*. (12 points) Two waves (same wavelength and frequency) are given by:

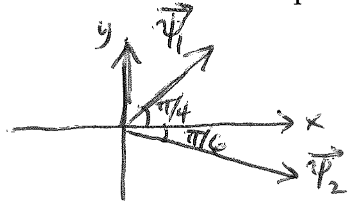
$$\psi_1(x,t) = \psi_{1m}\cos(kx-\omega t+\phi_1)$$
$$\psi_2(x,t) = \psi_{2m}\cos(kx-\omega t+\phi_2)$$

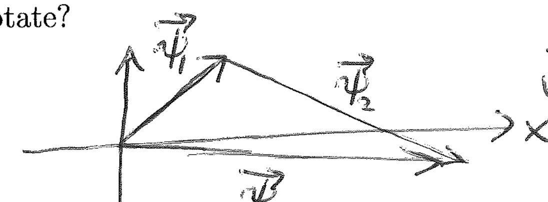
where

$$\psi_{1m}=4.0 \qquad \psi_{2m}=7.0 \qquad \phi_1=\pi/4\,\text{rad} \qquad \phi_2=-\pi/6\,\text{rad}$$

Assume that the units on ψ_{1m} and ψ_{2m} are appropriate for the type of wave involved.

a. (3 points) Draw a phasor diagram showing the two waves at $x=0$ and $t=0$, and how they would combine and interfere (assuming that the waves meet at $x=0$). As time elapses, which direction would the phasor diagram rotate?





$\circlearrowright$, cw (clockwise)

b. (3 points) Determine the complex amplitudes, ψ_{1mc} and ψ_{2mc}, both in polar form ($re^{i\theta}$) and cartesian form ($a+bi$).

$$\psi_{1mc} = 4.0\,e^{i\pi/4} = 4.0\cos\pi/4 \,\cancel{\ } + 4.0\sin\pi/4\, i = 2.828 + 2.828i$$

$$\psi_{2mc} = 7.0\,e^{-i\pi/6} = 7.0\cos(-\pi/6) + 7.0\sin(-\pi/6)i = 6.062 - 3.500i$$

c. (3 points) Determine the complex amplitude of the combined wave, ψ_{mc}, both in cartesian and polar form. Identify the amplitude and phase of the combined wave.

$$\psi_{mc} = \psi_{1mc} + \psi_{2mc} = 8.891 - 0.672i = \psi_m e^{i\phi} = 8.916\,e^{-i\,0.0754}$$

$$\psi_m = \sqrt{8.891^2+0.672^2} = \underset{\text{amplitude}}{8.916} \qquad \phi = \arctan\frac{-0.672}{8.891} = \underset{\text{phase}}{-0.0754\,\text{rad}}$$

$(= -4.320°)$

d. (3 points) Calculate $I/(I_1+I_2)$, where I_1 and I_2 represent the intensities of the individual waves and I represents the intensity of the combined wave.

$$\frac{I}{I_1+I_2} = \frac{\propto \psi_m^2}{\propto \psi_{1m}^2 + \propto \psi_{2m}^2} = \frac{8.916^2}{4.0^2+7.0^2} = \frac{79.494}{65} = 1.223$$

more constructive than destructive

$(|\Delta\phi| < \pi/2)$