

Homework #12

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Problem 1

In 1932, James Chadwick fired neutrons of unknown speed and mass into different substances. He found that protons (of mass 1 u) were given a speed 7.5 times that given to nitrogen nuclei (of mass 14 u). If the collisions were elastic and head on, what can you deduce about the mass of the neutron?

Solution

From the conservation of momentum, we have two equations, knowing that only the neutron is moving initially.

$$m_1 v_{1i} = m_1 v_{1f1} + m_2 v_{2f} \quad (1)$$

$$m_1 v_{1i} = m_1 v_{1f2} + m_3 v_{3f} \quad (2)$$

Subtracting equation (2) from equation (1), we get equation (3).

$$0 = m_1(v_{1f1} - v_{1f2}) + m_2 v_{2f} - m_3 v_{3f} \quad (3)$$

Solving (3) for m_1 , we get (5).

$$m_1(v_{1f1} - v_{1f2}) = m_3 v_{3f} - m_2 v_{2f} \quad (4)$$

$$m_1 = \frac{m_3 v_{3f} - m_2 v_{2f}}{v_{1f1} - v_{1f2}} \quad (5)$$

Following equations we learned in class, we can get equations (7) and (8).

$$v_{1f} + v_{1i} = v_{2f} + v_{2i} \quad (6)$$

$$v_{1f1} + v_{1i} = v_{2f} + v_{2i} \quad (7)$$

$$v_{1f2} + v_{1i} = v_{3f} + v_{3i} \quad (8)$$

We can subtract (8) from (7) to get something we can substitute into (5). We can then recall that the other particles are initially at rest.

$$v_{1f1} + v_{1i} = v_{2f} + v_{2i} \quad (7)$$

$$v_{1f2} + v_{1i} = v_{3f} + v_{3i} \quad (8)$$

$$v_{1f1} - v_{1f2} = v_{2f} + v_{2i} - v_{3f} - v_{3i} = v_{2f} - v_{3f} \quad (9)$$

$$m_1 = \frac{m_3 v_{3f} - m_2 v_{2f}}{v_{2f} - v_{3f}} \quad (10)$$

Knowing the ratios from above, we can substitute values in.

$$m_1 = \frac{14m_2v_{3f} - m_2 * 7.5v_{3f}}{7.5 * v_{3f} - v_{3f}} \quad (11)$$

$$m_1 = \frac{6.5m_2v_{3f}}{6.5 * v_{3f}} \quad (12)$$

$$m_1 = m_2 \quad (13)$$

Since $m_1 = m_2$ and $m_2 = 1\text{u}$, then by the transitive property, $\boxed{m_1 = 1\text{u}}$.

Problem 2

A one-dimensional inelastic collision may be characterized by a coefficient of restitution e that relates the relative velocities before and after the collision: $(v_{1,f} - v_{2,f}) = -e(v_{1,i} - v_{2,i})$. Show that the final velocities are:

$$v_{1,f} = \frac{(m_1 - em_2)v_{1,i} + m_2(1 + e)v_{2,i}}{m_1 + m_2} \quad (1)$$

$$v_{2,f} = \frac{m_1(1 + e)v_{1,i} + (m_2 - em_1)v_{2,i}}{m_1 + m_2} \quad (2)$$

(Hint, kinetic energy is not conserved. You will need two equations so solve for the two unknowns.)

Solution

I have manually numbered the above equations for my own convenience.

We know that momentum is conserved in an inelastic collision, which we have an equation for.

$$m_1v_{1,i} + m_2v_{2,i} = m_1v_{1,f} + m_2v_{2,f} \quad (3)$$

We can also use the other equation given.

$$(v_{1,f} - v_{2,f}) = -e(v_{1,i} - v_{2,i}) \quad (4)$$

We can remark that equation (1) does not have $v_{2,f}$, and to get to a point where we can get to this, we can solve equation (4) for $v_{2,f}$ and substitute that in equation (3), then solve for $v_{1,f}$.

$$v_{2,f} = v_{1,f} + e(v_{1,i} - v_{2,i}) \quad (5)$$

$$m_1v_{1,i} + m_2v_{2,i} = m_1v_{1,f} + m_2v_{1,f} + m_2e(v_{1,i} - v_{2,i}) \quad (6)$$

$$v_{1,f}(m_1 + m_2) = m_1v_{1,i} + m_2v_{2,i} - m_2e(v_{1,i} - v_{2,i}) \quad (7)$$

$$v_{1,f} = \frac{m_1v_{1,i} - m_2ev_{1,i} + m_2v_{2,i} + m_2ev_{2,i}}{m_1 + m_2} \quad (8)$$

$$v_{1,f} = \frac{(m_1 - em_2)v_{1,i} + m_2(1 + e)v_{2,i}}{m_1 + m_2} \quad (9)$$

