MATHEMATICAL QUALITY OF INSTRUCTION (MQI)
4-POINT VERSION

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SEGMENT CODES

Classroom Work is Connected to Mathematics	
Score here for whether the focus is on *mathematical content* during half or more of the segment (3.75 minutes or more total for a 7.5-minute segment).	
No	**Yes**
Focus for majority of the segment (at least 3.75 minutes for a 7.5-minute segment) is on non-mathematical topics, or student activities that have no clear connections to developing mathematical content. Examples: • Gathering or distributing materials, other administrative issues • Disciplinary issues that severely impinge upon instructional time • Students doing an activity (cutting, pasting, coloring) that is not clearly connected to mathematics (“bad reform”)	Focus is on mathematical content for majority of the segment (at least 3.75 minutes for a 7.5-minute segment). Examples: • Teacher reviewing content from a prior lesson • Teacher introducing content • Students practicing content • Students working on a warm-up problem while teacher takes attendance

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Richness of the Mathematics

This dimension attempts to capture the depth of the mathematics offered to students. The codes within this dimension are grouped into two broad categories: codes that capture the extent to which instruction focuses on the *meaning of facts and procedures* (Linking Between Representations, Explanations, and Mathematical Sense-Making), and codes that capture the degree to which instruction focuses on *key mathematical practices* (Multiple Procedures or Solution Methods, Patterns and Generalizations, and Mathematical Language).

For all codes within this dimension, the aspect of instruction must be substantially correct to count as Low, Mid or High. Richness elements that are not correct should be ignored (though the segment can be still credited for other correct elements within the same code).

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Linking Between Representations

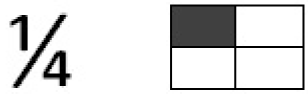
This code refers to teachers' and students' explicit linking and connections between different representations of a mathematical idea or procedure. To count, these links must occur across different representational "families" e.g., a linear graph and a table both capturing a linear relationship. So, two different representations that are both in the symbolic family (e.g., 1/4 and 0.25) are not candidates for being linked.

For Linking Between Representations to be scored above a Not Present:

- At least one representation must be visually present
- The explicit linking between the two representations must be communicated out loud

For Linking Between Representations to be scored Mid or High, two conditions must be satisfied:

- Both representations must be visually present
- The correspondence between the representations must be explicitly pointed out in a way that focuses on meaning (e.g., pointing to the numerator in 1/4, then commenting that you can see that one in the figure, pointing to the four in the denominator, pointing to the four partitions in the whole. "You can see the 1 in the 1/4 corresponds to the upper left-hand box, which is shaded, showing one piece out of four total pieces...")



For geometry, we do not count shapes as a representation that can be linked—we consider those to be the "thing itself." However, links can be scored in geometry if the manipulation of geometric objects is linked to a computation, e.g., showing that two 45-degree angles can be combined to get a 90 degree angle and linking that to the symbolic representation 45 + 45 =90.

Note: If links are made but underlying representation/idea is incorrect, do NOT count as linking between representations.

Not Present	Low	Mid	High
No linking occurs. Representations may be present, but no connections are actively made.	Links are present in a pro forma way; For example, the teacher may show the above figure and state that one quarter is one part out of four. These links will not be very explicit or detailed; both representations need not be present.	Links and connections have the features noted under High, but they occur as an isolated instance in the segment.	Links and connections are present with extended, careful work characterized by one of the following features: • Explicitness about how two or more representations are *related* (e.g., pointing to specific areas of correspondence) OR • Detail and elaboration about the relationship between two mathematical representations (e.g., noting meta-features; providing information about under what conditions the relationship occurs; discussing implications of relationship) These links will be a characterizing feature of the segment, in that they may in fact be the focus of instruction. They need not take up the majority or even a significant portion of the segment; however, they will offer significant insight into the mathematical material.

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Explanations

Mathematical explanations focus on why, e.g.:

- Why a procedure works (or doesn't work)
- Why a solution method is appropriate (or inappropriate)
- Why an answer is true (or not true)
- In geometry: justification using a definition, why an object is symmetrical, why a second figure is a transformation of the first
- In data analysis: why you would choose a specific graph to represent a set of data, why median is different than mode or mean of a dataset, etc.

Do NOT count *"how"* e.g., simply providing descriptions of steps (first I did x, then I did y) or definitions unless meaning is also attached.

Note: Do NOT count incorrect or incomplete explanations as explanations.

Not Present	Low	Mid	High
No mathematical explanations are offered by the teacher or students or the "explanations" provided are simply descriptions of steps of a procedure.	A mathematical explanation occurs as an isolated instance in the segment.	Two or more brief mathematical explanations occur in the segment OR an explanation is more than briefly present but not the focus of instruction.	One or more mathematical explanation(s) is a focus of instruction in the segment. The explanation(s) need not be most or even a majority of the segment; what distinguishes a High is the fact that the explanation(s) are a major feature of the teacher-student work (e.g., working for 2-3 minutes to elucidate the simplifying example above).

Scoring Help - Explanations

Examples of explanations:

- Explaining the reason for steps in simplifying fractions (dividing by 2/2 is same as dividing by 1; anything divided by 1 is still itself)
- Explaining why particular steps in a complex problem are justified or work to achieve the solution
- Classifying triangles as polygons because they are closed and made up of line segments that do not cross
- Explaining why a formula can be used to find an outcome (why ℓ x w works to find area)

Note that when scoring, you can count the build-up to an explanation as part of the explanation. Ask yourself: Was the point of the instruction to provide the explanation, even if it only emerged at the end? If so, you may score that clip as a High.

To help understand the difference between the Explanations code and the Mathematical Sense-Making code, see the Scoring Help for Sense Making.

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Mathematical Sense-Making

This code captures the extent to which the teacher or students attend to one or more of the following:

- The meaning of numbers
- Understanding relationships between numbers
- The relationships between contexts and the numbers or operations that represent them
- Connections between mathematical ideas or between ideas and representations
- Giving meaning to mathematical ideas
- Whether the modeling of and answers to problems make sense

Examples include:

- Focusing on value of quantities (e.g., "7/8 is close to 1")
- The meaning of quantities (e.g., "the six represents the number of groups")
- Discussing reasonableness of an expression, solution method, or answer
- Using estimation or number sense
- Giving meaning to procedures (e.g., "1/4 x 2/3 means taking 1/4 *of* 2/3 *of* a whole")
- Giving meaning to expressions or equations

For word problems, score for activities like explaining why an operation is called for by a problem, why certain numbers are used in the operation, reasonableness of answer, reasonableness of solution method, etc.

