Ay190 – Worksheet 4
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1 Root Finding

a

In this problem, we want to numerically calculate the root of the equation

$$E - \omega t - e \sin E = 0$$

for a given angular frequency ω, time t, and eccentricity e. To do this, we choose the Secant method, in which points are recursively chosen via the relation

$$x_{n+1} = x_n - f(x_n)\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

until the fractional error of some x_k is below the threshold ϵ, which we choose to be $\epsilon = 10^{-10}$. To begin the process, we must first make two guesses at the root. Since e is small, it is sensible to choose $x_0 = \omega t$ as the first guess of the root (ignoring the small $e \sin x$ term), and then take a point near that, say $x_1 = 0.9x_0$ as the second guess. After applying the Secant method, we get the results for three different times, shown below.

time (days)	E	# of iterations	(x, y) (AU)
91.0	1.58209228899	4	(-0.0112957219731, 0.99965732909)
182.0	3.13096420068	3	(-0.999943518526, 0.0106252886903)
273.0	4.67948910053	4	(-0.0328939450239, -0.999180108689)

We see that the three times correspond to the Earth being 1/4 along, 2/4 along, and 3/4 along its orbit. The convergence is very quick, taking a maximum of 4 iterations to be within our 10^{-10} threshold.

b

Here, we increase the eccentricity drastically, from 0.0167 to 0.99999, meaning the orbit is now essentially a straight line on the x-axis. The table of results below now shows the Secant method taking longer to find the root, especially for the 1/4 and 3/4 orbit times.

time (days)	E	# of iterations	(x, y) (AU)
91.0	2.30664638749	7	(-0.671217514443, 1.48251347872e-05)
182.0	3.13618964107	4	(-0.999985403763, 1.08059184419e-07)
273.0	4.67948910053	6	(-0.680720102446, -1.46507989671e-05)

It may be possible, in this case, to decrease the number of iterations necessary by using the exact derivative, since we can calculate it analytically. Doing so does indeed reduce the number of iterations for the 1/4 and 3/4 orbit times by 1. Since $e \approx 1$, ignoring the sin term no longer makes as much sense, so choosing a better first guess would also accelerate the convergence for this high eccentricity orbit calculation.