## Characterization of a Measure

A measure on a set is a systematic way to assign to each subset a non-negative real scalar number, commonly interpreted as the size, length, area, or volume of the subset. The measure of the union of any two disjoint elements equals the sum of the measures of each individual element. And, if the measure of the union of all elements sums to one, then it is called a probability measure. The following discussion applies to general measures, not necessarily probability measures, on the set of permissible trajectories defined by a dynamic model.

It is important to distinguish between a trajectory measure and its characterization. In order to characterize trajectory measures, a scorecard will be presented as a table with rows corresponding to algorithms and columns corresponding to datasets. A measure will receive a score for each of several characteristics based on each sample of data from our industry partners. Scores can only be derived from data sets that are accompanied by ground truth.

A relevant example of a scorecard can be seen on the Middlebury stereo vision evaluation website[1]. It offers a table with rows corresponding to algorithms and columns corresponding to datasets. Several datasets are provided with ground truth, from which to characterize measures related to stereo depth reconstruction. Each data set has only a few data elements, which in this case are stereo image pairs.

We plan to construct a similar scorecard. A measure will receive a score for each of several characteristics based on samples of data from the public domain and from AFRL. Scores will be computed by perturbing the ground truth trajectory and evaluating costs. Characteristics of interest include:

Hessian – This is the inverse of covariance in the case of normally-distributed measurement error and it is related to Circular Error Probable (CEP) by a simple transformation. It is computed by second order perturbations of the cost functional in the vicinity of ground truth. We also compute the Eigen vectors, Eigen values, and Eigen value ratios of this matrix.

**Jacobian** – Sensitivity of the cost function in the vicinity of ground truth.

**Cost** – The value of the cost function in the vicinity of ground truth.



Whereis the final cost produced by the measure when passed , the ground truth provided by the data container, and data .



**Granularity** – Distance in meters or radians until cost increases, similar to dilution of precision.

such that



Where is the minimum value necessary to produce a different cost.



**Monotonicity** – Distance in meters or radians until cost begins to decrease.

**Computation** – Equivalent number of floating point operations required to compute cost.

This analysis will help a system designer to create an overall objective function for navigation, based on a combination of sensors and algorithms that together meet the requirements of a given mission.

**References**

1. Middlebury College. *Stereo Vision Research Page*. http://vision.middlebury.edu/stereo/eval/