In geometry, include making sense of definitions (what counts as a polygon, what does not count as a polygon), formulas, by elaborating them, applying them, finding counter-examples, etc. rather than just stating/executing them. Do not count "Give me examples of a circle" – instead, count cases where the definition or formula has meaning made around it.

If sense-making is partially correct and partially incorrect, only score the portion that is correct (e.g., would be a High, but vague for parts, thus receives a Mid).

Not Present	Low	Mid	High
Not present or incorrect.	Teacher and/or students focus briefly on meaning. For instance, a student may remark that 7/8 is "almost 1" or attends to reasonableness of the solution method.	Teacher and/or students focus on meaning more than briefly (e.g., several instances within the segment or one somewhat long instance), but this work is not sustained or substantial.	Teacher and/or students focus on meaning in sustained way during segment. Need not be the entire segment, but must be substantial.

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Scoring Help - Mathematical Sense-Making
In many cases, Sense-Making overlaps with events already scored in Explanations. For example, a teacher may provide the explanation that dividing both the numerator and denominator by 4 is in essence dividing by 4/4. And because dividing by 4/4 is the same as dividing by 1, dividing by 4/4 actually does not change the value of the original fraction. This explanation would also count as sense-making, as the teacher is giving meaning to the fraction 4/4 and the procedure of making equivalent fractions. While many explanations will also qualify as Sense-Making, some will not. For example, a teacher who walks through an algebraic/geometric proof may get credit for explanations for explaining why a solution is true without meaning to the mathematical ideas. There also are instances of Sense-Making that do not count under Explanations. For example, attention to any of the following may be scored as Sense-Making without meeting the criteria for Explanations: • The value or meaning of quantities • The reasonableness of an expression or answer • Using estimation or number sense • Making sense of word problems Finally, it is important to note that instances that count under both Sense-Making and Explanations won't necessarily earn the same score point. For both codes, we ask raters to assess the *quantity* of the code, i.e., whether it occurs at all, is brief, or is more extended. However, a High for Explanations means that the instance is *the main feature of the segment*, whereas under Sense-Making it just needs to be *sustained*. Additionally, when scoring Explanations, you can count the build-up as a part of the explanation, even if that build-up is procedural. When scoring Sense-Making, only count time in which sense is actually being made; we do not count related procedural work or build-up as sense-making.

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Multiple Procedures or Solution Methods

Multiple procedures or solution methods occur or are discussed in the segment:

- Multiple solution methods for a single problem (including shortcuts)
- Multiple procedures for a given problem type

Defined as, e.g.:

- Taking different mathematical approaches to solving a problem (e.g., comparing fractions by finding a common denominator AND comparing fractions by finding a common numerator)
- Solving or discussing how to solve a word problem using two different strategies.

If the initial strategy or strategies occurred in a prior segment, score Multiple Procedures in the subsequent segment (i.e., no need to go back and adjust your score in the initial segment).

Note: Do NOT count incorrect procedures or solution methods.

Not Present	Low	Mid	High
No evidence of multiple procedures or solution methods for single problem or a given problem type.	Teacher or student briefly mentions a second procedure or method, but the method is not discussed at length or enacted ("we also showed yesterday that you can do it XYZ").	Multiple procedures or solution methods occur or are discussed in the segment (e.g., solving division problems in two ways), but does not include the special features listed in High, or feature these only momentarily (e.g., "this method is easier than the other" without explicit discussion of why).	Multiple procedures or solution methods occur or are discussed in the segment, and include special features: • Explicit comparison of multiple procedures or solution methods for efficiency, appropriateness, ease of use, or other advantages and disadvantages • Explicit discussion of features of a problem that cues the selection of a particular procedure • Explicit connections between multiple procedures or solution methods (e.g., how one is like or unlike the other)

Scoring Help - Multiple Procedures and Solution Methods

You will need to use some judgment when deciding whether to count two methods as distinct from one another. We consider methods distinct when they feature two different mathematical paths to the solution. For instance, in the case of comparing fractions, we would NOT consider it distinct if student A compares 3/5 and 7/10 by finding a common denominator of 10, and student B finds a common denominator of 50. However, we would consider finding common numerators and finding common denominators to be distinct methods.

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Patterns and Generalizations

This code is meant to capture instruction during which the class *first* examines instances or examples, *then* uses this information to develop or work on a mathematical generalization; to notice, extend or generalize a mathematical pattern; to derive a mathematical property; or to build and test definitions.

Examples of this activity include:

- Examining particular cases and then noticing and extending a pattern (e.g., looking at the sum of the angles in 3, 4, 5, and 6-sided regular polygons and extending the pattern or generalizing to an n-sided regular polygon)
- Saying whether mathematical procedures work in all cases
- "Building up" a mathematical definition or deriving a mathematical property (e.g., defining "polygons" after considering different examples and non-examples of polygons)

Notes:

- Patterns, generalizations and definitions must be based on *at least two examples* (either explicitly worked on or referred to)
- Do NOT count incorrect generalizations, incorrect pattern noticing, or incorrect definition building
- Do NOT count when teachers and/or students *state* generalizations, patterns, or definitions without first developing them from examples

Not Present	Low	Mid	High
No generalizations are developed or worked on; no patterns are noticed or extended; no definitions are built or tested.	There is brief work on developing a generalization or building a definition, but this work is undeveloped and/or is not the primary focus of the segment. OR Teachers and/or students engage in pattern-noticing and/or extending. This is done in a pro forma way (e.g. red, blue, blue, red, blue, blue, ??, blue blue)	There is work on developing a generalization, extending a pattern or building a definition, but the work is not finalized. For instance, a pattern may be noticed, extended, or reasoned about but not codified ("it looks like when we increase the coefficient, the line might get steeper"). OR Teachers and/or students develop a generalization, extend a pattern, or build a definition, but the work is not complete, clear or detailed.	The pattern or generalization is codified, AND the work is complete, clear and detailed. For instance, the teacher and/or students may carefully develop a generalization from examples in detail; or summarize and codify a pattern by describing how the pattern is generated.

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Mathematical Language

This code captures how fluently the teacher (and students) use mathematical language and whether the teacher supports students' use of mathematical language.

