*. (20 points) A Carnot heat engine operates on a four step cycle as follows:

1. Isothermal expansion at temperature T_3, where heat Q_H flows into the system from an external temperature reservoir T_4 ($T_4 \geq T_3$). This step is reversible if $T_4 = T_3$.
2. Adiabatic expansion (no heat flow), where the temperature drops from T_3 to T_2 ($T_2 < T_3$). This step is always reversible (as long as it is quasi-static).
3. Isothermal compression at temperature T_2, where heat Q_C flows out of the system to an external temperature reservoir T_1 ($T_1 \leq T_2$). This step is reversible if $T_1 = T_2$.
4. Adiabatic compression (no heat flow), where the temperature rises from T_2 back up to T_3. This step is always reversible.

The efficiency of this cycle is given by $e = 1 - T_2/T_3$, and is maximized when $T_3 = T_4$ and $T_2 = T_1$ (reversible), although the work output rate in that case is zero (heat flowrates are zero for steps 1 and 3). Allowing $T_1 < T_2 < T_3 < T_4$ enables us to consider a real heat engine that would run at a finite rate.

Consider such a heat engine with $T_4 = 600\,\mathrm{K}$, $T_3 = 500\,\mathrm{K}$, $T_2 = 400\,\mathrm{K}$, and $T_1 = 300\,\mathrm{K}$. Suppose also that the heat conductance for the rods connecting the system to T_4 during step 1 and to T_1 during step 3 are each $10\,\mathrm{W/K}$, and that the adiabatic steps (2 and 4) are both very rapid (essentially zero time).

a. (6 points) Calculate the efficiency of this heat engine. If $Q_H = 100\,\mathrm{J}$ for one cycle of this heat engine, how much heat flows into the cold reservoir (Q_C) and how much work is output (W) for each cycle?

$$e = 1 - T_2/T_3 = 1 - \frac{400K}{500K} = 1 - 0.80 = 0.20$$

$$W = e\,Q_H = 20\,J$$

$$Q_C = Q_H - W = 80\,J$$

b. (6 points) Calculate the net change in entropy during one cycle. During which steps does the positive entropy change occur?

step 1: $\Delta S = \frac{+Q_H}{T_3} + \frac{-Q_H}{T_4} = \frac{100J}{500K} - \frac{100J}{600K} = +0.033\bar{3}\ J/K$

step 3: $\Delta S = \frac{+Q_C}{T_1} + \frac{-Q_C}{T_2} = \frac{80J}{300K} - \frac{80J}{400K} = +0.066\bar{6}\ J/K$

steps 2, 4: $\Delta S = 0$

overall: $\Delta S = 0.033\bar{3} + 0.0\bar{6}\bar{6}$
$= \boxed{+0.10\ J/K}$

c. (8 points) Determine the rate of work production for this engine (work/time). (Hint: calculate the total time for one cycle).

step 1: $\frac{dQ}{dt} = K\Delta T = (10W/K)(600K - 500K) = 1000\ W$

$\Rightarrow t_1 = \frac{Q_H}{dQ/dt} = \frac{100J}{1000W} = 0.10s$

step 3: $\frac{dQ}{dt} = K\Delta T = (10W/K)(400K - 300K) = 1000\ W$

$\Rightarrow t_3 = \frac{Q_C}{dQ/dt} = \frac{80J}{1000W} = 0.080s$

steps 2, 4; $t_2 = t_4 \approx 0$

Total time $= 0.10s + 0.08s = 0.18s$

$\left(\frac{dW}{dt}\right)_{avg} = \frac{\text{work}}{\text{time}} = \frac{20J}{0.18s} = \boxed{111\ W}$