Find the mass in kilograms of 7.50×10^{24} atoms of arsenic, which has a molar mass of $74.9 \mathrm{g/mol}$.

1.1 Solution

Convert atoms to moles.

$$\frac{7.50 \times 10^{24} \text{ atoms}}{6.02 \times 10^{23} \text{ atoms/mol}} = 12.458 \text{mol}$$
 (1)

Using the molar mass, convert moles to grams.

$$12.458 \text{mol} * 74.9 \text{g/mol} = 933.140 \text{g} = \boxed{0.933 \text{kg}}$$
 (2)

Oxygen gas having a volume of 1000cm^3 at 40.0°C and $1.01 \times 10^5\text{Pa}$ expands until its volume is 1500cm^3 and its pressure is $1.06 \times 10^5\text{Pa}$. Find (a) the number of moles of oxygen present and (b) the final temperature of the sample.

2.1 Solution (a)

We can use the ideal gas law for this. We apply it to the first case, converting the 40.0°C to Kelvin.

$$40^{\circ}\text{C} = 313.15\text{K}$$
 (3)

$$1000 \text{cm}^3 = 1000 \times 10^{-6} \text{m}^3 = 1 \times 10^{-3} \text{m}^3 \tag{4}$$

$$pV = nRT (5)$$

$$n = \frac{pV}{RT} = \frac{1.01 \times 10^5 \text{Pa} * 10^{-3} \text{cm}^3}{8.31 \text{J/mol} \cdot \text{K} * 313.15 \text{K}}$$
(6)

$$= \frac{1.01 \times 10^2 \text{N} \cdot \text{m}}{2602.2765 \text{J/mol}} = \boxed{0.038812 \text{mol}}$$
 (7)

2.2 Solution (b)

The ideal gas law (or an equivalent) will be used here. The number of moles does not change here, neither does the gas constant R. We can use the to solve for the final value of the temperature.

$$\frac{p_1 V_1}{n_1 T_1} = \frac{p_2 V_2}{n_2 T_2} \tag{8}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \tag{9}$$

$$T_2 = T_1 * \frac{p_2 V_2}{p_1 V_1} \tag{10}$$

We can substitute in values now.

$$T_2 = T_1 * \frac{p_2 V_2}{p_1 V_1} = 313.15 * \frac{1.06 \times 10^5 * 1500}{1.01 \times 10^5 * 1000}$$
 (11)

$$= 313.15 * \frac{1.06 * 1.5}{1.01} = \boxed{492.97 \text{K} \approx 220^{\circ} \text{C}}$$
 (12)

The best laboratory vacuum has a pressure of about 1.00×10^{-18} atm, or 1.01×10^{-13} Pa. How many gas molecules are there per cubic centimeter in such a vacuum at 293 K?

3.1 Solution

Use the ideal gas law, the version with Boltzmann's constant. We can solve for $\frac{N}{V}$.

$$pV = NkT \tag{13}$$

$$\frac{N}{V} = \frac{p}{kT} \tag{14}$$

From here, we can just plug and chug, so to speak.

$$\frac{N}{V} = \frac{1.01 \times 10^{-13} \text{N/m}^2}{1.38 \times 10^{-23} \text{N} \cdot \text{m/K} * 293 \text{K}} = \frac{1.01 \times 10^{10}}{404.34} \text{m}^{-3}$$
(15)

$$= 24978978.09 \times \text{m}^{-3} = 24.979 \text{cm}^{-3}$$
 (16)

Suppose 1.80 mol of an ideal gas is taken from a volume of $3.00~\mathrm{m}^3$ to a volume of $1.50~\mathrm{m}^3$ via an isothermal compression at $30^{\circ}\mathrm{C}$. (a) How much energy is transferred as heat during the compression, and (b) is the transfer to or from the gas?

