COMP3366 cheatsheet.

1. Hermitian matrix: $H\in\mathbb{C}^d$, iff $H^\dagger=H$

2. Unitary matrix: $H\in\mathbb{C}^d$, iff $H^\dagger H=I$

3. $|a\rangle \to$ "ket", $\langle a| = |a\rangle^\dagger$ "bra"

4. $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

5. $\langle\psi|\phi\rangle^* = \langle\phi|\psi\rangle$;

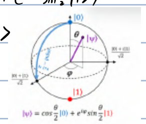
6. Arbitrary Qubit State, $|\psi\rangle = \begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha|0\rangle+\beta|1\rangle$

$\alpha,\beta\in\mathbb{C}$, $|\alpha|^2+|\beta|^2=1$, 被称作: 振幅 amplitude.

7. Bloch Sphere $|\psi'\rangle$

$|\psi\rangle=\alpha|0\rangle+\beta|1\rangle = e^{i\gamma}(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle)$

i.e. $|\psi'\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$



8. 常用的basis.

① Computation basis: → Z-basis $\{|0\rangle,|1\rangle\} \xrightarrow{d维} \{|j\rangle\}_{j=0}^{d-1}$ i.e. 到门转换

② Hadamard basis / X-basis: $\{|+\rangle, |-\rangle\}$ ($H|i\rangle$ 为 $\frac{|0\rangle+(-1)^i|1\rangle}{\sqrt2}$) $\xrightarrow{d维} \{H^{\otimes d}|j\rangle\}_{j=0}^{d-1}$ (e.g. $H^{\otimes 2}|01\rangle = |+-\rangle$)

③ Circular/Y-basis: $\{|i+\rangle, |i-\rangle\}$, 复数同

④ Bell Basis: $|\Phi^\pm\rangle = \frac{|00\rangle\pm|11\rangle}{\sqrt2}$, $|\Psi^\pm\rangle = \frac{|01\rangle\pm|10\rangle}{\sqrt2}$

9. 特殊地, 用Hermitian Matrix H 表示可观测量. H 的 eigenvalue $h_i \to \mathbb{R}$, 测得结果.

H 的谱形式: $H=\sum_i h_i|\varphi_i\rangle\langle\varphi_i|$ → ONB

(Spectral Decomposition) (Orthogonal Normalized complete Basis)

10. Born's rule

$|\psi\rangle$ 得到 $|\varphi_i\rangle$ 的概率 $P(\varphi_i) = |\langle\varphi_i|\psi\rangle|^2$.

Partial Measurement:

考虑两体系统 AB: $|\psi\rangle = |\psi_A\rangle|\psi_B\rangle$, $|\psi_A\rangle$ 的 ONB $\{|a_i\rangle_A\}$

$P_i = |a_i\rangle\langle a_i|_A \otimes I_B$.

则 $P(C_i) = \langle\psi_{AB}|(|a_i\rangle\langle a_i|\otimes I_B)|\psi_{AB}\rangle$

$= |\langle\psi_A|a_i\rangle|^2$

State 变为: $\frac{1}{\sqrt{P(C_i)}}|a_i\rangle|\psi_B\rangle$

11. Quantum Gate → Unitary Matrix

① $UU^\dagger=I$ ② $U^{-1}=U^\dagger$ ③ $UV\neq VU$

12. 定义一个Z门 ① 矩阵形式 ② 对basis的变换

13. 量子门 ⟺ Bloch Sphere 中对$|\psi\rangle$的旋转

14. 常见的量子门: $|0\rangle, |1\rangle$ 变为:

① identity $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $(|0\rangle,|1\rangle)$ 也就是NOT门

② Pauli, $X=\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $(|1\rangle,|0\rangle) = iR_x(\pi)$

$Y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}$, $(i|1\rangle,-i|0\rangle) = iR_y(\pi)$

$Z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $(|0\rangle,-|1\rangle) = iR_z(\pi)$

$XY=iZ=-YX$, $YZ=iX$, $ZX=iY$ $H=\frac{X+Z}{\sqrt2}$

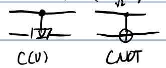
⑤ Hadamard $H=\frac{1}{\sqrt2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$, $(|+\rangle,|-\rangle)$ → 绕x+z轴转180°

⑥ 相位门

⑤ $\frac{\pi}{2}$-gate $S=\begin{pmatrix}1&0\\0&i\end{pmatrix}$. $(|0\rangle, i|1\rangle)$, $S^2=Z$

⑥ $\frac{\pi}{4}$-gate $T=\begin{pmatrix}1&0\\0&e^{i\pi/4}\end{pmatrix}=\begin{pmatrix}1&0\\0&\frac{1+i}{\sqrt2}\end{pmatrix}$ $(|0\rangle, e^{i\pi/4}|1\rangle)$

⑦ C(U)-gate



C(U) CNOT $= (|0\rangle\langle0|\otimes I + |1\rangle\langle1|\otimes U)$

15. $u=\begin{pmatrix}u_1\\\vdots\\u_n\end{pmatrix}$, $u\otimes v = \begin{pmatrix}u_1v\\\vdots\\u_nv\end{pmatrix}$

① $c(|\psi\rangle\otimes|\phi\rangle) = (c|\psi\rangle)\otimes|\phi\rangle = |\psi\rangle\otimes(c|\phi\rangle)$

② $(|\psi\rangle+|\psi'\rangle)\otimes|\phi\rangle = |\psi\rangle\otimes|\phi\rangle + |\psi'\rangle\otimes|\phi\rangle$

③ $(\langle\psi|\otimes\langle\psi'|)(|\phi\rangle\otimes|\phi'\rangle) = \langle\psi|\phi\rangle\langle\psi'|\phi'\rangle$

④ $(A\otimes B)(|\psi\rangle\otimes|\phi\rangle) = A|\psi\rangle\otimes B|\phi\rangle$

⑤ $(A\otimes B)^\dagger = A^\dagger\otimes B^\dagger$, 对*, T, A^T 同理.

