

## PHYS 4A Exam 2 Cheat Sheet (with L<sup>A</sup>T<sub>E</sub>X)

### Write Units

### Kinematic Equations

$$v_{avg} = \frac{\Delta x}{\Delta t}; s_{avg} = \frac{distance}{time}; v = \frac{dx}{dt}$$

$$a_{avg} = \frac{\Delta v}{\Delta t}; a = \frac{dv}{dt} = \frac{d^2x}{dt^2}; (1) v(t) = v_0 + at$$

$$(2) x = x_0 + v_0 t + \frac{1}{2}at^2; (3) v^2 = v_0^2 + 2a\Delta x$$

When doing a problem, account for all the variables you know the values of and all those you don't know the value of.

### Freefall

Object is in freefall iff only force acting on it is gravity

Kinematic eq'ns apply to freefall

Unless stated otherwise, gravitational acceleration  $g = -9.81 \text{ m/s}^2$

### Vectors

$$\vec{a} \cdot \vec{b} = ab \cos(\theta); ||\vec{a} \times \vec{b}|| = ab \sin(\theta)$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y \dots; \vec{a} \times \vec{b} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{pmatrix}$$

Vectors work as their separate parts for kinematic eq'ns

### Project

Motion in 2D+ (uses vectors)

Generally, vertical motion is freefall, horizontal motion is constant

x-value = magnitude times cosine of angle

y-value = magnitude times sine of angle

$$R = x - x_0 = \frac{v_0^2 \sin(2\theta)}{g}; t = \frac{R}{v_0 \cos(\theta)}$$

$$\Delta y = \tan \theta \Delta x - \frac{g \Delta x^2}{2(v_0 \cos \theta)^2}$$

### Uniform Circular Motion

$$\vec{x}(t) = x \cos \theta \hat{i} + x \sin \theta \hat{j}; a_c = \frac{v^2}{r}; F_c = \frac{mv^2}{r}$$

### Force

Force on an object is always represented on a FBD as starting from that object

Force on an object is calculated from that object's mass and consequent acceleration

$$F_{net} = ma | F_{AB} = -F_{BA}$$

There is no technical equation for the tension force. Treat it as an unknown when it is included.

Work Mechanical energy transfer to or from a system;  $W = \vec{F} \cdot \vec{d} = \int F(x) dx = \int \vec{F}(\vec{r}) \cdot d\vec{r}$ .

Kinetic Energy  $K = \frac{1}{2}mv^2$ ;  $W_{net} = \Delta K$

### Friction

$$f_s \leq \mu_s F_N; f_k = \mu_k F_N$$

At all points,  $0 < \mu < 1$ .  $\mu_s$  is for unmoving,  $\mu_k$  is for moving. When unmoving,  $f_s = F_{app}$ . Energy lost from it is thermal and uses  $W = f_k \cdot \vec{d}$ .

### Spring force

$$\vec{F}_s = -k\Delta\vec{d}; W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Power Rate at which work is done/energy changes

$$P = \frac{W}{\Delta t}$$

### Potential energy

Conservative force rules:  $W_{ab} = -W_{ba}$ ; Path does not matter; Net work done on closed path is 0

*Gravitational*:  $U = mgy$  so  $\Delta U = mg\Delta y$

*Spring*:  $U = \frac{1}{2}kx^2$  (nonnegative)

### Mechanical Energy

If only conservative forces are used,

$$E_{mech} = K - U = \text{Constant}$$

Center of mass For any dimension  $x$

$$x_{com} = \frac{\int x dm}{M} = \frac{\int x dV}{V}$$

Linear momentum  $\vec{p} = m\vec{v}$

$$\frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{net}$$

Impulse  $\vec{J} = \Delta\vec{p} = \int \vec{F} dt$

$\vec{p}$  is constant for a closed system w/o external forces

### Collisions

Momentum and total energy always conserved

Elastic is perfect bounce, KE conserved

Inelastic is imperfect bounce, KE not conserved

Perfectly inelastic move together, KE not conserved

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

### Elastic Collision Equations

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$

### Angular Kinematics (Basically normal kinematics just in circles)

$$\theta = \frac{S}{r}; \omega = \frac{d\theta}{dt}; \alpha = \frac{d^2\theta}{dt^2}; (1) \omega(t) = \omega_0 + \alpha t$$

$$(2) \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2; (3) \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$v_t = \omega r; a_t = \alpha r; a_c = \omega r^2; T = \frac{2\pi r}{\omega}$$