boldsymbol{\varphi}_1$ through $\boldsymbol{\varphi}_m$ are 5×5 coefficient matrices. The necessary and sufficient condition for stability is that all characteristic roots lie outside the unit circle.

3.3. ARIMA models

In contrast to the VAR and ST models that incorporate information from upstream and downstream locations into the prediction process, the ARIMA model solely relies on the on-site observations. In this paper, we use this model as the benchmark model.

A nonseasonal ARIMA model can be referred to as an ARIMA (p, d, q) model, in which: p is the number of autoregressive terms, d is the number of nonseasonal differences, and q is the number of lagged forecast errors. An ARIMA model is a generalization of autoregressive moving average (ARMA) model. The mathematical representation of an ARMA (p, q) process is as follows:

$$x_t - \varnothing_1 x_{t-1} - \varnothing_2 x_{t-2} - \cdots - \varnothing_p x_{t-p} = \omega_t + \theta_1\omega_{t-1} + \theta_2\omega_{t-2} + \cdots + \theta_q\omega_{t-q} \tag{7}$$

in which $\{x_t\}$ is stationary, ω_t is a normal white noise series with mean zero and variance σ^2_w, $\{\varnothing_1, \ldots, \varnothing_p\}$ and $\{\theta_1, \ldots, \theta_p\}$ are parameters for the autoregressive and the moving average terms, and the polynomials $(1 - \phi_1 z - \cdots - \phi_p z^p)$ and $(1 + \theta_1 z - \cdots - \theta_q z^q)$ have no common factors.

By letting $B^k x_t = x_{t-k}$, $\varnothing(B) = 1 - \varnothing_1 B - \varnothing_2 B^2 - \cdots - \varnothing_p B^p$, and $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_p B^q$, the model can be written in a more concise form

$$\phi(B)X_t = \theta(B)Z_t \tag{8}$$

The ARMA model requires stationary of the data series. As many time series are non-stationary, it is necessary to transform the original data to a stationary series. The ARIMA model is proposed to model the data which does not show evidence of an ARMA model, but proper transformation of the original data can fit an ARMA model. In the ARIMA model, the integrated part with order d, denoted as I(d), means the dth difference of the original data. The mathematical equation of an ARIMA (p, d, q) model is

$$\left(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p\right)(1 - B)^d X_t = \left(1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q\right)Z_t \tag{9}$$

This can also be rewritten in a more concise form

$$\phi(B)(1 - B)^d X_t = \theta(B)Z_t, \{Z_t\} \sim WN(0, \sigma^2), \tag{10}$$

in which $\phi(B)$ and $\theta(B)$ are polynomials of degrees p and q respectively, and $\phi(B) \neq 0$ for $|B| \leq 1$, other notations are the same as explained earlier.

3.4. Hybrid prediction models

Conventional speed forecasting methods normally use the past speed values as the only inputs for predicting the current speed values. However, as demonstrated in Section 2, the observed speed values often have a daily periodic pattern. Thus, one alternative way is to assume the freeway speed consists of two parts. One is the cyclic and deterministic component that represents the periodic trend of speed for the regular work days, and the other is the irregular component. Under this assumption, the following model is used to describe freeway speed:

(11) $$C_t = M_t + C_t^r$$

where C_t is the speed at time t at station C, M_t is the periodic component, C_t^r is the residual part after removing the periodic component.

From Fig. 4, we can see the daily pattern in observed speed values. To model the periodic component in eq. (11), we used a combination of sinusoids and cosinusoids, which is referred to as the trigonometric regression function. This approach can describe regular cyclical patterns or periodic variations and has been used in various time series data analysis (e.g., Adorf 1995). Previously, Gneiting et al. (2006) found that adding trigonometric polynomials in the ST models can improve the prediction accuracy for the wind speed with a diurnal cycle.

Using the observed 5 min average freeway speed values, the daily average 5 min speed at each station is calculated by using $S_t = \frac{1}{30}\sum_{d=1}^{30} s_t^d$, where S_t is daily average 5 min average speed at time t; s_t^d is 5 min average speed at time t on day d; t = 1, 2, …, 288; and d = 1, 2, …, 30 is the number of days. For the daily average 5 min speed, a least squares estimation method is used to determine the parameters in eq. (12). The number of trigonometric polynomials can affect the prediction performance of the hybrid models and the selection of number of trigonometric polynomials is discussed in this paper.

(12) $$M_u = m_0 + m_1 \sin\left(\frac{2\pi u}{288}\right) + m_2 \cos\left(\frac{2\pi u}{288}\right) + m_3 \sin\left(\frac{4\pi u}{288}\right) + m_4 \cos\left(\frac{4\pi u}{288}\right) + \ldots + m_{2n-1} \sin\left(\frac{2n\pi u}{288}\right) + m_{2n} \cos\left(\frac{2n\pi u}{288}\right)$$

where M_u is the estimated periodic component at time u; u = 1, 2, …, 288; n is the number of trigonometric polynomials; $m_0, m_1, \ldots, m_{2n}$ are the coefficients.

