Robust innovation-based adaptive Kalman filter for INS/GPS land navigation

Conference Paper · November 2013

DOI: 10.1109/CAC.2013.6775762

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Robust Innovation-Based Adaptive Kalman Filter for INS/GPS Land Navigation

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***Abstract*—The integration of Inertial Navigation System (INS) and Global Positioning System (GPS) is a most frequent method for land navigation. Conventional Kalman Filter (CKF) is an optimal estimation algorithm widely used in INS/GPS integration. CKF assumes that the covariance of the system process noise and measurement noise are given and constant. The performance of the CKF degrades seriously, when the GPS measurement noise changes. Researchers introduced an Innovation-based Adaptive Estimation Adaptive Kalman Filter (IAE-AKF) algorithm to keep the filter stable. However, under some extreme condition, the measurement noise may vary tremendously, which will lead to the degradation and divergence of the IAE-AKF. A robust IAE-AKF algorithm is presented in this paper, which evaluates the innovation sequence with Chi-square test and revises the abnormal innovation vector. Simulation and vehicle experiment results show that the new algorithm performs higher accuracy and robustness, and also has the ability to prevent the filtering from being diverged even in a rigorous GPS measurement environment.**

Keywords—Innovation-based Adaptive Estimation; Adaptive Kalman Filter; Land Navigation

I. INTRODUCTION

It is very important to obtain the vehicle's position, velocity and attitude precisely for land navigation. INS provides those information with high frequency, but the accuracy of INS deteriorates with time due to the accumulation of inertial sensors biases and noises [1-3]. GPS provides accurate position and velocity at low frequency, and its error does not increase with time. However, the GPS accuracy will decrease due to the satellite signal blockage. Because of their complementary characteristics to each other, the integration of INS and GPS has become a widely-used navigation system [4, 5].

As an optimal estimator, CKF has been applied to INS/GPS integration navigation system for decades [6-10]. However, the optimality of the CKF needs good priori knowledge about the system process noise and the measurement noise [11]. The statistical properties of system process noise and measurement noise depend on the real INS/GPS integration system and environment, which are difficult to obtain accurately. What's more, in a highly kinematic application the statistical properties are not always fixed but changeable. Insufficiently knowing a priori filter statistics may lead to the practical divergence of the filter [6, 12].

In order to solve these problems, many improved KF algorithms have been proposed. For instance, Multiple-Model based Adaptive Estimation (MMAE) [13], Covariance Scaling [7, 14] and innovation-based adaptive estimation (IAE) [6, 8, 12, 15]. In the past few years, researchers have concentrated on the IAE. The IAE utilizes the innovation sequence to estimate the system process and/or measurement noise covariance matrix (Q and R, respectively) based on Maximum-Likelihood (ML) criterion [6]. In practice, the IAE-AKF always leads to divergence when both Q and R were unknown. A novel IAE-AKF [8, 12] which can improve the filter's stability was proposed to overcome the problem. However, all the IAE-AKF needs a moving estimation window of innovation sequence. The estimated covariance matrix may not reflect the reality when the estimation window includes some acute outliers or disturbed innovations. This may reduce the accuracy of the filter or even cause filter divergence.

In this work, a Robust IAE-AKF (RIAE-AKF) algorithm is presented. The algorithm utilizes the Chi-square test to evaluate the reliability of innovation vectors, and revises the abnormal innovation vectors. The simulation and vehicle experiments are carried out to compare the effects of the RIAE-AKF, IAE-AKF and CKF, respectively. The test results demonstrate that, in rigorous measurement environment, the RIAE-AKF is more accurate and robust.

