Ay190 – Worksheet 16
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1D Finite Volume Hydro Code

1

This code solves a bunch of local Riemann problems by interpolating the midpoint of a pair of gridpoints using some kind of "reconstruction" and then calculating flux differences and integrating forwards in time.

For convenience in keeping track of all of the variables, the program defines a class called "mydata". In it, it distinguishes between primitive variables (density ρ, velocity v, internal energy per mass ϵ, and pressure P) and conserved variables ρ, ρv, and $\rho\epsilon + \frac{1}{2}\rho v^2$. The equation of state used is $P = (\gamma - 1)\rho\epsilon$.

First, the program sets up the grid with three ghost points on either side to let it calculate local Riemann problems on the edges. It then sets the initial conditions. The function "apply_bcs" applies boundary values to the ghost points by simply setting them equal to the last interior point.

It then calculates the smallest possible time step by taking the distance between two gridpoints, dividing it by the maximum of the sum and difference of that grid point's velocity and speed of sound, and choosing the minimum of the resulting Δt_i's.

There are three different types of reconstruction the program defines to interpolate the values of the variables at the interface. Piecewise constant, the simplest, sets the left and right values of the variables at the interface to be their values at the grid points. Total Variational Diminishing minmod reconstruction uses an estimate of the slope at the interface to calculate what the values there should be. If the slope at the two grid points around it are different signs, it sets the slope to 0 and this method reduces to the piecewise constant method at that point. The monotonized central method is a higher order method and uses a more accurate estimate of the slope. It should theoretically give the most accurate result.

Finally, at each iteration, the program calculates the flux at each interface by first finding the min and max characteristic speeds (s_{min} and s_{max} respectively) there (from the left and right grid points). The characteristic speeds are eigenvalues of the euler equations given by

$$\lambda_1 = v - c_s$$
$$\lambda_2 = v$$
$$\lambda_3 = v + c_s$$

It then needs to calculate the numerical fluxes F_j^L and F_j^R (from the left and right) with the code

```
for i in range(1,hyd.n-2):
    fluxl[0,i] = hyd.qp[0,i]*hyd.velp[i]
    fluxl[1,i] = hyd.qp[1,i]*hyd.velp[i] + hyd.pressp[i]
    fluxl[2,i] = hyd.velp[i]*(hyd.qp[2,i] + hyd.pressp[i])
```

```
fluxr[0,i] = hyd.qm[0,i+1]*hyd.velm[i+1]
fluxr[1,i] = hyd.qm[1,i+1]*hyd.velm[i+1] + hyd.pressm[i+1]
fluxr[2,i] = hyd.velm[i+1]*(hyd.qm[2,i+1] + hyd.pressm[i+1])
```

which are equations taken from IV.5.40 in the notes. With the numerical fluxes and characteristic speeds, the flux at each interface $x_{i+1/2}$ is given by

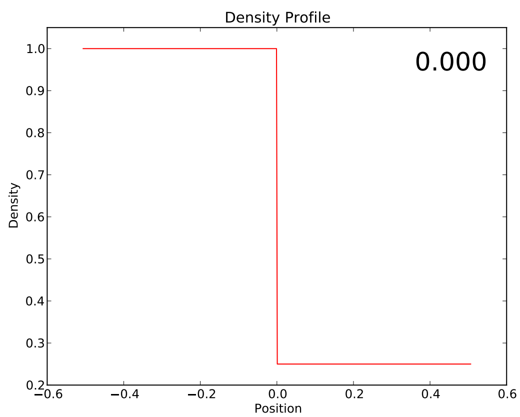
$$f_j(x_{i+1/2}) = \frac{s_{max} F_j^L - s_{min} F_j^R + s_{min} s_{max} (q_j^R - q_j^L)}{s_{max} - s_{min}}$$

where j goes from 1 to 3 and represents that number Euler equation, and q_j is the conserved quantity corresponding to the j^{th} Euler equation. Finally, it calculates the flux difference (FD) at each grid point using the values at the interfaces:

