*. (10 points) One of the fundamental principles of Classical Mechanics is that if the positions and velocities of all particles in an isolated system are known at some initial time and the internal interactions are completely understood (i.e., all force laws are known), then it should be possible (at least in theory) to determine the motion of the entire system for all subsequent time.

Suppose we have an infinite string with given tension T and mass/length μ and it is known at time $t = 0$ that $y(x, 0) = Y(x)$ and that all parts of the string are completely at rest. Determine $y(x, t)$ for the string (do *not* assume that the wave must travel in one direction or the other). Does your result make sense?

The following procedure is recommended.

- Start with the general expression $y(x, t) = f(x - vt) + g(x + vt)$.
- Derive an expression for $\partial y/\partial t$.
- Set $t = 0$ and construct two equations involving the two unknown functions $f(x)$ and $g(x)$.
- Solve those equations for $f(x)$ and $g(x)$ separately.
- Substitute into the expression for $y(x, t)$.

$$y(x,t) = f(x-vt) + g(x+vt)$$

$$\frac{\partial y}{\partial t} = -v f'(x-vt) + v g'(x+vt)$$

$$t=0 \Rightarrow \quad f(x) + g(x) = Y(x)$$

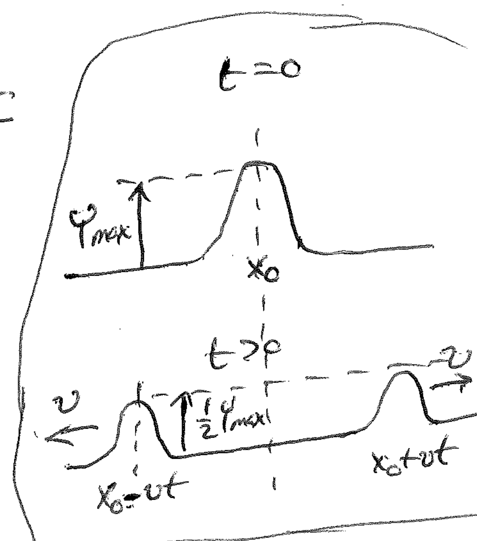
$$v(-f'(x) + g'(x)) = 0 \quad \Rightarrow f'(x) = g'(x)$$

$$\Rightarrow f(x) = g(x) + C$$

$$\Rightarrow 2g(x) + C = Y(x)$$

$$g(x) = \tfrac{1}{2}(Y(x) - C)$$

$$f(x) = g(x) + C = \tfrac{1}{2}(Y(x) + C)$$



$$\Rightarrow y(x,t) = f(x-vt) + g(x+vt)$$

$$= \tfrac{1}{2}[Y(x-vt) + C] + \tfrac{1}{2}[Y(x+vt) - C]$$

$$= \tfrac{1}{2}Y(x-vt) + \tfrac{1}{2}Y(x+vt)$$

makes sense.