Residual series without a periodic component can be obtained after removing the least squares fit from the speed series at all stations. Let A_t^r, B_t^r, C_t^r, D_t^r, and E_t^r denote the 5 min residual series at time t at stations A, B, C, D, and E, respectively. The hybrid ST, VAR and ARIMA models are introduced here.

For the hybrid ST (HST) model, we model the predictive center, μ_{t+p}, as a combination of the periodic component M_{t+p} and residual series μ_{t+p}^r.

(13) $$\mu_{t+p} = M_{t+p} + \mu_{t+p}^r$$

M_{t+p} is the fitted periodic component at target station C and the residual μ_{t+p}^r is fitted by a linear combination of the present and past values of the residual series at all stations. For example, when p = 1 (i.e., 5 min ahead prediction),

(14) $$\mu_{t+1}^r = \alpha_0 + \alpha_1 C_t^r + \alpha_2 C_{t-1}^r + \alpha_3 B_t^r + \alpha_4 D_t^r + \alpha_5 D_{t-1}^r + \alpha_6 A_t^r + \alpha_7 A_{t-1}^r + \alpha_8 E_{t-1}^r$$

where A_t^r, B_t^r, C_t^r, D_t^r and E_t^r are the 5 min residual series at time t at stations A, B, C, D, and E, respectively; $\alpha_0, \alpha_1, \ldots, \alpha_8$ are model coefficients.

For the hybrid VAR (HVAR) model, we model the speed at time $t+1$ at station C, C_{t+1}, as a combination of the periodic component M_{t+1} and residual series C_{t+1}^r.

(15) $$C_{t+1} = M_{t+1} + C_{t+1}^r$$

The residual C_{t+1}^r is fitted by using the present and past values of the residual series at all stations.

Table 1. The MAE, MAPE and RMSE values of ST, VAR and ARIMA models for different forecasting steps ahead.

		Number of forecasting steps ahead			
	Model	1	3	6	12
MAE	ST	**5.13**	**8.25**	**11.04**	**14.61**
	VAR	5.26	8.30	11.44	16.41
	ARIMA	5.22	8.40	11.44	15.74
MAPE (%)	ST	**8.75**	15.46	21.71	30.03
	VAR	8.81	**14.82**	**20.99**	**29.59**
	ARIMA	9.15	16.14	22.93	31.68
RMSE	ST	**8.09**	**13.98**	**17.88**	**21.59**
	VAR	8.46	15.16	20.35	26.97
	ARIMA	8.72	14.85	19.16	24.01

Note: Bold values indicate the smallest MAE, MAPE, and RMSE values.

(16) $$C_{t+1}^r = \varphi_{30} + \varphi_{31,1}A_t^r + \varphi_{32,1}B_t^r + \varphi_{33,1}C_t^r + \varphi_{34,1}D_t^r + \varphi_{35,1}E_t^r + \cdots + \varphi_{31,m}A_{t-m+1}^r + \varphi_{32,m}B_{t-m+1}^r + \varphi_{33,m}C_{t-m+1}^r + \varphi_{34,m}D_{t-m+1}^r + \varphi_{35,m}E_{t-m+1}^r$$

where A_t^r, B_t^r, C_t^r, D_t^r, and E_t^r are the 5 min residual series at time t at stations A, B, C, D, and E, respectively; $\varphi_{30}, \varphi_{31,1}, \ldots, \varphi_{35,m}$ are model coefficients.

For the hybrid ARIMA (HARIMA) model, we model speed, C_{t+1}, as a combination of the periodic component M_{t+1} and residual series C_{t+1}^r.

(17) $$C_{t+1} = M_{t+1} + C_{t+1}^r$$

The residual C_{t+1}^r is fitted by using the present and past values of the residual series at station C.

(18) $$C_{t+1}^r = \frac{\left(1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q\right)Z_t}{\left(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p\right)(1 - B)^d}$$

