Worksheet #6

PHYS 4C: Waves and Thermodynamics

Donald Aingworth IV

September 22, 2025

1 Problem

A wave in a string is described by the following function:

$$y(x,t) = (0.34\,\mathrm{mm})\cos((24\,\mathrm{rad/m})x - (960\,\mathrm{rad/s})t) \tag{1}$$

The mass per unit length of the string is $1.2\,\mathrm{g/m}$. Identify or calculate the following (include correct units).

a) Amplitude:

b) Wavenumber:

c) Wavelength:

d) Angular frequency:

e) Cycle frequency:

f) Period:

g) Wave velocity (include direction):

h) Tension in the string:

i) Maximum speed of some specific part of the string:

j) Kinetic energy density (per unit length), time-averaged:

k) Potential energy density, time-averaged:

l) Rate of energy transfer across some specific point, time-averaged:

m) Total energy in a 100 m-long section:

n) How much time does it take for the amount of energy calculated in part m to cross a specific point in the string?

1.1 Solution (a)

For the first four parts (except (c)), we will find the answers in the structure of the wave equation and the general wave equation.

$$\psi(x,t) = \underline{\psi_m} \cos(kx - \omega t) \tag{2}$$

$$y(x,t) = (\underline{0.34\,\mathrm{mm}}) \cos((24\,\mathrm{rad/m})x - (960\,\mathrm{rad/s})t) \tag{3}$$

$$\boxed{\psi_m = 3.4 \times 10^{-4}\,\mathrm{m}} \tag{4}$$

1.2 Solution (b)

For the first four parts (except (c)), we will find the answers in the structure of the wave equation and the general wave equation.

$$\psi(x,t) = \psi_m \cos(\underline{k}x - \omega t) \tag{5}$$

$$y(x,t) = (0.34\,\mathrm{mm}) \cos((\underline{24\,\mathrm{rad/m}})x - (960\,\mathrm{rad/s})t) \tag{6}$$

$$\boxed{k = 24\,\mathrm{rad/m}} \tag{7}$$

1.3 Solution (c)

The wavelength is inversely proportional to the wave number.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{24\,\mathrm{rad/m}} = \frac{\pi}{12}\,\mathrm{m} = \boxed{0.2618\,\mathrm{m}} \tag{8}$$

1.4 Solution (d)

For the first four parts (except (c)), we will find the answers in the structure of the wave equation and the general wave equation.

$$\psi(x,t) = \psi_m \cos(kx - \underline{\omega} t) \tag{9}$$

$$y(x,t) = (0.34\,\mathrm{mm})\cos((24\,\mathrm{rad/m})x - (\underline{960\,\mathrm{rad/s}})t) \tag{10}$$

$$\boxed{\omega = 960\,\mathrm{rad/s}} \tag{11}$$

1.5 Solution (e)

The cycle frequency is proportional to the angular frequency.

$$f = \frac{\omega}{2\pi} = \frac{960\,\mathrm{rad/s}}{2\pi} = \frac{480\,\mathrm{rad/s}}{\pi} = \boxed{152.8\,\mathrm{Hz}} \tag{12}$$

1.6 Solution (f)

The period is the reciprocal of the frequency.

$$T = \frac{1}{f} = \frac{\pi}{480\,\mathrm{rad/s}} = \boxed{6.545\,\mathrm{ms}} \tag{13}$$

1.7 Solution (g)

The wave speed is determined by a fraction.

$$v = \frac{\lambda}{T} = \frac{\frac{\pi}{12}}{\frac{\pi}{480}} = \frac{480}{12} = \boxed{40\,\mathrm{m/s}} \tag{14}$$

1.8 Solution (h)

The tension is measured in therms of the density and the wave speed.

$$\mu = 1.2\,\mathrm{g/m} = 1.2 \times 10^{-3}\,\mathrm{kg/m} \tag{15}$$

$$v = \sqrt{\frac{\tau}{\mu}} \tag{16}$$

$$v^2 = 40^2 = 1600\,\mathrm{m^2/s^2} = \frac{\tau}{\mu} \tag{17}$$

$$\tau = \mu v^2 = 1.2 \times 10^{-3}\,\mathrm{kg/m} * 1600\,\mathrm{m^2/s^2} \tag{18}$$

$$= 1.2 * 1.6\,\mathrm{kg\,m^2/s^2\,m} = \boxed{1.92\,\mathrm{N}} \tag{19}$$

1.9 Solution (i)

We can first find the formula for the velocity. By nature, the velocity would only be vertical since the wire would not be moving horizontally.