We can do the same for equation (2), except by substituting in $v_{1,f}$ and solving for $v_{2,f}$.

$$v_{1,f} = v_{2,f} - e(v_{1,i} - v_{2,i}) \quad (10)$$

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{2,f} - m_1 e(v_{1,i} - v_{2,i}) + m_2 v_{2,f} \quad (11)$$

$$(m_1 + m_2)v_{2,f} = m_1 v_{1,i} + m_2 v_{2,i} + m_1 e v_{1,i} - m_1 e v_{2,i} \quad (12)$$

$$v_{2,f} = \frac{m_1 v_{1,i} + m_1 e v_{1,i} + m_2 v_{2,i} - m_1 e v_{2,i}}{m_1 + m_2} \quad (13)$$

$$v_{2,f} = \frac{m_1(1 + e)v_{1,i} + (m_2 - e m_1)v_{2,i}}{m_1 + m_2} \quad (14)$$

Problem 3

An alpha particle (${}^4\text{He}$) undergoes an elastic collision with a stationary uranium nucleus (${}^{235}\text{U}$). What percent of the kinetic energy of the alpha particle is transferred to the uranium nucleus? Assume the collision is one dimensional.

Solution

In the conservation of momentum, we know this (knowing that $v_{2i} = 0$).

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad (1)$$

From the conservation of kinetic energy from collisions, we know this.

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (2)$$

We also know an in-class equation values of velocities.

$$v_{1f} + v_{1i} = v_{2f} + v_{2i} \quad (3)$$

$$v_{1f} + v_{1i} = v_{2f} \quad (4)$$

We also have an equation about elastic collisions.

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \quad (5)$$

The portion of the kinetic energy can be represented as a fraction.

$$\frac{K_{2f}}{K_{1i}} = \frac{m_2 v_{2f}^2}{m_1 v_{1i}^2} = \frac{m_2 \left(\frac{2m_1}{m_1 + m_2} v_{1i} \right)^2}{m_1 v_{1i}^2} = \frac{m_2 4m_1^2 v_{1i}^2}{(m_1 + m_2)^2 m_1 v_{1i}^2} = \frac{4m_1 m_2}{(m_1 + m_2)^2} \quad (6)$$

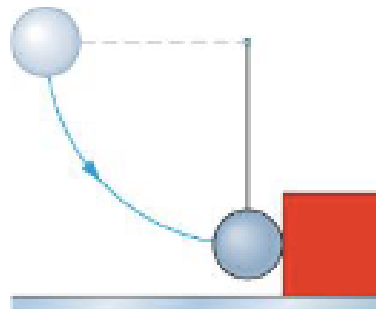
Assuming that ${}^4\text{He}$ weighs about 4 amu and ${}^{235}\text{U}$ weighs about 235 amu, we can plug in numbers to the above equation.

$$\frac{K_{2f}}{K_{1i}} = \frac{4m_1 m_2}{(m_1 + m_2)^2} = \frac{4 * 4 * 235}{(4 + 235)^2} = \frac{3760}{57121} = 0.0658 \quad (7)$$

We can convert this to a percentage to get $\boxed{6.58\%}$.

Problem 4

A steel ball of mass 0.500 kg is fastened to a cord that is 70.0 cm long and fixed at the far end. The ball is then released when the cord is horizontal. At the bottom of its path, the ball strikes a 2.50 kg steel block initially at rest on a frictionless surface. The collision is elastic. Find (a) the speed of the ball and (b) the speed of the block, both just after the collision.



Solution

Through the conservation of energy, we can know what the kinetic energy is at the bottom.

$$K_i + U_i = K_f + U_f$$
$$\Delta U = K_i - K_f$$

Since there is no initial velocity, we know that $K_i = 0$. We can now use these to expand the equations and solve for the velocity of the ball just before the collision.

$$\Delta U = K_i - K_f$$
$$mg\Delta y = -\frac{1}{2}mv_{1i}^2$$
$$v_{1i}^2 = -2g\Delta y$$
$$v_{1i} = \sqrt{-2g\Delta y}$$

In elastic collisions, the kinetic energy is conserved, so we could use that. We can here instead use the formulae for post-collision velocities. Remember that $v_{2i} = 0$.

