*. (20 points) A monatomic ideal gas expands from 3.0 l to 4.0 l along a process defined by

$$P = a/V^2 \qquad a = 10.0\,\text{atm}\,\text{l}^2 \qquad (\text{l} = \text{liter})$$

The initial temperature of the ideal gas is 300 K.

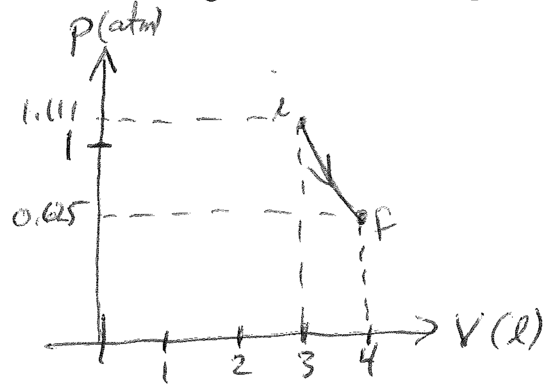
a. (2 points) Express 1 atm l in terms of SI units.

$$\left.\begin{array}{l} 1\,\text{atm} = 1.013\times10^5\,\text{Pa} \\ 1\,\ell = 10^{-3}\,\text{m}^3 \end{array}\right] \Rightarrow 1\,\text{atm}\cdot\ell = (1.013\times10^5\,\text{N/m}^2)(10^{-3}\,\text{m}^3) = 101.3\,\text{J}$$

b. (4 points) Determine the initial and final pressure of the gas and sketch the process in a P-V diagram.

$$P_i = a/V_i^2 = \frac{10.0\,\text{atm}\cdot\ell^2}{(3.0\,\ell)^2} = 1.11\bar{1}\,\text{atm}$$

$$P_f = a/V_f^2 = \frac{10.0\,\text{atm}\cdot\ell^2}{(4.0\,\ell)^2} = 0.625\,\text{atm}$$



c. (2 points) Calculate the number of molecules of the gas. How many moles does that correspond to?

$$P_i V_i = N k T_i \Rightarrow N = \frac{P_i V_i}{k T_i} = \frac{(1.111\,\text{atm})(3.0\,\ell)\left[101.3\,\text{J}/\text{atm}\cdot\ell\right]}{(1.381\times10^{-23}\,\text{J/K})(300\,\text{K})}$$

$$N_A = 6.022\times10^{23}/\text{mole} \qquad = 8.150\times10^{22}\ \text{molecules} = 0.1353\ \text{moles}$$

d. (2 points) Determine the final temperature of the gas.

$$P_f V_f = N k T_f \Rightarrow T_f = \frac{P_f V_f}{N k} = \frac{(0.625\,\text{atm})(4.0\,\ell)\left[101.3\,\text{J}/\text{atm}\cdot\ell\right]}{(8.150\times10^{22})(1.381\times10^{-23}\,\text{J/K})}$$

$$= 225\,\text{K}$$

ALT:
$$\left.\begin{array}{l} P_f V_f = N k T_f \\ \hline P_i V_i = N k T_i \end{array}\right] \Rightarrow \frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i} = \frac{(a/V_f^2)\,V_f}{(a/V_i^2)\,V_i} = \frac{a/V_f}{a/V_i} = \frac{V_i}{V_f} = \frac{3}{4}$$

$$\Rightarrow T_f = \tfrac{3}{4} T_i = 225\,\text{K}.$$

e. (2 points) Calculate the change in the internal energy of the gas.

$$\Delta E_{int} = \frac{3}{2} Nk\Delta T = \frac{3}{2}(8.150\times10^{22})(1.381\times10^{-23}\,J/K)(225K - 300K)$$

$$= -126.625\,J$$

f. (4 points) Determine the net work done on / by the gas during this process.

$$W = -\int_{V_i}^{V_f} P\,dV = -\int_{V_i}^{V_f}\frac{a}{V^2}dV = -\left(-\frac{a}{V}\right)\Big|_{V_i}^{V_f} = -\left(\frac{a}{V_i} - \frac{a}{V_f}\right)$$

$$= -\left(\frac{10.0\,atm\cdot\ell^2}{3.0\,\ell} - \frac{10.0\,atm\cdot\ell^2}{4.0\,\ell}\right) = -0.83\bar{3}\,atm\cdot\ell = -84.41\bar{6}\,J$$

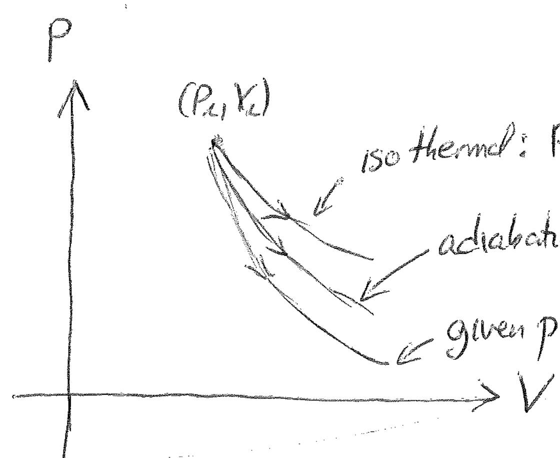
work done by system on environment.

g. (4 points) Determine the net heat flow into / out of the gas during this process. Does the direction of heat flow make sense? (Compare the given process to adiabatic and isothermal processes.)

$$\Delta E_{int} = Q + W \Rightarrow Q = \Delta E_{int} - W = -126.625\,J - (-84.41\bar{6}\,J)$$

$$= -42.208\bar{3}\,J$$

heat flows out of ideal gas system.



isothermal: $PV^1 = \text{constant} \Rightarrow W<0 \Rightarrow Q>0$ (since $\Delta E_{int} = 0$)

adiabatic: $PV^{5/3} = \text{constant} \Rightarrow Q = 0$

given process: $PV^2 = \text{constant} \Rightarrow Q<0$ as calculated.

This makes sense → P drops ~~faster~~ as a function of V faster than it does for an adiabatic process, so T drops further as well. Heat must be removed from the gas as it expands under this process.