PHYS 4A Exam 3 Cheat Sheet (with LaTeX)

Write Units

Kinematic Equations

$$v_{avg} = \frac{\Delta x}{\Delta t}; s_{avg} = \frac{distance}{time}; v = \frac{dx}{dt}$$
$$a_{avg} = \frac{\Delta v}{\Delta t}; a = \frac{dv}{dt} = \frac{d^2x}{dt^2}; (1)\ v(t) = v_0 + at$$
$$(2)\ x = x_0 + v_0 t + \frac{1}{2}at^2; (3)\ v^2 = v_0^2 + 2a\Delta x$$

When doing a problem, account for all the variables you know the values of and all those you don't know the value of.

Freefall

Object is in freefall iff only force acting on it is gravity

Kinematic eq'ns apply to freefall

Unless stated otherwise, gravitational acceleration $g = -9.81 m/s^2$

Vectors

$$\vec{a} \cdot \vec{b} = ab\cos(\theta); ||\vec{a} \times \vec{b}|| = ab\sin(\theta)$$
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y \ldots; \vec{a} \times \vec{b} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{pmatrix}$$

Vectors work as their separate parts for kinematic eq'ns

Project

Motion in 2D+ (uses vectors)

Generally, vertical motion is freefall, horizontal motion is constant

x-value = magnitude times cosine of angle

y-value = magnitude times sine of angle

$$R = x - x_0 = \frac{v_0^2 * \sin(2\theta)}{g}; t = \frac{R}{v_0 \cos(\theta)}$$
$$\Delta y = \tan\theta \Delta x - \frac{g * \Delta x^2}{2(v_0 \cos\theta)^2}$$

Uniform Circular Motion

$$\vec{x}(t) = x * \cos\theta \hat{i} + x * \sin\theta \hat{j}; a_c = \frac{v^2}{r}; F_c = \frac{mv^2}{r}$$

Force

Force on an object is always represented on a FBD as starting from that object

Force on an object is calculated from that object's mass and consequent acceleration

$$F_{net} = ma | F_{AB} = -F_{BA}$$

There is no technical equation for the tension force. Treat it as an unknown when it is included.

Work Mechanical energy transfer to or from a system; $W = \vec{F} \cdot \vec{d} = \int F(x)dx = \int \vec{F}(\vec{r}) \cdot d\vec{r}$.

Kinetic Energy $K = \frac{1}{2}mv^2$; $W_{net} = \Delta K$

Friction

$$f_s \leq \mu_s F_N; f_k = \mu_k F_N$$

At all points, $0 < \mu < 1$. μ_s is for unmoving, μ_k is for moving. When unmoving, $f_s = F_{app}$. Energy lost from it is thermal and uses $W = \vec{f_k} \cdot \vec{d}$.

Spring force

$$\vec{F_s} = -k\Delta\vec{d}\ ;\ W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Power Rate at which work is done/energy changes $P = \frac{W}{\Delta t}$

Potential energy

Conservative force rules: $W_{ab} = -W_{ba}$; Path does not matter; Net work done on closed path is 0

Gravitational: $U = mgy$ so $\Delta U = mg\Delta y$

Spring: $U = \frac{1}{2}kx^2$ (nonnegative)

Mechanical Energy

If only conservative forces are used,
$E_{mech} = K - U = Constant$

Center of mass For any dimension x

$$x_{com} = \frac{\int x\ dm}{M} = \frac{\int x\ dV}{V}$$

Linear momentum $\vec{p} = m\vec{v}$

$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{net}$

Impulse $\vec{J} = \Delta\vec{p} = \int \vec{F}\ dt$

$\vec{p}$ is constant for a closed system w/o external forces

Collisions

Momentum and total energy always conserved

Elastic is perfect bounce, KE conserved

Inelastic is imperfect bounce, KE not conserved

Perfectly inelastic move together, KE not conserved

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

Elastic Collision Equations

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$
$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}$$
$$v_{1f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$

Angular Kinematics (Basically normal kinematics just in circles)

$$\theta = \frac{S}{r}; \omega = \frac{d\theta}{dt}; \alpha = \frac{d^2\theta}{dt^2}; (1)\ \omega(t) = \omega_0 + \alpha t$$
$$(2)\ \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2; (3)\ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$
$$v_t = \omega r; a_t = \alpha r; a_c = \omega r^2; T = \frac{2\pi}{\omega}$$

Inertia: Resistance to change in motion (like mass). Rotational inertia represented with I. All following I are about center.

$$K = \frac{1}{2}I\omega^2; I = \sum_i m_i r_i^2 = \int r(m)^2 dm$$
$$I_{rod} = \frac{1}{12}ML^2; I_{ring} = MR^2; I_{disc} = \frac{1}{2}MR^2$$

Inerita about point h away from midpoint (Parallel Axis Theorem): $I = I_{com} + Mh^2$

Torque (τ)

$$\vec{\tau} = \vec{r} \times \vec{F}_T = rF\sin(\phi) = Fr\sin(\phi) = I\vec{\alpha}$$
$$r_t = r\sin(\phi) = \text{moment arm}$$
$$W = \int \tau(\theta)d\theta; P = \vec{\tau} \cdot \vec{\omega}$$

Rolling

Static friction applies, not kinetic friction. Gravity also applies.

$$\theta = \frac{s}{r}; \omega = \frac{v}{r}; \alpha = \frac{a}{r}$$
$$a_{com;x} = \frac{mg\sin(\theta)}{m + I_{com}/R^2}$$

Angular Momentum

ℓ is angular momentum of single particle
L is angular momentum of group of particles

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) = mvr\sin(\theta)$$
$$\vec{L} = \Sigma\ell_i; \vec{L} = I\vec{\omega}; \vec{p} = m\vec{v}$$

Simple Harmonic Motion

x_m = amplitude; ϕ = phase shift
ω = Angular frequency = $2\pi f$
f = frequency (unit Hz)
T = Period = $\frac{1}{f} = \frac{2\pi}{\omega}$

$$x(t) = x_m\cos(\omega t + \phi)$$
$$v(t) = -\omega x_m\sin(\omega t + \phi)$$
$$a(t) = -\omega^2 x_m\cos(\omega t + \phi)$$
$$v_{max} = \omega x_{max}$$

Spring: $\omega^2 = \frac{k}{m}$
Torsional pendulum: $\omega^2 = \frac{\kappa}{I}$. κ is for torsional pendulums what k is for springs.
Simple pendulum: $\omega^2 = \frac{g}{L}$ for small θ
Physical Pendulum: $\omega^2 = \frac{mgL}{I}$ (don't forget Parallel Axis Theorem)

Damped SHM

b = damping constant

$$F_d = -bv; \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$
$$x(t) = e^{-\frac{bt}{2m}}\cos(\omega' t + \phi)$$

ω'	Dampening
$\Re$	Underdamped
0	Critically damped
$!\Re$	Overdamped

ω'	Behavoir
$\Re$	Oscillates at decreasing amplitude
0	Goes back to origin as fast as possible
$!\Re$	Goes back to origin without oscillating