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Tutorial - 1

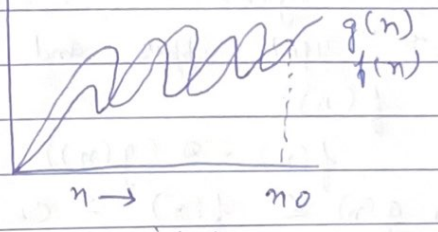
1. What do you understand by Asymptotic notations. Define different notation with examples?

Sol Asymptotic Notations:- They are the mathematical notations used to describe the running time of an algorithm when the input ends towards a particular value or a limiting value.

Different asymptotic notations -

i) Big O (n)

$f(n) = O(g(n))$



$f(n) = O(g(n))$

iff $f(n) \leq c\,g(n)$

$\forall\ n \geq n_0$

for some constant, $c > 0$

$g(n)$ is "tight" upper bound of $f(n)$.

ex. $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \leq c\,n^3$

$n^2 + n = O(n^3)$

(ii) Big Omega (Ω)

$f(n) = \Omega(g(n))$

$g(n)$ is "tight" Lower bound of function $f(n)$.

$f(n) = \Omega(g(n))$

iff $f(n) \geq c\,g(n)$

$\forall\ n \geq n_0$

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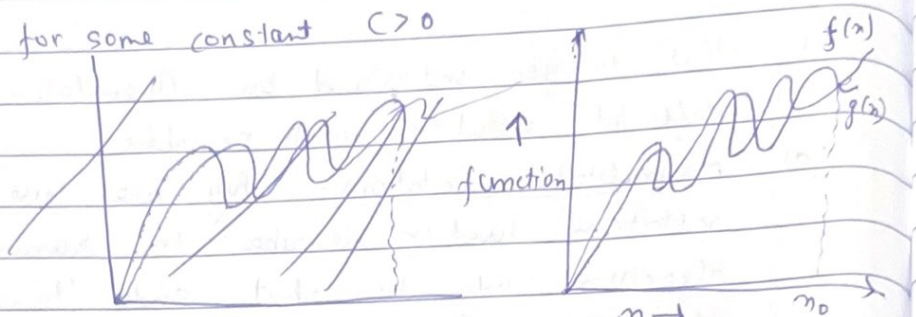
for some constant $C > 0$



ex. $f(n) = n^3 + 4n^2$

$g(n) = n^2$

$= n^3 + n^2 - \Omega(n^2)$

(iii) Big Theta (Θ)

$f(n) = \Theta(g(n))$

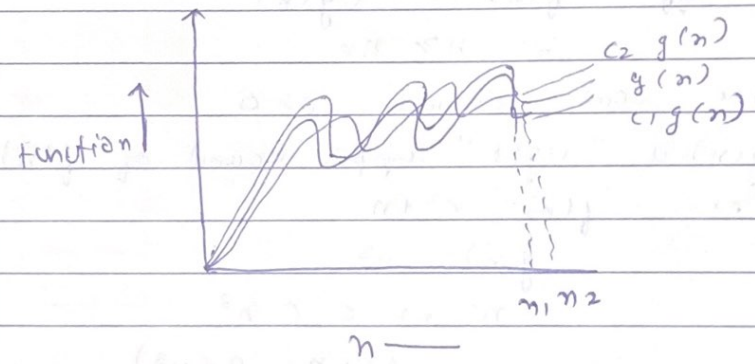
$g(n)$ is both "tight" upper and "lower bound" of function $f(n)$.

$f(n) = \Theta(g(n))$

iff $c_1 g(n) \leq f(n) \leq c_2 g(n)$

$\forall\ n \geq \max(n_1, n_2)$

for some constant $c_1 > 0$ and c_2 0



Ex

$3n+2 = O(n)$ as $3n+2 \geq 3n$ and

$3n+2 \leq 4(n)$ for $n, k_1 = 3, k_2 = 4$ & $n_0 = 2$

iv) Small $\Theta(\theta)$ -

$f(n) = o(g(n))$

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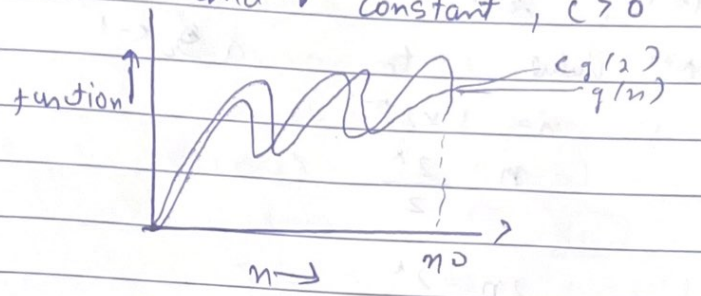
$g(n)$ is upper bound of function $f(n)$.