II. KALMAN FILTER AND INNOVATION-BASED ADAPTIVE ESTIMATION

A. Conventional Kalman Filter

Considering a linear discrete model of the INS/GPS integrated system:

$$X_{k+1} = \Phi_{k+1/k} X_k + W_k \tag{1}$$

$$Z_{k+1} = H X_{k+1} + V_{k+1} \tag{2}$$

Where X_k is the state vector at epoch k; $\Phi_{k+1/k}$ is the state transition matrix from epoch k to epoch $k+1$; W_k is the system process noise vector at epoch k; H is the measurement matrix; V_{k+1} is the measurement noise vector at epoch $k+1$; Z_{k+1} is the observation vector at epoch $k+1$. W_k and V_{k+1} are uncorrelated zero-mean white Gaussian noise sequence with covariance:

$$\begin{aligned}&\mathrm{cov}\left[W_k, V_j\right] = 0\\&\mathrm{cov}\left[W_k, W_j\right] = Q_k \delta_{kj}\\&\mathrm{cov}\left[V_k, V_j\right] = R_k \delta_{kj}\end{aligned} \tag{3}$$

Where cov is the function of calculating the covariance matrix; Q_k is the system process noise covariance matrix and R_k is the measurement noise covariance matrix; δ_{kj} is the Dirichlet function.

From the above linear discrete system, the CKF equations are as follows:

$$X_{k+1/k} = \Phi_{k+1/k} X_{k/k} \tag{4}$$

$$P_{k+1/k} = \Phi_{k+1/k} P_{k/k} \Phi^T_{k+1/k} + Q_k \tag{5}$$

$$K_{k+1} = P_{k+1/k} H^T_{k+1} [H_{k+1} P_{k+1/k} H^T_{k+1} + R_{k+1}]^{-1} \tag{6}$$

$$P_{k+1/k+1} = [I - K_{k+1} H_{k+1}] P_{k+1/k} \tag{7}$$

$$X_{k+1/k+1} = X_{k+1/k} + K_{k+1} v_k \tag{8}$$

Where $X_{k/k}$ is the estimated state vector at epoch k; $X_{k+1/k}$ is the predicted state vector at epoch $k+1$; $P_{k/k}$ is the covariance matrix for $X_{k/k}$; $P_{k+1/k}$ is the covariance matrix for $X_{k+1/k}$; K_{k+1} is the Kalman Filter (KF) gain matrix at epoch $k+1$; v_k is the innovation vector, which is defined as:

$$v_k = Z_k - H X_{k+1/k} \tag{9}$$

B. Innovation-Based Adaptive Estimation.

The covariance of innovation can be estimated as [6]:

$$\hat{C}_{v_k} = \frac{1}{N} \sum_{j=k-N+1}^{k} v_j v_j^T \tag{10}$$

Where, N is the size of the moving window. To improve the real-time performance, the above equation can be rewritten in a recursive way, that is,

$$\hat{C}_{v_k} = \hat{C}_{v_{k-1}} + \frac{1}{N}\left(v_k v_k^T - v_{k-N+1} v^T_{k-N+1}\right) \tag{11}$$

Comparing the innovation covariance's theoretic value with the estimated value, we can get the estimation of measurement noise covariance matrix, that is

$$\hat{R}_{k+1} = \hat{C}_{v_k} - H P_{k+1/k} H^T \tag{12}$$

The estimated system process noise covariance matrix can also be obtained [6, 16] as:

$$\hat{Q}_k = K_k \hat{C}_{v_k} K_k^T + P_{k/k} - \Phi P_{k-1/k-1} \Phi^T \approx K_k \hat{C}_{v_k} K_k^T \tag{13}$$

In practice, it is hard to estimate the Q and R simultaneously when both of them are unknown. An improved IAE algorithm was proposed, which utilizes the innovation sequence to calculate the KF gain matrix directly [8, 12] .

$$K_{k+1} = P_{k+1/k} H^T_{k+1} \hat{C}^{-1}_{v_k} \tag{14}$$

Where, $\hat{C}^{-1}_{v_k}$ is the inverse of the matrix $\hat{C}_{v_k}$.

Obviously, it is an essential step for all of the IAE algorithms to estimate the innovation covariance matrix. However, when the used innovation vectors include acute disturbed noise or outliers, the estimated matrix $\hat{C}_{v_k}$ is disturbed as well. And this disturbance will last N epochs. The disturbed $\hat{C}_{v_k}$ may reduce the accuracy or even cause filter divergence if we can't deal with the disturbed innovation vector properly.

