

# Homework #4, 4B

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## 1 Question 1

A surface that has the area vector  $\vec{A} = (2\hat{i} + 3\hat{j}) \text{ m}^2$ . What is the flux of a uniform electric field that is (a)  $\vec{E} = 4\hat{i}\text{N/C}$  and (b)  $\vec{E} = 4\hat{k}\text{N/C}$ ?

### a Solution

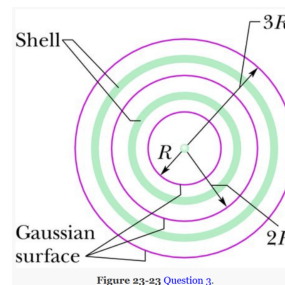
$$\Phi = \oint \vec{E} \cdot d\vec{A} = (4\hat{i}\text{N/C}) \cdot (2\hat{i} + 3\hat{j}) \text{ m}^2 = \boxed{8\text{Nm}^2/\text{C}}$$

### b Solution

$$\Phi = \oint \vec{E} \cdot d\vec{A} = (4\hat{k}\text{N/C}) \cdot (2\hat{i} + 3\hat{j}) \text{ m}^2 = \boxed{0\text{Nm}^2/\text{C}}$$

## 2 Question 3

Figure 23-23 shows, in cross section, a central metal ball, two spherical metal shells, and three spherical Gaussian surfaces of radii  $R$ ,  $2R$ , and  $3R$ , all with the same center. The uniform charges on the three objects are: ball,  $Q$ ; smaller shell,  $3Q$ ; larger shell,  $5Q$ . Rank the Gaussian surfaces according to the magnitude of the electric field at any point on the surface, greatest first.



**Solution:**  $E_1 = E_2 = E_3$

In all three cases, the area is the same formula and the electric field is the same at every point on the individual Gaussian surface. As such, we can develop a generalized formula for the electric field.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \rightarrow E = \frac{q_{enc}}{4\pi R^2 \epsilon_0}$$

For the smallest shell:

$$E_1 = \frac{q_{enc}}{4\pi r^2 \epsilon_0} = \frac{Q}{4\pi R^2 \epsilon_0}$$

For the middle shell:

$$E_2 = \frac{q_{enc}}{4\pi r^2 \epsilon_0} = \frac{Q + 3Q}{4\pi (2R)^2 \epsilon_0} = \frac{4Q}{4\pi 4R \epsilon_0} = \frac{Q}{4\pi R^2 \epsilon_0} \rightarrow E_1 = E_2$$

For the largest shell:

$$E_3 = \frac{q_{enc}}{4\pi r^2 \epsilon_0} = \frac{Q + 3Q + 5Q}{4\pi (3R)^2 \epsilon_0} = \frac{9Q}{4\pi 9R \epsilon_0} = \frac{Q}{4\pi R^2 \epsilon_0} \rightarrow \boxed{E_1 = E_2 = E_3}$$

### 3 Problem 6

Three infinite nonconducting sheets, with uniform positive surface charge densities  $\sigma$ ,  $2\sigma$ , and  $3\sigma$ , are arranged to be parallel like the two sheets in [Fig. 23-19a](#). What is their order, from left to right, if the electric field  $\vec{E}$  produced by the arrangement has magnitude  $E = 0$  in one region and  $E = 2\sigma/\epsilon_0$  in another region?

#### Solution

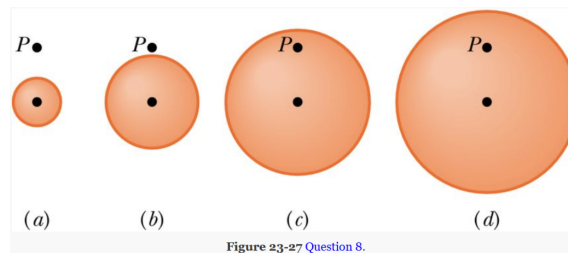
For an infinite nonconducting sheet of density  $\sigma$ , the electric field from it is equal to  $E = \sigma/2\epsilon_0$ . We can use this to provide a system of equations for electric field strengths  $(a, b, c)$ , which have unique magnitudes in the set  $(\sigma/2\epsilon_0, 2\sigma/2\epsilon_0, 3\sigma/2\epsilon_0)$  or alternatively  $(E, 2E, 3E)$ .