4.1 Solution (a)

Energy transferred can be thought of as work. We have a formula for work done by an ideal gas.

$$W = nRT \ln \left(\frac{V_f}{V_i}\right) \tag{17}$$

We can plug and chug into this.

$$T_K = T_C + 273.15 \text{K} = 30^{\circ} \text{C} + 273.15 \text{K} = 303.15 \text{K}$$
 (18)

$$W = (1.80 \text{mol})(8.31 \text{J/mol})(303.15 \text{K}) \ln \left(\frac{1.5 \text{m}^3}{3.0 \text{m}^3}\right)$$
(19)

$$= 4534.5177J * (-0.693147) = -3143.088J$$
 (20)

The energy transferred is the absolute value of this, which would be $\boxed{3143.088J}$.

4.2 Solution (b)

This is a volume compression process. The total energy in the system would remain constant, so Q = W by the first law of thermodynamics. This means Q < 0, so the energy is transferred from the gas as heat.

An automobile tire has a volume of $1.64 \times 10^{-2} \text{m}^3$ and contains air at a gauge pressure (pressure above atmospheric pressure) of 165 kPa when the temperature is 0.00°C . What is the gauge pressure of the air in the tires when its temperature rises to 27.0°C and its volume increases to $1.67 \times 10^{-2} \text{m}^3$? Assume atmospheric pressure is $1.01 \times 10^{5} \text{Pa}$.

5.1 Solution

We can here use the ideal gas law.

$$pV = nRT (21)$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \tag{22}$$

With our equation, we solve for p_f .

$$p_f = p_i \frac{V_i T_f}{V_f T_i} \tag{23}$$

This can be "plugged-and-chugged" to find our final answer. We should also bear in mind that the gauge pressure should be added to the atmospheric pressure to find the overall pressure. Do not forget to convert to Kelvin.

$$T_{iK} = 0^{\circ} \text{C} + 273 \text{K} = 273 \text{K}$$
 (24)

$$T_{fK} = 27^{\circ}\text{C} + 273\text{K} = 300\text{K}$$
 (25)

$$p_i = p_{gauge} + p_{atm} = 165 \times 10^3 \text{Pa} + 1.01 \times 10^5 \text{Pa} = 266 \times 10^3 \text{Pa}$$
 (26)

$$p_f = p_i \frac{V_i T_f}{V_f T_i} = 266 \times 10^3 \text{Pa} * \frac{(1.64 \times 10^{-2} \text{m}^3)(300 \text{K})}{(1.67 \times 10^{-2} \text{m}^3)(273 \text{K})}$$
(27)

$$= 266 \times 10^{3} \text{Pa} * \frac{492}{455.91} = 287 \times 10^{3} \text{Pa}$$
 (28)

This can in turn be turned into the gauge pressure (our final answer) by subtracting the atmospheric pressure.

$$p_{gauge;f} = 287 \times 10^{3} \text{Pa} - 1.01 \times 10^{5} \text{Pa} = \boxed{1.86 \times 10^{5} \text{Pa}}$$
 (29)

Air that initially occupies 0.140m^3 at a gauge pressure of 103.0kPa is expanded isothermally to a pressure of 101.3kPa and then cooled at constant pressure until it reaches its initial volume. Compute the work done by the air. Gauge pressure is the difference between the actual pressure and atmospheric pressure.

6.1 Solution

Isothermal expansion means that there is no net change in energy, so $Q_{in} = W_{out}$ (a.k.a. $Q_{in} = -W_{in}$) as volume increases. There is a formula for work we can use.

$$W = \int_{V_i}^{V_f} p(V) \, dV \tag{30}$$

The problem outright states that it is separated into two parts: the isothermal expansion at changing pressure and the cooling at constant pressure. We can separate our equation into these two parts and find useful equations for each. The integrals will be separated at a point of $V = V_{max}$ since that is the point where the volume reachest its highest. We can call the final pressure value of 101.3kPa the value p_r , with no meaning to its name.

