Homework #8

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Problem 1

A 0.315-kg particle moves from an initial position $\vec{r_1} = 2.00\hat{\imath} - 1.00\hat{\jmath} + 3.00\hat{k}$ m to a final position $\vec{r_2} = 4.00\hat{\imath} - 3.00\hat{\jmath} - 1.00\hat{k}$ m while a force $\vec{F} = 2.00\hat{\imath} - 3.00\hat{\jmath} + 1.00\hat{k}$ N acts on it. What is the work done by the force on the particle?

Solution

We know the formula for work is $W = \vec{F}\cdot\vec{d}$. We can then apply the change in position for the distance traveled. We can then substitute in values to find the answer.

$$\begin{aligned}
\vec{d} &= \vec{r_2} - \vec{r_1} \\
&= (4.00\hat{\imath} - 3.00\hat{\jmath} - 1.00\hat{k}) - (2.00\hat{\imath} - 1.00\hat{\jmath} + 3.00\hat{k}) \text{ m} \\
&= 2.00\hat{\imath} - 2.00\hat{\jmath} - 4.00\hat{k} \text{ m} \\
W &= \vec{F} \cdot \vec{d} \\
&= (2.00\hat{\imath} - 3.00\hat{\jmath} + 1.00\hat{k} \text{ N}) \cdot (2.00\hat{\imath} - 2.00\hat{\jmath} - 4.00\hat{k} \text{ m}) \\
&= 2 * 2 + (-3) * (-2) + 1 * (-4) \text{ J} = (4 + 6 - 4) \text{ J} = \boxed{6 \text{ J}}
\end{aligned}$$

Problem 2

Compute the kinetic energy for each of the cases below. Through what distance would a 800-N force have to act to stop each object? (a) A 150-g baseball moving at 40 m/s; (b) a 13-g bullet from a rifle moving at 635 m/s; (c) a 1500-kg Corvette moving at 250 km/h; (d) a 1.8×10^5 kg Concorde airliner moving at 2240 km/h.

Solution

In every case, we start with the formula of the work-kinetic energy thorem. We then substitute in formulae for work and kinetic energy, keeping in mind that the final velocity is zero (so the final kinetic energy $K_f = \frac{1}{2}mv_f^2$ is zero). We also keep in mind that the force would be applied in a direction opposite the kinetic energy.

$$\begin{aligned} W &= K_f - K_i \\ F \cdot d &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ d &= \frac{-\frac{1}{2}mv_i^2}{F} \end{aligned}$$

Section (a)

$$\begin{aligned} K &= \frac{1}{2}mv^2 = \frac{1}{2}0.15 * 40^2 = \boxed{120\text{J}} \\ d &= \frac{-\frac{1}{2}0.15 * 40^2\text{J}}{-800\text{N}} = \frac{0.15 * 800}{800}\text{m} = \boxed{0.15\text{m}} \end{aligned}$$

Section (b)

$$\begin{aligned} K &= \frac{1}{2}mv^2 = \frac{1}{2}0.013 * 635^2 = \boxed{2621\text{J}} \\ d &= \frac{-\frac{1}{2}0.013 * 635^2\text{J}}{-800\text{N}} = \frac{0.013 * 403225}{1600}\text{m} = \boxed{3.28\text{m}} \end{aligned}$$

Section (c)

$$v_i = 250\text{km/h} * \frac{1000\text{m}}{1\text{km}} * \frac{1\text{h}}{3600\text{s}} = \frac{625}{9}\text{m/s}$$
$$K = \frac{1}{2}mv^2 = \frac{1}{2}1500 * \frac{625}{9}^2 = \boxed{3616898\text{J}}$$
$$d = \frac{-\frac{1}{2}1500 * \frac{625}{9}^2\text{J}}{-800\text{N}} = \frac{1500 * \frac{390625}{81}}{1600}\text{m} = \boxed{\frac{1953125}{432}\text{m} = 4521\text{m}}$$

Section (d)

$$v_i = 2240\text{km/h} * \frac{1000\text{m}}{1\text{km}} * \frac{1\text{h}}{3600\text{s}} = \frac{5600}{9}\text{m/s}$$
$$K = \frac{1}{2}mv^2 = \frac{1}{2}1.8 * 10^5 * \frac{5600}{9}^2 = \boxed{3.484 \times 10^{10}\text{J}}$$
$$d = \frac{-\frac{1}{2}1.8 * 10^5 * \frac{5600}{9}^2\text{J}}{-800\text{N}} = \frac{1.8 * 10^9 * \frac{3136}{81}}{1600}\text{m} = \boxed{\frac{39200000000}{9}\text{m} = 4.36 \times 10^9\text{m}}$$

Problem 3

Compute the kinetic energies for each of the following. What force would be required to stop each object in 1.00 km? (a) The 8.00×10^7kg carrier Nimitz moving at 55 km/h; (b) a 3.4×10^5kg Boeing 747 moving at 1000 km/h; (c) the 270-kg Pioneer 10 spacecraft moving at 51,800 km/h.

