Homework #5

PHYS 4D: Modern Physics

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1 Questions

1.1 Question 1

Give the operators corresponding to the momentum and energy of a particle.

1.1.1 Answer

$$\hat{p} = -i\hbar\frac{\partial}{\partial x}; \hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial t^2} + V(x,t) \tag{1}$$

1.2 Question 2

Give the general form of an eigenvalue equation.

1.2.1 Answer

For operator $\hat{A}$, eigenfunction ψ, and eignenvalue A.

$$\hat{A}\psi = A\psi \tag{2}$$

1.3 Question 3

What significance do the eigenvalues have?

1.3.1 Answer

The eigenvalue is the formula for or value of the component being searched for within the bounds of the eigenfunction.

1.4 Question 4

What significance do the eigenfunctions have?

1.4.1 Answer

The eigenfunctions serve as the conditions governing the system.

1.5 Question 5

Write down the momentum eigenvalue equation.

1.5.1 Answer

Ths is a *solved* momentum eigenvalue equation.

$$-i\hbar\frac{\partial}{\partial x}\psi = \hbar k\psi \tag{3}$$

1.6 Question 6

Is the function $\cos(kx)$ an eigenfunction of the momentum operator?

1.6.1 Answer

No. The first derivative of $\cos(kx)$ is $-k\sin(kx)$, which does not contain $\cos(kx)$ to the point that the sets of terms in the two equations are practically disjoint.

1.7 Question 7

Is the function $\cos(kx)$ an eigenfunction of the operator corresponding to the kinetic energy?

1.7.1 Answer

Yes. The energy operator involves a second derivative. Taking the second derivative of $\cos(kx)$, we get $-k^2\cos(kx)$. This does contain $\cos(kx)$, so it is an eigenfunction.

1.8 Question 8

An electron is described by the wave function $\psi(x) = Ae^{i\alpha x}$, where α denotes the Greek letter alpha. What is the momentum of the electron?

1.8.1 Answer

Use the equation with the momentum operator.

$$\hat{p}\psi = p\psi \tag{4}$$

Plug in the momentum operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ and $\psi = Ae^{i\alpha x}$.

$$\hat{p}\psi = -i\hbar\frac{\partial}{\partial x}Ae^{i\alpha x} = \hbar\alpha\, Ae^{i\alpha x} \tag{5}$$

This does contain ψ, so we can divide that out of the equation.

$$p\psi = \hbar\alpha\, Ae^{i\alpha x} \tag{6}$$

$$\boxed{p = \hbar\alpha} \tag{7}$$

1.9 Question 9

How would you determine whether or not a particular wave function represented a state of the system having a definite value of the momentum?

1.9.1 Answer

Use the eigenfunction equation for the momentum ($\hat{p}\psi$). If the result is a multiplel of ψ, then the wave function did represent a state of the system with a definitive momentum value.

1.10 Question 10

Calculate the dot product of the following two vectors.

$$A = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}; B = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \end{pmatrix} \tag{8}$$

1.10.1 Answer

$$A \cdot B = 4 * 1 + 3 * 1 + 2 * 2 + 1 * 2 \tag{9}$$

$$= 4 + 6 + 4 + 2 = \boxed{16} \tag{10}$$

1.11 Question 11

For the two vectors, A and B, given in the preceding question, evaluate item by item BA and from there $A \cdot BA$.

1.11.1 Answer

$$BA = \begin{pmatrix} 4*1 \\ 3*1 \\ 2*2 \\ 1*2 \end{pmatrix} = \boxed{\begin{pmatrix} 4 \\ 3 \\ 4 \\ 2 \end{pmatrix}} \quad A \cdot BA = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ 4 \\ 2 \end{pmatrix} = 16 + 9 + 8 + 2 = \boxed{35} \tag{11}$$

1.12 Question 12

Suppose a function f(x) is defined in an interval between 0 and 10 and an equally spaced grid is created by the command,
`x = linspace(0.0,10.0,10)` and suppose the values of the function at the two Gauss quadrature points, ξ_{i1} and ξ_{i2}, within each interval are known. Write down a formula for determining the value of the following integral in terms of the values of the function at the Gauss points.

$$\int_0^{10} f(x)\, dx \tag{12}$$

1.13 Question 13

Suppose a differential equation is solved using spline collocation on a grid created by the command `x = linspace(0.0,10.0,10)`. How many rows and columns do the A, B, and C matrices have?

