

Introduction to Artificial Intelligence

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1 Agents

Definition 1.1 (Reflex Agent). A reflex agent chooses actions based on its current perception of the world.

Definition 1.2 (Planning Agent). A planning agent chooses actions based on hypothesized consequences of actions.

2 General Search Problems

2.1 Heuristics

Definition 2.1 (Heuristic). A heuristic $h(n)$ is a function that estimates the distance from state n to the goal state for a particular search problem. It is often solutions of relaxed problems.

1. A heuristic is admissible if $0 \leq h(n) \leq h^*(n)$ where h^* is the true cost to goal state;
2. A heuristic is consistent if $h(n) - h(n+1) \leq c(n, n+1)$ where c is the cost between states n and $n+1$.

Remark. Consistency necessarily implies admissibility.

2.2 Search Algorithms

Table 1: Search algorithms.

	Fringe	Complete	Optimal	Time	Space
Depth-First Search	Stack	<i>iff</i> no cycle	No	$O(b^m)$	$O(bm)$
Breadth-First Search	Queue	Yes	<i>iff</i> uniform cost	$O(b^s)^1$	$O(b^s)^1$
Uniform Cost Search	PQ $(g(n))^2$	<i>iff</i> positive cost	Yes	$O(b^{c^*/\epsilon})^3$	$O(b^{c^*/\epsilon})^3$
Greedy Search	PQ $(h(n))$	-	No	-	-
A^* Tree Search	PQ $(h(n) + g(n))$	-	<i>iff</i> $h(n)$ admissible	-	-
A^* Graph Search ⁴	PQ $(h(n) + g(n))$	-	<i>iff</i> $h(n)$ consistent	-	-

¹ s = depth of solution.

² $g(n)$ = cumulative path cost.

³ c^*/ϵ = effective solution depth (c^* = cost of the cheapest solution; ϵ = minimum cost of cost-contour arcs).

⁴ Compared to tree search, graph search keeps a closed set of expanded states to check against to prevent duplicate expansions.

Remark. Implementation of search algorithms differ only in fringe strategies.

3 Constrained Satisfaction Problems

Definition 3.1 (Constrained Satisfaction Problems). Constrained Satisfaction Problems (CSPs) are a type of **identification problem** defined by variable X_0, \dots, X_n with values from a domain D that satisfies a set of constraints.

Algorithm 1: Backtracking search.

Input: A constraint satisfaction problem P .

Output: A complete assignment A .

```

1 Function BS( $P$ ):
2   return EXPLORE ( $P$ , {})
3 Function EXPLORE( $P$ ,  $A$ ):
4   if  $A$  is complete then
5     return  $A$ 
6    $unassigned \leftarrow$  an unassigned VARIABLES( $P$ )
7   foreach  $value \in$  DOMAIN( $unassigned$ ) do
8     if  $value$  is consistent with all CONSTRAINTS( $P$ ) then
9       add { $unassigned \leftarrow value$ } to  $A$ 
10       $attempt \leftarrow$  EXPLORE( $P$ ,  $A$ )
11      if  $attempt$  failed then
12        remove { $unassigned \leftarrow value$ } from  $A$ 
13      else
14        return  $attempt$ 
15   return failed

```

3.1 Filtering

Definition 3.2 (Arc Consistency).

Arc $X \rightarrow Y$ is consistent \Leftrightarrow
 $(\forall x \in D_x)(\exists y \in D_y)(y \text{ can be assigned to } Y \text{ without violating a constraint.})$

Algorithm 2: Arc consistency filtering.

