Homework #4

PHYS 4D: Modern Physics

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1 Questions

1.1 Question 4

Denoting the wave function of a particle by $\psi(x)$, write down an expression for the probability that the particle will be found between a and b.

1.1.1 Solution

Use an integral and the relationship between $\psi(x)$ and probability. $\psi^\star(x)$ denotes the complex conjugate of $\psi(x)$.

$$P(a < x < b) = \int_a^b \psi^\star(x)\psi(x)\,dx \tag{1}$$

1.2 Question 5

Denoting the wave function of a particle by $\psi(x)$, write down an equation for the average value of x.

1.3 Question 6

Suppose that a particle, which is confined to move in one-dimension between 0 and L, is described by the wave function, $\psi(x) = Ax(L - x)$. What condition could be imposed upon the wave function $\psi(x)$ to determine the constant A?

1.4 Question 7

Suppose that a perfectly elastic ball were bouncing back and forth between two rigid walls with no gravity. Which of the variables, p, $|p|$, E, would have a constant value?

1.5 Question 8

Sketch the form of the wave functions corresponding to the three lowest energy levels of a particle confined to an infinite potential well.

1.6 Question 10

What is the value of the kinetic energy of a particle at the classical turning points of an oscillator?

1.7 Question 12

Suppose that a harmonic oscillator made a transition from the $n = 3$ to the $n = 2$ state. What would be the energy of the emitted photon?

1.8 Question 13

Describe in qualitative terms the form of the wave functions of the harmonic oscillator between the classical turning points?

1.9 Question 14

How does the form of the wave function of the harmonic oscillator change as x increase beyond the classical turning point.

1.10 Question 18

Describe the wave functions obtained by multiplying the stationary wave Ae^{ikx} by the function $e^{-i\omega t}$.

2 Problem 3

An electron in a 10 nm-wide infinite well makes a transition from the $n = 3$ to the $n = 2$ state emitting a photon. Calculate (a) the energy of the photon and (b) the wavelength of the light.

2.1 Solution (a)

The equation of the energy in a well is given in equation (2.17).

$$E = \frac{n^2 h^2}{8mL^2} \tag{2.17}$$

This can be used to calculate the change in energy.

$$\begin{aligned}
\Delta E = E_2 - E_3 &= \frac{2^2 h^2}{8mL^2} - \frac{3^2 h^2}{8mL^2} = (2^2 - 3^2)\frac{h^2}{8mL^2} && (2)\\
&= -5\frac{h^2}{8mL^2} = -\frac{5 * (6.626 \times 10^{-34})^2}{8(9.109 \times 10^{-31})(10 \times 10^{-9})^2} && (3)\\
&= -3.01 \times 10^{-21}\ \mathrm{J} && (4)
\end{aligned}$$

The energy of the photon would be the negative of this.

$$E_{\text{photon}} = -\Delta E = \boxed{3.01 \times 10^{-21}\ \mathrm{J}} \tag{5}$$

2.2 Solution (b)

Turn energy to wavelength.

$$\lambda = \frac{hc}{E} = \frac{1.986 \times 10^{-25}\ \mathrm{J \cdot m}}{3.01 \times 10^{-21}\ \mathrm{J}} = \boxed{65.9\,\mu\mathrm{m}} \tag{6}$$

3 Problem 4

Show by direct substitution that the wave function (7) satisfies Eq. (2.32) for the harmonic oscillator. Calculate the corresponding energy.

$$\psi(x) = Ae^{-\frac{m\omega x^2}{2\hbar}} \tag{7}$$

$$-\frac{\partial^2\psi}{\partial x^2} + \left(\frac{m\omega^2 x^2}{\hbar^2}\right)\psi = \left(\frac{2mE}{\hbar^2}\right)\psi \tag{2.32}$$

4 Problem 5

Determine the constant A in the preceding problem by requiring that the wave function be normalized. Hint: For an arbitrary value of the constant a, the integral that arises in doing this problem may be evaluated using equation (8).

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \tag{8}$$

5 Problem 6

A particle is described by the below wave function where A and a are constants.

$$\psi(x) = \begin{cases} Ax\,e^{-ax}, & x > 0 \\ 0, & x \leq 0 \end{cases} \tag{9}$$

(a) Sketch the wave function.

(b) Use the normalization condition to determine the constant A.

(c) Find the most probable position of the particle.

(d) Calculate the average value of the position of the particle.

6 Problem 7

(a) For a particle moving in the potential well shown in Fig. 2.7, write down the Schrödinger equations for the region where $0 \leq x \leq L$ and the region where $x \geq L$. (b) Give the general form of the solution in the two regions. (c)

Assuming that the potential is infinite at $x = 0$, impose boundary conditions that are natural for this problem and derive an equation that can be used to find the energy levels for the bound states.

7 Problem 8

Show that the wave function of a traveling wave (2.41) satisfies the time-dependent Schrödinger equation (2.47).

$$\psi(x,t) = Ae^{ikx} \cdot e^{-i\omega t} \tag{2.41}$$

$$\left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)\right]\psi(x,t) = i\hbar\frac{\partial \psi(x,t)}{\partial t} \tag{2.47}$$

8 Problem 9

Show how the wave function of the even states of a particle in an infinite well extending from $x = -L/2$ to $x = L/2$ evolve in time.

8.1 Solution

There exist three sectons we can use for the potential energy of the particle.

$$V(x) = \begin{cases} \infty & x > \frac{L}{2} \\ 0 & -\frac{L}{2} \leq x \leq \frac{L}{2} \\ \infty & x < -\frac{L}{2} \end{cases} \tag{10}$$

The central prong of this is what interests us. It gives us a version of the Schrödinger time-dependant Equation where $V = 0$.

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t} \tag{11}$$