Examples:

- Fluent use of technical language
- Explicitness about mathematical terminology
- Encouraging students to use mathematical terms

Not Present	Low	Mid	High
Score here when NO mathematical terms are used. Teacher uses non-mathematical terms to describe mathematical ideas and procedures AND/OR teacher talk is characterized by sloppy/incorrect use of mathematical terms.	Low density of mathematical language. Not necessarily an indication that teacher is not "fluent" in mathematics, but simply a segment where few mathematical terms are used, or the same term is used over and over without features of High. Also score as Low when segment has middling density, but sloppy use.	Teacher uses mathematical language as a vehicle for conveying content, with middling density. However, the segment has few or none of the special features listed under High. Also score as Mid when segment has both features of High but includes some linguistic sloppiness or low density.	Teacher uses mathematical language correctly and *fluently*. Can be achieved in two ways: 1. Density of mathematical language is high during periods of teacher talk. 2. Moderate density, but also explicitness about terminology, reminding students of meaning, pressing students for accurate use of terms, encouraging student use of mathematical language. Instances of students using sophisticated mathematical vocabulary can also count toward a High.

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Overall Richness of the Mathematics

This code captures the depth of the mathematics offered to students.

Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of richness.

Not Present	Low	Mid	High
Elements of richness are present but are all incorrect OR Elements of rich mathematics are not present.	Elements of rich mathematics are minimally present. Note that there may be isolated Mid scores in the codes of this dimension.	Elements of rich mathematics are more than minimally present but the overall richness of the segment does not rise to the level of a High. For example, a segment may be characterized by some Mid scores in the codes of this dimension or by an isolated High along with substantial procedural focus, etc.	Elements of rich mathematics are present, and either: a) There is a combination of elements that together saturate the segment with rich mathematics either through meaning or mathematical practices. OR b) There is truly outstanding performance in one or more of the elements.

Scoring Help - Overall Richness of the Mathematics

In scoring Overall Richness, we assign a score of Not Present when there are no elements of richness present in the segment, or the components of richness that are present are all incorrect. For this code, we do not consider middling density of Mathematical Language to be an element of richness. That is, a segment could get a score of Low or Mid for Mathematical Language and still get a score of Not Present for Overall Richness.

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Working with Students and Mathematics

This dimension captures whether teachers can understand and respond to students' mathematical contributions (utterances or written work) or mathematical errors. Student contributions include, but are not limited to, questions, claims, explanations, solution methods, ideas, etc. By students' mathematical errors, we mean those incorrect student contributions that offer opportunities for addressing student difficulty.

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Remediation of Student Errors and Difficulties

With this code, we mean to record instances of remediation in which student misconceptions and difficulties with the content are addressed.

Conceptual remediation gets at the root of student misunderstandings, rather than repairing just the procedure or fact. Conceptual remediation includes:

- *Identifying/addressing the source of student errors or misconceptions*: "I noticed that some of you seem to think that 1.024 is a larger number than 1.1. I think you were noticing the number of digits to the right of the decimal point rather than thinking about the place value."
- *Pointing to underlying meaning when responding to errors*: "I noticed that some of you seem to think that 1.024 is a larger number than 1.1. Both numbers start with 1. But what value do they have in the tenths place? Zero tenths, one tenth."

Procedural remediation corrects student problems with procedures (e.g., re-demonstrating the procedure for addition of fractions with unlike denominators without reference to why the procedure works or sense-making around the quantities). To score an instance of procedural remediation, there must be *more than a simple correction* of a student mistake.

Examples such as "no, that is not correct" or "you should have gotten 9" should be considered simple *corrections* rather than remediation because they do not address student difficulty. Examples of corrections could include correcting a misunderstanding about a definition ("This is an expression." "No, it's an equation.") or correcting the result of the calculation without talking about the calculation.

If some portion of the remediation muddles the mathematics, the score may be adjusted downward.

Notes:

- Remediation can occur during active instruction or small group/partner/individual work time.
- Remediation must have mathematical content.
- If teacher prompts a student to remediate another student, it can be scored as present as long as the remediation is correct.
- Pre-remediation (calling students' attention to a common error) counts as a Mid or High, depending upon the amount of detail and clarity. It demonstrates teacher familiarity with student thinking.

Not Present	Low	Mid	High
No remediation occurs for any of the following reasons: • There are no student misunderstandings or difficulties with the content • Remediation does not go beyond correcting students' answers • The teacher chooses not to remediate • The teacher remediation is confusing or off-track	Brief conceptual remediation occurs. OR Brief or moderate procedural remediation occurs.	Moderate (neither brief nor at length) conceptual remediation or extensive procedural remediation occurs. OR Brief pre-remediation occurs.	Teacher engages in conceptual remediation *systematically* and *at length*. Examples include: • Identifying the source of student errors or misconceptions • Discussing how student errors illustrate broader misunderstanding and then addressing those errors • Extended pre-remediation

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Scoring Help - Remediation of Student Errors and Difficulties
In scoring this code, it is helpful to first identify whether any student difficulty exists in the segment. If there is any student difficulty, then the teacher's response can be categorized according to whether or not it was remediation and if so, what type.

Examples - Remediation of Student Errors and Difficulties			
Not Present	**Low**	**Mid**	**High**
[correction, not remediation] Teacher notices student has gotten the wrong answer and says, "No, that is not correct. You should have gotten 9."	[brief conceptual remediation] "Remember, you need to keep both sides of your equation equivalent to each other, so you can't perform an operation on only one side."	[moderate conceptual remediation] "I noticed that some of you forgot to multiply both sides of the equation by *x*. What happens if you multiply one side by *x* and not the other?" A few students offer reasons, and the teacher summarizes their ideas by saying, "The sides wouldn't be equivalent anymore."	[systematic conceptual remediation] "I noticed that some of you forgot to multiply both sides of the equation by *x*. What happens if you multiply one side by *x* and not the other?" The class continues to discuss at length why you need to multiply on both sides.

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<table>
<tr><th colspan="4">Teacher uses Student Mathematical Contributions</th></tr>
<tr><td colspan="4">This item captures the extent to which teacher uses student mathematical contributions to move instruction forward. Contributions can include, but are not limited to, student answers to questions (including one-word answers), comments, mathematical ideas, explanations, representations, generalizations, questions to the teacher, and student work. If some portion of the response to students muddles the mathematics, the score may be adjusted downward. This code can be used in whole-group or small-group/individual time segments.</td></tr>
<tr><th>Not Present</th><th>Low</th><th>Mid</th><th>High</th></tr>
<tr><td>No or very few student responses and only pro forma use of student ideas to develop the mathematics. For example, class may be dominated by teacher talk with very few student comments.

OR

Teacher uses student contributions but in a way that muddles or confuses the mathematics of the lesson.

