# Problem Set #9

CHEM101A: General College Chemistry

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October 17, 2025

# 10 Topic E Problem 10

Calculate the energy of the fourth energy level in:
a) a hydrogen atom
b) a $C^{5+}$ ion (note that this is a one-electron ion)

## 10.1 Solution (a)

The atomic number of Hydrogen is 1. We can apply this to the Rydberg constant equation.

$$E = -R_y \frac{Z^2}{n^2} = -2.180 \times 10^{-18}\ \mathrm{J}\ \frac{1^2}{4^2} \tag{1}$$

$$= -1.3625 \times 10^{-19}\ \mathrm{J} = \boxed{-1.363 \times 10^{-19}\ \mathrm{J}} \tag{2}$$

## 10.2 Solution (b)

The atomic number of Carbon (C) is 6. This is a one-electron atom.

$$E = -R_y \frac{Z^2}{n^2} = -2.180 \times 10^{-18}\ \mathrm{J}\ \frac{6^2}{4^2} \tag{3}$$

$$= -4.905 \times 10^{-18}\ \mathrm{J} = \boxed{-4.905 \times 10^{-18}\ \mathrm{J}} \tag{4}$$

# 11 Topic E Problem 11

Both hydrogen atoms and $Be^{3+}$ ions have an allowed energy level at -7.77 kJ/mol.

a) What is the value of n for this level in the hydrogen atom?

b) What is the value of n for this level in the $Be^{3+}$ ion?

## 11.1 Solution (a)

We can use the equation with the Rydberg constant.

$$E = -R_y \frac{Z^2}{n^2} \tag{5}$$

$$n^2 = -R_y \frac{Z^2}{E} = -1313\,\mathrm{kJ/mol}\, \frac{1^2}{-7.77\,\mathrm{kJ/mol}} \tag{6}$$

$$= 168.9 \approx 169 \tag{7}$$

$$n = \sqrt{169} = \boxed{13} \tag{8}$$

## 11.2 Solution (b)

The atomic number of Beryllium (Be) is 4. We can use the equation with the Rydberg constant.

$$E = -R_y \frac{Z^2}{n^2} \tag{9}$$

$$n^2 = -R_y \frac{Z^2}{E} = -1313\,\mathrm{kJ/mol}\, \frac{4^2}{-7.77\,\mathrm{kJ/mol}} = 2704 \tag{10}$$

$$n = \sqrt{2704} = \boxed{52} \tag{11}$$

# 12 Topic E Problem 12

a) Calculate the wavelength of the light that is emitted when the electron in a hydrogen atom drops from n = 9 to n = 6.

b) What wavelength would be emitted if this electron were in a $B^{4+}$ ion instead of a hydrogen atom? Calculate the wavelength.

## 12.1 Solution (a)

Use the Rydberg Equation.

$$\Delta E = R_y Z^2 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = 2.180 \times 10^{-18}\,\mathrm{J} * 1^2 \left(\frac{1}{9^2} - \frac{1}{6^2}\right) \tag{12}$$

$$= 2.180 \times 10^{-18}\,\mathrm{J} \left(-\frac{5}{324}\right) = -3.364 \times 10^{-20}\,\mathrm{J} \tag{13}$$

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\,\mathrm{J\,s})(2.998 \times 10^{8}\,\mathrm{m/s})}{3.364 \times 10^{-20}\,\mathrm{J}} \tag{14}$$

$$= \frac{1.986 \times 10^{-25}\,\mathrm{J\,m}}{3.364 \times 10^{-20}\,\mathrm{J}} = \boxed{5.905 \times 10^{-6}\,\mathrm{m}} \tag{15}$$

## 12.2 Solution (b)

The atomic number of Boron is 5.

$$\Delta E = R_y Z^2 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = 2.180 \times 10^{-18}\,\mathrm{J} * 5^2 \left(\frac{1}{9^2} - \frac{1}{6^2}\right) \tag{16}$$

$$= 2.180 \times 10^{-18}\,\mathrm{J} \left(-\frac{25}{324}\right) = -1.682 \times 10^{-19}\,\mathrm{J} \tag{17}$$

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\,\mathrm{J\,s})(2.998 \times 10^{8}\,\mathrm{m/s})}{1.682 \times 10^{-19}\,\mathrm{J}} \tag{18}$$

$$= \frac{1.986 \times 10^{-25}\,\mathrm{J\,m}}{1.682 \times 10^{-19}\,\mathrm{J}} = \boxed{1.181 \times 10^{-6}\,\mathrm{m}} \tag{19}$$

# 13 Topic E Problem 13

The emission spectrum of hydrogen has a line at a wavelength of 2871 nm.

a) Calculate the energy change for the electron transition that corresponds to this line.

b) One of the energy levels involved in this transition has n = 5. What is the value of n for the other energy level?

c) Is the value of n you calculated in part b the initial value, or the final value?