16. Product State: $|\psi\rangle$ 可被分解为 $|\psi_1\rangle\otimes|\psi_2\rangle$, 如 $|00\rangle$

Entangled State: $|\psi\rangle$ 不可 ..., 如 $|\beta_0\rangle = \frac{1}{\sqrt2}(|00\rangle+|11\rangle)$

$|\beta_0\rangle \Leftrightarrow |\Phi^+\rangle$, $|\beta_1\rangle = |\Phi^-\rangle = X|\beta_0\rangle$, $|\beta_2\rangle = |\Psi^+\rangle = Y|\beta_0\rangle$ ...

Product/Entangled gate 定义类似.

17. SWAP $\geq CNOT_{1,2}\,CNOT_{2,1}\,CNOT_{1,2}$

18. Universal → 用一组 gates 近似 表示任意 gate.

19. ① Fidelity, $F(|\alpha\rangle,|\phi\rangle) = |\langle\psi|\phi\rangle| \to P(|\psi\rangle \text{ in } |\phi\rangle)$

② Bures Distance, $d(|\psi\rangle,|\phi\rangle) = \min_\gamma \| |\psi\rangle - e^{i\gamma}|\phi\rangle \|$

$= \sqrt{2(1-\sqrt F)}$

③ Gate Dist, $d(U,V) = \max_{|\psi\rangle} d(U|\psi\rangle, V|\psi\rangle)$

$\max_{|\psi\rangle}\min_\gamma \|(V-e^{i\gamma}U)|\psi\rangle\|$

U and V is ε-closed ⟺ $d(U,V)\leq\varepsilon$

20. 重要事实: ① $\forall\, \vec u \nparallel \vec v$, ∀ rotation can be composed by rotation around $\vec u$ and $\vec v$ at ∀ precision. "欧拉角"

i.e. ∀ rotation ⟺ $R_z(\gamma)R_y(\beta)R_z(\alpha)$, (α,β,γ) 为欧拉角

结论: 1-qubit Gate 可被 Z-Y-Z Euler Angle Parameterization

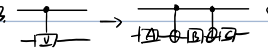
② $\forall\,\theta$, 无 $\varepsilon>0$, it's multiple can approach by ∀ precision

21. 我们令 $\begin{cases} U=THTH \to 绕 (\cos\frac{\pi}{8},\sin\frac{\pi}{8},\cos\frac{\pi}{8}) 转 2\arccos(\cos^2\frac{\pi}{8}) \\ V=HTHT \to 绕 (\cos\frac{\pi}{8},-\sin\frac{\pi}{8},\cos\frac{\pi}{8}) 转 2\arccos(\cos^2\frac{\pi}{8}) \end{cases}$

i.e. $\{T,H\}$ 为 1-qubit Universality Set.

22. Vatan & Williams: ∀ 2-qubit gate $\xrightarrow{分解为}$ 3CNOT + 单比特门]

23.



$\begin{cases} e^{i\alpha}AXBXC = U \\ ABC = I \end{cases}$

若 $U\to(\alpha,\beta,r) \Rightarrow A=R_z(\beta)R_y(\frac{\gamma}{2})$ $B=R_y(-\frac{\gamma}{2})R_z(-\frac{\alpha+\beta}{2})$

$C=R_z(\frac{\alpha-\beta}{2})$ i.e. ∀ C(U) → 2CNOT + 3单比特门]

24. Solovay-Kitaev Algorithm:

∀ gate U, 以精度ε approach U 所需要的门数不超 $\sim(\log(\frac{1}{\varepsilon}))^{3.97}$

理论上, 最优实现下限不低于 $\log(\frac{1}{\varepsilon})$

25. ① quantum circuit complexity: 使用universal set 中 gate 数量

② Query complexity: Query 次数.

26. Query: 考虑 $f:\{0,1,\cdots,M-1\}\to\{0,1,\cdots,N-1\}$

$Q_f: |x\rangle|q\rangle \to |x\rangle|q+f(x) \pmod N\rangle$

27. phase kick-back: qubit 间相位差可测, $|x\rangle(-|-\rangle) \Leftrightarrow -|x\rangle|-\rangle$

28. Q-classical fairness: 若 $f: x\mapsto f(x)$ 可被经典电路实现, QC 相似复杂度.

29. Complexity Class: → 有相同复杂度范围的集合.

Efficiency → 不超过 多项式

P → Efficiency solve; NP → Efficiency verify

NP-complete → NP中最难的 NP-hard: 不希望 NP已

Bounded-error Q-Polynomial: 误差有界, 可被 QC efficiency solve

i.e. P for QC, ≈ feasible via QC

Q-Merlin-Arthur ≈ NP for QC

30. Schrodinger Picture: 随时间追踪 state 变化, 指数级别

31. Pauli Group: 形如 $\otimes_{k=1}^n c_kA_k$, $c_k\in\{\pm1,\pm i\}$, $A_k\in\{I,X,Y,Z\}$

Prop: ① 乘法封闭 ② P_n 有 4^{n+1} 种; ③ $P^2=\pm I$, 西 ④ $[P_n,P_n']$ or $\{P_n,P_n'\}=0$

32. Clifford Group: $C_n = \{C \mid \forall P\in P_n,\ C^\dagger PC\in P_n\}$

33. 对满足 ① 两两对易 ② 线性无关 ③ 取+1 eigenvalue $\}$ n个 Pauli operation,

其+1 eigenstate 是唯一确定的 ($|0\rangle^{\otimes n} \to \{Z_k\}_{k=1}^n$ 的 joint +1 eigenstate)

34. Heisenberg Picture: 我们追踪一组 operator set $\{U_k\}_{k=1}^n$,

此时 $V|\psi\rangle$ 是 $\{V^\dagger U_kV\}_{k=1}^n$ 的 +1 eigenstate.