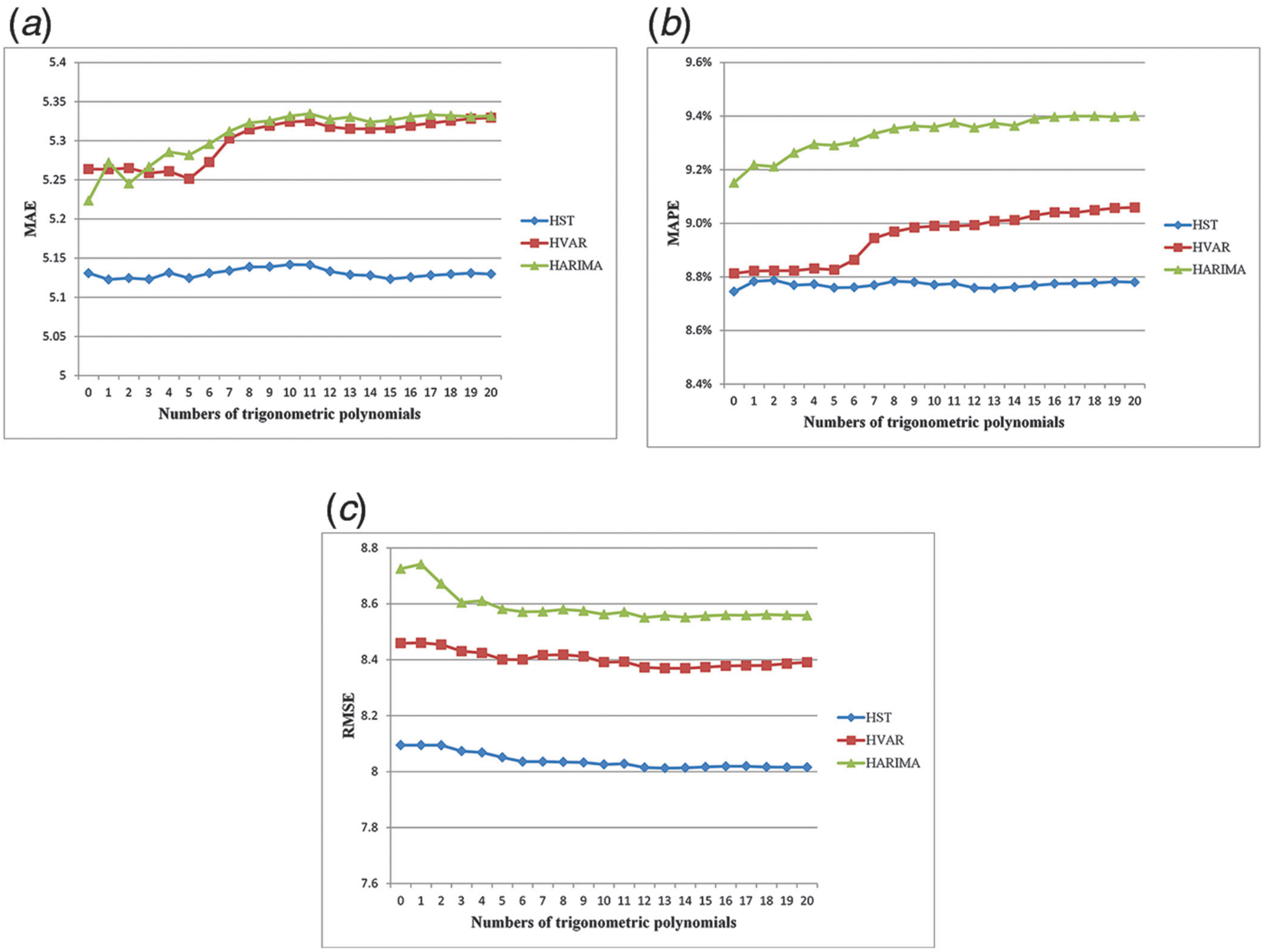
where $\{\varnothing_1, \ldots, \varnothing_p\}$ and $\{\theta_1, \ldots, \theta_p\}$ are parameters for the autoregressive and the moving average terms, $B^k x_t = x_{t-k}$, Z_t is a normal white noise series with mean zero and variance σ_z^2, p is the number of autoregressive terms, d is the number of nonseasonal differences, and q is the number of lagged forecast errors.

To estimate the coefficients in the prediction models, the classical maximum likelihood estimation and the minimum continuous ranked probability score (CRPS) estimation are adopted in this study. For ARIMA and VAR models, the parameters are estimated using the maximum likelihood estimation available in forecast and vars packages in R, respectively. For the ST model, we use minimum CRPS estimation to numerically estimate the parameters. Gneiting et al. (2006) found that the minimum CRPS estimation can give better results than the maximum likelihood estimation when applying the ST model. For more details about the minimum CRPS estimation approach, interested readers can see Gneiting et al. (2006). The parameter estimation for all prediction models (i.e., ST, VAR, ARIMA and hybrid models) was implemented in the Software R.

4. Results and discussion

In this section, the prediction performance of the ST, VAR, ARIMA, and hybrid models is evaluated using the speed data observed at station C. The testing period is 6:00 AM to 8:00 PM from 1 May to 31 May (23 weekdays). We chose the speed data in May 2013 as the testing period, so a large range of training periods can

Fig. 5. One-step ahead forecasting results for HST, HVAR and HARIMA models. Prediction results for ST, VAR, and ARIMA models are the data points where the number of trigonometric polynomials equals 0. (*a*) MAE values for 1-step ahead prediction; (*b*) MAPE values for 1-step ahead prediction; (*c*) RMSE values for 1-step ahead prediction.



be considered. Different training periods were compared in this study and we found that 30 weekdays are appropriate for capturing the periodic pattern of freeway speeds. Thus, a sliding training period that consists of the 30 most recent weekdays prior to the interested prediction time interval is adopted for the ST, VAR, and ARIMA models. For example, the parameters of ST, VAR and ARIMA models for speed prediction on 10 May 2013 were estimated using the 30 most recent weekdays (i.e., May (7 days), April (22 days), and March (1 day)).

