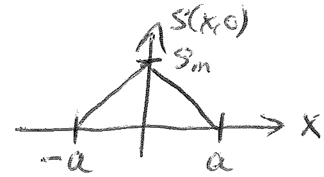
*. (20 points) The speed of sound in steel is 5941 m/s. Steel has a density of around 7900 kg/m^3 (depends somewhat on the alloy content).

a. (2 points) Based on this information, what is the bulk modulus of steel?

$$v = \sqrt{B/\rho} \Rightarrow B = \rho v^2 = (7900\,\mathrm{kg/m^3})(5941\,\mathrm{m/s})^2 = 2.788 \times 10^{11}\,\mathrm{Pa}$$

b. (8 points) A steel rod of cross-sectional area A is placed on the x-axis. The following sound pulse is sent through the steel rod in the $+x$ direction:

$$f(x) = s(x,0) = \begin{cases} s_m(1 - |x|/a) & |x| < a \\ 0 & |x| \geq a \end{cases}$$



Determine the total energy of this pulse.

$$\rho_u = \frac{1}{2} B \left(\frac{\partial s}{\partial x}\right)^2 = \frac{1}{2} B (\pm S_m/a)^2 = \frac{1}{2} B S_m^2/a^2 \quad (-a \leq x - vt \leq +a)$$

$$\rho_K = \frac{1}{2}\rho\left(\frac{\partial s}{\partial t}\right)^2 = \frac{1}{2}\rho\left(-v\frac{\partial s}{\partial x}\right)^2 = \frac{1}{2}\underbrace{\rho v^2}_{B}\left(\frac{\partial s}{\partial x}\right)^2 = \rho_u$$

$$\rho_E = \rho_u + \rho_K = B S_m^2/a^2 \quad (-a \leq x - vt \leq a)$$

$$E = \int \rho_E \, dV = \rho_E (2a)(A) = 2 B S_m^2 A/a$$

c. (10 points) Now suppose a sinusoidal sound wave with frequency 440 Hz is sent through steel with a sound level of 100 dB. Calculate s_m and Δp_m for this sound wave.

$$\omega = 2\pi f = 2764.6 \text{ rad/s}$$

$$\cancel{\langle I \rangle} = \beta = (10\,\text{dB}) \log_{10} \left(\langle I \rangle / I_0\right) = 100\,\text{dB}$$

$$\Rightarrow \langle I \rangle = \underset{\substack{\uparrow \\ 10^{-12}\,\text{W/m}^2}}{I_0}\, 10^{10} = 10^{-2}\,\text{W/m}^2$$

$$\langle I \rangle = 10^{-2}\,\text{W/m}^2 = \frac{1}{2}\rho\omega^2 s_m^2 v$$

$$\Rightarrow s_m = \sqrt{\frac{2\langle I \rangle}{\rho\omega^2 v}} = \sqrt{\frac{2(10^{-2}\,\text{W/m}^2)}{(7900\,\text{kg/m}^3)(2764.6\,\text{rad/s})^2(5941\,\text{m/s})}}$$

$$= 7.4669 \text{ nm}$$

$$\Delta p_m = B s_m k = \rho v^2 s_m k = \rho \omega s_m v$$

$$= (7900\,\text{kg/m}^3)(2764.6\,\text{rad/s})(7.4669\times 10^{-9}\,\text{m})(5941\,\text{m/s})$$

$$= 968.85 \text{ Pa} \quad (\sim 1\%\ \text{atm})$$