Geometric Cellular Automata Wave Front Propogation

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Introduction

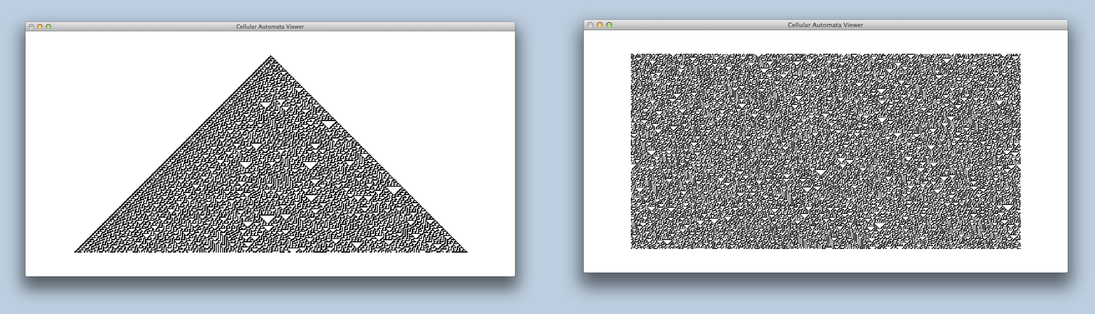
In Wolfram's 1-D Elementary Cellular Automata, propagation of information is based upon state transition rules for mapping all 8 states (with a neighborhood radius of 1) to either a 0 or 1 played out on a grid where states expand vertically downward over time and vertical edges are toroid in nature (they wrap around horizontally). Classification based on the observed global behavior is then characterized into four classes. For this study we extend this system to a 2-D space where start state will be in the form of a geometric shape with expansion happening perpendicular to each side of the geometric shape and interactions occurring around edges in convex geometries or between sides in concave geometries. The study attempts to compare and contrast start states and information propagation between such geometries as an equilateral triangular configuration, a square configuration, and possibly a star configuration (all shapes have equal length sides) to examine how information is propagated and perturbed by different surface interactions.

Methods

A 2-D Geometric Cellular Automata system was constructed in C++ and OpenGL to model the behavior of information propagation over geometric surfaces and edges. The program is configurable to allow for input of geometric shape type, surface length, grid size, initial distribution, neighborhood radius, and rule. Multiple runs with different shapes over various rule types were conducted and classified based on the four classes of Wolfram's system. An attempt to correlate the 2-D classification to the 1-D classification is explored, with the findings of each Wolfram Class type given.

