**1. UTA Method**

* 1. **Additive Utility Functions**

Let denote the relation of preference and of indifference on a set of alternatives . Each alternative from the set is evaluated by a set of criteria in such a way that the aggregation of those criteria () represents the overall preference or score of such alternative for a specific Decision Maker (DM).

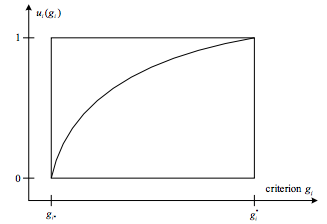
If the DM prefers alternative over we have the following relationship

If the DM is indifferent of and then the relationship of the utility of each alternative is represented as

The additive utility function is de sum of all the marginal utilities for each criterion in relation to its alternative

where is the marginal utility and is the weight or tradeoff associated with the criterion , subject to normalization constraints:

where is the best alternative for the criterion and is the worst *(Figure 1)*



*Figure 1 – Normalized marginal utility function*

Assuming the hypothesis of a non-decreasing preference for each criterion, then the marginal utility functions are monotonous.

* 1. **Modeling**

Before describing the model, it is necessary to define what we are trying to predict or measure. The original method [REF] was developed to learn the preference relation of a set of criteria based on the DM’s ranking of the alternatives *a priori*.

**1.3 Linearization**

To do so, each marginal utility function is estimated in a piecewise linear fashion. Let be the extreme values of the criterion. We split the interval into equal interval so that the end points are described by (6)

(6)

So the marginal value of an alternative or action is approximated by the linear interpolation so that is

Therefore, if and then the marginal utility is .

**1.4 The objective function**

The prediction of the utility function for the action is

Where is the error relative to our utility function. As our model aims to predict the utility function, we want to minimize the error for every action. So our objective function is

**1.5 Model Restrictions**

Our models have X restrictions. The problem space restrictions – the preference relation between actions and monotonicity – as well as the linearization and normalization restrictions.

The preference and indifference relation between two actions can be rewritten as follows:

Then one of the following holds

Where is a small real number with .

For monotonicity, the marginal utility of criterion must be always increasing, such that

The normalization constraints are described in (5) so the linear programming model is

**1.6 Sensibility Analysis**

AINDA NÃO SEI COMO FAZER ISSO NO MEU MODELO. PRECISO DAR UMA ESTUDADA MELHOR

2. UTASTAR

SHOULD I WRITE ABOUT THIS ALGORITHM AS WELL? IT MIGHT BE INTERESTING.

**2. Practical Examples**

The implementation of the algorithm is illustrated in the original article [REF]. Siskos and Yannacopoulos (1985) also have an example. They used it for their UTASTAR algorithm. Here, we will use both problems for the UTA algorithm.

2.1 Choosing a car

This example uses data from a real experiment where subjects were asked to rank cars from a list of 28 cars and 6 criteria. The data analyzed here consists of a single DM and his top 10 cars

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| car | rank | max speed | consumption town | consumption road | horse power | space | price |
| Peugeot 505 GR | 1 | 173 | 11.4 | 10.01 | 10 | 7.88 | 49500 |
| Opel Record 2000 LS | 2 | 176 | 12.3 | 10.48 | 11 | 7.96 | 46700 |
| Citroen Visa Super E | 3 | 142 | 8.2 | 7.30 | 5 | 5.65 | 32100 |
| VW Golf 1300 GLS | 4 | 148 | 10.5 | 9.61 | 7 | 6.15 | 39150 |
| Citroen CX 2400 Pallas | 5 | 178 | 14.5 | 11.05 | 13 | 8.06 | 64700 |
| Mercedes 230 | 6 | 180 | 13.6 | 10.40 | 13 | 8.47 | 75700 |
| BMW 520 | 7 | 182 | 12.7 | 12.26 | 11 | 7.81 | 68593 |
| Volvo 244 DL | 8 | 145 | 14.3 | 12.95 | 11 | 8.38 | 55000 |
| Peugeot 104 ZS | 9 | 161 | 8.6 | 8.42 | 7 | 5.11 | 35200 |
| Citroen Dyane | 10 | 117 | 7.2 | 6.75 | 3 | 5.81 | 24800 |

|  |  |  |  |
| --- | --- | --- | --- |
| **Criteria** |  |  |  |
| **max\_speed** | 110 | 190 | 5 |
| **consumption\_town** | -15 | -7 | 4 |
| **consumption\_road** | -13 | -6 | 4 |
| **horse\_power** | 3 | 13 | 5 |
| **space** | 5 | 9 | 4 |
| **price** | -80000 | -20000 | 5 |

The first step is to calculate the marginal utilities. Using the scales provided in Table 2, we have the intervals that will be used for the linear interpolation:

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| --- | --- |
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|  |  |

Now, the utility of each alternative (car) can be rewritten as

The table with all marginal utilities can be found at Appendix 1