## 13.1 Solution (a)

If the energy change is from the hydrogen atom emitting a photon, the energy will be negative.

$$E = -\frac{hc}{\lambda} = -\frac{1.9864748 \times 10^{-25}\,\mathrm{J\,m}}{2871 \times 10^{-9}\,\mathrm{m}} = \boxed{-6.9194 \times 10^{-20}\,\mathrm{J}} \tag{20}$$

## 13.2 Solution (b)

We can calculate the energy of a Hydrogen electron at level n = 5.

$$E = -\frac{R_y Z^2}{n^2} = -\frac{(2.180 \times 10^{-18}\,\mathrm{J})1^2}{5^2} = -8.72 \times 10^{-20}\,\mathrm{J} \tag{21}$$

At this point, we could either add or subtract our value of $E$ from part (a) from this. Such will determine whether the level is higher or lower than n = 5. Start with adding.

$$E_1 = -8.72 \times 10^{-20}\,\mathrm{J} - 6.9194 \times 10^{-20}\,\mathrm{J} \tag{22}$$

$$= -15.6394 \times 10^{-20}\,\mathrm{J} = -\frac{R_y Z^2}{n^2} \tag{23}$$

$$n = \sqrt{-\frac{R_y Z^2}{E}} = \sqrt{\frac{2.180 \times 10^{-18}\,\mathrm{J}}{15.6394 \times 10^{-20}\,\mathrm{J}}} \tag{24}$$

$$= \sqrt{13.939} = 3.73\,\text{(not an integer)} \tag{25}$$

Now subtracting.

$$E_1 = -8.72 \times 10^{-20}\,\mathrm{J} + 6.9194 \times 10^{-20}\,\mathrm{J} \tag{26}$$

$$= -1.8006 \times 10^{-20}\,\mathrm{J} = -\frac{R_y Z^2}{n^2} \tag{27}$$

$$n = \sqrt{-\frac{R_y Z^2}{E}} = \sqrt{\frac{2.180 \times 10^{-18}\,\mathrm{J}}{1.8006 \times 10^{-20}\,\mathrm{J}}} \tag{28}$$

$$= \sqrt{121} = 11\,\text{(an integer)} \tag{29}$$

The latter is an integer, so the answer would be the latter.

$$\boxed{n = 11}$$

### 13.3 Solution (c)

Since the atom would be emitting a photon, as mentioned in part (a), this would be the initial value.

# 14 Topic E Problem 14

The average kinetic energy of an electron in a ground-state helium atom is $2.4{\times}10^3$ kJ/mol.

a) What is the corresponding electron velocity?

b) If an experiment is able to measure this velocity with an uncertainty of 10%, what is the minimum uncertainty in the position of the electron for this experiment?

c) The effective radius of a helium atom is 130 pm. Is the uncertainty you calculated in part b a significant fraction of this effective radius?

## 14.1 Solution (a)

We can rewrite kinetic energy to have units of kJ/mol.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mv^2 N_A \times 10^{-3} \tag{30}$$

We can solve this for the velocity and find it from there.

$$v = \sqrt{\frac{2K \times 10^3}{mN_A}} = \sqrt{\frac{2 * 2.4 \times 10^6}{9.109 \times 10^{-31} * 6.022 \times 10^{23}}} \tag{31}$$

$$= \sqrt{\frac{4.8 \times 10^6}{5.485 \times 10^{-7}}} = \sqrt{8.75 \times 10^{12}} = \boxed{3.0 \times 10^6\,\mathrm{m/s}} \tag{32}$$

## 14.2 Solution (b)

If the uncertainty is 10%, we can find the raw value of the uncertainty.

$$\Delta v = 3.0 \times 10^6\,\mathrm{m/s} \times 0.1 = 3.0 \times 10^5\,\mathrm{m/s} \tag{33}$$

Use this in Heisenberg's Uncertainty Principle.

$$\Delta x \Delta p \leq \frac{h}{4\pi} \tag{34}$$

$$\Delta x \leq \frac{h}{4\pi\,\Delta p} = \frac{6.626 \times 10^{-34}\,\mathrm{J\,s}}{4\pi\,(9.109 \times 10^{-31}\,\mathrm{kg})(3.0 \times 10^5\,\mathrm{m/s})} \tag{35}$$

$$\Delta x \leq \frac{6.626 \times 10^{-34}\,\mathrm{J\,s}}{\underline{3.4}34 \times 10^{-24}\,\mathrm{kg\,m/s}} = \underline{19}2.95 \times 10^{-12}\,\mathrm{m} = \boxed{190\,\mathrm{pm}} \tag{36}$$

## 14.3 Solution (c)

It is a significant fraction of the effective radius. More than that, it's more than the effective radius. Beyond that, it's two thirds of the effective diameter.