OR

Student contributions occur but the teacher ignores them</td><td>Students contribute and the teacher responds in a pro forma way.</td><td>The teacher uses student contributions to some degree in the development of the mathematics.

Teacher may engage in features listed under High briefly, but instruction generally proceeds without strong use of student mathematical ideas.</td><td>Students’ mathematical ideas are woven at length into the development of mathematical ideas during the segment. Teacher “hears” what students are saying, mathematically, and responds appropriately during instruction.

In particular, teacher may comment on students’ mathematical ideas, elicit further student clarification of ideas, ask other students to comment on ideas, expand on and reinforce student utterances, etc.

Other markers include:
• Identifying key ideas in student statement (“Mark had an interesting idea…”)
• Highlighting key features of student questions (“Mark was asking about whether this would work in all cases…”)
• Identifying a student with an idea (“Mark’s method”)</td></tr>
</table>

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Scoring Help - Teacher uses Student Mathematical Contributions
This code is intended to measure how the teacher responds to and uses the mathematics that students contribute, regardless of the quality of the student contribution. That is, student contributions can be brief and procedural in nature; what we are looking for in this code is the length and quality of teacher uptake and use. Note that this is not a quantity code. If a teacher responds to multiple student contributions throughout the segment, but always does so in a pro forma way, score as a Low. Several types of teacher responses may qualify as pro forma: • The students regularly contribute basic calculations or answer-bounded questions during instruction, and the teacher acknowledges correct responses and uses them in the course of instruction, perhaps to move a calculation forward on the board. • Different students contribute many solutions or explanations, but the teacher doesn't make use of them beyond acknowledging students who are correct. • A student provides a contribution, and the teacher recognizes that it is interesting but decides not to take it up at that moment. Do not count use of "student" ideas that are not truly coming from the students as a feature of High (e.g., calling Claire's repeating of the addition procedure "Claire's idea" when the teacher just really wants to talk about the addition algorithm.)

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Overall Working with Students and Mathematics

This code provides an overall evaluation of the teacher-student interactions around the content.

Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of the teachers' interactions with the students around the content.

If some portion of the response to students or remediation muddles the mathematics, the score may be adjusted downward.

Not Present	Low	Mid	High
No or few interactions between teacher and students. There is no remediation and little use of student ideas OR Student mathematical contributions or difficulties occur, but teacher does not respond to or use those contributions. OR Teacher responses to student contributions are unclear or lead the segment off-track.	Teacher and students interact over content, but teacher responses are pro forma – moving instruction along with limited input from students. AND/OR There may be brief remediation.	Teacher and student interaction goes beyond pro forma exchanges to feature some use of student ideas, moderate conceptual remediation or extended procedural remediation. Portions of the clip may also feature a mix of strong and weak elements, or less-than-skillful use of student ideas.	Teacher weaves student ideas into the development of the mathematics and/or conceptually addresses misconceptions for clip. This must be done with some level of teacher skill at "hearing," understanding, and appropriately responding to student contributions or difficulties.

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Errors and Imprecision

This dimension is intended to capture teacher errors or imprecision in language and notation, or the lack of clarity/precision in the teacher's presentation of the content.

Do NOT count errors that are noticed and corrected within the segment.

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Mathematical Content Errors

The code is intended to capture events in the segment that are mathematically incorrect. For example:

- Solving problems incorrectly
- Defining terms incorrectly
- Forgetting a key condition in a definition
- Equating two non-identical mathematical terms

Mathematical errors that are made by students and endorsed by the teacher (e.g., leaving it on the board, saying it is correct, adopting an incorrect definition of fractions) should be counted here. Also score here if the teacher evaluates a correct solution method as incorrect.

Do not count

- Intentional errors (teacher following a wrong student idea or doing a procedure incorrectly to make a point)
- Errors that are corrected within the segment

Not Present	Low	Mid	High
None.	A brief content error. Does not obscure the mathematics of the segment.	Content errors occur in part(s) of the segment. OR Error(s) obscure the mathematics, but for only part of the segment.	Content errors occur in most or all of the segment. OR The errors obscure the mathematics of the segment.

Examples - Mathematical Content Errors

Not Present	Low	Mid	High
	When solving a multi-step problem, the teacher makes a calculation error in the last step, which results in an incorrect answer. Other similar problems are solved correctly.	The teacher's discussion of the solution to a problem is incorrect. This discussion is more than brief, but correct mathematics also occurs more than briefly during the segment.	The teacher uses an inappropriate metaphor for most of the segment (e.g., in a graph comparing distance and time, the teacher refers to the upward slope as runner going up the hill, flat slope as runner running straight).

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Imprecision in Language or Notation

This code is intended to capture problematic uses of mathematical language or notation. For example:

- Errors in notation (mathematical symbols)
- Errors in mathematical language
- Errors in general language

Definitions

- ***Notation*** includes conventional mathematical symbols (such as +, -, =) or symbols for fractions and decimals, square roots, angle notation, functions, probabilities, exponents, etc. Errors in notation might include inaccurate use of the equals sign, parentheses, or division symbol. By "conventional notation," we do not mean use of numerals or mathematical terms.
- ***Mathematical language*** includes technical mathematical terms, such as "angle," "equation," "perimeter," and "capacity." If the teacher uses these terms incorrectly, record as an error. When the focus is on a particular term or definition, also score errors in spelling or grammar.
- Teachers often use "***general language***" to convey mathematical concepts (i.e., explaining mathematical ideas or procedures in non-technical terms). General language also includes analogies, metaphors, and stories. Appropriate use of terms includes care in distinguishing everyday meanings different from their mathematical meanings. If the teacher is unclear in his/her general talk about mathematical ideas, terms, concepts, or procedures, record as an error.

Not Present	Low	Mid	High
None.	Brief instance of imprecision. Does not obscure the mathematics of the segment.	Imprecision occurs in part(s) of the segment. OR Imprecision obscures the mathematics, but for only part of the segment.	Imprecision occurs in most or all of the segment. OR Imprecision obscures the mathematics of the segment.

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Scoring Help - Imprecision in Language or Notation

We have specifically identified some commonly used imprecise terms and phrases that raters should be on the lookout for. These fall into two categories: phrases whose usage *automatically* results in a score of (at least) a Low, and "gray area" phrases whose isolated usage should not necessarily result in a score above Not Present, but can be considered in conjunction with other language use. If gray area phrases occur once or twice, we typically ignore them. However, if they occur repeatedly, or if they occur in combination with other linguistic or notational imprecision, we do consider them when scoring Imprecision.

