IML 2024 Fall documentation (Schilling Series 40)

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Contents

0.1 [Template: Model title, Model number]

Template that you can copy and paste. Delete this line.
Aspects to consider:

- Describe the black lines: e.g. if it is a triangle (the type of the triangle), if it is a star . . . , dashed lines are extension of the original sides of triangle),
- the outer curve (if it is simple, briefly say how you may construct the curve),
- red lines
- Symmetry (if any)

Mathematical description [1-2 paragraph]

Construction [optional]

Related models [optional]

Chapter 1: General Info

1.1 Description of K-curves, C-Curves and C'-curves

Rephrase and reorder this section.

- C-Curves: A (plane) curve of constant width, a convex curve that runs in the finite range such that the distance between two parallel supporting lines, i.e. the curve straight between enclosing parallel lines, is constant. (e.g. the Reuleaux triangle)
- K-curves: the geometric location of the centers of curvature or the evolute of the curve constant width, (e.g. the equilateral triangle for the case of a Reuleaux triangle)
- C'-curves: the geometric location of the centers for the double normal
- The tangent line of the K-curve has rotated by an angle of π if the point of contact has just passed around the curve once;
- Scal of the model: 12 cm
- support line:
- double normal:

Reference: Schilling 1911 catalog, description of Series 40 in Part 1 (pages 106-107)

1.2 Construction

Type 1

Type 2

Type 3

Chapter 2: Cathy

2.1 Reuleaux polygon composed of circular arcs (4)

Mathematical Description The black equilateral triangle is the $K-Curve$. The $C-Curve$ is the outermost curve, consisting of three segments of circular arcs. Each circular arc is centered at a vertex of the equilateral triangle, with radius equal to the side length of the triangle. The $C'-Curve$ is the inner red curves, consisting of three circular arcs. Each arc is centered at a vertex of the triangle, with radius equal to $\frac{1}{2}$ of the side length of the triangle.

2.2 Outermost C-curve to a general triangle with one kinked side (16)

Mathematical Description The $K - Curve$ is the black concave quadrilateral, which is made of joining two right triangles at their right angles, with the right-angle sides aligned along the same line. The $C - Curve$ is the outermost closed curve, consisting of seven circular arcs. Label the four vertices of the $K - Curve$ as A, B, C and D in counterclockwise order, with A representing the vertices where the $K - Curve$ and the $C - Curve$ intersect. Starting from A, we go in a clockwise order. The first arc is centered at B with radius $|AB|$, ranging from A to the first dashed line intersecting the $C - Curve$. The second arc is centered at C with radius equal to the distance between C and the endpoint of the previous curve, ranging from first dashed line to the second dashed line. The $C' - Curve$ is the inner red curve, which is composed by four circular arcs. Four arcs are centered at four vertices of the $K - Curve$ respectively, with points along the arc located at the midpoints of the chords.

2.3 Outermost C-curve to an equilateral triangle with sides bent in the middle (17)

Mathematical Descriptions The $K - Curve$ is an equilateral triangle with sides folded in the middle.

2.4 C-curve with two ellipse quadrants to a K-curve consisting of its evolute arcs and two segments (19)

Mathematical Descriptions Suppose we have two ellipses with equations

$$\begin{cases} x = a_1 \cos t \\ y = b_1 \sin t \end{cases} \quad \textit{and} \quad \begin{cases} x = a_2 \cos s \\ y = b_2 \sin s \end{cases},$$

where $t \in \left[\pi, \dfrac{3\pi}{2}\right]$, $s \in \left[\dfrac{3\pi}{2}, 2\pi\right]$, $a_1 > b_1$, $a_2 > b_2$, $a_1 + a_2 = 12cm$ and $b_1 = b_2$. Take the intersecting point of the horizontal line and the vertical line as the origin. Then the $C - Curve$ can be divided into 3 parts. Quadrant III is a quarter ellipse with major axis $2a_1$ and minor axis $2b_1$, and Quadrant IV is a quarter ellipse with major axis $2a_2$ and minor axis $2b_2$. The $C - Curve$ in Quadrants I and II is constructed by taking each point on the two quarter ellipses in Quadrants III and IV and extending it along the direction of the normal vector toward the concave side of the ellipse. Each point is extended by $12cm$ along this normal direction to create a corresponding point in Quadrants I and II. The $K - Curve$ is the inner black curve, consisting of four parts: The curve in Quadrant II is the evolute of the quarter ellipse in Quadrant III. The curve in Quadrant I is the evolute of the quarter ellipse in Quadrant IV. Respectively, the equations are

$$\begin{cases} x = \dfrac{a_1^2 - b_1^2}{a_1} \cos^3 t \\ y = \dfrac{b_1^2 - a_1^2}{b_1} \sin^3 t \end{cases} \quad \textit{and} \quad \begin{cases} x = \dfrac{a_2^2 - b_2^2}{a_2} \cos^3 s \\ y = \dfrac{b_2^2 - a_2^2}{b_2} \sin^3 s \end{cases},$$

where $t \in \left[\pi, \dfrac{3\pi}{2}\right]$, $s \in \left[\dfrac{3\pi}{2}, 2\pi\right]$. The rest two parts of the $K - Curve$ are two line segments: a horizontal line segment connecting $\left(-\dfrac{a_1^2 - b_1^2}{a_1}, 0\right)$ and $\left(\dfrac{a_2^2 - b_2^2}{a_2}, 0\right)$, and a vertical line segment connecting $\left(0, \dfrac{a_1^2 - b_1^2}{b_1}\right)$ and $\left(0, \dfrac{a_2^2 - b_2^2}{b_2}\right)$.

