Homework #16

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1 Problem 1

Two disks are mounted (like a merry-go-round) on low-friction bearings on the same axle and can be brought together so that they couple and rotate as one unit. The first disk, with rotational inertia 3.30 $\text{kg} * \text{m}^2$ about its central axis, is set spinning counterclockwise at 450 rev/min. The second disk, with rotational inertia 6.60 $\text{kg} * \text{m}^2$ about its central axis, is set spinning counterclockwise at 900 rev/min. They then couple together. (a) What is their angular speed after coupling? If instead the second disk is set spinning clockwise at 900 rev/min, what are their (b) angular speed and (c) direction of rotation after they couple together?

1.1 Solution

1.1.1 Section (a)

We have a concept called conservation of angular momentum.

$$L_i = L_f \tag{1}$$

$$L_f = l_1 + l_2 = I_1\omega_1 + I_2\omega_2 \tag{2}$$

$$\omega_f = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{3.3 * 450 + 6.6 * 900}{3.3 + 6.6} \tag{3}$$

$$= \frac{1485 + 5940}{9.9} = \boxed{750\text{rev/min}} \tag{4}$$

1.1.2 Section (b)

We just need to change a positive to a negative.

$$\omega_f = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{3.3 * 450 - 6.6 * 900}{3.3 + 6.6} \tag{5}$$

$$= \frac{1485 - 5940}{9.9} = \boxed{-450\text{rev/min}} \tag{6}$$

1.1.3 Section (c)

Since the magnitude is negative and negative angular velocity corresponds to clockwise motion, the angular motion is $\boxed{\textit{clockwise}}$.

2 Problem 2

The Sun's mass is 2.0 $\times 10^{30}$ kg, its radius is 7.0 $\times 10^5$ km, and it has a rotational period of approximately 28 days. If the Sun should collapse into a white dwarf of radius 3.5 $\times 10^3$ km, what would its period be if no mass were ejected and a sphere of uniform density can model the Sun both before and after?

2.1 Solution

We can calculate the angular frequency of the sun by using the period formula $T = \frac{2\pi}{\omega}$.

$$T = \frac{2\pi}{\omega} \tag{7}$$

$$\omega = \frac{2\pi}{T} \tag{8}$$

Next, we can use the conservation of angular momentum and the formula for the inertia of the dwarf sun to find a formula for the final angular velocity and then final period.

$$L_f = L_i \tag{9}$$

$$I_f \omega_f = I_i \omega_i \tag{10}$$

$$I_f \frac{2\pi}{T_f} = I_i \frac{2\pi}{T_i} \tag{11}$$

$$\frac{I_f}{I_i} \cdot \frac{2\pi}{2\pi} = \frac{T_f}{T_i} \tag{12}$$

$$\frac{I_f}{I_i} * T_i = T_f \tag{13}$$

$$\frac{\frac{2}{5}MR_f^2}{\frac{2}{5}MR_i^2} * T_i = \frac{R_f^2}{R_i^2} * T_i = \frac{(3.5 \times 10^3)^2}{(7.0 \times 10^5)^2} * 28\text{days} = T_f \tag{14}$$

$$\frac{12.25 \times 10^6}{49.0 \times 10^{10}} * 28\text{days} = \frac{28\text{days}}{4 \times 10^4} = 7 \times 10^{-4}\text{days} = T_f \tag{15}$$

This means that the period is $\boxed{7 \times 10^{-4} \text{ days}}$.

3 Problem 3

The displacement from equilibrium of a particle is given by $x(t) = A\cos\left(\omega t - \frac{\pi}{3}\right)$. Which, if any, of the following are equivalent expressions:

$$a)\ x(t) = A\cos\left(\omega t + \frac{\pi}{3}\right) \tag{16}$$

$$b)\ x(t) = A\cos\left(\omega t + \frac{5\pi}{3}\right) \tag{17}$$

$$c)\ x(t) = A\cos\left(\omega t + \frac{\pi}{6}\right) \tag{18}$$

$$d)\ x(t) = A\cos\left(\omega t - \frac{5\pi}{6}\right) \tag{19}$$

3.1 Solution

4 Problem 4

In a block and spring system $m = 0.250\text{kg}$ and $k = 4.00\text{N/m}$. At $t = 0.150\text{s}$, the velocity is $v = -0.174\text{m/s}$ and the acceleration $a = +0.877\text{m/s}^2$. Write an expression for the displacement as a function of time, $x(t)$. (Hint, remember that the inverse tan function only returns the principal value, but there is a secondary value as well.)

4.1 Solution

We have some formulas for velocity and acceleration that we can use.

$$v(t) = -\omega x_m \sin(\omega t + \phi) \quad \rightarrow v(0) = -0.174\text{m/s} = -\omega x_m \sin(\phi) \tag{20}$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \rightarrow a(0) = \ 0.877\text{m/s}^2 = -\omega^2 x_m \cos(\phi) \tag{21}$$

$$\frac{a(0)}{v(0)} = \frac{\omega^2 x_m \cos(\phi)}{\omega x_m \sin(\phi)} = \omega \frac{\cos(\phi)}{\sin(\phi)} = \frac{\cos(\phi)}{\sin(\phi)} \sqrt{\frac{k}{m}} \tag{22}$$

$$\tan(\phi) = \frac{\sin(\phi)}{\cos(\phi)} = \frac{v(0)\sqrt{k}}{a(0)\sqrt{m}} \tag{23}$$

$$\phi = \arctan\left(\frac{v(0)\sqrt{k}}{a(0)\sqrt{m}}\right) = \arctan\left(\frac{-0.174\sqrt{4.0}}{0.877\sqrt{0.25}}\right) \tag{24}$$

$$= \arctan\left(\frac{-0.174 * 2}{0.877 * 0.5}\right) = \arctan\left(-\frac{0.696}{0.877}\right) = \begin{matrix} 141.6^\circ \\ 321.6^\circ \end{matrix} \tag{25}$$

One of these is in the second quadrant, the other is in the fourth quadrant. Knowing that ω is positive and trusting that x_m is positive, since the negative cosine is positive and the negative sine is negative, the cosine is negative and the sine is positive, so ϕ is in the second quadrant. This means $\phi = 141.6^\circ$. Last we just neeed to find the value of x_m, which we will find using the value of $a(0)$.

$$a(0) = -\omega^2 x_m \cos(\omega * 0 + \phi) \tag{26}$$

$$x_m = -\frac{a(0)}{\omega^2 \cos(\phi)} = \frac{0.877 * 0.250}{4 * 0.7833} = 0.06998\text{m} \tag{27}$$

Lastly, we find the value of ω and use that to finalize the formula for $x(t)$.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4.0}{0.25}} = 4\text{s}^{-1} \tag{28}$$

$$x(t) = 0.06998 * \cos(4t + 141.6^\circ) \tag{29}$$

$$\boxed{x(t) = 0.06998 * \cos(4t + 141.6^\circ)} \tag{30}$$