# 15 Topic E Problem 15

In the Schrödinger equation $\mathcal{H}\Psi = \mathrm{E}\Psi$, what do the symbols E and $\psi$ stand for?

## 15.1 Solution

E stands for the total energy of every possible quantized state (every electron). It acts a lot like an eigenvalue for the hamiltonian $\mathcal{H}$. $\Psi$ refers to the wave function corresponding to the energy. It acts a lot like an eigenvector for the hamiltonian $\mathcal{H}$. It can be used for the radial distribution of the electron as well.

# 16 Topic E Problem 16

What is the difference between a radial node and an angular node?

## 16.1 Solution

A radial node is active at every point along a specific radius, so you would find no electrons at that radius. An angular node is active at every point along a plane at a specific angle with the z-axis, so you would find no electrons in that specific plane.

## 17 Topic E Problem 17

Complete the following table. The first row is completed for you as an example.

| Orbital | Value of n | Value of $\ell$ | Possible values of $m_\ell$ | Number of nodes | Number of radial nodes | Number of angular nodes |
|---|---|---|---|---|---|---|
| **2p** | **2** | **1** | **-1,0,1** | **1** | **0** | **1** |
| 5d | 5 | 2 | -2, -1, 0, 1, 2 | 4 | 2 | 2 |
| 6p | 6 | 1 | -1, 0, -1 | 5 | 4 | 1 |
| 5f | 5 | 3 | -3, -2, -1, 0, 1, 2, 3 | 4 | 1 | 3 |
| 7d | 7 | 2 | -2, -1, 0, 1, 2 | 6 | 4 | 2 |
| 4p | 4 | 1 | -1, 0, 1 | 3 | 2 | 1 |
| 9f | 9 | 3 | -3, -2, -1, 0, 1, 2, 3 | 8 | 5 | 3 |

Note to self: do this problem again later to drive it in.

# 18 Topic E Problem 18

Calculate the energy of the 5p$_x$ orbital in a hydrogen atom.

## 18.1 Solution

The quantum number n of this orbital is 5. The atomic number of hydrogen is 1. Use the Rydberg constant equation.

$$E = -R_y \frac{Z^2}{n^2} = -\frac{2.180 \times 10^{-18}\,\mathrm{J}(1)}{5^2} = \boxed{-8.72 \times 10^{-20}\,\mathrm{J}} \tag{37}$$

# 19 Topic E Problem 19

a) How many 2p orbitals are there?

b) How many 5f orbitals are there?

## 19.1 Solution (a)

The textbook tells us that the total number of orbitals in a shell is $2\ell + 1$.