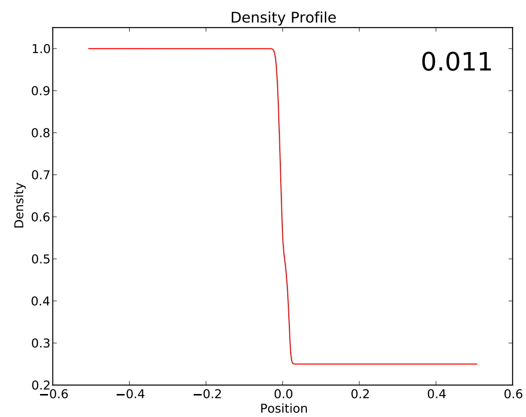
$$FD_i = \frac{1}{\Delta x_i}[f(x_{i+1/2}) - f(x_{i-1/2})].$$

Every 10 iterations, the program plots the density profile of the solution. Some of these plots are shown below in figures 1 to 5 which use piecewise constant reconstruction.

Just like ws15, we see the expected zones: the first and fifth are constant with the initial densities as the value, and there is a rarefaction zone as well as clear contact and shock close-to-vertical lines.



Piecewise Constant (PC) Density profile: Iteration 0

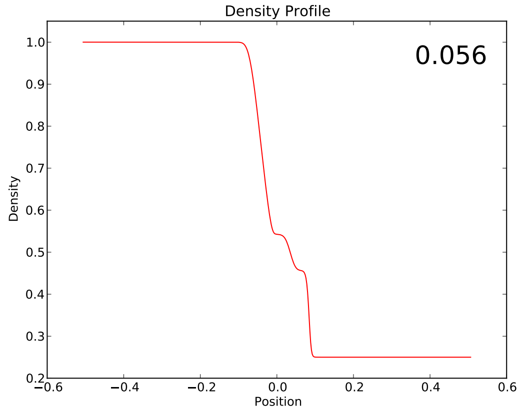


Iteration 80

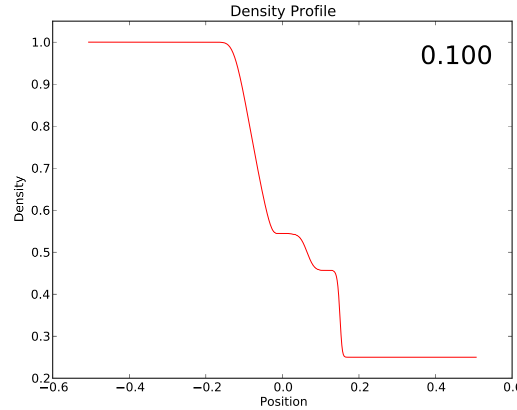
Figure 1

2

Plotting the density at $t = 0.2$ in figure 6 for the three different reconstruction methods, we see that as expected, the monotonized centered reconstruction produces the best result. The lines at about $x = 0.15$ and $x = 0.3$, which are supposed to be vertical, are most vertical using that reconstruction method.

