

“Doubt is not a pleasant condition, but certainty is absurd.”
-Voltaire (Not a physicist...somehow)

1 Question 2

Figure 24-25 shows three sets of cross sections of equipotential surfaces in uniform electric fields; all three cover the same size region of space. The electric potential is indicated for each equipotential surface. (a) Rank the arrangements according to the magnitude of the electric field present in the region, greatest first. (b) In which is the electric field directed down the page?

2 Question 3

Figure 24-26 shows four pairs of charged particles. For each pair, let $V = 0$ at infinity and consider V_{net} at points on the x axis. For which pairs is there a point at which $V_{net} = 0$ (a) between the particles and (b) to the right of the particles? (c) At such a point is E_{net} due to the particles equal to zero? (d) For each pair, are there off-axis points (other than at infinity) where $V_{net} = 0$?

a Solution (a): 1 and 2

For voltage, the formula is $V = \frac{kq}{r}$, and it has no direction. Thus, for each of the cases, since V has the same sign as q , we can know that there can only be a point at which the net voltage is zero when there is both a positive and a negative charge to influence the voltage. The ratio of the magnitudes of the distances (radii) must also be the same as the ratio of the charges in order to make the sum equal to zero.

$$\begin{aligned} 0 = V_1 + V_2 &= \frac{k|q_1|}{r_1} - \frac{k|q_2|}{r_2} \\ \frac{k|q_1|}{r_1} &= \frac{k|q_2|}{r_2} \\ \frac{|q_1|}{|q_2|} &= \frac{r_1}{r_2} \end{aligned}$$

Between the two points, there must be a point at which the ratio holds true. This means that it holds true for 1 and 2

b Solution (b): None

Since there is no point at which $V = 0$ on either (3) or (4), explained in part (a), we can rule them out. On both (1) and (2), the charge on the right is the stronger charge. The aforementioned ratio will start with the ratio either too large or too small, and will keep heading away from an equal ratio ad infinitum as it heads further to the right. This leaves it with none.

3 Problem 8

a Solution: 30V

We can define the function. $E(x) = \begin{cases} 0 \leq x < 2 & | \quad \frac{-20}{2}x \\ 2 \leq x < 4 & | \quad \frac{40}{2} - 60 \\ 4 \leq x < 6 & | \quad 20 \end{cases}$ We will only be working with one dimension.

$$\Delta V = - \int_0^2 E(x) dx = \int_0^2 10x dx = 5x^2 \Big|_0^2 = 20 + 0 = 20V$$
$$V_f = V_i + \Delta V = 10V + 20V = \boxed{30V}$$

b Solution: 40V

Since the electric field is the negative derivative of the electric potential difference, the point at which the electric potential difference would be maximum would be where the electric field would be zero crossing from negative into positive. Aside from the origin, the only spot where this is the case is at $x = 3$.

$$\begin{aligned} \Delta V &= - \int_0^3 E(x) dx \\ &= - \int_0^2 E(x) dx - \int_2^3 E(x) dx \\ &= \int_0^2 10x dx - \int_2^3 20x - 60 dx \\ &= 5x^2 \Big|_0^2 - [10x^2 - 60x]_2^3 \\ &= 20V - (90 - 40 - 60)V \\ &= 20 + 10V = 30V \\ V_f &= V_i + \Delta V = 10V + 30V = \boxed{40V} \end{aligned}$$

c Solution: $x=5.5\text{m}$

From $x=0$ to $x=2$, the electric potential increases until $V = 30\text{V}$. Between $x=2$ and $x=4$, the voltage increases, then decreases, canceling itself out. Due to properties of integration, we can find the value of x for which $20(x - 4) = 30$. The value for which that is the case, or the solution is $x=5.5$. Adding units, we get $\boxed{x = 5.5\text{m}}$.

