Worksheet #6

PHYS 4C: Waves and Thermodynamics

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1 Problem

A wave in a string is described by the following function:

$$y(x,t) = (0.34\,\mathrm{mm})\cos((24\,\mathrm{rad/m})x - (960\,\mathrm{rad/s})t) \tag{1}$$

The mass per unit length of the string is $1.2\,\mathrm{g/m}$. Identify or calculate the following (include correct units).

a) Amplitude:

b) Wavenumber:

c) Wavelength:

d) Angular frequency:

e) Cycle frequency:

f) Period:

g) Wave velocity (include direction):

h) Tension in the string:

i) Maximum speed of some specific part of the string:

j) Kinetic energy density (per unit length), time-averaged:

k) Potential energy density, time-averaged:

l) Rate of energy transfer across some specific point, time-averaged:

m) Total energy in a -long section:

n) How much time does it take for the amount of energy calculated in part m to cross a specific point in the string?

1.1 Solution (a)

For the first four parts (except (c)), we will find the answers in the structure of the wave equation and the general wave equation.

$$\psi(x,t) = \underline{\psi_m} \cos(kx - \omega t) \tag{2}$$
$$y(x,t) = (\underline{0.34\,\mathrm{mm}}) \cos((24\,\mathrm{rad/m})x - (960\,\mathrm{rad/s})t) \tag{3}$$
$$\boxed{\psi_m = 3.4 \times 10^{-4}\,\mathrm{m}} \tag{4}$$

1.2 Solution (b)

For the first four parts (except (c)), we will find the answers in the structure of the wave equation and the general wave equation.

$$\psi(x,t) = \psi_m \cos(\underline{k}x - \omega t) \tag{5}$$
$$y(x,t) = (0.34\,\mathrm{mm}) \cos((\underline{24\,\mathrm{rad/m}})x - (960\,\mathrm{rad/s})t) \tag{6}$$
$$\boxed{k = 24\,\mathrm{rad/m}} \tag{7}$$

1.3 Solution (c)

The wavelength is inversely proportional to the wave number.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{24\,\mathrm{rad/m}} = \frac{\pi}{12}\,\mathrm{m} = \boxed{0.2618\,\mathrm{m}} \tag{8}$$

1.4 Solution (d)

For the first four parts (except (c)), we will find the answers in the structure of the wave equation and the general wave equation.

$$\psi(x,t) = \psi_m \cos(kx - \underline{\omega} t) \tag{9}$$

$$y(x,t) = (0.34\,\mathrm{mm})\cos((24\,\mathrm{rad/m})x - (\underline{960\,\mathrm{rad/s}})t) \tag{10}$$

$$\boxed{\omega = 960\,\mathrm{rad/s}} \tag{11}$$

1.5 Solution (e)

The cycle frequency is proportional to the angular frequency.

$$f = \frac{\omega}{2\pi} = \frac{960\,\mathrm{rad/s}}{2\pi} = \frac{480\,\mathrm{rad/s}}{\pi} = \boxed{152.8\,\mathrm{Hz}} \tag{12}$$

1.6 Solution (f)

The period is the reciprocal of the frequency.

$$T = \frac{1}{f} = \frac{\pi}{480\,\mathrm{rad/s}} = \boxed{6.545\,\mathrm{ms}} \tag{13}$$

1.7 Solution (g)

The wave speed is determined by a fraction.

$$v = \frac{\lambda}{T} = \frac{\frac{\pi}{12}}{\frac{\pi}{480}} = \frac{480}{12} = \boxed{40\,\mathrm{m/s}} \tag{14}$$

1.8 Solution (h)

The tension is measured in therms of the density and the wave speed.

$$\mu = 1.2\,\mathrm{g/m} = 1.2 \times 10^{-3}\,\mathrm{kg/m} \tag{15}$$

$$v = \sqrt{\frac{\tau}{\mu}} \tag{16}$$

$$v^2 = 40^2 = 1600\,\mathrm{m^2/s^2} = \frac{\tau}{\mu} \tag{17}$$

$$\tau = \mu v^2 = 1.2 \times 10^{-3}\,\mathrm{kg/m} * 1600\,\mathrm{m^2/s^2} \tag{18}$$

$$= 1.2 * 1.6\,\mathrm{kg\,m^2/s^2\,m} = \boxed{1.92\,\mathrm{N}} \tag{19}$$

1.9 Solution (i)

We can first find the formula for the velocity. By nature, the velocity would only be vertical since the wire would not be moving horizontally.

$$v = \frac{\mathrm{d}}{\mathrm{d}t}\left((0.34\,\mathrm{mm})\cos((24\,\mathrm{rad/m})x - (960\,\mathrm{rad/s})t)\right) \tag{20}$$
$$= -3.4 \times 10^{-4} * 960\,\mathrm{rad/s}\sin(\cos((24\,\mathrm{rad/m})x - (960\,\mathrm{rad/s})t)) \tag{21}$$

For the maximum value of this velocity, the sine would have to be equal to one (or negative one) so we can set it to that.

$$v = -3.4 \times 10^{-4}\,\mathrm{m} * 960\,\mathrm{rad/s} * (-1) = \boxed{0.3264\,\mathrm{m/s}} \tag{22}$$

1.10 Solution (j)

We have an equation for the kinetic energy density.

1.11 Solution (k)

1.12 Solution (l)

1.13 Solution (m)

1.14 Solution (n)