1.14 Question 14

Sketch the wave function for an electron incident upon a potential step when the energy E of the electron is greater than the step height V_0.

1.15 Question 15

Sketch the wave function for an electron incident upon a potential step when the energy E of the electron is less than the step height V_0.

1.16 Question 16

Is the wavelength of a particle that has passed over a potential barrier greater than or less than the wavelength of the incident particle?

1.17 Question 17

Sketch the wave function of an electron which tunnels through a potential barrier located between 0 and L.

1.18 Question 18

Write down the Heisenberg uncertainty relations for the position and the momentum and for the time and the energy.

1.19 Question 19

Use the Heisenberg relation for the time and the energy to describe how the energy profile of an excited atomic state depends upon the lifetime of the state.

1.20 Question 20

How would the wave function $\psi(x)$ of a particle change as the width of the Fourier transform increases?

1.21 Question 21

Suppose that a particle localized between a and b is described by the wave function $\psi(x)$. Write down an equation for the average value of the kinetic energy of the particle.

2 Problem 1

Evaluate the product of the momentum operator and each of the two functions

$$\phi_1(x) = \cos\, kx \tag{13}$$

$$\phi_2(x) = \sin\, kx \tag{14}$$

Are these functions eigenfunctions of the momentum operator?

2.1 Solution (1)

The momentum operator is $\hat{p} = -i\hbar\dfrac{\partial}{\partial x}$, which we can apply here.

$$\hat{p}\,\phi_1(x) = -i\hbar\frac{\partial}{\partial x}(\cos\,kx) = i\hbar k \sin\,kx \tag{15}$$

This does not contain ϕ_1, so it is not an eigenfunction of $\hat{p}$.

2.2 Solution (2)

The momentum operator is $\hat{p} = -i\hbar\dfrac{\partial}{\partial x}$, which we can apply here.

$$\hat{p}\,\phi_2(x) = -i\hbar\frac{\partial}{\partial x}(\sin\,kx) = -i\hbar k \cos\,kx \tag{16}$$

This does not contain ϕ_2, so it is not an eigenfunction of $\hat{p}$.

3 Problem 2

Find a linear combination of the functions, $\phi_1(x)$ and $\phi_2(x)$, defined in the previous problem, which is an eigenfunction of the momentum operator.

3.1 Solution

We know that Euler's identity is an eigenfunction of the momentum operator.

$$e^{ix} = \cos(x) + i\sin(x) \tag{17}$$

We can change x into kx.

$$e^{ikx} = \cos(kx) + i\sin(kx) \tag{18}$$

This is a linear combination of $\phi_1(x)$ and $\phi_2(x)$, just a complex one. We can check if it is an eigenfunction of the momentum operator.

$$\hat{p}\,\psi(x) = -i\hbar\frac{\partial}{\partial x}(\cos(kx) + i\sin(kx)) = -i\hbar\frac{\partial}{\partial x}(e^{ikx}) \tag{19}$$

$$= -i\hbar(ike^{ikx}) = \hbar k e^{ikx} = \hbar k \psi(x) \quad \text{TENA} \tag{20}$$

4 Problem 8

For the scattering problem illustrated in Fig. 1, a particle is incident upon a potential step with the energy of the incident particle being less than the step height. Using the notation for this problem given in the text, derive expressions the ratios A/C and B/C and show that $R = 1$.

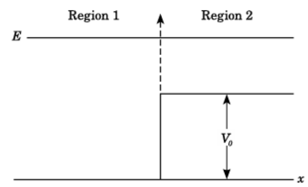

Figure 1: Potential Step

4.1 Solution

5 Problem 10

Using the Heisenberg uncertainty principle, estimate the momentum of an electron confined to a 1.0 nm well.

5.1 Solution