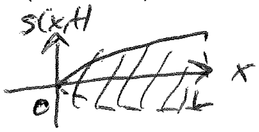
*. (10 points) A half-open organ pipe is tuned to A(440) (i.e., the fundamental frequency is 440 Hz). Air has a density of $1.21\,\mathrm{kg/m^3}$ and a speed of sound of 343 m/s at 20°C.

a. (3 points) What is the length of the pipe?



$$\lambda_0 = 4L \qquad f_0 = \frac{v}{4L} = 440\,\mathrm{Hz}$$

$$\Rightarrow L = \frac{v}{4f_0} = \frac{343\,\mathrm{m/s}}{4(440\,\mathrm{Hz})} = 0.19489\,\mathrm{m}$$

b. (4 points) What is the maximum kinetic energy density (per unit volume) at the open end of the pipe if $s_m = 2.0\,\mu\mathrm{m}$? At the closed end?

$$s(x,t) = s_m \sin kx \sin \omega t$$

$$\frac{\partial s}{\partial t} = s_m \omega \sin kx \cos \omega t$$

$$\left(\frac{\partial s}{\partial t}\right)_{max} = s_m \omega |\sin kx|$$

$$[\omega = 2\pi f = 2764.6\,\mathrm{rad/s}]$$

At closed end, $\sin kx = 0$ $\Rightarrow \rho_K = 0$ always,

At open end, $\sin kx = \pm 1$

so $\left(\frac{\partial s}{\partial t}\right)_{max} = s_m \omega$

$$\Rightarrow \rho_K = \frac{1}{2}\rho\left(\frac{\partial s}{\partial t}\right)^2 = \frac{1}{2}\rho s_m^2 \omega^2$$

$$= \frac{1}{2}(1.21\,\mathrm{kg/m^3})(2.0\times10^{-6}\,\mathrm{m})^2 \cdot (2764.6\,\tfrac{\mathrm{rad}}{\mathrm{s}})^2$$

$$= 18.496\,\mu\mathrm{J/m^3}$$

c. (3 points) If the ambient temperature were raised from 20°C to 40°C, what would be the new fundamental frequency of the pipe (ignore changes in the length of the pipe due to the temperature change)?

$$T_i = 20^\circ\mathrm{C} + 273 = 293\,\mathrm{K}$$

$$T_f = 40^\circ\mathrm{C} + 273 = 313\,\mathrm{K}$$

$$v = \sqrt{\frac{\gamma k T}{m}}$$

$$\Rightarrow \frac{v_f}{v_i} = \sqrt{\frac{T_f}{T_i}} = 1.0336$$

$$\Rightarrow v_f = (1.0336)(343\,\mathrm{m/s}) = 354.51\,\mathrm{m/s}$$

$$f_0 = \frac{v}{4L} \Rightarrow \frac{f_f}{f_i} = \frac{v_f}{v_i} = 1.0336$$

$$\Rightarrow f_f = (440\,\mathrm{Hz})(1.0336)$$

$$= 454.77\,\mathrm{Hz}$$

$$12 \log_2\left(\frac{454.77}{440}\right) = 0.57 \text{ half-steps} \Rightarrow \gtrsim \text{quarter-tone high!}$$