Worksheet #13

PHYS 4C: Waves and Thermodynamics

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November 17, 2025

1 Problem 1

Without looking anything up, determine what a light year is in meters.

1.1 Solution

I assume we know the speed of light to be $c = 2.998 \times 10^8$ m/s. We first find the conversion factor of seconds per year.

$$1 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ yr}}{365.25 \text{ day}} = \frac{1 \text{ yr}}{31557600} \tag{1}$$

Multiply the speed of light by the reciprocal of the conversion factor to find the distance light travels in a year.

$$c = 2.998 \times 10^8 \text{ m/s} \times 31557600 \text{ s/yr} = 9.46 \times 10^{15} \text{ m/yr} \tag{2}$$

Multiply this by 1 year to find that one lightyear is equal to $\boxed{9.46 \times 10^{15} \text{ m}}$

2 Problem 2

Unpolarized light travelling in the z-direction is incident upon three polaroid filters with pass directions given by $+35°$, $-40°$, and $+25°$ counter-clockwise from $+x$. Determine the fraction of the original light which passes through the three filters.

2.1 Solution

Divide it into four parts with four intensities for the light: initially (I_0), then after passing through the first (I_1), second (I_2), and third (I_3) filters. First, the the first filter will filter out half the light's intensity.

$$I_1 = \frac{1}{2} I_0 \tag{3}$$

Next, the second filter is $35° - (-40°) = 75°$ away from the first polarization direction. This can be used to find I_2.

$$I_2 = I_1 \cos^2(75°) = \frac{1}{2} \cos^2(75°) I_0 \tag{4}$$

Last, the third filter is $-40° - 25° = -65°$ away from the first polarization direction. This can be used to find I_3.

$$\begin{aligned} I_3 &= I_2 \cos^2(-65°) = \frac{1}{2} \cos^2(-65°) \cos^2(75°) I_0 && (5) \\ &= 0.00598 I_0 && (6) \end{aligned}$$

This tells is that the amount that passes through is $\boxed{0.00598}$.

3 Problem 3

(10 points) An electromagnetic wave has an electric field amplitude with a magnitude of $E_m = 12\text{V/m}$.

a) (1 point) Calculate the magnetic field amplitude.

b) (3 points) Calculate the (time averaged) energy density, intensity, momentum density (magnitude), and momentum current density of this wave.

c) (3 points) If this light is normally incident upon a $1.0\,\text{m}^2$ surface, determine the rate of energy absorption and force acting on this surface. Answer this question for both a perfectly reflecting surface and a perfectly absorbing surface.

d) (3 points) Assume that this electromagnetic wave is located at the point (1000 m, 0, 0) and is generated by a dipole antenna located at (0, 0, 0) and oriented along the z-axis. If the oscillation frequency is 106 Hz, determine the electric dipole amplitude (qz_m) of this antenna.

3.1 Solution (a)

The magnitude of the magnetic field is calculatable from the magnitude of the electric field and the speed of light in a vacuum.

$$\frac{E_m}{B_m} = c \rightarrow B_m = \frac{E_m}{c} = \frac{12\,\mathrm{V/m}}{2.998\,\mathrm{m/s}} = \boxed{4.003 \times 10^{-8}\,\mathrm{T}} \tag{7}$$

3.2 Solution (b)

Energy density first.

$$\begin{aligned} \langle u \rangle &= \frac{1}{2}\varepsilon_0 E_m^2 = \frac{1}{2} * 8.854 \times 10^{-12}\,\mathrm{F/m}(12\,\mathrm{V/m})^2 && (8) \\ &= \boxed{6.327 \times 10^{-10}\,\mathrm{J/m^3}} && (9) \end{aligned}$$

Next the intensity.

$$\langle I \rangle = \frac{1}{2} c\varepsilon_0 E_m^2 = c\,\langle u \rangle = \boxed{0.191\,\mathrm{W/m^2}} \tag{10}$$

Third the momentum density.

$$\begin{aligned} \langle g \rangle &= \frac{\langle u \rangle}{c} = \frac{6.327 \times 10^{-10}\,\mathrm{J/m^3}}{2.998 \times 10^{8}\,\mathrm{m/s}} && (11) \\ &= \boxed{2.1104 \times 10^{-18}\,\mathrm{W/m^4}} && (12) \end{aligned}$$

Last the momentum current density. Suppose that the wave is traveling in only one direction, the $+x$ direction.

$$\begin{aligned} \left\langle \underset{\sim}{\sigma} \right\rangle &= u\hat{r} \otimes \hat{r} = u \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} u & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} && (13) \\ &= \boxed{\begin{bmatrix} 6.327 \times 10^{-10} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathrm{N/m^2}} && (14) \end{aligned}$$

3.3 Solution (c)

3.3.1 Perfectly Reflecting

A perfectly reflecting surface would absorb none of the energy. This would merely redirect the light, resulting in a force equivalent to twice the energy density times the area affected.

$$F = 2\langle u\rangle A = 2 * 6.327 \times 10^{-10}\,\mathrm{N/m^2} * 1.0\,\mathrm{m^2} = \boxed{1.2654 \times 10^{-9}\,\mathrm{N}} \tag{15}$$

3.3.2 Perfectly Absorbing

The perfectly absorbing surface absorbs all energy. Due to that, we can find the rate of absorption by multiplying the intensity by the area covered.

$$\frac{\mathrm{d}E}{\mathrm{d}t} = IA = 0.191\,\mathrm{W/m^2} * 1\,\mathrm{m^2} = \boxed{0.191\,\mathrm{W}} \tag{16}$$

The force taken would be equal to the energy density times the area.

$$F = \langle u\rangle A = 6.327 \times 10^{-10}\,\mathrm{N/m^2} * 1.0\,\mathrm{m^2} = \boxed{6.327 \times 10^{-10}\,\mathrm{N}} \tag{17}$$

3.4 Solution (d)

qz_m can be related to the electric field by a sinusoidal equation.

$$\vec{E}(\vec{r}, t) = \frac{1}{4\pi\varepsilon_0} \times \frac{qz_m\omega^2}{c^2 r}\sin(\theta)\cos(\vec{k}\cdot\vec{r} - \omega t) \tag{18}$$

For the time being, let's set $\sin(\theta)\cos(\vec{k}\cdot\vec{r} - \omega t) = 1$. Due to that, the value of the rest would be equal to the magnitude of the electric field (12 V/m in this case). We can use that.

We can also use the frequency to find the value of ω.

$$\omega = f * 2\pi = 212\pi\,\mathrm{rad/s} \tag{19}$$

$$E_m = \frac{1}{4\pi\varepsilon_0} * \frac{qz_m\omega^2}{c^2 r} \tag{20}$$

$$qz_m = \frac{4\pi\varepsilon_0 * E_m * c^2 r}{\omega^2} \tag{21}$$

$$= \frac{4\pi\varepsilon_0 * 12\,\mathrm{V/m} * (2.998 \times 10^8\,\mathrm{m/s})^2 * 1000\,\mathrm{m}}{(212\pi\,\mathrm{rad/s})^2} \tag{22}$$

$$= \boxed{270412\,\mathrm{C\cdot m}} \tag{23}$$