$$W = \int_{V_i}^{V_f} p(V) \, dV = \int_{V_i}^{V_{max}} p_1(V) \, dV + \int_{V_{max}}^{V_f} p_2(V) \, dV \tag{31}$$

$$= \int_{V_i}^{V_{max}} \frac{nRT_i}{V} \, dV + \int_{V_{max}}^{V_f} p_r \, dV \tag{32}$$

$$= nRT_i \ln\left(\frac{V_{max}}{V_i}\right) + \int_{V_{max}}^{V_f} p_r \, dV \tag{33}$$

$$= p_i V_i \ln \left(\frac{V_{max}}{V_i}\right) + p_r (V_f - V_{max}) \tag{34}$$

This requires the initial and maximum magnitudes of the volume, of which we only have one. We can use the ideal gas law to find that which we need. First solve for the final volume, given n, R, and T are constant. Do not forget to convert gauge pressure to overall pressure by adding the atmospheric pressure $(1.01 \times 10^5 \text{Pa})$. We can assume that the final pressure

is not gauge pressure.

$$p_i = p_1 + p_{atm} = 103.0 \times 10^3 + 1.01 \times 10^5$$
Pa = 2.04×10^5 Pa (35)

$$p_f = p_2 = 101.3 \times 10^3 \text{Pa} \tag{36}$$

$$pV = nRT (37)$$

$$p_i V_i = p_f V_f = p_f V_{max} \tag{38}$$

$$V_{max} = V_i * \frac{p_i}{p_f} = 0.140 \text{m}^3 * \frac{2.04 \times 10^5 \text{Pa}}{1.013 \times 10^5 \text{Pa}} = 0.281935 \text{m}^3$$
 (39)

This can in turn be applied to our above equation for the work done by the gas.

$$W = p_i V_i \ln \left(\frac{V_{max}}{V_i}\right) + \int_{V_{max}}^{V_f} p_r \, dV \tag{40}$$

$$=0.140 \text{m}^3*2.04\times 10^5 \text{Pa}*\ln \left(\frac{0.281935 \text{m}^3}{0.140 \text{m}^3}\right) + \int_{0.281935 \text{m}^3}^{0.140 \text{m}^3} 1.013\times 10^5 \text{Pa} \, dV \quad (41)$$

$$= 28560 * 0.7000 + 101300 * (-0.1419) = 19992.96J - 14378J$$

$$(42)$$

$$= \boxed{5614.95J} \tag{43}$$

A sample of an ideal gas is taken through the cyclic process abca shown in Fig. 19-20. The scale of the vertical axis is set by $p_b = 7.5 \,\mathrm{kPa}$ and $p_{\mathrm{ac}} = 2.5 \,\mathrm{kPa}$. At point $a, T = 200 \,\mathrm{K}$. (a) How many moles of gas are in the sample? What are (b) the temperature of the gas at point b, (c) the temperature of the gas at point c, and (d) the net energy added to the gas as heat during the cycle?

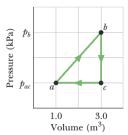


Figure 19-20 Problem 13.

7.1 Solution (a)

We can use the ideal gas law. We need to solve for n at point a.

$$pV = nRT \to n = \frac{pV}{RT} \tag{44}$$

From here, we plug in values to find n.

$$n = \frac{(2.5 \,\text{kPa}) * (1.0 \,\text{m}^3)}{R(200 \,\text{K})} = \frac{2.5 \times 10^3 \,\text{J}}{1662 \,\text{J/mol}} = \boxed{1.504 \,\text{mol}}$$
(45)

7.2 Solution (b)

We can use the ideal gas law again, this time comparing two situations and finding the final temperature.