Chapter 3: Darya

3.1 Outermost C-curves of an Isosceles Triangle with a Base Bent in the Middle, the Legs of which are Equal to, Smaller or Larger than the Base (13)

Mathematical Description

- Draw an isosceles triangle where the 2 equal sides are set to the desired constant width. The base of the isosceles triangle will be slightly shorter.
- At each endpoint of the base, use the vertices of isosceles triangle
-
-
-

Chapter 4: Eva

4.1 Equidistant C-Curve of the Releaux Circular Arc Polygon (5)

Geometric Properties The K-Curve for this C-Curve is an equilateral triangle. The edges are extended to make each line come to the desired 12 cm width of the C-Curve. Let the edges of the triangle be of length a. The C-Curve is made up of six circular arcs. These arcs are separated by the dashed lines seen on the model. The three longer arcs are each arcs of the circle with its center point at the opposite vertex of the triangle, and with a radius of $\frac{12+a}{2}$ cm. The three shorter arcs are each arcs of the circle with its center point at the nearest vertex of the triangle, and with a radius of $\frac{12-a}{2}$ cm. Thus each of the three straight lines seen on the model are lines of the width of the model.

Related Models Releaux Triangle (Model 4)

4.2 Outermost C-Curve of a Regular Three-Pointed Circular Arc Polygon (7)

Geometric Properties

Related Models

4.3 Outermost C-Curve of an Isosceles Triangle with Side Lengths Larger Than the Base (11)

Geometric Properties The K-Curve for this C-Curve is an isosceles triangle with side lengths longer than the base. At the vertices touching the base of the triangle, the edges of the triangle are extended to 12 cm, the desired width of the C-Curve. Let the legs of the triangle be of length a cm and the base be of length b cm. Let the model be oriented such that the base is at the bottom of the model, parallel to the x-axis. The C-Curve is made up of five circular arcs. The model has reflectional symmetry down the center, so there are two sets of equal, reflected arcs, as well as one arc that spans both sides of the center line. Each of the uppermost reflected arcs is an arc of the circle with its center point at the opposite vertex of the triangle and with a radius of length a cm. Each of the bottommost reflected arcs is an arc of the circle with its center point at the nearest vertex of the triangle and with a radius of $12 - a$ cm. The final arc, at the bottom of the model, is an arc of the circle that has its its center at the top vertex of the triangle and has a radius of 12 cm. Each of the three straight lines on the model are of the width of the model.

Related Models Outermost C-Curve of an Isosceles Triangle with Side Lengths Shorter Than the Base (Model 10)

4.4 Outermost C-Curve of a Special Equiangular Star (18)

Geometric Properties

Related Models

Chapter 5: Jeffrey

5.1 [Outermost C-curve to an isosceles triangle with smaller legs than the base, 10]

Geometric properties The C-curve in the figure consists of four arcs, each extending to meet the required 12 cm width. At the center, there is an equilateral triangle with legs shorter than its base. The boundaries of these arcs are defined by the dashed lines on the model and the two legs of the triangle. Three of the arcs are formed around the vertices of the triangle, and the uppermost arc is created by extending the two legs as indicated by the dashed lines in the figure

Related models

5.2 [Outermost C-curve to a general triangle , 15]

Mathematical description [Equations]

Geometric properties

Related models [content]

5.3 [Outermost C-curve to Steiner's hypocycloid, 6]

Mathematical description [Equations]

$$x = h(\phi)\cos\phi - h'(\phi)\sin\phi$$

$$y = h(\phi)\sin\phi + h'(\phi)\cos\phi$$

where $0 \leq \phi < 2\pi$.

$h(\phi)$ is the (positive) distance from the origin to the corresponding supporting line, and $h'(\phi)$ is the oriented distance between the foot of the perpendicular from the origin to the supporting line and the point of tangency between the supporting line and the curve.

Geometric properties

Related models [content]

5.4 [Outermost C-curve of an isosceles triangle with a base bent in the middle, the legs of which are equal to, smaller or larger that the base, 12]

Geometric Properties This geometric model is an isosceles triangle with a base that bends in the middle, and legs that may equal, be less than the base in length, extending to a consistent width of 12 cm for the outermost C-Curve. Oriented so the bent base lies parallel to the x-axis, the model comprises five circular arcs that demonstrate reflective symmetry across the center line. The topmost arcs derive from circles centered at the triangle's far vertex with a radius of a cm, the lower arcs from the nearest vertex with a radius of 12a cm, and the central bottom arc from the top vertex with a radius of 12 cm, ensuring uniform width across the structure.

Related models model 6