Section (a)

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} = \frac{m_1 - m_2}{m_1 + m_2}\sqrt{-2g\Delta y}$$
$$= \frac{0.500 - 2.50}{0.50 + 2.50}\sqrt{2 * 9.81 * 0.70} = -\frac{2.00}{3.00}\sqrt{9.81 * 1.40}$$
$$= -\frac{2.00\sqrt{13.734}}{3.00} = \boxed{-2.470\text{m/s}}$$

Section (b)

$$\begin{aligned}v_{2f} &= \frac{2m_2}{m_1 + m_2}v_{1i} + \frac{m_1 - m_2}{m_1 + m_2}v_{2i} = \frac{2m_2}{m_1 + m_2}\sqrt{-2g\Delta y} \\&= \frac{2 * 2.50}{0.50 + 2.50}\sqrt{2 * 9.81 * 0.70} = \frac{5.00}{3.00}\sqrt{9.81 * 1.40} \\&= \frac{5.00\sqrt{13.734}}{3.00} = \boxed{6.177\text{m/s}}\end{aligned}$$

Problem 5

The platter in a belt drive turntable is driven by a belt that wraps around a hub, of radius 3.00 cm, below the platter and around the shaft of the motor. If the platter rotates at 33.3 rpm and the motor rotates at 60 rpm, what is the required radius of the shaft?

Solution

The same distance along the belt would be covered by both discs at the furthest point. This means that we can first convert from rpm to cm/min, with the radius of the shaft being d .

$$60\text{rpm} = (2 * \pi * 3.00 \text{ cm/r}) * 33.3\text{rpm} = 199.8\pi \text{ cm/min}$$

$$33.3\text{rpm} = (2 * \pi * d \text{ cm/r}) * 60\text{rpm} = 120\pi * d \text{ cm/min}$$

$$120\pi * d \text{ cm/min} = 199.8\pi \text{ cm/min}$$

$$d = \frac{199.8\pi}{120\pi} = \frac{1998}{1200} = \frac{333}{200} = \boxed{1.665\text{cm}}$$

Problem 6

At $t = 0$, a flywheel has an angular velocity of 4.70 rad/s , a constant angular acceleration of -0.24 rad/s^2 , and a reference line at $\theta_0 = 0$. (a) Through what maximum angle θ_{max} will the reference line turn in the positive direction? What are the (b) first and (c) second times the reference line will be at $\theta = \frac{1}{2}\theta_{max}$? At what (d) negative time and (e) positive time will the reference line be at $\theta = -10.5 \text{ rad}$? (f) Graph θ versus t , and indicate your answers.

Solution

Whenever possible, I will be omitting the radian unit.

Section (a)

First plug the values into the angular equation for position. Include the fact that at the end the angular velocity will be zero.

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ 0 &= 4.70 \text{ rad/s} - 0.24 \text{ rad/s}^2 t \\ 0.24 \text{ rad/s}^2 t &= 4.70 \text{ rad/s} \\ t &= \frac{470}{24} \text{ s}\end{aligned}$$

We can here plug in our values into the angular equation for position, velocity, and acceleration.

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 = 0 + 4.70 \text{ rad/s} \frac{470}{24} \text{ s} - 0.12 \text{ rad/s}^2 \left(\frac{470}{24} \text{ s} \right)^2 \\ &= \frac{2209}{24} - \frac{2209}{48} = \frac{2209}{48} = \boxed{46.02}\end{aligned}$$

Section (b) and (c)

As much as I'd like to use energy here, I can't, so I'll use the quadratic formula on the angular equation for position, velocity, and acceleration.