$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f} \to T_f = T_i * \frac{p_f V_f}{p_i V_i}$$

$$\tag{46}$$

From here, we plug in values to find the final temperature.

$$T_f = T_i * \frac{p_f V_f}{p_i V_i} = 200 \,\mathrm{K} * \frac{7.5 \,\mathrm{kPa} * 3.0 \,\mathrm{m}^3}{2.5 \,\mathrm{kPa} * 1.0 \,\mathrm{m}^3}$$
 (47)

$$= 200 \,\mathrm{K} * 9 = \boxed{1800 \,\mathrm{K}} \tag{48}$$

7.3 Solution (c)

Use the ideal gas law again. This time, the pressure does not change.

$$\frac{V_i}{T_i} = \frac{V_f}{T_f} \to T_f = T_i * \frac{V_f}{V_i} \tag{49}$$

Now, we plug in to find T_f .

$$T_f = T_i * \frac{V_f}{V_i} = 1800 \,\mathrm{K} * \frac{1.0 \,\mathrm{m}^3}{3.0 \,\mathrm{m}^3} = \boxed{600 \,\mathrm{K}}$$
 (50)

7.4 Solution (d)

Let's start by calculating the total work done by the gas. We can separate it based on the phases. The phase between points b and c has no change in volume, so no work is done.

$$W = \int p(V) \, dV \tag{51}$$

$$= \int_{1.0 \,\mathrm{m}^3}^{3.0 \,\mathrm{m}^3} V * 2500 \,\mathrm{Pa/m}^3 \,dV + \int_{3.0 \,\mathrm{m}^3}^{3.0 \,\mathrm{m}^3} p_{\mathrm{bc}} \,dV + \int_{3.0 \,\mathrm{m}^3}^{1.0 \,\mathrm{m}^3} 2500 \,\mathrm{Pa} \,dV \quad (52)$$

$$= (1250 \,\mathrm{Pa/m^3}) \left(\frac{(3.0 \,\mathrm{m^3})^2 - (1.0 \,\mathrm{m^3})^2}{2} \right) + 2500 \,\mathrm{Pa} * (-2.0 \,\mathrm{m^3})$$
 (53)

$$= 4 * 2500 J - 2 * 2500 J = 10000 - 5000 J = 5000 J$$
(54)

Since the this is a cyclical process, we can apply the first law of thermodynamics.

$$\Delta E_{\rm int} = 0 = Q_{\rm in} - W_{\rm out} \tag{55}$$

$$Q_{\rm in} = W_{\rm out} = \boxed{5000\,\mathrm{J}}\tag{56}$$

Container A in Fig. 19-22 holds an ideal gas at a pressure of 5.0×10^5 Pa and a temperature of 300 K. It is connected by a thin tube (and a closed valve) to container B, with four times the volume of A. Container B holds the same ideal gas at a pressure of 1.0×10^5 Pa and a temperature of 400 K. The valve is opened to allow the

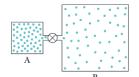


Figure 19-22 Problem 17.

pressures to equalize, but the temperature of each container is maintained. What then is the pressure?

8.1 Solution

Let's put together a formulas for the number of moles of the ideal gas in each container (A and B). We can use the Ideal Gas Law for this.

$$n_A = \frac{p_A V_A}{R T_A} = \frac{5.0 \times 10^5 \,\mathrm{Pa} * V_A}{R * 300 \,\mathrm{K}} = \frac{5 \times 10^3}{3} * \frac{V_A}{R}$$
 (57)

$$n_B = \frac{p_B V_B}{R T_B} = \frac{1.0 \times 10^5 \,\text{Pa} * 4V_A}{R * 400 \,\text{K}} = 1 \times 10^3 * \frac{V_A}{R}$$
 (58)

We can combine these to find the total number of moles.

$$n_{\Sigma} = n_A + n_B = \left(\frac{5}{3} + 1\right) \times 10^3 * \frac{V_A}{R} = \frac{8}{3} \times 10^3 * \frac{V_A}{R}$$
 (59)

We also know the total volume that there is.

$$V_{\Sigma} = \sum V_i = V_A + V_B = V_A + 4V_A = 5V_A \tag{60}$$

We can also calculate the average temperature of the cobined system, weighted by the volumes.