35. Fourier Transform: Time Domain $\longrightarrow$ Frequency Domain

函数 → 正/余弦波叠加

① CFT: $X(f) = \int_{-\infty}^{\infty} x(t)\cdot e^{-2\pi fti}\,dt$

$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{2\pi fti}\,df$

② DFT: 对 $\{x_0,\cdots x_{n-1}\}$, $X_k = \sum_{j=0}^{n-1} x_j e^{-\frac{2\pi kj}{n}i}$, $k=0,\cdots,n-1$

36. QFT. 在 d维下, 我们定义 $(w=e^{\frac{2\pi i}{d}})$

$|e_k\rangle = \frac{1}{\sqrt d}\sum_{j=0}^{d-1} e^{\frac{2\pi kj}{d}}|j\rangle \xrightarrow{简化} \frac{1}{\sqrt d}\sum_{j=0}^{d-1} w^{kj}|j\rangle$

QFT $\sum_{j=0}^{d-1}|e_j\rangle\langle j|$ → 对 computational basis 变换.

$|e_k\rangle = \otimes_{l=1}^{d}\left(\frac{|0\rangle+e^{2\pi i(\ )}|1\rangle}{\sqrt2}\right)$ $(0.k_{d-l+1}k_{d-l+2}\cdots k_d)_2$ → k的binary

37. QFT complexity: QFT → $O(n^2)$; QHT → $O(n)$

DFT → $O(4^n)$, 快速DFT → $O(n\cdot2^n)$

38. Implement QFT.

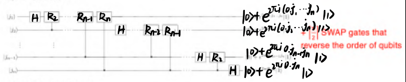
令 $R_k = |0\rangle\langle0| + e^{2\pi i\cdot2^{-k}}|1\rangle\langle1|$, 有 $R_k|+\rangle = \frac{1}{2}(|0\rangle+e^{2\pi i\cdot 2^{-k}}|1\rangle)$

i.e. $R_1=Z$, $R_2=S$, $R_3=T$, $R_n=R_Z(\frac{\pi}{2^n})$

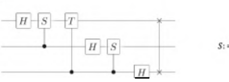
step 1. 用 $H^{\otimes n}$ step 2. 对 $k=2\cdots n-j+1$, $C(R_k)_{j+k-1,j}$

step3. 对整个 register 进行 相位反转 (SWAP i 与 $n-i+1$)

• QFT circuit =



• QFT for 3 qubits:



39. Q-Phase Estimation

∀ Unitary Matrix 可被对角化为: $U=\sum_l e^{2\pi i\varphi_l}|u_l\rangle\langle u_l|$ → ONB

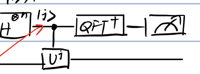
QPE: 对 U 及其本征态 $|\psi\rangle$, $U|\psi\rangle = e^{2\pi i\varphi}|\psi\rangle$, 可估 $\varphi\in[0,1]$

40. Kitaev's QPE: input: $C(U), |\psi\rangle$ output: $\tilde\varphi\approx\varphi$.

$|\tilde\varphi-\varphi|<2^{-n}$ 且失败概率 $\leq\varepsilon$, 使用 $t=n+\lceil\log_2(2+\frac{1}{2\varepsilon})\rceil$ 个 qubit

注意到 $C(U)|+\rangle|u\rangle = \frac{1}{\sqrt2}(|0\rangle+e^{2\pi i\varphi}|1\rangle)|u\rangle$ → 类似 QFT 形式

41. QPE circuit



$|+\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}}\sum_{j=0}^{2^n-1}|j\rangle$

42. $P(j) = \left|\langle e_j|(QFT^\dagger)\sum_{k=0}^{2^n-1}\frac{e^{2\pi i\varphi k}}{\sqrt{2^n}}|k\rangle\right|^2$, let $\delta\varphi_j = \varphi - j$

$= \left|\sum_k \frac{e^{2\pi ik\delta\varphi_j}}{2^n}\right|^2 = \frac{1}{2^{2t}}\left(\frac{\sin(\pi\cdot2^t\cdot\delta\varphi_j)}{\sin(\pi\cdot\delta\varphi_j)}\right)^2 \leq \frac{1}{2^{2t}}\left(\frac{1}{\delta\varphi_j}\right)^2$

43. Shor's Algorithm → 找到大数 $N=p\cdot q$ 质因子 p 和 q.

→ 快速求出 r 对 N 的阶 (if $a^r\equiv1 \pmod N$, $(a^{\frac r2}+1)(a^{\frac r2}-1)\,|\,N$) → 阶

Oracle $O: |y\rangle \to |xy \pmod N\rangle$. $|u_s\rangle = \frac{1}{\sqrt r}\sum_{k=0}^{r-1}e^{2\pi i(-\frac{ks}{r})}|x^k \pmod N\rangle$ → ONB

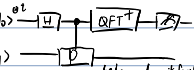
有 $O|u_s\rangle = e^{2\pi i\frac{s}{r}}|u_s\rangle$ 可通过经典~求出 r

Note that $|1\rangle = \sum_{s=0}^{r-1}\frac{1}{\sqrt r}|u_s\rangle$ ✓. Step 1. 通过 QPE 求出 $\frac{s}{r}$

Step 2. 通过 $O((\log N)^3)$ 连分数算法 求出 r Step 3. 若 $x^r\equiv1$ ✓.

