Ay190 – Worksheet 13
Daniel DeFelippis
Date: March 2, 2014

Direct Summation N-Body Code

We are solving an N-body gravitational system:

$$\ddot{\mathbf{x}}_i = -G \sum_{j=1, i\neq j}^{N} \frac{m_j}{|\mathbf{x}_{ij}|^2} \hat{\mathbf{x}}_{ij}$$

using direct summation code. The data file has six rows: three coordinates (xyz) and three velocities (xyz). I edited the code skeleton to create a function which computes the right hand side (RHS) of the resulting three equations for each object, as well as an RK4 integrator.

1

By running the code for the Sun-Earth system, we confirm that the code works: the Earth rotates the Sun at a visually constant radius over a year. However, if the step size is too small, going for multiple years results in the Earth spiraling inwards and being flung out of orbit fairly quickly. To see why this happens, I wrote a function which computes the total energy of the N-body system given by

$$E_{tot} = \sum_{i=1}^{N} \frac{1}{2} m_i |\mathbf{v}_i|^2 - G \sum_{i,j=1, i\neq k}^{N} \frac{m_i m_j}{|\mathbf{x}_{ij}|}.$$

Graphing the total energy over time with different values of the number of steps, we see that the energy doesn't necessarily remain constant. Sometimes it spikes up to a very large positive value, corresponding to the Earth being flung out of orbit. Even when that doesn't happen though, for larger numbers of time steps, the total energy oscillates, as shown by the green line in figure 1. Energy doesn't remain constant here probably because the orbit is discretized compared to what actually happens, causing small errors in the location of the Earth and therefore errors in the energy.

2

Next, we use our code on a more complicated system, a 13-star system orbiting a black hole. If I used too many steps, the code became unbearably slow, so I limited myself to 200 steps over 10 years. If we plot the energy here, we again see that the stars are flung out of orbit even more quickly than in the Sun-Earth system, as shown in figure 2. They didn't all get flung out at the same time: the horizontal sections correspond to a particular star maintaining an "orbit" for a short period of time longer than the rest of the stars.

Getting rid of the unit conversions so we can just watch the stars evolve on a scale of arcseconds, we see that they quickly exceed the maximum initial value of each coordinate very quickly, contrary to the actual data from the data file from the uchicago website. In that actual data, the stars