Solution

In every case, we start with the formula of the work-kinetic energy thorem. We then substitute in formulae for work and kinetic energy, keeping in mind that the final velocity is zero (so the final kinetic energy $K_f = \frac{1}{2}mv_f^2$ is zero). We also keep in mind that the force would be applied in a direction opposite the kinetic energy.

$$\begin{aligned} K_i &= \frac{1}{2}mv_i^2 \\ F \cdot d = W = K_f - K_i &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ F &= \frac{-\frac{1}{2}mv_i^2}{d} \end{aligned}$$

Section (a)

$$\begin{aligned} v &= 55\text{km/h} * \frac{1000\text{ m} * \text{h}}{3600\text{ km} * \text{s}} = \frac{275}{18}\text{m/s} \\ K_i &= \frac{1}{2}mv_i^2 = \frac{1}{2} * 8.00 \times 10^7\text{kg} * \left(\frac{275}{18}\text{m/s}\right)^2 = 9.34 \times 10^9\text{J} \\ F &= \frac{-\frac{1}{2}mv_i^2}{d} = \frac{-9.34 \times 10^9\text{J}}{1000\text{m}} = -9.34 \times 10^6\text{N} \end{aligned}$$

This means that the kinetic energy of the Nimitz is $\boxed{9.34 \times 10^9\text{J}}$ in one direction, while the force required would be $\boxed{9.34 \times 10^6\text{N}}$ in the opposite direction.

Section (b)

$$
\begin{aligned}
v &= 1000\text{km/h} * \frac{1000 \text{ m} * \text{h}}{3600 \text{ km} * \text{s}} = \frac{2500}{9}\text{m/s} \\
K_i &= \frac{1}{2}mv_i^2 = \frac{1}{2} * 3.40 \times 10^5\text{kg} * \left(\frac{2500}{9}\text{m/s}\right)^2 = 1.31 \times 10^{10}\text{J} \\
F &= \frac{-\frac{1}{2}mv_i^2}{d} = \frac{-1.31 \times 10^1 0\text{J}}{1000\text{m}} = -1.31 \times 10^7\text{N}
\end{aligned}
$$

This means that the kinetic energy of the Boeing 747 is $\boxed{1.31 \times 10^{10}\text{J}}$ in one direction, while the force required would be $\boxed{1.31 \times 10^7\text{N}}$ in the opposite direction.

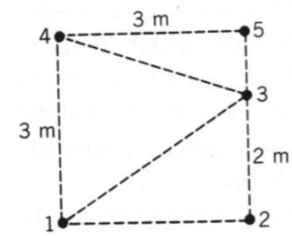
Section (c)

$$
\begin{aligned}
v &= 51800\text{km/h} * \frac{1000 \text{ m} * \text{h}}{3600 \text{ km} * \text{s}} = \frac{129500}{9}\text{m/s} \\
K_i &= \frac{1}{2}mv_i^2 = \frac{1}{2} * 270.0\text{kg} * \left(\frac{129500}{9}\text{m/s}\right)^2 = 2.80 \times 10^{10}\text{J} \\
F &= \frac{-\frac{1}{2}mv_i^2}{d} = \frac{-2.80 \times 10^{10}\text{J}}{1000\text{m}} = -2.80 \times 10^7\text{N}
\end{aligned}
$$

This means that the kinetic energy of the Pioneer 10 is $\boxed{2.80 \times 10^{10}\text{J}}$ in one direction, while the force required would be $\boxed{2.80 \times 10^7\text{N}}$ in the opposite direction.

Problem 4

A 1.50-kg block is moved at constant speed in a vertical plane from position 1 to position 3 via several routes shown in the figure. Compute the work done by gravity on the block for each segment indicated, where W_{ab} means work done from a to b. (a) W_{13}, (b) $W_{12} + W_{23}$ (c) $W_{14} + W_{43}$, (d) $W_{14} + W_{45}$ + W_{53}.



Solution

For each section, we use the formula of gravitational work, which is $W_g = mgd\cos(\phi)$, ϕ_g being the angle between the vertical force of gravity and the motion itself. We can derive from the cosine ratio (SOH**CAH**TOA) that $\cos(\phi) = \frac{\text{height}}{\text{distance}} = \frac{h}{d}$. Applying this to the formula for work, we get $W_g = mgh$.