Circuit: Control Register S: $|0\rangle^{\otimes t}$

Goal ~ L: $|1\rangle$



44. Deutsch Game: $f:\{0,\ldots,2^n-1\}\to\{0,1\}$ is $\begin{cases}\text{balanced, 一半} f(x) 为 0\\ \text{constant, 只输出 0/1}\end{cases}$

经典: $O(2^{n-1})$; Step 1. $|\psi_0\rangle = |0\rangle^{\otimes n}|1\rangle$ Step 2. $H^{\otimes n+1}$, $|\psi_1\rangle = \frac{1}{\sqrt{2^n}}\sum|x\rangle|-\rangle$

Step 3. $U_f=\sum_x(-1)^{f(x)}|x\rangle\langle x|$, $|\psi_2\rangle = \frac{1}{\sqrt{2^n}}\sum_x(-1)^{f(x)}|x\rangle|-\rangle$ Step 4. $H^{\otimes n+1}$

Step 5. 测所有 qubit, iff $|0\ldots0\rangle$ → constant. $O(1)$ query.

45. 线性方程: $A\vec x=\vec b$, 经典方法 高斯消元 $O(N^3)$, 共轭梯度 $O(Ns\kappa\log(\frac{1}{\varepsilon}))$

s: (non)sparsity, 非0 eigenvalue 数, κ: 条件数 $\frac{最大}{最小}$ eigenvalue.

46. Harrow Hassidim Lloyd. $\tilde O((\log N)\cdot s^2\cdot\kappa^3\cdot\frac{1}{\varepsilon})$ 解~.

Input Hermitian ~ A, $|b\rangle$ → $\vec b$. Output: $|x\rangle\propto A^{-1}|b\rangle$

Step 1. $U=e^{iAt}$ (unitary, any). 若 A 非 Hermitian, 则用 $\begin{pmatrix}0&A\\A^\dagger&0\end{pmatrix}x' = \begin{pmatrix}\vec b\\0\end{pmatrix}$

Step 2 QPE, $|b\rangle = \sum_j\beta_j|u_j\rangle$ → A的本征 对 U QPE 得到 $|\psi\rangle = \sum_j\beta_j|\tilde\lambda_j\rangle|u_j\rangle$ → λ_j binary 估计 → 将本征值编入 state

Step 3. Rejection Sampling, $\sum_j\beta_j|u_j\rangle \to \sum_j\frac{\beta_j}{\lambda_j}|u_j\rangle$

① 引入 ancilla $|0\rangle$ ② 对 $\forall|\tilde\lambda_j\rangle$, 执行 $|0\rangle \to \sqrt{1-\frac{C^2}{\tilde\lambda_j^2}}|0\rangle + \frac{C}{\tilde\lambda_j}|1\rangle$

③ 测量 ancilla. $|0\rangle$ → 重做, $|1\rangle$ → 状态塌缩为 $\propto\sum_j\frac{\beta_j}{\lambda_j}|\tilde\lambda_j\rangle|u_j\rangle$, 成功率 $O(\frac{1}{\kappa^2})$

Step 4. Uncomputation, $\sum_j\frac{\beta_j}{\lambda_j}|\tilde\lambda_j\rangle|u_j\rangle \xrightarrow{QPE^\dagger} |x\rangle\propto\sum_j\frac{\beta_j}{\lambda_j}|u_j\rangle = A^{-1}|b\rangle$

局限 ① QRAM 需 $O(N)$ 成本 ② 若可 SQ-access, 则有 polylog(N) 经典算法

优势: SVM → $\begin{pmatrix}0&1^T\\1&K+\gamma I\end{pmatrix}\begin{pmatrix}b\\a\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$, 量子 SVM

47. Grover. $O(N)\to O(\sqrt N)$. Oracle: $f(x) = \begin{cases}1 & x=x^*\\0 & \text{otherwise.}\end{cases}$

$O: |x\rangle|q\rangle \to |x\rangle|q\oplus f(x)\rangle$, let $|q\rangle=|-\rangle$, $\begin{cases}|x\rangle|q\rangle, \text{ if } f(x)=0\\ -|x\rangle|q\rangle, \text{ if } f(x)=1\end{cases}$, $O=I-2|x^*\rangle\langle x|$

Step 1. $|\psi_0\rangle = |0\rangle^{\otimes n}|-\rangle$ Step 2. $H^{\otimes n}$, $|\psi_1\rangle = \frac{1}{\sqrt N}\sum_{x=0}^{N-1}|x\rangle|-\rangle$, $N=2^n$.

Step 3. 重复 WO k次, $W=H^{\otimes n}(2|0\rangle\langle0|-I)H^{\otimes n}$ Step 4. 测量主寄存器.

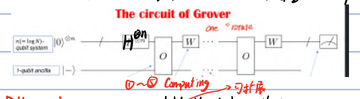
① 单目标, $k\approx\frac{\pi}{4}\sqrt N$, $P\geq1-\frac{1}{N}$; ② 多目标, $k=\frac{\pi}{4}\sqrt{\frac{N}{M}}$, $P\approx1-\frac{M}{N}$

③ 未知目标数, 尝试不同 k 值.