Note that these are not exhaustive lists of all phrases that might count towards score above Not Present.

Automatically score as an imprecision:

- referring to "bigger" or "smaller" equivalent fractions
- different variations on "you can't subtract a bigger number from a smaller"
- different variations on "multiplication makes a number bigger"
- misuse of "expression" and "equation"
- misuse of equals sign
- "reducing" fractions (instead of simplifying)

"Gray area" phrases:

- "timesing", "minusing"
- "top" and "bottom" for numerator and denominator
- "alligator mouth" for greater-than and less-than symbols
- "carrying"
- "canceling"
- "borrowing" (instead of regrouping)
- "line" instead of line segment
- brief reference that pi is 3.14 without mentioning this is an approximation.

Examples - Imprecision in Language or Notation

Not Present	Low	Mid	High
	1) The teacher misuses "expression" or "equation" once or twice in a lesson on representing patterns. 2) Teacher uses term like "reduce" instead of "simplify", and this does not obscure the mathematics being taught.	1) The teacher uses the word "expression" instead of "equation" one or two times during a segment specifically about equations or the nature of equality. 2) The teacher uses terms like "reduce" and tells students that reducing makes fractions smaller.	The teacher's language and notation is sloppy throughout the segment.

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Lack of Clarity in Presentation of Mathematical Content

This code is intended to capture when a teacher's utterances cannot be understood. For example:

- Mathematical point is muddled, confusing, or distorted
- Language or major errors make it difficult to discern the point
- Teacher neglects to clearly solve the problem or explain content

Teacher's launch of a task/activity lacks clarity (the "launch" is the teacher's effort to get the mathematical tasks/activities into play). If the launch is problematic, score for the launch plus amount of time students are confused/off-task/engaging in non-productive explorations

Not Present	Low	Mid	High
None.	Brief lack of clarity. Does not obscure the mathematics of the segment.	Lack of clarity occurs in part(s) of the segment. OR Lack of clarity obscures the mathematics, but for only part of the segment.	Lack of clarity occurs in most or all of the segment. OR Lack of clarity obscures the mathematics of the segment.

Scoring Help - Lack of Clarity

Definition: You have to ask: "What, mathematically, was the teacher trying to say?"

Examples:

- Discussion of why 7 + -3 = 4 heads toward "-4 is too small to be the answer"
    - This is not wrong, but the mathematical point is not clear.
- Teacher endorses conflicting definitions for same concept
    - "The area is a number of square units needed to cover the figure, and we've talked before about the box like a gift that somebody gives you. The box itself and everything inside the box is the area, but the wrapping paper around it would be like surface area and we talked about that and we talked about the perimeter is walking around the fence around an area."
- Talking through a division problem and alternating back and forth between "making 3 groups" and "making groups of 3."
- Garbling a task launch, e.g., by asking initially "How much TV is watched in the US?" when students really must draw a graph to show "How many TVs in US vs. Europe vs. rest of the world?"

Examples -

Not Present	Low	Mid	High
	The launch of task is unclear, but the teacher clarifies quickly. A sentence or phrase is unclear, but the main mathematical point is not affected.	To introduce inverse operations, teacher explains that multiplication and division are "best friends" and "if you know something about one, you know something about the other." Examples later in the segment make the point clearer.	Teacher states that the lesson is going to be on surface area and volume. When students are asked to describe a cardboard box using math terms, the teacher endorses correct and incorrect student suggestions. The teacher then tries to define volume by asking whether a twelve foot TV would fit into the box. Surface area is mentioned numerous times but never defined. It is unclear if the teacher is using surface area as a synonym for volume or whether the term is simply never defined.

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Overall Errors and Imprecision			
This code intends to capture the overall presence of teacher errors in doing and talking about mathematics. Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of the errors and imprecision in instruction.			
Not Present	**Low**	**Mid**	**High**
No errors occur. Do not score as Not present if Low, Mid or High is marked in any category above.	Small, momentary error(s) occur. For example, small slips in language, a brief lack of clarity, or a minor error in solving an exercise. These typically do not obscure the mathematics of the segment.	One or more errors, for example, persistent misuse of language, a lack of clarity in a portion of the segment and/or mathematical errors, but these typically obscure the math for part of the segment.	Either there are many small errors, a consistent lack of clarity**,** or one large error that obscures the mathematics of the segment.

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Common Core Aligned Student Practices (CCASP)

This dimension attempts to capture evidence of students' involvement in tasks that ask them to "do" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning. During active instructional segments, this mainly occurs through student mathematical statements, including reasoning, explanations, and asking questions. During small group/partner/individual work times, this mainly occurs through work on a non-routine task.

The CCASP dimension captures the same kind of student meaningful engagement with mathematics envisioned in the eight Standards of Mathematical Practices listed in the Common Core State Standards for Mathematics[1], which say that students should:

1. Make sense of problems and persevere in solving problems
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Although there is not a one-to-one correspondence between the CCASP codes and the 8 Common Core Standards, the CCASP dimension includes many of the observable student behaviors contained in the Common Core. For instance, the Common Core practice "Model with mathematics" is addressed in the MQI code Students Work with Contextualized Problems.

1 National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards Mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers

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Students Provide Explanations

Students provide a mathematical explanation for an idea, procedure, or solution.

Examples:
- Students explain why a procedure works
- Students explain the procedure they used to solve a particular problem by attending to the meaning of the steps involved in this procedure rather than simply listing those steps
- Students explain what an answer means
- Students explain why a solution method is suitable or better than another method
- Students explain an answer based on an estimate or other number-sense reasoning

Notes:
- Explanations can be initiated by the teacher or self-initiated
- Explanations can be co-constructed with the teacher or constructed individually
- Explanations do not have to be complete or correct
- If a student's explanation meets the criteria for the Explanations code in Richness, it should be counted in both places
- Only give credit for things you actually hear students say

Not Present	Low	Mid	High
No instances of student explanation are present.	One or two brief student explanations are present.	Student explanations are more sustained or more frequent, but they are not characteristic of the segment.	Student explanations characterize much of the segment.

Scoring Help – Students Provide Explanations

When students are working independently or in groups and you cannot hear anything they say, assign the segment a score of Not Present. If you can hear student explanations or reasoning during these times, score them as usual.

