Primer on Waves for Physics 4D

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The purpose of this is to preview or review the basic physics of waves. The advanced nature of the topics in the course heavily depend on students having a firm foundation in how physics is quantitatively modeled with the wave concept.

Before you read this primer, please do the Seeing the Moving Waves in Desmos exercises.

The Super Simple Very Conceptual Idea

Waves are traveling disturbances of a medium. Here are some examples.

As you read the descriptions of the phenomena, can you pick out:

- the source of the waves
- the medium that carries the waves
- the waves themselves
- and the sense in which the medium's equilibrium state has a disturbance that is moving because the medium itself is pushing it along?

1. A hand pulls a rope taut along an x axis. Then the hand shakes the rope up and down sending ripples away from the hand. The ripples are in a **transverse** direction to the x axis. This is an example of so-called transverse waves.
2. Vocal chords in a singer's throat touch a quiet atmosphere. Then the singer forces air past the vocal chords causing them to vibrate and push and pull on the atmosphere in an x direction. The atmosphere carries these compressions and rarefactions away from the vocal chords. These compressions and rarefactions are actual squeezes and evacuations **longitudinal** to (along) the x axis. These disturbances also travel in the x direction. This is an example of so-called longitudinal waves.
3. Electrons in an antenna are standing still. The electric and magnetic fields in the space surrounding the antenna have some shape and are unchanging (ie static). But then the electrons in the antenna are accelerated up and down (call this the y direction) as a current. The fields in the space surrounding the antenna change suddenly in time and in strength and direction. And these changes near the antenna then cause changes further from the antenna, which cause changes even further from the antenna, and etcetera .... and these changes in the fields race away from the antenna at the speed of light (call this direction away from the y

axis as the r (radial) direction).
4. Can you fill in the details for a rain drop that falls into a calm puddle?

The Quantitative Idea Behind the Above Concepts

In each of the examples above, some sort of oscillator is connected to some sort of medium such that the oscillator can pass energy to the medium. Of course, as the oscillator does this, the conservation of energy enforces that the energy gained by the medium must be lost by the oscillator. And this enforcement is a quantitative piece of physics.

But there is quantitative physics that dictates how the medium handles distortion from equilibrium.

Examples 1 and 2 are of Mechanical Waves Governed by Newton's Laws

In the examples 1 & 2 above, which are mechanical waves, these are based upon Newton's Laws for all the small points of mass of a medium connected via forces- that is, Newton's Laws of Motion. Newton's Laws of motion can be used to gain a quantitative description of the disturbance that will end up looking like this:

$$\frac{\partial^2}{\partial x^2} f(x,t) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2} f(x,t)$$

For the case of example 1, $f(x,t)$ is the height of the rope at position x and time t. And v is the speed of the deformation.

For the case of example 2, $f(x,t)$ would instead represent the pressure of the atmosphere at the position x at the time t. And v is the speed of the sound.

Example 3 Above Is Not Mechanical but Rather Electromagnetic

Example 3 above is not mechanical. It is instead governed by the physics of electricity and magnetism (E&M). The physics of E&M is summarized completely by four equations referred to as the Maxwell's Equations. You can find many sources where someone shows the derivation of the wave equation for electric E&M waves so I won't repeat that here.

But the wave equation that follows from Maxwell's Equations looks like this:

$$\frac{\partial^2}{\partial x^2}\vec{E}(x,y,z,t) = \epsilon_o \mu_o \frac{\partial^2}{\partial t^2}\vec{E}(x,y,z,t) \text{ and}$$

$$\frac{\partial^2}{\partial x^2}\vec{B}(x,y,z,t) = \epsilon_o \mu_o \frac{\partial^2}{\partial t^2}\vec{B}(x,y,z,t)$$

Note that *it's a **pair** of wave equations*; one for the electric field and one for the magnetic field. Also note that if you compare the meaning of v from examples 1 and 2 with the coefficient of the double time derivatives you will see that

$\epsilon_o \mu_o$ must be the same as the inverse of speed squared ($\frac{1}{v^2}$) of this traveling disturbance.

