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Abstract. Let R be a unique factorization domain of characteristic zero with maximal ideal generated by a prime element π having a finite residue field $R/(\pi)$. Let $f \in R[x_1, \ldots, x_n]$. For each $m \geq 1$, let c_m denote the number of solutions to the congruence $f(x_1, \ldots, x_n) \equiv 0 \bmod \pi^m$. The Poincaré series of f is the formal power series $P_f(y) = 1 + \sum_{m=1}^{\infty} c_m y^m$. In this paper we compute $P_f(y)$ for an arbitrary diagonal polynomial f given by $f(x_1, \ldots, x_n) = \epsilon_1 x_1^{t_1} + \cdots + \epsilon_n x_n^{t_n} + b$ where $\epsilon_1, \cdots, \epsilon_n \in R$, $t_1, \ldots, t_n$ are positive integers and $b \in R$. We show explicitly that $P_f(y)$ is a rational function extending results of J. Wang and Q. Han.

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