对于 多目标, $|\psi\rangle = \frac{1}{\sqrt N}\sum|x\rangle = \frac{\sqrt M}{\sqrt N}\frac{1}{\sqrt M}\sum|x^*\rangle + \frac{\sqrt{N-M}}{\sqrt N}\frac{1}{\sqrt{N-M}}|x\rangle = \sin\theta|x_0\rangle + \cos\theta|x_0^\perp\rangle$ (附加项)

O → 绕 $|x_0^\perp\rangle$ 旋转, W → 关于 $|\psi\rangle$ 的反射 → WO → 朝 $|x_0\rangle$ 旋转 2θ.

i.e. $WO|\psi\rangle = \sin3\theta|x_0\rangle + \cos3\theta|x_0^\perp\rangle$, 最优 k → $(2k+1)\theta$ 接近 $\frac{\pi}{2}$ → 概率最大.



48. DiVincenzo's criteria: ① scalable 物理系统, well characterized-qubits

② 能 initialize all qubits ③ 长 (de)coherence time → 保持叠加态 ④ 有 Universal set

⑤ 可以 measure ⑥ Interconvert stationary ⟺ flying qubit ⑦ Long dist transmission (⑥~⑦ Communication)

T_1: Longitudinal relaxation time, $(|1\rangle\to|0\rangle)$ T_2: transeverse r~ time $(|+\rangle \to \{|0\rangle, |1\rangle\})$

通常 $T_2\leq2T_1$, 可执行门数 $\approx\frac{T_2}{门时间}$

49. Nuclear Magnetic Resonance: 分子(molecule)中不同原子核(nucleus)的自旋(spin), $|0\rangle$ → 自旋与

外磁场同向(高能), $|1\rangle$ → 反向, Gate → Radio Frequency pulses = Bloswt

测量 → 类似MRI, It releases energy as electro-magnetic pulses when RF gone

问题: 初始化不完美, $S=\{q(x),|x\rangle\}$, → ensemble

50. Temporal Labeling: 在执行U前, 等概率执行 ① $I^{\otimes2}$ ② P

③ P^2 where $P|\omega\rangle=|\omega\rangle$, $|01\rangle\xrightarrow{P}|10\rangle$, $|10\rangle\xrightarrow{P}|11\rangle$, $|11\rangle\xrightarrow{P}|01\rangle$

i.e. $S\to\{(q_\omega,|\omega\rangle), (\frac{1-q_\omega}{3}, |x\rangle)_{x\neq|\omega\rangle}\}$

$P(y) = q_\omega|\langle\phi_y|U|\omega\rangle|^2 + \frac{1-q_\omega}{3}\sum_{x\neq\omega}|\langle\phi_y|U|x\rangle|^2$

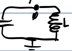
$= \frac{4q_\omega-1}{3}P_{ideal}(y) + \frac{1-q_\omega}{3}$ q_ω 可测.

当 qubit 多时, $q_{\omega\ldots0} = q_0^n \approx (\frac12)^n(1+n\varepsilon)$ 很小 → NMR不适合大规模

51. LC cricuit: 由 inductance L 与 capacitance 组成.

Energy level gaps 完全相同, i.e. 用 w force $|0\rangle\to|1\rangle$, 同样可能导致 $|1\rangle\to|2\rangle\ldots$, Overcooking

$E_n=\hbar w(n+\frac12)$, $\Delta E=\hbar w$



$E=\frac{\phi^2}{2L}+\frac{Q^2}{2C}$ gaps不同

52. Josephson Junction: 将L替换成JJ → Anharmonic energy

53. Superconducting QC: ① By connecting it to a resonate (in/out line 用 suitable microwave 驱动. ② 将 qubit couple 到 LC circuit上, 更灵敏.

54. No-cloning: 无 Unitary matrix U, s.t. $U(|\psi\rangle|0\rangle) = |\psi\rangle|\psi\rangle$ 若有, 则有:

$\frac{1}{\sqrt2}(|0\rangle|+\rangle+|1\rangle|+\rangle) = |+\rangle|+\rangle = U(|+\rangle|0\rangle) = \frac{1}{\sqrt2}(U|0\rangle|0\rangle+U|1\rangle|0\rangle) = \frac{1}{\sqrt2}(|00\rangle+|11\rangle)$ X.

55. X/Z Error: 某个qubit 可能会发生X/Z flip ③ Z错误出现在相位

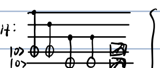
56. 考虑 3个一组, 采用多数的 qubit ① no-cloning ② 测量破坏叠加态

57. Q-Repetition Code: $|\psi\rangle|00\rangle \xrightarrow{CNOTs} \alpha|000\rangle+\beta|111\rangle$

58. Syndrome Detection: 引入2 Ancilla qubit, 测量 "stabilizers."

$S_1=Z_1Z_2$, $S_2=Z_2Z_3$ $\alpha|000\rangle+\beta|111\rangle$ 是 S_1,S_2 +1 本征态, 其余为 −1 本征态.