$$T_{\text{avg}} = \frac{\sum V_i * T_i}{\sum V_i} = \frac{V_A * T_A + V_B * T_B}{5V_A}$$
 (61)

$$= \frac{V_A * 300 \,\mathrm{K} + 4V_A * 400 \,\mathrm{K}}{5V_A} \tag{62}$$

$$= \frac{300 \,\mathrm{K} + 1600 \,\mathrm{K}}{5} = \frac{1900 \,\mathrm{K}}{5} = 380 \,\mathrm{K} \tag{63}$$

We have our final values for the number of moles, volume, and temperature, not to mention that R is a constant. We can put these into the ideal gas law to find the pressure at this point, assuming the pressure to be even.

$$pV = nRT (64)$$

$$p = \frac{nRT}{V} = \frac{\frac{8}{3} \times 10^3 * \frac{V_A}{R} * R * 380 \text{ K}}{5V_A}$$
 (65)

$$= \left(\frac{8}{15} \times 10^3 * 380\right) \text{ Pa} = \boxed{2.03 \times 10^5 \text{Pa}}$$
 (66)

(a) Compute the rms speed of a nitrogen molecule at 20.0° C. The molar mass of nitrogen molecules (N_2) is given in Table 19-1. At what temperatures will the rms speed be (b) half that value and (c) twice that value?

9.1 Solution (a)

As stated in table 19-1, the molar mass of Nitrogen gas (N_2) molecules is 28.0×10^{-3} kg/mol. We can use that to find the molecule's rms (root-mean-square) speed.

$$T_K = T_{^{\circ}\text{C}} + 273\text{K} = 293\text{ K}$$
 (67)

$$v_{\rm rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3*8.31 \,\text{J/mol} \cdot \text{K} * 293 \,\text{K}}{28.0 \times 10^{-3} \,\text{kg/mol}}}$$
 (68)

$$= \sqrt{\frac{7304.49}{28.0} \times 10^3 \,\mathrm{m}^2/\mathrm{s}^2} = \boxed{510.8 \,\mathrm{m/s}}$$
 (69)

9.2 Solution (b)

We can use the same equation, this time solving for the temperature.

$$\frac{v_{\rm rms}}{2} = \sqrt{\frac{3 * 8.31 \,\text{J/mol} \cdot \text{K} * T}{28.0 \times 10^{-3} \,\text{kg/mol}}}$$
 (70)

$$\frac{3*8.31 \text{ J/mol} \cdot \text{K} * 293 \text{ K}}{28.0 \times 10^{-3} \text{ kg/mol}} * \frac{1}{4} = \frac{3*8.31 \text{ J/mol} \cdot \text{K} * T}{28.0 \times 10^{-3} \text{ kg/mol}}$$
(71)

$$T = \frac{293 \,\mathrm{K}}{4} = 73.25 \,\mathrm{K} = \boxed{-199.75 \,^{\circ}\mathrm{C}}$$
 (72)

9.3 Solution (c)

The same equation can be used.

$$2v_{\rm rms} = \sqrt{\frac{3 * 8.31 \,\text{J/mol} \cdot \text{K} * T}{28.0 \times 10^{-3} \,\text{kg/mol}}}$$
 (73)

$$2v_{\rm rms} = \sqrt{\frac{3 * 8.31 \,\text{J/mol} \cdot \text{K} * T}{28.0 \times 10^{-3} \,\text{kg/mol}}}$$

$$\frac{3 * 8.31 \,\text{J/mol} \cdot \text{K} * 293 \,\text{K}}{28.0 \times 10^{-3} \,\text{kg/mol}} * 4 = \frac{3 * 8.31 \,\text{J/mol} \cdot \text{K} * T}{28.0 \times 10^{-3} \,\text{kg/mol}}$$
(74)

$$T = 4 * 293 \,\mathrm{K} = 1172 \,\mathrm{K} = 899 \,^{\circ}\mathrm{C}$$
 (75)

A beam of hydrogen molecules (H_2) is directed toward a wall, at an angle of 55° with the normal to the wall. Each molecule in the beam has a speed of $1.0 \,\mathrm{km/s}$ and a mass of $3.3 \times 10^{-24} \mathrm{g}$. The beam strikes the wall over an area of 2.0cm², at the rate of 10²³ molecules per second. What is the beam's pressure on the wall?