Input: A constraint satisfaction problem P .
 /* $O(|S||A|^3)$ runtime */

```

1 Function AC-3( $P$ ):
2    $Q \leftarrow$  empty Queue
3   enqueue all arcs  $\in P$ 
4   while  $Q$  is not empty do
5      $(X_i, X_j) \leftarrow$  dequeue from  $Q$ 
6     if FILTER( $X_i, X_j$ ) is successful then
7       foreach  $X_k \in \text{NEIGHBORS}(X_i)$  do
8         enqueue  $(X_k, X_i)$ 
9 Function FILTER ( $tail, head$ ):
10   $result \leftarrow$  false
11  foreach  $value \in \text{DOMAIN}(tail)$  do
12    if  $value$  violates some constraint with all values in  $\text{DOMAIN}(head)$  then
13      delete  $value$  from  $\text{DOMAIN}(tail)$ 
14       $result \leftarrow$  true
15  return  $result$ 

```

3.2 Ordering

Definition 3.3 (Minimum Remaining Values). The MRV policy chooses an unassigned variable that has the fewest valid remaining values in order to induce backtracking earlier and reduce potential node expansions.

Definition 3.4 (Least Constraining Value). The LCV policy chooses a value assignment that violates the least amount of constraints, which requires additional computation such as running arc consistency test on each value.

3.3 Structure

Given a tree-structured CSP, represent it as a directed acyclic graph. Enforcing arc consistency in reverse topological order then assigning in topological order ensures a runtime of $O(nd^2)$ (as opposed to $O(d^n)$ in the general case).

TODO: nearly tree-like CSPs and tree decomposition.

4 Local Search

Definition 4.1 (Local Search). A search strategy that improve a single option until no further improvements can be made. Typically, local search is faster and more memory efficient than other search algorithms but is generally neither complete nor optimal.

Definition 4.2 (Hill Climbing). A CSP strategy that randomly selects a conflicting variable and reassign values using min-conflicts heuristics.

Remark. Efficiency of the algorithm depends on $R = \frac{\text{number of constraints}}{\text{number of variables}}$; computation time is approximately constant time except when R approaches the *critical ratio*.

4.1 Simulated Annealing

Algorithm 3: Simulated annealing.

Input: A problem P and a schedule/mapping from time to "temperature" T .

Output: A solution state.

/* Escape local maxima by allowing downhill movement based on a "temperature"-dependent probabilistic function. */

```

1  $current \leftarrow$  initial state of  $P$ 
2 for  $t \leftarrow 1$  to  $\infty$  do
3    $temp \leftarrow T[t]$ 
4   if  $temp = 0$  then
5     return  $current$ 
6   else
7      $next \leftarrow$  a randomly selected successor of  $current$ 
8      $\Delta \leftarrow \text{VALUE}[next] - \text{VALUE}[current]$ 
9     if  $\Delta > 0$  then
10       $current \leftarrow next$ 
11     else
12       $currnet \leftarrow next$  with probability  $e^{\frac{\Delta}{temp}}$ 
```

4.2 Genetic Algorithm

Definition 4.3 (Genetic Algorithm). A strategy that keeps the best N hypothesis at each step based on a fitness function and generate next generation using pairwise cross-over operations (and, optionally, mutation operations).

5 Games

Definition 5.1 (Games). Games are multi-agent search problems that could be zero-sum (adversarial) or general sum.

5.1 Minimax

Definition 5.2 (Minimax). A zero-sum-game algorithm that assumes the opponent is an optimal adversary.

Algorithm 4: Minimax with alpha-beta pruning.

Input: A game state S
Output: The root minimax value.
 /* initialize $\alpha \leftarrow -\infty$ and $\beta \leftarrow \infty$ */

```

1 Function VALUE( $S, \alpha, \beta$ ):
2   if  $S$  is a terminal state then
3     return known terminal value
4   if the agent is maximizing then
5     return MAX-VALUE( $S, \alpha, \beta$ )
6   if the agent is minimizing then
7     return MIN-VALUE( $S, \alpha, \beta$ )

8 Function MAX-VALUE( $S, \alpha, \beta$ ):
9    $v \leftarrow -\infty$ 
10  foreach successor  $S'$  of  $S$  do
11     $v \leftarrow \text{MAX}(v, \text{VALUE}(S'))$ 
12    if  $v \geq \beta$  then
13      return  $v$ 
14     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
15  return  $v$ 

16 Function MIN-VALUE( $S, \alpha, \beta$ ):
17    $v \leftarrow \infty$ 
18  foreach successor  $S'$  of  $S$  do
19     $v \leftarrow \text{MIN}(v, \text{VALUE}(S'))$ 
20    if  $v \leq \alpha$  then
21      return  $v$ 
22     $\beta \leftarrow \text{MIN}(\beta, v)$ 
23  return  $v$ 