Circuit:



$\begin{cases}(0,0)\to 无错误\\(1,0)\to 1\text{ qubit }X\\(1,1)\to2\\(0,1)\to3\end{cases}$ (假设只有一个 X-error)

59. 对于 Z 错误. 注意到 $HZH=X$, 也即 z error $\xrightarrow{H}$ X error

令 $|\tilde0\rangle=|{+}{+}{+}\rangle$, $|\tilde1\rangle=|{-}{-}{-}\rangle$, 稳定子: $S_1=X_1X_2$, $S_2=X_2X_3$ (only z-error)

60. Shor Code: $|\tilde0\rangle = \left(\frac{|000\rangle+|111\rangle}{\sqrt2}\right)^{\otimes3}$, 也即块内防X-error, 块间防Z-error

8个 stabilizer: ① Z-type (防X-error) Z_1Z_2, Z_2Z_3, Z_4Z_5, Z_5Z_6, Z_7Z_8, Z_8Z_9 ② X-type $X_1X_2X_3X_4X_5X_6$, $X_4X_5X_6X_7X_8X_9$

(con)

对于Y-error, ∀单比特error: $E=aI+bX+cY+dZ$, Note that $Y=-iXZ$.

Shor Code 可以纠正∀单比特error, ∀U可以分作

61. Fault Tolerance: ①∀operation可能出错. ②错误会spread

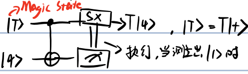
62. Transversal op~: logical $|\bar{0}\rangle=|0\rangle^{\otimes n}$ (→physical), $U_L=U^{\otimes n}$

63. Eastin-Knill Theorem: ∄一种QEC, s.t. can transversal all universal gate.

64. Steane Code: $|0_L\rangle=\frac{1}{2\sqrt{2}}\sum|x\rangle$ (→7-qubit), x有偶数个1; $|1_L\rangle$, 有奇数个1

H, CNOT 均可transversal, 但 T 不可以. $(T^{\otimes7}|0_L\rangle\neq|0_L\rangle)$

65. Fault-tolerant T: $|T\rangle$ (→Magic State)

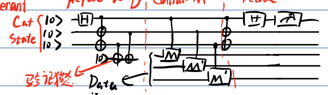


→$T|\psi\rangle$, $|T\rangle=T|+\rangle$

$|\psi\rangle$ → 执行, 当测出 $|1\rangle$ 时

66. FT-S: 用 $(ZS)^{\otimes7}$, 保证相位正确.

67. Fault-tolerant Measure:



68. 制备$|T\rangle$ → $|T\rangle$是 $e^{-\frac{i\pi}{4}}SX$ 的本征态, 且有 $|0\rangle=\frac{1}{\sqrt{2}}(|T\rangle+|T^{\perp}\rangle)$

Step1. 制备$|0\rangle$ Step2. 测量 $e^{-\frac{i\pi}{4}}SX$, 若为+1 → $|T\rangle$ ✓.

69. QEC两个问题: ① Error will Propagation → solution: {magic state($|T\rangle$), transversal gate}

② Error-correction 也会引入error → 纠错过程本身是FT的

70. Effective error-rate: $P_{logical}\leq C\cdot p^2$, C → constant rely on structure of QEC, C of Steane ~10^4

p → 物理门错误率.

71. Threshold Theorem: ∃threshold error rate $p^*>0$. ∀quantum circuit with f gates, 可被"有noise的FT物理电路模拟, ∀ε>0, 物理门: f·polylog f

72. Noisy Intermediate-Scale Q~ 含噪声中规模QC

qubit无纠错 / 几十~几百qubit / 且无法实现FT

NISQ限制: ① Circuit Depth 小 ② 无法运行 Shor, Grover ...

Solution: Hybird Algorithm: 经典计算机与量子混合

73. Max-Cut: → NP-complete. 我们∀顶点i, $s_i\in\{-1,1\}$ → 两个Cut Set

$L(s)=\frac12\sum_{(i,j)\in E}(1-s_is_j)$ → 求max → $H_{Max-Cut}\sum\frac12(I-Z_iZ_j)$

其max能量本征态 $|s\rangle$ 即为所求 (s的binstr表示分割方案.)

74. Variational Q-Eigensolver: Step1. Ansatz: $|\psi(\vec\theta)\rangle=U(\vec\theta)|0\rangle^{\otimes n}$

Step2. 期望能量 $C(\vec\theta)=\langle\psi(\vec\theta)|H_{target}|\psi(\vec\theta)\rangle$, 多次测定, 统计 → 概率

Step3. via Gradient Descend 调整 $\vec\theta$, let $H(\vec\theta)$ ↓.

75. Q-Approximate Optimization Algorithm: VQE在Max-Cut上具体实现

Step1. $|\psi_0\rangle=|+\rangle^{\otimes n}$ $H_C=H_{Max-Cut}$, $H_B=\sum_iX_i$

Step2. (p rounds) $|\psi(\vec\gamma,\vec\beta)\rangle=\left(\prod_{j=1}^{p}e^{-i\beta_jH_B}e^{-i\gamma_jH_C}\right)|\psi_0\rangle$

Step3. 测量 $\langle H_C\rangle$, 优化更新 $\vec\gamma,\vec\beta$. Step4. 得到 binstr $|s\rangle$

总深度: $O(p\delta)$, (→ 图的最大度数.)

