

# Homework #5

PHYS 4D: Modern Physics

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## 1 Questions

### 1.1 Question 1

Give the operators corresponding to the momentum and energy of a particle.

#### 1.1.1 Answer

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}; \hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \quad (1)$$

### 1.2 Question 2

Give the general form of an eigenvalue equation.

#### 1.2.1 Answer

For operator  $\hat{A}$ , eigenfunction  $\psi$ , and eigenvalue  $A$ .

$$\hat{A}\psi = A\psi \quad (2)$$

### 1.3 Question 3

What significance do the eigenvalues have?

#### 1.3.1 Answer

The eigenvalue is the formula for or value of the component being searched for within the bounds of the eigenfunction.

## 1.4 Question 4

What significance do the eigenfunctions have?

### 1.4.1 Answer

The eigenfunctions serve as the conditions governing the system.

## 1.5 Question 5

Write down the momentum eigenvalue equation.

### 1.5.1 Answer

This is a *solved* momentum eigenvalue equation.

$$-i\hbar \frac{\partial}{\partial x} \psi = \hbar k \psi \quad (3)$$

## 1.6 Question 6

Is the function  $\cos(kx)$  an eigenfunction of the momentum operator?

### 1.6.1 Answer

No. The first derivative of  $\cos(kx)$  is  $-k \sin(kx)$ , which does not contain  $\cos(kx)$  to the point that the sets of terms in the two equations are practically disjoint.

## 1.7 Question 7

Is the function  $\cos(kx)$  an eigenfunction of the operator corresponding to the kinetic energy?

### 1.7.1 Answer

Yes. The energy operator involves a second derivative. Taking the second derivative of  $\cos(kx)$ , we get  $-k^2 \cos(kx)$ . This does contain  $\cos(kx)$ , so it is an eigenfunction.

## 1.8 Question 8

An electron is described by the wave function  $\psi(x) = Ae^{i\alpha x}$ , where  $\alpha$  denotes the Greek letter alpha. What is the momentum of the electron?

### 1.8.1 Answer

Use the equation with the momentum operator.

$$\hat{p}\psi = p\psi \quad (4)$$

Plug in the momentum operator  $\hat{p} = -i\hbar \frac{\partial}{\partial x}$  and  $\psi = Ae^{i\alpha x}$ .

$$\hat{p}\psi = -i\hbar \frac{\partial}{\partial x} Ae^{i\alpha x} = \hbar\alpha Ae^{i\alpha x} \quad (5)$$

This does contain  $\psi$ , so we can divide that out of the equation.

$$p\psi = \hbar\alpha Ae^{i\alpha x} \quad (6)$$

$$\boxed{p = \hbar\alpha} \quad (7)$$

## 1.9 Question 9

How would you determine whether or not a particular wave function represented a state of the system having a definite value of the momentum?

### 1.9.1 Answer

Use the eigenfunction equation for the momentum ( $\hat{p}\psi$ ). If the result is a multiple of  $\psi$ , then the wave function did represent a state of the system with a definitive momentum value.

## 1.10 Question 10

Calculate the dot product of the following two vectors.

$$A = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}; B = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \end{pmatrix} \quad (8)$$

### 1.10.1 Answer

$$A \cdot B = 4 * 1 + 3 * 1 + 2 * 2 + 1 * 2 \quad (9)$$

$$= 4 + 6 + 4 + 2 = \boxed{16} \quad (10)$$

### 1.11 Question 11

For the two vectors, A and B, given in the preceding question, evaluate item by item  $BA$  and from there  $A \cdot BA$ .

#### 1.11.1 Answer

$$BA = \begin{pmatrix} 4 * 1 \\ 3 * 1 \\ 2 * 2 \\ 1 * 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 4 \\ 2 \end{pmatrix} \quad A \cdot BA = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ 4 \\ 2 \end{pmatrix} = 16 + 9 + 8 + 2 = \boxed{35} \quad (11)$$

### 1.12 Question 12

Suppose a function  $f(x)$  is defined in an interval between 0 and 10 and an equally spaced grid is created by the command,

`x = linspace(0.0, 10.0, 10)` and suppose the values of the function at the two Gauss quadrature points,  $\xi_{i1}$  and  $\xi_{i2}$ , within each interval are known. Write down a formula for determining the value of the following integral in terms of the values of the function at the Gauss points.

$$\int_0^{10} f(x) dx \quad (12)$$

### 1.13 Question 13

Suppose a differential equation is solved using spline collocation on a grid created by the command `x = linspace(0.0, 10.0, 10)`. How many rows and columns do the A, B, and C matrices have?

### 1.14 Question 14

Sketch the wave function for an electron incident upon a potential step when the energy  $E$  of the electron is greater than the step height  $V_0$ .

### 1.15 Question 15

Sketch the wave function for an electron incident upon a potential step when the energy  $E$  of the electron is less than the step height  $V_0$ .

### 1.16 Question 16

Is the wavelength of a particle that has passed over a potential barrier greater than or less than the wavelength of the incident particle?

### 1.17 Question 17

Sketch the wave function of an electron which tunnels through a potential barrier located between 0 and L.

### 1.18 Question 18

Write down the Heisenberg uncertainty relations for the position and the momentum and for the time and the energy.

### 1.19 Question 19

Use the Heisenberg relation for the time and the energy to describe how the energy profile of an excited atomic state depends upon the lifetime of the state.

### 1.20 Question 20

How would the wave function  $\psi(x)$  of a particle change as the width of the Fourier transform increases?

### 1.21 Question 21

Suppose that a particle localized between a and b is described by the wave function  $\psi(x)$ . Write down an equation for the average value of the kinetic energy of the particle.