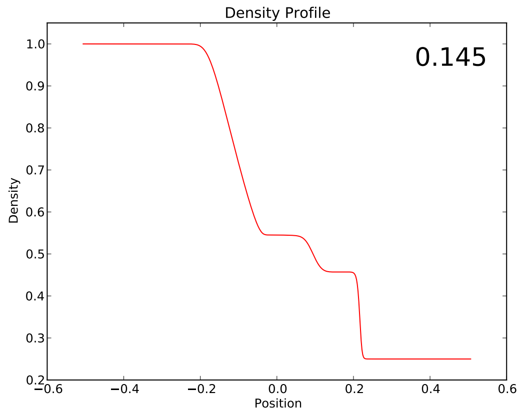


Iteration 160

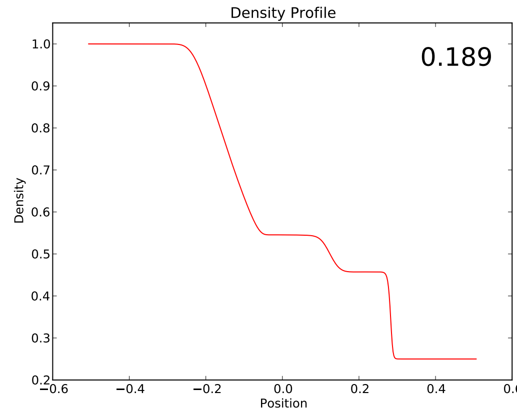


Iteration 240

Figure 2



Iteration 320

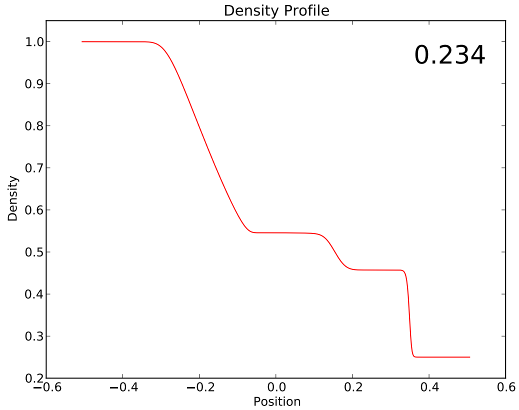


Iteration 400

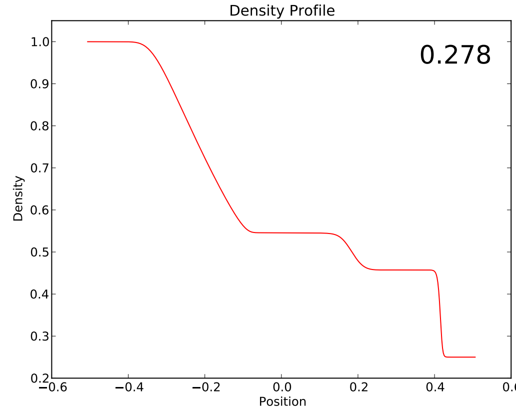
Figure 3

3

We now compare the best reconstruction method (MC) to the results from the previous worksheet (the SPH solver and the exact solver). We plot the density, velocity, and pressure profiles in figures 7, 8, and 9 respectively. The MC reconstruction method is indeed very good, as it almost completely covers up the exact solution at $t = 0.2$. The only location that seems to be innacurate is the beginning of the rarefaction zone around $x = -0.2$. Notably, there are no curved edges like in the SPH solution, and there's no pressure instability spike at $x = 0.15$.

