We start with the equation for the voltage at a given point.

$$dV = \frac{k\,dq}{r} \tag{1}$$

We have a known value for dq.

$$dq = \lambda\,d\ell \tag{2}$$

ℓ is going to be the individual point along the circle that we will reference. ℓ will always be a distance a from the center of the circle and it will always make an angle with the line from the center of the circle to the point P (call the angle θ). Using θ, we can formulate a value for the distance from the angle θ along the circle to the point P. This involves the law of cosines.

$$x^2 = b^2 + c^2 - 2bc\cos(\alpha) \tag{3}$$

Here, we can designate the line from the center to ℓ as b, the line from the center to point P as c, and the ine from ℓ to point P as x. Following from this, we would have $\theta = \alpha$. Furthermore, we already know the value of b and c to be the radius of the circle a.

$$x^2 = a^2 + a^2 - 2a^2\cos(\theta) = 2a^2(1-\cos(\theta)) \tag{4}$$

$$x = \sqrt{2a^2(1-\cos(\theta))}$$

For $x = r$, we can apply this to the earlier differential equation.

$$dV = \frac{k\,dq}{r} = \frac{k\lambda\,d\ell}{\sqrt{2a^2(1-\cos(\theta))}}$$

ℓ can be expressed in terms of θ. This makes use of radians.

$$d\ell = a\,d\theta \tag{5}$$

We can add this to our differential equation. It allows us to cancel out some terms.

$$dV = \frac{k\lambda a\,d\theta}{a\sqrt{2(1-\cos(\theta))}} = \frac{k\lambda\,d\theta}{\sqrt{2(1-\cos(\theta))}}$$

Our last step is to set up an integral. Our boundaries, due to the shape of the insulator, will be between θ_0 and $2\pi - \theta_0$. This is because the insulator is in a circle around its center. One boundary is located at an angle of θ_0 counterclockwise from the normal radius (horizontal radius starting at the center and going to the right). The other boundary is located at the same angle θ_0 clockwise from the normal radius. Since circles and sines go through the same set of values every 2π, we can add 2π to the second boundary to get $2\pi - \theta_0$.

$$\boxed{V = \int_{\theta_0}^{2\pi-\theta_0} \frac{k\lambda}{\sqrt{2(1-\cos(\theta))}}\,d\theta} \tag{6}$$

Note: k can be transformed into something involving ε_0 if necessary.