Results

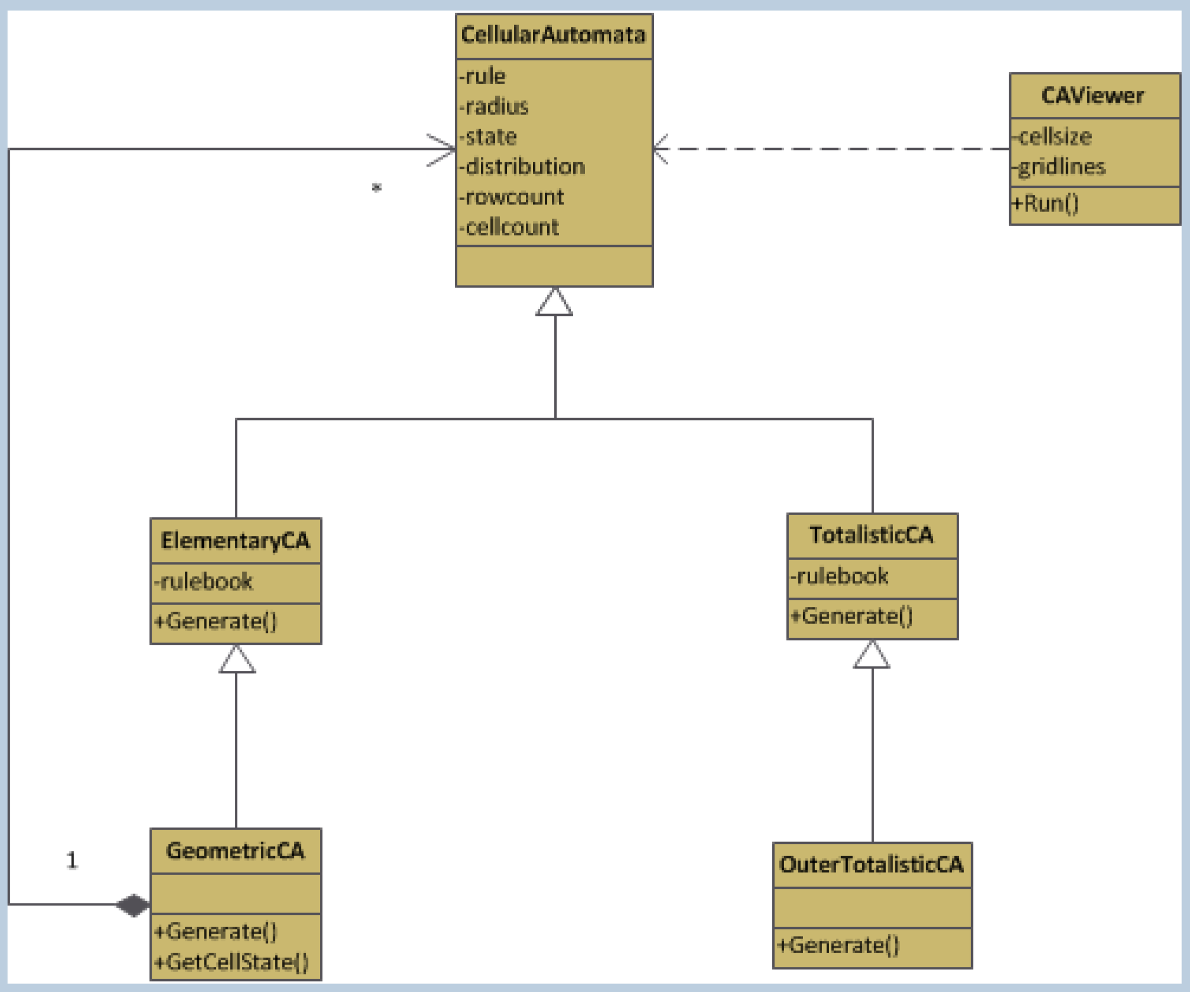
A visual comparison of Elementary CAs with Geometric CAs sampled from each of the four Wolfram Classes show that the class behavior is retained regardless of geometric configuration or point propagation. Though only a simple Geometric CA was used for the experiments, we hypothesize that the results will extend to any geometric front propagation. The reason is that the edge stitching or collision effects, though temporarily disruptive, are resilient in the same manner as each point of a side propagation is temporarily disruptive but still retains the distinguishing behaviors of both the rule and Wolfram Class.



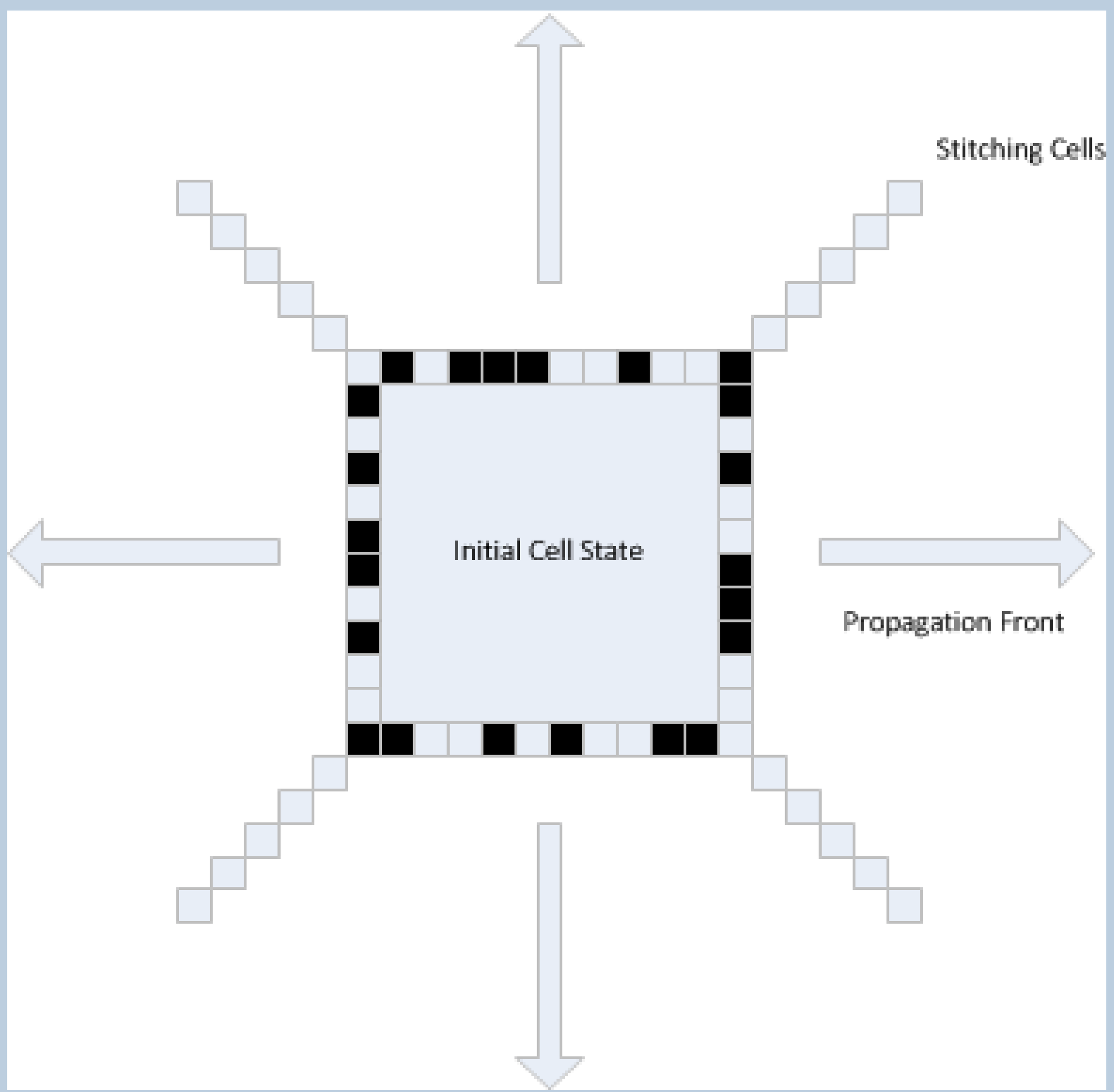
The above figures show a point propagation for rule 30 on the left compared to a geometric front propagation for rule 30 on the right. As can be seen, the essential elements of its Wolfram Class are retained in both.