When predicting each future speed value, the best order of the ARIMA model is determined by the Akaike information criterion (AIC) values using the most recent 30-weekday speed data. Similarly, we implemented the VAR model using a maximal order of 10. For each prediction step, the best order of the VAR model is also selected based on the AIC values using the differenced speed data. Both ARIMA and VAR models are used to predict several time steps ahead into the future. For example, for 12-step ahead prediction, the ARIMA and VAR models consider the $t + 1$'s prediction as an observed value and use it to predict speed at time $t + 2$. The procedure is repeated up to time $t + 12$.

To evaluate the multi-step prediction performance of ST, VAR, ARIMA, and hybrid models, three performance measures, the mean absolute error (MAE), the mean absolute percentage error (MAPE), and the root mean square error (RMSE) are considered. Note that the unit of the MAE and RMSE is kph. The equations for calculating MAE, MAPE, and RMSE are as follows:

$$\text{MAE} = \frac{\sum_{i=1}^{N} |\overline{v_i} - v_i|}{N} \tag{19}$$

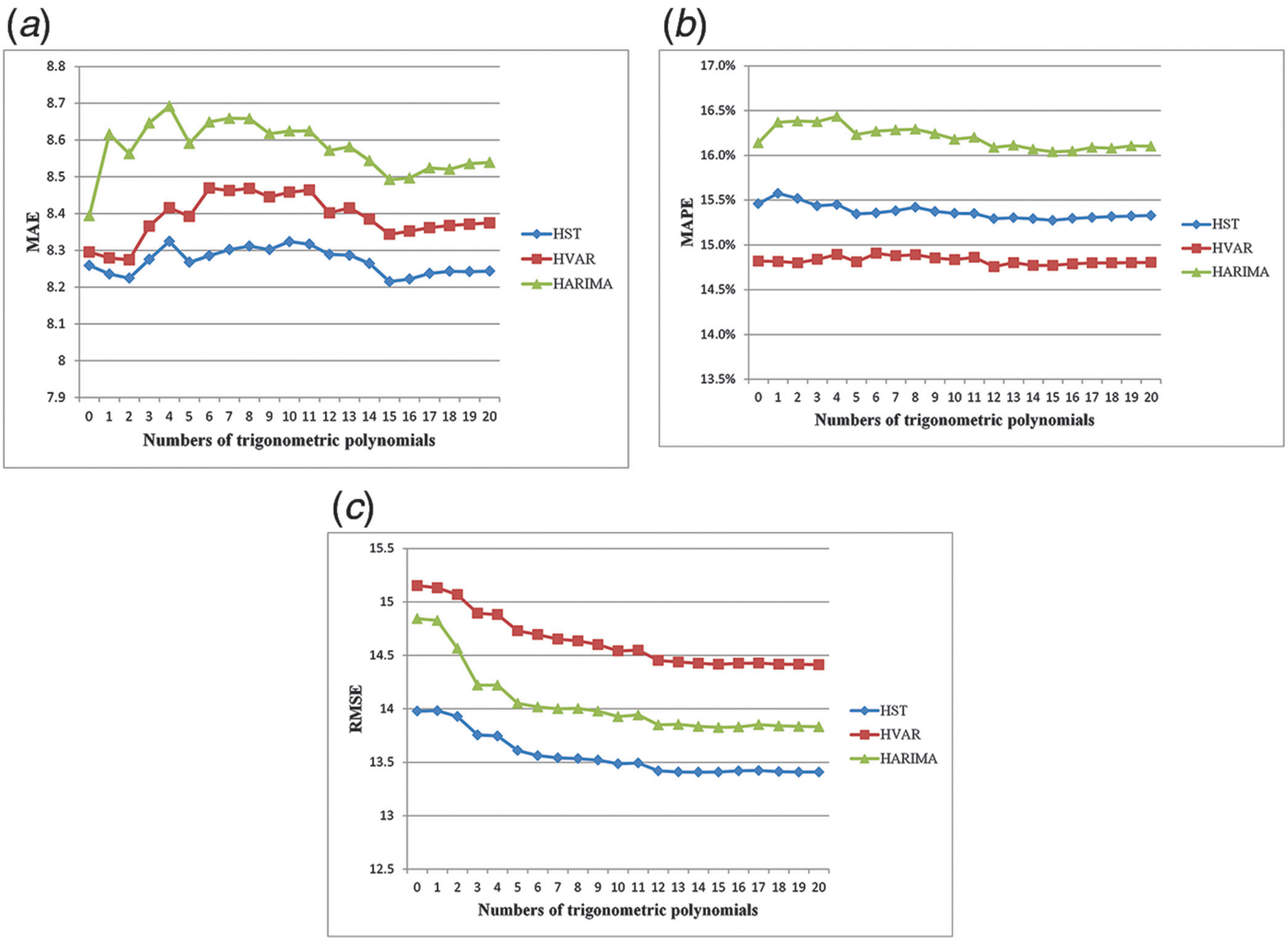
$$\text{MAPE} = \frac{1}{N}\sum_{i=1}^{N}\frac{|\overline{v_i} - v_i|}{v_i} \times 100\% \tag{20}$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(\overline{v_i} - v_i)^2}{N - 1}} \tag{21}$$

