Homework #7

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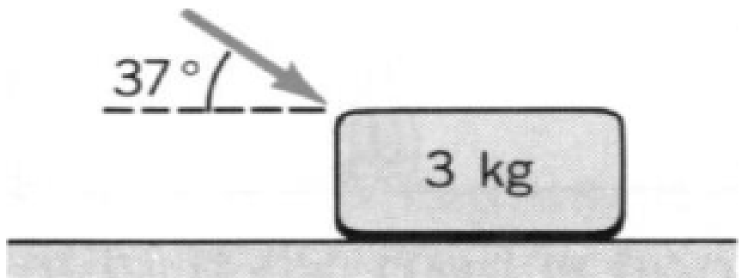
Problem 1

A 3.15-kg block is acted on by a 24.0-N force that acts at 37.0° below the horizontal, as shown in the figure. Take $\mu_k = 0.200$ and $\mu_s = 0.500$. (a) Does the block move if it is initially at rest? (b) If it is initially moving to the right, what is the blocks acceleration?



Solution

Section (a)

For the block to move, it must be that $F_x > f_s$.

$$\begin{aligned} F_x &= F_{app,x} = F_{app} * \cos(\theta) = 24\text{N} * \cos(37°) = 19.167\text{N} \\ f_s &= \mu_s * F_N = \mu_s * (F_g + F_{app,y}) = \mu_s * (m * g + F_{app} * \sin(\theta)) \\ &= 0.50 * (3.15\text{kg} * 9.81\text{m/s}^2 + 24\text{N} * \sin(37°)) = 22.672\text{N} \end{aligned}$$

Since 22.672N > 19.167N, $F_x < f_s$, so it does not move.

Section (b)

$$\begin{aligned} F_{net} &= m * a = F_x - f_k = F_{app}\cos(\theta) - \mu_k F_N \\ a &= \frac{F_{app} * \cos(\theta) - \mu_k * (m * g + F_{app} * \sin(\theta))}{m} \\ &= \frac{19.167\text{N} - 0.20 * (3.15\text{kg} * 9.81\text{m/s}^2 + 24\text{N} * \sin(37°)))}{3.15\text{kg}} \\ &= \boxed{3.206\text{m/s}^2} \end{aligned}$$

Problem 2

A block is released at the top of a 25° incline. Determine the coefficient of kinetic friction given that it slides 2.30 m in 3.15 s.

Solution

Here we merely use the formulae for the force in different directions and solve for μ_k.

$$
\begin{aligned}
F_N &= F_{gy} = F_g \cos(\theta) \\
\Sigma F_x &= ma = F_g \sin(\theta) - F_N \mu_k \\
a &= \frac{F_g \sin(\theta) - F_N \mu_k}{m} \\
\Delta x &= \frac{1}{2} a t^2 && = \frac{t^2(mg\sin(\theta) - mg\mu_k)}{2m} \\
\frac{2\Delta r}{t^2} &= g\cos(\theta) - g\sin(\theta)\mu_k \\
g\sin(\theta)\mu_k &= \frac{g\cos(\theta)t^2 - 2\Delta x}{t^2} \\
\mu_k &= \frac{g\cos(\theta)t^2 - 2\Delta x}{g\sin(\theta)t^2} = \boxed{0.414}
\end{aligned}
$$

Problem 3

A circular off ramp has a radius of 57.0 m and a posted speed limit of 50.0 km/h. If the road is horizontal, what is the minimum coefficient of friction required?

Solution

The maximum coefficient would require the centripital force to be equal to the static frictional force.

$$\begin{aligned} F_c &= f_k \\ \frac{mv^2}{r} &= \mu_s F_N \\ \mu_s &= \frac{mv^2}{mgr} = \frac{v^2}{gr} = \frac{\frac{125}{9}^2}{57 * 9.81} = \boxed{0.345} \end{aligned}$$

Problem 4

A car travels at speed v around a frictionless curve of radius r that is banked at an angle θ to the horizontal. Show that the proper angle of banking is given by $\tan(\theta) = \frac{v^2}{rg}$. (Hint, this is easier if you don't rotate the coordinate system like most other incline problems, and treat the x-axis as the horizontal direction, and the y-axis as the vertical direction. This is because the centripetal force is horizontal.)