III. ROBUST INNOVATION-BASED ADAPTIVE ESTIMATION

When the innovation v_k is the zero-mean white noise with Gaussian distribution, the IAE-AKF will be stable. That means

$$v_k \sim N\left(0, \hat{C}_{v_{k-1}}\right) \tag{15}$$

Then we have

$$\kappa_k = v_k^T \hat{C}^{-1}_{v_{k-1}} v_k \sim \chi^2(m) \tag{16}$$

Where, κ_k is a statistic variable which is used for the test; $\chi^2(m)$ represents a Chi-square distribution with m degrees of freedom, and m is the number of innovation vector's elements. According to the Chi-square hypothesis test, giving a confidential level α , we obtain

$$\begin{cases} if \quad \kappa_k \geq \chi^2_\alpha(m) \quad then \quad \text{abnormal} \\ if \quad \kappa_k < \chi^2_\alpha(m) \quad then \quad \text{normal} \end{cases} \tag{17}$$

Where $\chi^2(m)$ is the threshold value corresponding to the Chi-square distribution table and the given confidential level α . The sketch map of them is shown in Fig. 1 (Left). In the figure, x denotes the Chi-square statistic and $f(x)$ denotes its probability density function.

Using (17), we can judge whether a new innovation vector is normal or not. However, the elements of innovation vector may be uncorrelated and have different dimensions, so (17) may not be accurate. A particular method is to judge the element of the innovation vector separately. For every element of the innovation, we have [7]

$$\kappa_k(i) = \frac{v_k^2(i)}{C_{v_{k-1}}(i,i)} \sim \chi^2(1) \tag{18}$$

Where, $v_k(i)$ is the ith element of the vector v_k and $C_{v_{k-1}}(i,i)$ is the ith diagonal element of the matrix $\hat{C}_{v_k}$.

In order to reduce the influence of the abnormal innovation vector, the abnormal element is revised as follows:

$$\tilde{v}_k(i)=\begin{cases} v_k(i), & 0\le \kappa_k(i)<\chi^2_\alpha(1) \\ v_k(i)\exp\left(\dfrac{-\left(\kappa_k(i)-\chi^2_\alpha(1)\right)}{\chi^2_\alpha(1)}\right), & \kappa_k(i)\ge\chi^2_\alpha(1) \end{cases},\ i=1,2,\ldots,m \tag{19}$$

Where, $\exp(x)$ is the exponential of x.

The sketch map is shown in Fig. 1 (Right). When the element of an innovation vector is normal, we don't revise it, otherwise we multiply with a small factor to weaken the effect of the abnormal element of the innovation vector.

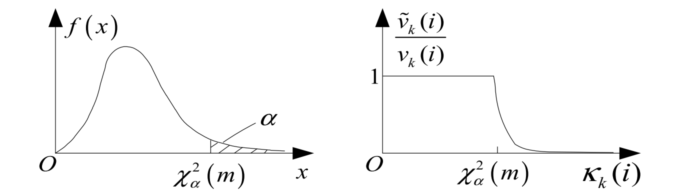

Fig.1 Sketch map Chi-square test

Substituting (19) into (11), we obtain the matrix $\hat{C}_{v_k}$. Then, the RIAE-AKF algorithm is as follows:

- KF state prediction: (4) and (5);
- Robust innovation covariance estimation: (19) and (11);
- KF measurement update: (14), (7) and (8).

IV. Performance Evaluations

A. System Model

The model of INS/GPS integrated navigation system consists of the error state equation and measurement equation. Choosing the states of the system as:

$$X=[\psi_N,\psi_U,\psi_E,\delta V_N,\delta V_U,\delta V_E,\delta L,\delta\lambda,\delta h,\varepsilon_x,\varepsilon_y,\varepsilon_z,\nabla_x,\nabla_y,\nabla_z] \tag{20}$$