$f(n) = O(g(n))$

when $f(n) < cg(n)$

$\forall \quad n > n_0$

and $\forall$ constant, $c > 0$



Ex - $f(n) = n^2$

$g(n) = n^3$

$n^2 = O(n^3)$

(v) Small Omega (n)

$f(n) \geq \omega(g(n))$

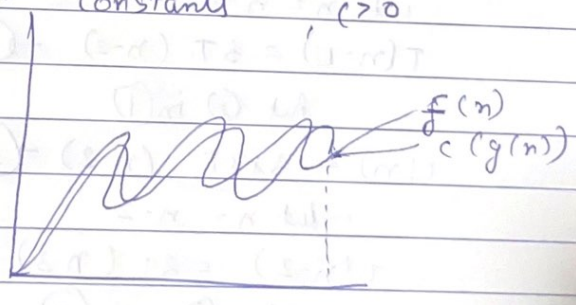
$g(n)$ is lower bound of $f(n)$

$f(n) = \omega(g(n))$

When $f(n) \geq c\,g(n)$

$\forall \quad n > n_0$

and $\forall$ constants, $c > 0$



$f(n) = 4n + 6 \qquad g(n) = (1)$

2. What should be time complexity of

for(i=1 - ln) { i = i * 2; }

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Sol. for (i=1 to n)
& i = i * 2 3

i = 1, 2, 4, 8, 16, ---- n g.p

= O(k)

a = 1, r = 2,

GP kth Value = $t_k = a r^{k-1}$

$n = 1 \times 2^{k-1}$

$n = \frac{2^k}{2}$

$2n = 2^k$

$\log(2n) = k \log 2$

$k = \log_2 2n$

$k = \log_2 2 + \log_2 n$

$k = 1 + \log n$

Time comp = $O(1 + \log_2 n)$

$= O(\log_2 n)$

3. $T(n) = \{ 3T(n-1)$ if $n > 0$, otherwise $1 \}$

$T(n) = 3T(n-1) \rightarrow (1)$

let $n = n-1$

$T(n-1) = 3T(n-2) \rightarrow (2)$

Put (2) in (1)

$T(n) = 3 \times 3T(n-2) \rightarrow (3)$

let $n = n-2$

$T(n-2) = 3T(n-3) \rightarrow (4)$

Put (4) in (3)

$T(n) = 3 \times 3 \times 3T(n-3) \rightarrow (5)$

$T(n) = 3^n T(n-n)$

$= 3^n (T(0))$

$= 3^n$

$= O(3^n)$

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4. $T(n) = \begin{cases} 2T(n-1)-1 & \text{if } n>0 \\ 1 & \text{otherwise} \end{cases}$

$T(n) = \begin{cases} 2T(n-1)-1 & , n>0 \\ 1 & , \text{otherwise} \end{cases}$

$T(n) = 2T(n-1) - 1 \quad ---(i)$

replacing n with $n-1$

$T(n-1) = 2T(n-2) - 1 \quad ---(ii)$

replacing n with $n-2$

$T(n-2) = 2T(n-3) - 1 \quad ---(iii)$

from (i), (ii), (iii) we get

$T(n) = 2(2T(n-2)-1) - 1$

$T(n) = 2^2 T(n-2) - 2 - 1$ using (ii)

$T(n) = 2^2[2T(n-3)-1] - 2 - 1$

$T(n) = 2^3 T(n-3) - 2^2 - 2 - 1$, using (iii)

$T(n) = 2^k T(n-k) + 2^{k-1} + 2^{k-2} + \cdots 2 + 1$

we know that

$T(0) = 1$

$n-k = 0$

$\boxed{k = n}$

on putting $k = n$, we get

$T(n) = 2^n T(0) + 2^{n-1} + 2^{n-2} + \cdots 2 + 1$

$= 2^n + 2^{n-1} + 2^{n-2} + \cdots + 2 + 1$

(G.P)

$T(n) = \frac{1 \cdot (2^{n+1}-1)}{2-1}$

$T(n) = 2^{n+1} - 1$

$\boxed{\text{Time complexity} = O(2^n)}$

5. what should time complexity of

```
int i = 1, s = 1;
while (s <= n){
    i++; s = s + i;
    print ("#");
```

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Sol. s depends on i, so we make case.