where N is the number of observations, v_i is the actual speed at time i at station C, and $\overline{v_i}$ is the predicted speed.

To evaluate the performance of the three models, both one-step and multi-step ahead prediction (i.e., 3-step (15 min), 6-step (30 min), and 12-step (60 min)) are considered. Table 1 provides the MAE, MAPE, and RMSE values of ST, VAR, and ARIMA models for different

Fig. 6. Three-step ahead forecasting results for HST, HVAR and HARIMA models. (*a*) MAE values for 3-step ahead prediction; (*b*) MAPE values for 3-step ahead prediction; (*c*) RMSE values for 3-step ahead prediction.

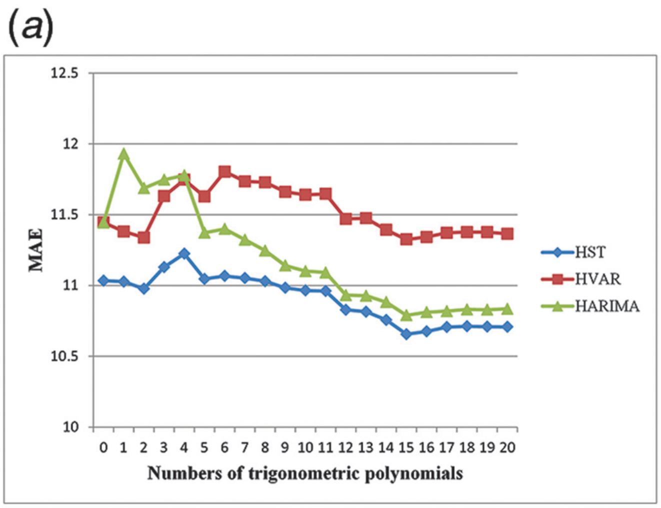


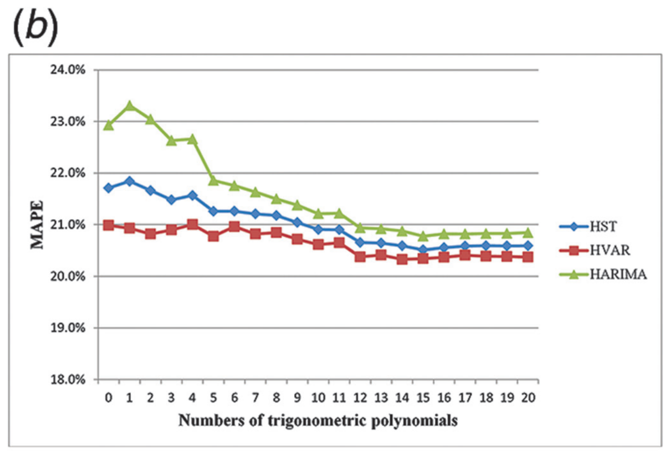
forecasting horizons. Note that in Table 1, bold values indicate the smallest MAE, MAPE, and RMSE values.

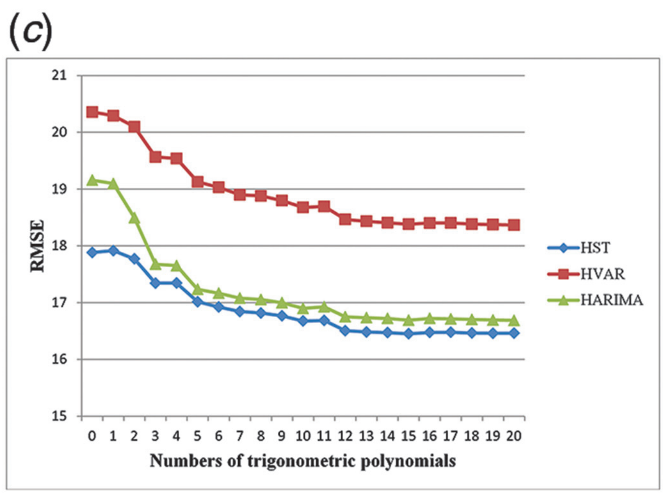
Several interesting findings can be observed from Table 1. First, as expected, the prediction accuracy of freeway speed deteriorates as the prediction time steps increase for all models. The MAE, MAPE, and RMSE values for 12-step ahead forecasting are significantly larger than the results of 1-step ahead. Second, when predicting 5 min ahead into the future, based on the results in Table 1, the performance measures differ slightly among ARIMA, VAR and ST models, and ST and VAR models are slightly preferred over the ARIMA model. Third, for the multi-step ahead forecasting, as the time step increases, the difference in prediction performance among the three models becomes larger and the ST model can consistently provide the lowest MAE and RMSE values. For example, if we look at RMSE values for 1 h ahead prediction, the ST model can improve the prediction results by 20% and 10% when compared with VAR and AMRIA models. It can be observed that the ST model is preferred over the VAR and ARIMA models when predicting multiple time points into the future. The possible reason is that the ST model uses spatial and temporal information from neighboring stations observed at time t to directly predict the future speed value at time $t + p$. On the contrary, the ARIMA and VAR models consider the $t + 1$'s prediction as an observed value and use it to predict speed at time $t + 2$ and this procedure is repeated p times to forecast speed values at time $t + p$. Thus, the prediction error may accumulate after multiple steps when using ARIMA and VAR models.

