Chapter 36 End-of-Chapter Problems

Halliday & Resnick, 10th Edition

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Hit me where it Matters

1 Problem 1

The distance between the first and fifth minima of a single-slit diffraction pattern is 0.35 mm with the screen 40 cm away from the slit, when light of wavelength 550 nm is used. (a) Find the slit width. (b) Calculate the angle θ of the first diffraction minimum.

1.1 Solution (a)

Estimate $\sin\theta \approx \theta \approx \tan\theta = \frac{y}{D}$. $D = 40\,\text{cm}$ and never changes in this case. y does change, so we can create a Δy.

$$a\sin\theta = a\frac{y}{D} = m\lambda \tag{1}$$

We can create a Δy on one side and Δm on the other, for a separation between the first and fifth fringes.

$$a\frac{y}{D} = m\lambda \tag{2}$$

$$a\frac{\Delta y}{D} = \Delta m\lambda \tag{3}$$

$$a = \frac{\Delta m}{\Delta y}\lambda D = \frac{4}{0.35\,\text{mm}} * 550\,\text{nm} * 0.4\,\text{m} = \boxed{2.51\,\text{mm}} \tag{4}$$

1.2 Solution (b)

Divide the distance between fringes by four.

$$\Delta y = \frac{0.35\,\text{mm}}{4} = 87.5 \times 10^{-6}\,\text{m} \tag{5}$$

Divide this by the distance to the screen to find the approximate angle.

$$\theta = \frac{87.5 \times 10^{-6}\,\text{m}}{0.4\,\text{m}} = \boxed{2.2 \times 10^{-4}} \tag{6}$$

2 Problem 5

A single slit is illuminated by light of wavelengths λ_a and λ_b, chosen so that the first diffraction minimum of the λ_a component coincides with the second minimum of the λ_b component. (a) If $\lambda_b = 350\,\text{nm}$, what is λ_a? For what order number m_b (if any) does a minimum of the λ_b component coincide with the minimum of the λ_a component in the order number (b) $m_a = 2$ and (c) $m_a = 3$?

2.1 Solution (a)

We can define a relationship between y_a and y_b.

$$y_a = y_b \tag{7}$$

Approximating $\sin\theta \approx \theta \approx \tan\theta$, we have a relationship equation between y and λ for both a and b.

$$a\sin\theta = a\frac{y_a}{D} = m_a\lambda_a \tag{8}$$

$$a\frac{y_b}{D} = m_b\lambda_b \rightarrow y_a = y_b = \frac{m_b\lambda_b D}{a} \tag{9}$$

$$a\frac{\frac{m_b\lambda_b D}{a}}{D} = m_a\lambda_a = 2m_b\lambda_b \tag{10}$$

In this case, m_a would be equal to 1, while m_b would be equal to 2.

$$\lambda_a = 2\lambda_b = 2 * 350\,\text{nm} = \boxed{700\,\text{nm}} \tag{11}$$

2.2 Solution (b)

4

2.3 Solution (c)

6

3 Problem 9

A slit 1.00 mm wide is illuminated by light of wavelength 589 nm. We see a diffraction pattern on a screen 3.00 m away. What is the distance between the first two diffraction minima on the same side of the central diffraction maximum?

3.1 Solution

Use the single slit diffraction equation. Use Δy and Δm, the latter of which will be equal to 1. Also use the small angle approximation.

$$a \sin\theta \approx a\frac{y}{D} = m\lambda \tag{12}$$

$$y = \frac{m\lambda D}{a} \tag{13}$$

$$\Delta y = \frac{\Delta m \lambda D}{a} = \frac{1 * 589\,\mathrm{nm} * 3.00\,\mathrm{m}}{1.00\,\mathrm{mm}} = \boxed{1.767\,\mathrm{mm}} \tag{14}$$

4 Problem 13

Monochromatic light with wavelength 538 nm is incident on a slit with width 0.025 mm. The distance from the slit to a screen is 3.5 m. Consider a point on the screen 1.1 cm from the central maximum. Calculate (a) θ for that point, (b) α, and (c) the ratio of the intensity at that point to the intensity at the central maximum.