Section (a)

$$W_{13} = mg(h_3 - h_1) = 1.50 * -9.81 * (2 - 0)\text{J} = \boxed{-29.43\text{J}}$$

Section (b)

$$\begin{aligned} W_{12} + W_{23} &= mg(h_2 - h_1) + mg(h_3 - h_2) \\ &= 1.50 * -9.81 * (0 - 0)\text{J} + 1.50 * -9.81 * (2 - 0)\text{J} = \boxed{-29.43\text{J}} \end{aligned}$$

Section (c)

$$\begin{aligned} W_{14} + W_{43} &= mg(h_4 - h_1) + mg(h_3 - h_4) \\ &= 1.50 * -9.81 * (3 - 0)\text{J} + 1.50 * -9.81 * (2 - 3)\text{J} \\ &= 4.5 * -9.81 - 1.5 * -9.81 = 3.0 * -9.81 = \boxed{-29.43\text{J}} \end{aligned}$$

Section (d)

$$\begin{aligned} W_{14} + W_{45} + W_{53} &= mg(h_4 - h_1) + mg(h_5 - h_4) + mg(h_3 - h_5) \\ &= 1.50 * g * (3 - 0)\text{J} + 1.50 * g * (3 - 3)\text{J} + 1.50 * g * (2 - 3)\text{J} \\ &= 1.50 * -9.81 * 2 \text{ J} = \boxed{-29.43\text{J}} \end{aligned}$$

Problem 5

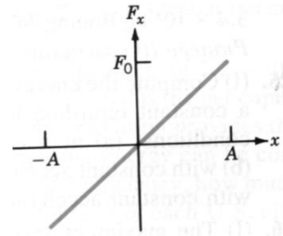
What is the work needed to lift 14.7 kg of water from a well 11.0 m deep. Assume the water has a constant upward acceleration of 0.700 m/s^2.

Solution

Here, the net force is equal to the gravitational force plus the applied force. We can then use the work formulae.

$$
\begin{aligned}
F_{net} = ma = F_{app} + F_g &= F_{app} - m * 9.81\text{m/s}^2 \\
F_{app} &= m(a + 9.81\text{m/s}^2) - 14.7(0.7 + 9.81)\text{N} = 154.497\text{m/s}^2 \\
W &= 154.497 * 11 = \boxed{1699.497\text{J}}
\end{aligned}
$$

Problem 6



The variation of a force with position is shown in the figure below. Find the work from (a) x = 0 to x = -A (b) x = +A to x = 0

Solution

a) $\frac{1}{2}F_0A$
We here use the integral for work along a curve.

$$F(x) = \frac{F_0}{A}x$$

$$W_1 = \int_{x_i}^{x_f} F(x)dx = \int_0^{-A} \frac{F_0}{A}x\ dx = \left(\frac{F_0}{2A}x^2\right)_0^{-A} = \frac{F_0(-A)^2}{2A} = \boxed{\frac{F_0A}{2}}$$

b)$-\frac{F_0A}{2}$
We use the same thng here.

$$W_2 = \int_{x_i}^{x_f} F(x)dx = \int_A^{0} \frac{F_0}{A}x\ dx = \left(\frac{F_0}{2A}x^2\right)_A^{0} = -\frac{F_0(-A)^2}{2A} = \boxed{-\frac{F_0A}{2}}$$

Problem 7

Consider a particle on which several forces act, one of which is known to be constant in time: $\vec{F}_1 = 3.00\ \hat{\imath} + 4.00\ \hat{\jmath}$ N. As a result, the particle moves along a straight path from a Cartesian coordinate of (0.00 m, 0.00 m) to (5.00 m, 6.00 m). What is the work done by $\vec{F}_1$?

Solution

We here use the dot product.

$$W = \vec{F} \cdot \vec{d} = \begin{pmatrix} 3\text{N} \\ 4\text{N} \end{pmatrix} \cdot \begin{pmatrix} 5\text{m} \\ 6\text{m} \end{pmatrix} = 3 * 5 + 4 * 6 \text{ J} = 15 + 24 \text{ J} = \boxed{39\text{J}}$$

Problem 8

A bungee cord exerts a nonlinear elastic force of magnitude $F(x) = k_1 x + k_2 x^3$, where x is the distance the cord is stretched, $k_1 = 204\text{N/m}$ and $k_2 = -0.233\text{N/m}^3$. How much work must be done on the cord to stretch it 16.7 m?

Solution

We here use the integral for work doen by a spring.

$$
\begin{aligned}
W_s &= \int_{x_i}^{x_f} F_x \; dx = \int_0^{16.7} k_1 x + k^2 x^3 \; dx = \int_0^{16.7} 204x - 0.233x^3 \; dx \\
&= \left(\frac{204x^2}{2} - \frac{0.233x^4}{4} \right)_0^{16.7} = \left(10216.7^2 - \frac{0.233 * 16.7^4}{4} \right) \text{J} \\
&= \boxed{23916.116\text{J}}
\end{aligned}
$$