Homework #2

PHYS 4D: Modern Physics

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1 Question 2

Which physical effect or experiment shows that light has a wave nature?

1.1 Solution

One experiment that demonstrates the wave nature of light is the double slit experiment. It results in multiple points of constructive and destructive interference, which occur for waves and not particles.

2 Question 3

Express the kinetic energy KE of a particle in terms of its momentum $\mathbf{p}$.

2.1 Solution

There are two answers to this. The first assumes we express it in terms of both momentum and mass.

$$KE = \boxed{\frac{\vec{p} \cdot \vec{p}}{2m}} \tag{1}$$

The second requires we assume momentum has a velocity component to it and uses an antiderivative or an integral.

$$KE = \frac{\mathrm{d}^{-1} p}{\mathrm{d}v^{-1}} = \int p(v)\, dv \tag{2}$$

3 Question 4

What would be the wavelength of a wave described by the function $u(x,t) = A\sin(2x/\mathrm{cm} - 10\mathrm{s})$?

3.1 Solution

There is a formula relating λ and k.

$$k = \frac{2\pi}{\lambda} \tag{3}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\,\mathrm{cm}^{-1}} = \boxed{\pi \times 10^2\,\mathrm{m}} \tag{4}$$

4 Question 5

What would be the frequency of a wave described by the function $u(x,t) = A\sin(2x/\mathrm{cm} - 10t/\mathrm{s})$?

4.1 Solution

$$\omega = 2\pi f \tag{5}$$

$$f = \frac{\omega}{2\pi} = \frac{10}{2\pi} = \boxed{\frac{5}{\pi}\,\mathrm{Hz}} \tag{6}$$

5 Question 6

What would be the phase velocity of a wave described by the function $u(x,t) = A\sin(2x/\mathrm{cm} - 10t/\mathrm{s})$?

5.1 Solution

The phase velocity is just the speed of the wave in this case.

$$v = \frac{\omega}{k} = \frac{10\,\mathrm{s}^{-1}}{2 \times 10^{-2}\,\mathrm{m}^{-1}} = \boxed{500\,\mathrm{m/s}} \tag{7}$$

6 Question 7

Write down the exponential function corresponding to a traveling wave with a wavelength of 10 cm and a frequency of 10 Hz.

6.1 Solution

The exponential function would have a format involving k and ω.

$$\psi(x,t) = \psi_0 e^{i(kx-\omega t)} = \psi_0 e^{i\left(\frac{2\pi}{\lambda}x - 2\pi f\, t\right)} \tag{8}$$

We have values of f and λ, whch we can substitute in.

$$\psi(x,t) = \psi_0 e^{i\left(\frac{2\pi}{0.1\,\mathrm{m}}x - 2\pi * 10\,\mathrm{Hz} * t\right)} \tag{9}$$

$$\boxed{\psi(x,t) = \psi_0 e^{i((20\pi\,\mathrm{m})\,x - (20\pi\,\mathrm{Hz})\,t)}} \tag{10}$$

7 Question 8

Write down a trigonometric function describing a stationary wave with a wavelength of 10 cm.

7.1 Solution

I'll be using sine.

$$\boxed{\psi(x,t) = \psi_0 \sin\left((20\pi\,\mathrm{m})\,x\right)} \tag{11}$$

8 Question 11

Which forms of electromagnetic radiation have a wavelength shorter than visible light?

8.1 Solution

Looking at the textbook, those with shorter wavelength than visible light are ultraviolet, x-rays, and gamma rays.

9 Question 12

Calculate the frequency of electromagnetic radiation having a wavelength of 10 nm.

9.1 Solution

Frequency is inversely proportional to wavelength.

$$c = \lambda f \tag{12}$$

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{10 \times 10^{-9}\,\mathrm{m}} = \boxed{2.998 \times 10^{16}\,\mathrm{Hz}} \tag{13}$$

10 Question 13

Write down a formula expressing the energy of a photon in terms of the frequency of light.

10.1 Solution

All credit goes to a German guy named Albert. f refers to Planck's constant.

$$E = hf \tag{14}$$

11 Question 14

Write down a formula expressing the energy of a photon in terms of the wavelength of light.

11.1 Solution

We have two equations known: one for the energy in terms of frequency, and the other relating frequency and wavelength.

$$E = hf \tag{15}$$

$$c = \lambda f \rightarrow f = \frac{c}{\lambda} \tag{16}$$

We can substitute the latter into the former.

$$\boxed{E = \frac{hc}{\lambda}} \tag{17}$$

12 Question 15

What is the energy of the photons for light with a wavelength of 0.1 nm?