At	i =	1	2	3	4	----	n
	s =	1	3	6	10	----	k

at $i = n$, $s = k$ & k breaks the while cond.
so we can clearly see that s is just of sum of n natural no. so

$$\frac{k(k+1)}{2} > n$$

$\rightarrow$ dominating power

$$\frac{k^2 + k}{2} > n$$

$$\rightarrow k^2 \geq n$$

$$\boxed{k = \sqrt{n}}$$

$$\boxed{\text{time complikity} = O(\sqrt{n})}$$

6. Time complexity of

```
void function (int n) {
    int i, count = 0;
    for (i=1 ; i * i <= n ; i++)
        count++ ;
}
```

Sol. $i = 1, 2, 3, \dots n$

$i^2 = 1, 4, 9, \dots n$

so $i^2 <= n$ or $i <= \sqrt{n}$

$a_k = a + (k-1)d$

$a = 1$, $d = 1$

$a_k <= \sqrt{n}$

$\sqrt{n} = 1 + (k-1)1$

$\sqrt{n} = k$

$T(n) = O(\sqrt{n})$

7. Time complexity of

```
void function (int n) {
    int i, j, k, count = 0;
```

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```
for (i = n/2 ; i <= n ; i++)
    for (j = 1 ; j <= n ; j = j * 2)
        for (k = 1 ; k <= n ; k = k * 2)
            count ++ ;
```

All are independent loops,

So time complexity $= \frac{n}{2} * \log_2 n * \log_2 n$

time complexity $= O(n \log^2 n)$

8. Time complexity of

```
function (int n) {
    if (n == 1) return;
    for (i = 1 to n) {
        for (j = 1 to n) {
            print ("*");
        }
    }
    function (n-3);
}
```

$T(n) = T(n-3) + n^2$ —— (1)

$T(1) = 1$ —— (2)

put $n = n-3$ in (1)

$T(n-3) = T(n-6) + (n-3)^2$ —— (3)

Put (3) in (1)

$T(n) = T(n-6) + (n-3)^2 + n^2$ —— (4)

put $n = n-6$ in (1)

$T(n-6) = T(n-9) + (n-6)^2$ —— (5)

Put (5) in (4)

$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n$

Generalizing

$T(n) = T(n-3k) + (n-3(k-1))^2 + (n-3(k-2))^2 + \dots + n^2$

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Let $n-3k=1$

$$\frac{n-1}{3}=k$$

$$T(n)=T(1)+\left(n-3\left(\frac{n-1}{3}-1\right)\right)^2$$

$$+\left(n-3\left(n-\frac{1}{3}\right)\right)^2+\cdots n^2$$

$$T(n)=T(1)+(n-(n-1)-3]^2+[n-(n-(-6))]^2$$

$$+(n-(n-(-9))^2+\cdots n^2$$

$$T(n)=1+(3+1)^2+(6+1)^2+\cdots n^2$$

$$T(n)=1^2+4^2+7^2+\cdots n^2$$

$$T(n)=n^2+\cdots 1$$

$$T=O(n^2)$$

9. Time Complexity of -

```
void function (int n) {
    for (i=1 to n) {
        for (j=1; j<=n; j=j*2+1)
            printf ("*")
    }
}
```

for $i=1 \longrightarrow j=1$ to $n \longrightarrow n$ times

$i=2 \longrightarrow j=1$ to $n \longrightarrow n/2$ times

$i=3 \longrightarrow j=1$ to $n \longrightarrow n/3$ times

⋮

$i=n \longrightarrow j=1$ to $n \longrightarrow 1$ times

So total $= n+n/2+n/3+n/4+\cdots+1$

$= n(1+1/2+1/3+1/4+\cdots 1/n)$

$\log n$

$T(n)=n \log n$

Time complexity $=O(n \log n)$

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10. for the function, n^k and c^n, what is the asymptotic relationship b/w these function? Assume that $k >= 1$ and $c > 1$ constants. Find out the value of c and n_0 for which relations holds?

Sol as given n^k & c^n

relation b/w n^k & c^n is

$n^k = O(c^n)$

as $n^k \leq d\, c^n$

if $n >= n_0$ & some constant $a > 0$

for $n_0 > 1$

$c > 2$

$1^k \leq d_2$

$n_0 = 1$ & $c = 2$