Design and Implementation

Below is a class model of the Geometric CA application showing the inheritance and dependency chain of the main classes.



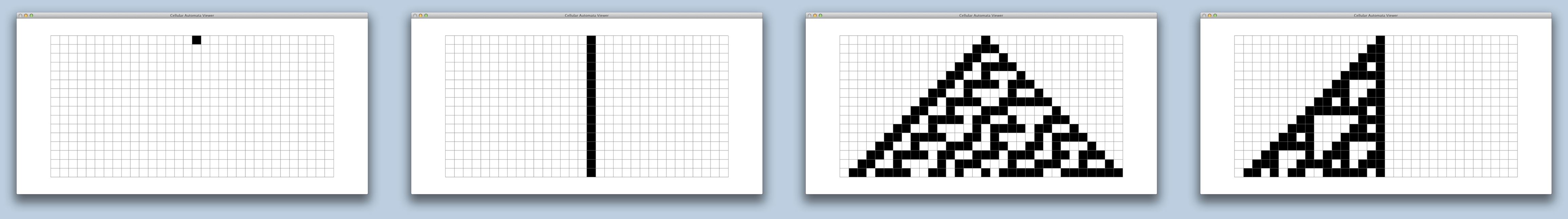
In order to accommodate the many variations of Cellular Automata types, we created a general purpose base class in C++ that facilitated a configurable neighborhood radius, multiple state cells, and various initial distribution of cells. This base class was then specialized into two distinct classes; one for Elementary CAs and the other for Totalistic CAs. A third specialization was necessary for the Geometric CA which was an aggregation of multiple CAs (one for each side of the geometric shape). This aggregation was key to allow stitching and propagation of edge states.



The above diagram illustrates the way in which propagation fronts expand outward from their initial configuration and the boundary cells shared by two sides as the sides expand outwards.

Validation

A validation was performed, comparing the results from our Cellular Automata application with the documented 1-D 256 rule Elementary Cellular Automata studies. All 256 results were generated through the application and were consistent with the published results of Wolfram's work.



The images above were created by the Geometric CA application to show that our results match the Wolfram rules exactly. The rules shown are for 160, 108, 30, and 110 from left to right.

Conclusion

Though further experimentation with other geometric structures is needed, we have shown that in the simple case of line based front propagation both class and rule behavior are retained regardless of perturbations due to interactions.

References

[1] Melanie Mitchell, *Complexity: A Guided Tour*. Oxford University Press, 2009.

[2] Stephen Wolfram, *Cellular Automata as Models of Complexity*. The Institute for Advanced Study, 1984.

[3] Wikipedia, *Cellular Automaton*. http://en.wikipedia.org/wiki/Cellular_automaton, 2013.