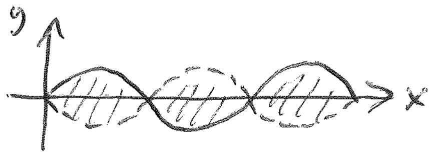
*. (8 points) Determine the total energy of a standing wave in a string in terms of T (tension), μ (mass/length), L (length), n (harmonic number), and y_m (amplitude).



$$y(x,t) = y_m \sin kx \sin \omega t$$

$$k = \frac{2\pi}{\lambda} = \frac{n\pi}{L} \quad (\sin kL = 0)$$

$$v = \omega/k = \sqrt{T/\mu}$$

Make note of, but do not substitute for k and ω too early, unless you enjoy headaches.

$$\mu_K = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 = \frac{1}{2}\mu y_m^2 \omega^2 \sin^2 kx \cos^2 \omega t$$

$$\mu_U = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2 = \frac{1}{2}T y_m^2 k^2 \cos^2 kx \sin^2 \omega t \qquad L\langle \sin^2 kx\rangle$$

$$K = \int_0^L dx \, \frac{1}{2}\mu y_m^2 \omega^2 \sin^2 kx \cos^2 \omega t = \frac{1}{2}\mu y_m^2 \omega^2 (L/2) \cos^2 \omega t \leftarrow \text{add to } 1$$

$$U = \int_0^L dx \, \frac{1}{2}T y_m^2 k^2 \cos^2 kx \sin^2 \omega t = \frac{1}{2}T y_m^2 k^2 (L/2) \sin^2 \omega t$$

$$L\langle \cos^2 kx\rangle$$

$$E = K + U = \frac{1}{2}T y_m^2 k^2 (L/2)\left[\cos^2 \omega t + \sin^2 \omega t\right]$$

$$= \frac{1}{4}TL y_m^2 k^2$$

$$= \frac{1}{4}TL y_m^2 \frac{n^2\pi^2}{L^2}$$

$$= \frac{\pi^2 y_m^2 T n^2}{4L}$$