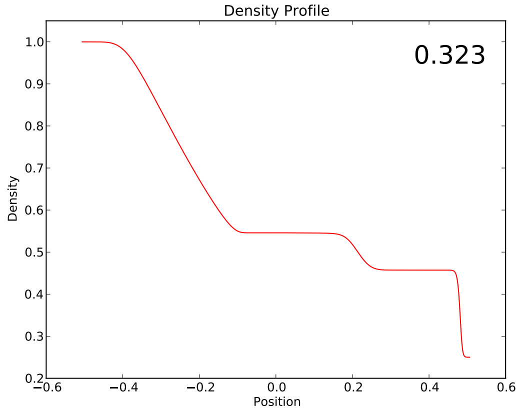


Iteration 480

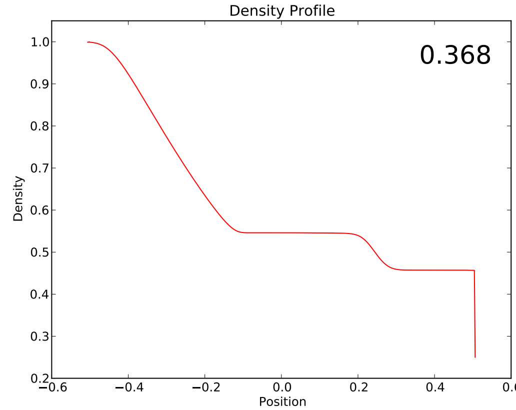


Iteration 560

Figure 4



Iteration 640



Iteration 720

Figure 5

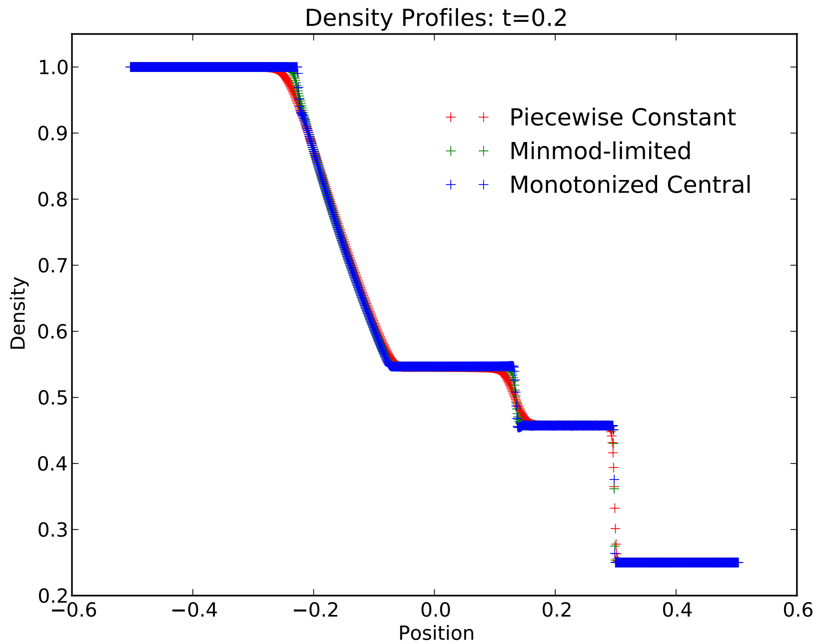


Figure 6: Density profiles of Piecewise Linear, TVD-minmod, and TVD-MC2 reconstruction methods at $t = 0.2$.

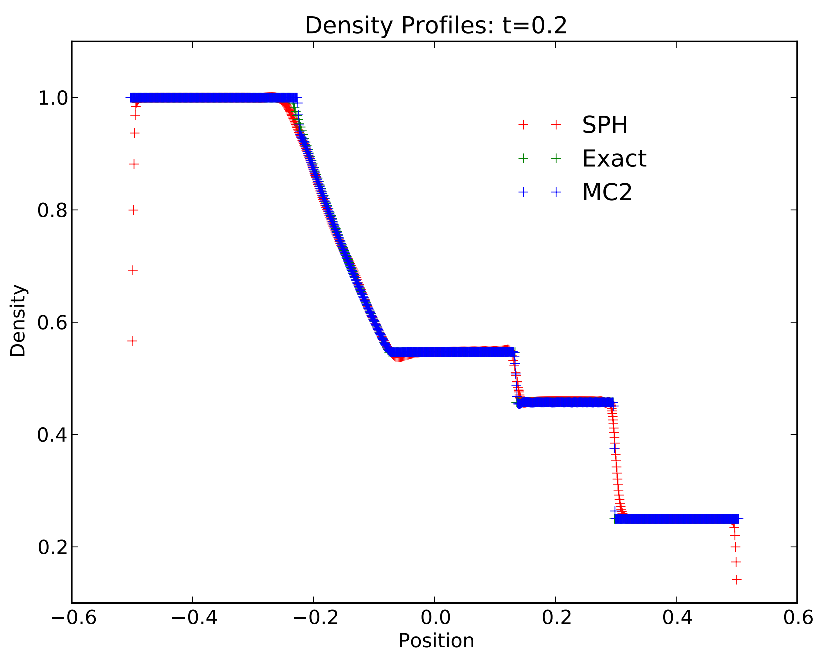


Figure 7: Density profiles of SPH, exact Riemann solver, and TVD-MC2 reconstruction at $t = 0.2$.