$$\begin{aligned}
v &= \frac{\mathrm{d}}{\mathrm{d}t}\left((0.34\,\mathrm{mm})\cos((24\,\mathrm{rad/m})x - (960\,\mathrm{rad/s})t)\right) && (20)\\
&= -3.4\times10^{-4} * 960\,\mathrm{rad/s} * \sin((24\,\mathrm{rad/m})x - (960\,\mathrm{rad/s})t) && (21)
\end{aligned}$$

For the maximum value of this velocity, the sine would have to be equal to one (or negative one) so we can set it to that.

$$v = -3.4\times10^{-4}\,\mathrm{m} * 960\,\mathrm{rad/s} * (-1) = \boxed{0.3264\,\mathrm{m/s}} \tag{22}$$

1.10 Solution (j)

We can start by finding the kinetic energy per unit length at any given time.

$$\begin{aligned}
K &= \frac{1}{2}mv^2 = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 && (23)\\
\psi(x,t) &= \psi_m\cos(kx-\omega t) && (24)\\
\frac{\partial\psi(x,t)}{\partial t} &= -\psi_m\omega\sin(kx-\omega t) && (25)\\
\left(\frac{\partial\psi}{\partial t}\right)^2 &= \psi_m^2\omega^2\sin^2(kx-\omega t) && (26)\\
\left(\frac{\partial y}{\partial t}\right)^2 &= y_m^2\omega^2\sin^2(kx-\omega t) && (27)\\
K &= \frac{1}{2}\mu y_m^2\omega^2\sin^2(kx-\omega t) && (28)
\end{aligned}$$

We can average this over time. The average of $\sin^2$ dependant on only one variable is always $\frac{1}{2}$, as is the average value of cosine.

$$\begin{aligned}
\langle K\rangle &= \frac{1}{2}\mu y_m^2\omega^2\left\langle\sin^2(kx-\omega t)\right\rangle = \frac{1}{4}\mu y_m^2\omega^2 && (29)\\
&= \frac{1}{4}(1.2\times10^{-3}\mathrm{kg/m})\left(3.4\times10^{-4}\mathrm{m}\right)^2(960\,\mathrm{rad/s})^2 && (30)\\
&= \boxed{3.196\times10^{-5}\,\mathrm{J/m}} && (31)
\end{aligned}$$

1.11 Solution (k)

The average kinetic energy would be equal to the average potential energy. Such is the nature of standing waves.

$$\langle U \rangle = \boxed{3.196 \times 10^{-5}\,\mathrm{J/m}} \tag{32}$$

1.12 Solution (l)

The average energy transfer would be the total energy multiplied by the wave speed.

$$\langle P \rangle = v * \langle E \rangle = v * \langle K + U \rangle = v * [\langle K \rangle + \langle U \rangle] \tag{33}$$

$$= v * \left(\frac{1}{2}\mu y_m^2 \omega^2\right) = (40\,\mathrm{m/s})(6.392 \times 10^{-5}\,\mathrm{J/m}) \tag{34}$$

$$= \boxed{0.00256\,\mathrm{J/s}} \tag{35}$$

1.13 Solution (m)

This can be found by multiplying the average energy per unit distance by the distance of 100 m.

$$E_{\text{total}} = \Delta x * E = 100\,\mathrm{m} * 6.392 \times 10^{-5}\,\mathrm{J/m} = \boxed{6.392\,\mathrm{mJ}} \tag{36}$$

1.14 Solution (n)

We can take the energy stored in the one hundred meters and divide it by the average rate at which energy is transfered to find the amount of time this transfer takes.

$$t = \frac{E_{\text{total}}}{\langle P \rangle} = \frac{6.392\,\mathrm{mJ}}{0.00256\,\mathrm{J/s}} = 2.496875\,\mathrm{s} \approx \boxed{2.50\,\mathrm{s}} \tag{37}$$