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Student Mathematical Questioning and Reasoning (SMQR)

Students engage in mathematical thinking that has features of important mathematical practices. There must be clear evidence of students engaging in such practices. Examples include but are not limited to:

- Students provide counter-claims in response to a proposed mathematical statement or idea (whether from another student, the teacher, or a text)
- Students ask mathematically motivated questions requesting explanations (e.g., "Why does this rule work?" "What happens if all the numbers are negative?")
- Students make conjectures about the mathematics discussed in the lesson (e.g., "I've been trying to make a triangle with two obtuse angles, and I don't think you can.")
- Students form conclusions based on patterns they identify or on other forms of evidence (e.g., "It looks like, for polygons, every time we add a side we add another 180 degrees.")
- Students engage in reasoning about a hypothetical or general case (e.g., "Because the sum of the angles of any triangle is 180 degrees, a triangle should have at least two acute angles.")
- Students use ideas from a different mathematical topic to reason about the content of the lesson (e.g., student uses ideas from symmetry to reason about equivalent fractions in a pie chart)
- Students make a connection between the topic of the lesson and another mathematical area (e.g., a student notes the connection between area models for multiplication and area in measurement)
- Students comment on the *mathematics* of one another's contributions (this must go beyond stating "I did it another way" or simply agreeing or disagreeing)

An explanation captured under the Student Explanations code should *also* be coded as SMQR only if the statement includes an additional SMQR element. For example, a conjecture and an explanation of the conjecture should be counted under both codes. (e.g., "I don't think that the output in that table will ever be 0 because all of the other outputs are odd numbers.")

Notes:
- Students' contributions do not have to be complete or correct
- Only give credit for things you actually hear students say

Not Present	Low	Mid	High
No instances of student mathematical questioning or reasoning are present.	One or two instances of brief student mathematical questioning or reasoning are present.	Student mathematical questioning or reasoning is more sustained or more frequent, but it is not characteristic of the segment.	Student mathematical questioning or reasoning characterizes much of the segment.

Scoring Help - SMQR

When students are working independently or in groups and you cannot hear anything they say, assign the segment a score of Not Present. If you can hear student explanations or reasoning during these times, score them as usual.

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Students Communicate about the Mathematics of the Segment

This item captures the extent to which students communicate their mathematical ideas during the course of the segment, either in whole-group or small group settings. Examples of *substantive* student contributions include, but are not limited to, students presenting solution methods publicly (with or without words), asking mathematical questions, describing the meaning of a term, offering an explanation, discussing solution methods, commenting on the reasoning of others, etc.

In cases in which students are working in pairs or small groups, code substantive student contributions when you can a) hear them (e.g., a student and teacher are talking as teacher circulates, or you can overhear pairs of students) or b) the teacher's directions are very clear, and we can reasonably expect students to be having a substantive exchange for the duration of the small group work (e.g., a turn and talk). However, if it is not clear what students are talking about in small groups/pair work, score as Not Present.

Not Present	Low	Mid	High
Not present or minimally present. Students may contribute a word or phrase infrequently during whole-group instruction, but the segment primarily features teacher talk.	Student contributions are very brief. For example, students offer one- or two-word answers to questions or a partial description of steps, and they occur regularly during the segment.	There are some substantive student contributions, but these do not characterize the segment.	Substantive student contributions characterize the segment.

Scoring Help - Students Communicate About The Mathematics Of The Segment

Note that the difference between Not Present and Low is the prevalence of brief, one- or two-word answers, and the difference between Mid and High is the prevalence of *substantive* student contributions. The difference between Not Present/Low and Mid/High is whether there exist *any* substantive student contributions (i.e. a segment with a single substantive student contribution must be scored at least a Mid, and a segment with no substantive student contributions may not score above a Low). For instance, a student may provide one step of a procedure, followed by the teacher giving the next step. This would count as a Low. If the student narrates a complete set of steps for a problem, it would be counted as a Mid.

Student explanations and SMQR-type responses count here. In addition, this code encompasses additional types of substantive student contributions under Mid and High, including descriptions of choices students made while solving word problems, definitions, and so forth.

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Task Cognitive Demand

This code captures student engagement in tasks in which they think deeply and reason about mathematics. This code refers to the *enactment* of the task, regardless of the initial demand of the curriculum/textbook task or how the teacher sets up the task for students.

Notes:

- Student confusion does not necessarily suggest that students are engaging with the content at a high cognitive level.
- Working on review tasks or on ideas discussed in previous lessons does not necessarily mean that students use lower order thinking skills.
- This code should not be confounded with the difficulty of the task or whether it is appropriate for a certain grade-level.
- Code a student presentation of a solution method at the same level of cognitive demand as the task itself was coded.

Not Present	Low	Mid	High
Students are engaged in cognitively undemanding activities. Examples of cognitively *undemanding* activities include: • Recalling and applying well-established procedures • Recalling or reproducing known facts, rules, or formulas • Listening to a teacher presentation with limited student input • Going over homework with little additional student work (e.g., reporting numerical answers) • Unsystematic exploration (i.e., students do not make *systematic and sustained progress in developing mathematical strategies or understanding*)	There is a brief example of a cognitively demanding activity, e.g. • A momentary think-pair-share where students define a term • Direct instruction with one or two examples of student explanations or SMQR • Tasks with a momentary high cognitive demand element • Tasks that are not completely routine, but are heavily scaffolded for students with hints or directions	Segment features mix of demanding and undemanding tasks and activities, e.g. • Tasks with variable enactment (e.g., demanding tasks followed by a transition to undemanding tasks; or, when working in small groups, some groups work on a high-demand task while some groups work on an undemanding task) • Direct instruction with student explanations and/or SMQR input at certain points • Tasks with middling cognitive demand	Students engage with content at a *high* level of cognitive demand. Examples of cognitively *demanding* activities include when students: • Determine the meaning of mathematical concepts, processes, or relationships • Draw connections among different representations or concepts • Make and test conjectures • Look for patterns • Examine constraints • Explain and justify

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Students Work with Contextualized Problems			
Students work with contextualized problems (e.g., story problems, real-world applications, experiments that generate data). This includes solving such problems; discussing solutions to such problems; writing expressions or equations to represent contextualized situations; making sense of contextualized relationships through tables, graphs or other representations; or creating contextualized problems/situations to match expressions/equations. Note: Do not count when the teacher or student mentions a contextualized example for illustrative purposes (e.g., "you can think of 1/4 as a quarter and 1 whole as a dollar when you are converting fractions to decimals" or "remember yesterday when we solved the hat problem?"), but the students do not work on it. Note: This is not a duration code; the difference between a Low, Mid, and High is amount of teacher scaffolding, not length. In the case of two or more different tasks with different levels, score to the highest level.			
Not Present	**Low**	**Mid**	**High**
Students do not work with contextualized problems or a contextualized problem is mentioned but not worked on.	The contextualized problems are executed as mostly rote/routine exercises. Teacher heavily scaffolds the presentation, for example, by telling students which procedure is to be applied, helping them write out the expression or equation, and so forth. Also include here times where there is data collection *without* reference to the underlying relationships or shape of the data. For instance, students may be collecting and marking down ice cream preferences in preparation for later plotting the graph and discussing.	Some student reasoning about contextualized problems is required for at least a portion of the problem execution; however, solution paths may be co-constructed or scaffolded by teacher to some extent. For instance: • Students play some role in deciding how to solve the problem • The problem starts off as non-routine but teacher hints at a solution method	Students are allowed significant opportunities to think and reason mathematically about contextualized problems. Students might need to choose which operation to apply, decide which kind of graph is appropriate for their data, or figure out how to write an expression that represents a pattern. The characterizing feature of this segment is that the teacher will not be doing much of the cognitive work of solving the problem.