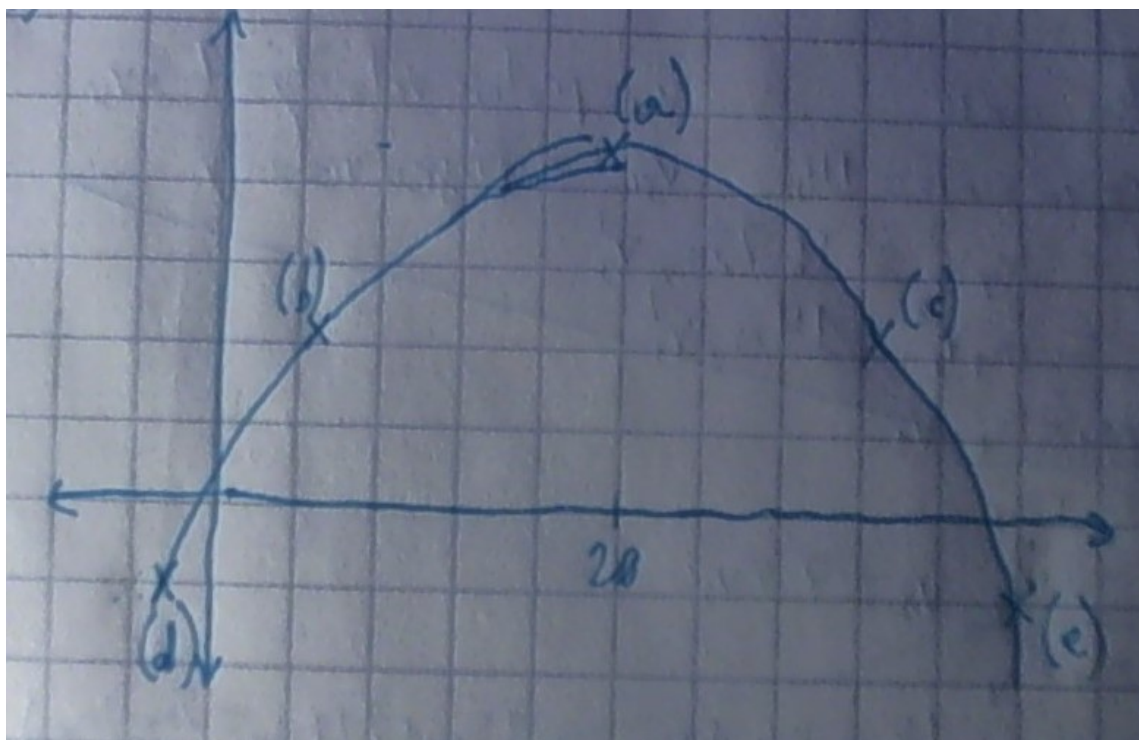
$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ \frac{1}{2} \theta_{max} &= \frac{2209}{96} = t * 4.70 \text{rad/s} - t^2 * 0.12 \text{rad/s}^2 \\ 0 &= t * 4.70 \text{rad/s} - t^2 * 0.12 \text{rad/s}^2 - \frac{2209}{96} \\ t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4.70 \pm \sqrt{4.70^2 - 4(0.12)(\frac{2209}{96})}}{-2 * 0.12} \\ &= \frac{-4.7 \pm \sqrt{22.09 - \frac{2209}{200}}}{-0.24} = \frac{4.7 \mp \sqrt{\frac{2209}{200}}}{0.24} \\ t_1 &= \frac{4.7 - \sqrt{\frac{2209}{200}}}{0.24} = \boxed{5.736\text{s}} \\ t_2 &= \frac{4.7 + \sqrt{\frac{2209}{200}}}{0.24} = \boxed{33.431\text{s}}\end{aligned}$$

Section (d) and (e)

We use the same strategy here.

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ -10.5 &= t * 4.70 \text{rad/s} - t^2 * 0.12 \text{rad/s}^2 \\ 0 &= 10.5 + t * 4.70 \text{rad/s} - t^2 * 0.12 \text{rad/s}^2 \\ t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4.70 \pm \sqrt{4.70^2 + 4(0.12)(10.5)}}{-2 * 0.12} \\ &= \frac{-4.7 \pm \sqrt{22.09 + 5.04}}{-0.24} = \frac{4.7 \mp \sqrt{28.13}}{0.24} \\ t_1 &= \frac{4.7 - \sqrt{28.13}}{0.24} = \boxed{-2.516\text{s}} \\ t_2 &= \frac{4.7 + \sqrt{28.13}}{0.24} = \boxed{41.682\text{s}}\end{aligned}$$

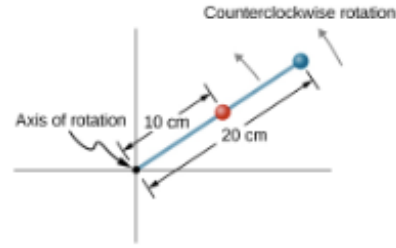
Section (f)



Identified answers as points from left to right: (d), (b), (a), (c), (e).

Problem 7

The angular position of a rod varies as $20.0 t^2$ radians starting from time $t = 0$. The rod has two beads on it as shown in the following figure, one at 10.0 cm from the rotation axis and the other at 20.0 cm from the rotation axis. (a) What is the instantaneous angular velocity of the rod at $t = 5.00$ s? (b) What is the angular acceleration of the rod? (c) What are the tangential speeds of the beads at $t = 5.00$ s? (d) What are the tangential accelerations of the beads at $t = 5.00$ s? (e) What are the centripetal accelerations of the beads at $t = 5.00$ s?



