

Homework #3

PHYS 4D: Modern Physics

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1 Questions

1.1 Question 1

How are the threshold frequency f_0 and wavelength λ_0 for the photoelectric effect related to the work function W of a metal?

1.1.1 Solution

The work function of a metal is related to the threshold frequency and wavelength separately, since they are dependant on each other.

$$W = hf_0 - (KE)_{\max} \quad (1)$$

$$W = \frac{hc}{\lambda_0} \quad (2)$$

1.2 Question 2

Write down an equation showing how the maximum kinetic energy of photoelectrons depends upon the wavelength of the incident light and the work function of the metal.

1.2.1 Solution

I'm taking this from the Basic Equations of this chapter.

$$(KE)_{\max} = \frac{hc}{\lambda} - W \quad (3)$$

1.3 Question 3

What would be the work function of a metal with threshold wavelength of 400 nm?

1.3.1 Solution

$$W = \frac{hc}{\lambda_0} = \frac{1240 \text{ eV} \cdot \text{nm}}{400 \text{ nm}} = \boxed{3.1 \text{ eV}} \quad (4)$$

1.4 Question 4

Would photoelectrons be emitted from a metal if the wavelength of the incident light were longer than the threshold wavelength?

1.4.1 Solution

The minimum in light is set by the frequency. This means that the maximum is given in the wavelength. This means that if it were longer, no photoelectrons would be emitted.

1.5 Question 6

What causes the dark lines in the solar spectra?

1.5.1 Solution

These would be the colors of light waves that are absorbed by atoms somewhere between the sun and the earth.

1.6 Question 7

Suppose that an atom makes a transition from an energy level E_2 to an energy level E_1 emitting light. How is the light's frequency f related to the difference of energy, $E_2 - E_1$?

1.6.1 Solution

The frequency is directly proportional to the difference of energy.

$$f = \frac{E_2 - E_1}{h} \quad (5)$$

1.7 Question 8

Write down a formula for the energy levels of hydrogen.

1.7.1 Solution

Let's use Rydberg's Rydberg's Equation.

$$E = -R \frac{hc}{n} = -\frac{13.6 \text{ eV}}{n^2} \quad (6)$$

1.8 Question 11

What is the energy of the hydrogen atom in the $n = 4$ state?

1.8.1 Solution

Use Rydberg's equation.

$$E_4 = -\frac{13.6 \text{ eV}}{4^2} = \boxed{-0.8375 \text{ eV}} \quad (7)$$

1.9 Question 12

What photon energy would be required to ionize a hydrogen atom in the $n = 5$ state?

1.9.1 Solution

Use Rydberg's equation.

$$-E_5 = \frac{13.6 \text{ eV}}{5^2} = \boxed{0.544 \text{ eV}} \quad (8)$$

1.10 Question 15

What model of light can be used to describe the *Compton effect*?

1.10.1 Solution

The particle model can be used to describe the Compton effect.

1.11 Question 16

What is Bragg's law for X-ray diffraction?

1.11.1 Solution

Bragg's law defines the requirements for constructive interference to occur between two X-rays of wavelength λ hitting two lattices of a crystal a distance d from each other at an angle θ with the perpendicular. It includes a coefficient $n \in \mathbb{N}$.

$$2d \sin(\theta) = n\lambda \quad (9)$$

1.12 Question 17

How are the momentum and the wavelength of a particle related?

1.12.1 Solution

The momentum of a particle is defined by its energy, which is defined by its frequency, which is directly related to its wavelength.

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h\nu}{c\lambda} = \frac{h}{\lambda} \quad (10)$$

1.13 Question 18

Express the momentum of an electron in terms of the angular wave vector k .

1.13.1 Solution

Use the equation from earlier and relate the wavelength and wave number.

$$\lambda = \frac{2\pi}{k}; \hbar = \frac{h}{2\pi} \quad (11)$$

$$p = \frac{h}{\lambda} = \frac{\hbar k}{1} = \hbar k \quad (12)$$

1.14 Question 19

How would the de Broglie wavelength of an electron change if its velocity were to double?

1.14.1 Solution

If the electron's velocity doubles, its momentum doubles. Since wavelength is inversely related to momentum, a doubled momentum would lead to a halved wavelength.

1.15 Question 20

Describe the interference pattern is produced when a beam of light having a single wavelength is incident upon two slits?

1.15.1 Solution

You would wind up with a pattern of light and dark spots (corresponding to where photons do and do not land). This is known as the two-slit experiment.

2 Problem 1

Calculate the energy of the photons for light having a wavelength $\lambda = 200 \text{ nm}$.

