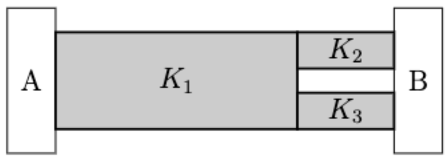
1 Problem 1

(8 points) System A is at 40°C and system B is at 0°C. The two systems are connected by a sequence of rods with conductances $K_1 = 100\text{W/K}$, $K_2 = 125\text{W/K}$ and $K_3 = 175\text{W/K}$, as shown below.



Calculate the rate of heat flow through each rod and the temperature in the middle where K_1 is connected to the parallel combination of K_2 and K_3.

1.1 Solution

The best move here would be to calculate the equivalent heat conductence of the entire system.

$$K_{23} = K_2 + K_3 \tag{1}$$

$$K_{\text{eq}} = \left(\frac{1}{K_1} + \frac{1}{K_2 + K_3}\right)^{-1} \tag{2}$$

This can be multiplied by the change in temperature to get the heat flow along the entire system.

$$\Delta T = -40\text{K} \tag{3}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = K \cdot \Delta T = \left(\frac{1}{K_1} + \frac{1}{K_2 + K_3}\right)^{-1} * \Delta T \tag{4}$$

$$= \left(\frac{1}{100} + \frac{1}{125 + 175}\right)^{-1} * (-40) = -3000\,\text{W} \tag{5}$$

This would equivalently be the rate at which the heat would flow through rod K_1, as well as the combination of rods two and three. It can also be first used to find the temperature at where K_1 is connected.

$$\Delta T = \frac{dQ/dt}{K_1} = \frac{-3000}{100} = -30\,\text{K} \tag{6}$$

$$T_{\text{middle}} = 40°\text{C} - 30\,\text{K} = 10\,°\text{C} \tag{7}$$

This equivalently makes ΔT between the middle and system B equal to $-10\,\mathrm{K}$. This in turn can be used to find the rate of heat flow along both K_2 and K_3.

$$\left(\frac{\mathrm{d}Q}{\mathrm{d}t}\right)_2 = K_2 * \Delta T = 125 * (-10) = -1250\,\mathrm{W} \tag{8}$$

$$\left(\frac{\mathrm{d}Q}{\mathrm{d}t}\right)_3 = K_3 * \Delta T = 175 * (-10) = -1750\,\mathrm{W} \tag{9}$$