12.1 Solution

Use the equation from Question 14.

$$E = \frac{hc}{\lambda} = \frac{6.62607015 \times 10^{-34} \cdot 2.998 \times 10^{8}}{10^{-10}} = \boxed{1.986 \times 10^{-15}\,\mathrm{J}} \tag{18}$$

13 Problem 1

Calculate the frequency of light having a wavelength $\lambda = 500\,\mathrm{nm}$.

13.1 Solution

Use the relationship between frequency and wavelength.

$$c = \lambda f \tag{19}$$

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^{8}}{500 \times 10^{-9}} = \boxed{5.996 \times 10^{14}\,\mathrm{Hz}} \tag{20}$$

14 Problem 2

Calculate the energy of the photons for light having a wavelength $\lambda = 500\,\mathrm{nm}$.

14.1 Solution

Use the relationhip between wavelength and energy.

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} * 2.998 \times 10^{8}}{500 \times 10^{-9}} = \boxed{3.973 \times 10^{-19}\,\mathrm{J}} \tag{21}$$

15 Problem 3

Suppose that a beam of light consists of photons having an energy of 5.4 eV. What is the wavelength of the light?

15.1 Solution

I will be using a new value of hc.

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV nm}}{5.4\,\text{eV}} = \boxed{229.6\,\text{nm}} \tag{22}$$

16 Problem 10

Using Euler's identity, Eq. (I.24) show that adding two waves given by $y_1 = Ae^{i(kx-\omega t)}$ and $y_2 = Ae^{i(kx+\omega t)}$ gives a new wave with time-dependent amplitude and position dependence $e^{i(kx)}$.

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{I.24}$$

16.1 Solution

Proof. Time to play the game! Take our two equations.

$$y_1 = Ae^{i(kx-\omega t)} \tag{23}$$

$$y_2 = Ae^{i(kx+\omega t)} \tag{24}$$

Add these two together in superposition.

$$y_\Sigma = y_1 + y_2 = Ae^{i(kx-\omega t)} + Ae^{i(kx+\omega t)} \tag{25}$$

To begin our manipulation, we can separate values of exponentials.

$$y_\Sigma = Ae^{i(kx)}e^{i(-\omega t)} + Ae^{i(kx)}e^{i(\omega t)} \tag{26}$$

Here, we combine like terms.

$$y_\Sigma = Ae^{i(kx)}\left(e^{i(-\omega t)} + e^{i(\omega t)}\right) \tag{27}$$

Using I.24, we can convert the values inside the parentheses to trigonometric values. We can then use trig identities and positive/negative functions to cancel out values.

$$\begin{aligned} y_\Sigma &= Ae^{i(kx)}\left(\cos(-\omega t)+\sin(-\omega t)+\cos(\omega t)+\sin(\omega t)\right) && (28)\\ &= Ae^{i(kx)}\left(\cos(\omega t)-\sin(\omega t)+\cos(\omega t)+\sin(\omega t)\right) && (29)\\ &= Ae^{i(kx)}\left(2\cos(\omega t)\right) = \boxed{2A\cos(\omega t)e^{i(kx)}} && (30) \end{aligned}$$

It's not pretty, but it works. This is indeed a wave with time-dependent amplitude with poposition-dependence $e^{i(kx)}$. TENA

17 Problem 11

A very sensitive detector measures the energy of a single photon from starlight at 2.5 eV and at the same time measures its wavelength at 495 nm. What is the value of Planck's constant at that far-away star?

17.1 Solution

Assume the speed of light to be constant everywhere. Radical, I know. Use this in the equation for the energy from wavelength.

$$E = \frac{hc}{\lambda} \tag{31}$$

Reorder this equation to isolate h.

$$\begin{aligned} h &= \frac{E\lambda}{c} = \frac{2.5\,\mathrm{eV}\cdot 495\,\mathrm{nm}}{2.998\times 10^{17}\,\mathrm{nm/s}} && (32)\\ &= \boxed{4.128\times 10^{-15}\,\mathrm{eV}\cdot\mathrm{s}} && (33) \end{aligned}$$

Doing the math with this, it is remarkably close to what the accepted value of Planck's constant is in our galaxy, off by $0.008\times 10^{-15}\,\mathrm{eV}\cdot\mathrm{s}$. Maybe our detector is just not precise enough for measuring Planck's constant from 500 light years away.