4.1 Solution (a)

Use the definition of the tangent, then use the arctangent.

$$\tan(\theta) = \frac{y}{D} \tag{15}$$

$$\theta = \arctan\left(\frac{y}{D}\right) = \arctan\left(\frac{0.011}{3.5}\right) = \boxed{0.003143\,\mathrm{rad}} \tag{16}$$

4.2 Solution (b)

Calculate α using its definition.

$$\alpha = \frac{\pi a}{\lambda}\sin\theta = \frac{\pi(25\times10^{-6}\,\mathrm{m})}{538\times10^{-9}\,\mathrm{m}}\sin(0.003143\,\mathrm{rad}) = \boxed{0.4588} \tag{17}$$

4.3 Solution

This is just calculating the intensity at that angle but foregoing the initial intensity.

$$I_{\mathrm{ratio}} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin(0.4588)}{0.4588}\right)^2 = \boxed{0.931} \tag{18}$$

5 Problem 15

The full width at half-maximum (FWHM) of a central diffraction maximum is defined as the angle between the two points in the pattern where the intensity is one-half that at the center of the pattern. (See Fig. 36-8b.) (a) Show that the intensity drops to one-half the maximum value when $\sin^2 \alpha = \alpha^2/2$. (b) Verify that $\alpha = 1.39\,\text{rad}$ (about $80°$) is a solution to the transcendental equation of (a). (c) Show that the FWHM is $\Delta\theta = 2\sin^{-1}(0.442\lambda/a)$, where a is the slit width. Calculate the FWHM of the central maximum for slit width (d) 1.00λ, (e) 5.00λ, and (f) 10.0λ.

5.1 Solution (a)

This is an intensity ratio case. Take the ratio of two cases: one at the maximum and one where the intensity is half the maximum. The ratio you would end up with is as follows.

$$\frac{1}{2} = \left(\frac{\sin(\alpha)}{\alpha}\right)^2 \tag{19}$$

Solve for $\sin^2(\alpha)$.

$$\sin^2(\alpha) = \frac{\alpha^2}{2} \tag{20}$$

TOA.

5.2 Solution (b)

Set $\alpha = 1.39\,\text{rad}$ and do the calculation.

$$\sin^2(\alpha) = \frac{\alpha^2}{2} \tag{21}$$

$$\sin^2(1.39) = \frac{1.39^2}{2} \tag{22}$$

$$0.9837^2 = \frac{1.9321}{2} \tag{23}$$

$$0.968 \approx 0.966 \tag{24}$$

TOA.

5.3 Solution (c)

Let's take the case of $\alpha = 1.39\,\text{rad}$ and find θ from the equation for α. Use $\frac{\theta}{2}$ instead of θ.

$$\alpha = \frac{\pi a}{\lambda} \sin \frac{\theta}{2} \tag{25}$$

$$\sin \frac{\theta}{2} = \frac{\lambda \alpha}{\pi a} \tag{26}$$

$$\frac{\theta}{2} = \arcsin\left(\frac{\lambda \alpha}{\pi a}\right) = \arcsin\left(\frac{\lambda\, 1.39}{\pi a}\right) = \arcsin\left(\frac{0.442\lambda}{a}\right) \tag{27}$$

$$\theta = 2 \arcsin\left(\frac{0.442\lambda}{a}\right) \tag{28}$$

TOA.

5.4 Solution (d)

Plug into the equation.

$$\frac{\Delta\theta}{2} = \arcsin\left(\frac{0.442\lambda}{a}\right) = \arcsin\left(\frac{0.442\lambda}{1.00\lambda}\right) \tag{29}$$

$$\Delta\theta = 2 \arcsin(0.442) = \boxed{0.9156} \tag{30}$$

5.5 Solution (e)

Plug into the equation.

$$\frac{\Delta\theta}{2} = \arcsin\left(\frac{0.442\lambda}{a}\right) = \arcsin\left(\frac{0.442\lambda}{5.00\lambda}\right) \tag{31}$$

$$\Delta\theta = 2 \arcsin\left(\frac{0.442}{5.00}\right) = \boxed{0.177} \tag{32}$$

5.6 Solution (d)

Plug into the equation.

$$\frac{\Delta\theta}{2} = \arcsin\left(\frac{0.442\lambda}{a}\right) = \arcsin\left(\frac{0.442\lambda}{10.00\lambda}\right) \tag{33}$$

$$\Delta\theta = 2 \arcsin\left(\frac{0.442}{10.00}\right) = \boxed{0.0884} \tag{34}$$

6 Problem 19

(a) How far from grains of red sand must you be to position yourself just at the limit of resolving the grains if your pupil diameter is 1.5 mm, the grains are spherical with radius 50 µm, and the light from the grains has wavelength 650 nm? (b) If the grains were blue and the light from them had wavelength 400 nm, would the answer to (a) be larger or smaller?

6.1 Solution (a)

7 Problem 21

7.1 Solution

8 Problem 37

8.1 Solution

9 Problem 39

9.1 Solution

10 Problem 45

10.1 Solution

11 Problem 47

11.1 Solution

12 Problem 49

12.1 Solution

13 Problem 59

13.1 Solution

14 Problem 63

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