Worksheet #5

PHYS 4C: Waves and Thermodynamics

Donald Aingworth IV

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1 Problem

(20 points) A Carnot heat engine operates on a four step cycle as follows:

1. Isothermal expansion at temperature T_3, where heat Q_H flows into the system from an external temperature reservoir T_4 ($T_4 \geq T_3$). This step is reversible if $T_4 = T_3$.

2. Adiabatic expansion (no heat flow), where the temperature drops from T_3 to T_2 ($T_2 < T_3$). This step is always reversible (as long as it is quasi-static).

3. Isothermal compression at temperature T_2, where heat Q_C flows out of the system to an external temperature reservoir T_1 ($T_1 \leq T_2$). This step is reversible if $T_1 = T_2$.

4. Adiabatic compression (no heat flow), where the temperature rises from T_2 back up to T_3. This step is always reversible.

The efficiency of this cycle is given by $e = 1 - T_2/T_3$, and is maximized when $T_3 = T_4$ and $T_2 = T_1$ (reversible), although the work output rate in that case is zero (heat flowrates are zero for steps 1 and 3). Allowing $T_1 < T_2 < T_3 < T_4$ enables us to consider a real heat engine that would run at a finite rate.

Consider such a heat engine with $T_4 = 600$ K, $T_3 = 500$ K, $T_2 = 400$ K, and $T_1 = 300$ K. Suppose also that the heat conductance for the rods connecting the system to T_4 during step 1 and to T_1 during step 3 are each 10 W/K, and that the adiabatic steps (2 and 4) are both very rapid (essentially zero time).

(a) (6 points) Calculate the efficiency of this heat engine. If $Q_H = 100$ J for one cycle of this heat engine, how much heat flows into the cold reservoir (Q_C) and how much work is output (W) for each cycle?

(b) (6 points) Calculate the net change in entropy during one cycle. During which steps does the positive entropy change occur?

(c) (8 points) Determine the rate of work production for this engine (work/time). (Hint: calculate the total time for one cycle).

1.1 Solution (a)

It is given that the heat engine would have the starting temperature of $T_3 = 500\text{K}$ and an ending temperature of $T_3 = 400\text{K}$. The