Problem 1

A spring gun with k = 90.0 N/m is compressed by 5 cm. What is the exit speed of a 2.10-g projectile?

Solution

$$
\begin{aligned}
W &= \int_{min}^{max} F(x)\ dx = \int_{-0.05}^{0} -kx\ dx = \left(-\frac{1}{2}kx^2\right) |_{-0.05}^{0} \\
&= \frac{1}{2}k * 0.05^2 = 45\text{N/m} * 0.0025\text{m}^2 = 0.1\text{J} \\
W &= \Delta K = K_f - K_i
\end{aligned}
$$

Since the spring on the block is unmoving at the start, then $v_i = 0$, so $K_i = \frac{1}{2}mv_i^2$ is also equal to zero. From there, we can determine the final kinetic energy and determine the velocity at the end.

$$
\begin{aligned}
K_f &= W = 0.1\text{J} = \frac{1}{2}mv_f^2 \\
\frac{2K_f}{m} &= v_f^2 \\
v_f &= \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2 * 0.1\text{J}}{0.0021\text{kg}}} = \sqrt{\frac{2 * 1000}{21}} \\
&= \boxed{\frac{20\sqrt{105}}{21}\text{m/s} \approx 9.759\text{m/s}}
\end{aligned}
$$

Problem 2

(a) The United States, with a population of 2.2×10^8 people, consumes 5×10^{19} J per year. What is the per capita consumption in watts? (b) The sun's radiation provides the earth with 1000 W/m^2. Assuming solar energy can be converted to electrical energy with a 20% efficiency, how much area is needed to serve the energy needs of each U.S. citizen?

Solution

Section (a)

The power is determined by the work (W) divided by the time, with a watt being a joule divided by a second. The per capita value is determined by division by the number of humans (c). Assuming a year of 365 days, we can calculate the number of seconds per year (t) first.

$$\begin{aligned}
t &= 1 \text{ years} * \frac{365 \text{ days}}{1 \text{ years}} * \frac{24 \text{ hours}}{1 \text{ days}} * \frac{3600\text{s}}{1 \text{ hours}} = 31536 \times 10^3\text{s} \\
P &= \frac{W}{t} = \frac{5 \times 10^{19}\text{J}}{(31536 \times 10^3\text{s})} = \frac{5 \times 10^{16}}{31536}\text{W} \\
&= 1.58549 \times 10^{12} \text{ W} \\
P_{per\ captia} &= \frac{1.58549 \times 10^{12}}{2.2 \times 10^8}\text{W} \\
&= \boxed{7206.77\text{W}}
\end{aligned}$$

Section (b)

Since there is only a 20% efficiency, the usable numbers of watts per square meter would be $Wa = 1000 \text{ W/m}^2 * \frac{20}{100} = 200 \text{ W/m}^2$. We can divide the total power necessary per citizen (which we calculated in part (a)) to get the area necessary.

$$A = \frac{P}{Wa} = \frac{7206.77\text{W}}{200 \text{ W/m}^2} = \boxed{36.03\text{m}^2}$$

Problem 3

A 0.595-kg object is released from a height of 3.60 m and lands on the ground. Find: (a) the work done by gravity; (b) the change in kinetic energy of the ball; (c) the speed just before it lands using energy methods. Ignore air resistance.

Solution

Section (a)

The object is released directly downward, so the angle with the vertical would be $\phi = 90° = \frac{\pi}{2}$, so $\cos(\phi) = \cos(\frac{\pi}{2}) = 1$. This simplifies the formula from $W_g = mgd\cos(\phi)$ to $W_g = mgd$.

$$W_g = mgd = 0.595\text{kg} * 9.81\text{m/s}^2 * 3.60\text{m} = \boxed{21.01302\text{J}}$$

Section (b)

The change in kinetic energy is equal to the work done by it. Since the object is in freefall, the only work done on it is the gravitational work.

$$\Delta K = W_g = \boxed{21.01302\text{J}}$$

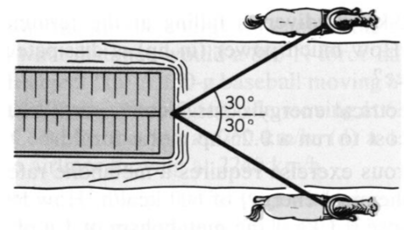
Section (c)

We can here use the formula for kinetic energy given the change in kinetic energy. The initial kinetic energy, since it starts from no velocity, is zero.

$$\Delta K = K_f - K_i = K_f - 0 = K_f$$

$$\begin{aligned}\Delta K = K_f &= \frac{1}{2}mv_f^2 \\ v_f^2 &= \frac{2K_f}{m} = \frac{2 * 21.01302\text{J}}{0.595\text{kg}} = 70.632\text{m}^2/\text{s}^2 \\ v_f &= \sqrt{70.632\text{m}^2/\text{s}^2} = \boxed{8.4043\text{m/s}}\end{aligned}$$

Problem 4



Two horses pull a barge along a canal at a steady 5.00 km/h, as shown in the figure. The tension in each rope is 420 N and each is at 30° to the direction of motion. What is the horsepower provided by the horses?

Solution

First, we convert units from km/h to m/s.

$$5.00\text{km/h} * \frac{1000\text{h} * \text{m}}{3600\text{km} * \text{s}} = \frac{25}{18}\text{m/s}$$

Now, we calculate from the tension the horizontal force exerted by each horse.

$$F_x = T\cos(\phi) = 420\text{N} * \cos(30^\circ) = 210 * \sqrt{3}\text{N}$$

Next, we multiply this force by the velocity of the horses to find the power of each individual horse, then multiply that by two to find the total horse power.

$$P = F_x * v = (210 * \sqrt{3}\text{N}) * (\frac{25}{18}\text{m/s}) = \frac{875\sqrt{3}}{3}\text{W}$$

$$2 * P = 2 * \frac{875\sqrt{3}}{3}\text{W} = \frac{1750\sqrt{3}}{3}\text{W} = \boxed{1010.36\text{W}}$$