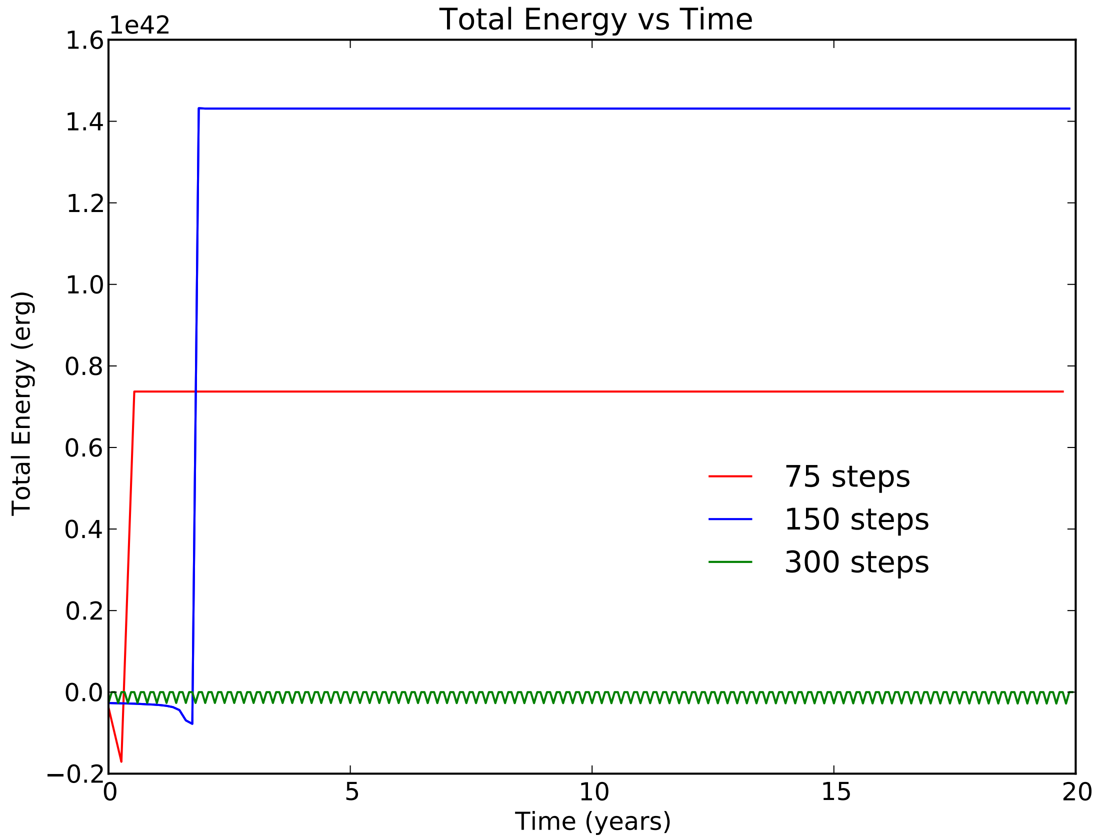

Figure 1: Plot of Total Energy vs time for Sun-Earth system

never have a coordinate with a larger magnitude than 3 arcseconds. My code however places the stars very much outside that range quickly, which can be seen by increasing value of "rmax" on the 3D plot and watching the stars *still* exceed the axes. So, as expected, this basic method of evolving N-body systems does not work very well for even $N = 14$.

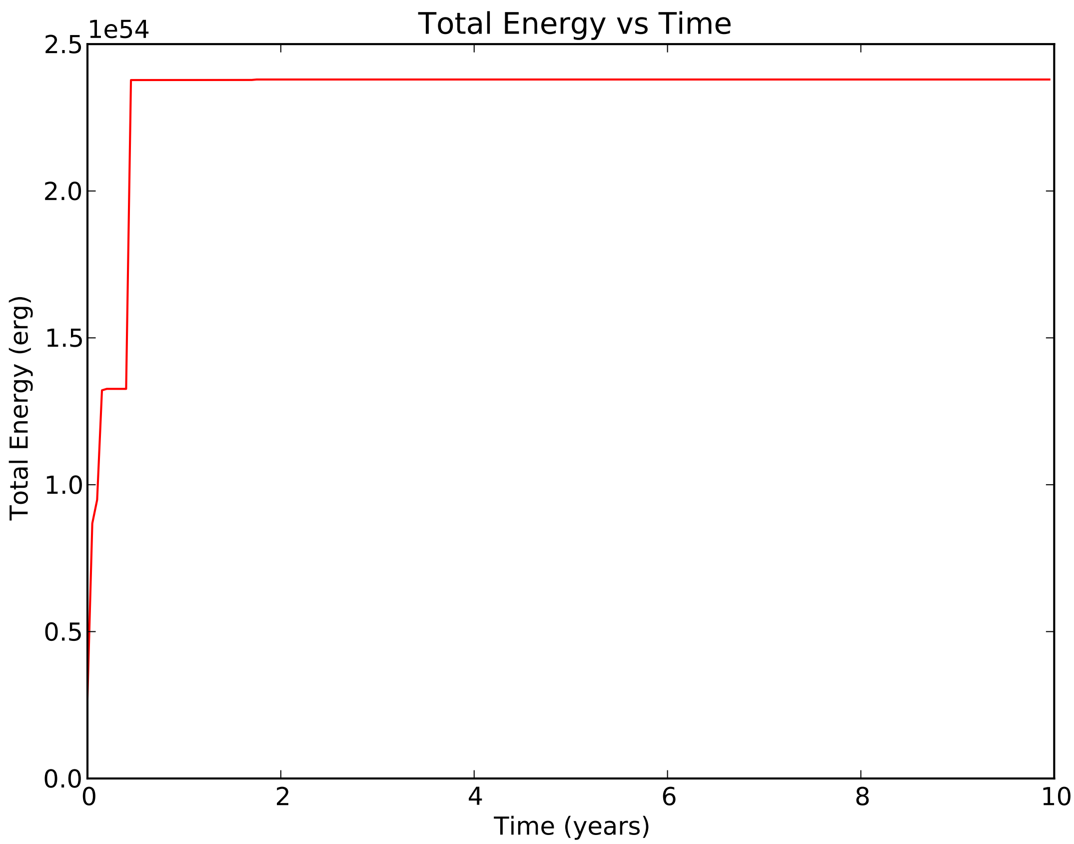

Figure 2: Plot of Total Energy vs time for 13 Stars and Black Hole