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Scoring Help - Students Work with Contextualized Problems
When scoring, first determine if a contextualized problem is present during the segment. When determining whether or not a problem is contextualized, it may be necessary to refer to previous segments to determine what problem or task was assigned. Special cases: • Probability experiments (such as rolling dice, spinning a spinner, or pulling colored chips out of a bag) are NOT contextualized problems, unless there is an additional context (such as pulling colored chips out of a bag that represent socks in a drawer). • Working with data is generally contextualized. If the data are completely void of context (for example, if students are asked to find the median of a set of numbers), it is not contextualized; but, if there is any meaning to the data involved (for example, students take a list of names, count the letters in each name, and then find the median), then it should be counted as contextualized. If there is a contextualized problem, determine whether students are working on it (they do not have to finish or 'solve' the problem, just work on it in some way). If students are working on a contextualized problem, determine how much scaffolding or support the students are given. It may be necessary to refer to previous tasks and segments in order to infer whether the task is routine or not. Use your best judgment.

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Overall Common Core Aligned Student Practices			
This code attempts to capture evidence of students' involvement in "doing" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning. • During ***active instruction segments***, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking. • During ***small group/partner/individual work time***, this mainly occurs through work on a non-routine task. Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of the student participation in meaning-making and reasoning.			
Not Present	**Low**	**Mid**	**High**
There are no examples of student involvement in cognitively demanding classroom work. Tasks are largely procedural in nature or heavily scaffolded by the teacher. For example, there may be inquiry-response-evaluation-type teacher lectures with no examples of student explanation, questioning, or reasoning. Also score as Not Present if there are unproductive explorations in which *the majority* of the students are off-track mathematically.	There are few examples of student engagement in mathematical practices such as explanation, questioning, and reasoning. Tasks may be largely procedural in nature, but *occasional* student participation or a brief cognitively demanding task occurs.	Students engage with content at *mixed level*. Students may provide substantive explanations or ask mathematically motivated questions. This may also include tasks with variable enactment (high and then low during segment). This can also include instances in which some students/groups are engaged in tasks at a high level and others are not. Students may also engage in a task with middling cognitive demand.	Students contribute substantially to the building of mathematical ideas through posing questions, offering explanations, looking for patterns, making conjectures, and engaging in other types of reasoning. Such contributions are a major feature of the segment, with many student contributions or extended work on a cognitively demanding task.

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WHOLE LESSON CODES

Lesson Time is Used Efficiently				
This code captures the extent to which lesson time is used efficiently; class is on task, and behavior issues do not disrupt the flow of the class.				
Not at all true of this lesson **1**	**2**	**(Default Score)** **3**	**4**	**Very true of this lesson** **5**
There is a fair to large amount of wasted time. For instance: • Teacher passes out materials at length while students are waiting. • Transitions take a long time and require lots of teacher repetition of instructions. • Teacher engages in off-topic conversation, distracting from the mathematics. • Behavior management takes away from instructional time. • Directions for launch are unclear, causing students to flounder mathematically until the teacher repeats them.		Lesson proceeds from one task/problem to another, yet there are minor distractions from the flow of the lesson. For instance: • Transitions are not highly efficient. • There is a brief off-task conversation. • Behavior management briefly takes away from instruction time.		The class is almost continuously working on mathematics. For instance: • Teacher hands out materials during a time in which students are already working on another task. • Transitions between groups and tasks are smooth and quick; students seem to know what to expect and get right down to business in their new stations. • Teacher does not allow excessive time for students to solve problems.

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Lesson is Mathematically Dense				
This code captures the amount of mathematics – problems, tasks or concepts – worked on relative to the length of the lesson.				
Not at all true of this lesson **1**	**2**	**(Default Score)** **3**	**4**	**Very true of this lesson** **5**
The class does not get through many problems/tasks/concepts, and the reason was not "rich" or "cognitively demanding" explorations of the mathematics. Also score here for mathematically vacuous tasks (e.g. coloring, cutting, pasting).		Teacher and students work through a reasonable number of problems, or cover a reasonable number of mathematical topics. (Fewer topics or problems covered in reasonable depth can also count here.)		The class works through many problems or concepts; teacher covers a lot of mathematical ground. OR The class works on a few problems or concepts in meaningful ways. For instance, they may have had a long discussion of a conceptual student error to a problem, or done an extended exploration with many connections to the underlying mathematics, etc.

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Students are Engaged				
This code describes the extent to which the classroom environment is characterized by student engagement. This is not about Task Cognitive Demand or other Common Core Aligned Student Practices; it is about engagement with *the lesson*.				
Not at all true of this lesson **1**	**2**	**(Default Score)** **3**	**4**	**Very true of this lesson** **5**
Students are not engaged with the lesson; many are off task for all or part of the lesson.		Students complete the requests made by the teacher, but do not appear eager to participate.		Students are eager to participate in the lesson. They raise their hands or call out answers. Most students are engaged in this fashion.

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Lesson Contains Rich Mathematics				
This code captures the depth of the mathematics offered to students. This can include a focus either on meaning or on mathematical practices, as described by the segment-level Richness codes. In all cases, ignore incorrect elements of richness in assigning a score.				
Not at all true of this lesson **1**	**2**	**(Default Score)** **3**	**4**	**Very true of this lesson** **5**
Elements of rich mathematics are not present, or are present but largely incorrect.		Some elements of rich mathematics are present. For instance, there may be occasional explanations, linked representations or comparisons of multiple solution methods. However, these may not be consistent or especially well executed. May include lessons in which elements of rich mathematics appear regularly but are of weaker quality.		Elements of rich mathematics are consistently present, with a strong focus on mathematical meaning and/or practices throughout the lesson. OR There is truly outstanding rich mathematics in a significant portion of the lesson.