```

5.2 Expectimax

Definition 5.3 (Expectimax). A zero-sum-game algorithm that assumes the opponent acts based on some probability distribution.

Algorithm 5: Expectimax.

Input: A game state S

Output: The root minimax value.

```

1 Function VALUE( $S$ ):
2   if  $S$  is a terminal state then
3     return known terminal value
4   if the agent is maximizing then
5     return MAX-VALUE( $S$ )
6   if the agent is randomizing then
7     return EXP-VALUE( $S$ )

8 Function MAX-VALUE( $S$ ):
9    $v \leftarrow -\infty$ 
10  foreach successor  $S'$  of  $S$  do
11     $v \leftarrow \text{MAX}(v, \text{VALUE}(S'))$ 
12  return  $v$ 

13 Function EXP-VALUE( $S$ ):
14   $v \leftarrow 0$ 
15  foreach successor  $S'$  of  $S$  do
16     $v \leftarrow \mathbb{E}[\text{VALUE}(S')] \text{ // expected value}$ 
17  return  $v$ 

```

6 Markov Decision Processes

Remark. Finite horizons (finite timestep before an agent terminates) and/or discount factors (γ) ensure an agent terminates in MDP.

Definition 6.1 (Transition Function). $T(s, a, s') = P(s' \mid s, a)$

6.1 Value Iteration

Algorithm 6: Value Iteration

Input: A MDP (S, A, R, T, γ) .

Output: The optimal policy $\pi^*(s)$ for all $s \in S$.

```

1 Function POLICY-EXTRACTION():
2    $V^* \leftarrow \text{VALUE-ITERATION()} \text{ // optimal value}$ 
3   foreach  $s \in S$  do
4      $\pi^*(s) \leftarrow \underset{a}{\operatorname{argmax}} \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V_k(s')]$ 
5   return  $\pi^*(s) \mid \forall s \in S$ 
6 Function VALUE-ITERATION():
7   //  $O(|S|^2|A|)$  runtime
8   Initialize  $V_0(s) \leftarrow 0$  for all  $s \in S$ 
9   while  $V_{k+1}(s) \neq V_k(s) \mid \forall s \in S$  do
10    // repeat until values converge
11     $V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V_k(s')]$ 
12  return  $V^*(s) \leftarrow V_{k+1}(s) \mid \forall s \in S$ 

```

6.2 Policy Iteration

Definition 6.2 (Policy Iteration). Define an initial policy π_0 (can be arbitrary, but ideality close to the optimal policy). Then, iteratively solve $\forall s \in S$

$$V^{\pi_i}(s) = \sum_{s'} T(s, \pi_i(s), s') [R(s, \pi_i(s), s') + \gamma V^{\pi_i}(s')] \quad (\text{policy evaluation})$$

$$\pi_{i+1}(s) \leftarrow \underset{a}{\operatorname{argmax}} \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^{\pi_i}(s')] \quad (\text{policy improvement})$$

until $\pi(s)$ converges for all $s \in S$ (yields π^*).

Remark. Policy evaluation solves a system of $|S|$ linear equations.

Remark. Policy iteration converges faster than value iteration.

7 Reinforcement Learning

Remark. Reinforcement learning operates on MDP problems where the T and R functions are unknown.

7.1 Model-Based Learning

Definition 7.1 (Model-Based RL). An algorithm that counts and normalizes sample outcomes s' for each s, a to construct $\hat{T}(s, a, s')$ and discovers each $\hat{R}(s, a, s')$ through exploration. The approximated MDP is then solved by value or policy iteration.