Where, ψ_N,ψ_U,ψ_E are attitude errors; $\delta V_N,\delta V_U,\delta V_E$ are velocity errors; $\delta L,\delta\lambda,\delta h$ are latitude error, longitude error and altitude error; $\varepsilon_x,\varepsilon_y,\varepsilon_z$ are gyro drifts; $\nabla_x,\nabla_y,\nabla_z$ are accelerometer biases; the subscripts N,U,E denote the north, up and east components in the n-frame, and the subscripts x,y,z denote the front, up and right components in the b-frame. Then the system model can be written as [2]:

$$\dot{X}(t)=A(t)X(t)+W(t) \tag{21}$$

Where, $W(t)$ is the system process noise vector. Simultaneously, choosing the position errors as the measurement vector, that is

$$Z(t)=\begin{bmatrix}\delta L\\ \delta\lambda\\ \delta h\end{bmatrix}=\begin{bmatrix}L^{SINS}-L^{GPS}\\ \lambda^{SINS}-\lambda^{GPS}\\ h^{SINS}-h^{GPS}\end{bmatrix} \tag{22}$$

The measurement equation is:

$$\begin{aligned}Z(t)&=HX(t)+V(t)\\ &=[0_{3\times3},I_{3\times3},0_{3\times6}]X(t)+V(t)\end{aligned} \tag{23}$$

Where, H is the measurement matrix, and $V(t)$ is the measurement noise vector.

B. Simulation

The proposed algorithm is applied to an INS/GPS integrated navigation system. The main hardware configuration is shown in Fig. 2 and the specifications are listed in Table I. The values in Table I are used to set the system process covariance matrix Q of KF.

TABLE I. SPECIFICATIONS OF INS/GPS

Sensors	**Specifications**
INS	Gyro: constant drift 0.01deg/h, white noise 0.005deg/h
	Accelerometer: constant bias 0.1mg, white noise 0.05mg
GPS	Position: 2.0 m(RMS)

The simulation test's data come from a static navigation, and the length is 2 hours. The data set was used to compare the performances of RIAE-AKF with CKF and AIE-AKF. The GPS measurement noise of latitude and longitude is enlarged intentionally after 3500s as is shown in Fig. 3. What's more, the squares in the figure denote the acute disturbed epochs. The window size for innovation covariance estimation is $N=30$, the Chi-square test's confidential level is $\alpha=0.005$, and the corresponding threshold value is $\chi^2_\alpha(1)=7.879$.

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Fig. 2. The hardware of the INS/GPS integrated navigation system

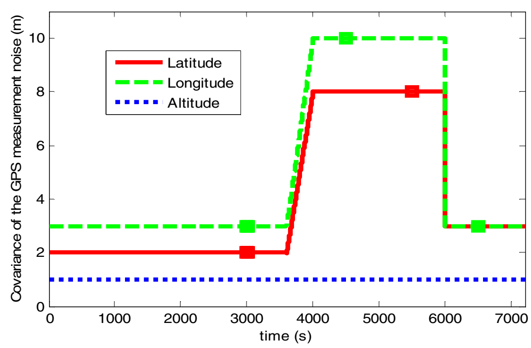


Fig. 3. Simulated covariance and disturbed epochs of the measurement

The horizontal 2D position errors are shown in Fig. 4. The result shows that, when the measurement noise was normal, all the algorithms can get the optimal values. However, when the measurement noise is enlarged, the CKF accuracy degrades obviously. What's more, for the acute disturbed epochs, the IAE-ACK is more sensitive than the RIAE-AKF, as shown in the zoomed in result in Fig. 4. At 3001s – 3005s, the CKF shows incapable to deal with the continued acute disturbance, while the RIAE can get the highest accuracy. At 6500s, there is an isolated outlier with an absolute error about 60m, which leads to the IAE becomes divergence at 6531s. The result shows that the RIAE outperforms the CKF and IAE with higher accuracy and robustness, especially in the unstable acute measurement environment.

Moreover, in order to compare innovation covariance estimation effects between RIAE and IAE, Fig. 5 shows the results. The IAE and RIAE almost have the same estimation results, when the measurement noise is stable or changes mildly. But when the innovation is abnormal, as is shown in the two zoomed in parts in Fig. 5, the IAE got wrong innovation covariance estimation $\hat{C}_{v_k}$ and the disturbance lasted N epochs. This may cause two disadvantages: Firstly, the two jumps of $\hat{C}_{v_k}$ may lead to the filter divergence, especially, when $\hat{C}_{v_k}$ jumps from high to low, the KF gain K_{k+1} will enlarge too much based on (14). Then the state covariance estimation $P_{k+1/k+1}$ may become negative definite matrix according to (7). That means the filter becomes divergence. Secondly, the long wrong value of $\hat{C}_{v_k}$ will reduce the accuracy of the filter. When the measurement becomes normal, the wrong $\hat{C}_{v_k}$ does not reflect the reality, and then the accuracy degrades. On the contrary, the RIAE adaptively adjust the abnormal innovation vector through (19). During the abnormal epochs, the innovation values are adjusted to normal values, and the matrix $\hat{C}_{v_k}$ is not affected, so the RIAE performs much better than IAE.

