Ay190 – Worksheet 14
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Advection Equation 2: Electric Burger-loo

We are solving Burger's equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0,$$

with the initial condition

$$\Psi(x, t = 0) = \frac{1}{8}\sin\left(\frac{2\pi x}{L}\right),$$

where $L = 100$ on the domain $[0, L]$. Like worksheet 11, we use outflow boundary conditions, represented by the function "apply_bcs(y)" defined by the code:

```
y[0] = y[1]
y[-1] = y[-2]
```

which uses the value of the function in the last interior grid points as the value at the boundary points.

There are two changes from this code and the code for worksheet 11. First, "dt" is defined using the maximum value of $\Psi(x, t = 0)$ because we want a constant dt for all grid points. This done with the line:

```
dt = cfl*(dx/max(np.abs(analytic(x, L))))
```

. The other change is in calculating the upwind gridpoints. The velocity is not always the same sign in this case, which means that the direction opposite of the velocity will change here as opposed to worksheet 11. Therefore, we need to check the sign of the velocity and adjust how we calculate the upwind gridpoints appropriately:

```
for j in range(1, n-1):
    if yold[j] > 0:
        # positive velocity: get data from the left
        y[j] = yold[j] - (yold[j]*dt/dx)*(yold[j] - yold[j-1])
    else:
        # negative velocity: get data from the right
        y[j] = yold[j] - (yold[j]*dt/dx)*(yold[j+1] - yold[j])
```

.

Running the code, we see that the two sides of the sin function, after starting out evenly (figure 1) start to push towards each other (figures 2 and 3). At $t = 120$ we see that the center of the time-evolved function looks almost vertical (figure 4). At about $t = 140$, a jump (the shock) appears, evidenced by the fact that the center of the function is devoid of any crosses indicating data points: they've all migrated to one side of the jump (figure 5). By $t = 160$ it is solidly there (figure 6). So, using the upwind scheme, we've demonstrated that around $t = 140$ a shock forms in the solution to Burger's equation for this sin wave.

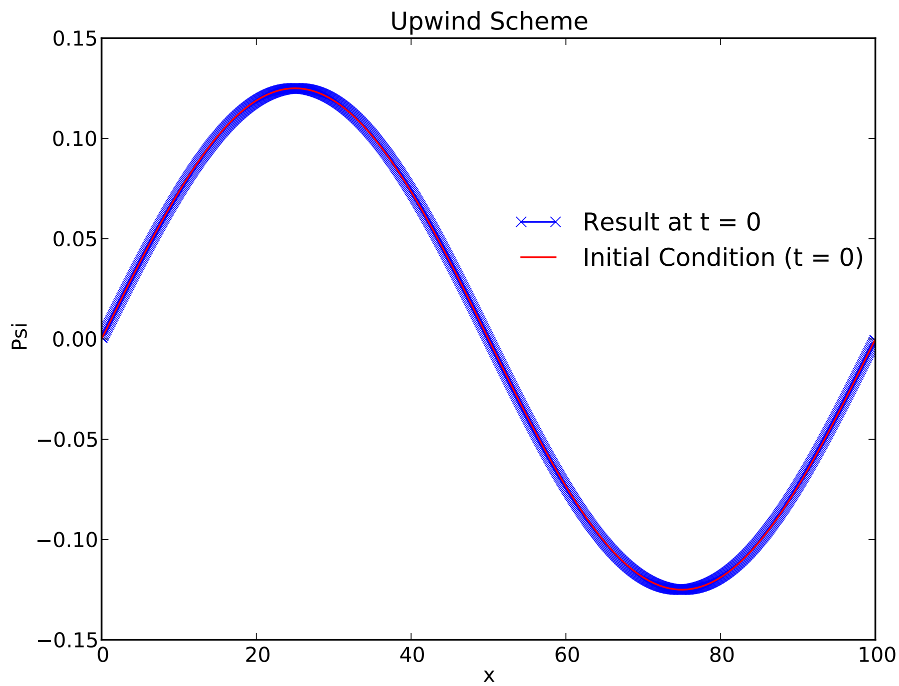

Figure 1: Plot of Ψ at $t = 0$.

