1 2

*. (14 points) Two sound sources are placed at $(0, +1.25\,\text{m}, 0)$ and $(0, -1.25\,\text{m}, 0)$. They both emit isotropic sound waves with a wavelength of 1.00 m at the same total power and in phase with one another. Let I represent the intensity of the sound wave emitted by either source by itself at 100 m distance.

a. (6 points) As a multiple of I, determine the intensity of the combined sound waves at $(100\,\text{m}, 0, 0)$, $(0, 100\,\text{m}, 0)$ and $(100/\sqrt{2}\,\text{m}, 100/\sqrt{2}\,\text{m}, 0)$. For this part, you may assume that the amplitude of each individual wave is equal to what it would be at 100 m distance.

A: $(100m, 0, 0)$: $\vec{\ell}_1 = (100\,m, -1.25m, 0)$ $\quad |\ell_1| = \sqrt{100^2+1.25^2}$ } equal
$\vec{\ell}_2 = (100\,m, +1.25m, 0)$ $\quad |\ell_2| = \sqrt{100^2+1.25^2}$

$\Rightarrow \Delta\ell = 0$ in phase. $\Rightarrow$ intensity $= 4I$

B: $(0, 100m, 0)$ $\vec{\ell}_1 = (0, 98.75m, 0)$ $\quad |\ell_1| = 98.75m$ } $\Delta\ell = 2.50m = 2.50\,\lambda$
$\vec{\ell}_2 = (0, 101.25m, 0)$ $\quad |\ell_2| = 101.25\,m$

$\Rightarrow 180°$ out of phase $\Rightarrow$ intensity $= 0$

C: $(\frac{100}{\sqrt{2}}m, \frac{100}{\sqrt{2}}m, 0)$ $\vec{\ell}_{1,2} = (\frac{100}{\sqrt{2}}m, \frac{100}{\sqrt{2}} \mp 1.25\,m, 0) \Rightarrow |\ell_1| = 99.12\,m$ $\quad |\ell_2| = 100.89\,m$

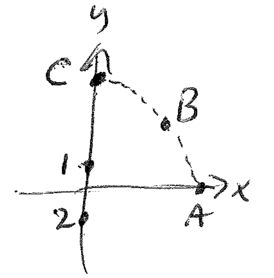
$(100/\sqrt{2} = 70.71)$ $\quad \Rightarrow \Delta\ell = 1.77m = 1.77\lambda$ $\quad \Delta\phi = 2\pi(1.77) = 11.11$ rad

intensity $= 2I(1+\cos\Delta\phi) = 2.22\,I$

b. (4 points) If you were to walk along a 90° arc in the x-y plane between $(100\,\text{m}, 0, 0)$ and $(0, 100\,\text{m}, 0)$, how many interference maxima would you encounter? Interference minima?

As one walks along the circular arc, $\Delta\ell$ varies from 0 to 2.50m
$\Rightarrow \Delta N$ varies from 0 to 2.50. Interference max occur at
$= \Delta\ell/\lambda$ $\quad \Delta N = 0, 1, 2$ (3 cases). Interference min occur at
$\Delta N = 1/2, 3/2, 5/2$ (3 cases).

c. (4 points) Now take into account the fact that the sound sources are not exactly 100 m away from the point $(0, 100\,\text{m}, 0)$ and the impact that has on the amplitude of the individual sound waves, and calculate the intensity of the combined wave at that point.



Let $I = \alpha \psi_{m,0}^2$ ← amplitude at 100m

$\psi_{m,1} = \psi_{m,0} \cdot \frac{100\,m}{101.25m}$ $\quad \psi_{m,2} = \psi_{m,0}\,\frac{100\,m}{98.75m}$

$\Delta\phi = 2\pi(2.50) = 5\pi$ rad, completely out of phase

$\Rightarrow \psi_m = \psi_{m,2} - \psi_{m,1} = \psi_{m,0}\left(\frac{100}{98.75} - \frac{100}{101.25}\right) = \psi_{m,0}(0.02500)$

↑ combined wave $\quad$ 1.01266 − 0.98765

$\Rightarrow$ intensity $= \alpha\psi_m^2 = (0.02500)^2 I = 6.25 \times 10^{-4}\,I$