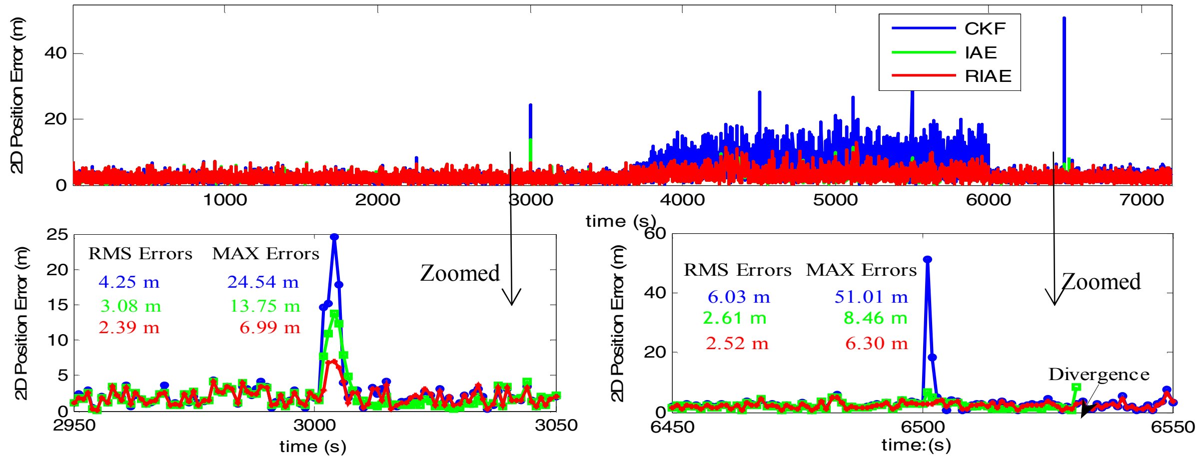


Fig. 4. Comparison of the 2D Position Errors

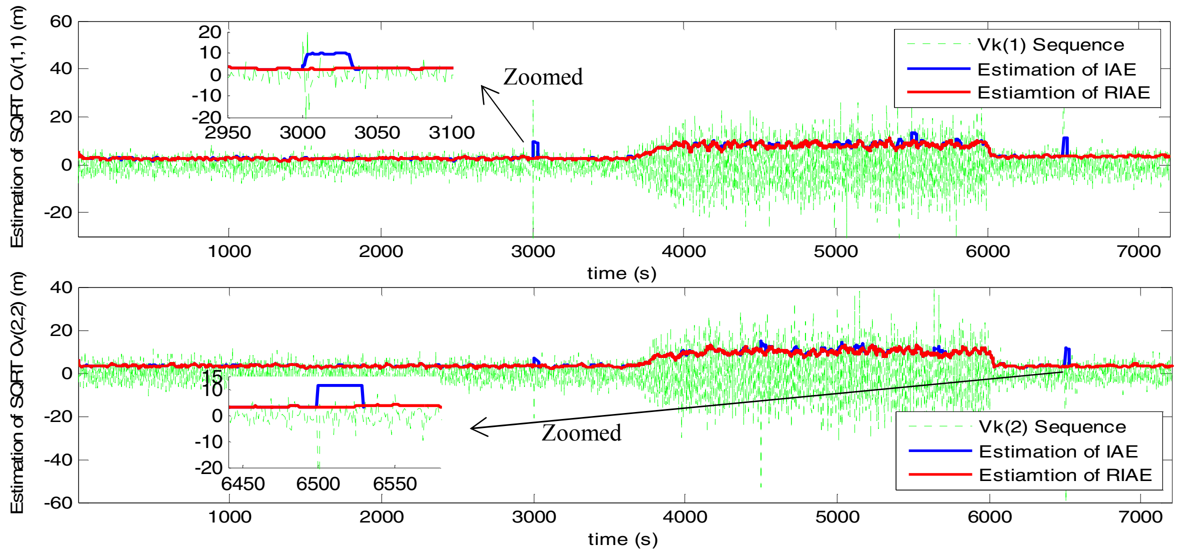


Fig. 5. Covariance Estimation of the Innovation Sequence

TABLE II. HORIZONTAL POSITION ERRORS AND VELOCITY ERRORS OVER THE TOTAL TRAJECTORY

Method	**RMS Position Error (m)**		**Max Position Error (m)**		**RMS Velocity Error (m/s)**		**Max Velocity Error (m/s)**	