Solution

Section (a)

We take the derivative of the angular position, then substitute in the value of time.

$$\omega(t) = \frac{d}{dt}\theta(t) = \frac{d}{dt}(20t^2) = 40t$$

$$\omega(5s) = (40 * 5)\text{Hz} = \boxed{200\text{Hz}}$$

Section (b)

We take the derivative of the angular velocity, then substitute in the value of time.

$$\alpha(t) = \frac{d}{dt}\omega(t) = \frac{d}{dt}(40t) = 40$$

$$\alpha(5s) = 40\text{Hz} = \boxed{40\text{Hz}^2}$$

Section (c)

To get this, we multiply the tangential velocities by the radii.

$$v_1 = \omega * r_1 = 200 * 10 = 2000\text{cm/s} = \boxed{20\text{m/s}}$$

$$v_2 = \omega * r_2 = 200 * 20 = 4000\text{cm/s} = \boxed{40\text{m/s}}$$

Section (d)

To get this, we multiply the tangential accelerations by the radii.

$$a_1 = \omega * r_1 = 40 * 10 = 400\text{cm/s} = \boxed{4\text{m/s}}$$

$$a_2 = \omega * r_2 = 40 * 20 = 800\text{cm/s} = \boxed{8\text{m/s}}$$

Section (e)

We recall that the formula for the centripital acceleration is $a_c = \frac{v^2}{r}$.

$$a_{c1} = \frac{v^2}{r} = \frac{4^2}{10}\text{m/s}^2 = \frac{16}{10}\text{m/s}^2 = \boxed{1.6\text{m/s}^2}$$

$$a_{c1} = \frac{v^2}{r} = \frac{8^2}{20}\text{m/s}^2 = \frac{32}{10}\text{m/s}^2 = \boxed{3.2\text{m/s}^2}$$

Problem 8

The angular acceleration of a rotating rigid body is given by $\alpha = (A - Bt)$, where $A = 2.00 \text{ rad/s}^2$ and $B = 3.00 \text{ rad}$. If the body starts rotating from rest at $t = 0$, (a) what is the angular velocity? (b) Angular position? (c) What angle does it rotate through in 10.0 s? (d) Where does the vector perpendicular to the axis of rotation indicating 0° at $t = 0$ lie at $t = 10.0 \text{ s}$?

Solution

Section (a)

We remark that since $\alpha = \frac{d\omega}{dt}$, we can manipulate it to know that $\omega = \int \alpha \, dt$. We also know that $\omega = 0$ at $t = 0$.

$$\begin{aligned}\alpha &= A - Bt \\ \omega &= \int \alpha \, dt = \int_0^t A - Bt' \, dt' = \left[At' - \frac{Bt'^2}{2} \right]_0^t \\ &= At - \frac{1}{2}Bt^2 = \boxed{2.00t - \frac{3}{2}t^2}\end{aligned}$$

Section (b)

We remark that since $\omega = \frac{d\theta}{dt}$, we can manipulate it to know that $\theta = \int \omega \, dt$. We also assume that $\theta = 0$ at $t = 0$.

$$\begin{aligned}\alpha &= A - Bt \\ \omega &= At - \frac{1}{2}Bt^2 \\ \theta &= \int \omega \, dt = \int_0^t At' - \frac{1}{2}Bt'^2 \, dt' = \left[\frac{At'^2}{2} - \frac{Bt'^3}{6} \right]_0^t \\ &= \frac{At^2}{2} - \frac{Bt^3}{6} = \boxed{t^2 - \frac{1}{2}t^3}\end{aligned}$$

Section (c)

If the question is what angle it travels to, the answer is as follows.

$$\theta(t) = t^2 - \frac{1}{2}t^3$$
$$\theta(10) = 10^2 - \frac{1}{2}10^3 = 100 - \frac{1}{2}1000 = 100 - 500 = \boxed{-400}$$

If we want the angle traveled, we can start by finding points where the angular velocity is zero.

$$\omega = 2.00t - \frac{3}{2}t^2 = t(2.00 - \frac{3}{2}t) = 2.00t(1 - \frac{3}{4}t)$$

This means that $\omega = 0$ at $t = 0$ and $t = \frac{4}{3}$.

Section (d)

We just translate radians to degrees from the previous problem.

$$\theta(10) = -400 = -400 * \frac{180^\circ}{\pi} = -\frac{72000}{\pi}$$