Worksheet #8

PHYS 4C: Waves and Thermodynamics

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1 Problem 1

The speed of sound in steel is 5941 m/s. Steel has a density of around 7900 kg/m^3 (depends somewhat on the alloy content).

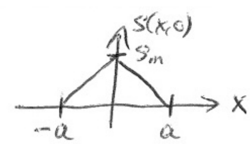
a. Based on this information, what is the bulk modulus of steel?

b. A steel rod of cross-sectional area A is placed on the x-axis. The following sound pulse is sent through the steel rod in the $+x$ direction:

$f(x) = s(x, 0) = s_m(1 - |x|/a)$, if $|x| < a$.

$f(x) = s(x, 0) = 0$, if $|x| \geq a$.

Determine the total energy of this pulse.



Sound pulse graph

c. Now suppose a sinusoidal sound wave with frequency 440 Hz is sent through steel with a sound level of 100 dB. Calculate s_m and Δp_m for this sound wave.

1.1 Solution (a)

The bulk modulus is used as part of an equation for the velocity.

$$v = \sqrt{\frac{B}{\rho}} \tag{1}$$

This can be solved for the bulk modulus.

$$B = \rho v^2 \tag{2}$$

We know all the values necessary, so we can solve this equation.

$$B = (7900\,\mathrm{kg/m^3})(5941\,\mathrm{m/s})^2 = \boxed{2.788 \times 10^{11}\,\mathrm{kg/m \cdot s^2}} \tag{3}$$

1.2 Solution (b)

If this is the initial point, there is no motion, so $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ at all points. There is an equation for the potential energy.

$$dU = \frac{1}{2}B\left(\frac{\partial s}{\partial x}\right)^2 A\,dx \tag{4}$$

At time $t = 0$, the partial derivative of s is calculatable, but it would have to be divided into two cases ($x \geq 0$ and $x < 0$). For the case of $x \geq 0$:

$$\frac{\partial s}{\partial x} = \frac{\partial}{\partial x}\left(s_m\left(1 - \frac{x}{a}\right)\right) = -\frac{s_m}{a} \tag{5}$$

For the case of $x < 0$:

$$\frac{\partial s}{\partial x} = \frac{\partial}{\partial x}\left(s_m\left(1 + \frac{x}{a}\right)\right) = \frac{s_m}{a} \tag{6}$$

Since the negative is the only thing differentiating these, it is safe to say that $\left(\frac{\partial s}{\partial x}\right)^2$ is identical for both.

These in turn can be plugged into our equation for the potential energy.

$$dU = \frac{1}{2}B\frac{s_m^2}{a^2}A\,dx \tag{7}$$

This in turn can be integrated between $-a$ and a. Technically, we would be integrating between $-\infty$ and ∞, but since $s(x,0) = 0$ everywhere outside the range of $(-a, a)$, all the other spots would result in zero to begin with.

$$\int_{-\infty}^{\infty} dU = \int_{-\infty}^{\infty} \frac{1}{2}BA\frac{s_m^2}{a^2}\,dx \tag{8}$$

$$U = \int_{-a}^{a} \frac{1}{2}BA\frac{s_m^2}{a^2}\,dx = \frac{1}{2}BA\frac{s_m^2}{a^2}\int_{-a}^{a} dx \tag{9}$$

$$= \frac{1}{2}BA\frac{s_m^2}{a^2}\left[x\right]_{-a}^{a} = \frac{1}{2}BA\frac{s_m^2}{a^2}(a - (-a)) = \boxed{BA\frac{s_m^2}{a}} \tag{10}$$

1.3 Solution (c)

We know the sound level. This can be used to calculate the intensity.

$$\beta = 100\,\mathrm{dB} = (10\,\mathrm{dB})\,\log_{10}\frac{I}{I_0} \tag{11}$$

$$10 = \log_{10}\frac{I}{I_0} \tag{12}$$

$$10^{10} = \frac{I}{10^{-12}\,\mathrm{W/m^2}} \tag{13}$$

$$10^{10} * 10^{-12} = 10^{-2}\,\mathrm{W/m^2} = I \tag{14}$$

Next, we know the bulk modulus (B), the speed of sound (v), and the frequency (f). The latter can be turned into the angular speed (ω) by arithmetic.

$$\omega = 2\pi f \tag{15}$$

We have an equation for the intensity that includes all of these values and the displacement amplitude, the latter of which can be solved for.

$$I = \frac{1}{2}\rho v\omega^2 s_m^2 \tag{16}$$

$$s_m = \sqrt{\frac{2I}{\rho v\omega^2}} = \sqrt{\frac{2I}{\rho v(2\pi f)^2}} = \sqrt{\frac{I}{2\rho v(\pi f)^2}} \tag{17}$$

$$= \sqrt{\frac{10^{-2}\,\mathrm{W/m^2}}{2(7900\,\mathrm{kg/m^3})(5941\,\mathrm{m/s})(\pi * 440\,\mathrm{Hz})^2}} \tag{18}$$

$$= \boxed{7.47 \times 10^{-9}\,\mathrm{m}} \tag{19}$$

The pressure amplitude is calculatable similarly.

$$\Delta p_m = (v\rho\omega)s_m \tag{20}$$

$$= (5941\,\mathrm{m/s})(7900\,\mathrm{kg/m^3})(2\pi * 440\,\mathrm{Hz})(7.47 \times 10^{-9}\,\mathrm{m}) \tag{21}$$

$$= \boxed{968.85\,\mathrm{Pa}} \tag{22}$$

2 Problem 2

A half-open organ pipe is tuned to A(440) (i.e., the fundamental frequency is 440 Hz). Air has a density of 1.21 kg/mol and a speed of sound of 343 m/s at 20°C.

a. What is the length of the pipe?

b. What is the maximum kinetic energy density (per unit volume) at the open end of the pipe if $s_m = 2.0\,\mu\text{m}$? At the closed end?

c. If the ambient temperature were raised from 20°C to 40°C, what would be the new fundamental frequency of the pipe (ignore changes in the length of the pipe due to the temperature change)?