

Homework #16

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December 11, 2024

1 Problem 1

Two disks are mounted (like a merry-go-round) on low-friction bearings on the same axle and can be brought together so that they couple and rotate as one unit. The first disk, with rotational inertia $3.30 \text{ kg} \cdot \text{m}^2$ about its central axis, is set spinning counterclockwise at 450 rev/min . The second disk, with rotational inertia $6.60 \text{ kg} \cdot \text{m}^2$ about its central axis, is set spinning counterclockwise at 900 rev/min . They then couple together. (a) What is their angular speed after coupling? If instead the second disk is set spinning clockwise at 900 rev/min , what are their (b) angular speed and (c) direction of rotation after they couple together?

1.1 Solution

1.1.1 Section (a)

We have a concept called conservation of angular momentum.

$$L_i = L_f \tag{1}$$

$$L_f = l_1 + l_2 = I_1\omega_1 + I_2\omega_2 \tag{2}$$

$$\omega_f = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{3.3 * 450 + 6.6 * 900}{3.3 + 6.6} \tag{3}$$

$$= \frac{1485 + 5940}{9.9} = \boxed{750 \text{ rev/min}} \tag{4}$$

1.1.2 Section (b)

We just need to change a positive to a negative.

$$\omega_f = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{3.3 * 450 - 6.6 * 900}{3.3 + 6.6} \tag{5}$$

$$= \frac{1485 - 5940}{9.9} = \boxed{-450 \text{ rev/min}} \tag{6}$$

1.1.3 Section (c)

Since the magnitude is negative and negative angular velocity corresponds to clockwise motion, the angular motion is clockwise.

2 Problem 2

The Sun's mass is 2.0×10^{30} kg, its radius is 7.0×10^5 km, and it has a rotational period of approximately 28 days. If the Sun should collapse into a white dwarf of radius 3.5×10^3 km, what would its period be if no mass were ejected and a sphere of uniform density can model the Sun both before and after?

2.1 Solution

We can calculate the angular frequency of the sun by using the period formula $T = \frac{2\pi}{\omega}$.

$$T = \frac{2\pi}{\omega} \quad (7)$$

$$\omega = \frac{2\pi}{T} \quad (8)$$

Next, we can use the conservation of angular momentum and the formula for the inertia of the dwarf sun to find a formula for the final angular velocity and then final period.

$$L_f = L_i \quad (9)$$

$$I_f \omega_f = I_i \omega_i \quad (10)$$

$$I_f \frac{2\pi}{T_f} = I_i \frac{2\pi}{T_i} \quad (11)$$

$$\frac{I_f}{I_i} \cdot \frac{2\pi}{2\pi} = \frac{T_f}{T_i} \quad (12)$$

$$\frac{I_f}{I_i} * T_i = T_f \quad (13)$$

$$\frac{\frac{2}{5}MR_f^2}{\frac{2}{5}MR_i^2} * T_i = \frac{R_f^2}{R_i^2} * T_i = \frac{(3.5 \times 10^3)^2}{(7.0 \times 10^5)^2} * 28\text{days} = T_f \quad (14)$$

$$\frac{12.25 \times 10^6}{49.0 \times 10^{10}} * 28\text{days} = \frac{28\text{days}}{4 \times 10^4} = 7 \times 10^{-4}\text{days} = T_f \quad (15)$$

This means that the period is $\boxed{7 \times 10^{-4} \text{ days}}$.

3 Problem 3

The displacement from equilibrium of a particle is given by $x(t) = A \cos(\omega t - \frac{\pi}{3})$. Which, if any, of the following are equivalent expressions:

$$a) x(t) = A \cos\left(\omega t + \frac{\pi}{3}\right) \quad (16)$$

$$b) x(t) = A \cos\left(\omega t + \frac{5\pi}{3}\right) \quad (17)$$

$$c) x(t) = A \cos\left(\omega t + \frac{\pi}{6}\right) \quad (18)$$

$$d) x(t) = A \cos\left(\omega t - \frac{5\pi}{6}\right) \quad (19)$$

3.1 Solution