To further investigate whether the proposed trigonometric regression function can improve the prediction performance, three hybrid models are considered to predict speed values at station C for the same testing period (6:00 AM to 8:00 PM from 1 May to 31 May). The best HARIMA and HVAR models are selected based on the AIC values using the most recent 30-day residual series without the periodic component. Note that ACF plots indicate that the residual speed series are non-stationary and thus first difference is conducted to ensure the stationary for the VAR model. Figures 5–8 show the MAE, MAPE, and RMSE values for three hybrid models from 1-step to 12-step ahead forecasting using different numbers of trigonometric polynomials. For 1-step ahead prediction, Fig. 5 indicates that the consideration of periodic component in the analysis may deteriorate the prediction performance for the hybrid models. For example, MAE and MAPE values suggest that the trigonometric polynomials deteriorate the prediction performance of HVAR and HARIMA models. For 3-step ahead prediction, the performance indexes are not consistent, and RMSE values indicate the hybrid models are slightly preferred. For 6-step and 12-step ahead prediction, based on the prediction results in Figs. 7 and 8, the periodic component can improve the prediction accuracy of speed over multiple time periods for all hybrid models. For example, for 12-step ahead forecasting, the lowest RMSE values for HST, HVAR, and HARIMA models are 11.51, 13.67, and 11.78, respectively; and the RMSE values for ST, VAR and ARIMA models are 13.42, 16.76 and 14.92. Thus, the forecasting accuracy can be improved by 14%, 18% and 21% when considering the periodic component in the

Fig. 7. Six-step ahead forecasting results for HST, HVAR and HARIMA models. (*a*) MAE values for 6-step ahead prediction; (*b*) MAPE values for 6-step ahead prediction; (*c*) RMSE values for 6-step ahead prediction.







prediction models. The improvement in forecasting demonstrates that the hybrid models can better capture the long-term traffic speed trend than the ST, VAR, and ARIMA models. This is because the considered trigonometric polynomials are capable of describing and capturing the observed cyclic pattern in speed data and thus improve the prediction accuracy.

The number of trigonometric polynomials can also influence the prediction results of the hybrid models. As demonstrated in Figs. 5–8, the performance of hybrid models with different number of trigonometric polynomials is not consistent. For 1-step and 3-step ahead forecasting, the ST, VAR, and ARIMA models can perform well and the periodical component is unnecessary for these scenarios. However, for 6-step and 12-step ahead forecasting, the MAE, MAPE, and RMSE values for three hybrid models have shown a strong declining trend when the number of trigonometric polynomials increases. This downward trend stabilizes after n (the number of trigonometric polynomials) reaches 15. It is also observed that the hybrid models with $n \leq 5$ do not show significant advantages over the ST, VAR, and ARIMA models. Thus, when implementing the proposed trigonometric regression function for multi-step (i.e., forecasting horizon ≥ 30 min) ahead freeway speed prediction, the selection of number of trigonometric polynomials for the periodical component should depend on the cyclical behaviors of the data. Note that we compared the computation time for different numbers of trigonometric polynomials, and it is found that the number of trigonometric polynomials has little impact on the computational time. For the speed data used in this study, it is found that 15 or more trigonometric polynomials should be included in the periodical component. Among the three hybrid models, the HARIMA model shows the most significant prediction improvement caused by the periodical component. In summary, the ST, VAR, and ARIMA models can perform well in 1-step and 3-step ahead forecasting and it is not necessary to consider the periodic component. However, the hybrid models demonstrate their advantages when the forecasting horizon is greater than 30 min. This is because the trigonometric polynomials successfully capture the long-term periodic trend observed in the speed data. Compared with HARIMA and HVAR models, the HST model can consistently provide the lowest MAE and RMSE values for all prediction steps.

