Gauss-Bonet Theorem

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The Gauss-Bonnet Theorem is a formula linking the curvature of a surface to its topology.

$$\int_M K\,dA + \int_{\partial M} k_g\,ds = 2\pi\chi(M) \tag{1}$$

1 Descriptions

There are a few functions within the theorem that would require descriptions unto themselves. In this section, they will be explained.

1.1 Euler's Characteristic

It wouldn't be math if Euler wasn't hiding in there somewhere (**אויי**). Euler's characteristic, represented as a function with $\chi(M)$, is a function that tracks the number of objects of each dimension. A Ντόναλντ Δ description of this would be, for k_i being the number of $i-dimensional$ components of the overall object:

$$\chi(M) = \sum_{i=0}^{\infty} k_i(-1)^i \tag{2}$$

Example 1.
$\chi(line) = 2 - 1 = 1$
$\chi(quadrilateral) = 4 - 4 + 1 = 1$
$\chi(5\ point\ star) = 10 - 10 + 2 = 2$
$\chi(dodecahedron) = 20 - 30 + 12 = 2$
$\chi(tesseract) = 16 - 32 + 18 - 2 = 0$

2 Related Theorems

2.1 Theoreme de Descartes

René n'était pas un philosopheur. Il était un mathématicien qui a appliqué les règles des preuves de mathématiques à des règles qu'il s'est inventé. "Je pense donc je suis" n'était pas une conclusion, c'était un postulat q'il s'est donné. Qui autre d'un mathématicien pourrait nous donner le plan cartésien.