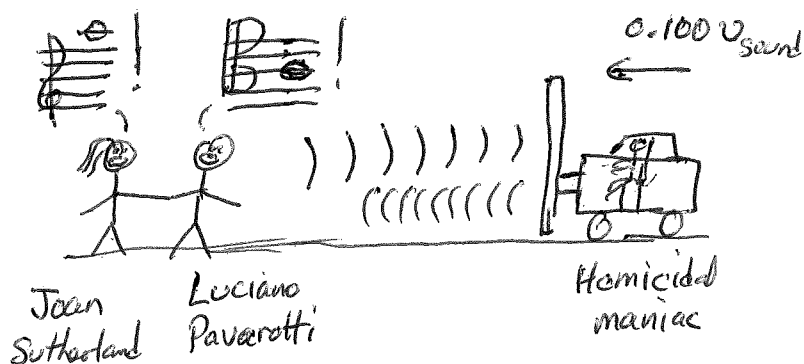*. (6 points) Two opera singers (standing still) are singing a high A ($f = 880\,\text{Hz}$) and low A ($f = 220\,\text{Hz}$), respectively, when a car carrying a sound-reflecting wall is travelling towards them at 0.100 times the speed of sound. Determine the frequencies of the reflected sound waves, as would be heard by the opera singers.

Note: No opera singers were harmed during the construction of this question or its solution.

f_S = frequency of source (singer) = $880\,\text{Hz}$ or $220\,\text{Hz}$

f_R = frequency as determined by a hypothetical detector at reflecting wall.

f_D = frequency at final detector (singers again)

$S \to R:\quad f_R/f_S = \dfrac{1 - v_R/v_w}{1 - v_S/v_w} = \dfrac{1 - (-0.100|v_w|)/|v_w|}{1 - 0} = 1.100$

$R \to D:$ (treat reflecting wall as a source)

$f_D/f_R = \dfrac{1 - v_D/v_w}{1 - v_R/v_w} = \dfrac{1 - 0}{1 - (-0.100|v_w|)/(-|v_w|)} = \dfrac{1}{0.900} = 1.11\overline{1}$

$\Rightarrow f_D = (1.100)\left(\dfrac{1}{0.900}\right) f_S = 1.22\overline{2}\, f_S = \begin{cases} 1075.\overline{5}\,\text{Hz} \\ 268.\overline{8}\,\text{Hz} \end{cases}$

$12 \log_2 (1.22\overline{2}) = 3.47$ half-steps

$\Rightarrow$ just about half-way between C♮ and C♯ above the respective A's.