$$\begin{aligned}a - b - c &= 0 \\a + b - c &= 2\sigma/\epsilon_0 = 4E \\0a + 2b + 0c &= 4E \\b &= 2E = 2\sigma/2\epsilon_0 \\a - 2E - c &= 0 \rightarrow a - c = 2E\end{aligned}$$

There is only one combination of the remaining two that this works for:  $a = 3E$  and  $c = E$ . Thus, the order is  $\boxed{\langle 3\sigma, 2\sigma, \sigma \rangle}$ .

## 4 Problem 8

Figure 23-27 shows four solid spheres, each with charge  $Q$  uniformly distributed through its volume. (a) Rank the spheres according to their volume charge density, greatest first. The figure also shows a point  $P$  for each sphere, all at the same distance from the center of the sphere. (b) Rank the spheres according to the magnitude of the electric field they produce at point  $P$ , greatest first.



### a Solution

All have the same charge total and the spheres are charged uniformly. The charge density  $\rho$  of the sphere will be equal to the charge  $Q$  divided by the volume  $V$ . The charge density can thus be ranked by the inverse of the volume and as such the inverse of the radius.  $(a) > (b) > (c) > (d)$

### b Solution

The electric field inside a solid sphere will be relevant for spheres (c) and (d). Know that anywhere on the sphere we are measuring the enclosure from, the electric field will be the same.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$E \int dA = \frac{\rho \frac{4}{3}\pi P^3}{\epsilon_0}$$

$$E * 4\pi P^2 = \frac{4\pi P^3 \rho}{3\epsilon_0}$$

$$E = \frac{P\rho}{3\epsilon_0}$$

The electric field outside a solid sphere (of radius  $R$ ) will be relevant for spheres (a) and (b). Know that anywhere on the sphere we are measuring the enclosure from,

the electric field will be the same.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E \int dA = \frac{\rho \frac{4}{3}\pi R^3}{\varepsilon_0}$$

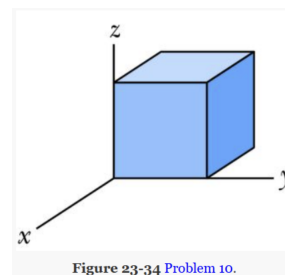
$$E * 4\pi R^2 = \frac{4\pi R^3 \rho}{3\varepsilon_0}$$

$$E = \frac{R^3 \rho}{3R^2 \varepsilon_0}$$

We have two equations, one for the electric field inside a uniformly charged sphere and other for the electric field outside the same. The value of P is the same for all four. Since  $\rho_c > \rho_d$ , we can conclude that  $E_c > E_d$ . Since  $R_a < R_b$  but that is concealed out by  $\rho_a > \rho_b$ , we can conclude that  $E_a = E_b$ . Additionally, since the electric field outside the sphere is a result of more charge than the electric field inside the sphere,  $E_a > E_c$ . As such,  $\boxed{E_b = E_a > E_c > E_d}$ .

## 5 Problem 10

Figure 23-34 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m. It lies in a region where the nonuniform electric field is given by  $\vec{E} = (3.00x + 4.00)\hat{i} + 6.00\hat{j} + 7.00\hat{k}$  N/C with  $x$  in meters. What is the net charge contained by the cube?



### a Solution

We have a formula given from Gauss' Law. We can apply these six for each of the six faces, then add them all up at the end.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

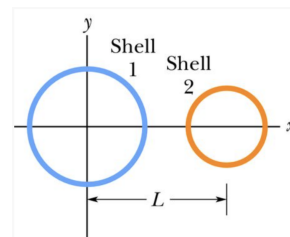
Face	Flux
$x = 0$	$\oint (3x + 4) dA = (3x + 4) \oint dA \rightarrow (3 * 0 + 4) * (-4) = -16$
$x = 2$	$\oint (3x + 4) dA = (3x + 4) \oint dA \rightarrow (3 * 2 + 4) * (4) = 40$
$y = 0$	$\oint 6 dA = 6 \oint dA \rightarrow 6 * (-4) = -24$
$y = 2$	$\oint 6 dA = 6 \oint dA \rightarrow 6 * 4 = 24$
$z = 0$	$\oint 7 dA = 7 \oint dA \rightarrow 7 * (-4) = -28$
$z = 2$	$\oint 7 dA = 7 \oint dA \rightarrow 7 * 4 = 28$
Sum	$40 - 16 + 24 - 24 + 28 - 28 = 24$

