Numerical Techniques 2024–2025

Student projects

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General idea

Starting from a simple model, apply some of the techniques from this course.

General remarks:

- no course on Python; no course on complex algebra;
  ask for help!
- open assignment: no exact solution + discussion is more important than results
- the purpose is you *learn* something
- try to trigger strange/unwanted phenomena, and discuss solutions.
- you can propose a topic yourself.

Shallow Water Equations

The linearized 1D shallow water equations (SWE) are given by:

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = 0$$
$$\frac{\partial h}{\partial t} + H\frac{\partial u}{\partial x} + U\frac{\partial h}{\partial x} = 0$$

where g is gravity, U and H are the constant basic-state velocity and water height, and $u(x,t)$ and $h(x,t)$ are the perturbations on the velocity and the water height.

The current model is very simple:

- Leapfrog time integration
- Second-order centered space differencing
- Periodic boundary conditions

Student projects:

1. Stagger u and h
2. Compare with nonlinear model
3. Transform into spectral model with Fourier decomposition
4. Transform into spectral model with Chebyshev decomposition
   J.P. Boyd, *Chebyshev and Fourier Spectral Methods*
5. Study Laplace transform integration
   Clancy and Lynch, 2011, QJRMS 137,
   Laplace transform integration of the shallow-water equations

Barotropic Vorticity Equation

Barotropic Vorticity Equation

- The barotropic vorticity equation BVE is given by:

$$\frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta = 0$$

where ζ is the vorticity, and $\mathbf{u}$ is the geostrophic wind, given by

$$\mathbf{u} = \mathbf{k} \times \nabla \psi \qquad\qquad \nabla^2 \psi = \zeta$$

- Writing everything in terms of the streamfunction ψ, the BVE becomes

$$\frac{\partial \nabla^2 \psi}{\partial t} + J(\psi, \nabla^2 \psi) = 0$$

where J is the Jacobian operator:

$$J(p, q) = \frac{\partial p}{\partial x}\frac{\partial q}{\partial y} - \frac{\partial p}{\partial y}\frac{\partial q}{\partial x}$$

Some background info

- This model was used for the first numerical weather prediction in 1950 on the ENIAC computer
- A 24-hour forecast took about 24h computing time

Original model

The original (1950) model is characterized by:

- a leapfrog time integration
- second-order centered space differences
- an expensive (pseudo-spectral) way to invert the Laplacian operator
- heuristic boundary conditions:
  - $\frac{\partial \psi}{\partial t} = 0$ on boundary
  - $\frac{\partial \nabla^2 \psi}{\partial t} = 0$ for entering fluid

Due to these boundary conditions and aliasing, the model is not stable.

The student's model is somewhat simplified:

- Spectral inversion of Laplacian
- Periodic boundary conditions
- Coriolis effect removed
- Projection impact removed

Improvements (student projects)

General remark: lots of interesting aspects in this model, but you'll have to dig deeper to find them.

Student projects:

6. Semi-Lagrangian scheme
7. High-resolution LAM nested in low-resolution LAM (coupling with Davies relaxation)
8. Spectral model and avoiding aliasing
9. Check energy cascade between large and small scales, and implement Arakawa Jacobian

Practical aspects

- Groups of 2/3 persons
- Pick a single topic (e.g. SWE-spectral); post your group + choice on Ufora forum.
- Jupyter notebooks for SWE and BVE on Ufora.
- More detailed background info (papers) on Ufora.
- Support sessions: **4 and 11 December**, 16h00-17h30; come prepared!
- Report (say 5–10 pp.): deadline Thursday 14 December.
- Presentation for other students: Monday 18 December.