This document has no purpose.

It is noted that there is a tension T in the string. This tension would be cumulative, held up over the length of the string. That means that each miniscule part of the string (call it $dx$) would have a tension acting on it (call it $T$) from each side. The difference between the two sides would be the angle at which it acts. Suppose that a tiny piece of the string $dx$ experiences a force from each side, one from the left and one from the right. The forces would have some nearly identical (albeit not identical) magnitude ($T$), but different directions ($\hat{T}_1$ and $\hat{T}_2$, respectively). Each of these would exert a force on the piece of string. We can apply Newton's second law to this as well.

$$\vec{F} = \hat{T}_1\,T + \hat{T}_2\,T = \vec{a}\,dm \tag{1}$$

We do not have a known mass, but we do have a known density of this string. We can get the mass of a tiny part of the string from that, which we can substitute into the above equation.

$$dm = \mu\,dx \tag{2}$$

$$\left(\hat{T}_1 + \hat{T}_2\right)\,T = \vec{a}\mu\,dx \tag{3}$$

The acceleration can also be expressed in terms of a second derivative.

$$\vec{a} = \frac{\partial^2 \vec{y}}{\partial t^2} \tag{4}$$

$$\left(\hat{T}_1 + \hat{T}_2\right)\,T = \frac{\partial^2 \vec{y}}{\partial t^2}\mu\,dx \tag{5}$$

This being a 2D wave, we can separate the tension vectors into both $x$ and $y$ components. These would be dependant on their separate vectors' angles with the horizontal. Since both are unit vectors, we only need to consider the sines and cosines, so no magnitudes.

$$\hat{T}_1 = \begin{pmatrix} \cos(\theta_1) \\ \sin(\theta_1) \end{pmatrix} \tag{6}$$

$$\hat{T}_2 = \begin{pmatrix} \cos(\theta_2) \\ \sin(\theta_2) \end{pmatrix} \tag{7}$$

$$\hat{T}_1 + \hat{T}_2 = \begin{pmatrix} \cos(\theta_1) \\ \sin(\theta_1) \end{pmatrix} + \begin{pmatrix} \cos(\theta_2) \\ \sin(\theta_2) \end{pmatrix} = \begin{pmatrix} \cos(\theta_1) + \cos(\theta_2) \\ \sin(\theta_1) + \sin(\theta_2) \end{pmatrix} \tag{8}$$

In the case of strings, the values of $\theta_1$ and $\theta_2$ are immeasurably small, so we can estimate and approximate the values of their sines and cosines.

$$\cos(\theta) \to 1; \sin(\theta) \to \theta \tag{9}$$

Bear in mind as well that the $x$-values of the tension forces are in opposite directions, so if ever these cosines are added together, we can set them both to be equal to 1 and cancel them out. Furthermore, the values of $\theta$ are approximately equal to the difference in $y$-value of the piece of string.

$$\hat{T}_1 + \hat{T}_2 = \begin{pmatrix} 1 - 1 \\ \theta_1 + \theta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ dy \end{pmatrix} \tag{10}$$

The values of $T$ also do change. The difference in tension would be directly proportional to the slope of the line at that specific point.

$$T' = \frac{\mathrm{d}y}{\mathrm{d}x} T \tag{11}$$