7.2 Model-Free Learning

Definition 7.2 (Direct Evaluation). An algorithm that fixes some policy π and empirically computes $V^\pi(s)$ for all $s \in S$ by averaging total sample utility for a given state.

Remark. Direct evaluation wastes information about state connections and will take a long time to learn.

Definition 7.3 (Temporal Difference Learning).

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha[R(s, \pi(s), s') + \gamma V^\pi(s')]$$

Remark. Can't extract policy.

Definition 7.4 (Q-Learning).

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + \alpha[R(s, a, s') + \gamma \max_{a'} Q(s', a')]$$

Remark. Q-learning is an example of off-policy learning, in which the algorithm can learn the optimal policy even by taking sub-optimal or random actions.

Definition 7.5 (Approximate Q-Learning). Represent q-values as weighted sums of features $Q(s, a) := \vec{w} \cdot \vec{f}(s, a)$. The update rule for Q-Learning then becomes

$$\begin{aligned} \Delta &\leftarrow [R'(s, a, s') + \gamma \max_{a'} Q(s', a')] - Q(s, a) \\ w_i &\leftarrow w_i + \alpha \cdot \Delta \cdot f_i(s, a) \end{aligned}$$

Definition 7.6 (ϵ -Greedy Policies). Define some probability $0 \leq \epsilon \leq 1$ to act randomly and explore. ϵ should be lowered over time to favor more exploitation as the learning becomes complete.

Definition 7.7 (Exploration Function).

$$\begin{aligned} Q(s, a) &\leftarrow (1 - \alpha)Q(s, a) + \alpha[R(s, a, s') + \gamma \max_{a'} f(s', a')] \\ f(s, a) &= Q(s, a) + \frac{\text{const.}}{\text{count}(Q(s, a))} \end{aligned}$$

Remark. $f(s, a)$ shown here is only a common example where the “bonus” for exploration diminishes as q-states are explored more and more.

8 Bayesian Network

Definition 8.1 (Conditional Independence). $\forall x, y, z : P(x|z, y) = P(x|z)$ (i.e., $x \perp\!\!\!\perp y \mid z$)

Definition 8.2 (Bayes' Net). Bayes' Net is a graphic model (DAG) that describes complex joint distributions using simple, local distributions (conditional probabilities)

- Nodes—variables (with domains) [assigned = observed, unassigned = unobserved]
- Arcs—interactions (i.e., encode conditional independence when lacking arrows) $\implies P(x_i|x_1, \dots, x_{i-1}) = P(x_i|\text{parents}(X_i))$
- A conditional distribution for each node

Remark. Bayes' Net does not imply causation, only the lack thereof

8.1 Inference

Definition 8.3 (Inference by Enumeration). General procedure:

1. Join Factors: $\forall r, t : P(r, t) = P(r)P(t|r)$
2. Eliminate: $P(T) = \sum_r P(r, T)$
3. Normalize: $P(Q|e_1, \dots, e_k) = \frac{P(Q, e_1, \dots, e_k)}{\sum_q P(Q, e_1, \dots, e_k)}$

Remark. Variable elimination is faster than inference by enumeration via interleaving joining and marginalizing.

8.2 D-Separation

Definition 8.4 (Causal Chains). $X \rightarrow Y \rightarrow Z$

$$P(x, y, z) = P(x)P(y|x)P(z|y) \implies X \perp\!\!\!\perp Z \mid Y$$

Definition 8.5 (Common Causes). $X \leftarrow Y \rightarrow Z$

$$P(x, y, z) = P(y)P(x|y)P(z|y) \implies X \perp\!\!\!\perp Z \mid Y$$

Definition 8.6 (Common Effect). $X \rightarrow Z \leftarrow Y \implies X \perp\!\!\!\perp Y$

Definition 8.7 (D-Separation). $X \perp\!\!\!\perp Y \mid Z$ iff X and Y are “d-separated” by Z (i.e., all undirected path from X to Z are inactive). A path is active if all triple it contains is active:

- Causal chain $A \rightarrow B \rightarrow C$ where B is unobserved
- Common cause $A \leftarrow B \rightarrow C$ where B is unobserved
- Common effect $A \rightarrow B \leftarrow C$ where B or one of its descendants is observed

8.3 Sampling

Algorithm 7: Sampling.

```

1 Function PRIOR-SAMPLING:
2   for  $i = 1, 2, \dots, n$  in topological order do
3     Sample  $x_i$  from  $P(X_i \mid \text{Parents}(X_i))$ 
4   return  $(x_1, x_2, \dots, x_n)$ 
5 Function REJECTION-SAMPLING:
6   // Will reject lots of samples if evidence is unlikely.
7   for  $i = 1, 2, \dots, n$  in topological order do
8     Sample  $x_i$  from  $P(X_i \mid \text{Parents}(X_i))$ 
9     if  $x_i$  inconsistent with evidence then
10      return // no sample generated
11   return  $(x_1, x_2, \dots, x_n)$ 
12 Function LIKELIHOOD-WEIGHTING-SAMPLING:
13    $w \leftarrow 1.0$ 
14   for  $i = 1, 2, \dots, n$  in topological order do
15     if  $X_i$  is an evidence variable then
16        $X_i \leftarrow$  observation  $x_i$  from  $X_i$ 
17        $w \leftarrow w \times P(x_i \mid \text{Parents}(X_i))$ 
18     else
19       Sample  $x_i$  from  $P(X_i \mid \text{Parents}(X_i))$ 
20   return  $(x_1, x_2, \dots, x_n), w$ 
21 Function GIBBS-SAMPLING:
22   /* keep track of a full instantiation  $x_1, \dots, x_n$ . Start with an
23      arbitrary instantiation consistent with the evidence. Sample
24      one variable at a time, conditioned on all the rest, but keep
25      evidence fixed. Keep repeating this for a long time. */

```

9 Decision Networks

Definition 9.1 (Decision Networks). A combination of both Bayes' nets and expectimax that has three types of nodes: 1) chance nodes, 2) action nodes (act as observed evidence and cannot have parents), and 3) utility nodes (depends on both action and chance nodes). The goal of decision networks is to identify the action a that yields the maximum expected utility (MEU) given evidence e and an instance/state s for the chance nodes

$$\text{MEU}(e) = \max_a \text{EU}(a|e) = \max_a \sum_s P(s|e) U(s, a)$$

Remark. A decision network can be represented as an outcome tree (TODO: example)

9.1 Value of Perfect Information

Definition 9.2 (Value of Perfect Information). The value of perfect information (VPI) quantifies the amount an agent's maximum expected utility is expected to increase if it observes some new evidence.

$$\text{VPI}(E'|e) = \text{MEU}(e, E') - \text{MEU}(e) = \sum_{e'} P(e'|e) \text{MEU}(e, e') - \text{MEU}(e)$$

VPI has several important properties:

- Nonnegativity:

$$(\forall E', e) \text{VPI}(E'|e) \geq 0$$

- Nonadditivity: (intuitively, observing E_j might change how much we care about E_k)

$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$

- Order-independence:

$$\text{VPI}(E_j, E_k|e) = \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j) = \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k)$$

10 Markov Models

Definition 10.1 (Markov Models). The Markov model is analogous to a chain-like, infinite-length Bayes' net that is time-dependent. The model assumes that:

1. The memoryless property holds at all timesteps, and
2. The transition model is stationary (i.e., for all values of i , $P(W_{i+1}|W_i)$ is identical)

Algorithm 8: Mini-forward algorithm.

1 $P(W_{i+1}) \leftarrow \sum_{w_i} P(W_{i+1}|w_i)P(w_i)$

Algorithm 9: Finding stationary distribution.