Lastly, we have to multiply it by the permittivity of free space to get our actual answer.

$$24\epsilon_0 = \boxed{2.124 \times 10^{-10} \text{C}}$$

## 6 Problem 12

Figure 23-36 shows two nonconducting spherical shells fixed in place. Shell 1 has uniform surface charge density  $+6.0\mu\text{C}/\text{m}$  on its outer surface and radius  $3.0\text{cm}$ ; shell 2 has uniform surface charge density  $+4.0\mu\text{C}/\text{m}$  on its outer surface and radius  $2.0\text{cm}$ ; the shell centers are separated by  $L = 10\text{cm}$ . In unit-vector notation, what is the net electric field at  $x = 2.0\text{cm}$ ?



### Solution

The field inside a uniform shell of charge (where we are measuring from) due to the shell is exactly zero. Thus, we need not concern ourselves with Shell 1 and only have to focus on Shell 2. Outside the shell, the formula for the electric field can be calculated, which gives us our answer with respect to Shell 2.

$$\begin{aligned}
 \vec{E} &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{\sigma * 4\pi R^2}{r^2} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \frac{\sigma R^2}{\epsilon_0 r^2} \begin{pmatrix} -1 \\ 0 \end{pmatrix} \\
 &= \frac{-4 \times 10^{-6} * (2 \times 10^{-2})^2}{8.85 \times 10^{-12} * (8 \times 10^{-2})^2} \hat{i} \text{N/C} = \frac{-16}{8.85 * 64} \times \frac{10^{-10}}{10^{-16}} \hat{i} \text{N/C} \\
 &= \frac{-1}{35.4} \times 10^6 \hat{i} \text{N/C} = -(2.825 \times 10^{-2}) \times 10^6 \hat{i} \text{N/C} = \boxed{-2.825 \times 10^4 \hat{i} \text{N/C}}
 \end{aligned}$$

## 7 Problem 18

The electric field just above the surface of the charged conducting drum of a photocopying machine has a magnitude  $E$  of  $2.3 \times 10^5 \text{N/C}$ . What is the surface charge density on the drum?

### Solution

We begin with the electric field of the electron at that point. We know the formula for the electric field along an infinite line of charge.

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\sigma = E\varepsilon_0 = (2.3 \times 10^5 \text{N/C}) * (8.85 \times 10^{-12}) = \boxed{2.0355 \times 10^{-12} \text{C/m}^2}$$



## 8 Problem 22

An electron is released 9.0cm from a very long nonconducting rod with a uniform  $6.0\mu\text{C}/\text{m}$ . What is the magnitude of the electron's initial acceleration?

### Solution

We begin with the electric field of the electron at that point. We know the formula for the electric field along an infinite line of charge.

$$\begin{aligned}E &= \frac{\lambda}{2\pi\epsilon_0 r} \\F &= qE = \frac{q\lambda}{2\pi\epsilon_0 r} \\a &= \frac{F}{m} = \frac{q\lambda}{2m\pi\epsilon_0 r}\end{aligned}$$

We can plug values into this.

$$a = \frac{q\lambda}{2m\pi\epsilon_0 r} = \frac{(-1.602 \times 10^{-19}\text{C})(6.0 \times 10^{-6}\text{C}/\text{m})}{2 * (9.109 \times 10^{-31}\text{kg}) * \pi\epsilon_0 * (9 \times 10^{-2}\text{m})}$$

## 9 Problem 28

A charge of uniform linear density  $2.0\text{nC/m}$  is distributed along a long, thin, non-conducting rod. The rod is coaxial with a long conducting cylindrical shell (inner radius =  $5.0\text{cm}$ , outer radius =  $10\text{cm}$ ). The net charge on the shell is zero. (a) What is the magnitude of the electric field  $15\text{cm}$  from the axis of the shell? What is the surface charge density on the (b) inner and (c) outer surface of the shell?