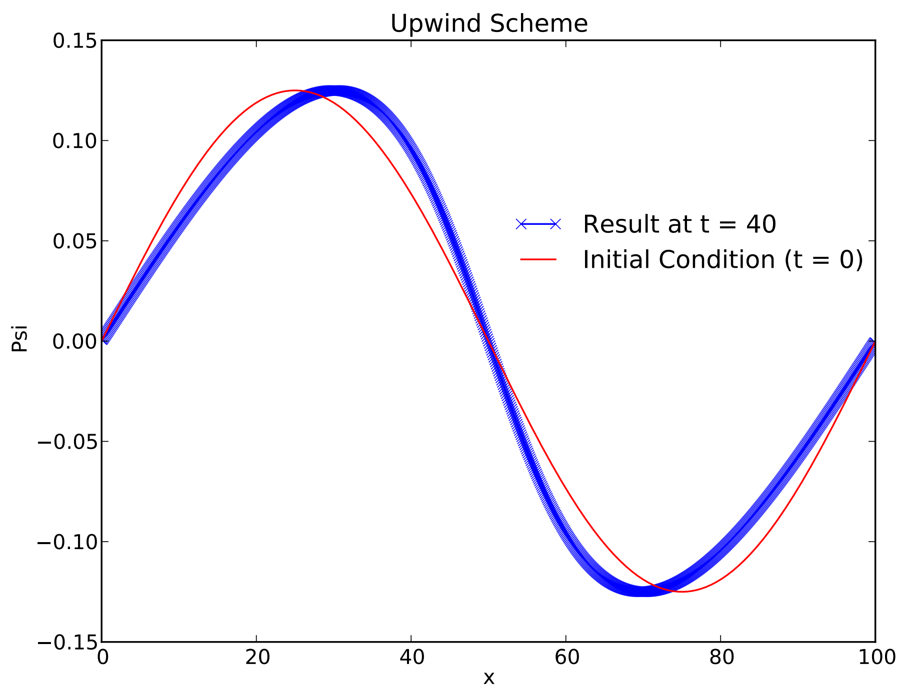

Figure 2: Plot of Ψ at $t = 40$ and initial condition.

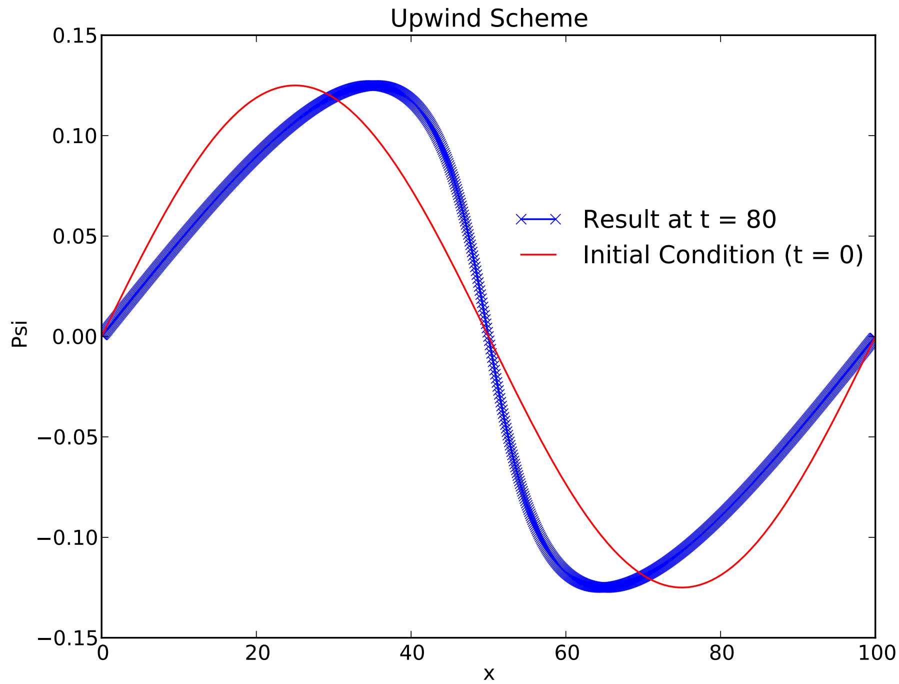

Figure 3: Plot of Ψ at $t = 80$ and initial condition.

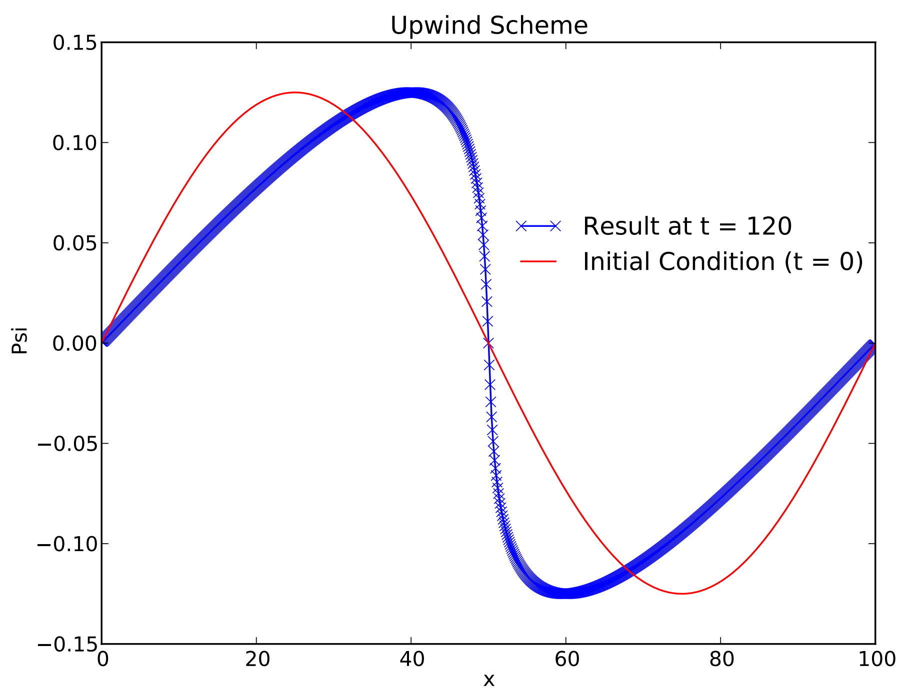

Figure 4: Plot of Ψ at $t = 120$ and initial condition.