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Teacher Attends to and Remediates Student Difficulty

This code describes the extent to which the teacher attends to student difficulty with the material.

We are not distinguishing here between conceptual and procedural remediation. This item should capture how attentive the teacher is to student difficulty, not how conceptual the remediation is.

Note: At the whole lesson level, attentive remediation can include cases in which the teacher notices student difficulty and chooses not to address it immediately, but ultimately does remediate.

Not at all true of this lesson **1**	**2**	**(Default Score)** **3**	**4**	**Very true of this lesson** **5**
Student responses indicate that many of them do not understand the material, but the teacher does not slow down to attend/remediate. Also score here if teacher attends to difficulty, but the instruction is confusing or off-track.		Teacher consistently attends to and remediates student difficulty, but in a pro forma way, e.g., correcting errors by asking again until a student gets the correct answer. May be brief instances of more substantive remediation. Also score here if there is no student difficulty.		Teacher is very attentive to student difficulty, and is correct and effective at remediating. Teacher addresses student difficulty by doing one or more of the following: • Remediating during individual/pair work time • Re-teaching material • Slowing down instruction • Understanding and addressing the root of student difficulty

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Teacher Uses Student Ideas

This code describes the extent to which the teacher uses student ideas and solutions to move the lesson forward.

Not at all true of this lesson **1**	**2**	**(Default Score)** **3**	**4**	**Very true of this lesson** **5**
Student talk does not include substantive contributions such as student explanations or reasoning. OR There are some substantive comments, but teacher either ignores them or cannot make use of them instructionally, including failed attempts to take up the comments.		Students contribute an occasional substantive idea to the lesson, but teacher use is pro forma. Pro forma use includes acknowledging or endorsing a correct student comment with nothing further.		Teacher weaves in substantive student ideas and solutions to help build the mathematical point(s) of the lesson.

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Mathematics is Clear and not Distorted

This code describes the extent to which the mathematics of the lesson is clear and not distorted. This is an overall judgment, but raters can use their thinking from the errors dimension. A lesson with consistent problems should be scored as a 1.

Not at all true of this lesson **1**	**2**	**3**	**4**	**Very true of this lesson** **5**
The mathematics of the lesson is severely undermined by errors, imprecision, poor task launches, lack of clarity, or other problems with the lesson.		Minor problems arise in the course of the lesson, but they do not significantly detract from the mathematics of the lesson.		No problems arise in the presentation of the mathematics, i.e., all segment level errors codes are scored as Not Present.

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Tasks and Activities Develop Mathematics

This code describes the extent to which the tasks and activities done by the class contribute to the development of the mathematics of the lesson. In other words, this code refers to the *architecture* of the lesson, i.e., whether the tasks/activities help move the mathematics along. More generally, does the lesson have *directionality*, in the sense that it is appropriately developing mathematical ideas?

Notes:

- We are defining "tasks and activities" very broadly here, including class discussions, teacher presentation of new content, etc.
- Do not score here for other problems with the lesson, such as slow pacing or minor to moderate mathematical errors.

Not at all true of this lesson **1**	**2**	**(Default Score)** **3**	**4**	**Very true of this lesson** **5**
Lesson architecture hinders development of the mathematics. For instance: • Tasks are so convoluted that they do not elucidate mathematical ideas • Poor examples (e.g., introducing slope and intercept with the equation y=x+1) obscure important points • Mathematical distortions are severe enough to significantly hinder the development of the math		Mathematics happens in the lesson. The lesson does not include the special features of a 5, and the lesson architecture does not hinder the development of the math. OR There is a mix of strong and weak features.		Tasks and activities are carefully designed to contribute to key mathematical points. Lesson includes both: • Tasks that build sequentially • Implementation of tasks that leads to an overarching mathematical point Lesson may also include: • Teacher making connections to larger mathematical ideas • Teacher drawing important points of the lesson together

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Lesson is Characterized by Common Core Aligned Student Practices

This code describes the extent to which students participate in the *mathematics* of the lesson in a substantive way, by engaging in the practices described by the codes in the Common Core Aligned Student Practices dimension.

For example:
- Students ask mathematically-motivated questions
- Students notice patterns and form conclusions based on them
- Students make connections across content areas
- Students provide mathematical explanations
- Students comment on the reasoning of others
- Students work on cognitively demanding tasks
- Students engage in sense-making

Not at all true of this lesson **1**	**2**	**(Default Score)** **3**	**4**	**Very true of this lesson** **5**
Classroom environment is one in which students never participate substantively in the mathematics. Student participation is limited to students answering fill-in-the-blank type questions or working on cognitively undemanding activities. Score here for lessons that *never* move beyond inquiry/response/ evaluation (IRE) instruction.		Students occasionally participate in the mathematics in a substantive way. However, much of the class may be characterized by inquiry/response/evaluation (IRE) instruction or students completing a low or medium cognitive demand task at length.		Lesson is characterized by mathematically substantive student participation.

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Overall MQI:

Whole-Lesson Mathematical Quality of Instruction				
This code is intended to capture the overall mathematical quality of instruction for the lesson (MQI) as suggested by the teacher's work during the lesson.				
Low **1**	**Low/Mid** **2**	**Mid** **3**	**Mid/High** **4**	**High** **5**
Instruction is characterized by one or more of the following: • Systematic teacher errors, imprecision, or lack of clarity around the mathematics • Major teacher conceptual error in a significant portion of the lesson • Unproductive teacher-student interactions around the content (e.g., teacher cannot effectively remediate) • Lack of directionality/ unsystematic exploration • Lack of connection of classroom activities to mathematical content		Instruction does not have characteristics of Low and is mostly error-free, but it lacks the mathematical richness, appropriate use and discussion of procedures, and the sharp mathematical focus detailed under High. Examples: • Mostly error-free instruction, perhaps with occasional but not consistent elements of richness • Mainly pro forma interactions with students (inquiry, response, evaluation-type discussion)		Instruction is error-free (save for *minor* slips) and characterized by combinations of the following: • Mathematical richness in terms of explanations and linking between representations • Focus on mathematical practices (developed generalizations, mathematical efficiency) that is sustained and detailed • Instruction has a clear and sharp mathematical focus and directionality that allows students to develop the important mathematical ideas under consideration • Instruction is characterized by at least some productive teacher-student interactions around the content (either working with student ideas/errors OR Common Core aligned student practices)

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