Solution

We start by noticing that the horizontal component of the normal force is equal to $F_N \sin(\theta)$ and the centripital force. We also note that the vertical force from the normal force is equal to $F_N \sin(\theta)$ and the gravitational force.

$$\begin{aligned}
F_c &= F_N \sin(\theta) \\
mg &= F_N \cos(\theta) \\
F_N &= \frac{mg}{\cos(\theta)} \\
F_c = \frac{mv^2}{r} &= F_N \sin(\theta) \\
\frac{mv^2}{r} &= mg\frac{\sin(\theta)}{\cos(\theta)} \\
&\boxed{\tan(\theta) = \frac{v^2}{gr}}
\end{aligned}$$

Problem 5

A button is at the rim of a turntable of radius 15.0 cm rotating at 45.0 rpm. What is the minimum coefficient of friction needed for it to stay on?

Solution

We start by converting to full SI units. 15 cm $\rightarrow$ 0.15 m. 45 rpm $\rightarrow$ 0.75 rps. We then find the centripital velocity in meters per second and then find the acceleration.

$$\begin{aligned}
v &= \text{circumference} * \text{rate} = 2\pi r * \text{rate} = 0.30\pi * 0.75\text{m/s} \\
&= \pi 0.225\text{m/s} = 0.7068\text{m/s} \\
a &= \frac{v^2}{r} = \frac{0.7068^2}{0.15} = 3.33099\text{m/s}^2
\end{aligned}$$

We can then apply the static frictional force and Newton's second law to this.

$$\begin{aligned}
f_s &= \mu_s F_N = \mu_s mg \\
F_{net} &= ma \\
\mu_s mg &= ma \\
\mu_s &= \frac{a}{g} = \frac{3.33099\text{m/s}^2}{9.81\text{m/s}^2} = \boxed{0.340}
\end{aligned}$$

Problem 6

A box is dropped onto a conveyor belt moving at 3.40 m/s. If the coefficient of friction between the box and the belt is 0.270, how long will it take before the box moves without slipping?

Solution

We begin wth the fully expanded formula for the maximum force of static friction and the application to Newton's second law.

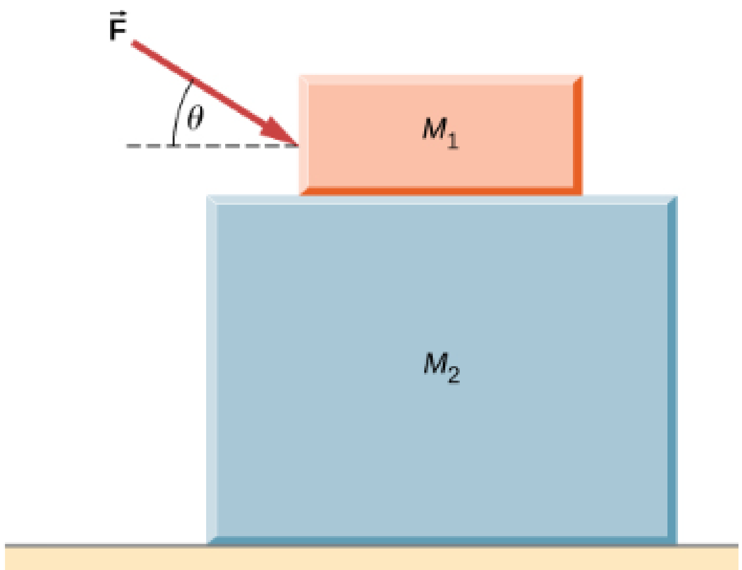
$$\begin{aligned} F_N &= \mu_s mg \\ ma &= \mu_s mg \\ a &= \mu_s g \end{aligned}$$

We can next apply the kinematic equation to solve for time and substitute in values.

$$v = v_0 + at \rightarrow t = \frac{v - v_0}{a} = \frac{v - v_0}{\mu_s g} = \frac{3.40 - 0}{0.270 * 9.81}\text{s} = \boxed{1.28\text{s}}$$

Problem 7

Two blocks are stacked as shown below, and rest on a frictionless surface. There is friction between the two blocks with a coefficient of friction μ_s. An external force is applied to the top block at an angle θ with the horizontal. What is the maximum force F that can be applied for the two blocks to move together?



Solution

$$\begin{aligned} f_s &= F_{app,x} \\ F_N \mu_s &= F_{app} \cos(\theta) \\ F_{app} \cos(\theta) &= \mu_s * mg + F_{app} \sin(\theta) \mu_s \\ F_{app} &= \boxed{\frac{\mu_s mg}{\cos(\theta) - \sin(\theta)\mu_s}} \end{aligned}$$