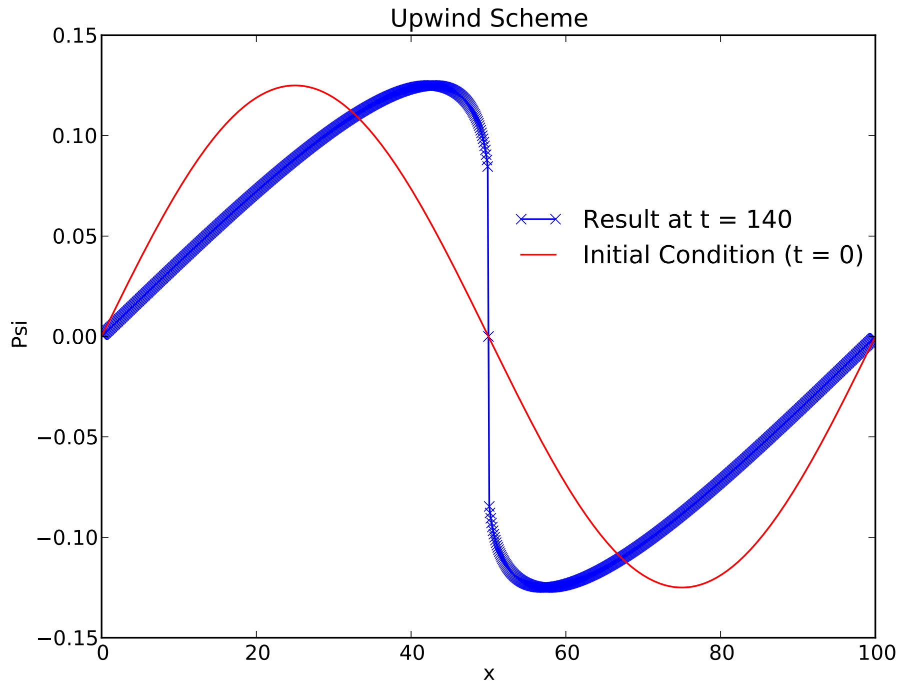

Figure 5: Plot of Ψ at $t = 140$ and initial condition.

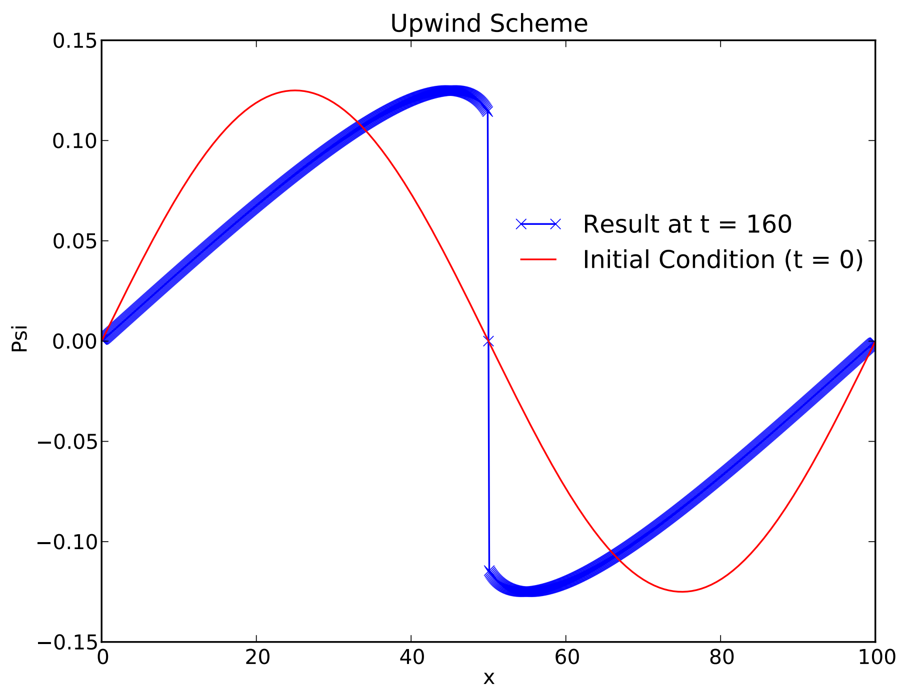

Figure 6: Plot of Ψ at $t = 160$ and initial condition.