PHYS 4 Exam 6 Cheat Sheet (with LaTeX)

Angular Kinematics

$$\theta = \frac{S}{r}; \omega = \frac{d\theta}{dt}; \alpha = \frac{d^2\theta}{dt^2}; (1)\ \omega(t) = \omega_0 + \alpha t$$
$$(2)\ \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2; (3)\ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$
$$v_t = \omega r; a_t = \alpha r; a_c = \omega r^2; T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Electric Fields and Forces

$$e = 1.602 \times 10^{-19}\mathrm{C}; \varepsilon_0 = 8.85 \times 10^{-12}\frac{\mathrm{C}^2}{\mathrm{Nm}^2}$$
$$k = 8.99 \times 10^9 \frac{\mathrm{Nm}^2}{\mathrm{C}^2} = \frac{1}{4\pi\epsilon_0}$$
$$\vec{F} = \frac{kq_1q_2}{r^2}\hat{r} = \frac{kq_1q_2}{r^3}\vec{r}; \vec{E} = \frac{kq}{r^2}\hat{r} = \frac{kq}{r^3}\vec{r}; F = qE$$

In a diagram, the direction of an electric field is represented by the direction of its arrows, while the strength of the field is represented by the proxmity of the lines.

$$\lambda = \frac{Q}{r}; \sigma = \frac{Q}{A}; \rho = \frac{Q}{V}$$
$$E = \int dE = \int \frac{k\ dq}{r^3}\vec{r} = \int \frac{k\lambda}{r^3}\vec{r}dr$$
$$\vec{E}_{ring}(z) = \frac{kqz}{(z^2+R^2)^{3/2}}\hat{k}$$

For a rod of length L, measured at a distance d from the close end from the rod of charge Q.

$$\vec{E}_{axis} = -\frac{kQ}{d(d-L)}\hat{i}$$

For a rod of length L, measured perpendicular to the rod at a distance d from the close end from the rod of charge Q.

$$\vec{E} = k\lambda\left[\frac{1}{z} - \frac{1}{L^2+z^2}\right]\hat{i} + \frac{k\lambda L}{z\sqrt{L^2+z^2}}\hat{j}$$
$$V = k\lambda \ln\left(\frac{L+\sqrt{L^2+d^2}}{d}\right)$$

$$\vec{E}_{arc} = \frac{k\lambda}{r}\begin{pmatrix} 2\sin(\frac{\theta}{2}) \\ 0 \end{pmatrix}$$
$$\vec{E}_{disc} = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

For a spherical shell of radius R.

$$\vec{E} = \begin{cases} 0 \text{ if } r < R \\ \frac{kq}{r^2}\hat{r} \text{ otherwise} \end{cases}$$

If $r < R$, $\Delta V = 0$. If $r \to \infty$, $V = 0$.
Solid sphere of radius R.

$$\vec{E} = \begin{cases} \frac{kqr}{R^3} \text{ if } r < R \\ \frac{kq}{r^2}\hat{r} \text{ otherwise} \end{cases}$$

Gauss' Law

$$\Phi = \frac{q_{enc}}{\varepsilon_0}; \Phi = \oint \vec{E} \cdot d\vec{A}$$

A must be a Gaussian surface. If $\vec{E}$ is constant on the surface, it can be simplified to $\Phi = E * A$. Conductors in an electric field have $\vec{E} = 0$ inside. Electrons move to ensure this. Inside, $\Phi = 0$.

Electrical Potential Difference

Path independent. For $\vec{E}(x, y, z)$:

$$\Delta V = \frac{\Delta U}{q} = -\int_i^f \vec{E} \cdot d\vec{x} = \int_i^f dV$$

Electric field lines go from more positive to more negative voltage.
Equipotential surface (ES): Surface with same V.
Conductors have equipotential volumes and $\vec{E} = 0$

$$V = \frac{kq}{r} = \int \frac{k\, dQ}{r}; \vec{E} = -\nabla V$$

Capacitance (C)

Relationship between charged separated and potential difference. $Q = C * \Delta V$ To find capacitance:
1. Draw a picture
2. Determine direction of $\vec{E}$
3. Determine $\vec{E}$ (Gauss' and determined distributions help), then $\Delta V = -\int \vec{E} \cdot d\vec{s}$
4. Calculate C with $C = \frac{Q}{\Delta V}$