10.1Solution

This solution may be difficult to understand because p represents both pressure and momentum. We need more letters. Maybe we could look at some language with a lot of different letters/characters. My Chinese teacher says their language has about 1500. We have a formula for pressure exerted by an ideal gas.

$$p = \frac{nMv_{\rm rms}^2}{3V} = \frac{F}{A} \tag{76}$$

We assume that any collision of a molecule with a wall is fully elastic, so we can find a formula for the change in the vertical momentum. This would only be for a single particle and we will only be considering the horizontal velocity.

$$v_i = v_{\rm rms} * \sin(55^\circ) \tag{77}$$

$$v_f = -v_i \tag{78}$$

$$\Delta p = p_f - p_i = mv_f - mv_i \tag{79}$$

$$=2mv_f = -2m * v_{\rm rms} * \cos(55^\circ) \tag{80}$$

This can be multiplied by the rate at which the molecules hit the wall to find the total force, which is itself constant.

$$F = p * \frac{\mathrm{d}N}{\mathrm{d}t} = -2m * v_{\mathrm{rms}} * \cos(55^{\circ}) * 10^{23} \,\mathrm{s}^{-1}$$
 (81)

We can apply this to our equation for the pressure. That would be the final equation that we can plug values into.

$$p = \frac{F}{A} = -\frac{2m * v_{\text{rms}} * \cos(55^{\circ}) * 10^{23} \text{s}^{-1}}{A}$$

$$= -\frac{2 * 3.3 \times 10^{-27} \text{ kg} * 1000 \text{ m/s} * \cos(55^{\circ}) * 10^{23} \text{ s}^{-1}}{2.0 \times 10^{-4} \text{ m}^{2}}$$
(82)

$$= -\frac{2 * 3.3 \times 10^{-27} \text{ kg} * 1000 \text{ m/s} * \cos(55^{\circ}) * 10^{23} \text{ s}^{-1}}{2.0 \times 10^{-4} \text{ m}^2}$$
(83)

$$= -1893$$
Pa (84)

Since pressure is always positive, we can finalize this by taking the absolute value to get our final answer. Said answer would be 1893 Pa.

Determine the average value of the translational kinetic energy of the gas's molecules of an ideal gas at temperatures (a) 0.00°C and (b) 100°C. What is the translational kinetic energy per mole of an ideal gas at (c) 0.00°C and (d) 100°C?

Water standing in the open at 32.0°C evaporates because of the escape of some of the surface molecules. The heat of vaporization (539 cal/g) is approximately equal to ϵ n, where ϵ is the average energy of the escaping molecules and n is the number of molecules per gram. (a) Find ϵ . (b) What is the ratio of ϵ to the average kinetic energy of H_2O molecules, assuming the latter is related to temperature in the same way as it is for gases?

In a certain particle accelerator, protons travel around a circular path of diameter 23.0 m in an evacuated chamber, whose residual gas is at 295 K and 1.00×10^{-6} torr pressure. (a) Calculate the number of gas molecules per cubic centimeter at this pressure. (b) What is the mean free path of the gas molecules if the molecu- lar diameter is 2.00×10^{-8} cm?

Ten particles are moving with the following speeds: four at 200m/s, two at 500m/s, and four at 600m/s. Calculate their (a) average and (b) rms speeds. (c) Is $v_{rms} > v_{avg}$?

The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0 C° without the pressure of the gas changing. The molecules in the gas rotate but do not oscillate. (a) How much energy is transferred to the gas as heat? (b) What is the change in the internal energy of the gas? (c) How much work is done by the gas? (d) By how much does the rotational kinetic energy of the gas increase?

When 1.0 mol of oxygen (O_2) gas is heated at constant pressure starting at 0°C, how much energy must be added to the gas as heat to double its volume? The molecules rotate but do not oscillate.

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