In addition, since hybrid models are preferred based on the prediction results for multi-step ahead prediction, we investigated the effect of including different upstream and downstream stations on the prediction results using HST and HVAR models. Tables 2 and 3 provide the 1-step to 12-step ahead prediction results with different input stations. As indicated in Tables 2 and 3, the omission of one or two adjacent stations generally has slight effect on the performance indexes. For the HST model in Table 3, if the information from the station D is excluded, this can result in minor prediction performance deterioration.

Fig. 8. Twelve-step ahead forecasting results for HST, HVAR and HARIMA. (*a*) MAE values for 12-step ahead prediction; (*b*) MAPE values for 12-step ahead prediction; (*c*) RMSE values for 12-step ahead prediction.

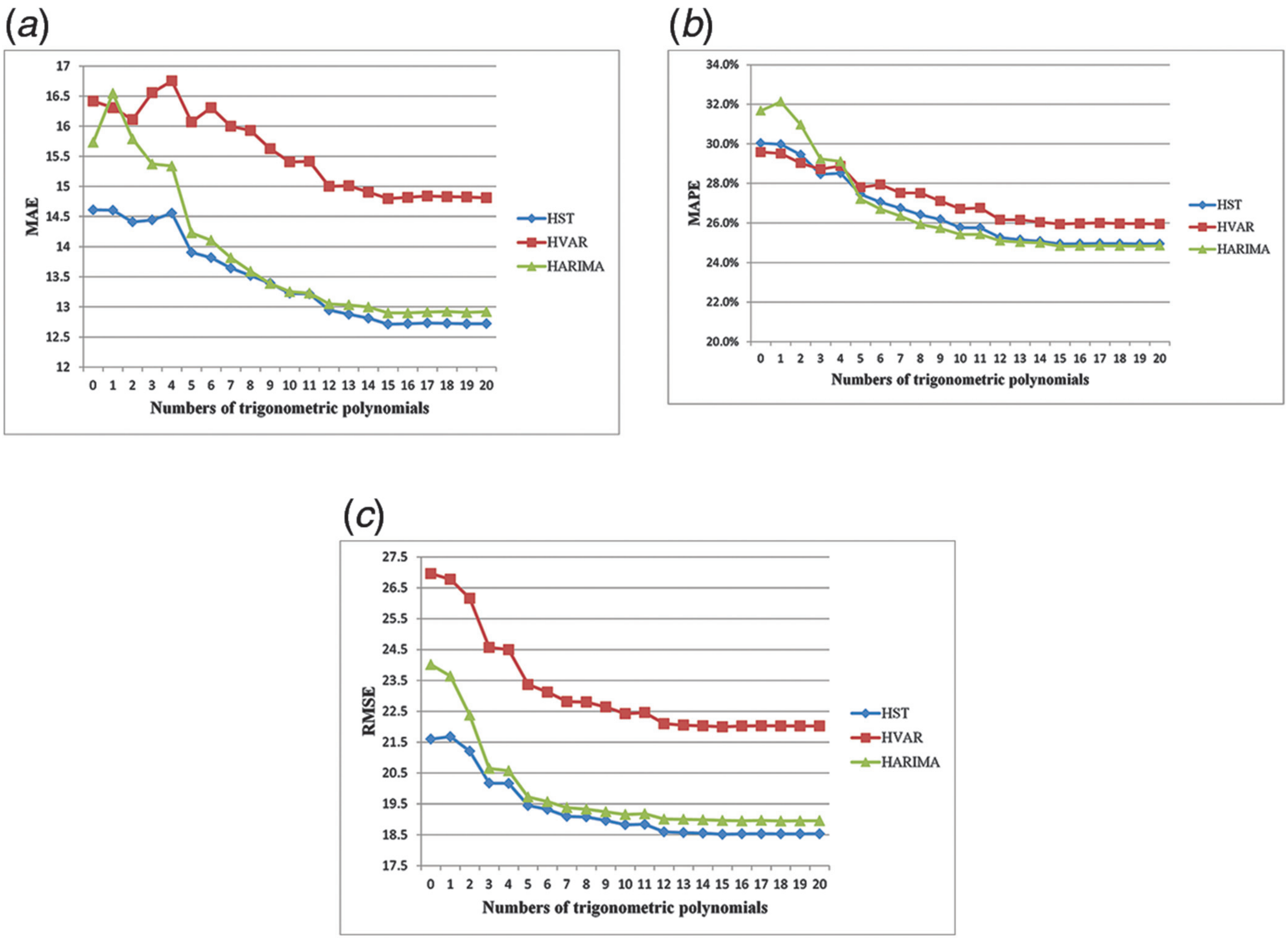


Table 2. Effect of different upstream and downstream stations on the multi-step ahead prediction performance for the HVAR model.