For parallel plates, $C = \frac{A\varepsilon_0}{d}$.
For cylindrical capacitor length L, $C = \frac{2\pi L\varepsilon_0}{\ln(b/a)}$.
Concentric spheres of radii a and b, $C = 4\pi\varepsilon_0\frac{ab}{b-a}$.
Isolated sphere of radius R, $C = 4\pi\varepsilon_0 R$.
Since $W = q\Delta V$, $\Delta U = \frac{1}{2}C{*}\Delta V^2 = \frac{q^2}{2C}$ (Electric Potential Energy)
$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{U}{Vol}$
A dielectric/material is in an electric field has a dielectric onstant κ. In it, ε_0 is replaced with $\kappa\varepsilon_0$. κ of metals is considered ∞. $\kappa(vaccum) = 1$
If you put a dielectric in a capacitor, treat it like a network of capacitors in a creative alignment.
Add a dielectric to charged capacitor:
$Q_\kappa = Q_0; V_\kappa < V_0; C_\kappa > C_0; U_\kappa < U_0$
Add a dielectric to battery-connected capacitor:
$V_\kappa = V_0; Q_\kappa > Q_0; C_\kappa > C_0; U_\kappa > U_0$

Current

$$I = \frac{dq}{dt}$$

Ohm's Law: $V = IR$
Junction rule: For any point on a circuit, $I_{in} = I_{out}$
Stored charge at junction slows down I_{in} & speeds up I_{out}

Resistors

For a resistor of resistance R, length L, cross section A, resistivity ρ.

$$R = \frac{V}{I} = \rho\frac{L}{A}; P = IV = I^2R = \frac{V^2}{R}; V = \frac{dW}{dq}$$

Current Density

For a cross-section $\vec{A}, dI = \vec{J} \cdot d\vec{A}$

$$\vec{J} = e * \vec{v}_d * n = \frac{\vec{E}}{\rho}$$

Circuits

Batteries keep ΔV constant

Long end of battery diagram is + side

	Series	Parallel
Capacitor	$\frac{1}{C} = \sum \frac{1}{C_i}$	$C = \sum C_i$
Resistor	$R = \sum R_i$	$\frac{1}{R} = \sum \frac{1}{R_i}$

RL circuit, current approaches equilibrium $\frac{\mathcal{E}}{R}$ (growth & decay resp).

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}); \tau = \frac{L}{R}; I = I_0 e^{-t/\tau}$$

Charge of charging capacitor (growth & decay):

$$q = C\mathcal{E}(1 - e^{-t/\tau}); q = q_0 e^{-t/RC}$$

Kirchoff's Loop Rule: The total voltage throughout a circuit must sum to zero. Ex: $\mathcal{E} = L\frac{dI}{dt} + IR$.

For an inductor, energy stored $U_B = \frac{1}{2}LI^2$.

Magnetism

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\vec{F} = I\vec{L} \times \vec{B}$$

↑ For electric field B, charged wire of length L. ↑

Hall effect: electrons traveling through a strip (length l) of conductor in a magnetic field B generate an internal capacitor. Charge Q, current I.

$$Q = \frac{BI}{Vl}$$

For conductor moving at v through B, area A, electron drift v_d.

$$V = vBd; v_d = \frac{I}{neA}$$

Free charge moving in magnetic field centers source at radius r.

$$qvB = \frac{mv^2}{r}; f = \frac{qB}{2\pi m}$$

Energy density in a magnetic field $u_B = \frac{B^2}{2\mu_0}$.

Current generates a magnetic field. Current rounding area A N times makes a magnetic dipole.

$$\tau = NI\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}$$

$$U(\theta) = -\vec{\mu} \cdot \vec{B}$$

A distance R from a straight wire, $B = \frac{\mu_0 I}{2\pi R}$. Along wire b from wire a: $B - \frac{\mu_0 I_a}{2\pi d}$.

At center of arc of angle ϕ of radius R, $B = \frac{\mu_0 I \phi}{4\pi R}$.

Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

Inside an ideal (infinite) sollenoid, $B = \mu_0 In = \mu_0 I\frac{N}{L}$.

Inside an ideal torroid, $B = \frac{\mu_0 IN}{2\pi R}$.

Along coil axis (coils create magnetic dipoles), $B = \frac{\mu_0 \vec{\mu}}{2\pi z^3}$.

Farraday's Law

In a conductor of area A, mag. field B.

$$\Phi_B = \int_S \vec{B} \cdot d\vec{A}; \mathcal{E} = \frac{d\Phi_B}{dt}$$

In a sollenoid, inductance $L = \frac{N\Phi_B}{I}$.

For length l, $\frac{L}{l} = \mu_0 n^2 A$.

Creates self-opposing emf of magnitude $\mathcal{E} = -L\frac{dI}{dt}$.

Electric Dipoles

$$\begin{aligned}\vec{E} &= \begin{cases} < 0 \text{ if } -\frac{d}{2} < z < \frac{d}{2} \\ > 0 \text{ otherwise} \end{cases} \\ &= \frac{2kQd}{z^3\left(1 - \frac{d^2}{4z^2}\right)^2}\hat{d}\end{aligned}$$

ESs are $\perp$ to $\vec{p}$. In an electric field:

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$W_{net} = \Delta K = -\Delta U$$

Maxwell's Equations

Gradient	**Integral**
$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$	$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$
$\nabla \cdot \vec{B} = 0$	$\oint \vec{E} \cdot d\vec{A} = 0$
$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\oint \vec{E} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$