*. (12 points) An electromagnetic wave has an electric field amplitude with a magnitude of $E_m = 12\,\mathrm{V/m}$.

$c = 3.00 \times 10^{8}\,\mathrm{m/s}$

a. (1 point) Calculate the magnetic field amplitude.

$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}$

$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$

$$B_m = E_m/c = 4.0 \times 10^{-8}\,\mathrm{T}$$

b. (4 points) Calculate the (time averaged) energy density, intensity, momentum density (magnitude), and momentum current density of this wave.

$$\langle u \rangle = \tfrac{1}{2}\varepsilon_0 \langle E^2 \rangle + \tfrac{1}{2\mu_0}\langle B^2 \rangle = \tfrac{1}{2}\varepsilon_0 (\tfrac{1}{2}E_m^2) \times 2 = 6.3\underline{22} \times 10^{-10}\,\mathrm{J/m^3}$$

equal

$$\langle I \rangle = \langle |\tfrac{1}{\mu_0}\vec{E}\times\vec{B}| \rangle = \langle u \rangle c = 0.19\underline{116}\,\mathrm{W/m^2}$$

$$\langle |\vec{g}| \rangle = \langle |\tfrac{1}{\mu_0 c^2}\vec{E}\times\vec{B}| \rangle = \langle u \rangle / c = 2.1\underline{24} \times 10^{-18}\,(\mathrm{kg\cdot m/s})/\mathrm{m^3}$$

$$\langle \text{Momentum flux density} \rangle = \langle |\vec{g}| \rangle c = \langle u \rangle = 6.3\underline{72} \times 10^{-10}\,\mathrm{N/m^2}$$

(Symbol?) Pa

time average.

c. (3 points) If this light is normally incident upon a $1.0\,\mathrm{m^2}$ surface, determine the rate of energy absorption and force acting on this surface. Answer this question for both a perfectly reflecting surface and a perfectly abosrbing surface.

Absorbing surface

Energy rate $= IA = 0.19\underline{116}\,\mathrm{W}$

Force $=$ momentum rate $= (\text{mom. flux dens.})(A) = 6.3\underline{72} \times 10^{-10}\,\mathrm{N}$

Reflecting surface

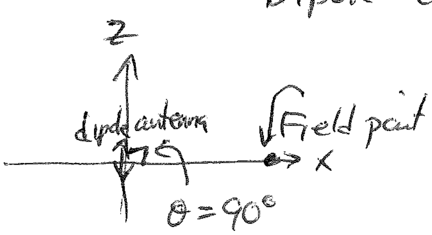
Energy rate $= 0$

Force $=$ mom. rate $= 2(\text{mom. flux dens.})A$

$= 1.2\underline{744} \times 10^{-9}\,\mathrm{N}$

d. (4 points) Assume that this electromagnetic wave is located at the point $(1000\,\mathrm{m}, 0, 0)$ and is generated by a dipole antenna located at $(0,0,0)$ and oriented along the z-axis. If the oscillation frequency is $10^6\,\mathrm{Hz}$, determine the electric dipole amplitude (qz_m) of this antenna.

Dipole emission: $$\vec{E}(\vec{r},t) = \underbrace{-\frac{1}{4\pi\varepsilon_0}\frac{qz_m\omega^2}{c^2 r}\sin\theta\,\hat{\theta}}_{\vec{E}_m}\cos(kr-\omega t)$$



$$E_m = \frac{1}{4\pi\varepsilon_0}\frac{qz_m\omega^2}{c^2 r}\sin\theta$$

$$\Rightarrow qz_m = \frac{4\pi\varepsilon_0 E_m c^2 r}{\omega^2 \sin\theta}$$

$$= \frac{4\pi(8.85\times10^{-12})(12)(3.00\times10^{8})^2(1000\,\mathrm{m})}{(2\pi\cdot10^{6}\,\mathrm{Hz})^2\sin 90^\circ}$$

$$= 3.0\underline{42} \times 10^{-3}\,\mathrm{C\cdot m}$$