	East	North	East	North	East	North	East	North
CKF	0.9595	1.4559	10.9677	26.6362	0.0794	0.1075	0.3353	0.3900
IAE	0.5101	0.8723	2.0359	5.8338	0.0794	0.1082	0.3349	0.3483
RIAE	0.4918	0.6730	1.7209	3.8999	0.0793	0.1073	0.3349	0.3483

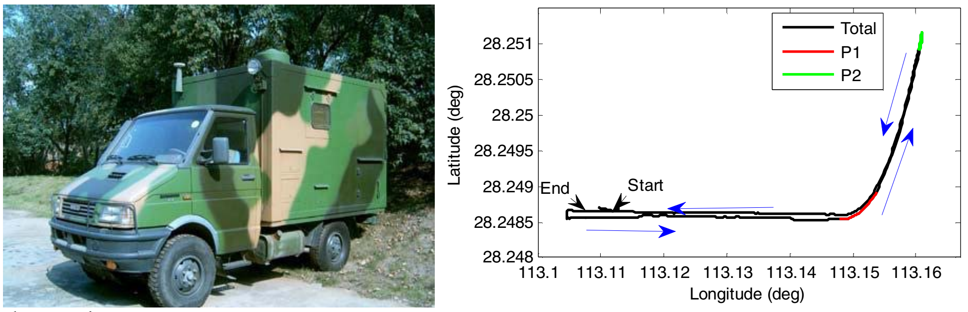
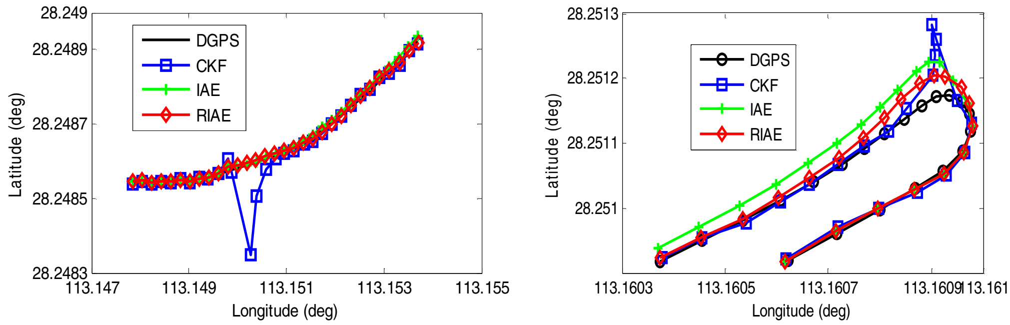