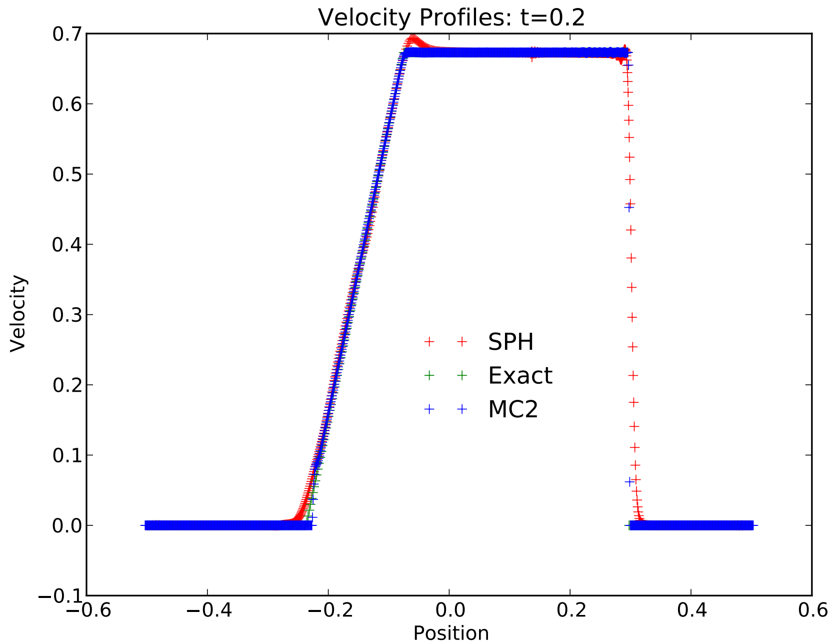


Figure 8: Velocity profiles of SPH, exact Riemann solver, and TVD-MC2 reconstruction at $t = 0.2$.

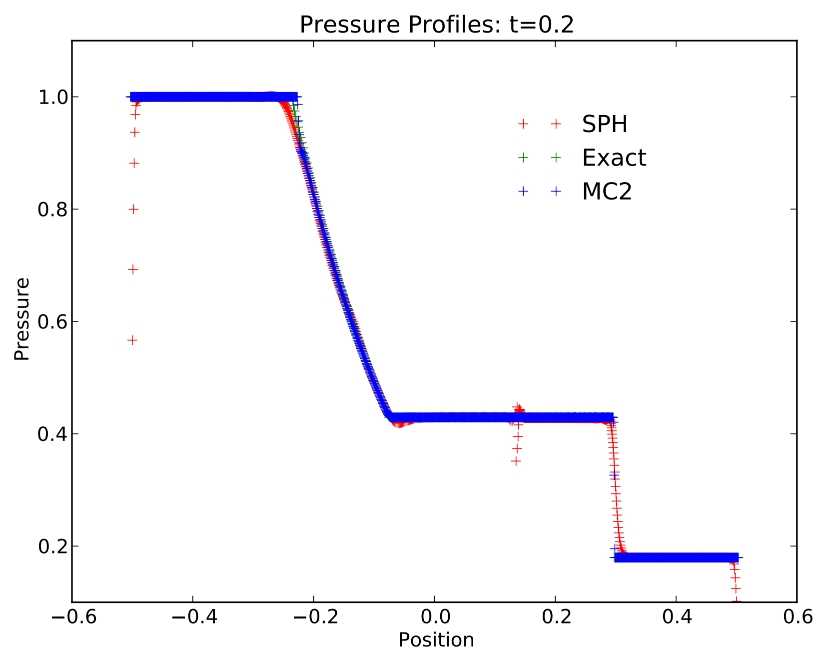


Figure 9: Pressure profiles of SPH, exact Riemann solver, and TVD-MC2 reconstruction at $t = 0.2$.