76. Adiabatic Thorem: 若H(t) varies slow enough, system remains ground/highest energy state.

77. Adiabatic QC: ① $H(t)=(1-s(t))H_0+s(t)H_p$, $s(0)=0$ $s(T)=1$

② initial state: H_0 的最高能态. ③ s(t) 从0→1 smoothly, H(t): $H_0\to H_p$

④ 当t=T时, 系统能态 ≈ H_p 基态

78. Trotter Formula: ∀Hermitian matrices A, B, t>0.

$\lim_{p\to\infty}\left(e^{\frac{iAt}{p}}e^{\frac{iBt}{p}}\right)^p=e^{i(A+B)t}$ ⇒ QAOA是AQC的Discretization

79. QAOA性能: p↑ → ①精确度↑ ② Circuit complexity↑ → error↑ (Simulation)

80. Evaluation of Objective: 在QAOA中, $L(\vec\beta,\vec\gamma)=\langle\psi(\vec\beta,\vec\gamma)|H_C|\psi(\vec\beta,\vec\gamma)\rangle$

考虑 locality: $L(\vec\beta,\vec\gamma)=|E|-\frac12\sum_{(i,j)\in E}f_{(i,j)}(\vec\beta,\vec\gamma)$, $\langle\psi|Z_iZ_j|\psi\rangle$

注意到 H_B, H_C 只与顶点i,j及其邻居有关, i.e. Neighbor(i,j) 影响 $f_{(i,j)}(\vec\beta,\vec\gamma)$

也即 $f_{(i,j)}$ 与整个问题规模无关, e.g. 6-qubit 可解决100个顶点问题

在3-reguler graph中, (i,j)邻域最多有6个qubit, 3种结构:

① 6 qubit, (i,j)有0共同邻居 ② 5, 1 ③ 4, 2 解决方案:

① initiaze $\vec\beta,\vec\gamma$ ② 在小型量子计算机上测量上述3种情况, 得到 f_1, f_2, f_3

③ ∀(i,j)∈E, 判断何种情况 ④ $L(\vec\beta,\vec\gamma)=|E|-\frac12\sum_{(i,j)\in E}f_{(i,j)}(\vec\beta,\vec\gamma)$ ⑤ 优化 $\vec\beta,\vec\gamma$

81. QAOA performance {对3-reguler graph, p=1时 0.6924 optimal size; p>1时, ↗ but complexity↑}

82. NISQ Algorithm → Q~ Complexity 与问题规模无关.

83. 标准VQA: 流程: ① chose Ansatz, $\vec\theta$ ② 运行circuit, 得到 $|\psi(\vec\theta)\rangle$

③ 测定, 计算 cost function $C(\vec\theta)$ (i.e. 在QAOA中是 $\langle\psi|H_C|\psi\rangle$) ④ 优化 $\vec\theta$ 至 $C(\vec\theta)$ 足够小

84. $C(\vec\theta)$: f(输入态{$|\psi\rangle$}, observables {O}, $U(\vec\theta)$) → R

3个标准: ① Trainability ② Faithfulness (min → 真实解) ③ Measurability

85. Ansatz: structure of parametric QC, 如下两种 ansatz 讨论:

① Problem-Inspired A~: 利用数学/物理结构 (如QAOA, 模拟AQC), 快 but 深度大, 难实现

② Hardward-Efficient A~: 结构: 交替使用 单bit门, 固定纠缠层,

电路浅, 抗noisy. 层数深, 可能局部极优

i.e. 快速收敛 → PIA, 易实现/抗noisy → HEA

86. Schrodinger's eq: $i\frac{\partial|\psi\rangle}{\partial t}=H|\psi\rangle$, we想找到H's 基态

87. Ground State P~: $H|\psi_j\rangle=E_j|\psi_j\rangle$, $E_0\leq E_1\leq\cdots\leq E_n$

we 想在NISQ上找出 $|\psi_0\rangle$ & E_0.

88. Fermions: 不可区分, 反对易. 若 $H=\sum_{ij}h_{ij}a_i^\dagger a_j+\sum_{ijkl}h_{ijkl}a_i^\dagger a_j^\dagger a_k a_l$

a_j / $a_j^\dagger$ → i-th orbital's annihilation/creation operator

有: $\{a_i,a_j\}=0$, $\{a_i,a_j^\dagger\}=\delta_{i,j}$. Partides(eletrons) 是F~

But Qubit → 可区分的 → 可通过物理手段区分不同state

89. 考虑 $|0\rangle$ → 无, $|1\rangle$ → orbital上有电子, 则 Jordan-Wigner 变换:

$a_j\to\left(\bigotimes_{k=0}^{j-1}Z_k\right)\frac{X_j-iY_j}{2}$, a_j → Q~ Operation

N个orbital → N qubit ≥已知数 (1个orbital = 空间orbital + state)

90. Rayleigh-Ritz 原理: $E_0=\min_{|\psi\rangle}\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$

VQE → $C(\vec\theta)=\frac{\langle\psi(\vec\theta)|H|\psi(\vec\theta)\rangle}{\langle\psi(\vec\theta)|\psi(\vec\theta)\rangle}$

91. $\langle H\rangle$ → expectation of H, 对于纯 $|\psi\rangle$, $\langle H\rangle=\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$

92. ∀Hamiltonian H → $\sum_i C_iP_i$ i.e. $\langle H\rangle=\sum_i C_i\langle P_i\rangle$

93. Deep learning classical workflow: ① build a neural network

model variational parameters (e.g. $f(x,\vec\theta)=\sigma(Wx+b)$, $\vec\theta=\{W,b\}$)

② Cost function: $C(\vec\theta)=\sum\|f(x,\vec\theta)-y\|^2$ ③ update parameters

eg. $\vec\theta_{t+1}=\vec\theta_t-\eta\nabla_{\vec\theta}C(\vec\theta)$ ④ test model

94. Data Classification: M data, N dimension 及对应的 labels (±1)

goal: 找到 classifier f, → separate data with separating hyperplanes.

