

Worksheet #12

PHYS 4C: Waves and Thermodynamics

Donald Aingworth IV

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1 Problem 1

Without looking anything up, determine what a light year is in meters.

1.1 Solution

I assume we know the speed of light to be $c = 2.998 \times 10^8 \text{ m/s}$. We first find the conversion factor of seconds per year.

$$1 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ yr}}{365.25 \text{ day}} = \frac{1 \text{ yr}}{31557600} \quad (1)$$

Multiply the speed of light by the reciprocal of the conversion factor to find the distance light travels in a year.

$$c = 2.998 \times 10^8 \text{ m/s} \times 31557600 \text{ s/yr} = 9.46 \times 10^{15} \text{ m/yr} \quad (2)$$

Multiply this by 1 year to find that one lightyear is equal to $9.46 \times 10^{15} \text{ m}$

2 Problem 2

Unpolarized light travelling in the z-direction is incident upon three polaroid filters with pass directions given by $+35^\circ$, -40° , and $+25^\circ$ counter-clockwise from $+x$. Determine the fraction of the original light which passes through the three filters.

2.1 Solution

Divide it into four parts with four intensities for the light: initially (I_0), then after passing through the first (I_1), second (I_2), and third (I_3) filters. First, the the first filter will filter out half the light's intensity.

$$I_1 = \frac{1}{2}I_0 \quad (3)$$

Next, the second filter is $35^\circ - (-40^\circ) = 75^\circ$ away from the first polarization direction. This can be used to find I_2 .

$$I_2 = I_1 \cos^2(75^\circ) = \frac{1}{2} \cos^2(75^\circ) I_0 \quad (4)$$

Last, the third filter is $-40^\circ - 25^\circ = -65^\circ$ away from the first polarization direction. This can be used to find I_3 .

$$I_3 = I_2 \cos^2(-65^\circ) = \frac{1}{2} \cos^2(-65^\circ) \cos^2(75^\circ) I_0 \quad (5)$$

$$= 0.00598 I_0 \quad (6)$$

This tells is that the amount that passes through is 0.00598.

3 Problem 3

(10 points) An electromagnetic wave has an electric field amplitude with a magnitude of $E_m = 12\text{V/m}$.

- (1 point) Calculate the magnetic field amplitude.
- (3 points) Calculate the (time averaged) energy density, intensity, momentum density (magnitude), and momentum current density of this wave.
- (3 points) If this light is normally incident upon a 1.0m^2 surface, determine the rate of energy absorption and force acting on this surface. Answer this question for both a perfectly reflecting surface and a perfectly absorbing surface.
- (3 points) Assume that this electromagnetic wave is located at the point $(1000\text{ m}, 0, 0)$ and is generated by a dipole antenna located at $(0, 0, 0)$ and oriented along the z-axis. If the oscillation frequency is 106 Hz, determine the electric dipole amplitude (qz_m) of this antenna.

3.1 Solution (a)

The magnitude of the magnetic field is calculatable from the magnitude of the electric field and the speed of light in a vacuum.

$$\frac{E_m}{M_m} = c \rightarrow M_m = \frac{E_m}{c} = \frac{12 \text{ V/m}}{2.998 \text{ m/s}} = \boxed{4.003 \times 10^{-8} \text{ T}} \quad (7)$$

3.2 Solution (b)

Energy density first.

$$\langle u \rangle = \frac{1}{2} \varepsilon_0 E_m^2 = \frac{1}{2} * 8.854 \times 10^{-12} \text{ F/m} (12 \text{ V/m})^2 \quad (8)$$

$$= \boxed{6.327 \times 10^{-10} \text{ J/m}^3} \quad (9)$$

Next the intensity.

$$I = \frac{1}{2} c \varepsilon_0 E_m = c \langle u \rangle = \boxed{0.191 \text{ W/m}^2} \quad (10)$$

Third the momentum density.

$$g = \frac{\langle u \rangle}{c} = \frac{6.327 \times 10^{-10} \text{ J/m}^3}{2.998 \times 10^8 \text{ m/s}} \quad (11)$$

$$= \boxed{2.1104 \times 10^{-18} \text{ W/m}^4} \quad (12)$$

Last the momentum current density. Suppose that the wave is traveling in only one direction, the $+x$ direction.

$$\sigma \underset{\sim}{=} u \hat{r} \otimes \hat{r} = u \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} u & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (13)$$

$$= \boxed{\begin{bmatrix} 6.327 \times 10^{-10} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ N/m}^2} \quad (14)$$

3.3 Solution (c)

3.3.1 Perfectly Reflecting

A perfectly reflecting surface would absorb none of the energy. This would merely redirect the light, resulting in a force equivalent to twice the energy density times the area affected.

$$F = 2 \langle u \rangle A = 2 * 6.327 \times 10^{-10} \text{ N/m}^2 * 1.0 \text{ m}^2 = [1.2654 \times 10^{-9} \text{ N}] \quad (15)$$

3.3.2 Perfectly Absorbing

The perfectly absorbing surface absorbs all energy. Due to that, we can find the rate of absorption by multiplying the intensity by the area covered.

$$\frac{dE}{dt} = IA = 0.191 \text{ W/m}^2 * 1 \text{ m}^2 = [0.191 \text{ W}] \quad (16)$$

The force taken would be equal to the energy density times the area.

$$F = \langle u \rangle A = 6.327 \times 10^{-10} \text{ N/m}^2 * 1.0 \text{ m}^2 = [6.327 \times 10^{-10} \text{ N}] \quad (17)$$

3.4 Solution (d)

qz_m can be related to the electric field by a sinusoidal equation.

$$\vec{E}(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \times \frac{qz_m\omega^2}{c^2 r} \sin(\theta) \cos(\vec{k} \cdot \vec{r} - \omega t) \quad (18)$$

For the time being, let's set $\sin(\theta) \cos(\vec{k} \cdot \vec{r} - \omega t) = 1$. Due to that, the value of the rest would be equal to the magnitude of the electric field (12 V/m in this case). We can use that.

We can also use the frequency to find the value of ω .

$$\omega = f * 2\pi = 212\pi \text{ rad/s} \quad (19)$$

$$E_m = \frac{1}{4\pi\epsilon_0} * \frac{qz_m\omega^2}{c^2 r} \quad (20)$$

$$qz_m = \frac{4\pi\epsilon_0 * E_m * c^2 r}{\omega^2} \quad (21)$$

$$= \frac{4\pi\epsilon_0 * 12 \text{ V/m} * (2.998 \times 10^8 \text{ m/s})^2 * 1000 \text{ m}}{(212\pi \text{ rad/s})^2} \quad (22)$$

$$= [270412 \text{ C} \cdot \text{m}] \quad (23)$$