HVAR*	1-step			3-step			6-step			12-step		
Input stations	MAE	MAPE (%)	RMSE	MAE	MAPE (%)	RMSE	MAE	MAPE (%)	RMSE	MAE	MAPE (%)	RMSE
A, B, C, D, E	5.32	9.03	8.37	8.34	14.77	14.42	11.33	20.35	18.38	14.80	25.95	22.00
A, B, C, D	5.32	9.04	8.38	8.34	14.77	14.41	11.32	20.35	18.38	14.80	25.95	22.00
B, C, D, E	5.32	9.04	8.39	8.35	14.78	14.43	11.32	20.33	18.38	14.80	25.95	22.00
A, C, D, E	5.31	9.02	8.37	8.34	14.78	14.41	11.33	20.34	18.38	14.80	25.94	22.00
A, B, C, E	5.33	9.13	8.59	8.33	14.74	14.39	11.32	20.33	18.37	14.81	25.97	22.02
B, C, D	5.33	9.05	8.41	8.34	14.77	14.43	11.32	20.33	18.38	14.80	25.96	22.01
A, C, E	5.34	9.13	8.61	8.34	14.77	14.41	11.34	20.35	18.41	14.81	25.97	22.02
A, B, C	5.33	9.13	8.60	8.33	14.75	14.39	11.32	20.33	18.38	14.81	25.97	22.02
C, D, E	5.32	9.03	8.40	8.34	14.78	14.42	11.32	20.33	18.38	14.80	25.96	22.00

*The number of trigonometric polynomials is set to 15 for all scenarios.

5. Conclusions

This paper evaluated the multi-step prediction performance of ST, VAR, and ARIMA models using the freeway speed data collected from five loop detectors located on I-394 in Twin Cities Metro area. To further consider the cyclical characteristics of freeway speed data, hybrid prediction approaches were proposed to decompose speed into two different components: a periodic trend and a residual part. A trigonometric regression function is used to capture the periodic component and the residual part is modeled by the ST, VAR, and ARIMA models. Comparisons among the ST, VAR, ARIMA and hybrid models demonstrated that modeling the periodicity and the residual part separately can provide promising multi-step forecasting results. The proposed hybrid models can better interpret the underlining structure of the speed data.

Based on the results from this study, several interesting conclusions can be drawn. First, for multi-step freeway speed prediction, as time step increases, the ST model demonstrates advantages over VAR and ARIMA models because its model structure can effectively incorporate the spatial and temporal information from neighboring stations to directly obtain future speed value at time $t + p$. Second, freeway speed data often demonstrate diurnal periodicity with peak-period traffic congestion caused by the morning and afternoon commute. For 1-step and 3-step ahead forecasting, ST, VAR and ARIMA models can perform well and it is unnecessary

Table 3. Effect of different upstream and downstream stations on the multi-step ahead prediction performance for the HST model.

HST*	1-step			3-step			6-step			12-step		
Input stations	MAE	MAPE (%)	RMSE	MAE	MAPE (%)	RMSE	MAE	MAPE (%)	RMSE	MAE	MAPE (%)	RMSE
A, B, C, D, E	5.12	8.77	8.02	8.22	15.27	13.41	10.66	20.51	16.45	12.71	24.95	18.52
A, B, C, D	5.13	8.78	8.02	8.22	15.27	13.40	10.66	20.51	16.46	12.68	24.87	18.53
B, C, D, E	5.14	8.78	8.06	8.21	15.25	13.46	10.65	20.50	16.47	12.72	24.97	18.53
A, C, D, E	5.12	8.77	8.01	8.21	15.24	13.41	10.66	20.48	16.45	12.72	24.95	18.53
A, B, C, E	5.17	8.91	8.36	8.21	15.30	13.72	10.70	20.65	16.68	12.84	25.27	18.79
B, C, D	5.15	8.79	8.06	8.21	15.26	13.46	10.63	20.47	16.45	12.66	24.85	18.51
A, C, E	5.17	8.93	8.36	8.21	15.28	13.72	10.71	20.63	16.68	12.85	25.26	18.79
A, B, C	5.16	8.89	8.39	8.17	15.23	13.79	10.66	20.58	16.77	12.75	25.18	19.03
C, D, E	5.14	8.75	8.08	8.21	15.18	13.52	10.66	20.47	16.52	12.76	24.99	18.56

*The number of trigonometric polynomials is set to 15 for all scenarios.

to take the periodic trend into consideration. However, when the forecasting horizon is greater than 30 min, the proposed hybrid prediction models can accommodate the periodic trends and provide more accurate prediction results. Third, among the three hybrid prediction models, the HST model consistently outperforms the HVAR and HARIMA models based on the MAE and RMSE values for all prediction steps. Fourth, when implementing the proposed hybrid modeling approach, the transportation researchers are recommended to select the number of trigonometric polynomials for the periodical component based on the cyclical behaviors of the data. For future research, since non-recurrent events (e.g., incidents, special events, etc.) may disturb the cyclical pattern of speed, it is useful to develop a regime-switching space time prediction model to take into account the alternating traffic regimes. In addition, it is also interesting to compare the prediction performance of the proposed hybrid models against machine-learning techniques.

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