4 Problem 10

Solution

By Gauss' Law, we know that the electric field from an infinite nonconducting plate is equal to $E = \frac{\sigma}{2\epsilon_0}$. The electric potential is equal to the integral $\Delta V = \int_i^f \vec{E} \cdot d\vec{s}$. We can separate that integral into two halves, one between the origin and the top plate, and the other between the top plate and $x = 80\text{cm}$. From that, we can compute the electric fields and subsequently the voltage. We can label the plate at $x = 50\text{cm}$ to be plate 1 and the plate at $x = -50\text{cm}$ to be plate 2. The vectors of the electric field and the motion are in the same direction, so we can just replace the dot product with multiplication. Check it out.

$$\begin{aligned}\Delta V &= \int_{0\text{m}}^{0.8\text{m}} \vec{E} \cdot d\vec{s} = \int_{0.5\text{m}}^{0.8\text{m}} \vec{E} \cdot d\vec{s} + \int_{0\text{m}}^{0.5\text{m}} \vec{E} \cdot d\vec{s} \\ &= \int_{0.5\text{m}}^{0.8\text{m}} E ds + \int_{0\text{m}}^{0.5\text{m}} E ds \\ &= (E_1 + E_2) \int_{0.5\text{m}}^{0.8\text{m}} ds + (E_1 - E_2) \int_{0\text{m}}^{0.5\text{m}} ds \\ &= (E_1 + E_2) * 0.3 + (E_1 - E_2) * 0.5 \\ &= \left(\frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} \right) * 0.3 + \left(\frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} \right) * 0.5 \\ &= 0.8 * \frac{\sigma_1}{2\epsilon_0} - 0.2 * \frac{\sigma_2}{2\epsilon_0} = \frac{0.4 * \sigma_1 - 0.1 * \sigma_2}{2\epsilon_0} \\ &= \frac{0.4 * (-50 \times 10^{-9}) - 0.1 * (25 \times 10^{-9})}{2 * (8.85 \times 10^{-12})} \\ &= \frac{-20 - 2.5}{17.7} \times \frac{10^{-9}}{10^{-12}} = -\frac{17.5}{17.7} \times 10^3 = -998.7\text{J/C} \\ |\Delta V| &= |-998.7\text{J/C}| = \boxed{998.7\text{J/C}}\end{aligned}$$

5 Problem 16

Solution: 2.214 J/C

Let's start with the corners. Suppose the distance between the corners and the center is a distance r_1 . Each of them is multiplied by q_1 . We can form a formula for the electric potential due to the four corners.

$$\begin{aligned} V_{\text{corners}} &= V_{11} + V_{12} + V_{21} + V_{22} \\ &= \frac{k * 2q_1}{r_1} + \frac{k * (-3q_1)}{r_1} + \frac{k * (-q_1)}{r_1} + \frac{k * 2q_1}{r_1} \\ &= \frac{kq_1}{r_1}(2 - 3 - 1 + 2) = \frac{kq_1}{r_1}(4 - 4) = \frac{kq_1}{r_1} * 0 = 0 \end{aligned}$$

Thus, we only need concern ourselves with the charges on the sides, each of which are only $\frac{a}{2}$ away from the center. We can use a like formula for the two.

$$\begin{aligned} V_{\text{sides}} &= V_{\text{top}} + V_{\text{bottom}} = \frac{k * 4q_2}{\frac{a}{2}} + \frac{k * 4q_2}{\frac{a}{2}} \\ &= \frac{2 * k * 4q_2}{a} + \frac{2 * k * 4q_2}{a} = \frac{16kq_2}{a} \\ &= \frac{16 * (8.99 \times 10^9) * (6.0 \times 10^{-12})}{39 \times 10^{-2}} \\ &= \frac{(143.84 \times 10^9) * (6.0 \times 10^{-12})}{39 \times 10^{-2}} \\ &= \frac{863.04 \times 10^{-3}}{39 \times 10^{-2}} = \boxed{2.214 \text{ J/C}} \end{aligned}$$