2.1 Solution

Energy of a photon is the quotient between hc (Planck's constant times the speed of light, $1240 \text{ eV} \cdot \text{nm}$) and the wavelength of the light.

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{200 \text{ nm}} = \boxed{6.2 \text{ eV}} \quad (13)$$

3 Problem 2

Suppose that a 100 Watt beam of light is incident upon a metal surface. If the light has a wavelength $\lambda = 200 \text{ nm}$, how many photons strike the surface every second? (Hint: Using the power of the beam, find how much energy is incident upon the surface every second.)

3.1 Solution

First find the energy absorbed in one second, assuming all photons are absorbed by the metal surface (don't ask me why or how).

$$P = \frac{E}{t} \quad (14)$$

$$E_{\text{bulb}} = Pt = 100 \text{ W} \times 1 \text{ s} = 100 \text{ J} \quad (15)$$

Convert the energy of each photon from the previous problem to joules.

$$E_{\text{photon}} = 6.2 \text{ eV} \times \frac{1.6022 \times 10^{-19} \text{ eV}}{1 \text{ J}} = 9.9335 \times 10^{-19} \text{ J} \quad (16)$$

Lastly, divide the energy absorbed by the energy per photon to find the number of photons.

$$n = \frac{E_{\text{bulb}}}{E_{\text{photon}}} = \frac{100 \text{ J}}{9.9335 \times 10^{-19} \text{ J}} = \boxed{1.007 \times 10^{20}} \quad (17)$$

4 Problem 4

What will the maximum kinetic energy of the emitted photoelectrons be when ultraviolet light having a wavelength of 200 nm is incident upon the following metal surfaces?

- a) Na
- b) Al
- c) Ag

4.1 Solution (a)

I will be using the same strategy for each of these. Take the work function value of the metal in question.

$$W_{\text{Na}} = 2.28 \text{ eV} \quad (18)$$

Plug this into the equation for the maximum kinetic energy. Also plug in the wavelength and the value of hc . Then compute that to find the answer.

$$(KE)_{\text{max}} = \frac{hc}{\lambda} - W = \frac{1240 \text{ eV} \cdot \text{nm}}{200 \text{ nm}} - 2.28 \text{ eV} \quad (19)$$

$$= 6.2 \text{ eV} - 2.28 \text{ eV} = \boxed{3.92 \text{ eV}} \quad (20)$$

4.2 Solution (b)

$$(KE)_{\text{max}} = \frac{hc}{\lambda} - W \quad (21)$$

$$= 6.2 \text{ eV} - 4.08 \text{ eV} = \boxed{2.12 \text{ eV}} \quad (22)$$

4.3 Solution (c)

$$(KE)_{\text{max}} = \frac{hc}{\lambda} - W \quad (23)$$

$$= 6.2 \text{ eV} - 4.73 \text{ eV} = \boxed{1.47 \text{ eV}} \quad (24)$$

5 Problem 6

Light having a wavelength of 460 nm is incident on a cathode, and electrons are emitted from the metal surface. It is observed that the electrons may be prohibited from reaching the anode by applying a stopping potential of 0.72 eV. What is the work function of the metal in the cathode?

5.1 Solution

The maximum KE of electrons emitted is equal to the electron charge times the stopping potential. However, since our stopping potential value is in units of electron volts and not volts, we can assume this refers to the kinetic energy and not the stopping potential.

$$(KE)_{\max} = 0.72 \text{ eV} \quad (25)$$

Plug this and the wavelength into the equation for maximum kinetic energy. Rounding in this problem will be to the closest hundredth of an electron-volt.

$$(KE)_{\max} = \frac{hc}{\lambda} - W \quad (26)$$

$$W = \frac{hc}{\lambda} - (KE)_{\max} = \frac{1240 \text{ eV nm}}{460 \text{ nm}} - 0.72 \text{ eV} \quad (27)$$

$$= 2.70 \text{ eV} - 0.72 \text{ eV} = \boxed{1.98 \text{ eV}} \quad (28)$$

6 Problem 7

By how much would the stopping potential in the previous problem increase if the wavelength of the radiation were reduced to 240 nm?

6.1 Solution

Let's calculate it. Keep the calculated value of W .

$$(KE)_{\max} = \frac{hc}{\lambda} - W = \frac{1240 \text{ eV nm}}{240 \text{ nm}} - 1.98 \text{ eV} \quad (29)$$

$$= 5.17 \text{ eV} - 1.98 \text{ eV} = \boxed{3.19 \text{ eV}} \quad (30)$$

7 Problem 11

An electron in the $n = 5$ state of hydrogen makes a transition to the $n = 2$ state. What are the energy and wavelength of the emitted photon?