95. Support Vector Machine: → create classifier hyperplanes.

$g(\vec x)=\vec w\cdot\vec x+b$, s.t. ∀testing data $\vec x$, $g(\vec x)\cdot y>0$

i.e. $g(\vec x)>0$ → 则label: $y=+1$, $g(\vec x)<0$ → $y=-1$, g 不唯一.

96. Margin → minimal ∀$\vec x$ 到 separating hyperplane 距离

i.e. minimal $|\vec w|$, 在约束: $g(\vec x)\cdot y\geq1$ 下.

97. hard-margin: 在 $y_i(\vec w\cdot\vec x_i+b)\geq1$ 下, $\min_{\vec w,b}\frac12 w\cdot w$.

KKT-condition: 对最优解, 有 $\vec w=\sum_{i=1}^{m}\alpha_i\vec x_i$, $\sum\alpha_i=0$.

① $\vec w\cdot\vec x_k+b=y_k$ → $(\vec x_k,y_k)$ in Support Vector set S ② $\alpha_k=0$

98. Support Vector: ∀$(\vec x_i,y_i)$, s.t. $\alpha_i>0$. $\vec w=\sum_{(\vec x_i,y_i)\in S}\alpha_i\vec x_i$.

i.e. 只需存储 S, 即可构建 $g(\vec x)$ → SVM compress data.

99. Kernel $g(\vec x)=\vec w\cdot\vec x+b=\sum_{i\in S}\alpha_iy_i(\vec x_i\cdot\vec x)+b=\sum\alpha_iy_iK(\vec x_i,\vec x)+b$

$K(\vec x,\vec x)$ → kernel function. → 加强 classifier

100. Kernel Trick: $K(x,x')$ = ① Linear (trivial) → $x\cdot x'$;

② Polynomial → $(1+x\cdot x')^p$ ③ Gaussian → $e^{-\frac{\|x-x'\|^2}{2\sigma^2}}$

④ Sigmoid → $\tanh(\beta_1x\cdot x'+\beta_2)$, 参数有"variational"

101. Q-Feature Map: $|\psi(\vec x)\rangle=U(\vec x)|0\rangle^{\otimes n}$

102. Data Encoding ① QRAM, $|x\rangle=x_0|0\rangle+x_1|1\rangle$ → 难实现

② $|\psi(\vec x)\rangle=\prod R_Y(x_i)_i|0\rangle^{\otimes n}$ → product state, 无优势

③ $U(\vec x)=H^{\otimes n}e^{i\sum_{S\subseteq[n]}\phi_S(\vec x)Z_S}H^{\otimes n}$, ϕ_S: phase of data

Z_S: $\bigotimes_{j\in S}Z_j$, 引入纠缠.

103. Q-Kernel: $K(x,x')=|\langle\psi(x)|\psi(x')\rangle|^2$, 三种方法去计算:

① $|\psi(x)\rangle=\cos\frac\theta2|0\rangle+e^{i\phi}\sin\frac\theta2|1\rangle$, 测定估算 θ, φ → 仅适小系统

② Swap Test: 引入 control qubit $|0\rangle$, $|\psi(x)\rangle$, $|\psi(x')\rangle$

$P(0)=\frac12(1+|\langle\psi(x)|\psi(x')\rangle|^2)$

③ By reversal: note that $\langle\psi(x)|\psi(x')\rangle=\langle0|^{\otimes n}U^\dagger(x)U(x')|0\rangle^{\otimes n}$

1. 准备 $|\psi(x')\rangle$ 2. 应用 $U^\dagger(x)$ 3. 测state 是否回到 $|0\dots0\rangle$ → P → K(x,x')

104. Q-Supremacy 量子霸权: 指一个算法, 只有QC能在合理时间完成

105. Elitzur-Vaidman Bomb Tester: B为两种可能: ①无bomb.

② 有bomb, → 测定 input qubit $|1\rangle$ → bomb, $|0\rangle$ → output $|0\rangle$. → B

考虑 R_δ, $\delta=\frac{\pi}{2N}$, i.e. $(BR_\delta)^N|0\rangle$, 当有 → $(BR_\delta)^n|0\rangle$

① 无bomb, 最终态: $(R_\delta)^N|0\rangle=|1\rangle$ ② 有, P(不爆) = $\cos^{2N}(\delta)\approx(1-\delta^2)^N$

$\approx e^{-T\delta^2}\approx e^{-\frac{\pi^2}{4N}}\approx1-O(\frac1N)$

106. ∀ Hermitian H, $e^{iH}=\sum_{k=0}^{\infty}\frac{(iH)^k}{k!}$

107. Matrix Function: 若 $H=U\begin{pmatrix}\lambda_1&&\\&\ddots&\\&&\lambda_n\end{pmatrix}U^\dagger$, $f(H)=U\begin{pmatrix}f(\lambda_1)&&\\&\ddots&\\&&f(\lambda_n)\end{pmatrix}U^\dagger$

14. ④ $R_X(\theta)=e^{-i\frac\theta2X}=\begin{pmatrix}\cos\frac\theta2&-i\sin\frac\theta2\\-i\sin\frac\theta2&\cos\frac\theta2\end{pmatrix}$

$R_Y(\theta)=e^{-i\frac\theta2Y}=\begin{pmatrix}\cos\frac\theta2&-\sin\frac\theta2\\\sin\frac\theta2&\cos\frac\theta2\end{pmatrix}$ $=\cos\frac\theta2I-i\sin\frac\theta2P$, $P\in\{X,Y,Z\}$

$R_Z(\theta)=e^{-i\frac\theta2Z}=\begin{pmatrix}e^{-i\frac\theta2}&0\\0&e^{i\frac\theta2}\end{pmatrix}$