### a Solution

Electric field is not affected by things of zero charge, but it is affected by conducting things. However, we can use Gauss's law here.

$$\frac{Q_{enc}}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A}$$

Since the enclosed charge would remain the same, it would be reasonably assumed that the electric field would also remain the same.

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2.0 \times 10^{-9}\text{C/m}}{2 * \pi * \epsilon_0 * (15 \times 10^{-2}\text{m})} = \boxed{239.781\text{N/C}}$$

### b Solution

The surface charge at any given circle of the inner wall of the cylinder would be equal to the charge of the rod. We know the linear charge density. We can divide this by the circumference ( $C$ ) of the inner circles to get the surface charge density. We also multiply this by negative one because of the opposition to the charge in rod.

$$\sigma = -\frac{\lambda}{C} = -\frac{2.0 \times 10^{-9}\text{C/m}}{2 \times \pi \times 5 \times 10^{-2}\text{m}} = \boxed{-6.366 \times 10^{-9}\text{C/m}^2}$$

### c Solution

We do the same thing, but for a larger radius and without multiplying by negative one.

$$\sigma = \frac{\lambda}{C} = \frac{2.0 \times 10^{-9}\text{C/m}}{2 \times \pi \times 10 \times 10^{-2}\text{m}} = \boxed{3.183 \times 10^{-9}\text{C/m}^2}$$

## 10 Problem 34

In Fig. 23-45, a small circular hole of radius  $R = 1.80\text{cm}$  has been cut in the middle of an infinite, flat, nonconducting surface that has uniform charge density  $\sigma = 4.50\text{pC/m}^2$ . A  $z$  axis, with its origin at the hole's center, is perpendicular to the surface. In unit-vector notation, what is the electric field at point  $P$  at  $z = 2.56\text{cm}$ ? (Hint: See Eq. 22-26 and use superposition.)

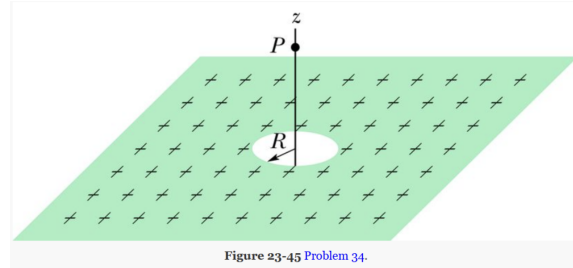


Figure 23-45 Problem 34.

### Solution

As suggested, we use an equation and superposition. We know the formula for an infinite charged plane and a charged plate. We can add these together, keeping in mind that the plate's charge would be opposite the plane's.

$$\begin{aligned} E_{\Sigma} &= E_{\text{plane}} - E_{\text{plate}} = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right) \\ &= \frac{\cancel{\sigma}}{2\epsilon_0} - \frac{\cancel{\sigma}}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \cdot \frac{z}{\sqrt{z^2 + R^2}} = \frac{\sigma}{2\epsilon_0} \cdot \frac{z}{\sqrt{z^2 + R^2}} \end{aligned}$$

We then can substitute in the known values.

$$\begin{aligned} E_{\Sigma} &= \frac{\sigma}{2\epsilon_0} \cdot \frac{z}{\sqrt{z^2 + R^2}} \\ &= \frac{4.50 \times 10^{-12}\text{C/m}}{2\epsilon_0} \cdot \frac{2.56 \times 10^{-2}\text{m}}{\sqrt{(2.56 \times 10^{-2}\text{m})^2 + (1.80 \times 10^{-2}\text{m})^2}} \\ &= \frac{4.50 \times 10^{-12}}{17.70 \times 10^{-12}} \cdot \frac{2.56 \times 10^{-2}}{\sqrt{9.7936 \times 10^{-4}}}\text{N/C} \\ &= \frac{4.50 * 2.56}{17.70 * 3.1294}\text{N/C} = \frac{11.52}{55.39}\text{N/C} = 0.208\text{N/C} \end{aligned}$$

Since it is in the  $+z$  direction, we would have the final answer be

$0.208\hat{k} \text{ N/C}$