Lecture1 Discrete Math

Theorem 1 Rational Root Theorem:

If p(x) is a monic polynomial with integer coefficients, then the roots of p(x) are either integers or irrational numbers.

Theorem2: If n is an integer, then nis either an integer or an irrational number

Lecture2 Discrete Math-- Propositional Logic

Propositions:

A Proposition is a statement that is either True (T) or False (F), but not both

T and F are the **truth values** of a Proposition.

Propositions must be **declarative** (must declare a fact) and must have a truth value.

Propositional Logic aka **Propositional Calculus**:

Negation (NOT):

Definition: Let p be a proposition. The negation of p denoted $\neg p$ or $\bar{p}$ is the statement "not p".

Conjunction (AND):

Definition: Let p and q be propositions. The conjunction of p and q denoted by $p \wedge q$ is the proposition "p and q".

The conjunction of two propositions is true if and only if both propositions are true and is false otherwise.

Disjunction (OR):

Definition: Let p and q be propositions. The disjunction of p and q denoted by $p \vee q$, is the proposition "p or q".

The disjunction of two propositions is false if and only if both propositions are false and is true otherwise.

Exclusive Or (XOR):

Definition: Let p and q be propositions. The 'exclusive or' of p and q denoted by $p \oplus q$ is the proposition "p xor q", that is true when exactly one of p and q is true and is false otherwise.

Implication (→):

Definition: Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition "if p, then q". The implication $p \rightarrow q$ is false when p is true and q is false, and true otherwise.

If (p → q), then:

p is sufficient for q

q is necessary for p

Converse 逆命题, Contrapositive 逆否命题 and Inverse 否命题：

Definition: The proposition $q \rightarrow p$ is called the converse of $p \rightarrow q$.

The proposition $\neg q \rightarrow \neg p$ is called the contrapositive of $p \rightarrow q$.

The proposition $\neg p \rightarrow \neg q$ is called the inverse of $p \rightarrow q$.

命题类型	形式	逻辑关系
原命题	$P \rightarrow Q$	原命题的真值决定逆否命题的真值。
逆命题	$Q \rightarrow P$	与原命题的真值无关。
否命题	$\neg P \rightarrow \neg Q$	与逆命题等价。
逆否命题	$\neg Q \rightarrow \neg P$	与原命题等价。

Biconditionals (↔)

Definition: Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q".

$p \leftrightarrow q$ is true when p and q have the same truth values and is false otherwise.

Precedence of Logical Operators：

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Methods of Propositional Logic:

Truth Tables for Compound Propositions

Logical Equivalences:

Tautology 重言式:

Definition: A compound proposition that is always true, irrespective of the truth values of the propositional variables in it, is called a Tautology.

Contradiction 矛盾:

Definition: A compound proposition that is always false, irrespective of the truth values of the propositional variables in it, is called a Contradiction.

Logically equivalent:

Definition: The compound propositions p and q are called logically equivalent (denoted by $p \equiv q$) if $p \leftrightarrow q$ is a tautology.

And sometimes we also use ⇔ in place of ≡.

p ≡ q is a statement, not a compound proposition

Identity laws, Domination laws, Idempotent laws, Negation laws:

Identity	Domination	Idempotent	Negation
$p \wedge T \equiv p$	$p \vee T \equiv T$	$p \vee p \equiv p$	$p \vee \neg p \equiv T$
$p \vee F \equiv p$	$p \wedge F \equiv F$	$p \wedge p \equiv p$	$p \wedge \neg p \equiv F$

Commutativity Rules:

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity Rules:

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Double Negation, Bi-Implication and Contrapositive rules:

$\neg(\neg p) \equiv p$

$(p \leftrightarrow q) \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ Also: $(p \leftrightarrow q) \equiv \neg p \leftrightarrow \neg q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$

Conditional-Disjunction Equivalence (aka Implication rule):

$(p \to q) \equiv (\neg p \vee q)$ Observe that $\neg p \vee q$ is false only when p is true and q is false.

$\neg(p \to q) \equiv (p \wedge \neg q)$

Distributive rule of Disjunction over Conjunction. Distributive rule of Conjunction over Disjunction:

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Also: $p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$

Also: $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

De Morgan's Laws:

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

In general: $\neg\left(\wedge_{j=1}^{n} p_j \right) \equiv \vee_{j=1}^{n} \neg p_j$

In general: $\neg\left(\vee_{j=1}^{n} p_j \right) \equiv \wedge_{j=1}^{n} \neg p_j$

Satisfiability:

Definition: A compound proposition is satisfiable if there is an assignment of truth values to its variables that makes it true.

Definition: An assignment of truth values that makes a compound proposition true is called a solution of the satisfiability problem.

We also can use Boolean Arithmetic to computing Truth Values

We set 0-False, 1-True and ⊕- addition modulo 2:

$\neg P: \quad 1 \oplus P$

$P \wedge Q: \quad P \oplus Q \oplus PQ$

$P \vee Q: \quad PQ$

$P \to Q: \quad (1 \oplus P)Q$

$P \leftrightarrow Q: \quad P \oplus Q$ (因为布尔代数中 XOR 是完备的)