Homework #1

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1. 7.83 $\frac{\text{L}}{100\text{ km}}$

$$\frac{30\text{ mi}}{1\text{ gal}} \cdot \frac{1\text{ gal}}{3.79\text{ L}} \cdot \frac{1\text{ km}}{0.62\text{ mi}} \cdot \frac{1}{100} \frac{100\text{ km}}{\text{km}} = \frac{30}{235} \frac{100\text{ km}}{\text{L}}$$

The reciprocal of this is:

$$\frac{235}{30} \frac{\text{L}}{100\text{km}} \approx \boxed{7.83 \frac{\text{L}}{100\text{km}}}$$

2. a) Dimensions are consistent

$$x = \frac{v^2}{2a}$$

Units:

$$\mathrm{m} = \frac{\mathrm{m}^2/\mathrm{s}^2}{\mathrm{m}/\mathrm{s}^2} = \frac{\mathrm{m}/\mathrm{s}^2}{1/\mathrm{s}^2} = \mathrm{m}$$

This lines up. The $\frac{1}{2}$ is ignored because it affects magnitude, not units.

b) Dimensions are not consistent

$$x = \frac{1}{2}at$$

Units:

$$\mathrm{m} \neq (\mathrm{m}/\mathrm{s}^2) * \mathrm{s} = \mathrm{m}/\mathrm{s}$$

This shows that the units are not consistent.

c) Dimensions are consistent

$$t = \sqrt{\frac{2x}{a}}$$

Units:

$$\mathrm{s} = \sqrt{\frac{\mathrm{m}}{\mathrm{m}/\mathrm{s}^2}} = \sqrt{\frac{\mathrm{m} * \mathrm{s}^2}{\mathrm{m}}} = \sqrt{\mathrm{s}^2} = \mathrm{s}$$

This lines up. The 2 is ignored because it affects magnitude, not units.

3. For partial cans, \$56.68. For full cans only, \$73.80

We first convert the units of the dimensions of the room from feet by feet by feet to meter by meter by meter, using a conversion rate of 1 ft = 0.3048 m. Next, we create a formula for the sum of the area of all four walls.

$$A = 2 \cdot (l \text{ m} \cdot h \text{ m} + w \text{ m} \cdot h \text{ m}) = 2h \cdot (l + w) \text{ m}^2$$

With that, we divide that by the price per square meter, represented by the variable r for ratio.

$$c = A \text{ m}^2 \cdot r \ \frac{\$}{\text{m}^2} = 2h \cdot (l + w) \text{ m}^2 \cdot r \ \frac{\$}{\text{m}^2} = \$ \ 2h \cdot (l + w) \cdot r$$

Substituting in values, we can find the solution.

$$c = \$ \ 2h \cdot (l + w) \cdot r = \$ \ 2 \cdot (8 \cdot 3.048) \cdot ((13 + 18) \cdot 3.048) \cdot \frac{24.60}{20} = \boxed{\$56.68}$$

Given that we are evaluating the number of cans, and it would be difficult to buy a fraction of a can, we round this up the the nearest multiple of \$24.60 above our current value. \$56.68 is greater than 2 * \$24.60 = \$49.20 , but less than 3 * \$24.60 = \$73.80. This leaves the realistic solution as $\boxed{\$73.80}$

4. 4.102 hr $\approx$ 4 hr 6 min

Assuming Car A and Car B have constant velocity, we first calculate the number of laps.

$$\frac{300\text{km}}{5\text{km/lap}} = \frac{\text{Total distance}}{\text{Lap length}} = 60 \text{ lap}$$

Next, we calculate the velocity of each car and related values like a reciprocal that will come into relevance.

$$\begin{aligned} v_A = \frac{60 \text{ lap}}{4 \text{ hr}} &= 15\frac{\text{lap}}{\text{hr}} \\ v_B &= \frac{58.5 \text{ lap}}{4 \text{ hr}} \\ \frac{1}{v_B} &= \frac{4 \text{ hr}}{58.5 \text{ lap}} \end{aligned}$$

Then, we calculate the time.

$$\begin{aligned} t_B &= D \cdot \frac{1}{v_B} \\ &= (60 \text{ lap}) \cdot \left(\frac{4}{58.5} \frac{\text{hr}}{\text{lap}}\right) \\ &= \frac{240}{58.5}\text{hr} \approx \boxed{4.102 \text{ hr} \approx 4 \text{ hr } 6 \text{ min}} \end{aligned}$$

5. 58.386 km/h

Converting 3 days, 10 hours, 40 minutes to hours, we get $\frac{247}{3} = 82.\bar{3}$ hours.
Multiplied by 65.5 km/h, we get $\frac{32357}{6}$ km $= 5392.8\bar{3}$ km, the total distance of the trip.
The time that the Queen Mary took is $82.\bar{3}$ hours plus 10 hours, 2 minutes, or $10 + \frac{1}{30}$ hours, or $\frac{301}{30}$ hours, totalling $\frac{2771}{30}$ hours.
Dividing the distance by the time, we get:

$$\frac{32357}{6}\text{km} \cdot \frac{30}{2771}\frac{1}{\text{h}} = \boxed{\frac{161785}{2771} \approx 58.386\frac{\text{km}}{\text{h}}}$$

6.

$a)$ 5 m/s $\quad b)$ 0 m/s $\quad c)$ -10 m/s $\quad d)$ -5 m/s $\quad e)$ 0 m/s

7. a) -2.547 m/s; b) 0.526 m/s

The average velocity would be the change in distance over the change in time.

$$\frac{x_2 - x_1}{t_2 - t_1} = \frac{-5.1 \text{ m} - 7 \text{ m}}{7 \text{ s} - 2.25 \text{ s}}$$
$$= -\frac{12.1 \text{ m}}{4.75 \text{ s}} \approx \boxed{-2.547 \text{ m/s}}$$

t	x	v
2.25s	7.00m	3.5 m/s
7.00s	-5.1m	6 m/s

Table of position and velocity at time t

The average acceleration would be the change in velocity over the change in time.

$$\frac{v_2 - v_1}{t_2 - t_1} = \frac{6 \text{ m/s} - 3.5 \text{ m}}{7 \text{ s} - 2.25 \text{ s}} = -\frac{2.5 \text{ m/s}}{4.75 \text{ s}} \approx \boxed{0.526 \text{ m/s}^2}$$

8. a) -0.640 m/s; b) -0.823 m/s; c) 0.274 m/s

a/ x(t) = 4.5 * $e^{-0.3t}$ m
x(2) = 4.5 * $e^{-0.6}$ m
x(3) = 4.5 * $e^{-0.9}$ m
$\frac{x(3) \text{ m} - x(2) \text{ m}}{3 \text{ s} - 2 \text{ s}} = \frac{4.5(e^{-0.9} - e^{-0.6})\text{m}}{1 \text{ s}} \approx \boxed{4.5 * (-0.142) \text{ m/s} \approx -0.640 \text{ m/s}}$

b/ x'(t) = -1.5 * $e^{-0.3t}$ m/s
x'(2) = -1.5 * $e^{-0.6}$ m/s ≈ -1.5 * 0.549 m/s ≈ $\boxed{-0.823 \text{ m/s}}$

c/ x"(t) = 0.5 * $e^{-0.3t}$ m/s^2
x"(2) = 0.5 * $e^{-0.6}$ m/s^2 ≈ 0.5 * 0.549 m/s^2 ≈ $\boxed{0.274 \text{ m/s}^2}$