Fig. 6. Vehicle and Test trajectory



(a) P1 trajectory (b). P2 trajectory

Fig. 7. Estimated trajectories with different algorithms

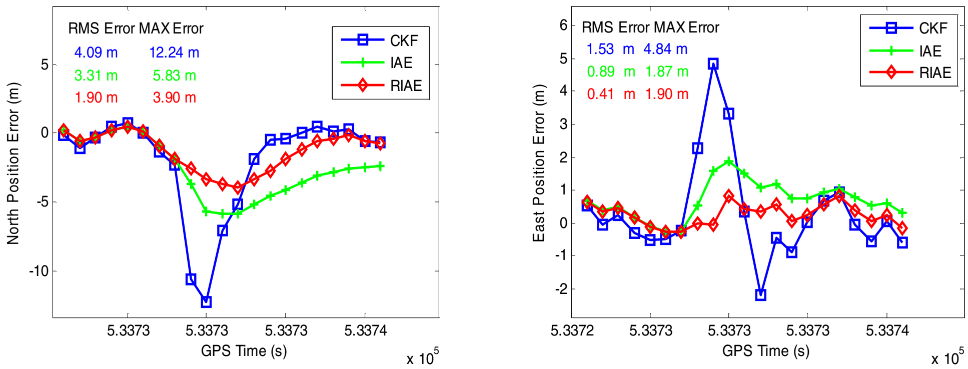


Fig. 8. Horizontal position errors over the P2 trajectory

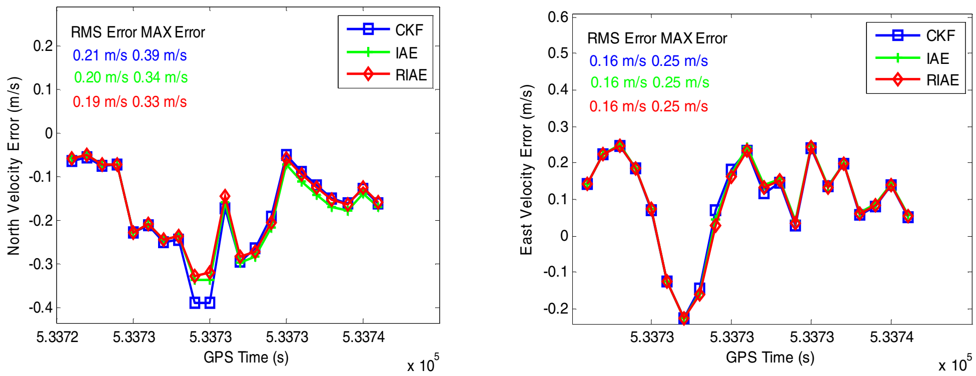


Fig. 9. Horizontal velocity errors over the P2 trajectory.

C. Vehicle Test

To verify the proposed algorithm for kinematic application, a vehicle test was designed. The specifications of INS/GPS integrated navigation system are listed in Table I. Moreover, a NovAtel DGPS is taken as a reference for the INS/GPS integrated system. It can reach such high accuracy that its position errors outperform 0.1m and velocity errors outperform 0.02 m/s.

The vehicle and the test trajectory are shown in Fig. 6. The total test time lasts 800 s, the speed of the vehicle is about 20 m/s, and there are several turnings in the trajectory. The GPS antenna of partial trajectory P1 and P2 has been sheltered for a few seconds.

For the total trajectory, the navigation results are listed in Table II. The results show that the accuracy of RIAE and IAE outperforms the CKF for the total trajectory. The RIAE and IAE have the similar accuracy for the total trajectory. The reason for it is that the total measurement is normal most of the time.

However, for the P1 and P2 trajectory, the RIAE and IAE perform differently. Fig. 7 shows the trajectory estimations of different algorithm. The detailed position and velocity over P2 trajectory are shown in Fig. 8 and Fig. 9. Obviously, the RIAE has higher accuracy and is smoother than the IAE. Therefore, the kinematic vehicle test evaluates the advantage of the proposed RIAE.

V. Conclusions

Unstable and acute disturbance measurement environment of INS/GPS integrated navigation system may degrade the performance of the CKF as well as the IAE-AKF. A robust innovation-based adaptive kalman filter has been proposed in this paper to overcome the shortcomings of the CKF and IAE-AKF. The algorithm utilized the Chi-square test to find the abnormal innovation vector and revises the abnormal innovation vector to weaken its bad influence.

The proposed algorithm has been applied to a static simulation and a vehicle test. The comparisons show that the RIAE-AKF has a much better performance than the CKF and IAE-AKF. Especially, when the measurement noise was changeable and rigorously disturbed, the proposed algorithm can get smooth innovation covariance estimation and prevent the divergence of the filter. The results demonstrate that RIAE-AKF is more robust and insensitive to the measurement noise variety, and has higher accuracy and more smooth process. With the results seen herein, the INS/GPS integrated navigation system, coupled with the RIAE-AKF algorithm, can be expected by more kinematic and complicated environments.

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