Output: The stationary distribution W_t

1 Given constraint $\sum_{w_t} = 1$, solve system of equations $P(w_i^t) = \sum_{w_j^t} P(w_i^t|w_j^t)P(w_j^t)$
 // $w^t = w^{t+1}$

Definition 10.2 (Hidden Markov Models). Hidden Markov Model

- Sensor model $P(F_i|W_i)$ is stationary
- Belief distribution: $B(W_i) = P(W_i|f_1, \dots, f_i)$, $B'(W_i) = P(W_i|f_1, \dots, f_{i-1})$

Algorithm 10: Forward algorithm.

1 $B'(W_{i+1}) \leftarrow \sum_{w_i} P(W_{i+1}|w_i)B(w_i)$ // time elapse update
 2 $B(W_{i+1}) \propto P(f_{i+1}|W_{i+1})B'(W_{i+1})$ // observation update

Definition 10.3 (Particle Filtering). Sample - i update - i calculate probability - i re-sample

11 Machine Learning

11.1 Naive Bayes

Definition 11.1 (Naive Bayes Classifier). A model for solving classification problems that makes the assumption that each feature F_i is conditionally independent of all other features given the class label, which is a trade-off between complexity and statistical efficiency.

$$\text{prediction}(F) = \underset{y_i}{\operatorname{argmax}} P(Y = y_i) \prod_j P(F_j = f_j | Y = y_i)$$

Definition 11.2 (Maximum Likelihood Estimation (MLE)). Assume all samples x_i are independent and identically distributed (i.i.d.) with a distribution parametrized by an unknown θ . Based on the assumptions, we wish to find the maximum likelihood

$$\underset{\theta}{\operatorname{argmax}} \mathcal{L}(\theta) = \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^N P_{\theta}(x_i)$$

which satisfies gradient optimization

$$\frac{\partial}{\partial \theta} \mathcal{L}(\theta) = 0$$

Under the Naive Bayes model with Bernoulli feature distributions, the MLE for any outcome x is

$$P_{MLE}(x) = \frac{\text{count}(x)}{N}$$

Definition 11.3 (Laplace Smoothing). A method for mitigating overfitting.

$$P_{LAP,k}(x) = \frac{\text{count}(x) + k}{N + k|X|}$$

$$P_{LAP,k}(x|y) = \frac{\text{count}(x, y) + k}{\text{count}(y) + k|X|}$$

11.2 Perceptron

Definition 11.4 (Linear Classifier). Method for binary classification (i.e., label has two possibilities positive and negative) using a linear combination of weighted features.

$$\text{activation}_w(x) = \langle w, f(x) \rangle = \|w\| \|f(x)\| \cos(\theta)$$

$$\text{classify}(x) = \begin{cases} + & \text{activation}_w(x) > 0 \\ - & \text{activation}_w(x) < 0 \end{cases} = \begin{cases} + & \cos(\theta) > 0 \\ - & \cos(\theta) < 0 \end{cases} = \begin{cases} + & \theta < \frac{\pi}{2} \\ - & \theta > \frac{\pi}{2} \end{cases}$$

Remark. $\text{activation}_w(x) = 0 \implies$ decision boundary (hyperplane)

Algorithm 11: Binary Perceptron

```

1 initialize  $\forall w_i \leftarrow 0$ 
2 while  $y = \text{classify}(x) \neq \text{true label } y^*$  do
3    $w \leftarrow w + \langle y^*, f \rangle$ 

```

Definition 11.5 (Multiclass Decision). Given a weight vector for w_y for each class y , the classification is

$$\text{classify}(x) = \underset{y}{\operatorname{argmax}} \langle w_y, f(x) \rangle$$

Algorithm 12: Multiclass Perceptron

```

1 initialize  $\forall w_{y,i} \leftarrow 0$ 
2 while  $y = \text{classify}(x) \neq \text{true label } y^*$  do
3    $w_y \leftarrow w_y - f(x)$ 
4    $w_{y^*} \leftarrow w_{y^*} + f(x)$ 

```
