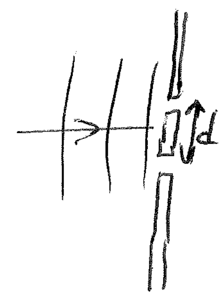
An electromagnetic plane wave ($\lambda = 632.8\,\text{nm}$) is incident upon a double slit with slit separation $d = 0.20\,\text{mm}$. An interference pattern is observed on a screen $D = 1.00\,\text{m}$ away from the double slit.

a. (3 points) Determine the separation (Δy) between adjacent bright fringes for the first few bright fringes on the screen.

$$d \sin\theta = m\lambda, \quad y = D\tan\theta$$

small angle $\Rightarrow$ $y = D\left(\frac{m\lambda}{d}\right) \Rightarrow \Delta y = D\frac{\lambda}{d}$

$$\Delta y = (1.00\,\text{m})\underbrace{\frac{632.8\,\text{nm}}{0.20\,\text{mm}}}_{0.003164 \ll 1} = 3.164\,\text{mm}$$



b. (3 points) What is the total number of bright fringes that can exist on the screen (assuming that the screen is large enough to accomodate them all). At what angle is the final fringe?

$$|m| = \frac{d}{\lambda}|\sin\theta| \le \frac{d}{\lambda} = \frac{0.20\,\text{mm}}{632.8\,\text{nm}} = 316.056$$

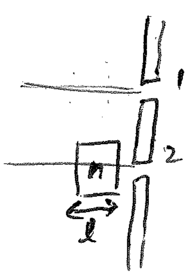
$m = 0, \pm 1, \ldots, \pm 316$

$2(316) + 1 = 633$ fringes

For $m = 316$, $\sin\theta = \frac{316\,(632.8\,\text{nm})}{0.20\,\text{mm}} = 0.999824$

$\Rightarrow \theta = 88.9^\circ$

c. (4 points) Suppose a small dielectric substance with $n = 1.70$ and length $l = 2.00\,\mu\text{m}$ is placed just before the second (bottom) slit. In what direction does the interference pattern shift, and by how many fringes?



optical path lengths

$$\Delta N_0 = \frac{\Delta L}{\lambda_0} = \frac{nl - l}{\lambda_0} = \frac{(n-1)l}{\lambda_0} = \frac{(1.70-1)(2.00\,\mu\text{m})}{632.8\,\text{nm}} = +2.212$$

(slit 2 emits "late")

λ_0: wavelength in vacuum

This results in a downward shift in the interference pattern by 2.21 fringes.

Indeed, $\frac{d\sin\theta}{\lambda} + \Delta N_0 = m \Rightarrow d\sin\theta = (m - \Delta N_0)\lambda$

$\Rightarrow$ $m=0$ fringe moves to where "$m = -2.21$" was before.