7.1 Solution

Use Rydberg's equation.

$$E_\gamma = 13.6 \text{ eV} \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) = 13.6 \text{ eV} \left(\frac{1}{2^2} - \frac{1}{5^2} \right) = \frac{13.6 \text{ eV}}{4} - \frac{13.6 \text{ eV}}{25} \quad (31)$$

$$= 3.4 \text{ eV} - 0.544 \text{ eV} = \boxed{2.9 \text{ eV}} \quad (32)$$

8 Problem 16

Ultraviolet light of wavelength 45.0 nm is incident upon a collection of hydrogen atoms in the ground state. Find the kinetic energy and the velocity of the emitted electrons.

8.1 Solution

Light will be able to transmit energy inversely relative to its wavelength.

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{45.0 \text{ nm}} = 27.6 \text{ eV} \quad (33)$$

This would be reduced by the energy necessary to ionize the atom. Said energy for reduction would be the energy from Rydberg's equation at ground state ($n = 1$).

$$KE = 27.6 \text{ eV} - 13.6 \text{ eV} = 14.0 \text{ eV} \quad (34)$$

Before we move on, convert electron-volts to joules.

$$14.0 \text{ eV} = 2.236 \times 10^{-18} \text{ J} \quad (35)$$

Multiply this by two, divide it by the mass of an electron, and take the

square root to find the velocity (in no particular direction).

$$K = 2.236 \times 10^{-18} \text{ J} \quad (36)$$

$$2K = 2 * 2.236 \times 10^{-18} \text{ J} = 4.472 \times 10^{-18} \text{ J} \quad (37)$$

$$\frac{2K}{m_e} = \frac{4.472 \times 10^{-18} \text{ J}}{9.109 \times 10^{-31} \text{ kg}} = 4.909 \times 10^{12} \text{ m}^2/\text{s}^2 \quad (38)$$

$$\sqrt{\frac{2K}{m_e}} = \sqrt{4.909 \times 10^{12} \text{ m}^2/\text{s}^2} = \boxed{2.216 \times 10^6 \text{ m/s}} \quad (39)$$

9 Problem 20

What potential difference must electrons be accelerated through to have the same wavelength as 40 keV X-rays?

9.1 Solution

First find the wavelength of the 40 keV X-rays.

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV nm}}{40 \text{ keV}} = 0.031 \text{ nm} \quad (40)$$

Plug this into the de Broglie wavelength equation.

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{0.031 \text{ nm}} = 2.137 \times 10^{-23} \text{ N} \cdot \text{s} \quad (41)$$

Lastly, use this to find the necessary kinetic energy.

$$E = \frac{p^2}{2m_e} = \frac{(2.137 \times 10^{-23} \text{ N} \cdot \text{s})^2}{2 \cdot 9.109 \times 10^{-31} \text{ kg}} = 2.507 \times 10^{-16} \text{ J} \quad (42)$$

Divide this by the charge of the electron to find the necessary potential.

$$V = \frac{E}{q} = \frac{2.507 \times 10^{-16} \text{ J}}{1.602 \times 10^{-19} \text{ C}} = \boxed{1.565 \text{ kV}} \quad (43)$$

10 Problem 21

Electrons, protons, and neutrons have wavelengths of 0.01 nm. Calculate their kinetic energies.

10.1 Solution

Combine (41) and (42). This will bring us an equation usable for each of these wave-particles.

$$E = \frac{h^2}{2m\lambda^2} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2m(0.01 \text{ nm})^2} = \frac{2.195 \times 10^{-45} \text{ J} \cdot \text{kg}}{m} \quad (44)$$

10.1.1 Electron

Use equation (44) with $m = 9.109 \times 10^{-31} \text{ kg}$.

$$E = \frac{2.195 \times 10^{-45} \text{ J} \cdot \text{kg}}{9.109 \times 10^{-31} \text{ kg}} = \boxed{2.41 \times 10^{-15} \text{ J}} \quad (45)$$

10.1.2 Proton

Use equation (44) with $m = 1.6726 \times 10^{-27} \text{ kg}$.

$$E = \frac{2.195 \times 10^{-45} \text{ J} \cdot \text{kg}}{1.6726 \times 10^{-27} \text{ kg}} = \boxed{1.3124 \times 10^{-18} \text{ J}} \quad (46)$$

10.1.3 Neutron

Use equation (44) with $m = 1.6749 \times 10^{-27} \text{ kg}$.

$$E = \frac{2.195 \times 10^{-45} \text{ J} \cdot \text{kg}}{1.6749 \times 10^{-27} \text{ kg}} = \boxed{1.3106 \times 10^{-18} \text{ J}} \quad (47)$$