Maxwell's Equations further enforce particular relationships between the magnitudes, directions, and relative phases of the electric and magnetic fields in a light wave. At any place and time in a light wave,

- The electric and magnetic parts of a light wave are exactly **perpendicular** to each other at every place in the wave.
- the relationship between the magnitudes is $|\vec{E}(x, y, z, t)| = c|\vec{B}(x, y, z, t)|$ (same but for the factor of the speed of light c).
- the direction of the vector cross product $\vec{E} \times \vec{B}$ is the direction in which the E&M wave is propagating, and
- the places in a wave where the electric field is zero are also the places in the wave where the magnetic field is zero. This means the electric and magnetic components of the wave are in phase and have the same wavelength.

A further piece of the physics of lightwaves is given by the following vector called the Poynting vector, $\vec{S}$:

- $\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B}$
- The dimensions of this vector are energy per time per area. This means that if you multiply the Poynting vector by an amount of time t and an area A, you will be left over with energy. $|\vec{S}| * t * A$ is energy
- Of course light delivers energy. Earth gets energy from the sun via light.

Exercise 1

The wave equations alluded to above come from using physics to describe a medium. Here is a wave equation for some sort of disturbance that we are calling h:

$$\frac{\partial^2}{\partial x^2} h(x, t) = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} h(x, t)$$

- Show by explicit differentiation and some algebra that this function solves the wave equation: $h(x, t) = h_{\text{max}} \sin\left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t + \phi\right)$ if:
  - $v = \lambda / T$

    - h_{max} is NOT x or t dependent, and
    - independent of the value of ϕ as long as it too is constant in space and time.
- How about a different wave shape like $h(x,t) = h_{\text{max}} \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t + \phi\right)$? Does it work?
- How about this wave shape? $h(x,t) = h_{\text{max}} \sin^2\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t + \phi\right)$
- Let's say you know that two different traveling waves, $h_1(x,t)$ and $h_2(x,t)$, both independently solve the wave equation. Show explicitly that any linear combination with coefficients that DO NOT have space or time dependence, ie. $h_{\text{superposition}} = ah_1 + bh_2$, is also a wave that the medium will propagate.

Exercise 2

Let's work with the specifics of EM waves. There are two different waves, one for the disturbance of electric field, and the other for the disturbance of the magnetic field. In a single light wave, both are present and the relationship between all the physical parameters (speed of each type of wave, wavelength, period, etc...) is tightly constrained as detailed above.

- If the amplitude of the magnetic field portion of a light wave is $B_{\text{max}} = 3\frac{N}{C\frac{m}{s}}$ then what must be the amplitude of the electric field component of this light wave? Note that the units of magnetic field strength have the dimensions of force per charge per speed. (This should not be a surprise because of the force law $\vec{F}_{\text{on charge by fields}} = q\vec{E} + q\vec{v} \times \vec{B}$ which means that to get magnetic force you have to multiply the magnetic field by both charge and the velocity of the charge.)
- At a place in a lightwave and at some moment in time the electric field points in the +x direction and the magnetic field points in the +y direction, what direction is the light wave traveling? What is the vector value of this wave's velocity?
- What if, at a place and time in a light wave, the magnetic field points in the -z direction and the wave is propagating in the x direction, what direction must the magnetic field point?
- There is a formula to calculate what is called the time-averaged intensity: $I = \frac{1}{2\mu_o c}E^2_{\text{max}}$. What is the intensity if the amplitude of the electric field portion of a lightwave is 5N/C? Answer this in units of Joules per second per meter squared (SI units of energy per time per area)
    - How long would it take this wave to deliver 3 J on a square cm?
    - Did this intensity and energy result depend on the frequency of the light (color of the light)?

New concepts: wave number and angular frequency

Let me introduce you to a couple of new quantities that describe a wave: the wave number and the angular frequency.

- The **wave number** is $k = \dfrac{2\pi}{\lambda}$ and described as some number of radians per length
- The **angular frequency** is $\omega = \frac{2\pi}{T} = 2\pi f$ (here I am using the idea that frequency f is one over the periodicity T. ω is described as some number of radians per time, or radians times frequency

Exercise 3

Let's summarize a lot of these ideas with thinking about a green lightwave:

- What is the wave number for a green light wave with wavelength of 500 nm?
- What is the frequency f of this green light wave?
- What is the angular frequency ω of this green lightwave?
- If the amplitude of the electric component of this green light wave is 2 N/C, write down a set of electric and magnetic field waves that represent one green light wave and travel in the positive +y direction.
- How much energy is delivered by this light wave on a square cm in 5 seconds?