$$2\ell + 1 = 2 * 1 + 1 = \boxed{3}$$

## 19.2 Solution (b)

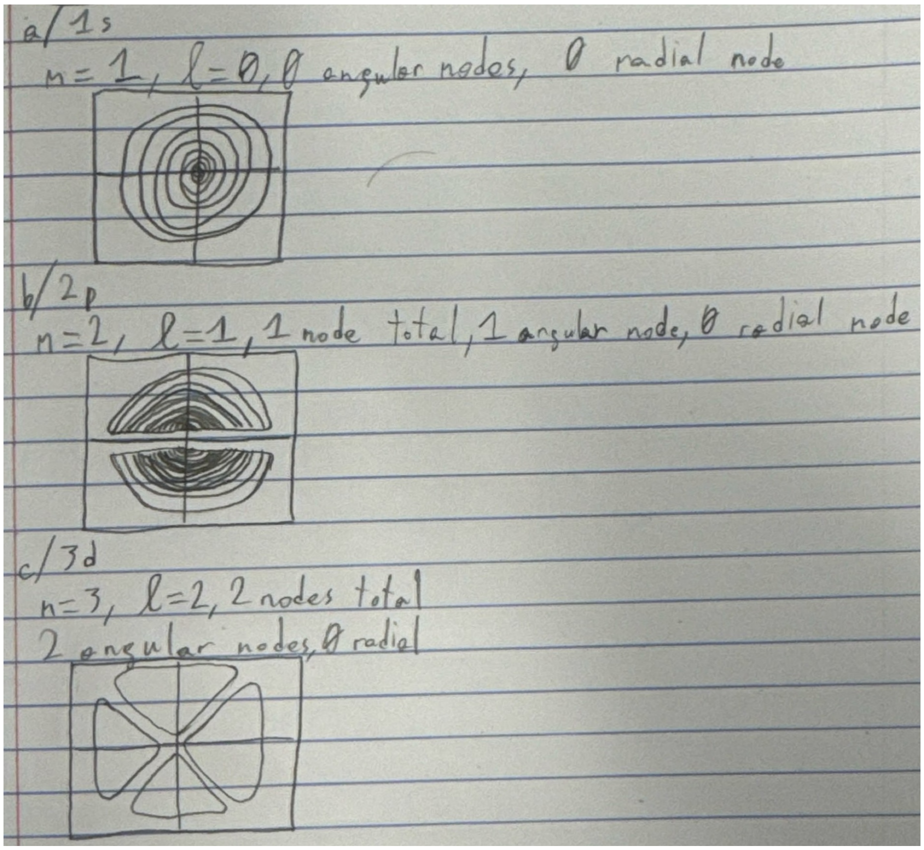
$$2\ell + 1 = 2 * 3 + 1 = \boxed{7}$$

# 20 Topic E Problem 20

Draw a picture of each of the following orbitals.

a)1s b)2p c)3d

## 20.1 Solution



# 21 Topic E Problem 21

a) How does a 1s orbital differ from a 2s orbital?

b) How does a 2s orbital differ from a 2p orbital?

c) How does a 2p$_x$ orbital differ from a 2p$_y$ orbital?

d) How does a 3p orbital differ from a 4d orbital?

## 21.1 Solution (a)

A 2s orbital is of greater radius than a 1s orbital due to a higher energy.

## 21.2 Solution (b)

The shape is different. A 2p orbital would be two lobes along the x/y/z axis, while the 2s orbital would have a spherical orbital.

## 21.3 Solution (c)

The 2p$_x$ orbital's lobes would be centered around the x-axis, while the 2p$_y$ orbital's lobes would be centered around the y-axis.

## 21.4 Solution (d)

The 4d orbital would be bigger due to a higher energy, and it would also be have a different shape with more lobes.

# 22 Topic E Problem 22

a) How many different orbitals have n = 7? Explain your answer briefly.

b) How many different orbitals have n = 9 and $\ell = 7$? Explain your answer briefly.

c) How many different orbitals have n = 8, $\ell = 5$, and $m_\ell$ = -3? Explain your answer briefly.

d) How many different orbitals have n = 6 and $m_\ell = 2$? Explain your answer briefly.

e) How many different orbitals have $\ell = 1$ and $m_\ell = 0$? Explain your answer briefly.

## 22.1 Solution (a)

For n = 7, $\ell$ has seven possible values: 0, 1, 2, and 3, 4, 5, and 6. For $\ell = 0$, there is 1 possible value of $m_\ell$. For $\ell = 1$, there is 3 possible values of $m_\ell$. For $\ell = 2$, there is 5 possible values of $m_\ell$. For $\ell = 3$, there is 7 possible values of $m_\ell$. For $\ell = 4$, there is 9 possible values of $m_\ell$. For $\ell = 5$, there is 11 possible values of $m_\ell$. For $\ell = 6$, there is 13 possible values of $m_\ell$. Adding these up, we get $1 + 3 + 5 + 7 + 9 + 11 + 13 = \boxed{49}$.

## 22.2 Solution (b)

For $\ell = 7$, there are $\boxed{15}$ possible values of $m_\ell$, the number of integers between $-\ell$ and $\ell$, inclusive.

## 22.3 Solution (c)

In this case, all bases are covered in terms of atomic orbitals and there is no room for variation. As such, there is $\boxed{1}$ orbital that fits this criteria.

## 22.4 Solution (d)

The varying value here is $\ell$. Having n = 6 sets an upper bound of the value of $\ell$ at 5. Meanwhile, having $m_\ell = 2$ sets a lower bound of $\ell = 2$. This gives $\boxed{4}$ orbitals.

## 22.5 Solution (e)

Since there is no upper bound of $n$ dependant on either $\ell$ or $m_\ell$, there are an <u>infinite</u> number of orbitals that fit this.

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